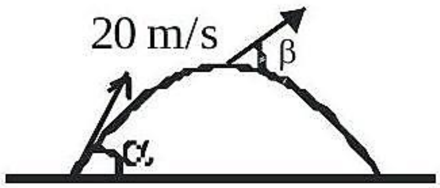
1. (b)



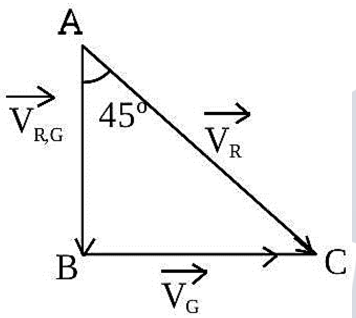
$v_x = u_x = 20\cos\alpha$

$v_y = 20\sin\alpha - 10 \times 10$

$\tan\beta = \dfrac{v_y}{v_x} = \dfrac{20\sin\alpha - 100}{20\cos\alpha}$

$= \tan\alpha - 5\sec\alpha$

2. (c)



$\mathrm{V} = \tan\theta = \dfrac{V_G}{V_{RG}}$

$1 = \dfrac{V_G}{V_{RG}} \Rightarrow 15\sqrt{2} = V_{RG}$

3. (a) $M = (0.6 \pm 0.006)g$

$r = (0.5 \pm 0.005)\mathrm{mm}$

$l = (4 \pm 0.04)\mathrm{cm}$

$\rho = \dfrac{m}{V}$

$\Rightarrow \dfrac{\Delta\rho}{\rho} = \dfrac{\Delta m}{m} + \dfrac{2\Delta r}{r} + \dfrac{\Delta l}{l}$

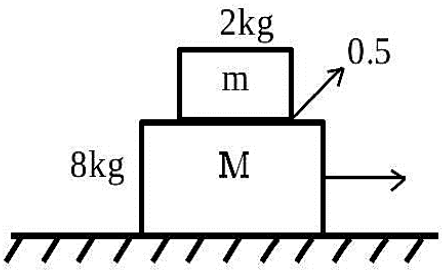
(Volume of cylinder $= \pi r^2 l$)

$= \dfrac{0.006}{0.6} + \dfrac{2 \times 0.005}{0.5} + \dfrac{0.04}{4}$

$100 \times \dfrac{\Delta\rho}{\rho} = 4 \times 10^{-2} \times 100$

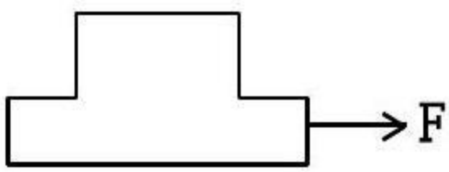
$\frac{\Delta\rho}{\rho} \times 100 = 4\%$

4. **(c)**



$(a_A)_{\max} = 0.5 \text{ g} = 4.9 \text{ m/s}^2$

For moving together



$F_{\max} = m_T a_A$

$= 10 \times 4.9$

$= 49 \text{ N}$

5. **(c)** $\Delta x_G = \frac{m_1 \Delta x_1 + m_2 \Delta x_2}{m_1 + m_2}$

$0 = \frac{10 \times 6 + 30(\Delta x_2)}{40}$

$\Delta x_2 = -2 \text{ cm}$

Block of mass 30 kg will to move towards 10 kg.

6. **(b)** $S = \frac{R_G}{\frac{I}{I_g} - 1}$

$8 = \frac{72}{\frac{I}{I_g} - 1}$

$\frac{I}{I_g} - 1 = 9$

$\frac{I}{I_g} = 10 \Rightarrow \frac{I_g}{I} = \frac{1}{10}$ $\%I = \frac{I_g}{I} \times 100 = 10\%$

7. **(a)** Since it is universal law so it hold good for any pair of bodies.

The value of **g** at centre is zero. So statement I and Statement II are true.

8. (c) Velocity after collision

$$V_2 = \frac{(m_2 - m_1)u_2 + 2m_1u_1}{m_1 + m_2}$$

$$V_2 = \frac{(5m - m)0 + 2m \cdot u_0}{m + 5m} = \frac{u_0}{3}$$

$$\%\Delta KE = \frac{\frac{1}{2}5m\left(\frac{u_0}{3}\right)^2 - 0}{\frac{1}{2}mu_0^2} \times 100$$

$$= \frac{5u_0^2}{9u_0^2} \times 100 = \frac{500}{9} = 55.6\%$$

9. (b) $F_V = mg - F_B$

Sol. $= mg - \left(\frac{m}{d_1} \times d_2\right)g$

$$= mg\left(1 - \frac{d_2}{d_1}\right)$$

10. (c) Susceptibility $\chi = 99$

$$\mu_r = \frac{\mu}{\mu_0} = 1 + \chi$$
$$\mu = \mu_0(1 + \chi)$$
$$= 4\pi \times 10^{-7}[1 + 99]$$
$$= 4\pi \times 10^{-5}$$

11. (d) $\omega = 120\pi = \frac{2\pi}{T} \Rightarrow T = \frac{1}{60}$ sec

time taken to reach peak value $= \frac{T}{4} = \frac{1}{240}$ s

12. (a)

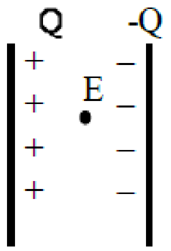
13. (b) $k = \frac{P^2}{2m} \Rightarrow P\alpha\sqrt{m}$

