1. (c) $P = \frac{\alpha}{\beta}\log_e\left(\frac{kt}{\beta x}\right)$

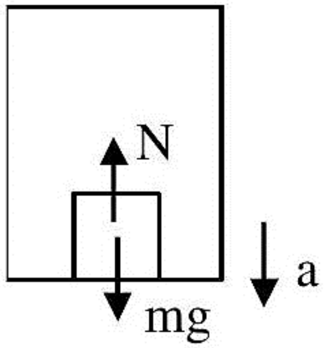
$$\frac{kt}{\beta x} = 1 \Rightarrow \beta = \frac{kt}{x} = \frac{ML^2\ T^{-2}}{L}$$

$$\left(\because E = \frac{1}{2}kt\right)$$

As P is dimensionless

$\Rightarrow [\alpha] = [\beta] = [MLT^{-2}]$

2. (b)



$mg - N = ma$

$\Rightarrow N = m(g - a)$

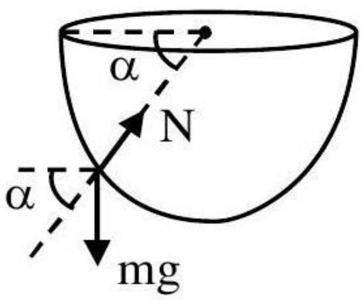
$\therefore$ Person experiences weightloss, when acceleration of lift is downward.

3. (a) At maximum height, $V = 0$

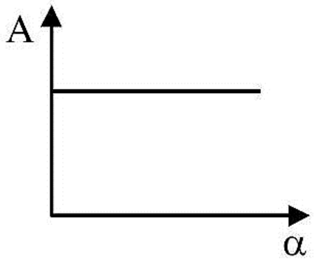
$\therefore$ Momentum of object is zero.

4. (c) $V = \sqrt{2gR\sin\alpha}$

$$N - mg\sin\alpha = \frac{mv^2}{R} = 2mg\sin\alpha$$



$$\frac{N}{2mg\sin\alpha} = \frac{1}{2} + 1 = \frac{3}{2}$$



$\Rightarrow A = \text{constant}$

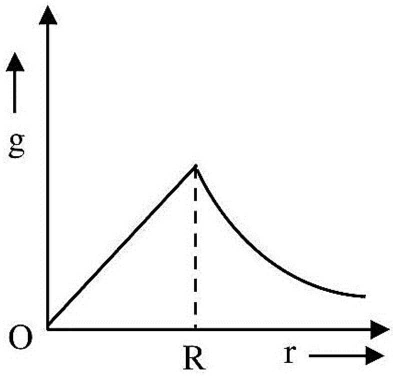
5. **(c)** Applying conservation of angular momentum

$MR^2\omega = (MR^2 + 2mR^2)\omega'$

$$\omega' = \frac{2M}{M+2m}$$

6. **(a)** $g = \begin{cases} \frac{GMr}{R^3}, r \leq R \\ \frac{GM}{r^2}, r \geq R \end{cases}$

$$T = 2\pi\sqrt{\frac{\ell}{g_{eff}}}$$



7. **(26.81)**

$$\eta = \left[1 - \frac{T_L}{T_n}\right] \times 100\%$$

$T_L = 0°C = 273\text{ K},\ T_n = 373\text{ K}$

$\therefore \eta = 26.809\%$

8. **(c)**

9. **(b)** $\frac{C_p}{C_v} = 1 + \frac{2}{F} = 1.4 \Rightarrow F = 5$

By conservation of energy

$$\frac{F}{2}nR\Delta T = \frac{1}{2}[\ nm]v^2$$

$$\Delta T = \frac{mv^2}{FR} = \frac{Mv^2}{5R}$$

10. **(a)** Charge on capacitor C_2

$$= \frac{C_2 \times Q_{total}}{C_{total}} = \frac{C_2[C_1 V]}{C_1 + C_2} = \frac{C_1 C_2 V}{C_1 + C_2}$$

11. **(c)** S1: In nonpolar molecules, centre of +ve charge coincides with centre of -ve charge, hence net dipole moment is comes to zero.

S2: When non polar material is placed in external field, centre of charges does not coincide, hence give non zero moment in field

12. **(a)** $\phi = 5t^3 + 4t^2 + 2t - 5$

$$|e| = \frac{d\phi}{dt} = 15t^2 + 8t + 2$$

$$\text{At } t = 2, |e| = 15 \times 2^2 + 8 \times 2 + 2$$

$$\Rightarrow e = 78\text{ V} \Rightarrow I = \frac{e}{R} = \frac{78}{5} = 15.60$$

13. **(c)** $R = \frac{\rho \ell}{A}$

$$\frac{\Delta R}{R} = \frac{\Delta \ell}{\ell} - \frac{\Delta A}{A}$$

$$\ell A = k$$

$$\frac{\Delta \ell}{\ell} + \frac{\Delta A}{A} = 0$$

$$\frac{\Delta R}{R} = \frac{2\Delta \ell}{\ell}$$

$$\frac{\Delta R}{R} = 2 \times 0.4 = 0.8\%$$

14. **(c)** $\frac{R_\alpha}{R_P} = \frac{M_\alpha}{M_P} \times \frac{q_P}{q_\alpha}$

$$\frac{R_\alpha}{R_P} = \frac{4}{1} \times \frac{1}{2} = 2$$

15. **(c)** $\vec{E} = 301.6\sin(kz - \omega t)(-\hat{a}_x) + 452.4\sin(kz - \omega t)\hat{a}_y$

$$\vec{B} = \frac{301.6}{C}\sin(kz - \omega t)(-\hat{a}_y) + \frac{452.4}{C}\sin(kz - \omega t)(-\hat{a}_x)$$

$$\vec{H} = \frac{\vec{B}}{\mu_0} = \frac{301.6}{\mu C}\sin(kz - \omega t)(-\hat{a}_y) + \frac{452.4}{\mu C}\sin(kz - \omega t)(-\hat{a}_x)$$

$$\vec{H} = -0.8\sin(kz - \omega t)\hat{a}_y - 1.2\sin(kz - \omega t)\hat{a}_x$$

For direction

$\vec{E} \times \vec{B}$ is direction of $\vec{C}$

For first part $\hat{E} = -\hat{i}, \hat{B} = ?$

$\hat{E} \times \hat{B} = \hat{k} \Rightarrow \hat{B} = -\hat{j}$

Similarly, for second

$\hat{E} = \hat{j}, \hat{B} = ?$

$\hat{E} \times \hat{B} = \hat{k} \Rightarrow \hat{B} = -\hat{i}$

16. (d) $\frac{a}{\lambda} = \frac{1}{100}$

For reflection size of obstacle must be much larger than wavelength, for diffraction size should be order of wavelength. Since the object is of size $\frac{\lambda}{100}$, much smaller than wavelength, so scattering will occur.

17. (b) $\lambda_e = \lambda_{photon}$

$$\frac{h}{mv} = \frac{h}{P_{photon}} \Rightarrow P_{photon} = mv$$

$$\frac{E_e}{E_{ph}} = \frac{\frac{1}{2}mv^2}{\frac{hc}{\lambda}} = \frac{1}{2}\frac{mv}{P_{Ph}C} \times v = \frac{v}{2C}$$

18. (d)

$$^{238}_{92}U \rightarrow \boxed{8}\,^{4}_{2}He + \boxed{6}\,^{0}_{-1}e + ^{206}_{82}Pb$$

8α particles and 6β particles are emitted.

