1. (d) $\vec{v} = \frac{d\vec{s}}{dt} = 4t\hat{j}$

At $t = 1 \sec \vec{v} = 4\hat{j}$

2. (b) Gases have less viscosity.

Due to insoluble impurities like detergent surface tension decreases

3. (a)

$$\cot\frac{A}{2} = \frac{\sin\left(\frac{A+\delta_{min}}{2}\right)}{\sin\frac{A}{2}}$$

$$\Rightarrow \cos\frac{A}{2} = \sin\left(\frac{A+\delta_{min}}{2}\right)$$

$$\frac{A+\delta_{min}}{2} = \frac{\pi}{2} - \frac{A}{2}$$

$$\delta_{min} = \pi - 2A$$

4. (c) Net force on particle must be zero i.e. $q\vec{E} + q\vec{V} \times \vec{B} = 0$

Possible cases are

(i) $\vec{E} \& \vec{B} = 0$

(ii) $\vec{V} \times \vec{B} = 0, \vec{E} = 0$

(iii) $q\vec{E} = -q\vec{V} \times \vec{B}$

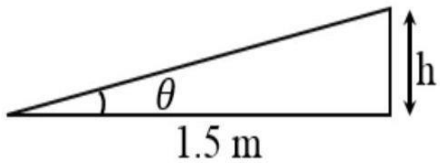
$\vec{E} \neq 0 \& \vec{B} \neq 0$

5. (d) $g = \frac{GM}{R^2} \Rightarrow g \propto \frac{1}{R^2}$

$$\frac{g_2}{g_1} = \frac{R_1^2}{R_2^2}$$

$$g_2 = 4\,g_1 \left(R_2 = \frac{R_1}{2}\right)$$

6. (b) $\tan\theta = \frac{v^2}{Rg} = \frac{12\times12}{10\times400}$



$$\tan\theta = \frac{h}{1.5}$$

$$\Rightarrow \frac{h}{1.5}=\frac{144}{4000}$$

$h = 5.4\ \text{cm}$

7. (d) P end should be at higher potential for forward biasing.

8. (b) Spherometer can be used to measure curvature of surface.

9. (c) $\frac{P_1^2}{2\ m_1}=\frac{P_2^2}{2\ m_2}$

$$\frac{P_1}{P_2}=\sqrt{\frac{m_1}{m_2}}=\frac{2}{5}$$

10. (c) $Q=\Delta U$ as work done is zero [constant volume] $\Delta U = ms\Delta T$

$= 0.08 \times (170 \times 4.18) \times 5$

$\simeq 284\ \text{J}$

11. (b) $r \propto \frac{n^2}{Z}$

$$\frac{r_4}{r_3}=\frac{4^2}{3^2}$$

$$r_4=\frac{16}{9}R$$

12. (c) Average emf $=\frac{\text{Change in flux}}{\text{Time}}=-\frac{\Delta\phi}{\Delta t}$

$$=-\frac{0-(4\times(2.5\times 2)\cos 60^\circ)}{10}$$

$= +1\ \text{V}$

13. (c) Potential difference $=\frac{KQ}{r_1}-\frac{KQ}{r_2}$

$$r_1=\sqrt{(\sqrt{3})^2+(\sqrt{3})^2}$$

$$r_2=\sqrt{(\sqrt{6})^2+0}$$

As $r_1 = r_2 = \sqrt{6}\ \text{m}$

So potential difference $= 0$

14. (d) As amount of energy incident on cell is same so current will remain same.

15. (d) Momentum will remain conserve

$1000 \times 6 = 1200 \times v$

$v = 5\ \text{m/s}$

16. (b) $I = \frac{1}{2}\varepsilon_0 E_0^2 \times c$

$$I = \frac{1}{2} \times 8.85 \times 10^{-12} \times 4 \times 10^4 \times 3 \times 10^8$$

$I = 53.1\ W/m^2$

17. (a)

$[h] = ML^2\ T^{-1}$

$[L] = ML^2\ T^{-1}$

$[P] = MLT^{-1}$

$[\tau] = ML^2\ T^{-2}$

(Here h is Planck's constant, L is angular momentum, P is linear momentum and τ is moment of force)

18. (c) For null point,

$$\frac{4.5}{60} = \frac{R}{40}$$

Also, $R = \frac{\rho\ell}{A} = \frac{\rho\ell}{\pi r^2}$

$$4.5 \times 40 = \rho \times \frac{0.1}{\pi \times 7 \times 10^{-8}} \times 60$$

$\rho = 66 \times 10^{-7}\Omega \times m$

19. (a) Resistance of each part $= \frac{R}{5}$

Total resistance $= \frac{1}{5} \times \frac{R}{5} = \frac{R}{25}$

20. (b) For monoatomic molecule degree of freedom = 3.

$$\therefore K_{avg} = \frac{3}{2} K_B T$$

$$T = \frac{0.414 \times 1.6 \times 10^{-19} \times 2}{3 \times 1.38 \times 10^{-23}}$$

$= 3200\ K$

21. (673)

