1. (c)

2. (b) Given: Frequency $f_1 = 6\text{MHz}$

Frequency $f_2 = 10\text{MHz}$

$\lambda_1 = \frac{C}{f_1}$

$\lambda_2 = \frac{C}{f_2}$

Wavelength bandwidth $= \lambda_2 - \lambda_1 = 20\text{ m}$

3. (c) At $t = 0$ disintegration rate $= 4250\text{dpm}$

At $t = 10$ disintegration rate $= 2250\text{dpm}$

$A = A_0 e^{-\lambda t}$

$2250 = 4250 e^{-\lambda(10)}$

$\Rightarrow \lambda(10) = \ln\left(\frac{4250}{2250}\right)$

$\Rightarrow \lambda = 0.063\text{ min}^{-1}$

4. (b) Wavelength of incident beam $\lambda = 900 \times 10^{-9}$ m Intensity of incident beam $= I = 100\text{ W/m}^2$

No. of photons crossing per unit sec

$= n = \frac{E_{net}}{E_{\text{single photon}}} = \frac{I A \lambda}{hc}$

$= \frac{(100)(1 \times 10^{-4})(900 \times 10^{-9})}{6.62 \times 10^{-34} \times 3 \times 10^{8}} = 4.5 \times 10^{16}$

5. (b) For a given light wavelength corresponding a medium of refractive index μ

$\lambda_{med} = \frac{\lambda_{vacuum}}{\mu}$

and we know that fringe width $\beta = \frac{\lambda D}{d}$

Therefore, $\beta_{med} = \frac{\beta_{vacuum}}{\mu} = \frac{12}{\frac{4}{3}} = 9\text{ mm}$

6. (c) $c = \frac{E_0}{B_0} \Rightarrow E_0 = cB_0$

$E_0 = (3 \times 10^8)(2 \times 10^{-8})$

$E_0 = 6\text{Vm}^{-1}$

As, $\vec{B} =$ along y-axis

$\vec{v}$ = along negative x-axis

hence $\vec{E}_0$ = along z-axis

7. **(b)** In case of L-R circuit

$Z = \sqrt{X_L^2 + R^2}$ & power factor

$P_1 = \cos\phi = \frac{R}{Z}$

As $X_L = R$

$\Rightarrow Z = \sqrt{2}R$

$\Rightarrow P_1 = \frac{R}{\sqrt{2}R} \Rightarrow P_1 = \frac{1}{\sqrt{2}}$

In case of L-C-R circuit

$Z = \sqrt{R^2 + (X_L - X_C)^2}$

As $X_L = X_C$

$\Rightarrow Z = R$

$\Rightarrow P_2 = \cos\phi = \frac{R}{R} = 1$

$\Rightarrow \frac{P_1}{P_2} = \frac{1}{\sqrt{2}}$

8. **(b)**

As $\vec{F} = q(\vec{v} \times \vec{B})$

$\vec{a} = \frac{q}{m}(\vec{v} \times \vec{B})$

So, $\vec{a}$&$\vec{B}$ are $\perp$ to each other

Hence, $\vec{a} \cdot \vec{B} = 0$

$$(\alpha\hat{i} - 4\hat{j}) \cdot (2\hat{i} + 3\hat{j}) = 0$$
$$\alpha(2) + (-4)(3) = 0$$
$$\alpha = \frac{12}{2} \Rightarrow \alpha = 6$$

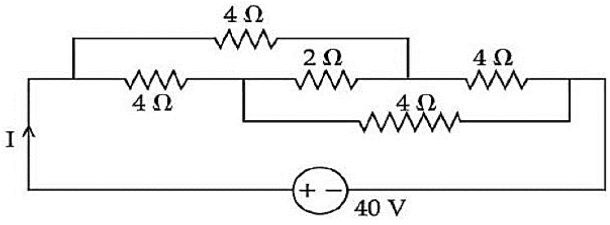
9. **(b)** At centre $B = N\left(\frac{\mu_0 i}{2R}\right)$

$B_x = 200\left(\frac{\mu_0 i}{2 \times 20\ \text{cm}}\right)$

$B_y = 400\left(\frac{\mu_0 i}{2 \times 20\ cm}\right)$

$\frac{B_x}{B_y} = \frac{1}{2}$

10. (a)



Given circuit is balanced wheat stone bridge

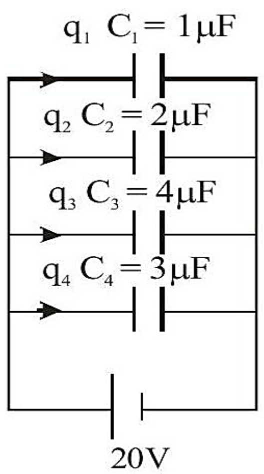
Hence 2Ω can be neglected

$R_{net} = 4\Omega$

$I = \frac{40}{4}$

$I = 10\ A$

11. (a)



Total charge $= q_1 + q_2 + q_2 + q_4$

$= 1 \times 20 + 2 \times 20 + 4 \times 20 + 3 \times 20 = 200\mu C$

12. (b) For a particle in SHM, its speed depends on position as

$v = \omega\sqrt{A^2 - x^2}$

Where ω is angular frequency and A is amplitude

Now $v^2 = \omega^2 A^2 - \omega^2 x^2$

So, $\frac{v^2}{(\omega A)^2} + \frac{x^2}{(A)^2} = 1$

So graph between v and x is elliptical

13. (b) For a quasi-static process the change in internal energy of an ideal gas is

$$\Delta U = nC_v\Delta T$$

$$= n \times \frac{3R}{2} \times \Delta T$$

[molar heat capacity at constant volume for mono atomic gas $= \frac{3R}{2}$]

$$\Delta U = 7 \times \frac{3}{2} \times 8.3 \times 40 = 3486 \text{ J}$$

14. (c) Constant entropy means process is adiabatic $PV^{\gamma} = \text{constant}$

$$V_2 = \frac{V_1}{8}$$

$$P_1 V_1^{\gamma} = P_2 V_2^{\gamma}$$

$$P_1 V_1^{\gamma} = P_2 \left(\frac{V_1}{8}\right)^{5/3}$$

$$P_1 V_1^{5/3} = \frac{P_2 V_1^{5/3}}{32}$$

$$\mathbf{P_2} = 32P_1$$

15. (c) Initial surface energy $= TA$

Where T is surface tension and A is surface area

$$U_i = \left(\frac{75 \times 10^{-5}}{10^{-2}} \frac{N}{m}\right) \times [4\pi(1 \times 10^{-2})^2]$$

$$= 75 \times 10^{-3} \times 4\pi \times 10^{-4} = 942 \times 10^{-7} \text{ J}$$