19. (b) $R = \frac{\Delta V}{\Delta i}$

$$\frac{R_1}{R_2} = \frac{\Delta v_1}{\Delta v_2}\frac{\Delta i_2}{\Delta i_1} = \frac{0.1}{0.2} \times \frac{50}{5} = 5$$

20. (c) In amplitude modulation the amplitude of high frequency carrier wave is varied in accordance with message signal

21. (60) Both should have same horizontal component of velocity

$200 = 400\cos\theta$

$\theta = 60°$

22. (5) $v^2 = u^2 + 2\,as$

$100 = 0 + 2(10)s$

$S = 5$ m

Height from ground $= 10 - 5 = 5$ m

23. (25) $y=\frac{\text{stress}}{\text{strain}}=2.0\times 10^{10}$

Energy density $=\frac{1}{2}$ stress $\times$ strain

$=\frac{1}{2}(\text{ strain })^2 y=\frac{1}{2}(5\times 10^{-4})^2\times 20\times 10^{10}$

$=25\times 10^2\times 10=25\frac{\text{kJ}}{\text{m}^3}\quad 25$

24. (6) $\Delta\ell\propto g$

$\frac{\Delta\ell_{\text{earth}}}{\Delta\ell_{\text{planet}}}=\frac{g_{\text{earth}}}{g_{\text{planet}}}=\frac{10^{-4}}{6\times 10^{-5}}$

$g_{\text{planet}}=6\ \text{m/s}^2$

25. (400)

$<\varepsilon>=\frac{\int\varepsilon dt}{\int dt}=\frac{\int(\text{ Ldi }/\text{dt})\text{dt}}{\int\text{dt}}=\frac{L\int\text{di}}{\int\text{dt}}$

$<\varepsilon>=\frac{\text{L}\Delta\text{i}}{\Delta\text{i}}$

$\text{i}_0=\frac{\text{V}}{\text{R}}=\frac{20}{10}=2\ \text{A, if i}=0\ \text{A}$

$\text{T}=100\mu s,\text{L}=20\text{mH}$

$<\varepsilon>=\frac{20\times 10^{-3}\times(2-0)}{100\times 10^{-6}}$

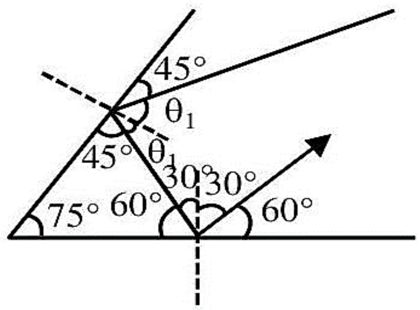
$=\frac{2\times 10^3}{5}$

$<\varepsilon>=400\ \text{V}$

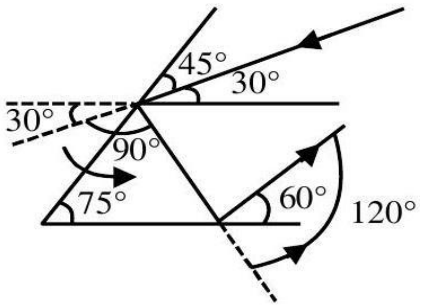
26. (210) $\delta_{\text{total}}=360^\circ-2\theta$

$=360^\circ-2\times 75^\circ$

$\delta_{\text{totakl}}=210^\circ$



$\theta_1=45^\circ$



$\delta = 120° + 90° = 210°$

27. (5) $20\text{MSD} = 1\text{ cm}$

$1\text{MSD} = \frac{1}{20}\text{ cm}$

$10\text{VSD} = 9\text{MSD}$

$1\text{VSD} = \frac{9}{10}\text{MSD}$

$= \frac{9}{10} \times \frac{1}{20}\text{ cm}$

$1\text{VSD} = \frac{9}{200}\text{ cm}$

$\text{VC} = 1\text{MSD} - 1\text{VSD}$

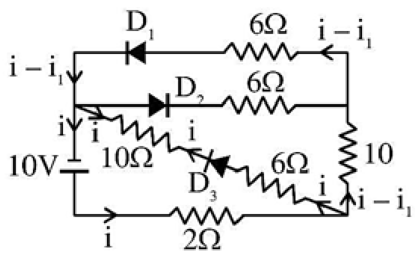
$= \frac{1}{20}\text{ cm} - \frac{9}{200}\text{ cm}$

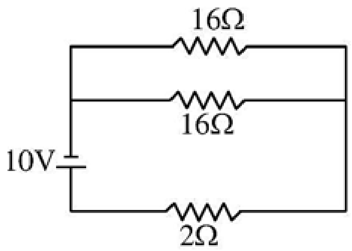
$= \frac{1}{200} \times 10\text{ mm}$

$\text{VC} = 5 \times 10^{-2}\text{ mm}$

5

28. (1)



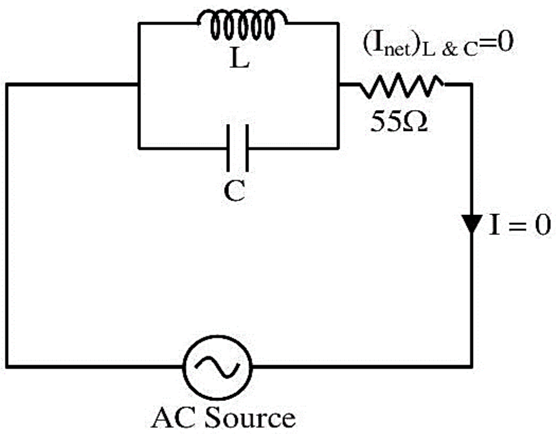


$V = IR_{net}$

$10 = I \times 10$

$I = 1\ A$

29. (0) At resonance $I_L = I_C$



Alternatively,

$$\frac{1}{Z} = \sqrt{\left(\frac{1}{X_L} - \frac{1}{X_C}\right)^2}$$

At resonance, $X_L = X_C \& Z \to \infty$

$\therefore Z_{\text{total circuit}} \to \infty$ i.e, $I = 0$

30. (363) From continuity equation

$$av_1 = \frac{a}{2}v_2$$

$v_2 = 2v_1$

From Bernoulli's theorem,

$$P_1 + \rho g\, h_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho g\, h_2 + \frac{1}{2}\rho v_2^2$$

$$P_1 - P_2 = \rho\left[\left(\frac{v_2^2 - v_1^2}{2}\right) + g(h_2 - h_1)\right]$$

$$4100 = 800\left[\left(\frac{4v_1^2 - v_1^2}{2}\right) + 10 \times (0 - 1)\right]$$

