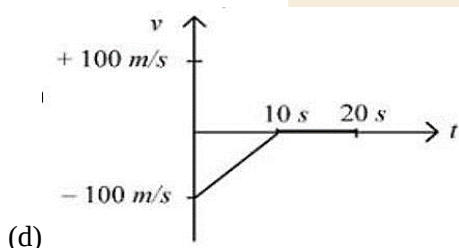
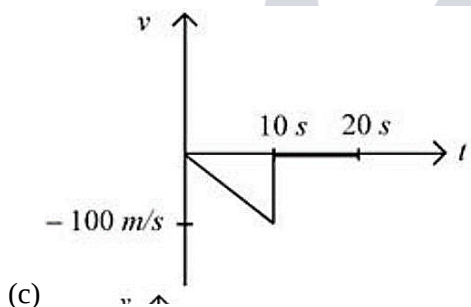
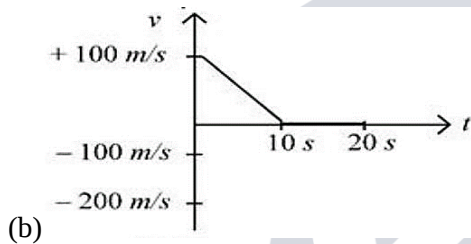
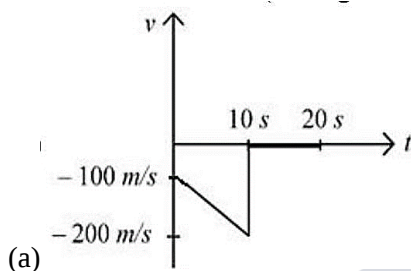


PART-I PHYSICS

SECTION-A

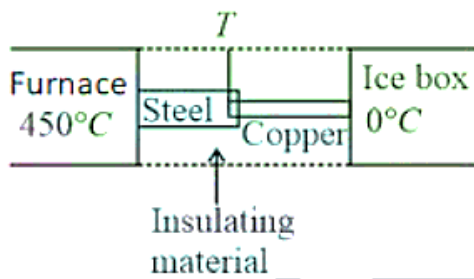
- A torque meter is calibrated to reference standards of mass, length and time each with 5% accuracy. After calibration, the measured torque with this torque meter will have net accuracy of:
(a) 15% (b) 25%
(c) 75% (d) 5%
- A bullet is shot vertically downwards with an initial velocity of 100 m/s from a certain height. Within 10 s, the bullet reaches the ground and instantaneously comes to rest due to the perfectly inelastic collision. The velocity-time curve for total time $t = 20$ s will be : (Take $g = 10 \text{ m/s}^2$)



- Sand is being dropped from a stationary dropper at a rate of 0.5 kgs^{-1} on a conveyor belt moving with a velocity of 5 ms^{-1} . The power needed to keep belt moving with the same velocity will be:
(a) 1.25 W (b) 2.5 W
(c) 6.25 W (d) 12.5 W
- A bag is gently dropped on a conveyor belt moving at a speed of 2 m/s. The coefficient of friction between the conveyor belt and bag is 0.4. Initially, the bag slips on the belt before it stops due to friction. The distance travelled by the bag on the belt during slipping motion is: [Take $g = 10 \text{ m/s}^2$]

- (a) 2 m (b) 0.5 m
(c) 3.2 m (d) 0.8 ms
5. Two cylindrical vessels of equal cross-sectional area 16 cm^2 contain water upto heights 100 cm and 150 cm respectively. The vessels are interconnected so that the water levels in them become equal. The work done by the force of gravity during the process, is [Take density of water = 10^3 kg/m^3 and $g = 10 \text{ ms}^{-2}$]
(a) 0.25 J (b) 1 J
(c) 8 J (d) 12 J
6. Two satellites A and B having masses in the ratio 4: 3 are revolving in circular orbits of radii $3r$ and $4r$ respectively around the earth. The ratio of total mechanical energy of A to B is:
(a) 9: 16 (b) 16: 9
(c) 1: 1 (d) 4: 3
7. If K_1 and K_2 are the thermal conductivities L_1 and L_2 are the lengths and A_1 and A_2 are the cross sectional areas of steel and copper rods respectively such that $\frac{K_2}{K_1} = 9, \frac{A_1}{A_2} = 2, \frac{L_1}{L_2} = 2$.

Then, for the arrangement as shown in the figure. The value of temperature T of the steel - copper junction in the steady state will be:



- (a) 18°C (b) 14°C
(c) 45°C (d) 150°C

8. Read the following statements:

- A. When small temperature difference between a liquid and its surrounding is doubled the rate of loss of heat of the liquid becomes twice.
- B. Two bodies P and Q having equal surface areas are maintained at temperature 10°C and 20°C . The thermal radiation emitted in a given time by P and Q are in the ratio 1: 1.15
- C. A carnot Engine working between 100 K and 400 K has an efficiency of 75%
- D. When small temperature difference between a liquid and its surrounding is quadrupled, the rate of loss of heat of the liquid becomes twice.

