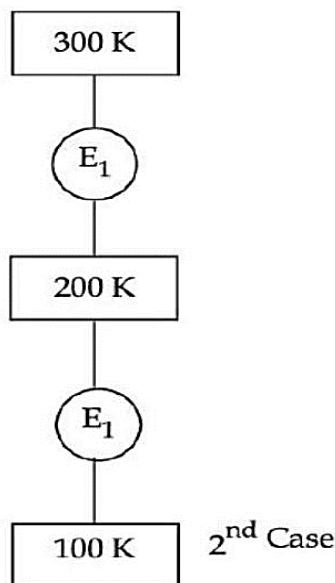


PART-I PHYSICS

SECTION-A

- An expression of energy density is given by $u = \frac{\alpha}{\beta} \sin\left(\frac{\alpha x}{kt}\right)$, where α, β are constants, x is displacement, k is Boltzmann constant and t is the temperature. The dimensions of β will be :
 (a) $[ML^2 T^{-2} \theta^{-1}]$ (b) $[M^0 L^2 T^{-2}]$
 (c) $[M^0 L^0 T^0]$ (d) $[M^0 L^2 T^0]$
- A body of mass 10 kg is projected at an angle of 45° with the horizontal. The trajectory of the body is observed to pass through a point (20, 10). If T is the time of flight, then its momentum vector, at time $t = \frac{T}{\sqrt{2}}$ is [Take $g = 10 \text{ m/s}^2$]
 (a) $100\hat{i} + (100\sqrt{2} - 200)\hat{j}$
 (b) $100\sqrt{2}\hat{i} + (100 - 200\sqrt{2})\hat{j}$
 (c) $100\hat{i} + (100 - 200\sqrt{2})\hat{j}$
 (d) $100\sqrt{2}\hat{i} + (100\sqrt{2} - 200)\hat{j}$
- A block of mass M slides down on a rough inclined plane with constant velocity. The angle made by the incline plane with horizontal is θ . The magnitude of the contact force will be:
 (a) Mg (b) $Mg \cos \theta$
 (c) $\sqrt{Mg \sin \theta + Mg \cos \theta}$ (d) $Mg \sin \theta \sqrt{1 + \mu}$
- A block 'A' takes 2 s to slide down a frictionless incline of 30° and length 'l', kept inside a lift going up with uniform velocity 'v'. If the incline is changed to 45° , the time taken by the block, to slide down the incline, will be approximately:
 (a) 2.66 s (b) 0.83 s
 (c) 1.68 s (d) 0.70 s
- The velocity of the bullet becomes one third after it penetrates 4 cm in a wooden block. Assuming that bullet is facing a constant resistance during its motion in the block. The bullet stops completely after travelling at $(4 + x)$ cm inside the block. The value of x is:
 (a) 2.0 (b) 1.0
 (c) 0.5 (d) 1.5
- A body of mass m is projected with velocity λv_e in vertically upward direction from the surface of the earth into space. It is given that v_e is escape velocity and $\lambda < 1$. If air resistance is considered to be negligible, then the maximum height from the centre of earth, to which the body can go, will be (R : radius of earth)
 (a) $\frac{R}{1+\lambda^2}$ (b) $\frac{R}{1-\lambda^2}$
 (c) $\frac{R}{1-\lambda}$ (d) $\frac{\lambda^2 R}{1-\lambda^2}$
- A steel wire of length 3.2 m ($Y_s = 2.0 \times 10^{11} \text{ Nm}^{-2}$) and a copper wire of length 4.4 m ($Y_c = 1.1 \times 10^{11} \text{ Nm}^{-2}$), both of radius 1.4 mm are connected end to end. When stretched by a load, the net elongation is found to be 1.4 mm. The load applied, in Newton, will be: (Given $\pi = \frac{22}{7}$)
 (a) 360 (b) 180
 (c) 1080 (d) 154
- In 1st case, Carnot engine operates between temperatures 300 K and 100 K. In 2nd case, as shown in the figure, a combination of two engines is used. The efficiency of this combination (in 2nd case) will be:



- (a) same as the 1st case
- (b) always greater than the 1st case
- (c) always less than the 1st case
- (d) may increase or decrease with respect to the 1st case

9. Which statements are correct about degrees of freedom?

A. A molecule with n degrees of freedom has n^2 different ways of storing energy.

B. Each degree of freedom is associated with $\frac{1}{2} RT$ average energy per mole.

C. A monoatomic gas molecule has 1 rotational degree of freedom whereas diatomic molecule has 2 rotational degrees of freedom

D. CH_4 has a total to 6 degrees of freedom Choose the correct answer from the option given below:

- (a) B and C only (b) B and D only
- (c) A and B only (d) C and D only

10. A charge of $4\mu\text{C}$ is to be divided into two. The distance between the two divided charges is constant. The magnitude of the divided charges so that the force between them is maximum, will be:

- (a) $1\mu\text{C}$ and $3\mu\text{C}$ (b) $2\mu\text{C}$ and $2\mu\text{C}$
- (c) 0 and $4\mu\text{C}$ (d) $1.5\mu\text{C}$ and $2.5\mu\text{C}$

11. The drift velocity of electrons decreases with the increase in the temperature of conductor.

B. The drift velocity is inversely proportional to the area of cross-section of given conductor.

C. The drift velocity does not depend on the applied potential difference to the conductor.

D. The drift velocity of electron is inversely proportional to the length of the conductor.

E. The drift velocity increases with the increase in the temperature of conductor.

Choose the correct answer from the options given below:

