**1. (d)**

$$\frac{\alpha[L]}{[ML^2\ T^{-2}]} = [M^0\ L^0\ T^0]$$

$$\alpha = [ML^1\ T^{-2}]$$

$$\frac{\alpha}{\beta} = \frac{[ML^2\ T^{-2}]}{[L^3]} \Rightarrow \beta = \frac{[ML^1\ T^{-2}][L^3]}{ML^2\ T^{-2}}$$

**2. (d)** $y = x - \frac{10x^2}{2u^2\left(\frac{1}{2}\right)} \Rightarrow 10 = 20 - \frac{(10)(100)}{u^2}$

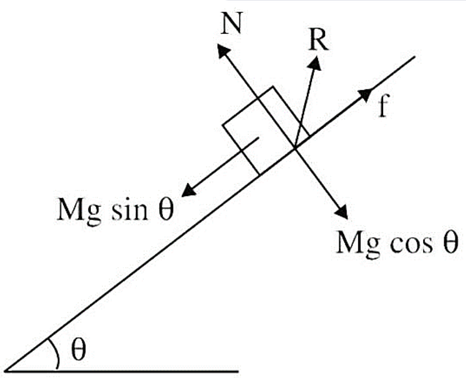
$$u = 20$$

$$T = \frac{(2)(20)}{\sqrt{2}(10)} = 2\sqrt{2}$$

$$\vec{v} = 10\sqrt{2}\hat{i} + (10\sqrt{2} - 10(2)]\hat{j}$$

Momentum $\vec{p} = M\vec{v} = 100\sqrt{2}\hat{i} + (100\sqrt{2} - 200)\hat{j}$

**3. (b)**



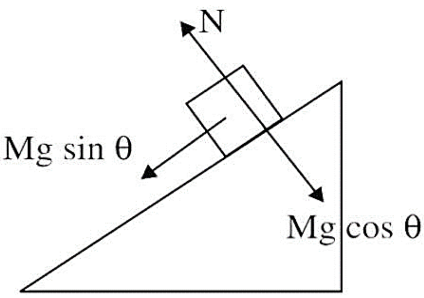

$$N = Mg\cos\theta$$

$$f = Mg\sin\theta$$

$$R = \sqrt{N^2 + f^2}$$

$$R = Mg$$

**4. (c)**



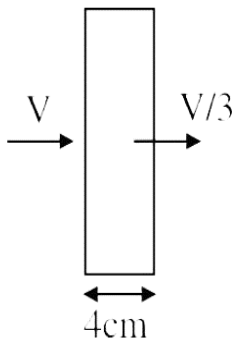

$$a = g\sin\theta$$

$$\ell=\frac{1}{2}\,g\sin 30^{\circ}(2)^{2}$$

$$\ell=\frac{1}{2}\,g\sin 45^{\circ}t^{2}$$

$$\left(\frac{1}{2}\right)(4)=\frac{1}{\sqrt{2}}t^{2}\Rightarrow t=\sqrt{2\sqrt{2}}\simeq 1.68$$

**5. (c)**




$$\left(\frac{V}{3}\right)^{2}=V^{2}-2a(4)\Rightarrow a=\frac{8\,V^{2}}{9(8)}=\frac{V^{2}}{9}$$

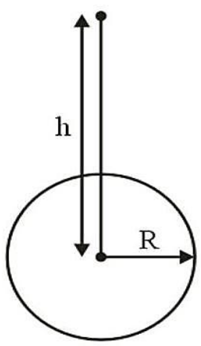
$$0=V^{2}-2a(4+x)$$

$$\Rightarrow\ V^{2}=2\left(\frac{V^{2}}{9}\right)(4+x)$$

$$4.5=4+x$$

$$x=0.5$$

**6. (b)**




$$-\frac{GMm}{R}+\frac{1}{2}\,m\lambda^{2}\,V_{e}^{2}=-\frac{GMm}{h}$$

$$-\frac{GMm}{R}+\frac{1}{2}\lambda^{2}\frac{2GMm}{R}=-\frac{GMm}{h}$$

$$\frac{\lambda^{2}}{R}-\frac{1}{R}=\frac{-1}{h}$$

$$\frac{1}{h}=\frac{1-\lambda^{2}}{R}$$

$$h=\frac{R}{1-\lambda^2}$$

**7. (d)**

$$3.2\text{ m} \mid y_{\text{steel}}=2\times10^{11}$$
$$4.4\text{ m} \mid y_{\text{cm}}=1.1\times10^{11}$$
$$\Delta\ell_1+\Delta\ell_2=\Delta\ell$$
$$\frac{F\ell_1}{A_1y_1}+\frac{F\ell_2}{A_2y_2}=\Delta\ell$$
$$F=\frac{\Delta\ell}{\frac{\ell_1}{A_1y_1}+\frac{\ell_2}{A_2y_2}}=1.54\times10^2=154$$

**8. (c)** First case: $\eta=1-\frac{100}{300}=\frac{2}{3}$

Second case: $\eta_{\text{net}}=\eta_1+\eta_2-\eta_1\eta_2$

$$\eta_1=1-\frac{200}{300}=\frac{1}{3}$$

$$\eta_2=1-\frac{100}{200}=\frac{1}{2}$$

$$\eta_{\text{net}}=\frac{1}{3}+\frac{1}{2}-\frac{1}{6}=\frac{2}{3}$$

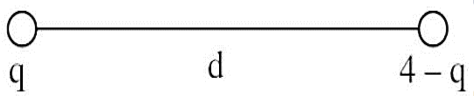
$\eta$ (first case) $=\eta$ (second case)

**9. (b)** Methane molecule is tetrahedron

Degree of freedom due to rotation = 3

Degree of freedom due to translation = 3

**10. (b)**




$$F=\frac{Kq(4-q)}{d^2}$$

$$\frac{dF}{dq}=\frac{K}{d^2}[4-2q]=0$$

$$q=2$$

**11. (b)** Drift velocity $=\left(\frac{e\tau}{m}\right)E$

$$v_d=\left(\frac{e\tau}{m}\right)\left(\frac{\Delta V}{\ell}\right)$$

$\Delta V$ = Potential difference applied across the wire

As temperature increases, relaxation time decreases, hence $V_d$ decreases.

As per formula, $V_d \propto \frac{1}{\ell}$

$v_d = \frac{I}{neA}$, as it is not mentioned that current is at steady state neither it is mentioned that n is constant for given conductor. So it can't be said that $v_d$ is inversely proportional to A.

$$I = neAv_d = \frac{V}{R} = \frac{V}{\rho\ell}A$$

$$v_d = \frac{V}{\rho\ell ne}\ \left(E = \frac{V}{\ell}\right)$$

$$v_d = \frac{eE\tau}{m}$$

$\tau$ decrease with temperature increase.

First and fourth statements are correct.