To get final radius of drops by volume conservation

$$\frac{4}{3}\pi R^3 = 729\left(\frac{4}{3}\pi r^3\right)$$

$R =$ Initial radius

$r =$ final radius

$$r = \frac{R}{(729)^{1/3}} = \frac{R}{9} = \frac{1}{9} \text{ cm}$$

Final surface energy

$$U_f = 729[TA]$$

$$= 729\left[\frac{75 \times 10^{-5}}{10^{-2}} \frac{N}{m}\right] \times \left[4\pi\left(\frac{1}{9} \times 10^{-2}\right)^2\right]$$

$$= 729\left[75 \times 10^{-3} \times \frac{4\pi \times 10^{-4}}{81}\right]$$

$$= 9[942 \times 10^{-7}\ \mathrm{J}]$$

Gain in surface energy

$$\begin{aligned}\Delta U &= 9 \times 942 \times 10^{-7} - 942 \times 10^{-7} \\ &= 8 \times 942 \times 10^{-7}\ \mathrm{J} = 7536 \times 10^{-7}\ \mathrm{J} \\ &= 7.5 \times 10^{-4}\ \mathrm{J}\end{aligned}$$

16. (a) Acceleration due to gravity at a height h << R is

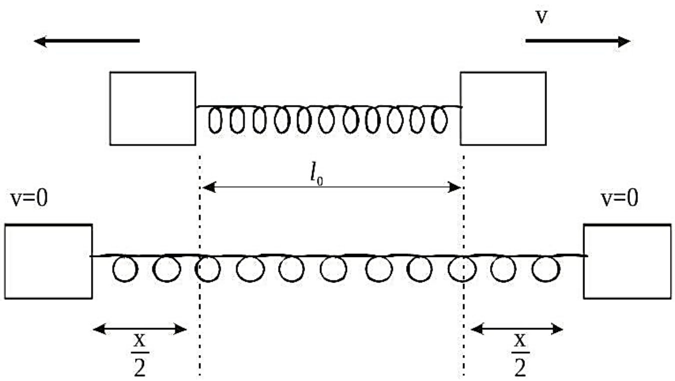
$$g' = g\left(1 - \frac{2h}{R}\right)$$

$$\therefore \frac{\Delta g}{g} = \frac{2\,h}{R}$$

$$\Rightarrow \frac{\Delta g}{g} \times 100 = \frac{2\,h}{R} \times 100$$

$$= 2 \times \frac{32}{6400} \times 100 = 1\%$$

17. (b)



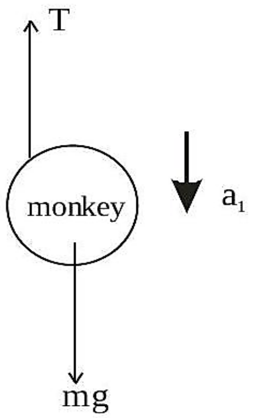
using energy conservation

$$\frac{1}{2}mv^2 \times 2 = \frac{1}{2}kx^2$$

$$\Rightarrow \frac{1}{4}v^2 = \frac{1}{2} \times 2 \times x^2$$

$$\therefore x = \frac{V}{2}$$

18. (c) F.B.D of monkey while moving downward

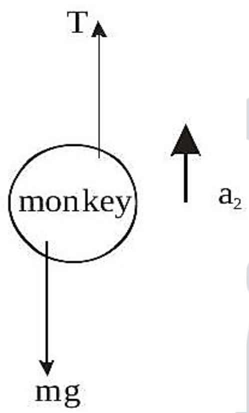


Using Newton's second law

$mg - T = ma_1$

$\therefore\ 500 - T = 50 \times 4 \Rightarrow T = 300\text{ N}$

F.B.D of monkey while moving up



Using Newton's second law of motion

$T - mg = ma_2$

$\Rightarrow\ T - 500 = 50 \times 5$

$\Rightarrow T = 750\text{ N}$

Breaking strength of string $= 350$ N

$\therefore$ String will break while monkey is moving upward

19. (c) Time taken to reach maximum height

$$t = \frac{u\sin\theta}{g}$$

$$\therefore \frac{u_1\sin\theta_1}{g} = \frac{u_2\sin\theta_2}{g}$$

$$\Rightarrow u_1\sin 30 = u_2\sin 45$$

$\Rightarrow \frac{u_1}{u_2} = \frac{1/\sqrt{2}}{1/2} = \frac{\sqrt{2}}{1}$

20. (b) L.C. $= \frac{P}{N} = \frac{0.5\text{ mm}}{50} = 0.01$ mm

Length of wire $= 6.8$ cm

Diameter of wire $= 1.5$ mm $+ 7 \times$ L.C

$= 1.5$ mm $+ 7 \times .01 = 1.57$ mm

Curved surface area $= \pi D\ell$

$= 3.14 \times 6.8 \times 1.57 \times 10^{-1}$ cm^2

$= 3.352$ cm$^2 = 3.4$ cm^2

21. (5) $u_x = 1$

$y = 5x(1-x)$

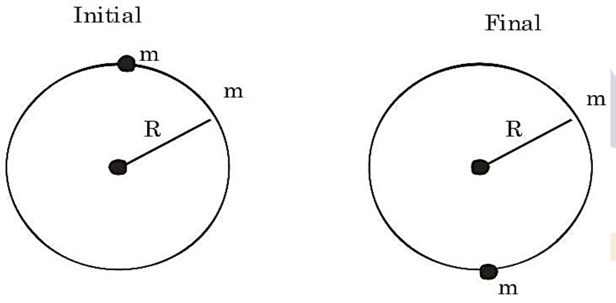
$\frac{dy}{dt} = 5\frac{dx}{dt} - 10x\frac{dx}{dt}$

For initial y-component of velocity

$u_y = \left(\frac{dy}{dt}\right)_{x=0} \Rightarrow 5(1) = 5$

$\vec{u}_y = 5\hat{j}$

22. (5)



using conservation of mechanical energy $mg\,2R = \frac{1}{2}I_{disc}\,\omega^2 + \frac{1}{2}I_{particle}\,\omega^2$