- (a) A and B only (b) A and D only
- (c) B and E only (d) B and C only

12. A compass needle of oscillation magnetometer oscillates 20 times per minute at a place P of dip 30° . The number of oscillations per minute become 10 at another place Q of 60° dip. The ratio of the total magnetic field at the two places ($B_Q : B_P$) is:
- (a) $\sqrt{3}:4$ (b) $4:\sqrt{3}$
 (c) $\sqrt{3}:2$ (d) $2:\sqrt{3}$
13. A cyclotron is used to accelerate protons. If the operating magnetic field is 1.0 T and the radius of the cyclotron 'dees' is 60 cm, the kinetic energy of the accelerated protons in MeV will be: [use $m_p = 1.6 \times 10^{-27}$ kg, $e = 1.6 \times 10^{-19}$ C]
- (a) 12 (b) 18
 (c) 16 (d) 32
14. A series LCR circuit has $L = 0.01$ H, $R = 10\Omega$ and $C = 1\mu\text{F}$ and it is connected to ac voltage of amplitude (V_m) 50 V. At frequency 60% lower than resonant frequency, the amplitude of current will be approximately:
- (a) 466 mA (b) 312 mA
 (c) 238 mA (d) 196 mA
15. Identify the correct statements from the following descriptions of various properties of electromagnetic waves.

A. In a plane electromagnetic wave electric field and magnetic field must be perpendicular to each other and direction of propagation of wave should be along electric field or magnetic field.

B. The energy in electromagnetic wave is divided equally between electric and magnetic fields.

C. Both electric field and magnetic field are parallel to each other and perpendicular to the direction of propagation of wave.

D. The electric field, magnetic field and direction of propagation of wave must be perpendicular to each other.

E. The ratio of amplitude of magnetic field to the amplitude of electric field is equal to speed of light.

Choose the most appropriate answer from the options given below:

- (a) D only (b) B and D only
 (c) B, C and E only (d) A, B and E only

16. Two coherent sources of light interfere. The intensity ratio of two sources is 1: 4. For this interference pattern if the value of $\frac{I_{\max} + I_{\min}}{I_{\max} - I_{\min}}$ is equal to $\frac{2\alpha + 1}{\beta + 3}$, then $\frac{\alpha}{\beta}$ will be :

- (a) 1.5 (b) 2
 (c) 0.5 (d) 1

17. With reference to the observations in photo-electric effect, identify the correct statements from below:

A. The square of maximum velocity of photoelectrons varies linearly with frequency of incident light.

B. The value of saturation current increases on moving the source of light away from the metal surface.

C. The maximum kinetic energy of photo-electrons decreases on decreasing the power of LED (light emitting diode) source of light.

D. The immediate emission of photo-electrons out of metal surface cannot be explained by particle nature of light/electromagnetic waves.

E. Existence of threshold wavelength cannot be explained by wave nature of light/electromagnetic waves.

Choose the correct answer from the options given below:

- (a) A and B only (b) A and E only
(c) C and E only (d) D and E only

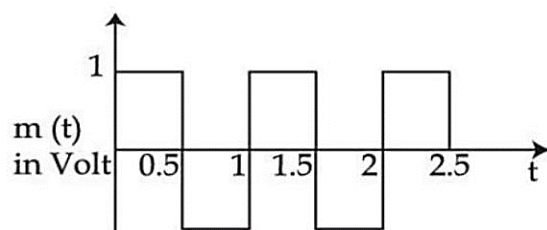
18. The activity of a radioactive material is 6.4×10^{-4} curie. Its half life is 5 days. The activity will become 5×10^{-6} curie after:

- (a) 7 days (b) 15 days
(c) 25 days (d) 35 days

19. For a constant collector-emitter voltage of 8 V, the collector current of a transistor reached to the value of 6 mA from 4 mA, whereas base current changed from $20 \mu\text{A}$ to $25 \mu\text{A}$ value. If transistor is in active state, small signal current gain (current amplification factor) will be:

- (a) 240 (b) 400
(c) 0.0025 (d) 200

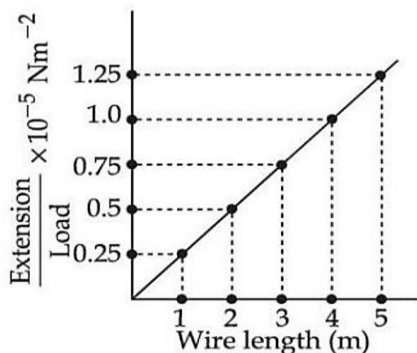
20. A square wave of the modulating signal is shown in the figure. The carrier wave is given by $C(t) = 5\sin(8\pi t)$ Volt. The modulation index is:



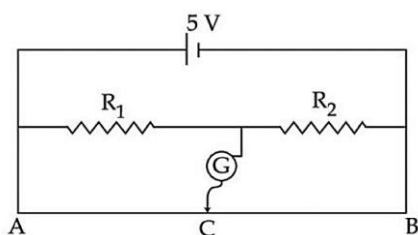
- (a) 0.2 (b) 0.1
(c) 0.3 (d) 0.4

SECTION-B

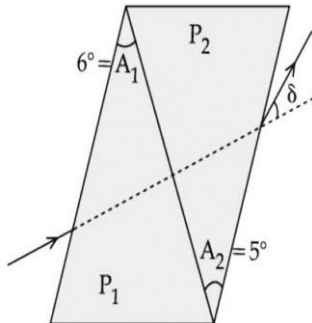
21. In an experiment to determine the Young's modulus, steel wires of five different lengths (1, 2, 3, 4 and 5 m) but of same cross section (2 mm^2) were taken and curves between extension and load were obtained. The slope (extension/load) of the curves were plotted with the wire length and the following graph is obtained. If the Young's modulus of given steel wires is $x \times 10^{11} \text{ Nm}^{-2}$, then the value of x is



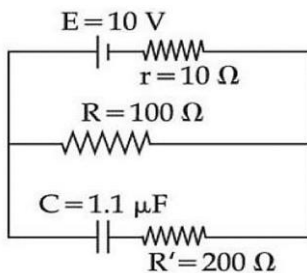
22. In the given figure of meter bridge experiment, the balancing length AC corresponding to null deflection of the galvanometer is 40 cm. The balancing length, if the radius of the wire AB is doubled, will be . cm.