**12. (a)** $T = 2\pi\sqrt{\frac{I}{B_H M}}$

$$T_1 = 3\text{sec} = 2\pi\sqrt{\frac{I}{(B_P\cos 30^\circ)M}}$$

$$T_2 = 6\text{sec} = 2\pi\sqrt{\frac{I}{(B_Q\cos 60^\circ)M}}$$

$$\frac{3}{6} = \sqrt{\frac{1}{\left(B_P\frac{\sqrt{3}}{2}\right)} \times (B_Q/2)}$$

$$\frac{3}{6} = \sqrt{\left(\frac{B_Q}{\sqrt{3}\,B_P}\right)}$$

$$\frac{\sqrt{3}}{4} = \frac{B_Q}{B_P}$$

$$B_Q : B_P = \sqrt{3} : 4$$

**13. (b)** Kinetic energy of electron in cyclotron

$$= \left[\frac{q^2\,B^2 r_0^2}{2\,m}\right]$$

$$= 18\text{MeV}$$

**14. (c)** Resonant frequency, $\omega_0 = \frac{1}{\sqrt{LC}} = 10^4\text{rad/sec}$

$\omega' = .4 \times 10^4 = 4000 \text{rad/sec}$

$$i_0 = \frac{V_0}{\sqrt{R^2 + (X'_C - X'_L)^2}} = 238 \text{ mA}$$

**15. (b)** Second and fourth statements are correct.

**16. (b)** $\frac{I_1}{I_2} = \frac{1}{4}$

$I_2 = 4I_1$

$I_{max} = I_1 + 4I_1 + 2\sqrt{I_1 4I_1} = 9I_1$

$I_{min} = I_1 + 4I_1 - 2\sqrt{I_1 4I_1} = I_1$

$$\therefore \frac{9I_1 + I_1}{9I_1 - I_1} = \frac{10}{8} = \frac{5}{4} = \frac{2\alpha + 1}{\beta + 1}$$

$\alpha = 2\ \beta = 1$

$$\therefore \frac{\alpha}{\beta} = \frac{2}{1} = 2$$

**17. (b)** $\frac{1}{2} m V_{max}^2 = hf - \phi$

Photoelectric effect can be explained by particle nature of light. Threshold λ is max wavelength at which emission takes place.

**18. (d)** $A_0 = 6.4 \times 10^{-4}$ Curie

$T_{1/2} = 5 \text{ days} = \frac{\ln 2}{\lambda}$

$A = A_0 e^{-\lambda t}$

$5 \times 10^{-6} = 6.4 \times 10^{-4} e^{-\lambda t}$

$$\frac{5}{6.4} \times 10^{-2} = e^{-\lambda t}$$

$7.8 \times 10^{-3} = e^{-\lambda t}$

$\log(7.8 \times 10^{-3}) = -\lambda t \ln e$

$$\ln(7.8 \times 10^{-3}) = -\frac{\lambda n2}{5} \cdot t$$

$\therefore \frac{5 \times 4.853}{0.693} = t = 35 \text{ days}$

**19. (b)** $V_{CE} = 8\text{ V} = I_C = 6 \text{ mA from } 4 \text{ mA},$

$I_B = 20\mu A \text{ to } 25\mu A$

Current gain $\beta_{av} = \frac{I_C{}' - I_C}{I_B{}' - I_B} = \frac{2 \text{ mA}}{5\mu A}$

$\beta_{av} = \frac{2}{5} \times 10^3 = \frac{2000}{5} = 400$

**20. (a)**

Modulation Index $\mu = \frac{A_m}{A_C} = \frac{1}{5} = 0.2$

$A_m$ = amp. of modulating signal

$A_C$ = amp. of carrier wave

**21. (2)** Slope $= \frac{\Delta l/w}{L} = \frac{\Delta l/L}{w} = \frac{1}{YA}$

$\Rightarrow Y = \frac{1}{(\text{ slope })A}$

$Y = \frac{1}{2 \times 10^{-6}(0.25 \times 10^{-5})}$

$Y = 2 \times 10^{11}\ N/m^2$

**22. (40)** Independent of area in case of uniform wire.

**23. (4)** $\delta = A(\mu_y - 1) - A'(\mu_y{}' - 1)$

$= 6(1.5 - 1) - 5(1.55 - 1) = \frac{1}{4}$

**24. (12)** $\phi = \vec{B} \cdot \vec{A}$

$= (3t^3\hat{j} + 3t^2\hat{k}) \cdot (\pi(1)^2\hat{k})$

$\phi = 3t^2\pi$

$\varepsilon_{IND} = \left|\frac{d\phi}{dt}\right| = 6t\pi$

at $t = 2, \varepsilon_{IND} = 12$

**25. (10)**

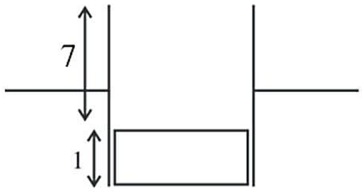
$q = CV_{100\Omega}$

$= (1.1 \times 10^{-6})\left(\frac{10}{R + r}R\right)$

$= 1.1 \times 10^{-6}\left(\frac{10}{110} \times 100\right)$

$= 10\mu C$

**26. (240)**




$$C_{eff}=\left[\frac{\varepsilon_0(7\times 4)}{4/10}+\frac{5\varepsilon_0(1\times 4)}{4/10}\right]\times 10^{-2}$$

$$C_{eff}=1.2\varepsilon_0$$

$$\text{Energy}=\frac{1}{2}C_{eff}V^2$$

$$=\frac{1}{2}(1.2)\varepsilon_0(20)(20)=240\varepsilon_0$$

**27. (3)** $\frac{nv}{0.6}=400\&\frac{(n+1)v}{0.6}=450$

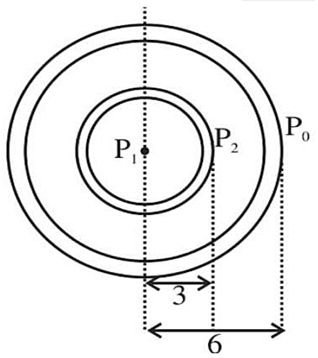
$$\Rightarrow\left[\frac{0.6\times 400}{v}+1\right]\frac{v}{0.6}=450$$

$$\Rightarrow = v=30$$

$$\Rightarrow\sqrt{\frac{T}{\mu}}=30$$

$$\Rightarrow\frac{2700}{\mu}=900=\mu=3$$

**28. (2)**




$$P_2-P_0=\frac{4\,T}{6}\&P_1-P_2=\frac{4\,T}{3}$$

$$\Rightarrow P_1-P_0=\frac{4\,T}{2}=2$$

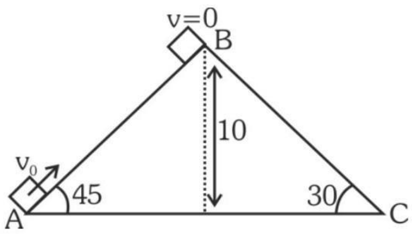
**29. (120)** From energy conservation

$$mgh=\frac{1}{2}mvv^2+\frac{1}{2}I\omega^2$$

$$\text{mgh} = \frac{1}{2}\text{mv}^2 + \frac{1}{2}\frac{\text{mR}^2}{2}\omega^2$$

$$10\text{ h} = \frac{16}{2} + \frac{16}{4} \Rightarrow \text{h} = 1.2\text{ m} = 120\text{ cm}$$

**30. (2)**




From E.C. $= \frac{1}{2}\text{mv}_0^2 = \text{mgh}$

$v_0 = 10\sqrt{2}$

For $A \rightarrow B$ at $B, v = 0$

$$a = -g\sin 45^\circ = \frac{-10}{\sqrt{2}}$$

$$v = u + at_1 \Rightarrow 0 = 10\sqrt{2} - \frac{10}{\sqrt{2}}t_1 \Rightarrow t_1 = 2\text{sec}$$

For $B \rightarrow C$

$$s = ut_2 + \frac{1}{2}at_2^2$$

$$\frac{10}{\sin 30^\circ} = \frac{1}{2}(10\sin 30^\circ)t_2^2$$

$t_2 = 2\sqrt{2}$

So total time

$T = t_1 + t_2$

$= 2\sqrt{2} + 2$

$= 2(\sqrt{2} + 1)\text{sec}$

**31. (a)**

(a) $n + \ell = 3 + 0 = 3$

(b) $n + \ell = 4 + 0 = 4$

(c) $n + \ell = 3 + 1 = 4$

(d) $n + \ell = 3 + 2 = 5$

Higher $n+\ell$ value, higher the energy & if same $n+\ell$ value, then higher n value, higher the energy. Thus: $D > B > C > A$.