$u_x = 5\ m/s\ a_x = 3\ m/s^2\ x = 84\ m$

$v_x^2 - u_x^2 = 2ax$

$v_x^2 - 25 = 2(c)(84)$

$V_x = 23\ m/s$

$v_x - u_x = a_x t$

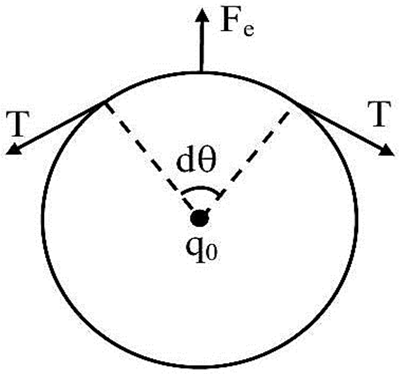
$t = \dfrac{23-5}{3} = 6\ s$

$v_y = 0 + a_y t = 0 + 2 \times (6) = 12\ m/s$

$v^2 = v_x^2 + v_y^2 = 23^2 + 12^2 = 673$

$v = \sqrt{673}\ m/s$

22. (3)



$$2\ T\sin\frac{d\theta}{2} = \frac{kq_0}{R^2} \cdot \lambda R d\theta$$

$$\left[\lambda = \frac{Q}{2\pi R}\right]$$

$$\Rightarrow T = \frac{Kq_0Q}{(R^2) \times 2\pi}$$

$$= \frac{(9 \times 10^9)(2\pi \times 30 \times 10^{-12})}{(0.30)^2 \times 2\pi}$$

$$= \frac{9 \times 10^{-3} \times 30}{9 \times 10^{-2}} = 3\ N$$

23. (2) $\phi = Mi = Mi_0 \sin \omega t$

$$EMF = -M\frac{di}{dt} = -0.002(i_0 \omega \cos \omega t)$$

$$EMF_{max} = i_0 \omega (0.002) = (5)(50\pi)(0.002)$$

$$EMF_{max} = \frac{\pi}{2}\ V$$

24. (31)

$$h_{app} = \frac{h_1}{\mu_1} + \frac{h_2}{\mu_2} = \frac{6}{3/2} + \frac{6}{8/5} = 4 + \frac{15}{4} = \frac{31}{4}\ cm$$

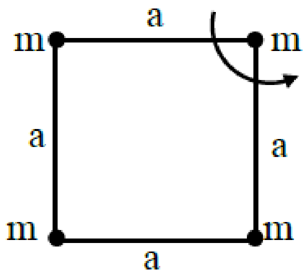
25. (236)

$Q = BE_{Product} - BE_{Rectant}$

$= 2(118)(8.6) - 236(7.6)$

$= 236 \times 1 = 236MeV$

26. (16)



$I = ma^2 + ma^2 + m(\sqrt{2}a)^2$

$= 4ma^2$

$= 4 \times 1 \times (2)^2 = 16$

27. (12) $V_{\text{at mean position}}$ $A\omega \Rightarrow 10 = 4\omega$

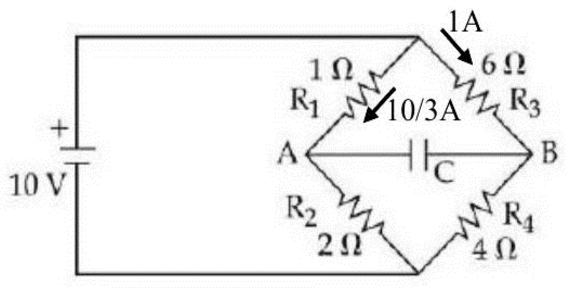
$$\omega = \frac{5}{2}$$
$$v = \omega\sqrt{A^2 - x^2}$$
$$5 = \frac{5}{2}\sqrt{4^2 - x^2} \Rightarrow x^2 = 16 - 4$$
$$x = \sqrt{12}\ \text{cm}$$

28. (160)

$$B = \left(\frac{\mu_0 i}{2\pi a}\right) \times 2 = \frac{4\pi \times 10^{-7} \times 10}{\pi \times \left(\frac{5}{2} \times 10^{-2}\right)}$$

$= 16 \times 10^{-5} = 160\mu T$

29. (400)



$$V_A + \frac{10}{3}(a) - 6(a) = V_B$$

$V_A - V_B = 6 - \frac{10}{3} = \frac{8}{3}$ volt

$Q = C(V_A - V_B)$

$= 150 \times \frac{8}{3} = 400\mu C$

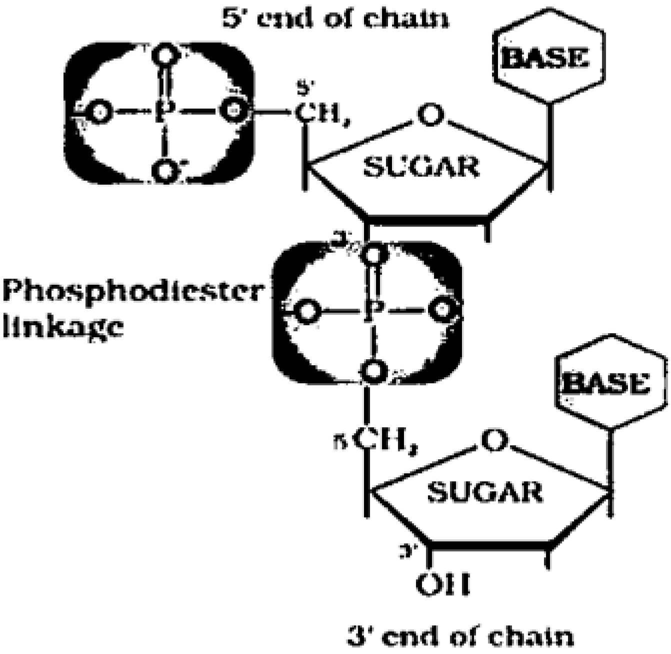
30. (2)

$B = -\frac{\Delta P}{\left(\frac{\Delta V}{V}\right)}$

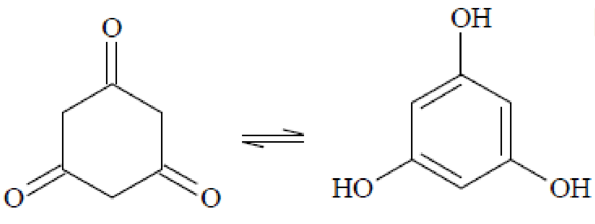
$-\left(\frac{\Delta V}{V}\right) = \frac{\rho g h}{B} = \frac{1000 \times 10 \times 4000}{2 \times 10^9}$