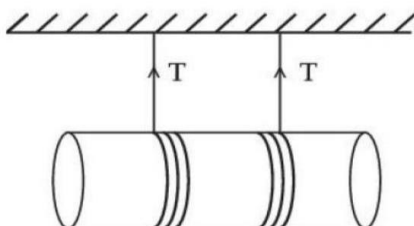
23. A thin prism of angle 6° and refractive index for yellow light (n_Y) 1.5 is combined with another prism of angle 5° and $n_Y = 1.55$. The combination produces no dispersion. The net average deviation (δ) produced by the combination is $\left(\frac{1}{x}\right)^\circ$. The value of x is.....



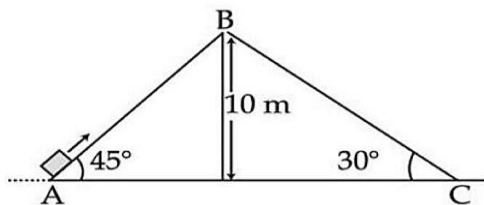
24. A conducting circular loop is placed in $X - Y$ plane in presence of magnetic field $\vec{B} = (3t^3\hat{j} + 3t^2\hat{k})$ in SI unit. If the radius of the loop is 1 m, the induced emf in the loop, at time, $t = 2$ s is $n\pi$ V. The value of n is
25. As show in the figure, in steady state, the charge stored in the capacitor is.... $\times 10^{-6}$ C.



26. A parallel plate capacitor with width 4 cm, length 8 cm and separation between the plates of 4 mm is connected to a battery of 20 V. A dielectric slab of dielectric constant 5 having length 1 cm, width 4 cm and thickness 4 mm is inserted between the plates of parallel plate capacitor. The electrostatic energy of this system will be ϵ_0 J. (Where ϵ_0 is the permittivity of free space)
27. A wire of length 30 cm, stretched between rigid supports, has it's n^{th} and $(n + 1)^{\text{th}}$ harmonics at 400 Hz and 450 Hz, respectively. If tension in the string is 2700 N, it's linear mass density is..... kg/m
28. A spherical soap bubble of radius 3 cm is formed inside another spherical soap bubble of radius 6 cm. If the internal pressure of the smaller bubble of radius 3 cm in the above system is equal to the internal pressure of the- another single soap bubble of radius r cm. The value of r is
29. A solid cylinder length is suspended symmetrically through two massless strings, as shown in the figure. The distance from the initial rest position, the cylinder should by unbinding the strings to achieve a speed of 4 ms^{-1} , is.....cm. (take $g = 10 \text{ ms}^{-2}$)



30. Two inclined planes are placed as shown in figure. A block is projected from the Point A of inclined plane AB along its surface with a velocity just sufficient to carry it to the top Point B at a height 10 m. After reaching the Point B the block slides down on inclined plane BC. Time it takes to reach to the point C from point A is $t(\sqrt{2} + 1)s$. The value of t is.... (use $g = 10 \text{ m/s}^2$)



PART-II CHEMISTRY

SECTION-A

31. The correct decreasing order of energy, for the orbitals having, following set of quantum numbers:

- (A) $n = 3, l = 0, m = 0$ (B) $n = 4, l = 0, m = 0$
 (C) $n = 3, l = 1, m = 0$ (D) $n = 3, l = 2, m = 1$
 (a) $(D) > (B) > (C) > (A)$ (b) $(B) > (D) > (C) > (A)$
 (c) $(C) > (B) > (D) > (A)$ (d) $(B) > (C) > (D) > (A)$

32. Match List-I with List-II

List-I

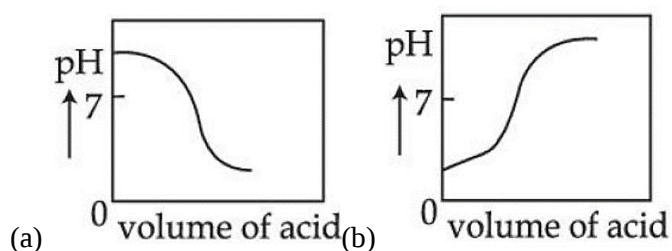
- (A) $\Psi_{MO} = \Psi_A - \Psi_B$
 (b) $\mu = Q \times r$
 (c) $\frac{N_b - N_a}{2}$

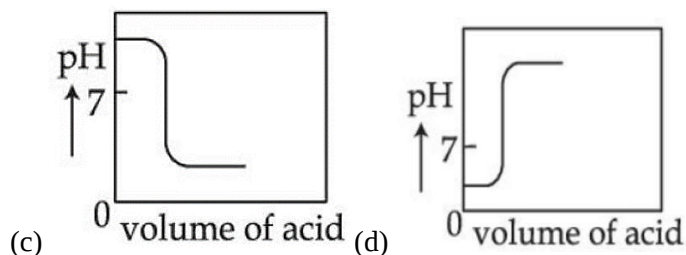
LIST-II

- (I) Dipole moment
 (II) Bonding molecular orbital
 (III) Anti-bonding molecular orbital
 (IV) Bond order

- (a) (A)-(II), (B)-(I), (C)-(IV), (D)-(III)
 (b) (A)-(III), (B)-(IV), (C)-(I), (D)-(II)
 (c) (A)-(III), (B)-(I), (C)-(IV), (D)-(II)
 (d) (A)-(III), (B)-(IV), (C)-(II), (D)-(I)

33. The Plot of pH-metric titration of weak base NH_4OH vs strong acid HCl looks like:





34. Given below are two statements:

Statement I: For KI, molar conductivity increases steeply with dilution.