Choose the correct answer from the options given below:

- (a) A, B, C only (b) A, B only
(c) A, C only (d) B, C, D only

9. Same gas is filled in two vessels of the same volume at the same temperature. If the ratio of the number of molecules is 1: 4, then
- A. The r.m.s. velocity of gas molecules in two vessels will be the same.
- B. The ratio of pressure in these vessels will be 1: 4
- C. The ratio of pressure will be 1: 1

D. The r.m.s. velocity of gas molecules in two vessels will be in the ratio of 1: 4

- (a) A and C only (b) B and D only
(c) A and B only (d) C and D only

10. Two identical positive charges Q each are fixed at a distance of ' $2a$ ' apart from each other. Another point charge q_0 with mass ' m ' is placed at midpoint between two fixed charges. For a small displacement along the line joining the fixed charges, the charge q_0 executes SHM. The time period of oscillation of charge q_0 will be:

- (a) $\sqrt{\frac{4\pi^3\epsilon_0 ma^3}{q_0 Q}}$ (b) $\sqrt{\frac{q_0 Q}{4\pi^3\epsilon_0 ma^3}}$
(c) $\sqrt{\frac{2\pi^2\epsilon_0 ma^3}{q_0 Q}}$ (d) $\sqrt{\frac{8\pi^3\epsilon_0 ma^3}{q_0 Q}}$

11. Two sources of equal emfs are connected in series. This combination is connected to an external resistance R . The internal resistances of the two sources are r_1 and r_2 ($r_1 > r_2$). If the potential difference across the source of internal resistance r_1 is zero then the value of R will be

- (a) $r_1 - r_2$ (b) $\frac{r_1 r_2}{r_1 + r_2}$
(c) $\frac{r_1 + r_2}{2}$ (d) $r_2 - r_1$

12. Two bar magnets oscillate in a horizontal plane in earth's magnetic field with time periods of 3 s and 4 s respectively. If their moments of inertia are in the ratio of 3: 2 then the ratio of their magnetic moments will be:

- (a) 2: 1 (b) 8: 3
(c) 1: 3 (d) 27: 16

13. A magnet hung at 45° with magnetic meridian makes an angle of 60° with the horizontal. The actual value of the angle of dip is

- (a) $\tan^{-1}\left(\sqrt{\frac{3}{2}}\right)$ (b) $\tan^{-1}(\sqrt{6})$
(c) $\tan^{-1}\left(\sqrt{\frac{2}{3}}\right)$ (d) $\tan^{-1}\left(\sqrt{\frac{1}{2}}\right)$

14. A direct current of 4 A and an alternating current of peak value 4 A flow through resistance of 3Ω and 2Ω respectively. The ratio of heat produced in the two resistances in same interval of time will be:

- (a) 3: 2 (b) 3: 1
(c) 3: 4 (d) 4: 3

15. A beam of light travelling along X-axis is described by the electric field $E_y = 900\sin \omega(t - x/c)$. The ratio of electric force to magnetic force on a charge q moving along Y-axis with a speed of $3 \times 10^7 \text{ ms}^{-1}$ will be: [Given speed of light = $3 \times 10^8 \text{ ms}^{-1}$]

- (a) 1: 1 (b) 1: 10
(c) 10: 1 (d) 1: 2

16. A microscope was initially placed in air (refractive index 1). It is then immersed in oil (refractive index 2). For a light whose wavelength in air is λ , calculate the change of microscope's resolving power due to oil and choose the correct option

- (a) Resolving power will be $\frac{1}{4}$ in the oil than it was in the air
(b) Resolving power will be twice in the oil than it was in the air.
(c) Resolving power will be four times in the oil than it was in the air.
(d) Resolving power will be $\frac{1}{2}$ in the oil than it was in the air.

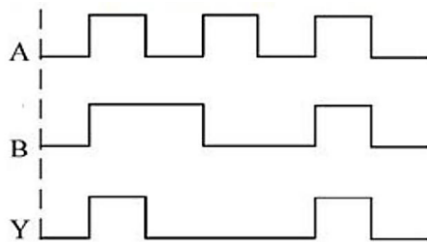
17. An electron (mass m) with an initial velocity $\vec{v} = v_0 \hat{i}$ ($v_0 > 0$) is moving in an electric field $\vec{E} = -E_0 \hat{i}$ ($E_0 > 0$) where E_0 is constant. If at $t = 0$ de Broglie wavelength is $\lambda_0 = \frac{h}{mv_0}$, then its de Broglie wavelength after time t is given by

- (a) λ_0 (b) $\lambda_0 \left(1 + \frac{eE_0 t}{mv_0}\right)$
(c) $\lambda_0 t$ (d) $\frac{\lambda_0}{\left(1 + \frac{eE_0 t}{mv_0}\right)}$

18. What is the half-life period of a radioactive material if its activity drops to $1/16^{\text{th}}$ of its initial value of 30 years?