$$\frac{41}{8} + 10 = \frac{3v_1^2}{2}$$

$$\frac{121}{8} \times \frac{2}{3} = v_1^2$$

$$v_1 = \sqrt{\frac{I21}{4 \times 3} \times \frac{3}{3}}$$

$$v_1 = \frac{\sqrt{363}}{6}\ \text{m/s}$$

$X = 363.$

31. (c) Let total volume $= 1000$ mL $= 1$ L

total mass of solution $= 1460$ g

mass of HCl $= \frac{35}{100} \times 1460$

moles of HCl $= \frac{35 \times 1460}{100 \times 36.5}$

So molarity $= \frac{35 \times 1460}{100 \times 36.5} = 14$M

32. (d) Mass of liquid $= 135 - 40 = 95$ g

Volume of liquid $= \frac{\text{mass}}{\text{density}} = \frac{95}{.95}$ mL

$= 100$ mL $= 0.1$ L

mass of ideal gas $= 40.5 - 40$ g $= 0.5$ g

$PV = nRT$

$$0.82 \times 0.1 = \left(\frac{0.5}{M}\right) \times 0.082 \times 250$$

$M = 125$

33. (b) $r = 0.529 \times \frac{n^2}{z}$ Å

$$r_3 = 0.529 \times \frac{3^2}{1}$$

$r_4 = 0.529 \times \frac{4^2}{1}$

$\frac{r_4}{r_3} = \frac{4^2}{3^2} = \frac{16}{9}$

$r_4 = \frac{16 r_3}{9}$

34. (a)

ion/molecule	Number of e^- in BMO	Number of e^- in ABMO	Bond order
O_2^+	10	5	2.5
O_2	10	6	2
O_2^-	10	7	1.5
O_2^{2-}	10	8	1

Bond order $O_2^{2-} < O_2^- < O_2 < O_2^+$

35. (c) A cell with less variation in EMF with temperature is preferred as reference electrode because it can be used for wider range of temperature without much derivation from standard value so a cell with less $\left(\frac{\partial E}{\partial T}\right)_P$ is preferred.

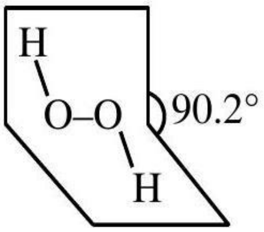
36. (a) Moving down the group stability of lower oxidation state increases

$Al < Ga < In < Tl$

37. (d) Metal oxide with lower ΔG° is more stable Statement II is correct

38. (a)

$2HSO_4^- \,^-(aq.) \xrightarrow[(2)\ \text{Hydrolysis}]{(1)\ \text{Electrolysis}} 2HSO_4^- + 2H^+ + \underset{(A)}{H_2O_2}$



Solid phase.

39. (d)

	M.P
Be	1560 K
Mg	924 K
Ca	1124 K
Sr	1062 K

40. (a)

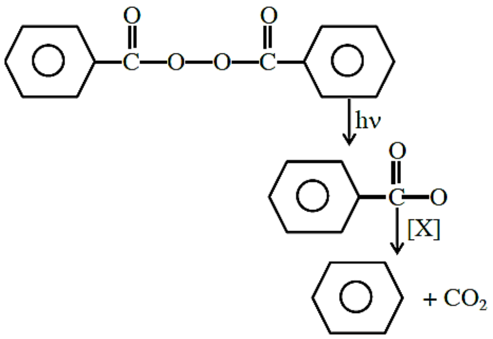
41. (b) Red P_4 + Alkali $\rightarrow H_4P_2O_6$ (No P-H bond)

42. (b) Polar stratospheric clouds provide surface on which hydrolysis of $ClONO_2$ takes place to form HOCl (Hypochlorous acid)

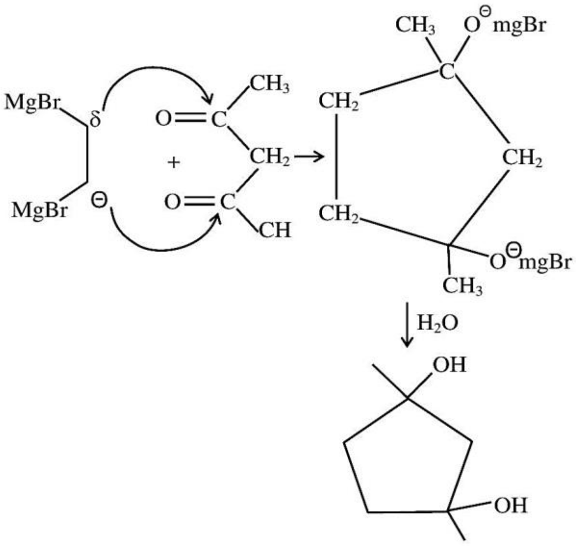
$$ClONO_2(g) + H_2O(g) \rightarrow HOCl(g) + HNO_3(g)$$

43. (a) Both statement I & statement II are correct.

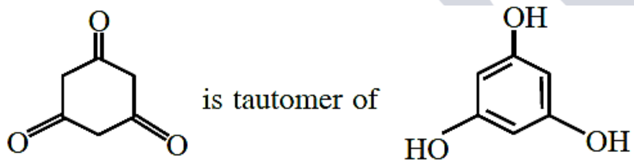
44. (d)



45. (a) Although Acetyl Acetone predominantly gives Acid base reaction with G.R due to Active methylene group but according to given option ans should be based on nucleophilic addition reaction (NAR).



46. (c)



Which is aromatic in nature.

47. (c) All these enamines are interconvertible through their resonating structures. So most stable form is ' C ' due to steric factor.

48. (a) Which of the following set are correct regarding polymer.

Bona - **5** is copolymer of butadiene + styrene

Nylon 6.6 is condensation polymer of adipic Acid and hexanediamine.

Nylon 6.6 is fiber

Terylene is fiber not themosetting polymer

Buna-N is copolymer nol Homopolymer

49. (c) Histamine (It is use for secretion of pepsin & HCl in stomach)

50. (b) Ring is formed due to formation of nitrosoferrous sulphate

51. (727) $\Delta U = -726 KJ/mol$

$$\Delta ng = 1 - 3/2 = \frac{-1}{2}$$

$$\Delta H = \Delta U + \Delta ngRT$$

$$= -726 - \frac{1}{2} \times \frac{8.3 \times 300}{1000}$$

$$= -727.245$$

52. (98) 0.5% solution of KCl

So $m = \frac{0.5}{74.6} \times \frac{1}{0.1}$

$$\Delta T_f = i \times m \times K_f$$

$$0.24 = i \times \frac{0.5}{74.6} \times \frac{1.80}{0.1}$$

$$i = \frac{0.24 \times 74.6}{0.5 \times 1.80} \times 0.1$$

$$= 1.989$$

$$1.989 = 1 + \alpha(n-1)$$

$$1.989 = 1 + \alpha$$

$$\alpha = .989$$

$$\%\alpha = 98.9\%$$

Ans 99%

If mass of $H_2O = 99.5$

$$m = \frac{0.5}{74.5} \times \frac{1}{.0995}$$

$$i = \frac{0.24 \times 74.6 \times .0995}{.5 \times 1.80}$$

$$= 1.979$$

$$1.979 = 1 + \alpha(n-1)$$

$$1.979 = 1 + \alpha$$

$$\alpha = .979$$

$$\%\alpha = 97.9\%$$

Ans 98%

53. (476) Moles of $CH_3COOH = 5$ m mole

moles of NaOH $= 2.5$ m mole

$NaOH + CH_3COOH \longrightarrow CH_3COONa + H_2O$

2.5 m mole 2.5mmole

0 2.5 m mole 2.5 m mole

so buffer is formed

$pH = pKa + \log\left(\frac{2.5/75}{2.5/75}\right) = pKa$

$pH = 4.76$

$= 476 \times 10^{-2}$

54. (200)