Now $\lambda = \frac{h}{p}$

So, $\lambda\alpha\frac{1}{p} \Rightarrow \lambda\alpha\frac{1}{\sqrt{m}}$

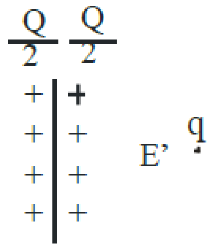
$$\frac{\lambda_\alpha}{\lambda_{C12}} = \frac{\sqrt{3}}{1}$$

14. (a)



$$F = qE = q\left(\frac{Q}{A\in_0}\right) = \frac{qQ}{A\in_0} = 10N$$

Now, when one plate is removed.



$$E' = \frac{Q}{2A\epsilon_0}$$

$$F = qE' = \frac{Qq}{2A\in_0} = 5N$$

15. (d) $X = A\sin\omega t\left(t = 3, X = \frac{A}{2}\right)$

$$\Rightarrow \frac{A}{2} = A\sin 3\omega$$

$$\Rightarrow \sin 3\omega = \frac{1}{2}$$

$$\Rightarrow 3\omega = \frac{\pi}{6}$$

$$\Rightarrow \omega = \frac{\pi}{18} = \frac{2\pi}{T}$$

$$\Rightarrow T = 36\text{ s}$$

16. (a) $f_0 = \left(\frac{v+v_0}{v}\right)f_s$

$$f_0 = \left(\frac{v+\frac{v}{5}}{v}\right)f_s$$

$$f_0 = \frac{6}{5}f_s$$

% change $= \frac{f_0 - f_s}{f_s} \times 100$

$= \frac{1}{5} \times 100 = 20\%$

17. (d) $i = 2r$

$\sin i \times n_1 = \sin r \times n_2$

Sol. $\sin i \times 1 = \sin\frac{i}{2} \times \sqrt{2n}$

$$\frac{\sin i}{\sin\frac{i}{2}} = \sqrt{2n}$$

$$\frac{2\sin\frac{i}{2}\cos\frac{i}{2}}{\sin\frac{i}{2}} = \sqrt{2n}$$

$$\cos\frac{i}{2} = \sqrt{\frac{n}{2}}$$

$$\frac{i}{2} = \cos^{-1}\left(\sqrt{\frac{n}{2}}\right)$$

$$i = 2\cos^{-1}\left(\sqrt{\frac{n}{2}}\right)$$

18. (b) $13.6\left(\frac{1}{1^2} - \frac{1}{n^2}\right) = 10.2$

$n = 2$

$$L_i = \frac{h}{2\pi} \times 1$$

$$L_F = \frac{2h}{2\pi}$$

$$\Delta L = L_F - L_i = \frac{h}{2\pi} = \frac{6.6 \times 10^{-34}}{2 \times \frac{22}{7}}$$

$= 1.05 \times 10^{-34}\ \mathrm{J-s}$

19. (b)

A	B	Y
1	1	0
0	0	1
0	1	1
1	0	1
1	1	0
0	0	1
0	1	1
1	0	1
NAND Gate		

$$Y = \overline{A.B}$$

20. (b) PV = nRT

$$n = \frac{100 \times 10^3 \times 2000 \times 10^{-6}}{\frac{25}{3} \times 300}$$

$$n = 80 \times 10^{-3}$$

$$n_1 + n_2 = 0.08$$

$$n_1 \times 2 + n_2 \times 32 = 0.76$$

$$(0.08 - n_2)2 + n_2(32) = 0.76$$

$$n_2 = 0.02$$

$$n_1 = 0.06$$

$$\frac{n_1}{n_2} = \frac{3}{1}$$

21. (16)

$$(T_2)T_{\sin k} = 200K$$

$$(T_1)T_{\text{Reservoir}} = 527 + 273 = 800 \text{ K}$$

$$W = 12000KJ = 12 \times 10^6 J$$

$$Q_1 = ?$$

$\eta = 1 - \frac{T_2}{T_1} = \frac{W}{Q_1} = 1 - \frac{200}{800} = \frac{12 \times 10^6}{Q_1}$

$\frac{3}{4} = \frac{12 \times 10^6}{Q_1} = Q_1 = 16 \times 10^6 \text{ J}$

22. (975)

$\text{P} = \text{Vi}$

$5 = 25\text{i}$

$i = \frac{1}{5}$

$V_R = iR$

$(220 - 25) = \frac{1}{5}R$

$R = 195 \times 5 = 975\Omega$

23. (450)

$d = 0.6 \times 10^{-3}$

1st Dark fringe $= \frac{D\lambda}{2d} = \frac{d}{2}, \lambda = \frac{d^2}{D}$

$= 450 \times 10^{-9}$ m

24. (114)

$\text{Z} = 3$

$\frac{1}{\lambda} = RZ^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$

$n_1 = 1, n_2 = 3,$

$\frac{1}{\lambda} = R(9)\left(\frac{1}{1} - \frac{1}{9}\right) = 8R$

$\lambda = \frac{1}{8R} = 114 \times 10^{-10}$ m

25. (8) $\frac{V_1}{V_2} = \frac{3}{2} = \frac{E - i_1 r}{E - i_2 r}$

$= \frac{E - \frac{E}{8 + r} \times r}{E - \frac{E}{4 + r} \times r}$

$\frac{3}{2} = \frac{8(4 + r)}{4(8 + r)}$

$24 + 3r = 16 + 4r$

$r = 8\Omega$

26. (48) $J = \frac{I}{A}$

$I = JA$

$= 4 \times 10^6 \times \left[\pi R^2 - \pi\left(\frac{R}{2}\right)^2\right]$

$= 4 \times 10^6 \times \pi R^2 \times \frac{3}{4}$

$= 4 \times 10^6 \times \pi \times (4 \times 10^{-3})^2 \times \frac{3}{4} = 48\pi A.$