$= 2 \times 10^{-2}$ [-ve sign represent compression]

31. (a) Phosphodiester linkage



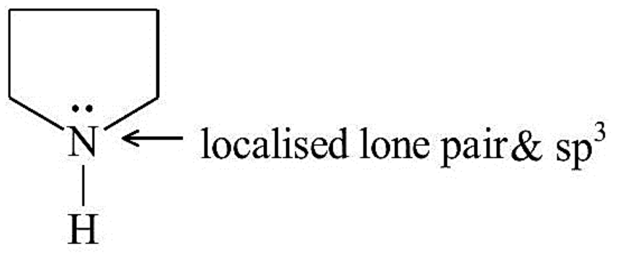
32. (b)



Aromatic

33. (d) Fluorine does not show variable oxidation state.

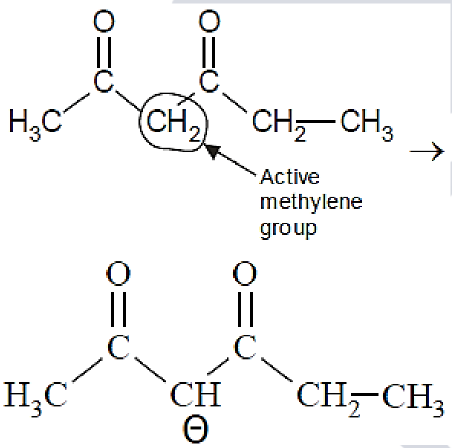
34. (d)



35. (d)

	$3\ d^7$	$3\ d^8$	$3\ d^3$	$3\ d^6$
No.of. unpaired e-	3	2	3	4
Spin only Magnetic moment	$\sqrt{15}$ BM	$\sqrt{8}$ BM	$\sqrt{15}$ BM	$\sqrt{24}$ BM

36. (d)



Conjugate base is more stable due to more resonance of negative charge.

37. (d) Solution with negative deviation has

$P_T < P_A 0 X_A + P_B 0 X_B$

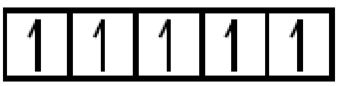
$P_A < P_A X_A$

$P_B < P_B 0 X_B$

If vapour pressure decreases so boiling point increases.

38. (c)

$[FeF_6]^{3-}: Fe^{+3}: [Ar]3\ d^5$

F : Weak field Ligand 

No. of unpaired electron's = 5

$\mu = \sqrt{5(5+2)}$

$\mu = \sqrt{35}$BM

$[V(H_2O)_6]^{+2} : V^{+2} : 3\ d^3$

↿	↿	↿		

No. of unpaired electron's = 3

$\mu = \sqrt{3(3+2)}$

$\mu = \sqrt{15}$BM

$[Fe(H_2O)_6]^{+2} : Fe^{+2} : 3\ d^6$

H_2O : Weak field Ligand

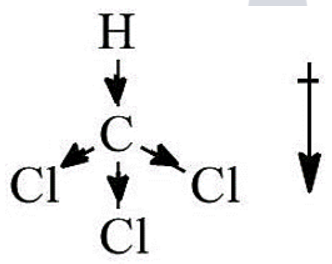
1	1	1	1	1

No. of unpaired electron's = 4

$\mu = \sqrt{4(4+2)}$

$\mu = \sqrt{24}$BM

39. (d)

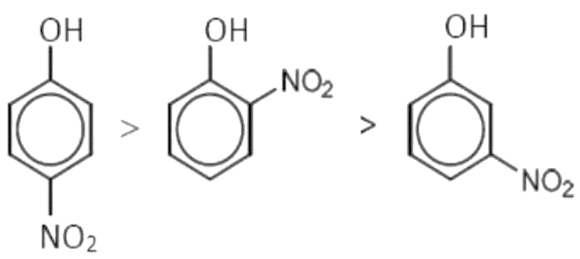


$\mu \neq 0$

$CHCl_3$ is polar molecule and rest all molecules are non-polar.

40. (c) s-block elements are highly reactive and found in combined state.

41. (c) Acidic strength

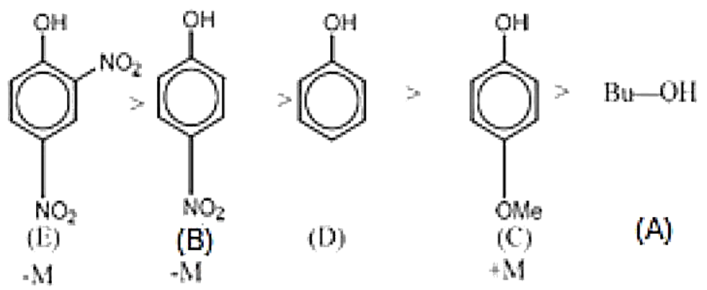


Ethanol give lucas test after long time

Statement (I) → correct

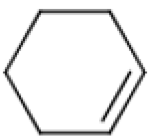
Statement (II) → incorrect

42. (d)



43. (b) Solid Boron has very strong crystalline lattice so its melting point unusually high in group 13 elements

44. (d)

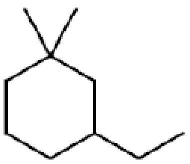
 is Alicyclic

45. (d) $PbCrO_4 + NaOH$ (hot excess) → $[Pb(OH)_4]^{-2} + Na_2CrO_4$

Dianionic complex with coordination number four

46. (a)

47. (b)