$mg\,2R = \frac{\omega^2}{2}\left[\frac{mR^2}{2} + mR^2\right]$

$mg\,2R = \frac{\omega^2}{2}\frac{3}{2}mR^2$

$\frac{3}{4}\omega^2 = \frac{2g}{R}$

$\omega^2 = \frac{8g}{3R}$

$\omega = \sqrt{\frac{80}{3R}}$

Given $\omega = 4\sqrt{\frac{x}{3R}}$

$16\frac{x}{3R} = \frac{80}{3R}$

$x = 5$

23. (2) $L = 1$ m

$\Delta L = 0.4 \times 10^{-3}$ m

$m = 1$ kg

$d = 0.4 \times 10^{-3}$ m

$\frac{F}{A} = Y\frac{\Delta L}{L}$

$Y = \frac{FL}{A\Delta L} = \frac{(mg)\cdot(1)}{\left(\frac{\pi d^2}{4}\right)0.4 \times 10^{-3}}$

$\Rightarrow \frac{10 \times 4}{\pi(0.4 \times 10^{-3})^2 \times 0.4 \times 10^{-3}}$

$Y = \frac{40}{\pi(0.4 \times 10^{-3})^3}$

$Y = \frac{40 \times 7}{22 \times 64 \times 10^{-3} \times 10^{-9}}$

$Y = 0.199 \times 10^{-12}\ N/m^2$

$\frac{\Delta Y}{Y} = \frac{\Delta F}{F} + \frac{\Delta L}{L} + \frac{\Delta A}{A} + \frac{\Delta(\Delta L)}{(\Delta L)}$

$= \frac{0.02}{0.4} + 2\frac{\Delta d}{d} = \frac{0.2}{4} + 2 \times \frac{0.01}{0.4}$

$= \frac{0.1}{2} + \frac{0.1}{2} = 0.1$

$\Rightarrow \Delta Y = 0.1 \times Y$

$= 0.199 \times 10^{11} = 1.99 \times 10^{10}$

24. (200)

$f_1 = 100 = f_0\left(\frac{C}{C - V_s}\right)$

C = speed of sound

V_S = speed of source

$$f_2 = 50 = f_0\left(\frac{C}{C+V_s}\right)$$

$$\frac{f_1}{f_2} = 2 = \frac{C+V_s}{C-V_s}\ 2C - 2\,V_s = C + V_s$$

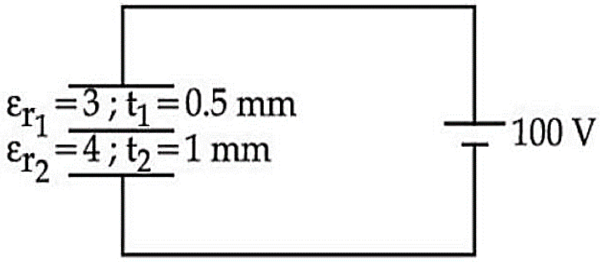
$$3\,V_s = C$$

$$V_s = \frac{C}{3}$$

$$100 = f_0\frac{C}{\frac{2C}{3}} = \frac{3}{2}f_0$$

$$f_0 = \frac{200}{3}$$

25. (60)



Capacitance of each capacitor

$$C_1 = \frac{A3\in_0}{\frac{1}{2}} = 6\,A\in_0$$

$$C_2 = A4\epsilon_0 = 4\,A\in_0$$

Equivalent capacitance

$$C_{eq} = \frac{C_1C_2}{C_1+C_2} \Rightarrow \frac{24}{10}\,A\in_0$$

$$q_{net} = C_{eq}\,(\Delta V) \Rightarrow 240\,A\in_0$$

$$\Delta V_2 = \frac{240\,A\in_0}{4\,A\in_0} = 60\text{ V}$$

(ΔV_2 = Potential drop across C_2)

$V_{foil} = 60$ V

26. (20) Initially, $\frac{P}{Q} = \frac{40\text{ cm}}{60\text{ cm}} = \frac{2}{3}$

Finally, $\frac{P+x}{Q} = \frac{80\text{ cm}}{20\text{ cm}} = \frac{4}{1}$

Divide (2) by (1)

$$\frac{P+x}{P} = 4 \times \frac{3}{2} = 6$$

$$\Rightarrow \ 1 + \frac{X}{P} = 6 \Rightarrow \frac{X}{P} = 5$$

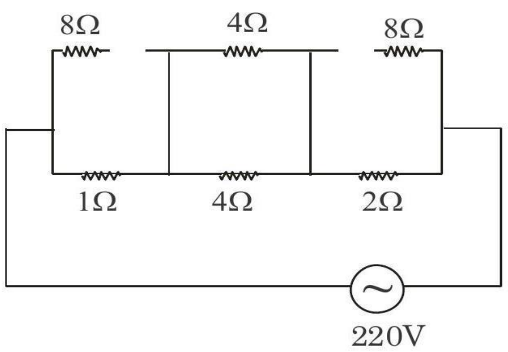
$\therefore \ x = 5P = 5 \times 4 = 20\Omega$

27. (44) At very high frequencies,

$$X_C = \frac{1}{\omega C} \approx 0$$

Also $X_L = \omega L \approx \infty$

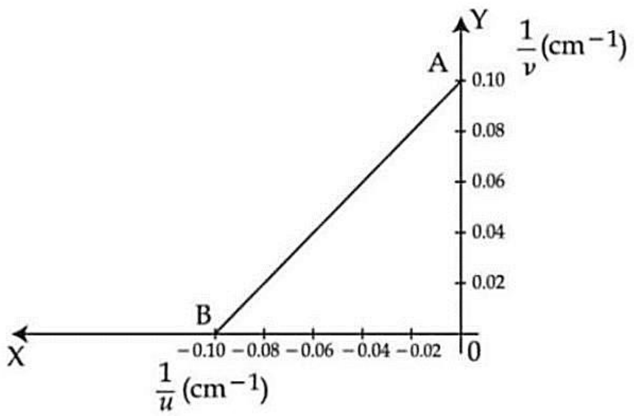
Thus, equivalent circuit can be redrawn as



$Z = 1 + 2 + 2 = 5\Omega$

$$I = \frac{220 \text{ V}}{5\Omega} = 44 \text{ A}$$

28. (10)