27. (125) Energy loss $= \frac{1}{2}\frac{C_1C_2}{C_1+C_2}(V_1 - V_2)^2$

$= \frac{1}{2}\frac{50 \times 50 \times 10^{-12} \times 10^{-12}}{(50+50)10^{-12}}(100-0)^2 = 125nJ$

28. (192)

$LOS = \sqrt{2Rh_T} + \sqrt{2Rh_R}$

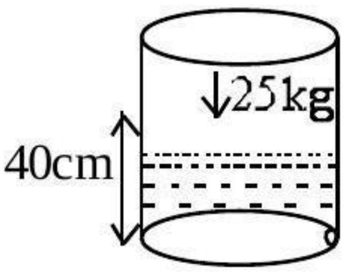
$= \sqrt{2R}\left(\sqrt{h_T} + \sqrt{h_R}\right)$

$= \sqrt{2 \times 64 \times 10^5}(\sqrt{25} + \sqrt{49})$

$= 192\sqrt{5} \times 10^2$ m.

$K = 192$

29. (300)



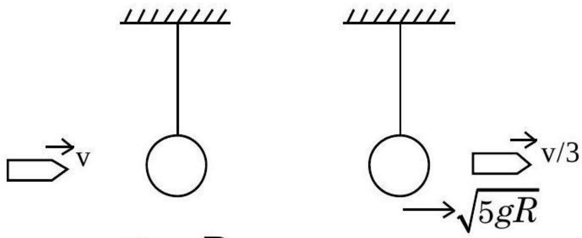
$P_0 + \frac{250}{0.5} + \rho g(40 \times 10^{-2}) = P_0 + \frac{1}{2}\rho v^2$

$500 + \frac{1000 \times 10 \times 40}{100} = \frac{1}{2} \times 1000 \times v^2$

$\mathrm{V} = 3$ m/s

$\mathrm{V} = 300$ cm/s

30. (10) Considering Only Horizontal direction



$\mathbf{P_i} = \mathbf{P_f}$

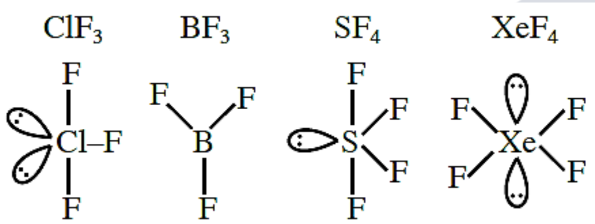
$(75v) + 0 = 50(\sqrt{5gR}) + 75\frac{v}{3}$

$75\left(v - \frac{v}{3}\right) = 50\sqrt{100}$

$v = 10$ m/s

31. (a) Molality is independent of temperature and hence both assertion and reason are true.

32. (b)



33. (b) (a) For a spontaneous process $\Delta G_{T,P} < 0$

(b) $\Delta P = 0 \rightarrow$ Isobaric process

$\Delta T = 0 \rightarrow$ Isothermal process

(c) $\Delta H_{reaction}$ = (Σ Bond energies of reactants) − (Σ bond energies of products)

(d) $\Delta H < 0$ is for exothermic reaction

34. (a)

(a) Protective colloids are lyophilic colloids

(b) Emulsions are liquid in liquid colloidal solutions

(c) $FeCl_3$ + hot water forms positively charged colloidal solution of hydrated ferric oxide.

(d) $FeCl_3 + NaOH$ forms negatively charged colloidal solution due to preferential adsorption of OH^-ions

35. (d) Ionic radius of O^{2-} is more than that of Mg^{2+} Both O^{2-} and Mg^{2+} are isoelectronic with 10 electrons

36. (b)

Gold is concentrated by cyanidation

Leaching of alumina is done by NaOH

Froth stabiliser is aniline

Blister copper has condensed SO_2 on the surface

37. (d) $BaO_2 + H_2SO_4 \rightarrow BaSO_4 + H_2O_2$

This is a common method to prepare hydrogen peroxide

38. (c) $2BeCl_2 + LiAlH_4 \rightarrow 2BeH_2 + LiCl + AlCl_3$

This is the method to prepare BeH_2

39. (d) $(CH_3)_4Si$ is a silane

$(CH_3)Si(OH)_3$ polymerise to form 2D silicone

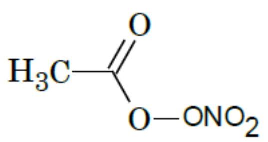
$(CH_3)_2Si(OH)_2$ polymerise to form chain silicone $(CH_3)_3Si(OH)$ form dimer $(CH_3)_3Si-O-Si(CH_3)_3$

40. (d) $P_4 + 3NaOH + 3H_2O \rightarrow 3NaH_2PO_2 + PH_3$

41. (c) Co^{3+} has maximum effective nuclear charge and $CN^-$is the strongest ligand in the given options

42. (a) Classical smog occurs in cool humid climate. It is a reducing mixture of smoke, fog and sulphur dioxide Photochemical smog has components, ozone, nitric oxide, acrolein, formaldehyde, PAN etc.