$k_A = \frac{\ln 2}{100}; k_B = \frac{\ln 2}{50}$

$A_t = A_0 \times e^{-k_A t}$

$A_t = A_0 \times e^{\left(\frac{-\ln 2}{100} \times t\right)}$

$B_t = B_0 \times e^{\left(\frac{-\ln 2}{50} \times t\right)}$

$A_0 = B_0$

$\& A_t = 4B_t$

$e^{-\frac{\ln 2}{100} \times t} = 4 \times e^{-\frac{\ln 2}{50} \times t}$

Final JEE-Main Exam June, 2022/26-06-2022/Morning Session

$e^{\frac{\ln 2}{100} \times t} = 4$

$e^{\frac{\ln 2}{100} \times t} = 4$

$\frac{\ln 2}{100} \times t = \ln 4 = 2\ln 2$

$t = 200 sec$

55. (9960) Volume of $H_2 = \frac{nRT}{p} = \frac{2}{2} \times \frac{0.083 \times 300}{1}$

$= 24.92$ L

$= 24900 \text{ mL}$

So 1 g platinum adsorb $= \frac{24900}{2.5} \text{mL} H_2$

$= 9960$

56. (3) Most basic oxide is V_2O_3

$\mathbf{V}^{+3} \rightarrow [A_r]3d^2$

$\mu = \sqrt{2(2+2)} = 2.84 \text{BM} \approx 3$

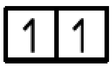
57. (3)

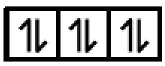
$CoCl_3.4NH_3 \rightarrow [Co(NH_3)_4Cl_2]Cl$

$NiCl_2 \cdot 6H_2O \rightarrow [Ni(H_2O)_6]Cl_2$

$PtCl_4 \cdot 2HCl \rightarrow H_2[PtCl_6]$

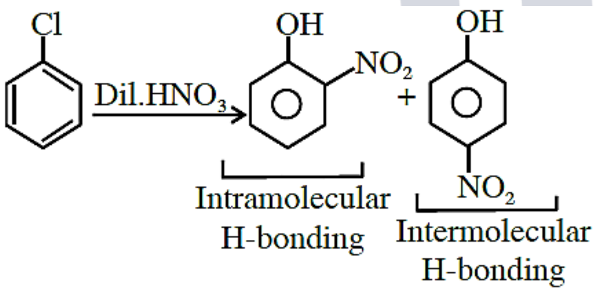
$[Ni(H_2O)_6]Cl_2 \xrightarrow{2AgNO_3} 2AgCl \downarrow + [Ni(H_2O)_6](NO_3)_2$

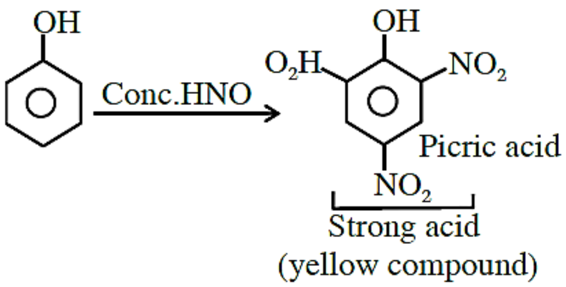




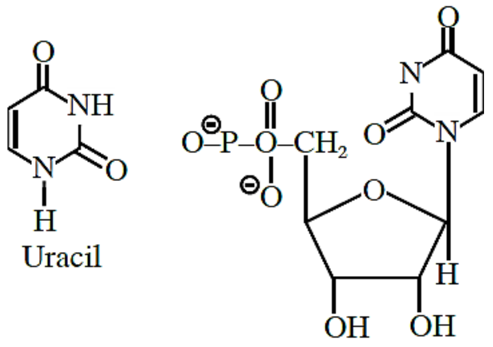
$\mu = \sqrt{2(2+2)}$ B

58. (18)
59. (7)





60. (9) Uracil is the base which only present is RNA.



Structure of nucleotides number of 0-9.

61. (b) $f(x)=\frac{x-1}{x+1}$

$$\Rightarrow f^{2}(x)=f(f(x))=\frac{\frac{x-1}{x+1}-1}{\frac{x-1}{x+1}+1}=-\frac{1}{x}$$

$$f^{3}(x)=f(f^{2}(x))=f\left(-\frac{1}{x}\right)=\frac{x+1}{1-x}$$

$$\Rightarrow f^{4}(x)=f\left(\frac{x+1}{1-x}\right)=-\frac{1}{x}$$

$$\Rightarrow f^{6}(x)=-\frac{1}{x}\Rightarrow f^{6}(6)=-\frac{1}{8}$$

$$f^{7}(x)=\left(-\frac{1}{x}\right)=\frac{x+1}{1-x}$$

$$\Rightarrow f^{7}(7)=\frac{8}{-6}=-\frac{4}{3}$$

$$\therefore -\frac{1}{6}+-\frac{4}{3}=-\frac{3}{2}$$

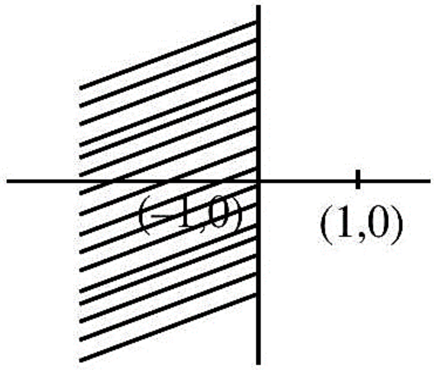
62. (b) Set A

$$\Rightarrow \left|\frac{z+1}{z-1}\right|<1$$

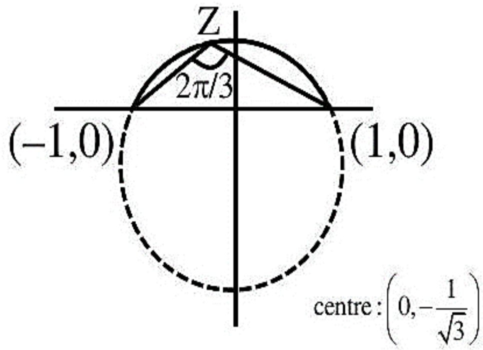
$$\Rightarrow |z+1|<|z-1|$$

$$\Rightarrow (x+1)^{2}+y^{2}<(x-1)^{2}+y^{2}$$

$$\Rightarrow x<0$$



Set B



$\Rightarrow \arg\left(\frac{z-1}{z+1}\right)=\frac{2\pi}{3}$

$\Rightarrow \tan^{-1}\left(\frac{y}{x-1}\right)-\tan^{-1}\left(\frac{y}{x+1}\right)=\frac{2\pi}{3}$

$\Rightarrow x^2+y^2+\frac{2y}{\sqrt{3}}-1=0$

$A \cap B$

$\Rightarrow$ Centre $\left(0,-\frac{1}{\sqrt{3}}\right)$

63. (c) $|\operatorname{adj}(24\,A)|=|\operatorname{adj} 3(\operatorname{adj} 2\,A)|$