Statement II: For carbonic acid, molar conductivity increases slowly with dilution.

In the light of the above statements, choose the correct answer from the options given below:

- (a) Both Statement I and Statement II are true
- (b) Both Statement I and Statement II are false
- (c) Statement I is true but Statement II is false
- (d) Statement I is false but Statement II is true

35. Given below are two statements: one is labelled as Assertion (a) and the other is labelled as Reason (R)

Assertion (a): Dissolved substances can be removed from a colloidal solution by diffusion through a parchment paper.

Reason (R): Particles in a true solution cannot pass through parchment paper but the colloidal particles can pass through the parchment paper.

In the light of the above statements, choose the correct answer from the options given below:

- (a) Both (a) and (R) are correct and (R) is the correct explanation of (a)
- (b) Both (a) and (R) are correct but (R) is not the correct explanation of (A)
- (c) (a) is correct but (R) is not correct
- (d) (a) is not correct but (R) is correct

36. Outermost electronic configurations of four elements A, B, C, D are given below:

(A) $3s^2$

(B) $3s^23p^1$

(C) $3s^23p^3$

(D) $3s^23p^4$

The correct order of first ionization enthalpy for them is:

- (a) $(A) < (B) < (C) < (D)$
- (b) $(B) < (A) < (D) < (C)$
- (c) $(B) < (D) < (A) < (C)$
- (d) $(B) < (A) < (C) < (D)$

37. An element A of group 1 shows similarity to an element B belonging to group 2. If A has maximum hydration enthalpy in group 1 then B is:

- (a) Mg (b) Be
(c) Ca (d) Sr

38. Given below are two statements: one is labelled as Assertion (a) and the other is labelled as Reason (R)

Assertion (a): Boron is unable to form BF_6^{3-}

Reason (R): Size of B is very small.

In the light of the above statements, choose the correct answer from the options given below:

- (a) Both (A) and (R) are true and (R) is the correct explanation of (A)
(b) Both (A) and (R) are true but (R) is not the correct explanation of (A)
(c) (A) is true but (R) is false
(d) (A) is false but (R) is true

39. In neutral or alkaline solution, MnO_4^- oxidises thiosulphate to:

- (a) $\text{S}_2\text{O}_7^{2-}$ (b) $\text{S}_2\text{O}_8^{2-}$
(c) SO_3^{2-} (d) SO_4^{2-}

40. Low oxidation state of metals in their complexes are common when ligands:

- (a) have good π -accepting character
(b) have good σ -donor character
(c) are having good π -donating ability
(d) are having poor σ -donating ability

41. Given below are two statements:

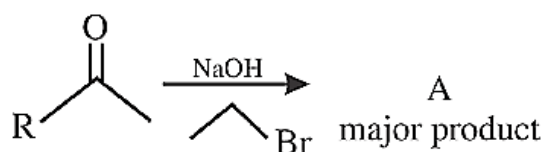
Statement I : The non-bio-degradable fly ash and slag from steel industry can be used by cement industry.

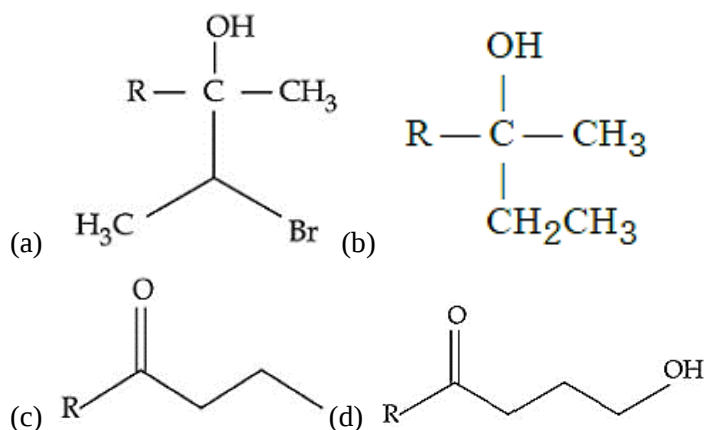
Statement II: The fuel obtained from plastic waste is lead free.

In the light of the above statements, choose the most appropriate answer from the options given below:

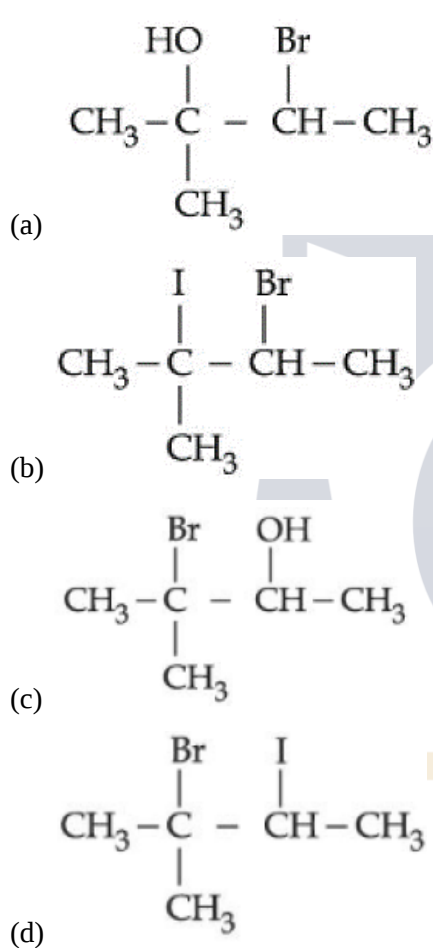
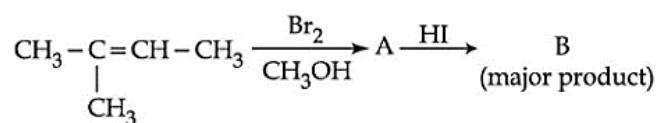
- (a) Both Statement I and Statement II are correct
(b) Both Statement I and Statement II are incorrect
(c) Statement I is correct but Statement II is incorrect
(d) Statement I is incorrect but Statement II is correct