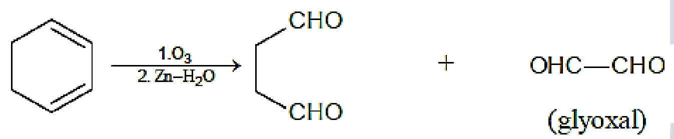
$$CH_4 + O_3 \rightarrow HCHO + H_2O + CH_2=CH-CHO +$$



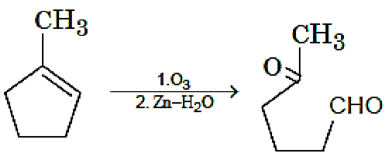
(PAN - peroxyacetyl nitrate)

43. (a) It is used to separate liquid-liquid mixture which is immiscible with different densities

44.(d)

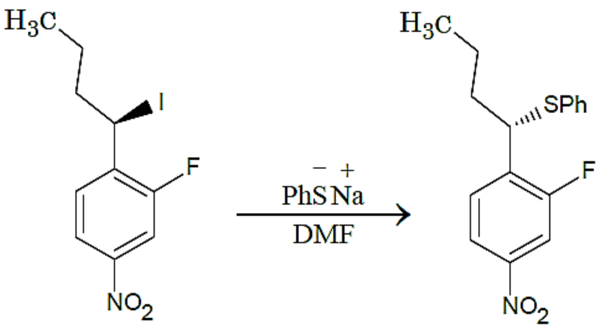


ethane-1, 2-dicarbaldehyde



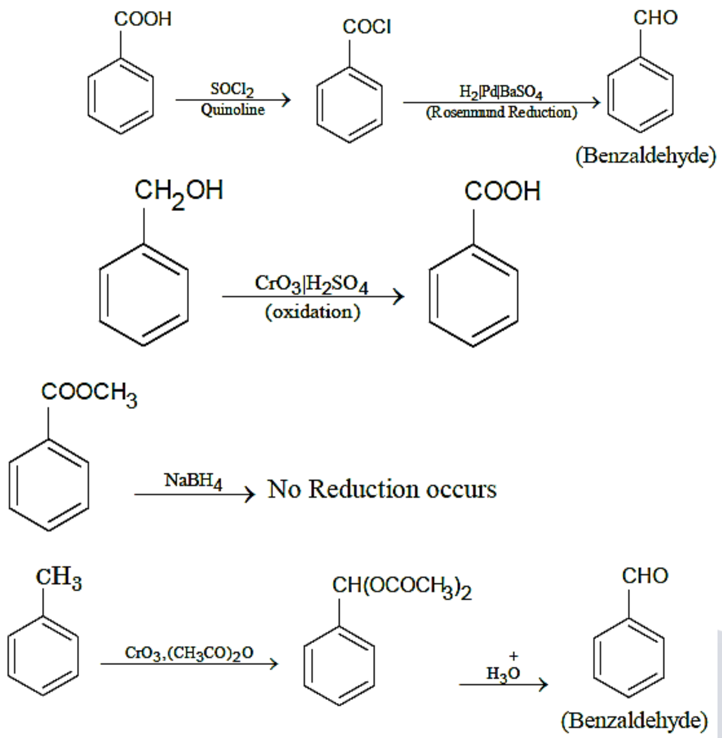
5-oxohexanal

45. (a)



It is bimolecular nucleophilic substitution (SN^2) which occur at benzylic carbon by inversion in contiguration. This reaction cannot undergo substitution at benzene ring

46.(c)



47. (d) $R-CO-NH_2+Br_2+NaOH \rightarrow$

$$R-NH_2+Na_2CO_3+NaBr+H_2O$$

$$R-CO-NH_2+\overset{-}{O}H \rightarrow R-CO-\overset{-}{N}H \xrightarrow{Br_2}$$

$$R-CO-NH-Br \xrightarrow{OH^-} R-CO-\overset{-}{N}-Br$$

$$\xrightarrow{\text{migration of } \overset{-}{R}} R-NCO \xrightarrow{2\overset{-}{O}H} RNH_2+CO_3^{2-}$$

In this reaction of alkyl as well as aryl group can migrate to electron deficient nitrogen atom.

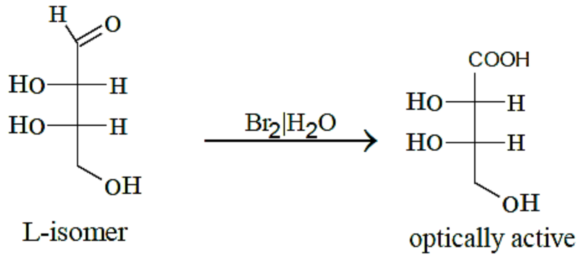
48. (a) Bakelite- It is thermosetting polymer used for making electrical switches.