$\Rightarrow |24a|^2=(3\operatorname{adj}(2\,A))^2$

$\Rightarrow (24^3|\,A|)^2=(3^3|\operatorname{adj}(2\,A)|)^2$

$=3^6(|2\,A|^2)^2$

$\Rightarrow 24^6|\,A|^2=(24^3|\,A|)^2=3^6\times 2^{12}|\,A|^4$

$\Rightarrow |A|^2=\frac{24^6}{3^6\times 2^{12}}=64$

64. (c)

$$\begin{vmatrix} 3 & -2 & 1 \\ 5 & -8 & 9 \\ 2 & 1 & a \end{vmatrix}=0$$

$3(-8a-9)+2(5a-18)+1(21)=0$

$\Rightarrow a=-3$

Also $\Delta_2=\begin{vmatrix}3 & -2 & b\\ 5 & 8 & 3\\ 2 & 1 & -1\end{vmatrix}^{\frac{1}{3}}$

If $b=\frac{1}{3}$

$\Delta_2=0$

So b must be equal to

$-\frac{1}{3}$

65. (c) $(2021)^{2023}=(7\lambda-2)^{2023}$

$= {}^{2023}C_0(7\,A)^{2023}-\cdots\ldots {}^{2023}C_{2023}2^{2023}$

$=7t-2^{2023}$

$\therefore -2^{2023}=-2\times 2^{2022}$

$=-2\times(2^3)^{674}$

$=-2(1+7\mu)^{674}$

$=-(7\alpha+2)$

$\Rightarrow$ remainder $=-2$ or $+5$

66. (d)

$$\lim_{x\to\frac{1}{\sqrt{2}}}\frac{\sin(\cos^{-1}x)-x}{1-\tan(\cos^{-1}x)}$$

$$\lim_{x\to\frac{1}{\sqrt{2}}}\frac{\sin\left(\sin^{-1}\sqrt{1-x^2}\right)-x}{1-\tan\left(\tan^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right)\right)}$$

$$\lim_{x\to\frac{1}{\sqrt{2}}}\frac{\sqrt{1-x^2}-x}{1-\left(\frac{\sqrt{1-x^2}}{x}\right)}$$

$$\lim_{x\to\frac{1}{\sqrt{2}}}(-x)=-\frac{1}{\sqrt{2}}$$

67. (d) $f(x)=\begin{cases}x+3 & ;\ x<-3\\ -(x+3) & ;\ -3\le x<0\\ e^x & ;\ x\ge 0\end{cases}$

$$g(x)=\begin{cases}x^2+k_1x & ; \quad x<0\\ 4x+k_2 & ; \quad x\geq 0\end{cases}$$

$$g(f(x))=\begin{cases}f(x)^2+k_1f(x) & ; f(x)<0\\ 4f(x)+k_2 & ; f(x)\geq 0\end{cases}$$

$$g(f(x))=\begin{cases}(x+3)^2+k_1(x+3) & ; \quad x<-3\\ (x+3)^2-k_1(x+3) & ; \quad -3\leq x<0\\ 4e^x+k_2 & ; \quad x>0\end{cases}$$

check continuity at $x=0$

$$gof(0)=g\left(f(0^-)\right)=g\left(f(0^+)\right)$$

$$4+k_2=9-3k_1=4+k_2$$

$$3k_1+k_2=5$$

differentiate

$$(g(f(x)))'=\begin{cases}2(x+3)+k_1 & ; \quad x<-3\\ 2(x+3)-k_1 & ; \quad -3\leq x<0\\ 4e^x & ; \quad x\geq 0\end{cases}$$

$$6-k_1=4$$

$$k_1=2$$

$$\therefore k_1=2, k_2=-1$$

$$gof(x)=\begin{cases}(x+3)^2+2(x+3) & ; \quad x<-3\\ (x+3)^2-2(x+3) & ; \quad -3\leq x<0\\ 4e^x-1 & ; \quad x\geq 0\end{cases}$$

$$gof(-4)+gof(4)=4e^4-2$$

$$\Rightarrow 2(2e^4-1)$$

68. (a)

$$f(x)=\begin{cases}x^2-4x-2, & \forall x\in\left(-1,\dfrac{3-\sqrt{17}}{2}\right)\\ -x^2+2x+2, & \forall x\in\left(\dfrac{3-\sqrt{17}}{2},2\right)\end{cases}$$

$f'(x)$ when $x\in\left(-1,\frac{3-\sqrt{17}}{2}\right)$

$$f'(x)=2x-4=0\Rightarrow x=2$$

$f'(x)=2(x-2)\ \Rightarrow f'(x)$ is always $\downarrow$

$$f(2)=2$$

$$f(-1)=3$$

$$f\left(\frac{3-\sqrt{17}}{2}\right)=\frac{\sqrt{17}-3}{2}$$

$f'(x)$ when $x \in \left(\frac{3-\sqrt{17}}{2}, 2\right)$

$f'(x)=-2x+2$

$f'(x)=-2(x-1)$

$f'(x)=0$ when $x=1$

$f(1)=3$

absolute minimum value $=\frac{\sqrt{17}-3}{2}$

absolute maximum value $=3$

Sum $=\frac{\sqrt{17}-3}{2}+3=\frac{\sqrt{17}+3}{2}$

69. (d) $\left(\frac{x}{a}\right)^n+\left(\frac{y}{b}\right)^n=2$

Slope of tangent at (a, b)

$n.\left(\frac{x}{a}\right)^{n-1}\cdot\frac{1}{a}+n\left(\frac{x}{b}\right)^{n-1}\cdot\frac{1}{b}\frac{dy}{dx}=0$

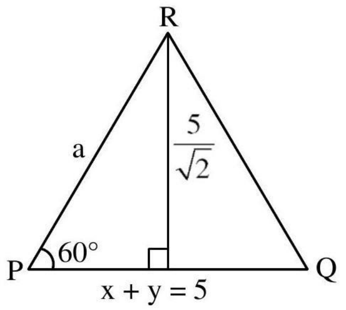
$$\left.\frac{dy}{dx}\right|_{(a,b)}=-\frac{b}{a}$$

$\therefore$ Equation of tangent

$$y-b=-\frac{b}{a}(x-a)$$

$$\frac{x}{a}+\frac{y}{b}=2 \forall n \in N$$

70. (d)
71. (d)

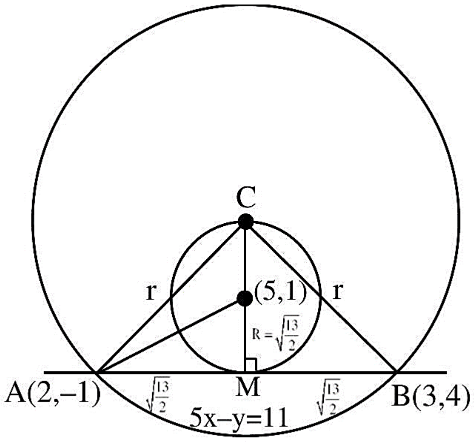


$$\sin 60^\circ=\frac{5/\sqrt{2}}{a}$$

$$a=\frac{5\sqrt{2}}{3}$$

Area of $\triangle$ PQR $= \frac{\sqrt{3}}{4}a^2 = \frac{25}{2\sqrt{3}}$

72. (b)