3-ethy 1, 1 -dimethylcyclohexane

48. (a)

$$NaCl + \text{conc. } H_2SO_4 + K_2Cr_2O_7 \rightarrow \underset{\substack{(B)\\ \text{Reddish brown}}}{CrO_2Cl_2} + KHSO_4 + NaHSO_4 + H_2O$$

$$CrO_2Cl_2 + NaOH \rightarrow \underset{\substack{(C)\\ \text{Yellow colour}}}{Na_2CrO_4} + NaCl + H_2O$$

49. (d)

SN^1 – Racemisation

SN^2 – Inversion

50. **(a)** Electronic configuration of $Nd(Z = 60)$ is; $[Xe]4f^4 6s^2$

51. **(107gm or 108)**

Eq. of Ag = Eq. of O_2

Let x gm silver displaced,

$$\frac{x \times 1}{108} = \frac{5.6}{22.7} \times 4$$

(Molar volume of gas at STP = 22.7 lit)

$x = 106.57gm$

Ans. 107

OR,

as per old STP data, molar volume = 22.4 lit

$\frac{x \times 1}{108} = \frac{5.6}{22.4} \times 4, x = 108gm.$

Ans. 108

52. **(2)**

Let, $R = k[HI]^n$

using any two of given data,

$$\frac{3 \times 10^{-3}}{7.5 \times 10^{-4}} = \left(\frac{0.01}{0.005}\right)^n$$

$n = 2$

53. **(8)**

$CH_4 + 2O_2 \rightarrow CO_2 + 2H_2O$

Moles of $CO_2 = \frac{22}{44} = 0.5$

So, required moles of $CH_4 = 0.5$

Mass $= 0.5 \times 16 = 8gm$

54. **(1200)**

Using, first law of thermodynamics,

$\Delta U = Q + W,$

$\Delta U = 0$: Process is isothermal

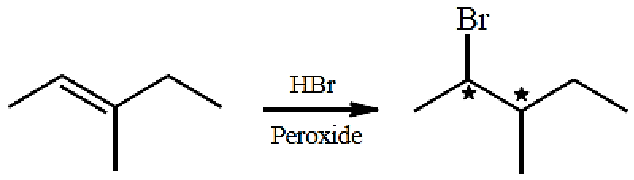
$Q = -W$

$W = -P_{ext}\Delta V$: Irreversible

$= -80 \times 10^3 (45 - 30) \times 10^{-3}$

$= -1200\ \text{J}$

55. (4)



2chiral centres

No. of stereoisomers = 4

56. (3)

57. (d)

$-NO_2, -C \equiv N, -COR, -COOH$

are meta directing.

58. (16) n = 4 can have,

	4s	4p	4d	4f
Total e^-	2	6	10	14
Total e with $S = +\frac{1}{2}$	1	3	5	7

So, Ans. 16

59. (6)

$CO \Rightarrow \quad \overset{-}{C} \equiv \overset{+}{O} \quad : \ BO = 3$

$NO^+ \Rightarrow \quad N \equiv O^+ \quad : \ BO = 3$

60. (3)

Compounds	SO_3	H_2SO_3	$SOCl_2$	SF_4	$BaSO_4$	$H_2S_2O_7$
O.S. of Sulphur:	+6	+4	+4	+4	+6	+6

61. (a) ${}^{n-1}C_r = (k^2 - 8)\,{}^{n}C_{r+1}$

$\underbrace{r + 1 \geq 0,\ r \geq 0}_{r \geq 0}$

$\frac{{}^{n-1}C_r}{{}^{n}C_{r+1}} = k^2 - 8$

$\frac{r+1}{n} = k^2 - 8$

$\Rightarrow k^2 - 8 > 0$

$(k - 2\sqrt{2})(k + 2\sqrt{2}) > 0$

$k \in (-\infty, -2\sqrt{2}) \cup (2\sqrt{2}, \infty)$

$\therefore n \geq r + 1, \frac{r+1}{n} \leq 1$

$\Rightarrow k^2 - 8 \leq 1$

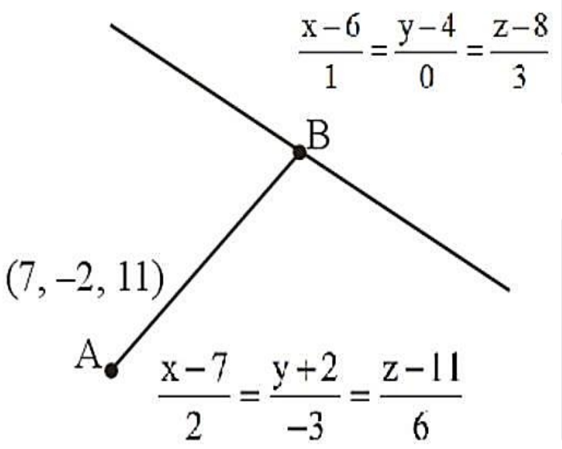
$k^2 - 9 \leq 0$

$-3 \leq k \leq 3$

From equation (I) and (II) we get

$k \in [-3, -2\sqrt{2}) \cup (2\sqrt{2}, 3]$

62. (b) $B = (2\lambda + 7, -3\lambda - 2, 6\lambda + 11)$



Point B lies on $\frac{x-6}{1} = \frac{y-4}{0} = \frac{z-8}{3}$

$\frac{2\lambda + 7 - 6}{1} = \frac{-3\lambda - 2 - 4}{0} = \frac{6\lambda + 11 - 8}{3}$

$-3\lambda - 6 = 0$

$\lambda = -2$

$B \Rightarrow (3, 4, -1)$

$AB = \sqrt{(7-3)^2 + (4+2)^2 + (11+1)^2}$
$= \sqrt{16 + 36 + 144}$
$= \sqrt{196} = 14$

63. (d)