- (a) 9.5 years (b) 8.5 years
(c) 7.5 years (d) 10.5 years

19. A logic gate circuit has two inputs A and B and output Y. The voltage waveforms of A, B and Y are shown below



The logic gate circuit is

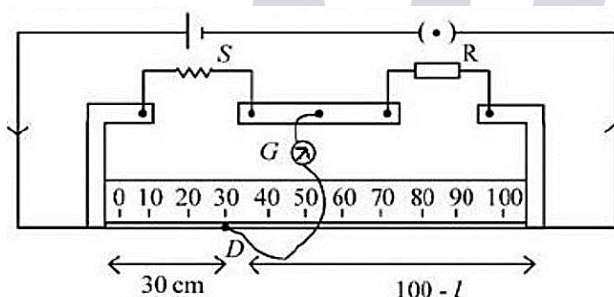
- (a) AND gate (b) OR gate
(c) NOR gate (d) NAND gate

20. At a particular station, the TV transmission tower has a height of 100 m. To triple its coverage range, height of the tower should be increased to

- (a) 200 m (b) 300 m
(c) 600 m (d) 900 m

SECTION-B

21. In meter bridge experiment for measuring unknown resistance 'S', the null point is obtained at a distance 30 cm from the left side as shown at point D. If R is $5.6\text{ k}\Omega$, then the value of unknown resistance 'S' will be Ω .



22. The one division of main scale of vernier callipers reads 1 mm and 10 divisions of Vernier scale is equal to the 9 divisions on main scale. When the two jaws of the instrument touch each other the zero of the Vernier scale lies to the right of zero of the main scale and its fourth division coincides with a main scale division. When a spherical bob is tightly placed between the two jaws, the zero of the Vernier scale lies in between 4.1 cm and 4.2 cm and 6th Vernier division coincides with a main scale division. The diameter of the bob will be 10^{-2} cm

23. Two beams of light having intensities I and 4I interfere to produce a fringe pattern on a screen. The phase difference between the two beams are $\pi/2$ and $\pi/3$ at points A and B respectively. The difference between the resultant intensities at the two points is xI. The value of x will be

24. To light, a 50 W, 100 V lamp is connected, in series with a capacitor of capacitance $\frac{50}{\pi\sqrt{x}} \mu\text{F}$, with 200 V, 50 Hz AC source. The value of x will be
25. A 1 m long copper wire carries a current of 1 A. If the cross section of the wire is 2.0 mm^2 and the resistivity of copper is $1.7 \times 10^{-8} \Omega\text{m}$. the force experienced by moving electron in the wire is $\times 10^{-23} \text{ N}$. (charge on electron = $1.6 \times 10^{-19} \text{ C}$)
26. A long cylindrical volume contains a uniformly distributed charge of density $\rho \text{ Cm}^{-3}$. The electric field inside the cylindrical volume at a distance $x = \frac{2\epsilon_0}{\rho} \text{ m}$ from its axis is Vm^{-1}
27. A mass 0.9 kg, attached to a horizontal spring, executes SHM with an amplitude A_1 . When this mass passes through its mean position, then a smaller mass of 124 g is placed over it and both masses move together with amplitude A_2 . If the ratio $\frac{A_1}{A_2}$ is $\frac{\alpha}{\alpha-1}$, then the value of α will be
28. A square aluminium (shear modulus is $25 \times 10^9 \text{ Nm}^{-2}$) slab of side 60 cm and thickness 15 cm is subjected to a shearing force (on its narrow face) of $18.0 \times 10^4 \text{ N}$. The lower edge is riveted to the floor. The displacement of the upper edge is μm .
29. A pulley of radius 1.5 m is rotated about its axis by a force $F = (12t - 3t^2) \text{ N}$ applied tangentially (while t is measured in seconds). If moment of inertia of the pulley about its axis of rotation is 4.5 kg m^2 , the number of rotations made by the pulley before its direction of motion is reversed, will be $\frac{K}{\pi}$. The value of K is
30. A ball of mass m is thrown vertically upward. Another ball of mass 2 m is thrown an angle θ with the vertical. Both the balls stay in air for the same period of time. The ratio of the heights attained by the two balls respectively is $\frac{1}{x}$. The value of x is

PART-II CHEMISTRY

SECTION-A

31. 250 g solution of D-glucose in water contains 10.8% of carbon by weight. The molality of the solution is nearest to
(Given: Atomic Weights are H, 1u; C, 12u; O, 16u)
(a) 1.03 (b) 2.06
(c) 3.09 (d) 5.40

32. Given below are two statements.

Statement I: O_2 , Cu^{2+} and Fe^{3+} are weakly attracted by magnetic field and are magnetized in the same direction as magnetic field.

Statement II: NaCl and H_2O are weakly magnetized in opposite direction to magnetic field.