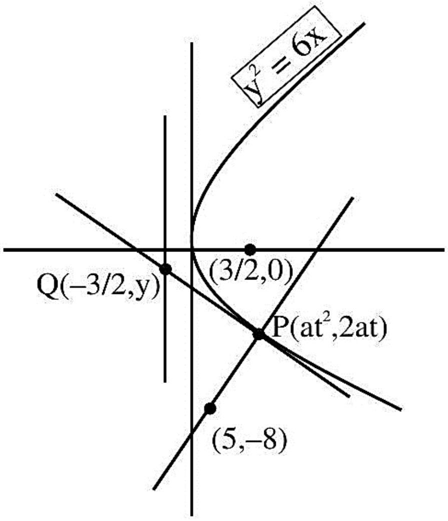
$AB = \sqrt{26}$

$r^2 = CM^2 + AM^2$

$$= \left(2 \times \sqrt{\frac{13}{2}}\right)^2 + \left(\sqrt{\frac{13}{2}}\right)^2$$

$r^2 = \frac{65}{2}$

73. (b)



Equation of normal : $y = -tx + 2at + at^3$ $\left(a = \frac{3}{2}\right)$

since passing through $(5, -8)$, we get $t = -2$

Co-ordinate of Q: $(6, -6)$

Equation of tangent at $Q: x + 2y + 6 = 0$

Put $x = \frac{-3}{2}$ to get $R\left(\frac{-3}{2}, \frac{-9}{4}\right)$

74. (b) $l_1: \frac{x-2}{3} = \frac{y+1}{-2} = \frac{z-2}{0}$

$$l_2: \frac{x-1}{1} = \frac{y+3/2}{\alpha/2} = \frac{z+5}{2}$$

$$l_3: \frac{x-1}{-3} = \frac{y-1/2}{-2} = \frac{z-0}{4}$$

$$l_1 \perp l_2 \Rightarrow \frac{|3-\alpha+0|}{\sqrt{13}\sqrt{1+\frac{\alpha^2}{4}+4}} = 0 \Rightarrow \alpha = 3$$

angle between $l_2 \& l_3$

$$\cos\theta = \frac{|1\times(-3)+(-2)(\alpha/2)+2\times 4|}{\sqrt{1+4+\frac{\alpha^2}{4}}\sqrt{9+16+4}}$$

$$\cos\theta = \frac{|-3-\alpha+8|}{\sqrt{5+\frac{\alpha^2}{4}}\sqrt{29}}$$

put $\alpha = 3$

$$\cos\theta = \frac{2}{\sqrt{\frac{29}{4}}\sqrt{29}} = \frac{4}{29}$$

$$\theta = \cos^{-1}\left(\frac{4}{29}\right) \Rightarrow \theta = \sec^{-1}\left(\frac{29}{4}\right)$$

75. (a) Let equation of rotated plane be:

$(2x+3y+z+20) + \lambda(x-3y+5z-8) = 0$

$(2+\lambda)x + (3-3\lambda)y + (1+5\lambda)z + 20 - 8\lambda = 0$

Above plane is perpendicular to $2x+3y+z+20=0$

So, $(2+\lambda)\cdot 2 + (3-3\lambda)\cdot 3 + (1+5\lambda)\cdot 1 = 0 \Rightarrow \lambda = 7$

$\Rightarrow$ Equation of rotated plane : $x - 2y + 4z - 4 = 0$

Mirror image of $A\left(2, \frac{-1}{2}, 2\right)$ in rotated plane is $B(a, b, c)$

Equation of AB: $\frac{x-2}{1} = \frac{y+1/2}{-2} = \frac{z-2}{4} = k$

Let coordinate of B be $\left(2+k, \frac{-1}{2}-2k, 2+4k\right)$ midpoint of AB is $\left(2+\frac{k}{2}, \frac{-1}{2}-k, 2+2k\right)$ which will lie on the plane $x - 2y + 4z - 4 = 0$

Hence $k = \frac{-2}{3}$

Therefore, B is $\left(\frac{4}{3}, \frac{5}{6}, \frac{-2}{3}\right) \equiv \left(\frac{8}{6}, \frac{5}{6}, \frac{-4}{6}\right)$

So, $\frac{a}{8} = \frac{b}{5} = \frac{c}{-4}$

76. (a) $\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c} = 3\vec{b} - \vec{c}$

$\vec{b} \times (\vec{c} \times \vec{a}) = (\vec{b} \cdot \vec{a})\vec{c} - (\vec{b} \cdot \vec{c})\vec{a} = \vec{c} - 2\vec{a}$

$\vec{c} \times (\vec{b} \times \vec{a}) = (\vec{c}.\vec{a})\vec{b} - (\vec{c} \cdot \vec{b})\vec{a} = 3\vec{b} - 2\vec{a}$

$[3\vec{b} - \vec{c}, \vec{c} - 2\vec{a}, 3\vec{b} - 2\vec{a}]$

$(3\vec{b} - \vec{c}) \cdot [(\vec{c} - 2\vec{a}) \times (3\vec{b} - 2\vec{a})]$

$(3\vec{b} - \vec{c}) \cdot [3(\vec{c} \times \vec{b}) - 2(\vec{c} \times \vec{a}) - 6(\vec{a} \times \vec{b})]$

$-6[\vec{b}\vec{c}\vec{a}] + 6[\vec{c}\vec{a}\vec{b}]$

77. (d) $P(H) = x, P(T) = 1 - x$

$P(4H, 1\ T) = P(5H)$

${}^5C_1(x)^4(1-x)^1 = {}^5C_5x^5$

$5(1-x) = x$

$6x = 5 = 0\ x = \frac{5}{6}$

P (atmost 2H)

$= P(OH, 5\ T) + P(1H, 4\ T) + P(2H, 3\ T)$

$= {}^5C_0\left(\frac{1}{6}\right)^5 + {}^5C_1\frac{5}{6} \cdot \left(\frac{1}{6}\right)^4 + {}^5C_2\left(\frac{5}{6}\right)^3\left(\frac{1}{6}\right)^3$

$= \frac{1}{6^5}(1 + 25 + 250) = \frac{276}{6^5}$

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$= \frac{46}{6^4}$

78. (a) $\sigma^2 = \frac{\sum_{i=1}^{5}(x_i - \bar{x})^2}{n}$

Mean = 6

$\frac{a + b + 8 + 5 + 10}{5} = 6$

$a + b = 7$

$b = 7 - a$

$$6.8=\frac{(a-6)^2+(b-6)^2+(8-6)^2+(5-6)^2+(10-6)^2}{5}$$

$34=(a-6)^2+(7-a-6)^2+4+1+18$

$a^2-7a+12=0 \Rightarrow a=4$ or $a=3$

$a=4$ $a=3$

$b=3$ $b=4$

$$M=\frac{\sum_{i=1}^{5}|x_i-x|}{n}$$

$$M=\frac{|a-6|+|b-6|+|8-6|+|5-6|+|10-6|}{5}$$

when $a=3, b=4$ when $a=4, b=3$

$M=\frac{3+2+2+1+4}{5}$ $M=\frac{2+3+2+1+7}{5}$

$M=\frac{12}{5}$ $M=\frac{12}{5}$

$25M=25\times\frac{12}{5}=60$

79. (b) $f'(x)=\frac{-2}{\sqrt{1-x^2}}-\frac{4}{1+x^2}-6x-2$

$$=-2\left[\frac{1}{\sqrt{1-x^2}}+\frac{2}{1+x^2}+3x+1\right]$$

$f'(x)<0 \Rightarrow f(x)$ is a dec. function

$f(1)=\pi+5$

$f(-1)=5\pi+5$

Range: $[a,b]\equiv[\pi+5, 5\pi+5]$

$a=\pi+5, b=5\pi+5 \Rightarrow 4a-b=11-\pi.$

80. (a) Case-I If $\Delta\equiv\nabla\equiv\wedge$

$(p\wedge q)\rightarrow((p\wedge q)\wedge r)$

it can be false if r is false,

so not a tautology

Case-II If $\Delta\equiv\nabla\equiv\vee$

$(p\vee q)\rightarrow((p\vee q)\vee r)\equiv$ tautology

then $(p\vee q)\vee r\equiv(p\Delta r)\vee q$

Case-III if $\Delta = \vee, \nabla = \wedge$

then $(p \wedge q) \rightarrow \{(p \vee q) \wedge r\}$

Not a tautology

(Check $\mathrm{p} \rightarrow \mathrm{T}, \mathrm{q} \rightarrow \mathrm{T}, \mathrm{r} \rightarrow \mathrm{F}$)