$\frac{dx}{dt} + ax = 0$

$\frac{dx}{x} = -adt$

$\int \frac{dx}{x} = -a\int dt$

$\ln|x| = -at + c$

at $t = 0, x = 2$

$\ln 2 = 0 + c$

$\ln x = -$ at $+ \ln 2$

$\frac{x}{2} = e^{-at}$

$x = 2e^{-2at}$

$\frac{dy}{dt} + by = 0$

$\frac{dy}{y} = -bdt$

$\ln|y| = -bt + \lambda$

$t = 0, y = 1$

$0 = 0 + \lambda$

$y = e^{-bt}$

According to question

$3y(a) = 2x(a)$
$3e^{-b} = 2(2e^{-a})$
$e^{a-b} = \frac{4}{3}$

For $x(t) = y(t)$

$\Rightarrow 2e^{-at} = e^{-bt}$

$2 = e^{(a-b)t}$

$2 = \left(\frac{4}{3}\right)^t$

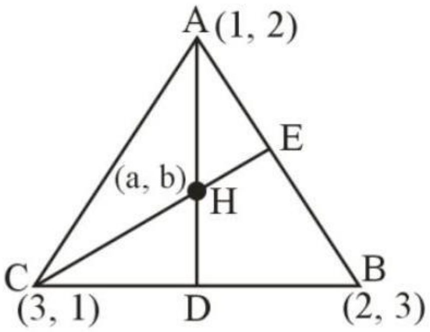
$\log_{\frac{4}{3}} 2 = t$

64. (a)

Equation of CE

$y-1=-(x-3)$

$x+y=4$



Ortho centre lies on the line $x+y=4$

so, $a+b=4$

$I_1=\int_a^b x\sin(x(4-x))dx$

Using king rule

$I_1=\int_a^b (4-x)\sin(x(4-x))dx$

(i) + (ii)

$2I_1=\int_a^b 4\sin(x(4-x))dx$

$2I_1=4I_2$

$I_1=2I_2$

$\frac{I_1}{I_2}=2$

$\frac{36I_1}{I_2}=72$

65. (a) Sum of coefficients in the expansion of $(1-3x+10x^2)^n=A$

then $A=(1-3+10)^n=8^n$(put $x=1$)

and sum of coefficients in the expansion of $(1+x^2)^n=B$

then $B=(1+1)^n=2^n$

$A=B^3$

66. (c)

4,9,14,19, ..., up to 25^{th} term

$T_{25}=4+(25-1)5=4+120=124$

3,6,9,12, ..., up to 37^{th} term

$T_{37} = 3 + (37 - 1)3 = 3 + 108 = 111$

Common difference of I^{st} series $d_1 = 5$

Common difference of II^{nd} series $d_2 = 3$

First common term $= 9$, and

their common difference $= 15$(LCM of d_1 and d_2)

then common terms are

9,24,39,54,69,84,99

67. **(c)** Equation of normal to parabola

$y = mx - 2m - m^3$

this normal passing through center of circle (2,8)

$$8 = 2m - 2m - m^3$$
$$m = -2$$

So point P on parabola $\Rightarrow (am^2, -2am) = (4,4)$

And $C = (2,8)$

$PC = \sqrt{4 + 16} = \sqrt{20}$

$d^2 = 20$

68. **(b)**

$\cdot \frac{x-4}{1} = \frac{y+1}{2} = \frac{z}{-3}$

$$\frac{x - \lambda}{2} = \frac{y + 1}{4} = \frac{z - 2}{-5}$$

the shortest distance between the lines

$$= \left| \frac{(\vec{a} - \vec{b}) \cdot (\vec{d_1} \times \vec{d_2})}{|\vec{d_1} \times \vec{d_2}|} \right|$$

$$= \left| \frac{\begin{vmatrix} \lambda - 4 & 0 & 2 \\ 1 & 2 & -3 \\ 2 & 4 & -5 \end{vmatrix}}{\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -3 \\ 2 & 4 & -5 \end{vmatrix}} \right|$$

$$= \left|\frac{(\lambda-4)(-10+12)-0+2(4-4)}{|2\hat{i}-1\hat{j}+0\hat{k}|}\right|$$

$$\frac{6}{\sqrt{5}} = \left|\frac{2(\lambda-4)}{\sqrt{5}}\right|$$

$3 = |\lambda - 4|$

$\lambda - 4 = \pm 3$

$\lambda = 7,1$

Sum of all possible values of λ is $= 8$

69. (d)

$$\int_0^1 \frac{1}{\sqrt{3+x}+\sqrt{1+x}}dx = \int_0^1 \frac{\sqrt{3+x}-\sqrt{1+x}}{(3+x)-(1+x)}dx$$

$$\frac{1}{2}\left[\int_0^1 \sqrt{3+x}dx - \int_0^1 (\sqrt{1+x})dx\right]$$

$$\frac{1}{2}\left[2\frac{(3+x)^{\frac{3}{2}}}{3} - \frac{2(1+x)^{\frac{3}{2}}}{3}\right]_0^1$$

$$\frac{1}{2}\left[\frac{2}{3}(8-3\sqrt{3}) - \frac{2}{3}\left(2^{\frac{3}{2}}-1\right)\right]$$

$$\frac{1}{3}[8-3\sqrt{3}-2\sqrt{2}+1]$$

$$= 3-\sqrt{3}-\frac{2}{3}\sqrt{2} = a + b\sqrt{2} + c\sqrt{3}$$

$$a = 3, b = -\frac{2}{3}, c = -1$$

$2a + 3b - 4c = 6 - 2 + 4 = 8$

70. (d) Let $S = \{1,2,3,\ldots,10\}$

$R = \{(A,B): A \cap B \neq \phi; A, B \in M\}$

For Reflexive,

M is subset of ' S '

So $\phi \in M$

for $\phi \cap \phi = \phi$

$\Rightarrow$ but relation is $A \cap B \neq \phi$

So it is not reflexive.