In the light of the above statements, choose the most appropriate answer form the options given below:

- (a) Both Statement I and Statement II are correct.
(b) Both Statement I and Statement II are incorrect.

(c) Statement I is correct but Statement II is incorrect.

(d) Statement I is incorrect but Statement II is correct

33. Given below are two statements. One is labelled as Assertion A and the other is labelled as Reason R.

Assertion A: Energy of 2 s orbital of hydrogen atom is greater than that of 2 s orbital of lithium.

Reason R: Energies of the orbitals in the same subshell decrease with increase in the atomic number.

In the light of the above statements, choose the correct answer from the options given below.

(a) Both A and R are true and R is the correct explanation of A.

(b) Both A and R are true but R is NOT the correct explanation of A.

(c) A is true but R is false.

(d) A is false but R is true.

34. Given below are two statements. One is labelled as Assertion A and the other is labelled as Reason R.

Assertion A: Activated charcoal adsorbs SO_2 more efficiently than CH_4 .

Reason R: Gases with lower critical temperatures are readily adsorbed by activated charcoal.

In the light of the above statements, choose the correct answer from the options given below.

(a) Both A and R are correct and R is the correct explanation of A.

(b) Both A and R are correct but R is NOT the correct explanation of A.

(c) A is correct but R is not correct.

(d) A is not correct but R is correct.

35. Boiling point of a 2% aqueous solution of a nonvolatile solute A is equal to the boiling point of 8% aqueous solution of a non-volatile solute B. The relation between molecular weights of A and B is.

(a) $M_A = 4M_B$

(b) $M_B = 4M_A$

(c) $M_A = 8M_B$

(d) $M_B = 8M_A$

36. The incorrect statement is

(a) The first ionization enthalpy of K is less than that of Na and Li

(b) Xe does not have the lowest first ionization enthalpy in its group

(c) The first ionization enthalpy of element with atomic number 37 is lower than that of the element with atomic number 38.

(d) The first ionization enthalpy of Ga is higher than that of the d-block element with atomic number 30.

37. Which of the following methods are not used to refine any metal?

(a) Liquation (b) Calcination

(c) Electrolysis (d) Leaching

(E) Distillation

Choose the correct answer from the options given below:

(a) B and D only (b) A, B, D and E only

(c) B, D and E only (d) A, C and E only

38. Given below are two statements:

Statement I: Hydrogen peroxide can act as an oxidizing agent in both acidic and basic conditions.

Statement II: Density of hydrogen peroxide at 298 K is lower than that of D_2O .

In the light of the above statements. Choose the correct answer from the options.

- (a) Both statement I and Statement II are true
- (b) Both statement I and Statement II are false
- (c) Statement I is true but Statement II is false
- (d) Statement I is false but Statement II is true

39. Given below are two statements:

Statement I: The chlorides of Be and Al have Cl-bridged structure. Both are soluble in organic solvents and act as Lewis bases.

Statement II: Hydroxides of Be and Al dissolve in excess alkali to give beryllate and aluminate ions. In the light of the above statements. Choose the correct answer from the options given below.

- (a) Both statement I and Statement II are true
- (b) Both statement I and Statement II are false
- (c) Statement I is true but Statement II is false
- (d) Statement I is false but Statement II is true

40. Which oxoacid of phosphorous has the highest number of oxygen atoms present in its chemical formula?

- (a) Pyro phosphorous acid
- (b) Hypo phosphoric acid
- (c) Phosphoric acid
- (d) Pyrophosphoric acid

41. Given below are two statements:

Statement I: Iron (III) catalyst, acidified $K_2Cr_2O_7$ and neutral $KMnO_4$ have the ability to oxidise I^- to I_2 independently.

Statement II: Manganate ion is paramagnetic in nature and involves $p\pi - p\pi$ bonding.

In the light of the above statements, choose the correct answer from the options.

- (a) Both statement I and Statement II are true
- (b) Both statement I and Statement II are false
- (c) Statement I is true but Statement II is false
- (d) Statement I is false but Statement II is true

42. The total number of Mn = O bonds in Mn_2O_7 is

- (a) 4
- (b) 5
- (c) 6
- (d) 3

43. Match List I with List II

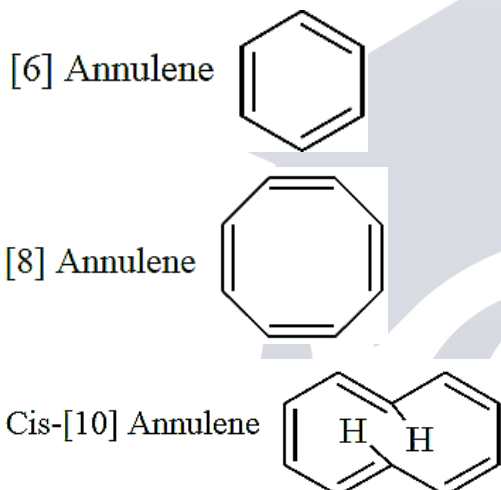
List I Pollutant	List II Disease /sickness
A. Sulphate (>500 ppm)	I. Methemoglobinemia
B. Nitrate (>50 ppm)	II. Brown mottling of teeth
C. Lead (> 50 ppb)	III. Laxative effect
D. Fluoride (>2 ppm)	IV. Kidney damage