42. The structure of A in the given reaction is:





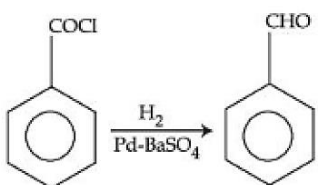
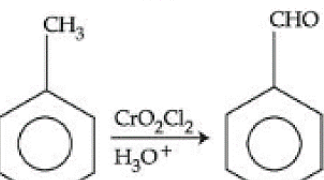
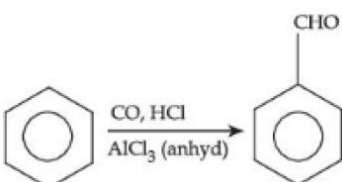
43. Major product 'B' of the following reaction sequence is



44. Match List-I with List-II.

LIST-I

LIST-II

- (A)  (I) Gatterman Koch reaction
- (B) $\text{CH}_3\text{-CN} \xrightarrow[\text{H}_3\text{O}^+]{\text{SnCl}_2/\text{HCl}} \text{CH}_3\text{-CHO}$ (II) Etard reaction
- (C)  (III) Stephen reaction
- (D)  (IV) Rosenmund reaction

Choose the correct answer from the options given below:

- (a) (A)-(IV), (B)-(III), (C)-(II), (D)-(I)
 (b) (A)-(I), (B)-(II), (C)-(III), (D)-(IV)
 (c) (A)-(II), (B)-(III), (C)-(IV), (D)-(I)
 (d) (A)-(III), (B)-(II), (C)-(I), (D)-(IV)

45. Match List-I with List-II.

List-I

(Polymer)

(A) Neoprene

(B) Teflon

(C) Acrilan

(D) Natural rubber

List-II

(Monomer)

(I) Acrylonitrile

(II) Chloroprene

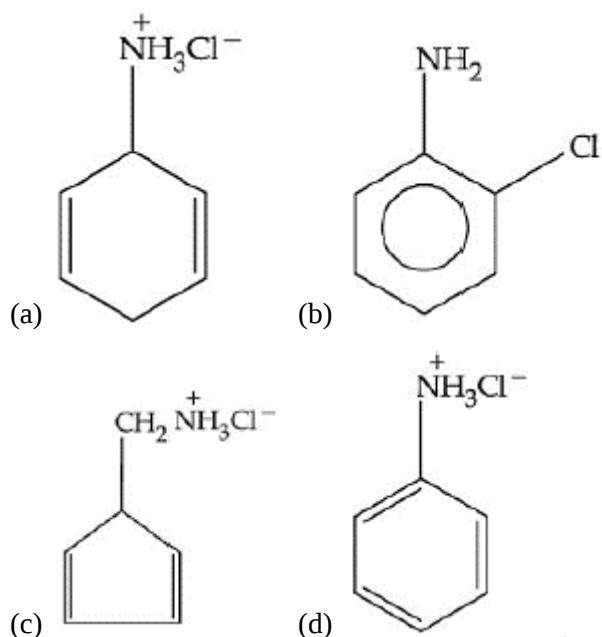
(III) Tetrafluoroethene

(IV) Isoprene

Choose the correct answer from the option given below:

- (a) (A)-(II), (B)-(III), (C)-(I), (D)-(IV)
 (b) (A)-(II), (B)-(I), (C)-(III), (D)-(IV)
 (c) (A)-(II), (B)-(I), (C)-(IV), (D)-(III)
 (d) (A)-(I), (B)-(II), (C)-(III), (D)-(IV)

46. An organic compound 'A' contains nitrogen and chlorine. It dissolves readily in water to give a solution that turns litmus red. Titration of compound 'A' with standard base indicates that the molecular weight of 'A' is 131 ± 2 . When a sample of 'A' is treated with aq. NaOH, a liquid separates which contains N but not Cl. Treatment of the obtained liquid with nitrous acid followed by phenol gives orange precipitate. The compound 'A' is:



47. Match List-I with List-II

List-I

(A) Glucose + HI

(B) Glucose + Br₂ water

(C) Glucose + acetic anhydride

(D) Glucose + HNO₃

List-II

(I) Gluconic acid

(II) Glucose pentacetate

(III) Saccharic acid

(IV) Hexane

Choose the correct answer from the options given below:

(a) (A)-(IV), (B)-(I), (C)-(II), (D)-(III)

(b) (A)-(IV), (B)-(III), (C)-(II), (D)-(I)

(c) (A)-(III), (B)-(I), (C)-(IV), (D)-(II)

(d) (A)-(I), (B)-(III), (C)-(IV), (D)-(II)

48. Which of the following enhances the lathering property of soap?

(a) Sodium stearate (b) Sodium carbonate

(c) Sodium rosinate (d) Trisodium phosphate

49. Match List-I with List-II

List-I

(Mixture)

(A) Chloroform & Aniline

(B) Benzoic acid & Napthalene

List-II

(Purification Process)

(I) Steam distillation

(II) Sublimation

- (C) Water & Aniline (III) Distillation
(D) Napthalene & Sodium chloride (IV) Crystallisation