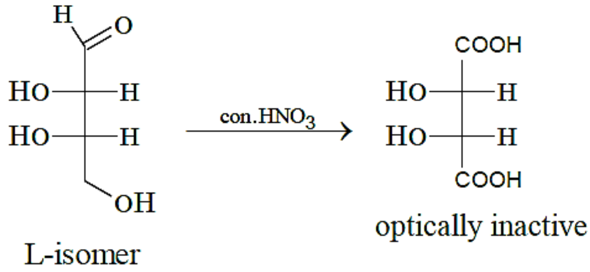
Glyptal - manufacture of paints and lacquers

PVC - manufacture of water pipes, rain coats, hand bags

Polystyrene - manufacture of radio and television cabinets

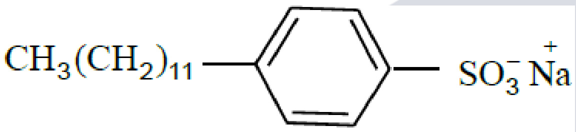
49. (a)





50. (b)

(A) $[CH_3(CH_2)_{15} - N(CH_3)_3]^+Br^-$ is cationic detergents used in hair conditioner



(B)

Is anionic detergent used in tooth pastes

(C) $C_{17}H_{35}COO^-Na^+ + Na_2CO_3 +$ Rosin ate is used as laundary soap

(D) $CH_3(CH_2)_{16}COO(CH_2CH_2O)_NCH_2CH_2OH$ is non-ionic detergents formed from stearic acid and poly ethylene glycol used as liquid dishwashing detergents

51. (85) In $Fe_{0.93}O$ for every 93Fe ions 14 are Fe^{+3} and $(93 - 14) = 79$ are Fe^{+2} ions

$\therefore \%Fe^{+2} = \frac{79}{93} \times 100 = 84.9\%$ nearest integer $= 85\%$

52. (22) $\Delta V = 2.4 \times 10^{-26}\ ms^{-1}$

$\Delta x = 10^{-7}$ m

$\because \Delta p. \Delta x = \frac{h}{4\pi} \therefore m\Delta V. \Delta x = \frac{h}{4\pi}$

$$\Rightarrow m \times 2.4 \times 10^{-26} \times 10^{-7} = \frac{6.626 \times 10^{-34}}{4 \times \pi}$$

$$m = \frac{6.626}{9.6 \times \pi} \times 10^{-1}$$

$m = 0.02198$ kg

$m = 21.98$gm

nearest integer $= 22$

53. (8) Given: $\frac{(K_b)_A}{(K_b)_B} = \frac{1}{8}$

$\therefore \frac{(\Delta T_B)_A}{(\Delta T_B)_B} = \frac{(K_b)_A \cdot m}{(K_b)_B \cdot m} = \frac{1}{8} = \frac{X}{y}$

$\because \frac{x}{y} = \frac{1}{8}$

$\therefore$ y = 8 (nearest integer)

54. (125) $2NOCl(g) \rightleftharpoons 2NO(g) + Cl_2(g)$

$t = 0\ 2M$

$t = t_{eq} (2-x)M\ xM\ \frac{x}{2}M$
$\because x = 0.4M$

$\therefore [NOCl]_{eq} = 1.6M$

$[NO]_{eq} = 0.4M$

$[Cl_2]_{eq} = 0.2M$

$\Rightarrow K_c = \frac{[NO]^2[Cl_2]}{[NOCl]^2} = \frac{[0.4]^2[0.2]}{[1.6]^2}$

$K_c = \frac{32}{2.56} \times 10^{-3}$

$K_c = 12.5 \times 10^{-3}$

$K_c = 125 \times 10^{-4}$

Integer answer is 125

55. (14) Given

(1) $\lambda_m^{\infty}(NaI) = 12.7 mSm^2\ mol^{-1}$

(2) $\lambda_m^{\infty}(NaNO_3) = 12.0 mSm^2\ mol^{-1}$

(3) $\lambda_m^{\infty}(AgNO_3) = 13.3 mSm^2\ mol^{-1}$

$\lambda_m^{\infty}(Ag\,I) = (1) + (3) - (2)$

$= 12.7 + 13.3 - 12.0$

$= 26.0 - 12.0$

$\lambda_m^{\infty}(AgI) = 14.0$

56. (166) $\ln k = \ln A - \frac{E_A}{RT}$

Given: $\ln k = 33.24 - \frac{2.0\times10^4}{T}$

$\therefore$ on comparing $\frac{E_A}{R} = 2.0 \times 10^4$

$\therefore E_A = 2.0 \times 10^4 \times R$

$\Rightarrow E_A = 2.0 \times 10^4 \times 8.3\ J$

$\Rightarrow E_A = 16.6 \times 10^4\ J = 166\ kJ$

57. (3) A, B, C are correct and D is incorrect because Fehling solution has Cu(II)
58. (6) $2KMnO_4 + 5H_2C_2O_4 + 3H_2SO_4 \rightarrow K_2SO_4 + 2MnSO_4 + 10CO_2 + 8H_2O$

Mn^{2+} has 5 unpaired electrons therefore the magnetic moment is $\sqrt{35}$BM

59. (2) Given: Molar mass of $A_2B = AB_3$

$\therefore (2A + B) = (A + 3B) \begin{bmatrix} A \rightarrow & \text{Atomic wt. of } A \\ B \rightarrow & \text{Atomic wt.of } B \end{bmatrix}$