Choose the correct answer from the options given below:

- (a) A-IV, B -I, C-II, D-III (b) A-III, B -I, C-IV, D-II
(c) A-II, B -IV, C-I, D-III (d) A-II, B -IV, C-III, D-I

44. Given below are two statements. One is labelled as Assertion A and the other is labelled as Reason R.

Assertion A: [6] Annulene. [8] Annulene and cis - [10] Annulene, are respectively aromatic, not-aromatic and aromatic.

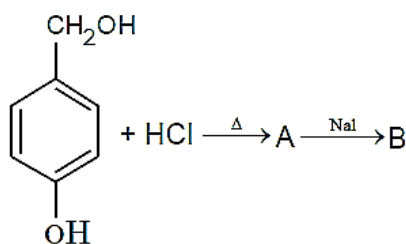


Reason R: Planarity is one of the requirements of aromatic systems.

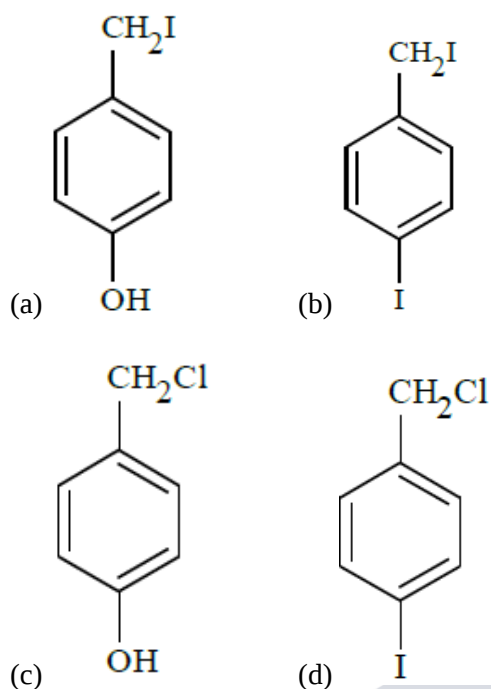
In the light of the above statements, choose the most appropriate answer from the options given below.

- (a) Both A and R are correct and R is the correct explanation of A.
(b) Both A and R are correct but R is NOT the correct explanation of A.
(c) A is correct but R is not correct.
(d) A is not correct but R is correct.

45.



In the above reaction product B is:



46. Match List I with List II

List I Polymers	List II Commercial names
A. Phenol-formaldehyde resin	I. Glyptal
B. Copolymer of 1,3-butadiene and styrene	II. Novolac
C. Polyester of glycol and phthalic acid	III. Buna-S
D. Polyester of glycol and terephthalic acid	IV. Dacron

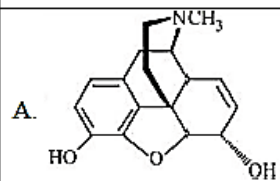
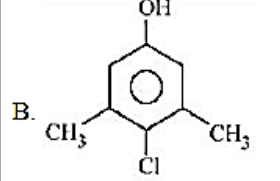
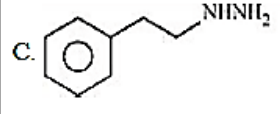
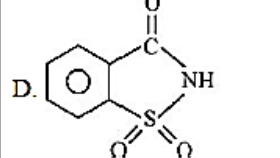
Choose the correct answer from the options given below:

- (a) A-II, B -III, C-IV, D-I (b) A-II, B -III, C-I, D-IV
(c) A-II, B -I, C-III, D-IV (d) A-III, B -II, C-IV, D-I

47. A sugar 'X' dehydrates very slowly under acidic condition to give furfural which on further reaction with resorcinol gives the coloured product after sometime. Sugar 'X' is

- (a) Aldopentose (b) Aldotetrose
(c) Oxalic acid (d) Ketotetrose

48. Match List I with List II

List I	List II
A. 	I. Anti-depressant
B. 	II. 550 times sweeter than cane sugar
C. 	III. Narcotic analgesic
D. 	IV. Antiseptic

Choose the correct answer from the options given below:

- (a) A-IV, B -III, C-II, D-I (b) A-III, B -I, C-II, D-IV
(c) A-III, B -IV, C-I, D-II (d) A-III, B -I, C-IV, D-II

49. In Carius method of estimation of halogen. 0.45 g of an organic compound gave 0.36 g of AgBr. Find out the percentage of bromine in the compound.