**32. (c)** (a) $\psi_{MO} = \psi_A - \psi_B$

(III) ABMO

(b) $\mu = Q \times r$

(I) Dipole moment

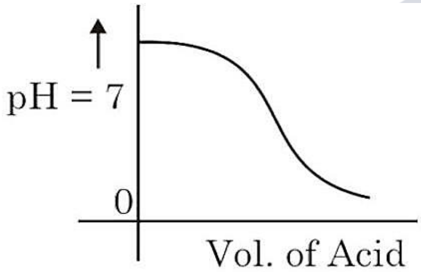
(c) $\frac{N_b - N_a}{2}$

(IV) Bond order

(d) $\psi_{MO} = \psi_A + \psi_B$

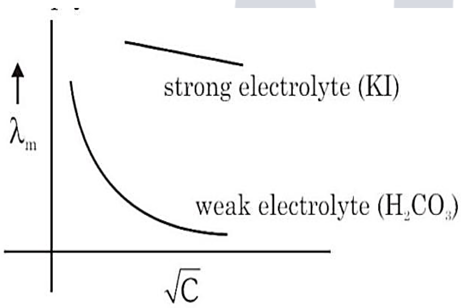
(II) BMO

**33. (a)** Titration curve of $NH_4OH$ vs $HCl(WB + SA)$.




**34. (b)** Statement I: KI is strong electrolyte thus almost constant on dilution.

Statement II: In weak electrolyte it increases, sharply.




**35. (c)**

Assertion (a): Correct.

Reason(R): Incorrect.

Particles of true solution pass through parchment paper thus answer is (c).

**36. (b)**

(a) $3\,s^2 \rightarrow Mg$

(b) $3\,s^2 3p^1 \rightarrow Al$

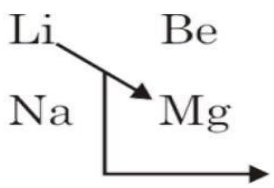
(c) $3\ s^2 3p^3 \rightarrow P$

(d) $3\ s^2 3p^4 \rightarrow S$

$\underset{\text{Half filled stability}}{P > S} > \underset{\text{Penetrating power of} > p.}{Mg > Al}$

$C > D > A > B.$

**37. (a)**




Diagonal relationship

$Li^+ \rightarrow$ Maximum hydration enthalpy in group 1 due to small size.

So ' B ' is Mg.

**38. (b)** Assertion (a): True

Reason (R): True but not correct explanation.

Correct explanation: Expansion of octet not possible for ' B '.

**39. (d)**

$$8MnO_4^- + 3S_2O_3^{2-} + H_2O \xrightarrow[\text{alk. solution}]{\text{neutral or}} 8MnO_2 + 6SO_4^{2-} + 2OH^-$$

**40. (a)** When metal is in low oxidation state then it forms complexes when ligands have good π-accepting character.

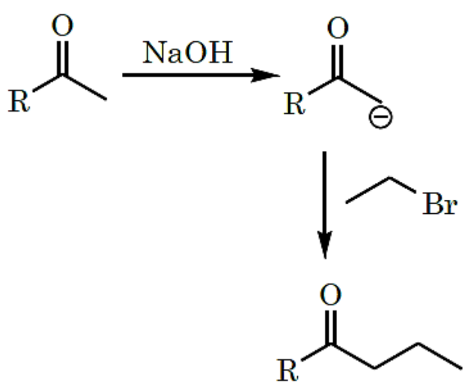
**41. (a)**

(I) Fly ash and slag from steel industry are utilised by cement industry.

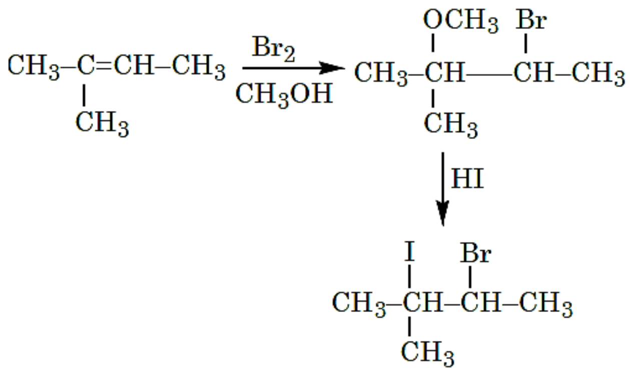
(II) Fuel obtained from plastic waste has high octane rating. It contains no lead and it is known as green fuel.

Both statement (I) & (II) are correct.

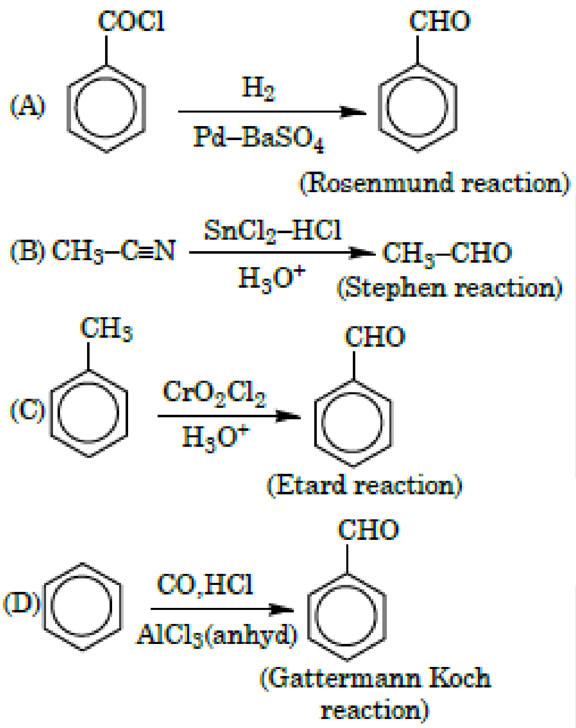
**42. (c)**



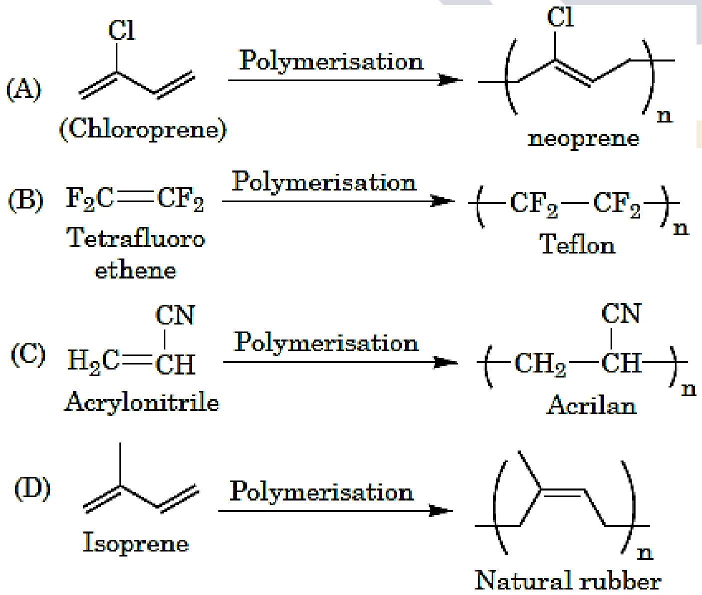

**43. (b)**


Br2
CH3OH
OCH3
Br
HI
I

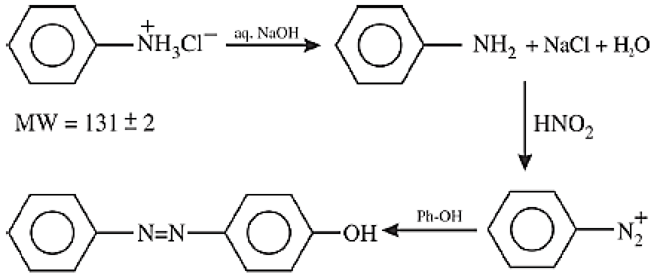
**44. (a)**


(Rosenmund reaction)
(Stephen reaction)
(Etard reaction)
(Gattermann Koch reaction)