For point B, $\frac{1}{u} = -0.10 \text{ cm}^{-1}, \frac{1}{v} = 0$

$\therefore$ Thus, $u = -10$ cm, $v = \infty$

i.e. $f = 10$ cm

$$\Rightarrow \frac{1}{10\text{ cm}} = (1.5-1)\left(\frac{2}{R}\right) = \frac{1}{R} \Rightarrow R = 10\text{ cm}$$

29. (5) For first line of Lyman

$$\frac{1}{\lambda} = R\left(1-\frac{1}{4}\right) = R\left(\frac{3}{4}\right)$$

$$\Rightarrow \lambda = \frac{4}{3R}$$

3^{rd} line (Paschen)

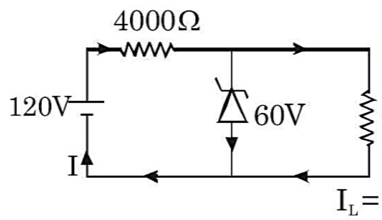
$$\frac{1}{\lambda_3} = R\left(\frac{1}{3^2}-\frac{1}{6^2}\right) = \frac{R}{9}\times\frac{3}{4}$$

2nd line (Balmer)

$$\frac{1}{\lambda_2} = R\left(\frac{1}{2^2}-\frac{1}{4^2}\right) = \frac{R}{4}\times\frac{3}{4}$$

Thus $a\lambda = \lambda_3 - \lambda_2 = \frac{12}{R} - \frac{16}{3R} = \frac{20}{3R}$ putting (1) $a\left(\frac{4}{3R}\right) = \frac{20}{3R} \Rightarrow a = 5$

30. (9) Consider input 120 V



$$I_L = \frac{60V}{10{,}000\Omega} = 0.006A$$

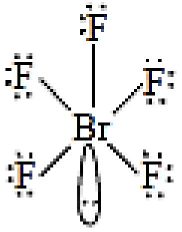
$$I = \frac{(120-60)V}{4000\Omega} = 0.015\text{ A}$$

Thus $I_2 = I - I_L$

$= 0.015 - 0.006 = 0.009\text{ A} = 9\text{ mA}$

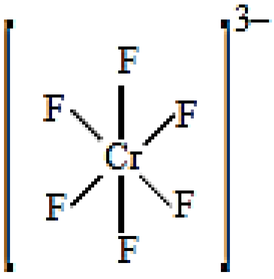
31. (c)

Sol.



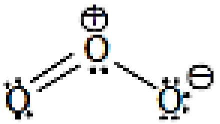
(Square pyramidal)

$[CrF_6]^{3-}$:



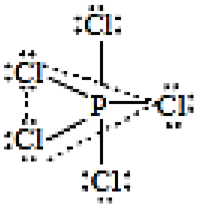
(Octahedral)

O_3 :



(Bent)

PCl_5 :



(Trigonal bipyramidal)

32.(b)

$$2SO_2(g) + O_2(g) \xrightarrow{V_2O_5} 2SO_3(g):$$

contact process

$$4NH_3(g) + 5O_2(g) \xrightarrow{Pt(s)-Rh(s)} 4NO(g) + 6H_2O(g)$$

Ostwald's process

$$N_2(g) + 3H_2(g) \xrightarrow{Fe(s)} 2NH_3(g); \text{Haber's process}$$

$$\text{Vegetable oil } (l) + H_2(g) \xrightarrow{Ni(s)} \text{vegetable ghee}$$

: Hydrogenation

33. **(d)** In Cl_2 molecule, the covalent radius is half of the internuclear distance, so statement(I) is false.
For the same element, anion has lower effective nuclear charge than atom ⇒ so anion is larger than atom. ⇒ statement (II) is correct.

34. **(a)** Liquation is used to purify metals having lower melting point than impurities present in them.

35. (a) Urea acts as stabiliser for H_2O_2.
36. (b) $2BeCl_2 + LiAlH_4 \rightarrow 2BeH_2 + LiCl + AlCl_3$
37. (b)

$3\ B_2H_6 + 6NH_3 \xrightarrow{\Delta} 2\ B_3\ N_3H_6 + 12H_2$

38. (c)

$2H_2\overset{-1}{O}_2 \longrightarrow 2H_2\overset{2-}{O} + \overset{0}{O}$: : Disproportionation

$2\ \overset{+4}{N}O_2 + H_2O \rightarrow H\overset{+5}{N}O_3 + \overset{+3}{H}N_2$: Disproportionation

$MnO_4^- + 4H^+ + 3e^- \rightarrow MnO_2 + 2H_2O$: reduction

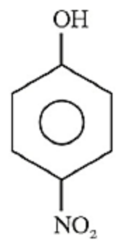
$3MnO_4^{-2-} + 4H^+ \rightarrow \overset{+77}{Mn}O_4^- + \overset{+4}{Mn}O_2 + 2H_2O$: Disproportionation

39. (a) In acidic medium,

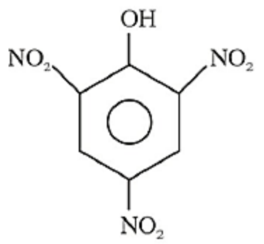
$MnO_4^- \rightarrow Mn^{+2}$

change in ox. no. = 5

40. (c) $Ci + CH_4 \longrightarrow \dot{C}H_3 + HCl$
41. (d)



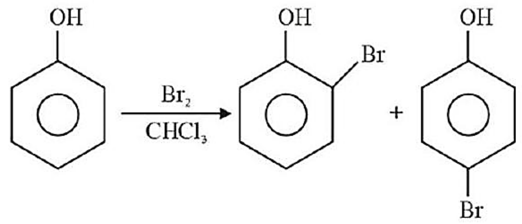
Shows inter molecular H-bonding

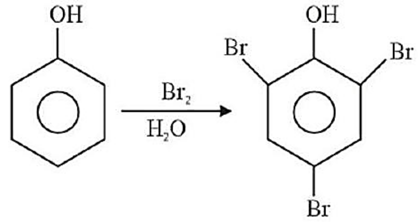


Shows intra molecular H-bonding

Solvent polarity has been related to R_f value of nitrocompounds.
100mg p-nitrophenol and picric acid have different R_f value on silica gel plate
∴ Preparative TLC is best to separate 100mg of para nitrophenol and picric acid

42.(b)





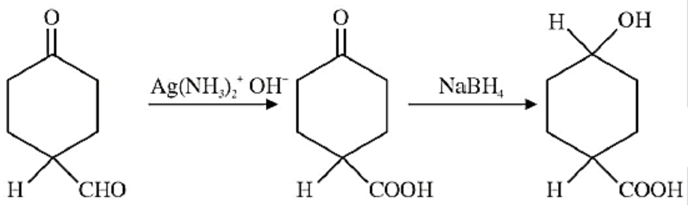
Difference in reactions is observed due to solvent polarity, which

(i) Ionizes phenol to make more reactive phenoxide ion

(ii) Increases electrophilicity of bromine.

