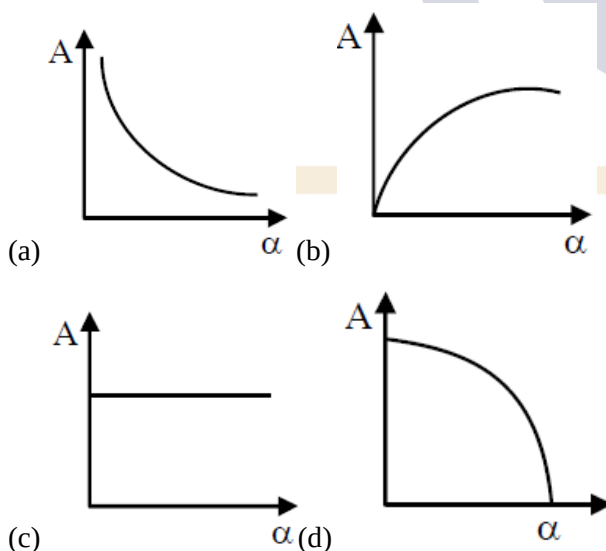
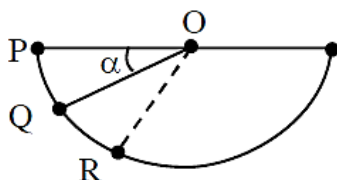


PART-I PHYSICS

SECTION-A

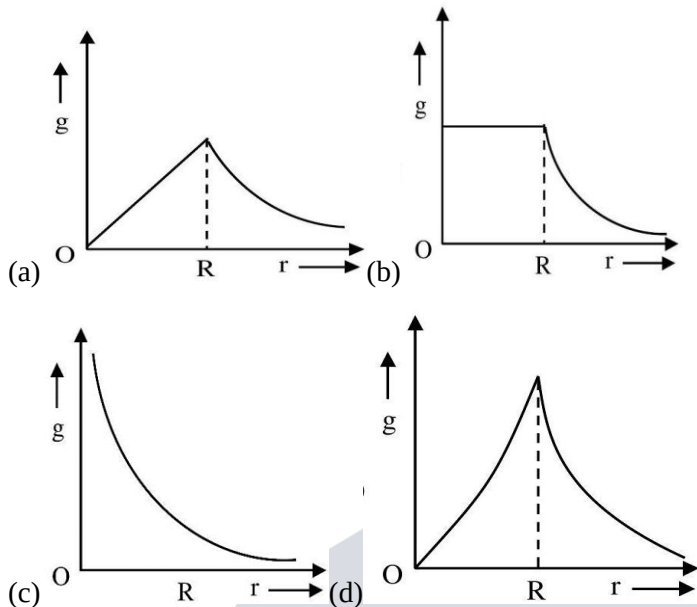
- An expression for a dimensionless quantity P is given by $P = \frac{\alpha}{\beta} \log_e \left(\frac{kt}{\beta x} \right)$; where α and β are constants, x is distance; k is Boltzmann constant and t is the temperature. Then the dimensions of α will be :
 (a) $[M^0 L^{-1} T^0]$ (b) $[ML^0 T^{-2}]$
 (c) $[MLT^{-2}]$ (d) $[ML^2 T^{-2}]$
- A person is standing in an elevator. In which situation, he experiences weight loss?
 (a) When the elevator moves upward with constant acceleration
 (b) When the elevator moves downward with constant acceleration
 (c) When the elevator moves upward with uniform velocity
 (d) When the elevator moves downward with uniform velocity
- An object is thrown vertically upwards. At its maximum height, which of the following quantity becomes zero?
 (a) Momentum (b) Potential energy
 (c) Acceleration (d) Force
- A ball is released from rest from point P of a smooth semi-spherical vessel as shown in figure. The ratio of the centripetal force and normal reaction on the ball at point Q is A while angular position of point Q is α with respect to point P . Which of the following graphs represent the correct relation between A and α when ball goes from Q to R ?



- A thin circular ring of mass M and radius R is rotating with a constant angular velocity 2rads^{-1} in a horizontal plane about an axis vertical to its plane and passing through the center of the ring. If two objects each of mass m be attached gently to the opposite ends of a diameter of ring, the ring will then rotate with an angular velocity (in rads^{-1}).

- (a) $\frac{M}{(M+m)}$ (b) $\frac{(M+2m)}{2M}$
(c) $\frac{2M}{(M+2m)}$ (d) $\frac{2(M+2m)}{M}$

6. The variation of acceleration due to gravity (g) with distance (r) from the center of the earth is correctly represented by: (Given R = radius of earth)



7. The efficiency of a Carnot's engine, working between steam point and ice point, will be:

- (a) 26.81% (b) 37.81%
(c) 47.81% (d) 57.81%

8. Time period of a simple pendulum in a stationary

lift is ' T '. If the lift accelerates with $\frac{g}{6}$ vertically upwards then the time period will be:

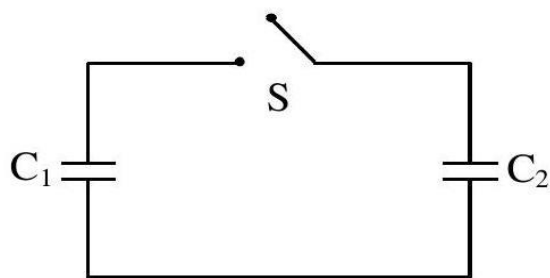
(where g = acceleration due to gravity)