**45. (a)**


Polymerisation
(Chloroprene)
neoprene
Tetrafluoro ethene
Teflon
Acrylonitrile
Acrilan
Isoprene
Natural rubber

**46. (d)**




p-hydroxy azobenzene
(Orange dye)

**47. (a)**

(A) $\text{Glucose} \xrightarrow{HI} \text{n-hexane}$

(B) $\text{Glucose} \xrightarrow[H_2O]{Br_2} \begin{array}{c} COOH \\ | \\ (CHOH)_4 \\ | \\ CH_2OH \end{array}$

Gluconic acid

(C) $\text{Glucose} \xrightarrow[\text{anhydride}]{\text{5 acetic}} \text{Glucose pentacetate}$

(D) $\text{Glucose} \xrightarrow{HNO_3} \begin{array}{c} COOH \\ | \\ (CHOH)_4 \\ | \\ COOH \end{array}$

Saccharic acid

**48. (c)** Rosin is added to soaps which forms sodium rosinate which lathers well.

**49. (d)**

(a) Chloroform + Aniline → (III) Distillation

(b) Benzoic acid + Napthalene → (IV) Crystallisation

(c) Water + Aniline → (I) Steam distillation

(d) Napthalene + Sodium chloride → (II) Sublimation

**50. (d)** $4Fe^{3+} + 3[Fe(CN)_6]^{-4} \longrightarrow Fe_4[Fe(CN)_6]_3$ Prussian Blue

**51. (1)** No. of equivalents of $H_2SO_4 = 100 \times 0.1 \times 2 = 20$

No. of equivalents of NaOH $= 50 \times 0.1 = 5$

No. of equivalents of $H_2SO_4$ left $= 20 - 5 = 15$

$\Rightarrow 150 \times x = 15$

$x = \frac{1}{10} = 0.1 \text{ N} = 1 \times 10^{-1} \text{ N}$

**52. (0.25)** For real gas under high pressure

$$Z = 1 + \frac{Pb}{RT} \Rightarrow b = \frac{RT}{P}$$

$$= \frac{0.083 \times 298}{99}$$

$$= 0.25 \times 10^{-2}\ \text{L mol}^{-1}$$

**53. (35)** Let xg is burnt

moles $= \frac{x}{280}$

heat released by $\frac{x}{280}$ mole $= 2.5 \times 0.45$ kJ

heat released by 1 mole $= \frac{2.5 \times 0.45 \times 280}{x}$ kJ

$\Delta H = \Delta U + \Delta ngRT$

$\Delta H \simeq \Delta U$

$$9 = \frac{2.5 \times 280 \times 0.45}{x}$$

$x = 35$ g

**54. (3)**

Diliute solution given:

$$\frac{P^0 - P_S}{P^0} \sim \frac{{}^{n}\text{ solute}}{{}^{n}\text{ solvent}}$$

$$\frac{P^0 - P^0/2}{P^0} = \frac{{}^{n}\text{ solute}}{{}^{n}\text{ solvent}}$$

$${}^{n}\text{ solute} \sim \frac{{}^{n}\text{ solvent}}{2} = \frac{100}{18 \times 2} = 2.78\ \text{mol}$$

More accurate approach:

$$\frac{P^0 - P_S}{P_S} = \frac{{}^{n}\text{ solute}}{{}^{n}\text{ solvent}}$$

$$\frac{P^0 - P^0/2}{P^0/2} = \frac{{}^{n}\text{ solute}}{{}^{n}\text{ solvent}}$$

$${}^{n}\text{ solute} = {}^{n}\text{ solvent} = \frac{100}{18} = 5.55\ \text{mol}$$

**55. (165)**

$$K = \frac{0.693}{t_{1/2}} = \frac{0.693}{70 \times 60}$$

$$= \frac{6930}{7 \times 6} \times 10^{-6}$$

$$= 165 \times 10^{-6}\ \text{s}^{-1}$$

**56. (4)**

Bauxite – $AlO_x(OH)_{3-2x}$( where $0 < x < 1$)

✓ Siderite – $FeCO_3$

Cuprite – $Cu_2O$

Calamine – $ZnCO_3$

✓ Haematite – $Fe_2O_3$

Kaolinite – $Al_2(OH)_4Si_2O_5$

Malachite – $CuCO_3 \cdot Cu(OH)_2$

✓ Magnetite – $Fe_3O_4$

Sphalerite – ZnS

✓ Limonite – $Fe_2O_3 \cdot 3H_2O$

Cryolite – $Na_3AlF_6$

**57. (4)**

$$2KMnO_4 + 3H_2O_2 \xrightarrow{\text{basic medium}} 2\overset{+4}{Mn}O_2 + 3O_2 + 2H_2O + 2KOH$$

**58. (6)**

| | | | |
|---|---|---|---|
| $SO_3$ | - | $sp^2$ | Planar |
| $BF_3$ | - | $sp^2$ | Planar |
| $NO_3^-$ | - | $sp^2$ | Planar |
| $SF_4$ | - | $sp^3d$ | Non-planar |
| $H_2O_2$ | - | $sp^3$ | Non-planar |
| $PCl_3$ | - | $sp^3$ | Non-planar |
| $[Al(OH)_4]^-$ | - | $sp^3$ | Non-planar |
| $XeF_4$ | - | $sp^3d^2$ | Planar |
| $eO_3$ | — | $sp^3$ | Non-planar |
| $H^+$ | - | $sp^3$ | Non-planar |

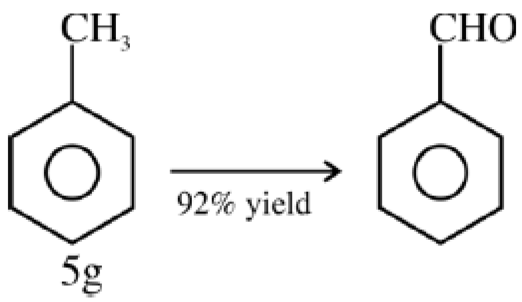
**59. (2)** Fehling solution is a complex of $Cu^{++}$

$Cu^{++} = 3d^9$

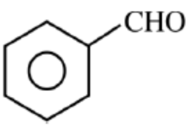
No. of unpaired $e^-$ = 1

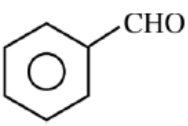
$M.M = \sqrt{1(1+2)} = \sqrt{3} = 1.73 BM$

**60. (530)**




moles $= \frac{5}{92}$

moles of  $= \frac{5}{92} \times \frac{92}{100} = 5 \times 10^{-2}$

mass of  $= 106 \times 5 \times 10^{-2} = 5.3$ g

$= 530 \times 10^{-2}$ g

**61. (c)**

$$f(x) = \sin^{-1}[2x^2 - 3] + \log_2\left(\log_{\frac{1}{2}}(x^2 - 5x + 5)\right)$$

$P_1: -1 \le [2x^2 - 3] < 1$

$\Rightarrow -1 \le 2x^2 - 3 < 2$

$\Rightarrow 2 < 2x^2 < 5$

$\Rightarrow 1 < x^2 < \frac{5}{2}$

$\Rightarrow P_1: x \in \left(-\sqrt{\frac{5}{2}}, -1\right) \cup \left(1, \sqrt{\frac{5}{2}}\right)$

$P_2: x^2 - 5x + 5 > 0$

$\Rightarrow \left(x - \left(\frac{5-\sqrt{5}}{2}\right)\right)\left(x - \left(\frac{5+\sqrt{5}}{2}\right)\right) > 0$