(Molar masses: $\text{AgBr} = 188 \text{ g mol}^{-1}$; $\text{Br} = 80 \text{ g mol}^{-1}$)

- (a) 34.04% (b) 40.04%
(c) 36.03% (d) 38.04%

50. Match List I with List II

List I	List II
A. Benzene sulphonyl chloride	I. Test for primary amines
B. Hoffmann bromamide reaction	II. Anti Saytzeff
C. Carbylamine reaction	III. Hinsberg reagent

D. Hoffmann orientation	IV. Known reaction of Isocyanates.
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Choose the correct answer from the options given below:

- (a) A-IV, B -III, C-II, D-I (b) A-IV, B -II, C-I, D-III
(c) A-III, B -IV, C-I, D-II (d) A-IV, B -III, C-I, D-II

SECTION-B

51. 20 mL of 0.02M $K_2Cr_2O_7$ solution is used for the titration of 10 mL of Fe^{2+} solution in the acidic medium. The molarity of Fe^{2+} solution is $\times 10^{-2}M$ (Nearest Integer)

52. $2NO + 2H_2 \rightarrow N_2 + 2H_2O$

The above reaction has been studied at 800°C. The related data are given in the table below

Reaction serial number	Initial pressure of H_2 / kPa	Initial Pressure of NO / kPa	Initial rate $\left(\frac{-dp}{dt}\right) / (kPa / s)$
1	65.6	40.0	0.135
2	65.6	20.1	0.033
3	38.6	65.6	0.214
4	19.2	65.6	0.106

The order of the reaction with respect to NO is

53. Amongst the following the number of oxide(s) which are paramagnetic in nature is

$Na_2O, KO_2, NO_2, N_2O, ClO_2, NO, SO_2, Cl_2O$

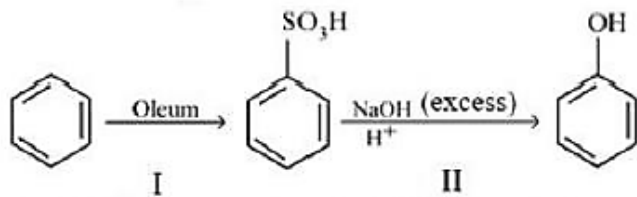
54. The molar heat capacity for an ideal gas at constant pressure is $20.785 J K^{-1} mol^{-1}$. The change in internal energy is 5000 J upon heating it from 300 K to 500 K. The number of moles of the gas at constant volume is [Nearest integer] (Given: $R = 8.314 J K^{-1} mol^{-1}$)

55. According to MO theory, number of species/ions from the following having identical bond order is $CN^-, NO^+, O_2, O_2^+, O_2^{2+}$

56. At 310 K, the solubility of CaF_2 in water is

$2.34 \times 10^{-3} g/100 mL$. The solubility product of CaF_2 is $\times 10^{-8} (mol/L)^3$. (Given molar mass: $CaF_2 = 78 g mol^{-1}$)

57. The conductivity of a solution of complex with formula $\text{CoCl}_3(\text{NH}_3)_4$ corresponds to 1: 1 electrolyte, then the primary valency of central metal ion is
58. In the titration of KMnO_4 and oxalic acid in acidic medium, the change in oxidation number of carbon at the end point is
59. Optical activity of an enantiomeric mixture is $+12.6^\circ$ and the specific rotation of (+) isomer is $+30^\circ$. The optical purity is_ %
60. In the following reaction



The % yield for reaction I is 60% and that of reaction II is 50%. The overall yield of the complete reaction is % [nearest integer]

PART-III MATHEMATICS

SECTION-B

61. Let R_1 and R_2 be two relations defined on $\mathbb{R}$ by $aR_1b \Leftrightarrow ab \geq 0$ and $aR_2b \Leftrightarrow a \geq b$, then
- (a) R_1 is an equivalence relation but not R_2
 - (b) R_2 is an equivalence relation but not R_1
 - (c) both R_1 and R_2 are equivalence relations
 - (d) neither R_1 nor R_2 is an equivalence relation
62. Let $f, g: \mathbb{N} - \{1\} \rightarrow \mathbb{N}$ be functions defined by $f(a) = \alpha$, where α is the maximum of the powers of those primes p such that p^α divides a , and $g(a) = a + 1$, for all $a \in \mathbb{N} - \{1\}$. Then, the function $f + g$ is
- (a) one-one but not onto
 - (b) onto but not one-one
 - (c) both one-one and onto
 - (d) neither one-one nor onto
63. Let the minimum value v_0 of $v = |z|^2 + |z - 3|^2 + |z - 6i|^2$, $z \in \mathbb{C}$ is attained at $z = z_0$. Then $|2zz_0^2 - \bar{z}_0^3 + 3|^2 + v_0^2$ is equal to
- (a) 1000
 - (b) 1024
 - (c) 1105
 - (d) 1196
64. Let $A = \begin{pmatrix} 1 & 2 \\ -2 & -5 \end{pmatrix}$. Let $\alpha, \beta \in \mathbb{R}$ be such that $\alpha A^2 + \beta A = 2I$. Then $\alpha + \beta$ is equal to -
- (a) -10
 - (b) -6
 - (c) 6
 - (d) 10
65. The remainder when $(2021)^{2022} + (2022)^{2021}$ is divided by 7 is
- (a) 0
 - (b) 1
 - (c) 2
 - (d) 6