- (a) $\sqrt{\frac{6}{5}} T$ (b) $\sqrt{\frac{5}{6}} T$
(c) $\sqrt{\frac{6}{7}} T$ (d) $\sqrt{\frac{7}{6}} T$

9. A thermally insulated vessel contains an ideal gas of molecular mass M and ratio of specific heats 1.4. Vessel is moving with speed v and is suddenly brought to rest. Assuming no heat is lost to the surrounding and vessel temperature of the gas increases by : (R = universal gas constant)

- (a) $\frac{Mv^2}{7R}$ (b) $\frac{Mv^2}{5R}$
(c) $2 \frac{Mv^2}{7R}$ (d) $7 \frac{Mv^2}{5R}$

10. Two capacitors having capacitance C_1 and C_2 respectively are connected as shown in figure. Initially, capacitor C_1 is charged to a potential difference V volt by a battery. The battery is then removed and the charged capacitor C_1 is now connected to uncharged capacitor C_2 by closing the switch S . The amount of charge on the capacitor C_2 , after equilibrium is:



- (a) $\frac{C_1 C_2}{(C_1 + C_2)} V$ (b) $\frac{(C_1 + C_2)}{C_1 C_2} V$
(c) $(C_1 + C_2)V$ (d) $(C_1 - C_2)V$

11. Assertion (a): Non-polar materials do not have any permanent dipole moment.

Reason (R): When a non-polar material is placed in an electric field, the centre of the positive charge distribution of its individual atom or molecule coincides with the centre of the negative charge distribution.

In the light of above statements, choose the most appropriate answer from the options given below.

- (a) Both (a) and (R) are correct and (R) is the correct explanation of (a).
(b) Both (a) and (R) are correct and (R) is not the correct explanation of (a).
(c) (a) is correct but (R) is not correct.
(d) (a) is not correct but (R) is correct.

12. The magnetic flux through a coil perpendicular to its plane is varying according to the relation $\phi = (5t^3 + 4t + 2t - 5)$ Weber. If the resistance of the coil is 5 ohm, then the induced current through the coil at $t = 2$ sec will be:

- (a) 15.6 A (b) 16.6 A
(c) 17.6 A (d) 18.6 A

13. An aluminium wire is stretched to make its length 0.4% larger. Then percentage change in resistance is:

- (a) 0.4% (b) 0.2%
(c) 0.8% (d) 0.6%

14. A proton and an alpha particle of the same mass enter in a uniform magnetic field which is acting perpendicular to their direction of motion. The ratio of the circular paths described by the alpha particle and proton is:

- (a) 1:4 (b) 4:1
(c) 2:1 (d) 1:2

15. If electric field intensity of a uniform plane electromagnetic wave is given as

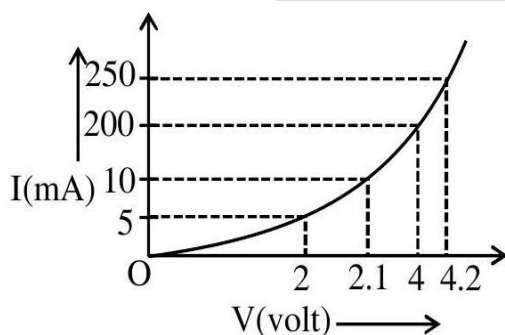
$$E = -301.6 \sin(kz - \omega t) \hat{a}_x + 452.4 \sin(kz - \omega t) \hat{a}_y \frac{V}{m}$$

Then, magnetic intensity H of this wave in $A m^{-1}$ will be:

[Given: Speed of light in vacuum $c = 3 \times 10^8 \text{ ms}^{-1}$, permeability of vacuum $\mu_0 = 4\pi \times 10^{-7} \text{ NA}^{-2}$]

- (a) $+0.8 \sin(kz - \omega t) \hat{a}_y + 0.8 \sin(kz - \omega t) \hat{a}_x$
(b) $+1.0 \times 10^{-6} \sin(kz - \omega t) \hat{a}_y + 1.5 \times 10^{-6} \sin(kz - \omega t) \hat{a}_x$
(c) $-0.8 \sin(kz - \omega t) \hat{a}_y - 1.2 \sin(kz - \omega t) \hat{a}_x$
(d) $-1.0 \times 10^{-6} \sin(kz - \omega t) \hat{a}_y - 1.5 \times 10^{-6} \sin(kz - \omega t) \hat{a}_x$

16. In free space, an electromagnetic wave of 3GHz frequency strikes over the edge of an object of size $\frac{\lambda}{100}$, where λ is the wavelength of the wave in free space. The phenomenon, which happens there will be:
- (a) Reflection (b) Refraction
(c) Diffraction (d) Scattering
17. An electron with speed v and a photon with speed c have the same de-Broglie wavelength. If the kinetic energy and momentum of electron are E_e and p_e and that of photon are E_{ph} and p_{ph} respectively. Which of the following is correct?
- (a) $\frac{E_e}{E_{ph}} = \frac{2c}{v}$ (b) $\frac{E_e}{E_{ph}} = \frac{v}{2c}$
(c) $\frac{p_e}{p_{ph}} = \frac{2c}{v}$ (d) $\frac{p_e}{p_{ph}} = \frac{v}{2c}$
18. How many alpha and beta particles are emitted when Uranium ${}_{92}\text{U}^{238}$ decays to lead ${}_{82}\text{Pb}^{206}$?
- (a) 3 alpha particles and 5 beta particles
(b) 6 alpha particles and 4 beta particles
(c) 4 alpha particles and 5 beta particles
(d) 8 alpha particles and 6 beta particles
19. The $I - V$ characteristics of a $p - n$ junction diode in forward bias is shown in the figure. The ratio of dynamic resistance, corresponding to forward bias voltages of 2 V and 4 V respectively, is:



- (a) 1: 2 (b) 5: 1
(c) 1: 40 (d) 20: 1

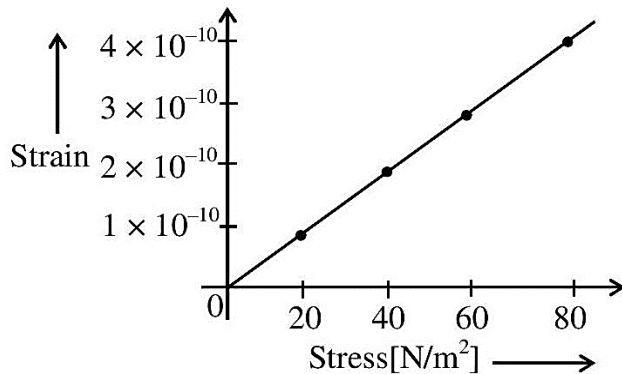
20. Choose the correct statement for amplitude modulation:

- (a) Amplitude of modulating is varied in accordance with the information signal.
(b) Amplitude of modulated is varied in accordance with the information signal.
(c) Amplitude of carrier signal is varied in accordance with the information signal.
(d) Amplitude of modulated is varied in accordance with the modulating signal.

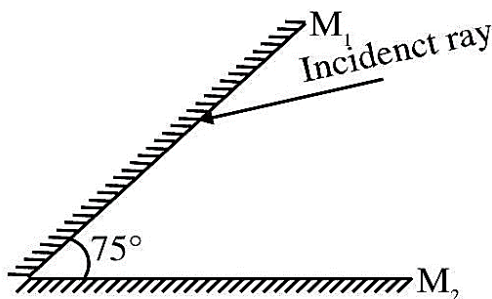
SECTION-B

21. A fighter jet is flying horizontally at a certain altitude with a speed of 200 ms^{-1} . When it passes directly overhead an anti-aircraft gun, bullet is fired from the gun, at an angle θ with the horizontal, to hit the jet. If the bullet speed is $400\frac{\text{m}}{\text{s}}$, the value of θ will be... _____°.
22. A ball of mass 0.5 kg is dropped from the height of 10 m . The height, at which the magnitude of velocity becomes equal to the magnitude of acceleration due to gravity, is m . (Use $g = 10\text{ m/s}^2$).

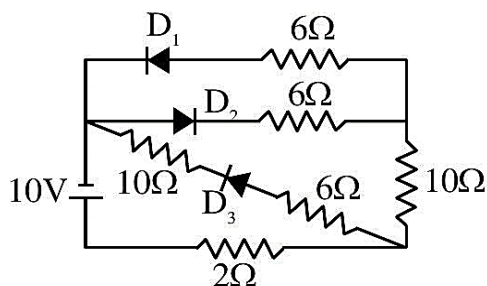
23. The elastic behaviour of material for linear stress and linear strain, is shown in the figure. The energy density for a linear strain of 5×10^{-4} is kJ/m^3 . Assume that material is elastic upto the linear strain of 5×10^{-4} .



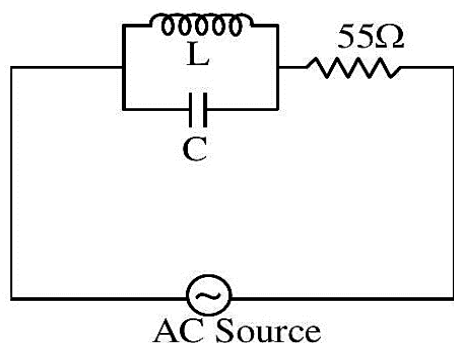
24. The elongation of a wire on the surface of the earth is 10^{-4} m. The same wire of same dimensions is elongated by 6×10^{-5} m on another planet. The acceleration due to gravity on the planet will be ms^{-2} . (Take acceleration due to gravity on the surface of earth = 10 m/s^{-2})
25. A 10Ω , 20mH coil carrying constant current is connected to a battery of 20 V through a switch is opened current becomes zero in $100\mu\text{s}$. The average emf induced in the coil is V .
26. A light ray is incident, at an incident angle θ_1 , on the system of two plane mirrors M_1 and M_2 having an inclination angle 75° between them (as shown in figure). After reflecting from mirror M_1 it gets reflected back by the mirror M_2 with an angle of reflection 30° . The total deviation of the ray will be degree.



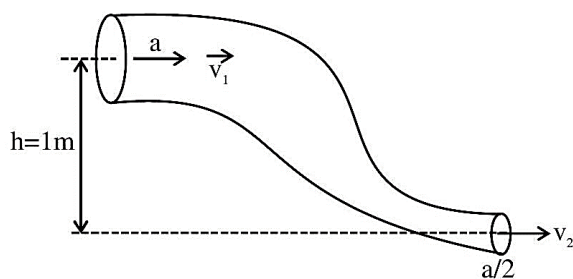
27. In a vernier callipers, each cm on the main scale is divided into 20 equal parts. If tenth vernier scale division coincides with ninth main scale division. Then the value of vernier constant will be $\times 10^{-2} \text{ mm}$.
28. As per the given circuit, the value of current through the battery will be A.