$\Rightarrow A = 2B$

$\therefore$ atomic wt. of A is 2 times of atomic wt. of B

Integer answer is 2

60. (6)
61. (a) $\Rightarrow$ Let $z = x + iy, x, y \in R$

Now $\bar{Z} = iz^2$

then $x - iy = i(x^2 - y^2 + 2xyi)$

$x - iy = i(x^2 - y^2) - 2xy$

$\Rightarrow x = -2xy \& -y = x^2 - y^2$

$\Rightarrow x(1 + 2y) = 0$

$x = 0$ or $y = -\frac{1}{2}$

Put $x = 0$ in $-y = x^2 - y^2$

We get $y = y^2$

$\Rightarrow y = 0,1$

Similarly

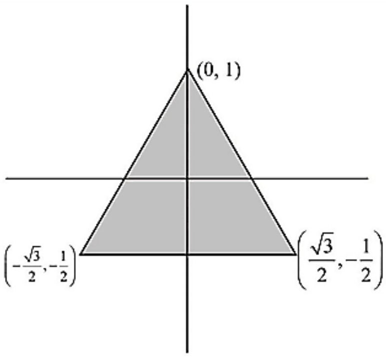
Put $y = -\frac{1}{2}$ in $-y = x^2 - y^2$

$\Rightarrow \frac{1}{2} = x^2 - \frac{1}{4}$

$\Rightarrow x^2 = \frac{3}{4}$

$x = \pm\frac{\sqrt{3}}{2}$

$z = \left(0, i, \frac{\sqrt{3}}{2} - \frac{1}{2}i, -\frac{\sqrt{3}}{2} - \frac{1}{2}i\right)$



$$\text{Area } = \frac{1}{2}\cdot(\sqrt{3})\left(\frac{3}{2}\right) = \frac{3\sqrt{3}}{4}$$

62. (d) $\Delta = \begin{vmatrix} 1 & 2 & 1 \\ 2 & 3 & -1 \\ -2 & 1 & 2 \end{vmatrix}$

$= (6 + y) - 2((2\alpha - \alpha) + 1(\alpha + 3\alpha)$

$= 7 - 2\alpha + 4\alpha$

$= 7 + 2\alpha$

$\Delta = 0 \Rightarrow \alpha = -\frac{7}{2}$

$\Delta_1 = \begin{vmatrix} 2 & 2 & 1 \\ \alpha & 3 & -1 \\ -\alpha & 1 & 2 \end{vmatrix}$

$= 14 + 2\alpha$

$\alpha = -x_2 = 7$

$\Delta_1 \neq 0$

63. (c) $x = 1 + a + a^2 =$

$x = \frac{1}{1-a} \Rightarrow a = 1 - \frac{1}{x}$

$y = \frac{1}{1-b} \Rightarrow b = 1 - \frac{1}{y}$

$z=\frac{1}{1-c} \Rightarrow c=1-\frac{1}{Z}$

a, b, c are in A.P.

$\Rightarrow 1-\frac{1}{x}, 1-\frac{1}{y}, 1-\frac{1}{z}$ are in A.P.

$\Rightarrow -\frac{1}{x}, -\frac{1}{y}, -\frac{1}{z}$ are in A.P.

$\Rightarrow \frac{1}{x}, \frac{1}{y}, \frac{1}{z}$ are in A.P.

64. (b) Let equation of circle is

$x^2+y^2+2gx+2fy+c=0$

$\Rightarrow \frac{dy}{dx}=\frac{-(2x+2g)}{(2y+2f)}$

Comparing with $\frac{dy}{dx}=\frac{ax-by+a}{bx+cy+a}$

$\Rightarrow \mathrm{b}=0, \mathrm{a}=-2, \mathrm{c}=2$

$\Rightarrow -2\ \mathrm{g}=-2 \Rightarrow \mathrm{g}=1\ 2\mathrm{f}=-2$

$\mathrm{f}=-1$

Now circle will be

$\mathrm{x}^2+\mathrm{y}^2+2\mathrm{x}-2\mathrm{y}+\mathrm{c}=0$

its passes through (2,5)

which will give $\mathrm{c}=-23$

so circle will be $\mathrm{x}^2+\mathrm{y}^2+2\mathrm{x}-2\mathrm{y}-23=0$

centre $\mathrm{C}=(-1,1)$

and radius 5

Now P is (11,6)

So minimum distance of P from circle will be

$=\sqrt{(11+1)^2+(6-1)^2}-5$

$=13-5$

$=8$

65. (a) $\lim_{x \rightarrow 7} \frac{18-[1-x]}{[x]-3a}$

L.H.L. $\lim_{x \rightarrow 7-} \frac{18-[1-x]}{[x]-3a}$

$= \dfrac{18-(-6)}{6-3a}$

$= \dfrac{24}{6-3a}$

R.H.L. $\lim_{x\to 7+} \frac{18-[1-x]}{[x]-3a}$

$= \dfrac{18-(-7)}{7-3a}$

$= \dfrac{25}{7-3a}$

Now L.H.L. = R.H.L.

$\dfrac{24}{6-3a} = \dfrac{25}{7-3a}$

$\Rightarrow 168 - 72a = 150 - 75a$

$\Rightarrow 18 = -3a$

$\Rightarrow \mathrm{a} = -6$

66. (b) Let $f(x) = x^4 - 4x + 1$

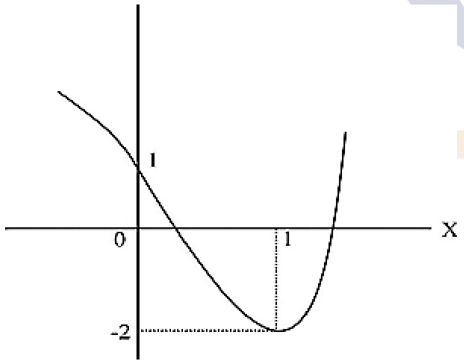
$\mathrm{f}'(\mathrm{x}) = 4\mathrm{x}^3 - 4$

$\mathrm{f}'(\mathrm{x}) = 0 \Rightarrow \mathrm{x} = 1$

$\mathrm{x} = 1$ is point of minima.