$P_3: \log_{\frac{1}{2}}(x^2 - 5x + 5) > 0$

$\Rightarrow x^2 - 5x - 5 < 1$

$\Rightarrow x^2 - 5x + 4 < 0$

$\Rightarrow P_3 : x \in (1,4)$

So, $P_1 \cap P_2 \cap P_3 = \left(1, \frac{5-\sqrt{5}}{2}\right)$

**62. (c)** $\pi < \alpha, \beta < 2\pi$

$\frac{1 - i\sin\alpha}{1 + i(2\sin\alpha)}$ = Purely imaginary

$\Rightarrow \frac{(1 - i\sin\alpha)(1 - i(2\sin\alpha))}{1 + 4\sin^2\alpha}$ = Purely imaginary

$$\Rightarrow \frac{1 - 2\sin^2\alpha}{1 + 4\sin^2\alpha} = 0$$

$$\Rightarrow \sin^2\alpha = \frac{1}{2}$$

$$\Rightarrow \alpha = \left\{\frac{5\pi}{4}, \frac{7\pi}{4}\right\}$$

& $\frac{1 + i\cos\beta}{1 + i(-2\cos\beta)}$ = Purely real

$\Rightarrow \frac{(1 + i\cos\beta)(1 + 2i\cos\beta)}{1 + 4\cos^2\beta}$ = Purely real

$$\Rightarrow 3\cos\beta = 0$$

$$\Rightarrow \beta = \frac{3\pi}{2}$$

$$\Rightarrow Z_{\alpha\beta} = \sin\frac{5\pi}{2} + i\cos 3\pi = 1 - i$$

or

$$Z_{\alpha\beta} = \sin\frac{7\pi}{2} + i\cos 3\pi = -1 - i$$

Required value $= \left[i(1 - i) + \frac{1}{i(1+i)}\right] + \left[i(-1 - i) + \frac{1}{i(-1+i)}\right]$

$$= i(-2i) + \frac{1}{i}\frac{2i}{(-2)} \Rightarrow 2 - 1 = 1$$

**63. (b)**

**64. (b)** The characteristic equation for A is $|A - \lambda I| = 0$

$$\Rightarrow \begin{vmatrix} 4 - \lambda & -2 \\ \alpha & \beta - \lambda \end{vmatrix} = 0$$

$$\Rightarrow (4 - \lambda)(\beta - \lambda) + 2\alpha = 0$$

$$\Rightarrow \lambda^2 - (\beta + 4)\lambda + 4\beta + 2\alpha = 0$$

Put $\lambda = A$

$A^2 - (\beta + 4)A + (4\beta + 2\alpha)I = 0$

On comparison

$-9(\beta + 4) = \gamma \& 4\beta + 2\alpha = 18$

and $|A| = 4\beta + 2\alpha = 18$

**65. (b)** $f(0) = \lim_{x \to 0} f(x)$

Limit should be $\frac{0}{0}$ form

So, $\sqrt[7]{p.729} - 3 = 0 \Rightarrow p.3^6 = 3^7 \Rightarrow p = 3$

Now, $f(0) = \lim_{x \to 0} \frac{\sqrt[7]{3(3^6+x)} - 3}{\sqrt[3]{3^6+qx} - 9}$

$$= \lim_{x \to 0} \frac{3\left[\left(1 + \frac{x}{3^6}\right)^{1/7} - 1\right]}{9\left[\left(1 + \frac{qx}{3^6}\right)^{1/3} - 1\right]} = \frac{3}{9} \times \frac{\frac{1}{7.3^6}}{\frac{q}{3.3^6}}$$

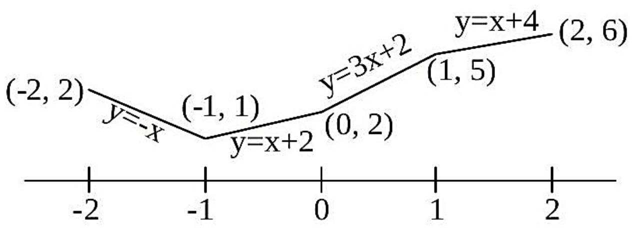
$$\Rightarrow f(0) = \frac{1}{3} \times \frac{3}{7q} = \frac{1}{7q}$$

$\Rightarrow 7qf(0) - 1 = 0$

$\Rightarrow 7 \cdot p^2 \cdot qf(0) - p^2 = 0$ (for option)

$\Rightarrow 63qf(0) - p^2 = 0$

**66.(d)**




(S1) : $f'\left(-\frac{3}{2}\right) + f'\left(-\frac{1}{2}\right) + f'\left(\frac{1}{2}\right) + f'\left(\frac{3}{2}\right) = 4$

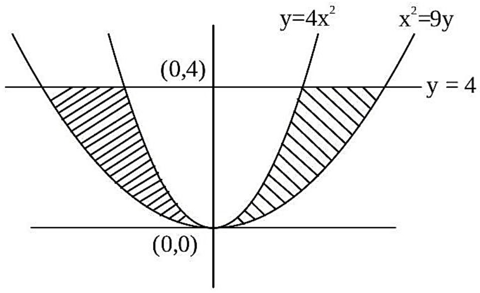
(S2): $\int_{-2}^{2} f(x)dx = 12$

**67. (a)** $S_{21} = \frac{21}{2}[20ar + 20.10ar^2]$

$= 21[10ar + 100ar^2]$

$= 21.a_{11}$

**68. (d)**




$\Delta = 2 \cdot \int_0^4 \left(3\sqrt{y} - \frac{\sqrt{y}}{2}\right) dy$

$= 2 \cdot \int_0^4 \frac{5}{2}\sqrt{y} dy = \frac{80}{3}$

**69. (b)**

$\int_0^2 |2x^2 - 3x| dx$

$$= \int_0^{\frac{3}{2}} (3x - 2x^2) dx + \int_{\frac{3}{2}}^2 (2x^2 - 3x) dx = \frac{19}{12}$$

$$\int_0^2 \left[x - \frac{1}{2}\right] dx = \int_{\frac{-1}{2}}^{\frac{3}{2}} [t] dt$$

$$= \int_{-\frac{1}{2}}^0 (-1) dt + \int_0^1 0 \cdot dt + \int_1^{\frac{3}{2}} 1 \cdot dt = 0.$$

**70. (c)** Given that $A_1 = 2 A_2$

from the graph $A_1 + A_2 = xy - 8$

$\Rightarrow \frac{3}{2} A_1 = xy - 8$

$\Rightarrow A_1 = \frac{2}{3} xy - \frac{16}{3}$

$\Rightarrow \int_4^x f(x) dx = \frac{2}{3} xy - \frac{16}{3}$

$\Rightarrow f(x) = \frac{2}{3}\left(x \frac{dy}{dx} + y\right)$

$\Rightarrow \frac{2}{3} x \frac{dy}{dx} = \frac{y}{3}$

$\Rightarrow 2 \int \frac{dy}{y} = \int \frac{dx}{x}$

$\Rightarrow 2\ell ny = \ln x + \ell nc$

$\Rightarrow y^2 = cx$

As $f(4) = 2 \Rightarrow c = 1$

so $y^2 = x$

slope of normal $= -6$

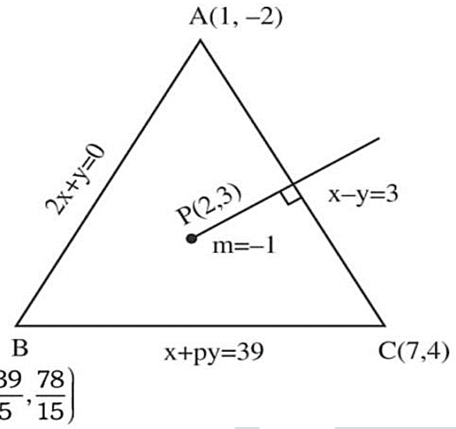
$$y = -6(x) - \frac{1}{2}(-6) - \frac{1}{4}(-6)^3$$

$$\Rightarrow y = -6x + 3 + 54$$

$$\Rightarrow y + 6x = 57$$

Now check options and (c) will not satisfy

**71. (d)**




Perpendicular bisector of AB

$$x + y = 5$$

Take image of A

$$\frac{x-1}{1} = \frac{y+2}{1} = \frac{-2(-6)}{2} = 6$$

(7,4)

$$7 + 4p = 39$$

$$p = 8$$

solving $x + 8y = 39$ and $y = -2x$

$$x = \frac{-39}{15}\ y = \frac{78}{15}$$

$$AC^2 = 72 = 9p$$

$$AC^2 + p^2 = 72 + 64 = 136$$

$$\Delta ABC = \frac{1}{2}\begin{vmatrix} 1 & -2 & 1 \\ 7 & 4 & 1 \\ \frac{-39}{15} & \frac{78}{15} & 1 \end{vmatrix}$$