29. A 110 V , 50 Hz , AC source is connected in the circuit (as shown in figure). The current through the resistance 55Ω , at resonance in the circuit, will be A.



30. An ideal fluid of density 800kgm^{-3} , flows smoothly through a bent pipe (as shown in figure) that tapers in cross-sectional area from a to $\frac{a}{2}$. The pressure difference between the wide and narrow sections of pipe is 4100 Pa . At wider section, the velocity of fluid is $\frac{\sqrt{x}}{6}\text{ ms}^{-1}$ for $x =$ (Given $g = 10\text{ m}^{-2}$)



PART-II CHEMISTRY

SECTION-A

31. A commercially sold conc. HCl is 35% HCl by mass. If the density of this commercial acid is 1.46 g/mL , the molarity of this solution is: (Atomic mass: $\text{Cl} = 35.5\text{amu}$, $\text{H} = 1\text{amu}$)
 (a) 10.2M (b) 12.5M
 (c) 14.0M (d) 18.2M
32. An evacuated glass vessel weighs 40.0 g when empty, 135.0 g when filled with a liquid of density 0.95 g mL^{-1} and 40.5 g when filled with an ideal gas at 0.82 atm at 250 K . The molar mass of the gas in gmol^{-1} is: (Given: $R = 0.082\text{ L atm K}^{-1}\text{ mol}^{-1}$)
 (a) 35 (b) 50
 (c) 75 (d) 125
33. If the radius of the 3rd Bohr's orbit of hydrogen atom is r_3 and the radius of 4th Bohr's orbit is r_4 . Then:
 (a) $r_4 = \frac{9}{16}r_3$ (b) $r_4 = \frac{16}{9}r_3$
 (c) $r_4 = \frac{3}{4}r_3$ (d) $r_4 = \frac{4}{3}r_3$
34. Consider the ions/molecule O_2^+ , O_2 , O_2^- , O_2^{2-}
 For increasing bond order the correct option is:
 (a) $\text{O}_2^{2-} < \text{O}_2^- < \text{O}_2 < \text{O}_2^+$ (b) $\text{O}_2^- < \text{O}_2^{2-} < \text{O}_2 < \text{O}_2^+$
 (c) $\text{O}_2^- < \text{O}_2^{2-} < \text{O}_2^+ < \text{O}_2$ (d) $\text{O}_2^- < \text{O}_2^+ < \text{O}_2^{2-} < \text{O}_2$

35. The $\left(\frac{\partial E}{\partial T}\right)_P$ of different types of half cells are as follows :

A	B	C	D
1×10^{-4}	2×10^{-4}	0.1×10^{-4}	0.2×10^{-4}

(Where E is the electromotive force)

Which of the above half cells would be preferred to be used as reference electrode?

- (a) A (b) B
(c) C (d) D

36. Choose the correct stability order of group 13 elements in their +1-oxidation state.

- (a) $\text{Al} < \text{Ga} < \text{In} < \text{Tl}$ (b) $\text{Tl} < \text{In} < \text{Ga} < \text{Al}$
(c) $\text{Al} < \text{Ga} < \text{Tl} < \text{In}$ (d) $\text{Al} < \text{Tl} < \text{Ga} < \text{In}$

37. Given below are two statements:

Statement I: According to the Ellingham diagram, any metal oxide with higher ΔG° is more stable than the one with lower ΔG° .

Statement II: The metal involved in the formation of oxide placed lower in the Ellingham diagram can reduce the oxide of a metal placed higher in the diagram.

In the light of the above statements, choose the most appropriate answer from the options given below:

- (a) Both Statement I and Statement II are correct.
(b) Both Statement I and Statement II are incorrect.
(c) Statement I is correct but Statement II is incorrect.
(d) Statement I is incorrect but Statement II is correct.

38. Consider the following reaction:



The dihedral angle in product A in its solid phase at 110 K is:

- (a) 104° (b) 111.5°
(c) 90.2° (d) 111.0°

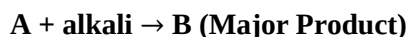
39. The correct order of melting point is:

- (a) $\text{Be} > \text{Mg} > \text{Ca} > \text{Sr}$
(b) $\text{Sr} > \text{Ca} > \text{Mg} > \text{Be}$
(c) $\text{Be} > \text{Ca} > \text{Mg} > \text{Sr}$
(d) $\text{Be} > \text{Ca} > \text{Sr} > \text{Mg}$

40. The correct order of melting points of hydrides of group 16 elements is:

- (a) $\text{H}_2\text{S} < \text{H}_2\text{Se} < \text{H}_2\text{Te} < \text{H}_2\text{O}$
(b) $\text{H}_2\text{O} < \text{H}_2\text{S} < \text{H}_2\text{Se} < \text{H}_2\text{Te}$
(c) $\text{H}_2\text{S} < \text{H}_2\text{Te} < \text{H}_2\text{Se} < \text{H}_2\text{O}$
(d) $\text{H}_2\text{Se} < \text{H}_2\text{S} < \text{H}_2\text{Te} < \text{H}_2\text{O}$

41. Consider the following reaction:



If B is an oxoacid of phosphorus with no P – H bond, then A is:

- (a) White P_4 (b) Red P_4
(c) P_2O_3 (d) H_3PO_3

42. Polar stratospheric clouds facilitate the formation of:

- (a) $ClONO_2$ (b) $HOCl$
(c) ClO (d) CH_4

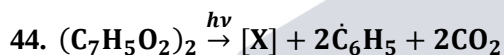
43. Given below are two statements:

Statement I: In 'Lassaigne's Test, when both nitrogen and sulphur are present in an organic compound, sodium thiocyanate is formed.