66. Suppose $a_1, a_2, \dots, a_n, \dots$ be an arithmetic progression of natural numbers. If the ratio of the sum of the first five terms of the sum of first nine terms of the progression is 5: 17 and $110 < a_{15} < 120$, then the sum of the first ten terms of the progression is equal to -
 (a) 290 (b) 380
 (c) 460 (d) 510
67. Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be a function defined as
 $f(x) = a \sin\left(\frac{\pi[x]}{2}\right) + [2 - x]$, $a \in \mathbb{R}$, where $[t]$ is the greatest integer less than or equal to t . If $\lim_{x \rightarrow -1} f(x)$ exists, then the value of $\int_0^4 f(x) dx$ is equal to:
 (a) -1 (b) -2
 (c) 1 (d) 2
68. $I = \int_{\pi/4}^{\pi/3} \left(\frac{8 \sin x - \sin 2x}{x} \right) dx$. Then
 (a) $\frac{\pi}{2} < I < \frac{3\pi}{4}$ (b) $\frac{\pi}{5} < I < \frac{5\pi}{12}$
 (c) $\frac{5\pi}{12} < I < \frac{\sqrt{2}}{3} \pi$ (d) $\frac{3\pi}{4} < I < \pi$
69. The area of the smaller region enclosed by the curves $y^2 = 8x + 4$ and $x^2 + y^2 + 4\sqrt{3}x - 4 = 0$ is equal to
 (a) $\frac{1}{3}(2 - 12\sqrt{3} + 8\pi)$ (b) $\frac{1}{3}(2 - 12\sqrt{3} + 6\pi)$
 (c) $\frac{1}{3}(4 - 12\sqrt{3} + 8\pi)$ (d) $\frac{1}{3}(4 - 12\sqrt{3} + 6\pi)$
70. Let $y = y_1(x)$ and $y = y_2(x)$ be two distinct solutions of the differential equation $\frac{dy}{dx} = x + y$, with $y_1(0) = 0$ and $y_2(0) = 1$ respectively. Then, the number of points of intersection of $y = y_1(x)$ and $y = y_2(x)$ is
 (a) 0 (b) 1
 (c) 2 (d) 3
71. Let $P(a, b)$ be a point on the parabola $y^2 = 8x$ such that the tangent at P passes through the centre of the circle $x^2 + y^2 - 10x - 14y + 65 = 0$. Let A be the product of all possible values of a and B be the product of all possible values of b . Then the value of $A + B$ is equal to:
 (a) 0 (b) 25
 (c) 40 (d) 65
72. Let $\vec{a} = \alpha\hat{i} + \hat{j} + \beta\hat{k}$ and $\vec{b} = 3\hat{i} - 5\hat{j} + 4\hat{k}$ be two vectors, such that $\vec{a} \times \vec{b} = -\hat{i} + 9\hat{j} + 12\hat{k}$. Then the projection of $\vec{b} - 2\vec{a}$ on $\vec{b} + \vec{a}$ is equal to
 (a) 2 (b) $\frac{39}{5}$
 (c) 9 (d) $\frac{46}{5}$
73. Let $\vec{a} = 2\hat{i} - \hat{j} + 5\hat{k}$ and $\vec{b} = \alpha\hat{i} + \beta\hat{j} + 2\hat{k}$. If $((\vec{a} \times \vec{b}) \times \hat{i}) \cdot \hat{k} = \frac{23}{2}$, then $|\vec{b} \times 2\hat{j}|$ is equal to
 (a) 4 (b) 5
 (c) $\sqrt{21}$ (d) $\sqrt{17}$
74. Let S be the sample space of all five- digit numbers. If p is the probability that a randomly selected number from S , is a multiple of 7 but not divisible by 5, then $9p$ is equal to
 (a) 1.0146 (b) 1.2085
 (c) 1.0285 (d) 1.1521
75. Let a vertical tower AB of height $2h$ stands on a horizontal ground. Let from a point P on the ground a man can see upto height h of the tower with an angle of elevation 2α . When from P , he moves a distance d in the direction of $\overrightarrow{AP}$, he can see the top B of the tower with an angle of elevation α . If $d = \sqrt{7}h$, then $\tan \alpha$ is equal to
 (a) $\sqrt{5} - 2$ (b) $\sqrt{3} - 1$
 (c) $\sqrt{7} - 2$ (d) $\sqrt{7} - \sqrt{3}$