$= \frac{1}{2}\left[4 - \frac{78}{15} + 2\left(7 + \frac{39}{15}\right) + 7\left(\frac{78}{15}\right) + \frac{4 \times 39}{15}\right]$

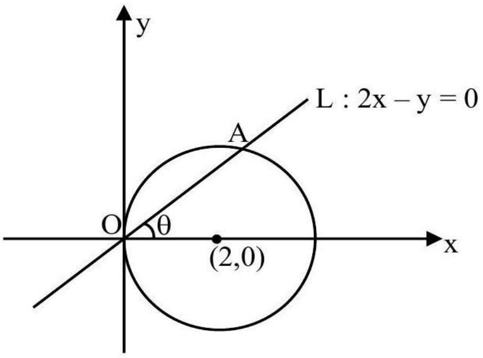
$= \frac{1}{2}\left[18 + 18 \times \frac{13}{5}\right]$

$= 9\left[\frac{18}{5}\right] = \frac{162}{5} = 32.4$

**72. (a)**

$C_1 : x + y - 4x = 0$

$\tan\theta = 2$




$C_2$ is a circle with OA as diameter.

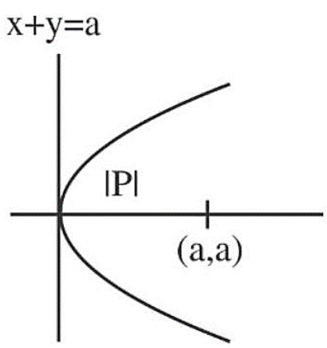
So, tangent at A on $C_2$ is perpendicular to OR

Let $OA = \ell$

$\therefore \frac{QA}{AP} = \frac{\ell\cot\theta}{\ell\tan\theta}$

$= \frac{1}{\tan^2\theta} = \frac{1}{4}$

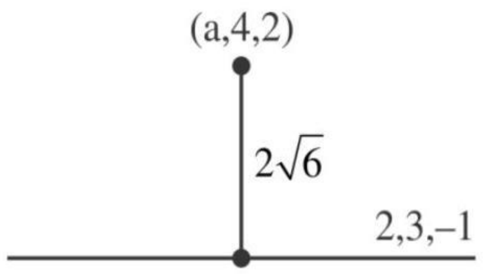
**73. (c)**




$|P| = \left|\frac{a}{\sqrt{2}}\right| = \frac{16}{4} = 4$

$|a| = 4\sqrt{2}$

**74. (b)**




$(2\lambda-1, 3\lambda+3, -\lambda+1)$

$(2\lambda-1-a)2+(3\lambda-1)3+(-\lambda-1)(-1)=0$

$\Rightarrow 4\lambda-2-2a+9\lambda-3+\lambda+1=0$

$\Rightarrow 14\lambda-4-2a=0$

$\Rightarrow 7\lambda-2-a=0$

and,

$(2\lambda-1-a)^2+(3\lambda-1)^2+(\lambda+1)^2=24$

$\Rightarrow (5\lambda-1)^2+(3\lambda-1)^2+(\lambda+1)^2=24$

$\Rightarrow 35\lambda^2-14\lambda-21=0$

$\Rightarrow (\lambda-1)(35\lambda+21)=0$

For, $\lambda=1 \ \Rightarrow a=5$

Let $(\alpha_1, \alpha_2, \alpha_3)$ be reflection of point P

$\alpha_1+5=2 \ \alpha_2+4=12 \ \alpha_3+2=0$

$\alpha_1=-3 \quad \alpha_2=8 \ \alpha_3=-2$

$a+\alpha_1+\alpha_2+\alpha_3=8$

**75. (b)** $\vec{n}=\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a & b & 0 \\ a & b & c \end{vmatrix}$

$=bc\hat{i}-ac\hat{j}$

Direction ratios of line are $\mathbf{b}, -\mathbf{a}, \mathbf{0}$

Direction ratios of normal of the plane are $\mathbf{0}, \mathbf{1}, -\mathbf{1}$

$\cos 60^\circ = \left|\frac{-a}{|\sqrt{2}|\sqrt{b^2+a^2}}\right| = \frac{1}{2}$

$\Rightarrow \left|\frac{a}{\sqrt{a^2+b^2}}\right| = \frac{1}{\sqrt{2}}$

$\Rightarrow b=\pm a$

So, D.R.'s can be $(\pm a, -a, 0)$

$\therefore$ D.C.'s can be $\pm\left(\frac{\pm 1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 0\right)$

**76. (b)** Let $\alpha$ = Mean & $\beta$ = Variance $(\alpha>\beta)$

So, $\alpha+\beta=24, \ \alpha\beta=128$

$\Rightarrow \alpha = 16 \,\&\, \beta = 8$

$\Rightarrow np = 16\ npq = 8 \Rightarrow q = \frac{1}{2}$

$\therefore p = \frac{1}{2}, n = 32$

$p(x > n-3) = \frac{1}{2^n}\left({}^nC_{n-2} + {}^nC_{n-1} + {}^nC_n\right)$

$\therefore k = {}^{32}C_{30} + {}^{32}C_{31} + {}^{32}C_{32} = \frac{32 \times 31}{2} + 32 + 1$

$= 496 + 33 = 529$

**77. (d)** Let $\frac{P(\text{a prime number})}{2} = \frac{P(\text{a composite})}{1} = \frac{P(1)}{3} = k$

So, $P(\text{ a prime number }) = 2k$,

$P(\text{a composite number }) = k$,

$\& \ P(1) = 3k$

$\& 3 \times 2k + 2 \times k + 3k = 1$

$\Rightarrow k = \frac{1}{11}$

$P(\text{ success }) = P(1 \text{ or } 4) = 3k + k = \frac{4}{11}$

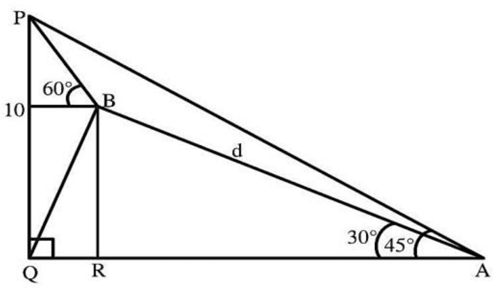
Number of trials, $n = 2$

$\therefore \text{mean} = np = 2 \times \frac{4}{11} = \frac{8}{11}$

**78. (a)** $QA = 10\ RA = d\cos 30^\circ = \frac{\sqrt{3}\,d}{2}$

$QR = 10 - \frac{\sqrt{3}\,d}{2}$

$BR = d\sin 30^\circ = \frac{d}{2}$




$\tan 60^\circ = \frac{PQ - BR}{QR} = \frac{10 - \frac{d}{2}}{10 - \frac{\sqrt{3}\,d}{2}}$