Case-IV if $\Delta = \wedge, \nabla = \vee$

$(\mathrm{p} \wedge \mathrm{q}) \rightarrow \{\overline{(\mathrm{p} \wedge \mathrm{q}) \vee \mathrm{r}}\}$

Not a tautology

81. (36) $x^4 - 3x^3 - 2x^2 + 3x + 1 = 10$

$\mathrm{x} = 0$ is not the root of this equation so divide it by x^2

$$x^2 - 3x - 2 + \frac{3}{x} + \frac{1}{x^2} = 0$$

$$\mathrm{x}^2 + \frac{1}{\mathrm{x}^2} - 2 + 2 - 3\left(\mathrm{x} - \frac{1}{\mathrm{x}}\right) - 2 = 0$$

$$\left(\mathrm{x} - \frac{1}{\mathrm{x}}\right)^2 - 3\left(\mathrm{x} - \frac{1}{\mathrm{x}}\right) = 0$$

$$\mathrm{x} - \frac{1}{\mathrm{x}} = 0,\ \mathrm{x} - \frac{1}{\mathrm{x}} = 3$$

$\mathrm{x}^2 - 1 = 0\ \mathrm{x}^2 - 3\mathrm{x} - 1 = 0$

$\mathrm{x} = \pm 1\ \gamma + \delta = 3$

$\alpha = 1, \beta = -1\ \gamma\delta = -1$

$\alpha^3 + \beta^3 + \gamma^3 + \delta^3$

$1 - 1 + (\gamma + \delta)((\gamma + \delta)^2 - 3\gamma\delta)$

$0 + 3(9 - 3(-1))$

$+3(12) = 36$

82. (1120) $\mathrm{n(b)} = 10$

$\mathrm{n(a)} = 5$

The number of ways of forming a group of 3 girls of 3 boys. $= {}^{10}\mathrm{C}_3 \times {}^{5}\mathrm{C}_3$

$$= \frac{10 \times 9 \times 8}{3 \times 2} \times \frac{5 \times 4}{2} = 1200$$

The number of ways when two particular boys B_1 of B_2 be the member of group together

$= {}^{8}\mathrm{C}_1 \times {}^{5}\mathrm{C}_3 = 8 \times 10 = 80$

Number of ways when boys B_1 of B_2 hot in the same group together

$= 1200 \times 80 = 1120$

83. (4) $x^2 + y^2 = \frac{9}{4}\ y = 4x$

Equation tangent in slope form

$$y = mx \pm \frac{3}{2}\sqrt{(1 + m^2)}$$

$$\mathrm{y} = \mathrm{mx} + \frac{1}{\mathrm{m}}$$

compare (1) & (2)

$$\pm\frac{3}{2}\sqrt{(1 + \mathrm{m}^2)} = \frac{1}{\mathrm{m}^2}$$

$$9m^2(1 + m^2) = 4$$

$$9m^4 + 9m^2 - 4 = 0$$

$$9m^4 + 12m^2 - 3m^2 - 4 = 0$$

$$3m^2(3m^2 + 4) - (3m^2 + 4) = 0$$

$\mathrm{m}^2 = -\frac{4}{3}$ (Rejected)

$$\mathrm{m}^2 = \frac{1}{3} \Rightarrow \mathrm{m} = \pm\frac{1}{\sqrt{3}}$$

Equation of common tangent

$$y = \frac{1}{\sqrt{3}}x + \sqrt{3}$$

on X axis $\mathrm{y} = 0$

$$0Q = -3$$

$$\mathrm{b} = |0Q| = 3$$

$$\mathrm{a} = 6$$

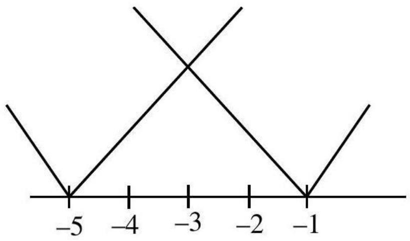
$$\mathrm{b}^2 = \mathrm{a}^2(1 - \mathrm{e}^2) \Rightarrow \mathrm{e}^2 = 1 - \frac{9}{36} = \frac{3}{4}$$

$$\mathrm{e} = \frac{2\,\mathrm{b}^2}{\mathrm{a}} = \frac{2 \times 9}{6} = 3$$

$$\frac{\mathrm{e}}{\mathrm{e}^2} = \frac{3}{3/4} = 4$$

84. (21)

$$\mathrm{f(x)} = \max\{|\mathrm{x} + 1|, |\mathrm{x} + 2|, |\mathrm{x} + 3|, |\mathrm{x} + 4|, |\mathrm{x} + 5|\}$$



$\int_{-6}^{0} f(x)dx = \int_{-6}^{-3} |x+1|dx + \int_{-3}^{0} |x+5|dx$

$= -\int_{-6}^{-3} (x+1)dx + \int_{-3}^{0} (x+5)dx$

$= -\left[\frac{x^2}{2} + x\right]_{-6}^{-3} + \left[\frac{x^2}{2} + 5x\right]_{-3}^{0}$

$= -\left[\left(\frac{9}{2} - 3\right) - (18 - 6)\right] + \left[0 - \left(\frac{9}{2} - 15\right)\right]$

$= -\left[\frac{3}{2} - 12\right] + \frac{21}{2} = \frac{21}{2} + \frac{21}{2} = 21$

85. (12)

$(4 + x^2)dy - 2x(x^2 + 3y + 4)dx\ (x^2 + 4)\frac{dy}{dx} = 2x^3 + 6xy + 8x$

$(x^2 + 4)\frac{dy}{dx} - 6xy = 2x^3 + 8x$

$\frac{dy}{dx} - \frac{6x}{x^2 + 4}y = \frac{2x^3 + 8x}{x^2 + y}$

L.I. $\frac{\mathrm{dy}}{\mathrm{dx}} + \mathrm{py} = \phi$

$\mathrm{p} = \frac{-6\mathrm{x}}{\mathrm{x}^2 + 4}\ \phi = \frac{2\mathrm{x}^3 + 8\mathrm{x}}{\mathrm{x}^2 + 4}$