76. $(p \wedge r) \Leftrightarrow (p \wedge (\sim q))$ is equivalent to $(\sim p)$ when r is
 (a) p (b) $\sim p$
 (c) q (d) $\sim q$
77. If the plane P passes through the intersection of two mutually perpendicular planes $2x + ky - 5z = 1$ and $3kx - ky + z = 5$, $k < 3$ and intercepts a unit length on positive x -axis, then the intercept made by the plane P on the y -axis is
 (a) $\frac{1}{11}$ (b) $\frac{5}{11}$
 (c) 6 (d) 7
78. Let $A(1, 1)$, $B(-4, 3)$, $C(-2, -5)$ be vertices of a triangle ABC , P be a point on side BC , and Δ_1 and Δ_2 be the areas of triangle APB and ABC . Respectively.
 If $\Delta_1 : \Delta_2 = 4 : 7$, then the area enclosed by the lines AP , AC and the x -axis is
 (a) $\frac{1}{4}$ (b) $\frac{3}{4}$
 (c) $\frac{1}{2}$ (d) 1
79. If the circle $x^2 + y^2 - 2gx + 6y - 19c = 0$, $g, c \in \mathbb{R}$ passes through the point $(6, 1)$ and its centre lies on the line $x - 2cy = 8$, then the length of intercept made by the circle on x -axis is
 (a) $\sqrt{11}$ (b) 4
 (c) 3 (d) $2\sqrt{23}$
80. Let a function $f: \mathbb{R} \rightarrow \mathbb{R}$ be defined as:

$$f(x) = \begin{cases} \int_0^x (5 - |t - 3|) dt, & x > 4 \\ x^2 + bx, & x \leq 4 \end{cases}$$

where $b \in \mathbb{R}$. If f is continuous at $x = 4$, then which of the following statements is NOT true?

- (a) f is not differentiable at $x = 4$
 (b) $f'(3) + f'(5) = \frac{35}{4}$
 (c) f is increasing in $(-\infty, \frac{1}{8}) \cup (8, \infty)$
 (d) f has a local minima at $x = \frac{1}{8}$

SECTION-B

81. For $k \in \mathbb{R}$, let the solutions of the equation

$$\cos \left(\sin^{-1} \left(x \cot \left(\tan^{-1} \left(\cos(\sin^{-1} x) \right) \right) \right) \right) = k, 0 < |x| < \frac{1}{\sqrt{2}}$$

be α and β , where the inverse trigonometric functions take only principal values. If the solutions of the equation $x^2 - bx - 5 = 0$ are $\frac{1}{\alpha^2} + \frac{1}{\beta^2}$ and $\frac{\alpha}{\beta}$, then $\frac{b}{k^2}$ is equal to

82. The mean and variance of 10 observations were calculated as 15 and 15 respectively by a student who took by mistake 25 instead of 15 for one observation. Then, the correct standard deviation is
83. Let the line $\frac{x-3}{7} = \frac{y-2}{-1} = \frac{z-3}{-4}$ intersect the plane containing the lines $\frac{x-4}{1} = \frac{y+1}{-2} = \frac{z}{1}$ and $4ax - y + 5z - 7a = 0 = 2x - 5y - z - 3$, $a \in \mathbb{R}$ at the point $P(\alpha, \beta, \gamma)$. Then the value of $\alpha + \beta + \gamma$ equals

84. An ellipse $E: \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ passes through the vertices of the hyperbola $H: \frac{x^2}{49} - \frac{y^2}{64} = -1$. Let the major and minor axes of the ellipse E coincide with the transverse and conjugate axes of the hyperbola H . Let the product of the eccentricities of E and H be $\frac{1}{2}$. If l is the length of the latus rectum of the ellipse E , then the value of 1131 is equal to
85. Let $y = y(x)$ be the solution curve of the differential equation $\sin(2x^2) \log_e(\tan x^2) dy + \left(4xy - 4\sqrt{2}x \sin\left(x^2 - \frac{\pi}{4}\right)\right) dx = 0$, $0 < x < \sqrt{\frac{\pi}{2}}$, which passes through the point $\left(\sqrt{\frac{\pi}{6}}, 1\right)$. Then $\left|y\left(\sqrt{\frac{\pi}{3}}\right)\right|$ is equal to
86. Let M and N be the number of points on the curve $y^5 - 9xy + 2x = 0$, where the tangents to the curve are parallel to x -axis and y -axis, respectively. Then the value of $M + N$ equals
87. Let $f(x) = 2x^2 - x - 1$ and $S = \{n \in \mathbb{Z} : |f(n)| \leq 800\}$. Then, the value of $\sum_{n \in S} f(n)$ is equal to
88. Let S be the set containing all 3×3 matrices with entries from $\{-1, 0, 1\}$. The total number of matrices $A \in S$ such that the sum of all the diagonal elements of $A^T A$ is 6 is
89. If the length of the latus rectum of the ellipse $x^2 + 4y^2 + 2x + 8y - \lambda = 0$ is 4, and l is the length of its major axis, then $\lambda + l$ is equal to
90. Let $S = \{z \in \mathbb{C} : z^2 + \bar{z} = 0\}$. Then $\sum_{z \in S} (\operatorname{Re}(z) + \operatorname{Im}(z))$ is equal to