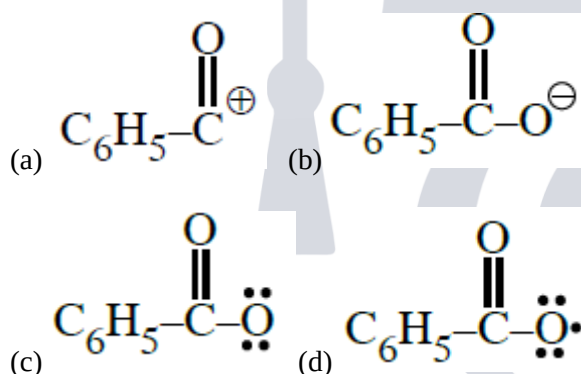
Statement II: If both nitrogen and sulphur are present in an organic compound, then the excess of sodium used in sodium fusion will decompose the sodium thiocyanate formed to give $NaCN$ and Na_2S .

In the light of the above statements, choose the most appropriate answer from the options given below:

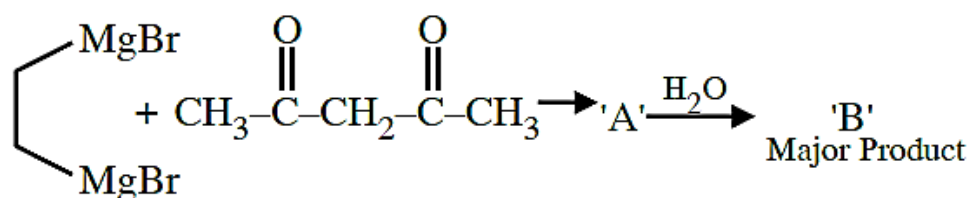
- (a) Both Statement I and Statement II are correct.
(b) Both Statement I and Statement II are incorrect.
(c) Statement I is correct but Statement II is incorrect.
(d) Statement I is incorrect but Statement II is correct.



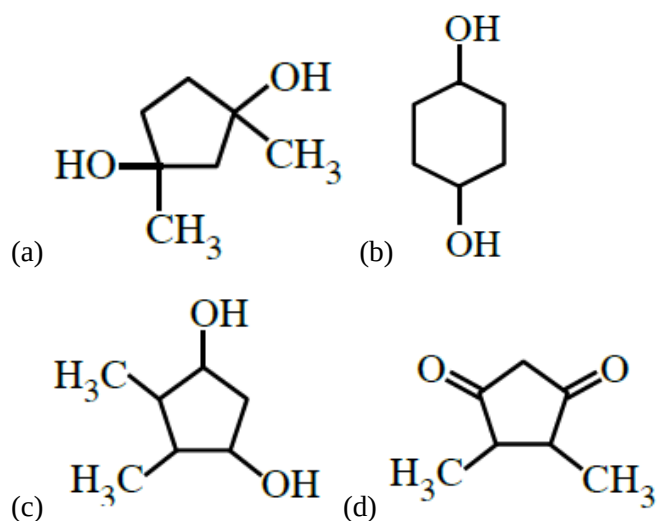
Consider the above reaction and identify the intermediate 'X'



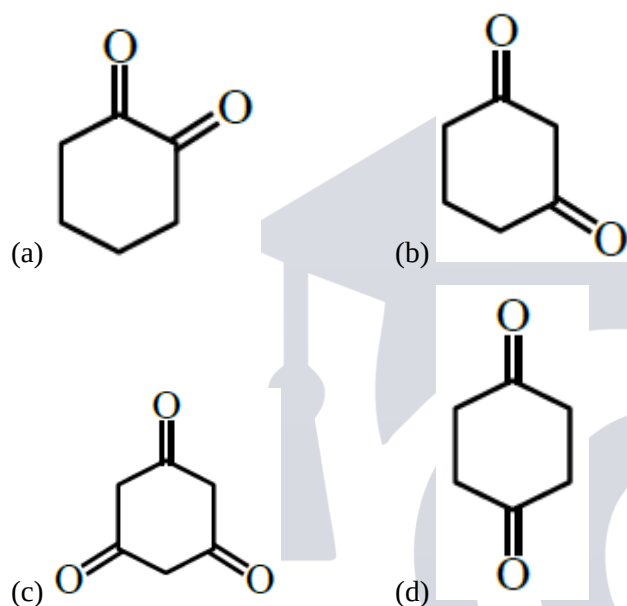
45.



Consider the above reaction sequence and identify the product B.