43. **(c)** [10] Annulene, although follow $(4n + 2)\pi$ electron rule, but it is non-aromatic due to its non planar nature. It is nonplanar due to repulsion of $C - H$ bonds present inside the ring.

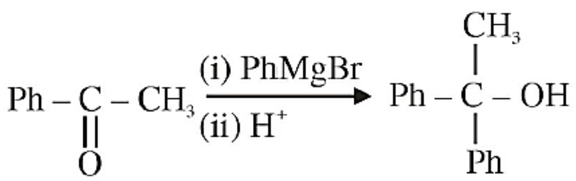
44. **(c)**



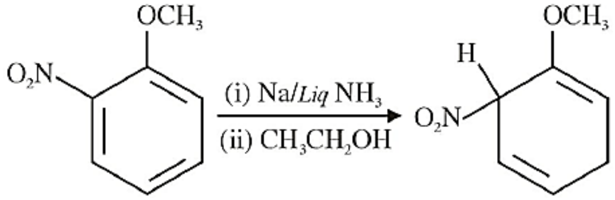
(A) (B)

$NaBH_4$ does not reduce carboxylic acid.

45.(d)

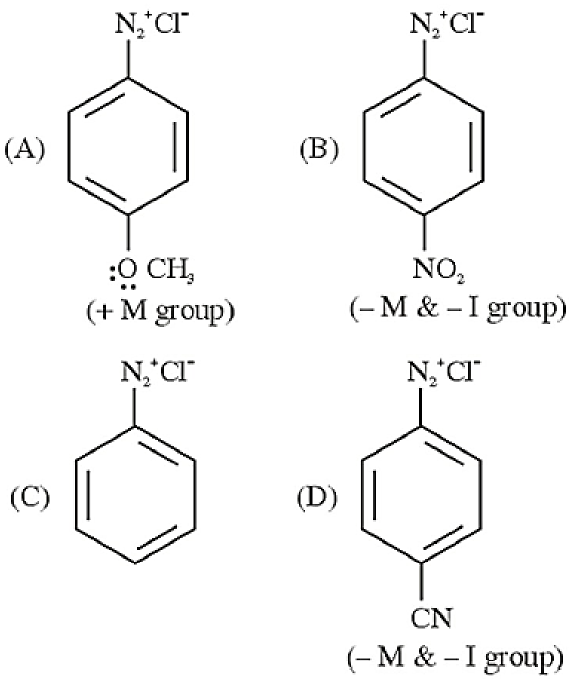


46. **(a)**



Given reaction is an example of birch reduction.

47. **(b)**

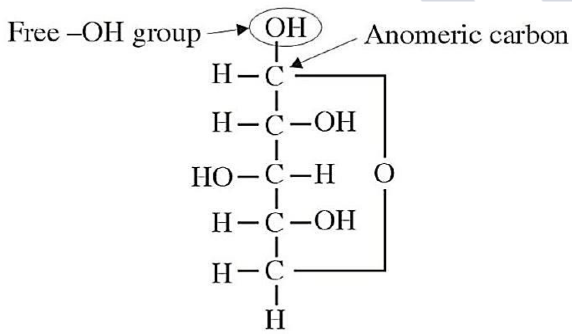


Since diazonium ion is a cation hence it is stabilized by electron donating groups and destabilized by electron withdrawing group. Hence Stability order should be A > C > D > B.

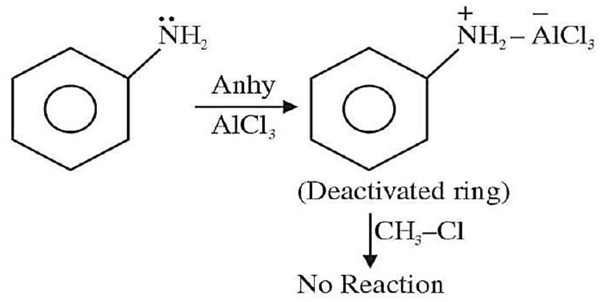
48. (d)

$$\underset{\text{Stearic acid}}{CH_3(CH_2)_{16}COOH} + \underset{\text{Polyethylene glycol}}{OH(CH_2CH_2O)_n CH_2CH_2OH} \xrightarrow{-H_2O} \underset{\text{Non-ionic detergent}}{CH_3(CH_2)_{16}-\overset{\overset{\displaystyle O}{\|}}{C}-O-(CH_2CH_2O)_n CH_2CH_2OH}$$

49.(a) If any sugar is having free $-OH$ group at anomeric carbon then it will be a reducing sugar



50. (c)



Friedel Craft Alkylation does not occur on this deactivated ring.

51. **(24)** ppm $= \frac{W_{Mg}}{V_{soln}} \times 10^6 = 48$

$\Rightarrow W_{Mg} = \frac{48 \times 2 \times 1000}{10^6}$

$= 48 \times 2 \times 10^{-3}$ g

$n_{Mg} = \frac{W_{Mg}}{24} = \frac{48 \times 2 \times 10^{-3}}{24}$

$= 4 \times 10^{-3}$

Number of Mg atoms $= 4 \times 10^{-3} \times 6.02 \times 10^{23}$

$= 4 \times 6.02 \times 10^{20}$

$= 24.08 \times 10^{20}$

$\therefore x = 24.08$

52. **(2)** Let $W_{H_2} = 40\text{ g} \Rightarrow n_{H_2} = \frac{40}{2} = 20$

$W_{O_2} = 60\text{ g} \Rightarrow n_{O_2} = \frac{60}{32} = \frac{15}{8}$

$P_{H_2} = \left(\frac{20}{20 + \frac{15}{8}}\right) \times 2.2$

$= \frac{20}{20 + 1.875} \times 2.2$

$= \frac{20}{21.875} \times 2.2$

$= 2.0114$

$\simeq 2.01\text{bar}$

53. **(1758)** $v_e = xv_N$

$\lambda_e = \lambda_N$

$$\Rightarrow \frac{h}{m_e v_e} = \frac{h}{m_N v_N}$$

$$v_e = \frac{m_N}{m_e} \cdot v_N$$

$$= \frac{1.6 \times 10^{-27}}{9.1 \times 10^{-31}} v_N$$

$$v_e = 1758.24 \times v_N$$

$$\therefore x = 1758.24$$

54. (200)