For symmetric,

ARB $A \cap B \neq \phi$,

$\Rightarrow$ BRA $\Rightarrow B \cap A \neq \phi$,

So it is symmetric.

For transitive,

If $A = \{(1,2), (2,3)\}$

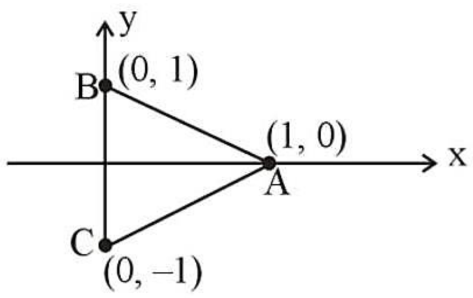
$B = \{(2,3), (3,4)\}$

$C = \{(3,4), (5,6)\}$

ARB & BRC but A does not relate to C

So it not transitive

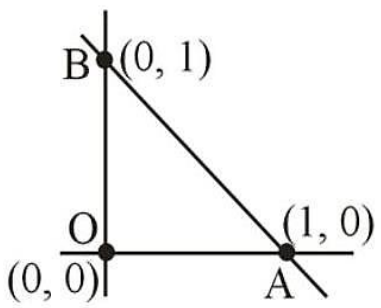
71. (a) $|z - i| = |z + i| = |z - 1|$



ABC is a triangle. Hence its circum-centre will be the only point whose distance from A, B, C will be same.

So $n(S) = 1$

72. (c) $(2k, 3k)$ will lie on circle whose diameter is AB.



$(x - 1)(x) + (y - 1)(y) = 0$

$x^2 + y^2 - x - y = 0$

Satisfy $(2k, 3k)$ in (i)

$(2k)^2 + (3k)^2 - 2k - 3k = 0$

$13k^2 - 5k = 0$

$k = 0, k = \frac{5}{13}$

hence $k=\frac{5}{13}$

73. (d) $f(3^-)=\frac{a}{b}\frac{(7x-12-x^2)}{|x^2-7x+12|}$ (for f(x) to be cont.)

$\Rightarrow f(3^-)=\frac{-a}{b}\frac{(x-3)(x-4)}{(x-3)(x-4)}; x<3 \Rightarrow \frac{-a}{b}$

Hence $f(3^-)=\frac{-a}{b}$

Then $f(3^+)=2^{\lim_{x\to 3^+}\left(\frac{\sin(x-3)}{x-3}\right)}=2$ and

$f(c)=b.$

Hence $f(c)=f(3^+)=f(3^-)$

$\Rightarrow b=2=-\frac{a}{b}$

$b=2, a=-4$

Hence only 1 ordered pair $(-4,2)$.

74. (b) $\sum_{k=1}^{10} a_k=50$

$a_1+a_2+\cdots+a_{10}=50$

$\sum_{\forall k<j} a_k a_j=1100$

If $a_1+a_2+\cdots+a_{10}=50$.

$(a_1+a_2+\cdots+a_{10})^2=2500$

$\Rightarrow \sum_{i=1}^{10} a_i^2+2\sum_{k<j} a_k a_j=2500$

$\Rightarrow \sum_{i=1}^{10} a_i^2=2500-2(1100)$

$\sum_{i=1}^{10} a_i^2=300$, Standard deviation ' σ '

$$=\sqrt{\frac{\sum a_i^2}{10}-\left(\frac{\sum a_i}{10}\right)^2}=\sqrt{\frac{300}{10}-\left(\frac{50}{10}\right)^2}$$

$=\sqrt{30-25}=\sqrt{5}$

75. (a) Equation of chord with given middle point.

$T=S_1$

$$\frac{x}{25}+\frac{y}{40}=\frac{1}{25}+\frac{1}{100}$$

$$\frac{8x+5y}{200}=\frac{8+2}{200}$$

$y = \frac{10-8x}{5}$

$\frac{x^2}{25} + \frac{(10-8x)^2}{400} = 1$ (put in original equation)

$$\frac{16x^2 + 100 + 64x^2 - 160x}{400} = 1$$

$4x^2 - 8x - 15 = 0$

$$x = \frac{8 \pm \sqrt{304}}{8}$$

$$x_1 = \frac{8 + \sqrt{304}}{8}; x_2 = \frac{8 - \sqrt{304}}{8}$$

Similarly, $y = \frac{10-18\pm\sqrt{304}}{5} = \frac{2\pm\sqrt{304}}{5}$

$$y_1 = \frac{2 - \sqrt{304}}{5}; y_2 = \frac{2 + \sqrt{304}}{5}$$

Distance $= \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

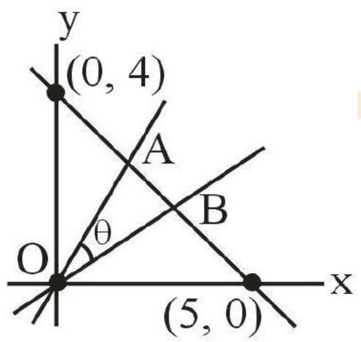
$$= \sqrt{\frac{4 \times 304}{64} + \frac{4 \times 304}{25}} = \frac{\sqrt{1691}}{5}$$

76. (d) Co-ordinates of A $= \left(\frac{5}{3}, \frac{8}{3}\right)$

Co-ordinates of B $= \left(\frac{10}{3}, \frac{4}{3}\right)$

Slope of OA $= m_1 = \frac{8}{5}$

Slope of OB $= m_2 = \frac{2}{5}$



$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

$$\tan\theta = \frac{\frac{6}{5}}{1 + \frac{16}{25}} = \frac{30}{41}$$

$\tan\theta = \frac{30}{41}$

77. (b)