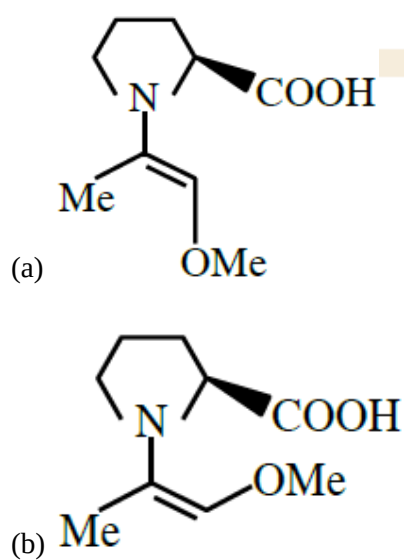


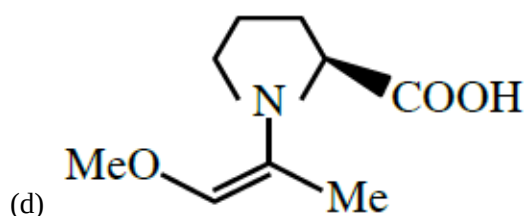
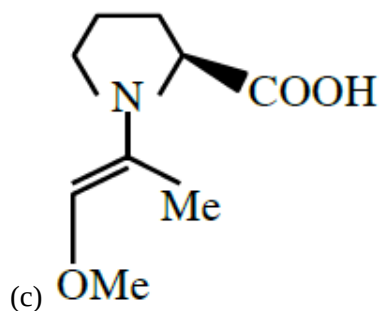
46. Which will have the highest enol content?



47. Among the following structures, which will show the most stable enamine formation?

(Where Me is $-\text{CH}_3$)





48. Which of the following sets are correct regarding polymer?

- (A) Copolymer: Buna-S
- (B) Condensation polymer: Nylon-6,6
- (C) Fibre : Nylon-6,6
- (D) Thermosetting polymer: Terylene
- (E) Homopolymer: Buna-N

Choose the correct answer from given options below:

- (a) (A), (B) and (C) are correct
- (b) (B), (C) and (D) are correct
- (c) (A), (C) and (E) are correct
- (d) (A), (B) and (D) are correct

49. A chemical which stimulates the secretion of pepsin is:

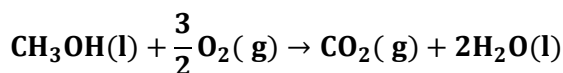
- (a) Anti histamine (b) Cimetidine
- (c) Histamine (d) Zantac

50. Which statement is not true with respect to nitrate ion test?

- (a) A dark brown ring is formed at the junction of two solutions.
- (b) Ring is formed due to nitroferrous sulphate complex.
- (c) The brown complex is $[\text{Fe}(\text{H}_2\text{O})_5(\text{NO})]\text{SO}_4$.
- (d) Heating the nitrate salt with conc. H_2SO_4 , light brown fumes are evolved.

SECTION-B

51. For complete combustion of methanol



the amount of heat produced as measured by bomb calorimeter is 726 kJ mol^{-1} at 27°C . The enthalpy of combustion for the reaction is $-x\text{kJmol}^{-1}$, where x is . (Nearest integer) (Given: $R = 8.3\text{JK}^{-1}\text{mol}^{-1}$)

52. A 0.5 percent solution of potassium chloride was found to freeze at -0.24°C . The percentage dissociation of potassium chloride is (Nearest integer) (Molal depression constant for water is $1.80 \text{ K kg mol}^{-1}$ and molar mass of KCl is 74.6 g mol^{-1})

53. 50 mL of 0.1M CH_3COOH is being titrated against 0.1M NaOH . When 25 mL of NaOH has been added, the pH of the solution will be $\times 10^{-2}$. (Nearest integer)

(Given: $\text{pK}_a(\text{CH}_3\text{COOH}) = 4.76$)

$$\log 2 = 0.30 \quad \log 3 = 0.48$$

$$\log 5 = 0.69$$

$$\log 7 = 0.84$$

$$\log 11 = 1.04$$

54. A flask is filled with equal moles of A and B. The half lives of A and B are 100 s and 50 s respectively and are independent of the initial concentration. The time required for the concentration of A to be four times that of B is s. (Given: $\ln 2 = 0.693$)

55. 2.0 g of H_2 gas is adsorbed on 2.5 g of platinum powder at 300 K and 1bar pressure. The volume of the gas adsorbed per gram of the adsorbent is mL. (Given: $R = 0.083 \text{ Lbar}^{-1}\text{mol}^{-1}$)

56. The spin-only magnetic moment value of the most basic oxide of vanadium among V_2O_3 , V_2O_4 and V_2O_5 is B.M. (Nearest Integer)

57. The spin-only magnetic moment value of an octahedral complex among $\text{CoCl}_3 \cdot 4\text{NH}_3$, $\text{NiCl}_2 \cdot 6\text{H}_2\text{O}$ and $\text{PtCl}_4 \cdot 2\text{HCl}$, which upon reaction with excess of AgNO_3 gives 2 moles of AgCl is B.M. (Nearest Integer)

58. On complete combustion 0.30 g of an organic compound gave 0.20 g of carbon dioxide and 0.10 g of water. The percentage of carbon in the given organic compound is (Nearest Integer)

59. Compound 'P' on nitration with dil. HNO_3 yields two isomers (a) and (b). These isomers can be separated by steam distillation. Isomers (a) and (b) show the intramolecular and intermolecular hydrogen bonding respectively. Compound (P) on reaction with conc. HNO_3 yields a yellow compound 'C', a strong acid. The number of oxygen atoms is present in compound 'C'

60. The number of oxygens present in a nucleotide formed from a base, that is present only in RNA is

PART-III MATHEMATICS

SECTION-A

61. Let $f(x) = \frac{x-1}{x+1}$, $x \in \mathbb{R} - \{0, -1, 1\}$. If $f^{n+1}(x) = f(f^n(x))$ for all $n \in \mathbb{N}$, then $f^6(6) + f^7(7)$ is equal to:

(a) $\frac{7}{6}$
(c) $\frac{7}{12}$

(b) $-\frac{3}{2}$
(d) $-\frac{11}{12}$

62. Let $A = \left\{ z \in \mathbb{C} : \left| \frac{z+1}{z-1} \right| < 1 \right\}$
and $B = \left\{ z \in \mathbb{C} : \arg \left(\frac{z-1}{z+1} \right) = \frac{2\pi}{3} \right\}$.