$f(1) = -2$

$\mathrm{f}(0) = 1$



Hence 2 solutions.

67. (10)
68. (d)

$$\cos^{-1}\left(\frac{y}{2}\right) = \log_e\left(\frac{x}{5}\right)^5$$

$$\cos^{-1}\left(\frac{y}{2}\right) = 5\log_e\left(\frac{x}{5}\right)$$

$$\frac{-1}{\sqrt{1-\frac{y^2}{4}}}\cdot\frac{y'}{2} = 5\cdot\frac{1}{\frac{x}{5}}\times\frac{1}{5}$$

$$\Rightarrow \frac{-y'}{\sqrt{4-y^2}} = \frac{5}{x}$$

$$-xy' = 5\sqrt{4-y^2}$$

$$-xy'' - y' = 5\cdot\frac{1}{2\sqrt{4-y^2}}(-2yy')$$

$$\Rightarrow xy'' + y' = \frac{5y'\cdot y}{\sqrt{4-y^2}}$$

$$xy'' + y' = 5\cdot\left(\frac{-5}{x}\right)y$$

$$x^2y'' + xy' = -25y$$

69. (b)

$$\int\left(\frac{x^2+1}{(x+1)^2}\right)e^x\cdot dx$$

$$= \int\left(\frac{x^2-1+2}{(x+1)^2}\right)e^x dx$$

$$= \int\left(\frac{x-1}{x+1}+\frac{2}{(x+1)^2}\right)e^x dx$$

$$= \int (f(x)+f'(x))e^x dx$$

$$= f(x)e^x + c$$

Where $f(x) = \frac{x-1}{x+1}$

$$f'(x) = \frac{2}{(x+1)^2}$$

$$f''(x) = \frac{-4}{(x+1)^3}$$

$$= \frac{12}{(x+1)^4}$$

$$f''(1) = \frac{12}{16}$$

$$= \frac{3}{4}$$

70. (d) $f(x) = \frac{|x^3+x|}{(e^{x|x|}+1)}dx$

$$\int_{-2}^{2} f(x)dx = \int_0^2 (f(x)+f(-x))dx$$

$$= \int_0^2\left(\frac{|x^3+x|}{(e^{x|x|}+1)}+\frac{|-x^3-x|}{(e^{-x|-x|}+1)}\right)dx$$

$= \int_0^2 \left(\frac{|x^3+x|}{(e^{x|x|}+1)} + \frac{|x^3+x|}{(e^{-x|x|}+1)} \right) dx$

$= \int_0^2 \left(\frac{x^3+x}{(e^{x^2}+1)} + \frac{x^3+x}{(e^{-x^2}+1)} \right) dx$

$I = \int_0^2 \left(\frac{x^3+x}{1+e^{x^2}} + \frac{e^{x^2}(x^3+x)}{1+e^{x^2}} \right) dx$

$= \int_0^2 (x^3+x) dx$

$= \left[\frac{x^4}{4} + \frac{x^2}{2} \right]_0^2$

$= 4 + 2 = 6$

71. (d) $\frac{dy}{dx} + \frac{2^{x-y}(2^y-1)}{2^x-1} = 0,$

$x, y > 0, y(1) = 1, y(2) = ?$

$\frac{dy}{dx} = -\frac{2^x(2^y-1)}{2^y(2^x-1)}$

$\int \frac{2^y}{2^y-1} dy = -\int \frac{2^x}{2^x-1} dx$

$\frac{1}{\ln 2} \int \frac{2^y \ln 2}{2^y-1} dy = -\frac{1}{\ln^2} \int \frac{2^x \ln 2}{2^x-1} dx$

$\frac{1}{\ln 2} \ln|2^y-1| = \frac{-1}{\ln 2} \ln|2^x-1| + C$

At $x = 1, y = 1$

Putting this values in above relation we get $C = 0$

$\ln|2^y-1| + \ln|2^x-1| = 0$

$(2^x-1)(2^y-1) = 1$

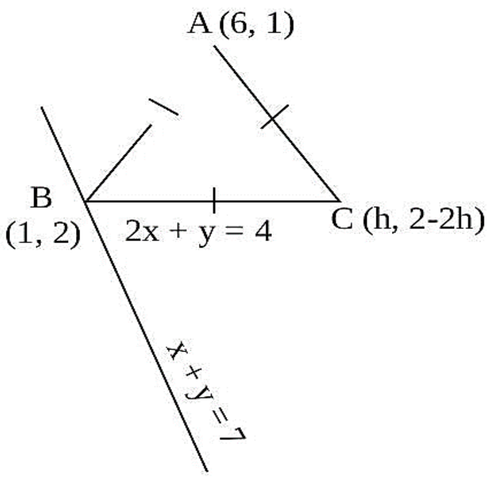
$2^y - 1 = \frac{1}{2^x+1}$

At $x = 2$

$2^y = \frac{1}{3} + 1 = \frac{4}{3}$

$y = \log_2 \frac{4}{3} = \log_2 4 - \log_2 3 = 2 - \log_2 3$

72. (c)



Point B (1,2)

Now let C be (h, 4 - 2h)

(As C lies on 2x + y = 4)