$\Rightarrow \sqrt{3} = \frac{20 - d}{20 - \sqrt{3}\, d}$

$\Rightarrow 20\sqrt{3} - 3\, d = 20 - d$

$\Rightarrow 2\, d = 20(\sqrt{3} - 1)$

$\Rightarrow d = 10(\sqrt{3} - 1)$

$ar(PQRB) = \alpha = \frac{1}{2}(PQ + BR) \cdot QR$

$= \frac{1}{2}\left(10 + \frac{d}{2}\right) \cdot \left(10 - \frac{\sqrt{3}\, d}{2}\right)$

$= \frac{1}{2}(10 + 5\sqrt{3} - 5)(10 - 15 + 5\sqrt{3})$

$= \frac{1}{2}(5\sqrt{3} + 5)(5\sqrt{3} - 5) = \frac{1}{2}(75 - 25) = 25$

**79. (c)** Let $\alpha = \theta + (m - 1)\frac{\pi}{6}$

$\& \beta = \theta + m\frac{\pi}{6}$

So, $\beta - \alpha = \frac{\pi}{6}$

Here, $\sum_{m=1}^{9} \sec\alpha \cdot \sec\beta = \sum_{m=1}^{9} \frac{1}{\cos\alpha \cdot \cos\beta}$

$= 2\sum_{m=1}^{9} \frac{\sin(\beta - \alpha)}{\cos\alpha \cdot \cos\beta} = 2\sum_{m=1}^{9} (\tan\beta - \tan\alpha)$

$= 2\sum_{m=1}^{9} \left(\tan\left(\theta + m\frac{\pi}{6}\right) - \tan\left(\theta + (m - 1)\frac{\pi}{6}\right)\right)$

$= 2\left(\tan\left(\theta + \frac{9\pi}{6}\right) - \tan\theta\right) = 2(-\cot\theta - \tan\theta) = -\frac{8}{\sqrt{3}}$

(Given)

$\therefore \tan\theta + \cot\theta = \frac{4}{\sqrt{3}}$

$\Rightarrow \tan\theta = \frac{1}{\sqrt{3}}$ or $\sqrt{3}$

So, $S = \left\{\frac{\pi}{6}, \frac{\pi}{3}\right\}$

$\sum_{\theta \in S} \theta = \frac{\pi}{6} + \frac{\pi}{3} = \frac{\pi}{2}$

**80. (d)** $X \Rightarrow Y$ is a false when X is true and Y is false

So, $P \rightarrow T, Q \rightarrow F, R \rightarrow F$

(a) $P \vee Q \rightarrow \sim R$ is T

(b) $R \vee Q \rightarrow \sim P$ is T

(c) $\sim (P \vee Q) \rightarrow \sim R$ is T

(d) $\sim (R \vee Q) \rightarrow \sim P$ is F

**81. (42)** $A = \begin{bmatrix} \alpha & \beta & \gamma \\ \alpha^2 & \beta^2 & \gamma^2 \\ \beta+\gamma & \gamma+\alpha & \alpha+\beta \end{bmatrix}$

$R_3 \rightarrow R_3 + R_1$

$\Rightarrow |A| = |\alpha+\beta+\gamma| \begin{vmatrix} \alpha & \beta & \gamma \\ \alpha^2 & \beta^2 & \gamma^2 \\ 1 & 1 & 1 \end{vmatrix}$

$\Rightarrow |A| = (\alpha+\beta+\gamma)(\alpha-\beta)(\beta-\gamma)(\gamma-\alpha)$

$\because |\operatorname{adj} A| = |A|^{n-1}$

$|\operatorname{adj}(\operatorname{adj} A)| = |A|^{(n-1)^2}$

$|\operatorname{adj}(\operatorname{adj}(\operatorname{adj}(\operatorname{adj} A)))| = |A|^{(n-1)^4} = |A|^{2^4} = |A|^{16}$

$\therefore (\alpha+\beta+\gamma)^{16} = 2^{32} \cdot 3^{16}$

$\Rightarrow (\alpha+\beta+\gamma)^{16} = (2^2 \cdot 3)^{16} = (12)^{16}$

$\Rightarrow \alpha+\beta+\gamma = 12$

$\because \alpha, \beta, \gamma \in N$

$(\alpha-1)+(\beta-1)+(\gamma-1) = 9$

number all tuples $(\alpha, \beta, \gamma) = {}^{11}C_2 = 55$

1 case for $\alpha = \beta = \gamma$

&12 case when any two of these are equal

So, No. of distinct tuples $(\alpha, \beta, \gamma)$

$= 55 - 13 = 42$

**82. (1440)**

$(x^2 - 10x + 9) \leq 0 \Rightarrow (x-1)(x-9) \leq 0$

$\Rightarrow x \in [1,9] \Rightarrow A = \{1,2,3,4,5,6,7,8,9\}$

$f(x) \leq (x-3)^2 + 1$

$x = 1: f(1) \leq 5 \Rightarrow 1^2, 2^2$

$x = 2: f(2) \leq 2 \Rightarrow 1^2$

$x = 3: f(3) \leq 1 \Rightarrow 1^2$

$x = 4: f(4) \leq 2 \Rightarrow 1^2$

$x = 5: f(5) \leq 5 \Rightarrow 1^2, 2^2$

$x = 6: f(6) \leq 10 \Rightarrow 1^2, 2^2, 3^2$

$x = 7: f(7) \leq 17 \Rightarrow 1^2, 2^2, 3^2, 4^2$

$x = 8: f(8) \leq 26 \Rightarrow 1^2, 2^2, 3^2, 4^2, 5^2$

$x = 9: f(9) \leq 37 \Rightarrow 1^2, 2^2, 3^2, 4^2, 5^2, 6^2$

Total number of such function

$= 2(6!) = 2(720) = 1440$

**83.(24)**

$(3+6x)^n = {}^nC_0 3^n + {}^nC_1 3^{n-1}(6x)^1 + \cdots$

$T_{r+1}\ {}^nC_r 3^{n-r} \cdot (6x)r = {}^nC_r 3^{n-r} \cdot 6^r \cdot x^r$

$= {}^nC_r 3^{n-r} \cdot 3^r \cdot 2^r \cdot \left(\frac{3}{2}\right)^r = {}^nC_r 3^n \cdot 3^r\ [\quad \text{for } x = \frac{3}{2}]$

$T_9$ is greatest of $x = \frac{3}{2}$

So, $T_9 > T_{10}$ and $T_9 > T_8$

(concept of numerically greatest term)

Here, $\frac{T_9}{T_{10}} > 1$ and $\frac{T_9}{T_8} > 1$

$\Rightarrow \frac{{}^nC_8 3^n \cdot 3^8}{{}^nC_9 3^n \cdot 3^9} > 1$ and $\frac{{}^nC_8 3^n \cdot 3^8}{{}^nC_7 3^n \cdot 3^7} > 1$

and $\frac{{}^nC_8}{{}^nC_7} > \frac{1}{3}$

and $\frac{n-7}{8} > \frac{1}{3}$

$\Rightarrow \frac{29}{3} < n < 11 \Rightarrow n = 10 = n_0$

So, in $(3+6x)^n$ for $n = n_0 = 10$

i.e., in $(3+6x)^{10}$, here $T_{r+1} = {}^{10}C_r 3^{10-r} 6^r x^r$

$T_7 = {}^{10}C_6 3^4 \cdot 6^6 \cdot x^6 = 210 \cdot 3^{10} \cdot 2^6 x^6$

$T_4 = {}^{10}C_3 3^7 6^3 x^3 = 120.3^{10} \cdot 2^3 x^3$

Ratio of coefficient of $x^6$ and coefficient of $x^3 = k$

$\therefore k = \dfrac{210 \cdot 3 \cdot {}^{10}2^6}{120.3^{10} \cdot 2^3} = \dfrac{7}{4} \times 2^3 = 14$