- (a) (A)-(IV), (B)-(III), (C)-(I), (D)-(II)
(b) (A)-(III), (B)-(I), (C)-(IV), (D)-(II)
(c) (A)-(III), (B)-(IV), (C)-(II), (D)-(I)
(d) (A)-(III), (B)-(IV), (C)-(I), (D)-(II)

50. Fe^{3+} cation gives a prussian blue precipitate on addition of potassium ferrocyanide solution due to the formation of:

- (a) $[\text{Fe}(\text{H}_2\text{O})_6]_2[\text{Fe}(\text{CN})_6]$ (b) $\text{Fe}_2[\text{Fe}(\text{CN})_6]_2$
(c) $\text{Fe}_3[\text{Fe}(\text{OH})_2(\text{CN})_4]_2$ (d) $\text{Fe}_4[\text{Fe}(\text{CN})_6]_3$

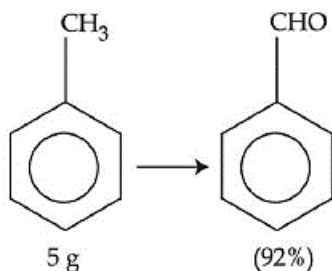
SECTION-B

51. The normality of H_2SO_4 in the solution obtained on mixing 100 mL of 0.1M H_2SO_4 with 50 mL of 0.1 MNaOH is $\times 10^{-1}$ N. (Nearest Integer)
52. for a real gas at 25°C temperature and high pressure (99 bar) the value of compressibility factor is 2, so the value of Vander Waal's constant ' b ' should be $\times 10^{-2}$ L mol $^{-1}$ (Nearest integer) (Given R = 0.083 L bar K $^{-1}$ mol $^{-1}$)
53. A gas (Molar mass = 280 g mol $^{-1}$) was burnt in excess O_2 in a constant volume calorimeter and during combustion the temperature of calorimeter increased from 298.0 K to 298.45 K. If the heat capacity of calorimeter is 2.5 kJ K $^{-1}$ and enthalpy of combustion of gas is 9 kJ mol $^{-1}$ then amount of gas burnt is g. (Nearest Integer)
54. When a certain amount of solid A is dissolved in 100 g of water at 25°C to make a dilute solution, the vapour pressure of the solution is reduced to one-half of that of pure water. The vapour pressure of pure water is 23.76mmHg. The number of moles of solute A added is (Nearest Integer)
55. [A] $\rightarrow$ B
Reactant Product
- If formation of compound [B] follows the first order of kinetics and after 70 minutes the concentration of [A] was found to be half of its initial concentration. Then the rate constant of the reaction is $x \times 10^{-6}$ s $^{-1}$. The value of x is.....(Nearest Integer)
56. Among the following ores Bauxite, Siderite, Cuprite, Calamine, Haematite, Kaolinite, Malachite, Magnetite, Sphalerite, Limonite, Cryolite, the number of principal ores if (of) iron is
57. The oxidation state of manganese in the product obtained in a reaction of potassium permanganate and hydrogen peroxide in basic medium is
58. The number of molecule(s) or ion(s) from the following having non-planar structure is

$\text{NO}_3^-, \text{H}_2\text{O}_2, \text{BF}_3, \text{PCl}_3, \text{XeF}_4,$
 $\text{SF}_4, \text{XeO}_3, \text{PH}_4^+, \text{SO}_3, [\text{Al}(\text{OH})_4]^-$

59. The spin only magnetic moment of the complex present in Fehling's reagent is B.M. (Nearest integer).

60.



In the above reaction, 5 g of toluene is converted into benzaldehyde with 92% yield. The amount of benzaldehyde produced is..... $\times 10^{-2}$ g. (Nearest integer)

PART-III MATHEMATICS

SECTION-A

61. The domain of the function

$$f(x) = \sin^{-1}[2x^2 - 3] + \log_2 \left(\log_{\frac{1}{2}}(x^2 - 5x + 5) \right),$$

where $[t]$ is the greatest integer function, is:

- (a) $\left(-\sqrt{\frac{5}{2}}, \frac{5-\sqrt{5}}{2}\right)$ (b) $\left(\frac{5-\sqrt{5}}{2}, \frac{5+\sqrt{5}}{2}\right)$
 (c) $\left(1, \frac{5-\sqrt{5}}{2}\right)$ (d) $\left[1, \frac{5+\sqrt{5}}{2}\right)$

62. Let S be the set of all (α, β) , $\pi < \alpha, \beta < 2\pi$, for which the complex number $\frac{1-i\sin\alpha}{1+2i\sin\alpha}$ is purely imaginary and $\frac{1+i\cos\beta}{1-2i\cos\beta}$ is purely real. Let $Z_{\alpha\beta} = \sin 2\alpha + i\cos 2\beta$, $(\alpha, \beta) \in S$.