$C(s) + O_2(g) \rightarrow CO_2(g); \Delta H = -xkJ/\text{ mole}$

$Q = C\Delta T = 20\text{ kJ} \times 2$

40 kJ heat is released for 2.4 g of C

For 1 mole ' C ' :

$$Q = \frac{40}{2.4} \times 12$$
$$= \frac{400}{24} \times 12 = 200\text{ kJ/mole}$$

$Q = \Delta E = \Delta H = 200\text{ kJ}(\because \Delta n_g = 0)$

$x = 200$

55. (54)

$n_{HNO_3} = 0.5 \times 0.8$

$= 0.4$ mole

$$(n_{HNO_3})_{\text{remains}} = 0.4 - \frac{11.5}{63}$$

$= 0.4 - 0.1825$

$= 0.2175$

Molarity $= \frac{0.2175}{400} \times 1000$

$$= \frac{0.2175}{0.4}$$

$= 0.5437$ mole/lit.

$\simeq 0.54$ mole/lit.

$= 54 \times 10^{-2}$ mol/ lit.

56. (2)

$$K'_{eq} = \frac{1}{\sqrt{K_{eq}}} = \frac{1}{\sqrt{2 \times 10^{15}}} = x \times 10^{-8}$$

$$\Rightarrow \frac{1}{\sqrt{20}} \times \frac{1}{10^{7}} = x \times 10^{-8}$$

$$\Rightarrow \frac{1}{\sqrt{20}} \times 10^{-7} = x \times 10^{-8}$$

$$\frac{10}{\sqrt{20}} = x$$

$$\Rightarrow x = \frac{\sqrt{10}}{\sqrt{2}} = \sqrt{5} = 2.236$$

$\simeq 2.24$

57. (8)

$$Fe_3O_4 \overset{+8e^-}{\rightarrow} 3Fe$$

Charge for 1 mole Fe $= 8/3$ F

$= 2.67$ F

58. (2) $t_{\frac{1}{2}} \propto \frac{1}{[A_0]^{n-1}}$

$$[100] \propto \frac{1}{(0.5)^{n-1}}$$

$$(50) \propto \frac{1}{(1)^{n-1}}$$

$$[2]^1 = \left[\frac{1}{0.5}\right]^{n-1}$$

$$[2]^1 = [2]^{n-1}$$

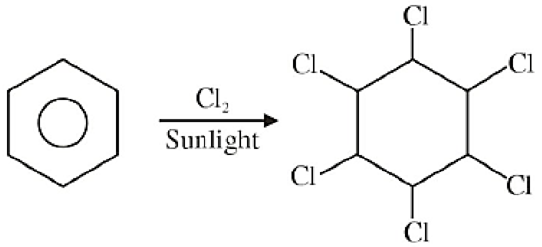
$n - 1 = 1$

$n = 2$

order $= 2$

59. (0)

60. (6)



61. (b) $f(x) - f(x/3) = x/3$

$f(x/3) - f(x/3^2) = x/3^2$

.... on adding

$$f(x) - \lim_{n\to\infty}\left(\frac{x}{3^n}\right) = x\left(\frac{1}{3} + \frac{1}{3^2} \ldots .\infty\right)$$

$$f(x) - f(0) = \frac{x}{2}$$

$f(8) = 7; f(0) = 3$

$f(x) = x/2 + 3$

$f(14) = 10$

62. (d) $AB = AO \cdot z^{-i\pi/2} = -2 + i$

So $OB = (-2 + i) + (1 + 2i)$

$z_2 = -1 + 3i$

$\therefore |2z_1 - z_2| = \sqrt{10}$

63. (d)

$$D = \begin{vmatrix} 8 & 1 & 4 \\ 1 & 1 & 1 \\ \lambda & -3 & 0 \end{vmatrix} = 0 \Rightarrow \lambda = 4$$

Also $D_1 = D_2 = D_3 = 0$

So $\mu = -2$

Point $\left(4, -2, -\frac{1}{2}\right)$

Distance from plane $= \frac{10}{3}$

64. (b) Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$; $ad - bc = -1$

$$|A + I||\operatorname{adj} A + I| = 4$$
$$\Rightarrow ad - bc + a + d + 1 = 2 \text{ or } -2$$
$$a + d = 2 \text{ or } -2$$

65. (b) $A = \int_1^3 y^a \cdot dy = \left.\frac{y^{a+1}}{a+1}\right|_1^3 = \frac{364}{3}$

$\Rightarrow a = 5$

66. (c) $\left((2^1 2^2 \ldots\ldots 2^{60})(4^1 . 4^2 \ldots\ldots 4^n)\right)^{\frac{1}{60+n}} = 2^{\frac{225}{8}}$

$$\left(2^{30\times 61} 4^{\frac{n(n+1)}{2}}\right)\frac{1}{60+n} = 2^{\frac{225}{8}}$$

$$2^{1830+n^2+n} = 2^{\frac{(225)(60+n)}{8}}$$

$= 8n^2 - 217n + 1140 = 0$

$$n = 20, \frac{57}{8}$$

$$\sum_{k=1}^{n} nk - k^2 = \frac{n^2(n+1)}{2} - \frac{n(n+1)(2n+1)}{6}$$

$= 1330$

67. (1)

$$\lim_{x\to 0} \frac{(\ln(1+x^2+x^4))\cos x}{1-\cos^2 x}$$

$$\lim_{x\to 0} \frac{\left(\frac{\ln(1+x^2+x^4)}{x^2+x^4}\right)x^2(1+x^2)\cos x}{\left(\frac{\sin^2 x}{x^2}\right)x^2} = 1$$

$\therefore k = 1$

68. (d)

$$f(x) = \begin{cases} x+a & ; x \le 0 \\ |x-4| & ; x > 0 \end{cases}; \quad g(x) = \begin{cases} x+1 & ; x < 0 \\ (x-4)^2 + b; & x \ge 0 \end{cases}$$

For continuity $a = 4$ and $b = -15$

$g(f(2)) + f(g(-2))$

$= g(2) + f(-1) = -8$

69. (c) $f(1) = 3$

For $x < 1, f'(x) = 3x^2 - 2x + 10 > 0$

$\Rightarrow f(x)$ is increasing

For $x > 1$, $f'(x) < 0$

$\Rightarrow$ function is decreasing.