$\vec{a}\cdot[(\vec{c}\times\vec{b}) - \vec{b} - \vec{c}]$

$\vec{a}\cdot(\vec{c}\times\vec{b}) - \vec{a}\cdot\vec{b} - \vec{a}\cdot\vec{c}$

given $\vec{a}\times\vec{c} = \vec{b}$

$\Rightarrow (\vec{a}\times\vec{c})\cdot\vec{b} = \vec{b}\cdot\vec{b} = |\vec{b}|^2 = 27$

$\Rightarrow \vec{a}\cdot(\vec{c}\times\vec{b}) = [\vec{a}\quad\vec{c}\quad\vec{b}] = (\vec{a}\times\vec{c})\cdot\vec{b} = 27$

Now $\vec{a}\cdot\vec{b} = 3 - 6 + 3 = 0$

$\vec{a}\cdot\vec{c} = 3$

By (i), (ii), (iii) & (iv)

$27 - 0 - 3 = 24$

78. (b) $a = \lim_{x\to 0}\frac{\sqrt{1+\sqrt{1+x^4}}-\sqrt{2}}{x^4}$

$$= \lim_{x\to 0}\frac{\sqrt{1+x^4}-1}{x^4\left(\sqrt{1+\sqrt{1+x^4}}+\sqrt{2}\right)}$$

$$= \lim_{x\to 0}\frac{x^4}{x^4\left(\sqrt{1+\sqrt{1+x^4}}+\sqrt{2}\right)\left(\sqrt{1+x^4}+1\right)}$$

Applying limit $a = \frac{1}{4\sqrt{2}}$

$$b = \lim_{x\to 0}\frac{\sin^2 x}{\sqrt{2}-\sqrt{1+\cos x}}$$

$$= \lim_{x\to 0}\frac{(1-\cos^2 x)(\sqrt{2}+\sqrt{1+\cos x})}{2-(1+\cos x)}$$

$b = \lim_{x\to 0}(1+\cos x)(\sqrt{2}+\sqrt{1+\cos x})$

Applying limits $b = 2(\sqrt{2}+\sqrt{2}) = 4\sqrt{2}$

Now, $ab^3 = \frac{1}{4\sqrt{2}}\times(4\sqrt{2})^3 = 32$

79. (d)

$$f(-x) = \begin{bmatrix}\cos x & \sin x & 0\\ -\sin x & \cos x & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$f(x)\cdot f(-x)=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}=I$$

Hence statement- I is correct

Now, checking statement II

$$f(y)=\begin{bmatrix}\cos y&-\sin y&0\\\sin y&\cos y&0\\0&0&1\end{bmatrix}$$

$$f(x)\cdot f(y)=\begin{bmatrix}\cos(x+y)&-\sin(x+y)&0\\\sin(x+y)&\cos(x+y)&0\\0&0&1\end{bmatrix}$$

$\Rightarrow f(x)\cdot f(y)=f(x+y)$

Hence statement-II is also correct.

80. (d) $f: N-\{1\}\to N$

$f(n)=$ The highest prime factor of n.

$f(b)=2$

$f(d)=2$

$\Rightarrow$ many one

4 is not image of any element

$\Rightarrow$ into

Hence many one and into

Neither one-one nor onto.

81. (5)

$$\cos\theta=\frac{(\alpha\hat{i}-2\hat{j}+2\hat{k})\cdot(\alpha\hat{i}+2\alpha\hat{j}-2\hat{k})}{\sqrt{\alpha^2+4+4}\sqrt{\alpha^2+4\alpha^2+4}}$$

$$\cos\theta=\frac{\alpha^2-4\alpha-4}{\sqrt{\alpha^2+8}\sqrt{5\alpha^2+4}}$$

$\Rightarrow \alpha^2-4\alpha-4>0$

$\Rightarrow \alpha^2-4\alpha+4>8 \Rightarrow (\alpha-2)^2>8$

$\Rightarrow \alpha-2>2\sqrt{2}$ or $\alpha-2<-2\sqrt{2}$

$\alpha>2+2\sqrt{2}$ or $\alpha<2-2\sqrt{2}$

$\alpha\in(-\infty,-0.82)\cup(4.82,\infty)$

Least positive integral value of $\alpha\Rightarrow 5$

82. (2890)

$f(x) - f(y) \geq \ln x - \ln y + x - y$

$\frac{f(x) - f(y)}{x - y} \geq \frac{\ln x - \ln y}{x - y} + 1$

Let $x > y$

$\lim_{y \to x} f'(x^-) \geq \frac{1}{x} + 1$

Let $x < y$

$\lim_{y \to x} f'(x^+) \leq \frac{1}{x} + 1$

$f^1(x^-) = f^1(x^+)$

$f^1(x) = \frac{1}{x} + 1$

$f'\left(\frac{1}{x^2}\right) = x^2 + 1$

$\sum_{x=1}^{20}(x^2 + 1) = \sum_{x-1}^{20} x^2 + 20$

$= \frac{20 \times 21 \times 41}{6} + 20$

$= 2890$

83. (29)