Then $\sum_{(\alpha,\beta) \in S} \left(iZ_{\alpha\beta} + \frac{1}{iZ_{\alpha\beta}} \right)$ is equal to:

- (a) 3 (b) 3i
 (c) 1 (d) 2 - i

63. If α, β are the roots of the equation $x^2 - \left(5 + 3\sqrt{\log_3 5} - 5\sqrt{\log_5 3}\right) + 3 \left(3^{(\log_3 5)^{\frac{1}{3}}} - 5^{(\log_5 3)^{\frac{2}{3}}} - 1\right) = 0$ then the equation, whose roots are $\alpha + \frac{1}{\beta}$ and $\beta + \frac{1}{\alpha}$,

- (a) $3x^2 - 20x - 12 = 0$ (b) $3x^2 - 10x - 4 = 0$
 (c) $3x^2 - 10x + 2 = 0$ (d) $3x^2 - 20x + 16 = 0$

64. Let $A = \begin{pmatrix} 4 & -2 \\ \alpha & \beta \end{pmatrix}$

If $A^2 + \gamma A + 18I = O$, then $\det(a)$ is equal to

- (a) -18 (b) 18
(c) -50 (d) 50

65. If for $p \neq q \neq 0$, then function

$f(x) = \frac{\sqrt[7]{p(729+x)}-3}{\sqrt[3]{729+qx}-9}$ is continuous at $x = 0$, then:

- (a) $7pqf(0) - 1 = 0$ (b) $63qf(0) - p^2 = 0$
(c) $21qf(0) - p^2 = 0$ (d) $7pqf(0) - 9 = 0$

66. Let $f(x) = 2 + |x| - |x - 1| + |x + 1|$, $x \in \mathbb{R}$.

Consider

(S1): $f'(-\frac{3}{2}) + f'(-\frac{1}{2}) + f'(\frac{1}{2}) + f'(\frac{3}{2}) = 2$

(S2): $\int_{-2}^2 f(x) dx = 12$

Then,

- (a) both (S1) and (S2) are correct
(b) both (S1) and (S2) are wrong
(c) only (S1) is correct
(d) only (S2) is correct

67. Let the sum of an infinite G.P., whose first term is a and the common ratio is r , be 5. Let the sum of its first five terms be $\frac{98}{25}$. Then the sum of the first 21 terms of an AP, whose first term is $10ar$, n^{th} term is a_n and the common difference is $10ar^2$, is equal to:

- (a) $21a_{11}$ (b) $22a_{11}$
(c) $15a_{16}$ (d) $14a_{16}$

68. The area of the region enclosed by $y \leq 4x^2$, $x^2 \leq 9y$ and $y \leq 4$, is equal to:

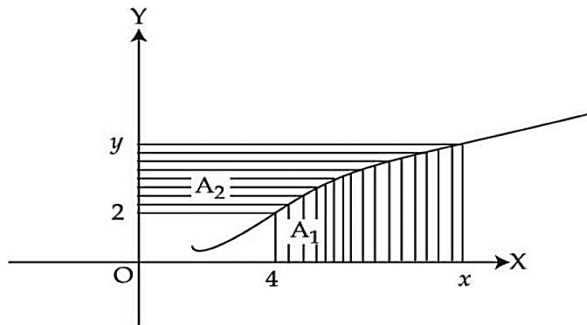
- (a) $\frac{40}{3}$ (b) $\frac{56}{3}$
(c) $\frac{112}{3}$ (d) $\frac{80}{3}$

69. $\int_0^2 \left(|2x^2 - 3x| + \left[x - \frac{1}{2} \right] \right) dx$,

where $[t]$ is the greatest integer function, is equal to:

- (a) $\frac{7}{6}$ (b) $\frac{19}{12}$
(c) $\frac{31}{12}$ (d) $\frac{3}{2}$

70. Consider a curve $y = y(x)$ in the first quadrant as shown in the figure. Let the area A_1 is twice the area A_2 . Then the normal to the curve perpendicular to the line $2x - 12y = 15$ does NOT pass through the point.



- (a) (6,21) (b) (8,9)
(c) (10,-4) (d) (12,-15)

71. The equations of the sides AB, BC and CA of a triangle ABC are $2x + y = 0$, $x + py = 39$ and $x - y = 3$ respectively and $P(2, 3)$ is its circumcentre. Then which of the following is NOT true:

- (a) $(AC)^2 = 9p$
(b) $(AC)^2 + p^2 = 136$
(c) $32 < \text{area}(\triangle ABC) < 36$
(d) $34 < \text{area}(\triangle ABC) < 38$

72. A circle C_1 passes through the origin O and has diameter 4 on the positive x-axis. The line $y = 2x$ gives a chord OA of a circle C_1 . Let C_2 be the circle with OA as a diameter. If the tangent to C_2 at the point A meets the x-axis at P and y-axis at Q, then QA: AP is equal to:

- (a) 1: 4 (b) 1: 5
(c) 2: 5 (d) 1: 3

73. If the length of the latus rectum of a parabola, whose focus is (a, a) and the tangent at its vertex is $x + y = a$, is 16, then $|a|$ is equal to :

- (a) $2\sqrt{2}$ (b) $2\sqrt{3}$
(c) $4\sqrt{2}$ (d) 4

74. If the length of the perpendicular drawn from the point $P(a, 4, 2)$, $a > 0$ on the line $\frac{x+1}{2} = \frac{y-3}{3} = \frac{z-1}{-1}$ is $2\sqrt{6}$ units and $Q(\alpha_1, \alpha_2, \alpha_3)$ is the image of the point P in this line, then $a + \sum_{i=1}^3 \alpha_i$ is equal to:

- (a) 7 (b) 8
(c) 12 (d) 14

75. If the line of intersection of the planes $ax + by = 3$ and $ax + by + cz = 0$, $a > 0$ makes an angle 30° with the plane $y - z + 2 = 0$, then the direction cosines of the line are :

- (a) $\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0$ (b) $\frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}, 0$
(c) $\frac{1}{\sqrt{5}}, -\frac{2}{\sqrt{5}}, 0$ (d) $\frac{1}{2}, -\frac{\sqrt{3}}{2}, 0$

76. Let X have a binomial distribution $B(n, p)$ such that the sum and the product of the mean and variance of X are 24 and 128 respectively. If $P(X > n - 3) = \frac{k}{2^n}$, then k is equal to

- (a) 528 (b) 529
(c) 629 (d) 630

77. A six faced die is biased such that $3 \times P(\text{a prime number}) = 6 \times P(\text{a composite number}) = 2 \times P(1)$.