$\lim_{x\to 1^+} f(x) = -2 + \log_2(b^2 - 4)$

For maximum value at $x = 1$

$3 \geq -2 + \log_2(b^2 - 4)$

$32 \geq b^2 - 4 > 0$

$b \in [-6, -2) \cup (2, 6]$

70. (c)

$$a = \frac{1}{n}\sum_{k=1}^{n} \frac{2}{1 + \left(\frac{k}{n}\right)^2} = \int_0^1 \frac{2}{1 + x^2} dx = \frac{\pi}{2}$$

$f(x) = \tan\left(\frac{x}{2}\right); x \in (0,1)$

$f\left(\frac{\pi}{4}\right) = \sqrt{2} - 1$

$f'\left(\frac{\pi}{4}\right) = \frac{1}{2}\sec^2\left(\frac{\pi}{8}\right) = \frac{\sqrt{2}}{\sqrt{2} + 1}$

$f'\left(\frac{\pi}{4}\right) = \sqrt{2} f\left(\frac{\pi}{4}\right)$

71. (a) $\frac{dy}{dx} + 2y\tan x = \sin x$

$I.F = e^{\int 2\tan x dx} = e^{\ln(\sec x)^2} = \sec^2 x$

$y(\sec^2 x) = \int \sin x \sec^2 x dx + C$

$y \cdot \sec^2 x = \sec x + C$

Put $x = \frac{\pi}{3}, y = 0$

$y = \cos x - 2\cos^2 x$

$= \frac{1}{8} - 2\left(\cos x - \frac{1}{4}\right)^2$

$\therefore y_{max} = \frac{1}{8}$

72. (b)

$(x-1)^2 + (y-2)^2 + (x+2)^2 + (y-1)^2 = 14$

$\Rightarrow x^2 + y^2 + x - 3y - 2 = 0$

Put $x = 0$

$\Rightarrow y^2 - 3y - 2 = 0$

$\Rightarrow y = \frac{3 \pm \sqrt{17}}{2}$

Put $y = 0$

$\Rightarrow x^2 + x - 2 = 0$

$(x+2)(x-1) = 0$

$\therefore A(-2,0), B(1,0), C\left(0, \frac{3+\sqrt{17}}{2}\right), D\left(0, \frac{3-\sqrt{17}}{2}\right)$

Area $= \frac{1}{2} \cdot 3 \cdot \sqrt{17} = \frac{3\sqrt{17}}{2}$

73. (d) Tangent at (α, β) has slope 1

$\beta^2 = 24\alpha$

Equation of tangent $y\beta = 12(x+\alpha), \frac{12}{\beta} = 1$

$\Rightarrow \alpha = 6, \beta = 12$

$\therefore (\alpha+4, \beta+4) = (10,16)$

Normal at $(10,16)$ to $\frac{x^2}{36} - \frac{y^2}{144} = 1$ is

$2x + 5y = 100$

74. (a) d.r's of the line $= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & -1 \\ 1 & -2 & 3 \end{vmatrix} = \hat{i} - 4\hat{j} - 3\hat{k}$

$\therefore$ equation of line is

$\vec{r} = \hat{i} + 2\hat{j} + 4\hat{k} + \lambda(\hat{i} - 4\hat{j} - 3\hat{k})$

Let $A(1,2,4)$ and P be $(1+\lambda, 2-4\lambda, 4-3\lambda)$

$\therefore \overrightarrow{PA} \cdot (\hat{i} - 4\hat{j} - 3\hat{k}) = 0$

$\lambda = \frac{1}{2}$

$\Rightarrow P\left(\frac{1}{2}, 2, \frac{-5}{2}\right)$

$|AP| = \sqrt{\frac{21}{2}}$

75. (d) $\vec{a} \times \vec{b} = (1-\alpha)\hat{i} + (\alpha^2 - 2)\hat{j} + (\alpha - 2)\hat{k}$

Projection of $\vec{a} \times \vec{b}$ on $-\hat{i} + 2\hat{j} - 2\hat{k}$

$= \frac{(\vec{a} \times \vec{b}) \cdot (-\hat{i} + 2\hat{j} - 2\hat{k})}{3} = 30$

$\Rightarrow 2\alpha^2 - \alpha - 91 = 0$

$\Rightarrow \alpha = 7, -\frac{13}{2}$

76. (c) $np = \alpha$

$npq = \alpha/3$

From (1) & (2)

$q = 1/3 \& p = 2/3$

${}^{n}C_1 q^{n-1} p^1 = \frac{4}{243}$

$\frac{n}{3^n} = \frac{2}{243}$

$n = 6$

$P(4 \text{ or } 5) = {}^{6}C_4 \left(\frac{2}{3}\right)^4 \left(\frac{1}{3}\right)^2 + {}^{6}C_5 \left(\frac{2}{3}\right)^5 \cdot \left(\frac{1}{3}\right)^0$

$= \frac{16}{27}$

77. (d) $0 \leq P(E_i) \leq 1$ for $i = 1,2,3$

$\Rightarrow -2/3 \leq p \leq 1$

$E_1 \& E_2 \& E_3$ are mutually exclusive

$P(E_1) + P(E_2) + P(E_3) \leq 1$

$\Rightarrow 2/3 \leq p \leq 1$

$p_1 = 1, p_2 = 2/3$

$p_1 + p_2 = 5/3$

78. (c) $8^{2\sin^2\theta} + 8^{2-2\sin^2\theta} = 16$

$y + \frac{64}{y} = 16$

$\Rightarrow y = 8$

$\Rightarrow \sin^2\theta = 1/2$

$n(S) + \sum_{\theta \in S} \frac{1}{\cos(\pi/4 + 2\theta)\sin(\pi/4 + 2\theta)}$

$= 4 + (-2) \times 4 = -4$

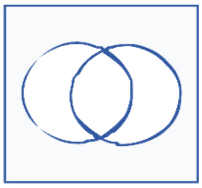
79. (b) $\tan\left(2\left(\tan^{-1}\frac{1}{5} + \tan^{-1}\frac{1}{8}\right) + \tan^{-1}\left(\frac{1}{2}\right)\right)$