Then $A \cap B$ is:

- (a) a portion of a circle centred at $\left(0, -\frac{1}{\sqrt{3}} \right)$ that lies in the second and third quadrants only
(b) a portion of a circle centred at $\left(0, -\frac{1}{\sqrt{3}} \right)$ that lies in the second quadrant only
(c) an empty set
(d) a portion of a circle of radius $\frac{2}{\sqrt{3}}$ that lies in the third quadrant only

63. Let A be a 3×3 invertible matrix. If $|\text{adj}(24A)| = \text{adj}(3\text{adj}(2A))|$, then $|A|^2$ is equal to:

- (a) 6^6 (b) 2^{12}
(c) 2^6 (d) 1

64. The ordered pair (a, b) , for which the system of linear equations

$$3x - 2y + z = b$$

$$5x - 8y + 9z = 3$$

$$2x + y + az = -1$$

has no solution, is:

- (a) $\left(3, \frac{1}{3} \right)$ (b) $\left(-3, \frac{1}{3} \right)$
(c) $\left(-3, -\frac{1}{3} \right)$ (d) $\left(3, -\frac{1}{3} \right)$

65. The remainder when $(2021)^{2023}$ is divided by 7 is:

- (a) 1 (b) 2
(c) 5 (d) 6

66. $\lim_{x \rightarrow \frac{1}{\sqrt{2}}} \frac{\sin(\cos^{-1} x) - x}{1 - \tan(\cos^{-1} x)}$ is equal to:

- (a) $\sqrt{2}$ (b) $-\sqrt{2}$
(c) $\frac{1}{\sqrt{2}}$ (d) $-\frac{1}{\sqrt{2}}$

67. Let $f, g: \mathbb{R} \rightarrow \mathbb{R}$ be two real valued functions defined as $f(x) = \begin{cases} -|x+3|, & x < 0 \\ e^x, & x \geq 0 \end{cases}$ and $g(x) =$

$$\begin{cases} x^2 + k_1x, & x < 0 \\ 4x + k_2, & x \geq 0 \end{cases}, \text{ where } k_1 \text{ and } k_2 \text{ are real constants. If } (g \circ f) \text{ is differentiable at } x = 0, \text{ then } (g \circ f)$$

$(-4) + (g \circ f)(4)$ is equal to:

- (a) $4(e^4 + 1)$ (b) $2(2e^4 + 1)$
(c) $4e^4$ (d) $2(2e^4 - 1)$

68. The sum of the absolute minimum and the absolute maximum values of the function $f(x) = |3x - x^2 + 2| - x$ in the interval $[-1, 2]$ is:

- (a) $\frac{\sqrt{17}+3}{2}$ (b) $\frac{\sqrt{17}+5}{2}$
(c) 5 (d) $\frac{9-\sqrt{17}}{2}$

69. Let S be the set of all the natural numbers, for which the line $\frac{x}{a} + \frac{y}{b} = 2$ is a tangent to the curve $\left(\frac{x}{a} \right)^n + \left(\frac{y}{b} \right)^n = 2$ at the point (a, b) , $ab \neq 0$. Then:

- (a) $S = \phi$ (b) $n(S) = 1$
(c) $S = \{2k; k \in \mathbb{N}\}$ (d) $S = \mathbb{N}$

70. The area bounded by the curve $y = |x^2 - 9|$ and the line $y = 3$ is :

- (a) $4(2\sqrt{3} + \sqrt{6} - 4)$
(b) $4(4\sqrt{3} + \sqrt{6} - 4)$
(c) $8(4\sqrt{3} + 3\sqrt{6} - 9)$
(d) None of these

71. Let R be the point (3, 7) and let P and Q be two points on the line $x + y = 5$ such that PQR is an equilateral triangle. Then the area of $\triangle PQR$ is:

- (a) $\frac{25}{4\sqrt{3}}$ (b) $\frac{25\sqrt{3}}{2}$
(c) $\frac{25}{\sqrt{3}}$ (d) $\frac{25}{2\sqrt{3}}$

72. Let C be a circle passing through the points A(2, -1) and B(3, 4). The line segment AB is not a diameter of C. If r is the radius of C and its centre lies on the circle $(x - 5)^2 + (y - 1)^2 = \frac{13}{2}$, then r^2 is equal to :

- (a) 32 (b) $\frac{65}{2}$
(c) $\frac{61}{2}$ (d) 30

73. Let the normal at the point P on the parabola $y^2 = 6x$ pass through the point (5, -8). If the tangent at P to the parabola intersects its directrix at the point Q, then the ordinate of the point Q is:

- (a) -3 (b) $-\frac{9}{4}$
(c) $-\frac{5}{2}$ (d) -2

74. If the two lines $l_1: \frac{x-2}{3} = \frac{y+1}{-2}, z = 2$ and $l_2: \frac{x-1}{1} = \frac{2y+3}{\alpha} = \frac{z+5}{2}$ perpendicular, then an angle between the lines l_2 and $l_3: \frac{1-x}{3} = \frac{2y-1}{-4} = \frac{z}{4}$ is :