So, $k + n_0 = 14 + 10 = 24$

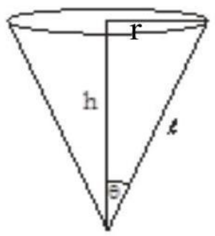
**84. (120)**

$T_n = \dfrac{2\sum_{r=1}^{n}(2r)^3 - \left(\sum_{r=1}^{2n} r^3\right)}{n(4n+3)}$

$\Rightarrow T_n = n$

So, $\sum_{n=1}^{15} T_n = 120$

**85. (5)**




$\tan\theta = \dfrac{3}{4} = \dfrac{r}{h}$

$\dfrac{dV}{dt} = 6$

$V = \dfrac{1}{3}\pi r^2 h = \dfrac{1}{3}\pi h^3 \tan^2\theta = \dfrac{9\pi}{48} h^3 = \dfrac{3\pi}{16} h^3$

$\Rightarrow \dfrac{dV}{dt} = \dfrac{3\pi}{16} \cdot 3h^2 \cdot \dfrac{dh}{dt} = 6 \Rightarrow \left(\dfrac{dh}{dt}\right)_{h=4} = \dfrac{2}{3\pi} m/hr$

Now, $S = \pi r \ell = \frac{15}{16}\pi h^2$

$\Rightarrow \dfrac{dS}{dt} = \dfrac{15\pi}{16} \cdot 2h \dfrac{dh}{dt}$

$\Rightarrow \left(\dfrac{dS}{dt}\right)_{h=4} = 5\ m^2/hr$

**86. (16)** $(\alpha, \alpha)$ lies on

$C: x^2 + y^2 - 3 + x^2 - y^2 - 1\ {}^5 = 0$

Put $(\alpha, \alpha), 2\alpha^2 - 3 + -1^5 = 0$

$\Rightarrow \alpha = \sqrt{2}$

Now, differentiate C

$2x + 2y \cdot y' + 5(x^2 - y^2 - 1)^4(2x - 2yy') = 0$

At $(\sqrt{2}, \sqrt{2})$
$\sqrt{2} + \sqrt{2}y' + 5(-1)^4(\sqrt{2} - \sqrt{2}y') = 0$

$\Rightarrow y' = \frac{3}{2}$

Diff. (1) w.r.t. x

Again, Diff. (1) w.r.t. x

$$1 + (y')^2 + yy'' + 20(x^2 - y^2 - 1)^3(x - yy')^2 \cdot 2$$
$$+ 5(x^2 - y^2 - 1)^4(1 - (y')^2 - yy'') = 0$$

At $(\sqrt{2}, \sqrt{2})$ and $y' = \frac{3}{2}$

We have,

$$\left(1 + \frac{9}{4}\right) + \sqrt{2}y'' - 40\left(\sqrt{2} - \sqrt{2} \cdot \frac{3}{2}\right)^2$$
$$+ 5(1)\left(1 - \frac{9}{4} - \sqrt{2}y''\right) = 0$$
$$\Rightarrow \; 4\sqrt{2}y'' = -23$$
$$\therefore \; 3y' - y^3y'' = \frac{9}{2} + \frac{23}{2} = 16$$

**87. (385)**

$f(x) = [x] - 10$

$\int_0^{10} f(x) \cdot dx = -10 - 9 - 8 - \cdots .. -1$

$= -\frac{10 \cdot 11}{2} = -55$

$\int_0^{10} (f(x))^2 dx = 10^2 + 9^2 + 8^2 + \cdots + 1^2$

$= \frac{10 \cdot 11 \cdot 21}{6} = 385$

$\int_0^{10} |f(x)| = 10 + 9 + 8 + \cdots . +1$

$= \frac{10 \cdot 11}{2} = 55$

$= -55 + 385 + 55 = 385$

**88. (12)** Let, $\frac{\lambda^2 x}{3} = t$

$\Rightarrow \frac{2\lambda x}{3} \, d\lambda = dt$

$\Rightarrow d\lambda = \frac{3}{2} \cdot \frac{1\sqrt{x}}{x \cdot \sqrt{3}\sqrt{t}} dt$

$\Rightarrow d\lambda = \frac{\sqrt{3}}{2} \cdot \frac{1}{\sqrt{x}} \cdot \frac{dt}{\sqrt{t}}$

So, $f(x)=\frac{1}{\sqrt{x}}\int_0^x \frac{f(t)}{\sqrt{t}}dt$

$\Rightarrow \sqrt{x}\cdot f'(x)+\frac{f(x)}{2\sqrt{x}}=\frac{f(x)}{\sqrt{x}}$

$\Rightarrow \sqrt{x}\cdot f'(x)=\frac{f(x)}{2\sqrt{x}}$

$\Rightarrow \frac{dy}{y}=\frac{dx}{2x}$

$\Rightarrow \ln y=\frac{1}{2}\ln x+c \Rightarrow f(x)=\sqrt{x}$

$\Rightarrow y=\sqrt{3x}$ as $f(1)=\sqrt{3}\}$

So, $f(x)=\sqrt{3x}$

Now, $f(\alpha)=6 \Rightarrow 36=3\alpha$

$\Rightarrow \alpha=12$

**89. (20)** Let common tangents are

$T_1: y=mx\pm\sqrt{4m^2+9}$

$\&T_2: y=mx\pm\sqrt{42m^2-13}$

So, $4\ m^2+9=42\ m^2-143$

$\Rightarrow 38\ m^2=152$

$\Rightarrow m=\pm2$

$\&c=\pm5$

For given tangent not pass through $4^{th}$ quadrant

$T: y=2x+5$

Now, comparing with $\frac{xx_1}{4}+\frac{yy_1}{9}=1$

We get, $\frac{x_1}{8}=-\frac{1}{5}\Rightarrow x_1=-\frac{8}{5}$

$\frac{xx_2}{42}-\frac{yy_2}{143}=1$

$2x-y=-5$ we have

$x_2=-\frac{84}{5}$

So, $|2x_1+x_2|=\left|\frac{-100}{5}\right|=20$

**90. (36)**

$\vec{a} \times \vec{b} = 4\vec{c} \Rightarrow \vec{a} \cdot \vec{c} = 0 = \vec{b} \cdot \vec{c}$

$\vec{b} \times \vec{c} = 9\vec{a} \Rightarrow \vec{a} \cdot \vec{b} = 0 = \vec{a} \cdot \vec{c}$

$\therefore \vec{a}, \vec{b}, \vec{c}$ are mutually $\perp$ set of vectors.

$\Rightarrow |\vec{a}||\vec{b}| = 4|\vec{c}|, |\vec{b}||\vec{c}| = 9|\vec{a}| \& |\vec{c}||\vec{a}| = \alpha|\vec{b}|$

$$\Rightarrow \frac{|\vec{a}|}{|\vec{c}|} = \frac{4}{9}\frac{|\vec{c}|}{|\vec{a}|}$$

$$\Rightarrow \frac{|\vec{c}|}{|\vec{a}|} = \frac{3}{2}$$

$\therefore$ If $|a| = \lambda, |c| = \frac{3\lambda}{2} \&| b| = 6$

Now $|a| + |b| + |c| = \frac{1}{36}$

$$\Rightarrow \frac{5}{2}\lambda + 6 = \frac{1}{36}, \lambda = \frac{-43}{18} = |a|$$

which gives negative value of $\lambda$ or $|a|$ which is NOT possible & hence data seems to be wrong.

But if $|\vec{a}| + |\vec{b}| + |\vec{c}| = 36$

$$\frac{5}{2}\lambda + 6 = 36$$

$\lambda = 12$

$$\alpha = \frac{|\vec{c}||\vec{a}|}{|\vec{b}|} = \frac{3 \times 12}{2} \times \frac{12}{6}$$

$\alpha = 36$