$= \tan\left[2\tan^{-1}\left(\frac{1}{3}\right) + \tan^{-1}\left(\frac{1}{2}\right)\right]$

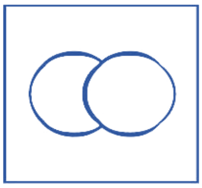
$= 2$

80. (d) $(\sim (p \Leftrightarrow \sim q)) \wedge q \equiv (p \Leftrightarrow q) \wedge q$

$(p \Leftrightarrow q) \wedge q \equiv p \wedge q$



p⇔q



p→q

81. (272)

$(\mathrm{px} - \mathrm{q})^2 + (\mathrm{qx} - \mathrm{r})^2 = 0$

$\Rightarrow \mathrm{x} = \frac{\mathrm{q}}{\mathrm{p}} = \frac{\mathrm{r}}{\mathrm{q}} = -4$

$\Rightarrow \frac{\mathrm{q}^2 + \mathrm{r}^2}{\mathrm{p}^2} = 272$

82. (180) $3 \times \frac{5!}{2!2!} + \frac{5!}{3!\times 2!} + \frac{5!}{2!} + \frac{5!}{3!} = 180$

83. (6993)

$\mathrm{S}_{11} = 3[101 + 102 + +121]$

$= \frac{3}{2}(222) \times 21 = 6993$

84. (3)

$\mathrm{x}^5(\mathrm{x}^3 - \mathrm{x}^2 - \mathrm{x} + 1) + \mathrm{x}(3\mathrm{x}^3 - 4\mathrm{x}^2 - 2\mathrm{x} + 4) - 1 = 0$

$\Rightarrow (\mathrm{x} - 1)^2(\mathrm{x} + 1)(\mathrm{x}^5 + 3\mathrm{x} - 1) = 0$

Let $\mathrm{f}(\mathrm{x}) = \mathrm{x}^5 + 3\mathrm{x} - 1$

$\mathrm{f}'(\mathrm{x}) > 0 \forall \mathrm{x} \in \mathrm{R}$

Hence 3 real distinct roots.

85. (23) Since coefficient of x is -3

$\Rightarrow {}^{\mathrm{p}}\mathrm{C}_1 - {}^{\mathrm{q}}\mathrm{C}_1 = -3$

$\Rightarrow \mathrm{p} - \mathrm{q} = -3$

Comparing coefficients of x^2

$- {}^{\mathrm{p}}\mathrm{C}_1^{\mathrm{q}}\mathrm{C}_1 + {}^{\mathrm{p}}\mathrm{C}_2 + {}^{\mathrm{q}}\mathrm{C}_2 = -5$

$-pq + \frac{p(p-1)}{2} + \frac{q(q-1)}{2} = -5$

Solving (1) and (2)

$p = 8, q = 11$

Coefficient of x^3 is

$- {}^{q}C_3 + {}^{p}C_3 + {}^{p}C_1 C_2 - {}^{p}C_2^{q} C_1$

$= - {}^{11}C_3 + {}^{8}C_3 + {}^{8}C_1\, {}^{11}C_2 - {}^{8}C_2\, {}^{11}C_1$

$= 23$

86. (24) Let $I_1 = \int_0^1 (1-x^n)^{2n} dx, I_2 = \int_0^1 (1-x^n)^{2n+1} dx$

$I_2 = \int_0^1 (1-x^n)^{2n+1} \cdot 1dx$

$= (1-x^n)^{2n+1} \cdot x|_0^1 - \int_0^1 (2n+1)(1-x^n)^{2n}(-nxn^{n-1})xdx$

$I_2 = -n(2n+1)\{I_2 - I_1\}$

$(2n^2 + n + 1)I_2 = n(2n+1)I_1$

$\frac{I_1}{I_2} = \frac{2n^2+n+1}{n(2n+1)} = \frac{1177}{n(2n+1)}$

$\Rightarrow 2n^2 + n - 1176 = 0 \Rightarrow n = 24$

87. (6) $x^4 = 3yx \cdot y' - 3y^2$

$\Rightarrow 3xy\frac{dy}{dx} = 3y^2 + x^4$

Put $y^2 = t, y\frac{dy}{dx} = \frac{1}{2}\frac{dt}{dx}$

$\frac{dt}{dx} - \frac{2}{x}t = \frac{2}{3}x^3$

$\therefore \frac{t}{x^2} = \frac{x^2}{3} + C$

$\Rightarrow \frac{y^2}{x^2} = \frac{x^2}{3} - 2$

Put $(3,3), C = -2$

$\therefore \frac{y^2}{x^2} = \frac{x^2}{3} - 2$

$3y^2 = x^4 - 6x^2$

$x^4 - 6x^2 = 1080$

$\therefore x = 6$

88. (3) Coordinates of $A(1, -2), B\left(\frac{15a}{1-2p}, \frac{-30a}{1-2p}\right)$ and orthocentre $H(2, a)$

Slope of AH $= p$

$a + 2 = p$

Slope of BH $= -1$

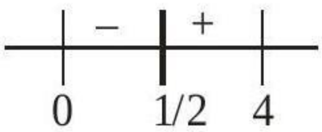
$31a - 2ab = 15a + 4p - 2$

From (1) and (2)

$a = 1 \& p = 3$

89. (45)

$$f'(x) = 4x - \frac{1}{x}$$



$$a = \frac{1}{2}$$

Let $P(x_1, y_1)$ be any point on $y^2 = 4ax$

$$\frac{1}{y_1} = \frac{3 - y_1}{4 - x_1} \Rightarrow y_1^2 - 6y_1 + 8 = 0$$

$y_1 = 2,4$

$\Rightarrow P(8,4)$ as $P(2,2)$ rejected

Equation of normal at P.

$y - 4 = -4(x - 8)$

$$\frac{x}{9} + \frac{y}{36} = 1$$

$\alpha = 9, \beta = 36$

$\alpha + \beta = 45$

90. (153)

Let $(2\lambda - 1, 3\lambda - 2, 2\lambda + 1)$ be any point on the line

$(2\lambda - 5)^2 + (3\lambda - 4)^2 + (2\lambda - 6)^2 = 26$

$\lambda = 1,3$

$Q(1,1,3); R(5,7,7); P(4,2,7)$

Area of triangle PQR $= 1/2|\overrightarrow{PQ} \times \overrightarrow{PR}|$

$= \sqrt{153}$