I.F. $= \mathrm{e}^{-\int \frac{6x}{x^2+4}dx} = e^{-3\log_e(x^2+4)}$

$= \mathrm{e}^{\log_e(\mathrm{x}^2+4)^{-3}} = \frac{1}{(\mathrm{x}^2 + 4)^3}$

$y \cdot \frac{1}{(x^2 + 4)^3} = \int \frac{2x^3 + 8x}{(x^2 + 4)^3(x^2 + 4)}dx$

$\frac{y}{(x^2 + 4)^3} = \int \frac{2x(x^2 + 4)}{(x^2 + 4)^3(x^2 + 4)}dx$

$\mathrm{x}^2 + 4 = \mathrm{t}$

$2\mathrm{xdx} = \mathrm{dt}$

$\frac{y}{(x^2 + 4)^3} = \int \frac{dt}{t^3}$

$$\frac{y}{(x^2+4)^3}=\frac{-1}{2(x^2+4)^2}+C$$

passes through origin (0,0)

$$0=\frac{-1}{2\times 16}+C$$

$$\frac{y}{(x^2+4)^3}=\frac{-1}{2(x^2+4)^2}+\frac{1}{32}$$

$$y=\frac{-(x^2+4)}{2}+\frac{(x^2+4)^3}{32}$$

$$y(2)=-\frac{8}{2}+\frac{8\times 8\times 8}{32}=12$$

86. (80) $\sin 10^\circ\left(\frac{1}{2}\cdot 2\sin 20^\circ \sin 40^\circ\right)\cdot \sin 10^\circ \sin(60^\circ-10^\circ)\sin(60^\circ+10^\circ)\ \sin 10^\circ \frac{1}{2}(\cos 20^\circ-\cos 60^\circ)\cdot \frac{1}{4}\sin 30^\circ$

$$\frac{1}{2}\cdot\frac{1}{4}\cdot\frac{1}{2}\cdot\sin 10^\circ\left(\cos 20^\circ-\frac{1}{2}\right)$$

$$=\frac{1}{32}(2\sin 10^\circ\cos 20^\circ-\sin 10^\circ)$$

$$=\frac{1}{32}(\sin 30^\circ-\sin 10^\circ-\sin 10^\circ)$$

$$=\frac{1}{32}\left(\frac{1}{2}-2\sin 10^\circ\right)$$

$$=\frac{1}{64}(1-4\sin 10^\circ)$$

$$=\frac{1}{64}-\frac{1}{16}\sin 10^\circ$$

Hence $\alpha=\frac{1}{64}$

$16+\alpha^{-1}=80$

87. (5264) Sum of elements in $A\cap B$

$$=\underbrace{(2+4+6+\cdots+200)}_{\text{Multiple of 2}}-\underbrace{(6+12+\cdots+198)}_{\text{Multiple of 2\&3 i.e. 6}}$$
$$-\underbrace{(10+20+\cdots+200)}_{\text{Multiple of 5\&2 i.e. 10}}+\underbrace{(30+60+\cdots+180)}_{\text{Multiple of 2,5 \textbackslash\& 3 i.e. 30}}$$
$$=5264$$

88. (6) $I=\frac{48}{\pi^4}\int_0^\pi x^2\left(\frac{3\pi}{2}-x\right)\frac{\sin x}{1+\cos^2 x}dx$

Apply king property $I=\frac{48}{\pi^4}\int_0^\pi(\pi-x)^2\left(\frac{\pi}{2}+x\right)\frac{\sin x}{1+\cos^2 x}dx$

(1) + (2)

$I=\frac{12}{\pi^3}\int_0^\pi \frac{\sin x}{1+\cos^2 x}[\pi^2+(\pi-2)\cdot x\cdot(\pi-2x)]dx$

Apply king again

$I=\frac{12}{\pi^3}\int_0^\pi \frac{\sin x}{1+\cos^2 x}[\pi^2+(\pi-2)(\pi-x)(2x-\pi)]dx$

(3) + (4)

$I=\frac{6}{\pi^2}\int_0^\pi \frac{\sin x}{1+\cos^2 x}[2\pi+(\pi-2)(\pi-2x)]dx$

Apply king

$I=\frac{6}{\pi^2}\int_0^\pi \frac{\sin x}{1+\cos^2 x}[2\pi+(\pi-2)(2x-\pi)]dx.$

(5) + (6)

$I=\frac{12}{\pi}\int_0^\pi \frac{\sin x}{1+\cos^2 x}dx$

Let $\cos x=t \Rightarrow \sin x dx=-dt$

$I=\frac{12}{\pi}\int_1^{-1}\frac{-dt}{1+t^2}=6$

89. (1100)

$A=\sum_{i=1}^{10}\sum_{j=1}^{10}\min\{i,j\}$

$B=\sum_{i=1}^{10}\sum_{j=1}^{10}\max\{i,j\}$

$A=\sum_{j=1}^{10}\min(i,1)+\min(j,2)+\cdots\min(i,10)$

$=\underbrace{(1+1+1+\cdots+1)}_{19\text{ times}}+\underbrace{(2+2+2\ldots+2)}_{17\text{ times}}+\underbrace{(3+3+3\ldots+3)}_{15\text{ times}}$

$+\cdots(1)\ 1$ times

$$B=\sum_{j=1}^{10}\max(i,1)+\max(j,2)+\cdots\max(i,10)$$
$$=\underbrace{(10+10+\cdots+10)}_{19\text{ times}}+\underbrace{(9+9+\cdots+9)}_{17\text{ times}}+\cdots+11\text{ times}$$
$$A+B=20(1+2+3+\cdots+10)$$
$$=20\times\frac{10\times 11}{2}=10\times 110=1100$$

90. (42) $\frac{dy}{dx}=\frac{1}{1+\sin 2x}$

$\int dy=\int\frac{dx}{(\sin x+\cos x)^2}$

$$\int dy = \int \frac{\sec^2 x}{(1+\tan x)^2}$$

$$y(x) = -\frac{1}{1+\tan x} + C$$

$$y\left(\frac{\pi}{4}\right) = \frac{1}{2} = -\frac{1}{2} + C$$

$C = 1$

$$y(x) = \frac{-1}{1+\tan x} + 1$$

$$y(x) = \frac{-1+1+\tan x}{1+\tan x}$$

$$y(x) = \frac{\tan x}{1+\tan x}$$

Solving with $y = \sqrt{2}\sin x$

$$\frac{\tan x}{1+\tan x} = \sqrt{2}\sin x$$

$$\sin x = 0,\ \frac{1}{\sqrt{2}} = \sin x + \cos x$$

$$x = \pi\ \frac{1}{2} = \sin\left(x + \frac{\pi}{4}\right)$$

$$\sin\frac{\pi}{6} = \sin\left(x + \frac{\pi}{4}\right)$$

$$x + \frac{\pi}{4} = \pi - \frac{\pi}{6}, 2\pi + \frac{\pi}{6}$$

$$x = \frac{5\pi}{6} - \frac{\pi}{4}, x = \frac{13\pi}{6} - \frac{\pi}{4}$$

$$x = \frac{7\pi}{12}, x = \frac{23\pi}{12}$$

sum of sol.

$$= \pi + \frac{7\pi}{12} + \frac{23\pi}{12}$$

$$= \frac{12\pi + 7\pi + 23}{12} = \frac{42\pi}{12} = \frac{k\pi}{12}$$

$\Rightarrow k = 42$