- (a) $\cos^{-1}\left(\frac{29}{4}\right)$ (b) $\sec^{-1}\left(\frac{29}{4}\right)$
(c) $\cos^{-1}\left(\frac{2}{29}\right)$ (d) $\cos^{-1}\left(\frac{2}{\sqrt{29}}\right)$

75. Let the plane $2x + 3y + z + 20 = 0$ be rotated through a right angle about its line of intersection with the plane $x - 3y + 5z = 8$. If the mirror image of the point $\left(2, -\frac{1}{2}, 2\right)$ in the rotated plane is B(a, b, c), then :

- (a) $\frac{a}{8} = \frac{b}{5} = \frac{c}{-4}$ (b) $\frac{a}{4} = \frac{b}{5} = \frac{c}{-2}$
(c) $\frac{a}{8} = \frac{b}{-5} = \frac{c}{4}$ (d) $\frac{a}{4} = \frac{b}{5} = \frac{c}{2}$

76. If $\vec{a} \cdot \vec{b} = 1, \vec{b} \cdot \vec{c} = 2$ and $\vec{c} \cdot \vec{a} = 3$, then the value of $[\vec{a} \times (\vec{b} \times \vec{c}), \vec{b} \times (\vec{c} \times \vec{a}), \vec{c} \times (\vec{b} \times \vec{a})]$ is :

- (a) 0 (b) $-6\vec{a} \cdot (\vec{b} \times \vec{c})$
(c) $12\vec{c} \cdot (\vec{a} \times \vec{b})$ (d) $-12\vec{b} \cdot (\vec{c} \times \vec{a})$

77. Let a biased coin be tossed 5 times. If the probability of getting 4 heads is equal to the probability of getting 5 heads, then the probability of getting atmost two heads is:

- (a) $\frac{275}{6^5}$ (b) $\frac{36}{5^4}$
(c) $\frac{181}{5^5}$ (d) $\frac{46}{6^4}$

78. The mean of the numbers a, b, 8, 5, 10 is 6 and their variance is 6.8. If M is the mean deviation of the numbers about the mean, then 25M is equal to:

- (a) 60 (b) 55
(c) 50 (d) 45

79. Let $f(x) = 2\cos^{-1}x + 4\cot^{-1}x - 3x^2 - 2x + 10, x \in [-1, 1]$. If [a, b] is the range of the function then $4a - b$ is equal to:

- (a) 11 (b) $11 - \pi$
(c) $11 + \pi$ (d) $15 - \pi$

80. Let $\Delta, \nabla \in \{\wedge, \vee\}$ be such that

$p \nabla q \Rightarrow ((p \Delta q) \nabla r)$ is a tautology.

Then $(p \vee q) \Delta r$ is logically equivalent to:

- (a) $(p \Delta r) \vee q$ (b) $(p \Delta r) \wedge q$
(c) $(p \wedge r) \Delta q$ (d) $(p \vee r) \wedge q$

SECTION-B

81. The sum of the cubes of all the roots of the equation $x^4 - 3x^3 - 2x^2 + 3x + 1 = 10$ is
82. There are ten boys $B_1, B_2, \dots, B_{10}$ and five girls $G_1, G_2, \dots, G_5$ in a class. Then the number of ways of forming a group consisting of three boys and three girls, if both B_1 and B_2 together should not be the members of a group, is
83. Let the common tangents to the curves $4(x^2 + y^2) = 9$ and $y^2 = 4x$ intersect at the point Q. Let an ellipse, centered at the origin O, has lengths of semi-minor and semi-major axes equal to OQ and 6, respectively. If e and l respectively denote the eccentricity and the length of the latus rectum of this ellipse, then $\frac{l}{e^2}$ is equal to
84. Let $f(x) = \max\{|x+1|, |x+2|, \dots, |x+5|\}$. Then $\int_{-6}^0 f(x) dx$ is equal to
85. Let the solution curve $y = y(x)$ of the differential equation $(4 + x^2)dy - 2x(x^2 + 3y + 4)dx = 0$ pass through the origin. Then $y(2)$ is equal to
86. If $\sin^2(10^\circ)\sin(20^\circ)\sin(40^\circ)\sin(50^\circ)\sin(70^\circ) = \alpha - \frac{1}{16}\sin(10^\circ)$, then $16 + \alpha^{-1}$ is equal to
87. Let $A = \{n \in \mathbb{N} : \text{H.C.F.}(n, 45) = 1\}$ and Let $B = \{2k : k \in \{1, 2, \dots, 100\}\}$. Then the sum of all the elements of $A \cap B$ is
88. The value of the integral $\frac{48}{\pi^4} \int_0^\pi \left(\frac{3\pi x^2}{2} - x^3 \right) \frac{\sin x}{1 + \cos^2 x} dx$ is equal to
89. Let $A = \sum_{i=1}^{10} \sum_{j=1}^{10} \min\{i, j\}$ and $B = \sum_{i=1}^{10} \sum_{j=1}^{10} \max\{i, j\}$. Then $A + B$ is equal to
90. Let $S = (0, 2\pi) - \left\{ \frac{\pi}{2}, \frac{3\pi}{4}, \frac{3\pi}{2}, \frac{7\pi}{4} \right\}$. Let $y = y(x), x \in S$, be the solution curve of the differential equation $\frac{dy}{dx} = \frac{1}{1 + \sin 2x}, y\left(\frac{\pi}{4}\right) = \frac{1}{2}$, if the sum of abscissas of all the points of intersection of the curve $y = y(x)$ with the curve $y = \sqrt{2}\sin x$ is $\frac{k\pi}{12}$, then k is equal to