Let X be a random variable that counts the number of times one gets a perfect square on some throws of this die. If the die is thrown twice, then the mean of X is:

- (a) $\frac{3}{11}$ (b) $\frac{5}{11}$
(c) $\frac{7}{11}$ (d) $\frac{8}{11}$

78. The angle of elevation of the top P of a vertical tower PQ of height 10 from a point A on the horizontal ground is 45° . Let R be a point on AQ and from a point B, vertically above R, the angle of elevation of P is 60° . If $\angle BAQ = 30^\circ$, $AB = d$ and the area of the trapezium PQRB is α , then the ordered pair (d, α) is :

- (a) $(10(\sqrt{3} - 1), 25)$ (b) $(10(\sqrt{3} - 1), \frac{25}{2})$
(c) $(10(\sqrt{3} + 1), 25)$ (d) $(10(\sqrt{3} + 1), \frac{25}{2})$

79. Let $S = \left\{ \theta \in \left(0, \frac{\pi}{2}\right) : \sum_{m=1}^9 \sec\left(\theta + (m-1)\frac{\pi}{6}\right) \sec\left(\theta + \frac{m\pi}{6}\right) = -\frac{8}{\sqrt{3}} \right\}$

Then

- (a) $S = \left\{ \frac{\pi}{12} \right\}$ (b) $S = \left\{ \frac{2\pi}{3} \right\}$
(c) $\sum_{\theta \in S} \theta = \frac{\pi}{2}$ (d) $\sum_{\theta \in S} \theta = \frac{3\pi}{4}$

80. If the truth value of the statement

$(P \wedge (\sim R)) \rightarrow ((\sim R) \wedge Q)$ is F, then the truth value of which of the following is F ?

- (a) $P \vee Q \rightarrow \sim R$ (b) $R \vee Q \rightarrow \sim P$
(c) $\sim (P \vee Q) \rightarrow \sim R$ (d) $\sim (R \vee Q) \rightarrow \sim P$

SECTION-B

81. Consider a matrix $A = \begin{bmatrix} \alpha & \beta & \gamma \\ \alpha^2 & \beta^2 & \gamma^2 \\ \beta + \gamma & \gamma + \alpha & \alpha + \beta \end{bmatrix}$, where α, β, γ are three distinct natural numbers. If

$\frac{\det(\text{adj}(\text{adj}(\text{adj}(\text{adj} A))))}{(\alpha - \beta)^{16}(\beta - \gamma)^{16}(\gamma - \alpha)^{16}} = 2^{32} \times 3^{16}$, then the number of such 3 - tuples (α, β, γ) is

82. The number of functions f , from the set

$A = \{x \in \mathbb{N} : x^2 - 10x + 9 \leq 0\}$ to the set

$B = \{n^2 : n \in \mathbb{N}\}$ such that $f(x) \leq (x - 3)^2 + 1$, for every $x \in A$, is

83. Let for the 9^{th} term in the binomial expansion of $(3 + 6x)^n$, in the increasing powers of $6x$, to be the greatest for $x = \frac{3}{2}$, the least value of n is n_0 . If k is the ratio of the coefficient of x^6 to the coefficient of x^3 , then $k + n_0$ is equal to:

84. $\frac{2^3 - 1^3}{1 \times 7} + \frac{4^3 - 3^3 + 2^3 - 1^3}{2 \times 11} + \frac{6^3 - 5^3 + 4^3 - 3^3 + 2^3 - 1^3}{3 \times 15} + \dots + \frac{30^3 - 29^3 + 28^3 - 27^3 + \dots + 2^3 - 1^3}{15 \times 63}$ is equal to

85. A water tank has the shape of a right circular cone with axis vertical and vertex downwards. Its semivertical angle is $\tan^{-1} \frac{3}{4}$. Water is poured in it at a constant rate of 6 cubic meter per hour. The rate (in square meter per hour), at which the wet curved surface area of the tank is increasing, when the depth of water in the tank is 4 meters, is

86. For the curve C: $(x^2 + y^2 - 3) + (x^2 - y^2 - 1)^5 = 0$, the value of $3y' - y^3 y''$, at the point (α, α) , $\alpha > 0$, on C, is equal to

87. Let $f(x) = \min\{[x-1], [x-2], \dots, [x-10]\}$ where $[t]$ denotes the greatest integer $\leq t$. Then $\int_0^{10} f(x) dx + \int_0^{10} (f(x))^2 dx + \int_0^{10} |f(x)| dx$ is equal to
88. Let f be a differentiable function satisfying $f(x) = \frac{2}{\sqrt{3}} \int_0^{\sqrt{3}} f\left(\frac{\lambda^2 x}{3}\right) d\lambda$, $x > 0$ and $f(1) = \sqrt{3}$. If $y = f(x)$ passes through the point $(\alpha, 6)$, then α is equal to
89. A common tangent T to the curves $C_1: \frac{x^2}{4} + \frac{y^2}{9} = 1$ and $C_2: \frac{x^2}{42} - \frac{y^2}{143} = 1$ does not pass through the fourth quadrant. If T touches C_1 at (x_1, y_1) and C_2 at (x_2, y_2) , then $|2x_1 + x_2|$ is equal to
90. Let $\vec{a}, \vec{b}, \vec{c}$ be three non-coplanar vectors such that $\vec{a} \times \vec{b} = 4\vec{c}$, $\vec{b} \times \vec{c} = 9\vec{a}$ and $\vec{c} \times \vec{a} = \alpha\vec{b}$, $\alpha > 0$. If $|\vec{a}| + |\vec{b}| + |\vec{c}| = \frac{1}{3\alpha}$, then α is equal to