$2x + 3y - 2 = t$

$4x + 6y - 4 = 2t$

$2 + 3\frac{dy}{dx} = \frac{dt}{dx}$

$4x + 6y - 7 = 2t - 3$

$\frac{dy}{dx} = \frac{-(2x + 3y - 2)}{4x + 6y - 7}$

$\frac{dt}{dx} = \frac{-3t + 4t - 6}{2t - 3} = \frac{t - 6}{2t - 3}$

$\int \frac{2t - 3}{t - 6} dt = \int dx$

$\int \left(\frac{2t - 12}{t - 6} + \frac{9}{t - 6}\right) \cdot dt = x$

$2t + 9\ln(t - 6) = x + c$

$2(2x + 3y - 2) + 9\ln(2x + 3y - 8) = x + c$

$x = 0, y = 3$

$c = 14$

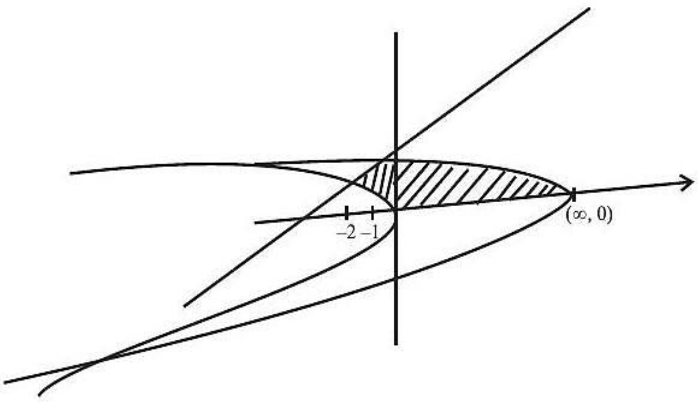
$4x + 6y - 4 + 9\ln(2x + 3y - 8) = x + 14$

$x + 2y + 3\ln(2x + 3y - 8) = 6$

$\alpha = 1, \beta = 2, \gamma = 8$

$\alpha + 2\beta + 3\gamma = 1 + 4 + 24 = 29$

84. (119)



$A = \int_0^1 [(8 - 4y^2) - (-2y^2)]dy +$

$\int_1^{3/2} [(8 - 4y^2) - (2y - 4)]dy$

$$= \left[8y - \frac{2y^3}{3}\right]_0^1 + \left[12y - y^2 - \frac{4y^3}{3}\right]_1^{3/2} = \frac{107}{12} = \frac{m}{n}$$

$\therefore m + n = 119$

85. (9)

$$8 = \frac{3}{1 - \frac{1}{4}} + \frac{p \cdot \frac{1}{4}}{\left(1 - \frac{1}{4}\right)^2}$$

(sum of infinite terms of A.G.P $= \frac{a}{1-r} + \frac{dr}{(1-r)^2}$)

$$\Rightarrow \frac{4p}{9} = 4 \Rightarrow p = 9$$

86. (12)

$a = P(X = 3) = \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6} = \frac{25}{216}$ $b = P(X \geq 3) = \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6} + \left(\frac{5}{6}\right)^3 \cdot \frac{1}{6} + \left(\frac{5}{6}\right)^4 \cdot \frac{1}{6} + \cdots \ldots$

$$= \frac{\frac{25}{216}}{1-\frac{5}{6}} = \frac{25}{216} \times \frac{6}{1} = \frac{25}{36}$$

$$P(X \geq 6) = \left(\frac{5}{6}\right)^5 \cdot \frac{1}{6} + \left(\frac{5}{6}\right)^6 \cdot \frac{1}{6} + \cdots \ldots$$

$$= \frac{\left(\frac{5}{6}\right)^5 \cdot \frac{1}{6}}{1-\frac{5}{6}} = \left(\frac{5}{6}\right)^5$$

$$c = \frac{\left(\frac{5}{6}\right)^5}{\left(\frac{5}{6}\right)^3} = \frac{25}{36}$$

$$\frac{b+c}{a} = \frac{\left(\frac{5}{6}\right)^2 + \left(\frac{5}{6}\right)^2}{\left(\frac{5}{6}\right)^2 \cdot \frac{1}{6}} = 12$$

87. (48)

$$\cos 2x + a \cdot \sin x = 2a - 7$$

$$a(\sin x - 2) = 2(\sin x - 2)(\sin x + 2)$$

$$\sin x = 2,\ a = 2(\sin x + 2)$$

$$\Rightarrow a \in [2,6]$$

$$p = 2\ q = 6$$

$$r = \tan 9^\circ + \cot 9^\circ - \tan 27 - \cot 27$$

$$r = \frac{1}{\sin 9 \cdot \cos 9} - \frac{1}{\sin 27 \cdot \cos 27}$$

$$= 2\left[\frac{4}{\sqrt{5}-1} - \frac{4}{\sqrt{5}+1}\right]$$

$$r = 4$$

$$p \cdot q \cdot r = 2 \times 6 \times 4 = 48$$

88. (202)

$$f(x) = x^3 + x^2 \cdot f'(a) + x \cdot f''(b) + f'''(c)$$

$f'(x) = 3x^2 + 2xf'(a) + f''(b)$
$f''(x) = 6x + 2f'(a)$
$f'''(x) = 6$
$f'(a) = -5, f''(b) = 2, f'''(c) = 6$
$f(x) = x^3 + x^2 \cdot (-5) + x \cdot (b) + 6$
$f'(x) = 3x^2 - 10x + 2$
$f'(10) = 300 - 100 + 2 = 202$

89. (28)

$$A = \begin{bmatrix} 2 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} B = [B_1,\ B_2,\ B_3]$$

$$B_1 = \begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix},\ B_2 = \begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix},\ B_3 = \begin{bmatrix} x_3 \\ y_3 \\ z_3 \end{bmatrix}$$

$$AB_1 = \begin{bmatrix} 2 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

90. (5)

$x^2 + x + 1 = 0 \Rightarrow x = \omega, \omega^2 = \alpha$

Let $\alpha = \omega$
Now $(1+\alpha)^7 = -\omega^{14} = -\omega^2 = 1 + \omega$
$A = 1, B = 1, C = 0$

$\therefore 5(3\text{ A} - 2\text{ B} - \text{C}) = 5(3 - 2 - 0) = 5$