$\because$ Δ is isosceles with base BC

$\therefore AB = AC$

$\sqrt{25+1} = \sqrt{(6-h)^2 + (2h-3)^2}$

$\sqrt{26} = \sqrt{36 + h^2 - 12h + 4h^2 + 9 - 12h}$

$26 = 5h^2 - 24h + 45 \Rightarrow 5h^2 - 24h + 19 = 0$

$\Rightarrow 5h^2 - 5h - 19h + 19 = 0$

$h = \frac{19}{5}$ or h = 1

Thus $C\left(\frac{19}{5}, \frac{-18}{5}\right)$

Centroid $\left(\frac{6+1+\frac{19}{5}}{3}, \frac{1+2-\frac{18}{5}}{3}\right)$

$\left(\frac{35+19}{15}, \frac{15-18}{15}\right)$

$\left(\frac{54}{15}, \frac{-3}{15}\right)$

$\alpha = \frac{54}{15}; \beta = \frac{-3}{15}$

$15(\alpha + \beta) = 51$

73. (b) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 a > b$

$e^2 = 1 - \frac{b^2}{a^2}$

$$\frac{1}{16} = 1 - \frac{b^2}{a^2}$$

$$\frac{b^2}{a^2} = 1 - \frac{1}{16} = \frac{15}{16} \Rightarrow b^2 = \frac{15}{16}a^2$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\frac{16 \times \frac{2}{5}}{a^2} + \frac{9}{b^2} = 1$$

$$\frac{32}{5a^2} + \frac{9}{b^2} = 1$$

$$\frac{32}{5a^2} + \frac{9}{\frac{15}{16}a^2} = 1$$

$$\frac{80}{5a^2} = 1$$

$$16 = \mathrm{a}^2$$

$$\mathrm{b}^2 = 15$$

74. (a) $1 + \mathrm{m} - \mathrm{n} = 0$

$$31^2 + \mathrm{m}^2 + \mathrm{cl}(1 + \mathrm{m}) = 0$$

$$\mathrm{n} = 1 + \mathrm{m}$$

$$31^2 + \mathrm{m}^2 + \mathrm{cl}^2 + \mathrm{clm} = 0$$

$$(3 + c)l^2 + c\mathrm{lm} + m^2 = 0$$

$$(3 + c)\left(\frac{l}{m}\right)^2 + c\left(\frac{l}{m}\right) + 1 = 0$$

$\because$ lies are parallel.

Roots of (1) must be equal

$$\Rightarrow D = 0$$

$$c^2 - 4(3 + c) = 0$$

$$c^2 - 4c - 12 = 0$$

$$(\mathrm{c} - 6)(\mathrm{c} + 2) = 0$$

$\mathrm{c} = 6$ or $\mathrm{c} = -2$

+ve value of $\mathrm{c} = 6$

75. (a) $\vec{a} = i + j - k$

$\vec{c} = 2i - 3j + 2k$

$\vec{b} \times \vec{c} = \vec{a}$

$|\vec{b}| \in \{1,2 \dots \dots .10\}$

$\because \vec{b} \times \vec{c} = \vec{a}$

$\Rightarrow \vec{a}$ is perpendicular to $\vec{b}$ as well as $\vec{a}$ is perpendicular to $\vec{c}$
Now $\vec{a} \cdot \vec{c} = 2 - 3 - 2 = -3 \neq 0$

This $\vec{b} \times \vec{c} = \vec{a}$ is not possible.

No. of vectors $\vec{b} = 0$

76. (c) No. of ways to select and arrange x_1, x_2, x_3, x_4, x_5 from 1,2,3.

$n(s) = {}^{18}C_5$

$x_1 \quad (x_2) \quad x_3 \quad (x_4) \quad x_5$

7 11

$n(E) = {}^{6}C_1 \times {}^{3}C_1 \times {}^{7}C_1$

$$P(E) = \frac{6 \times 3 \times 7}{{}^{18}C_5}$$

$$\frac{1}{17 \times 4} = \frac{1}{68}$$

77. (c) $B(7, p)$

$n = 7 \; p = p$

given

$P(x = 3) = 5P(x = 4)$
${}^{7}C_3 \times p^3(1-p)^4 = 5.\ {}^{7}C_4 p^4 (1-p)^3$

$$\frac{{}^{7}C_3}{5 \times {}^{7}C_4} = \frac{p}{1-p}$$

$1 - p = 5p$

$6p = 1$

$$p = \frac{1}{6} \Rightarrow q = \frac{5}{6}$$

$n = 7$

$\text{Mean} = np = 7 \times \frac{1}{6} = \frac{7}{6}$

$\text{Var} = npq = 7 \times \frac{1}{6} \times \frac{5}{6} = \frac{35}{36}$

Sum

$= \frac{7}{6} + \frac{35}{36}$

$= \frac{42+35}{36}$

$= \frac{77}{36}$

78. (b)

$\cos\frac{2\pi}{7} + \cos\frac{4\pi}{7} + \cos\frac{6\pi}{7}$

$= \frac{\sin\left(3\times\frac{\pi}{7}\right)}{\sin\frac{\pi}{7}} \times \cos\left(\frac{\frac{2\pi}{7}+\frac{6\pi}{7}}{2}\right)$

$= \frac{2\sin\left(\frac{3\pi}{7}\right)}{2\sin\frac{\pi}{7}} \times \cos\left(\frac{4\pi}{7}\right)$

$= \frac{\sin\left(\frac{7\pi}{7}\right) + \sin\left(\frac{-\pi}{7}\right)}{2\sin\frac{\pi}{7}}$

$= \frac{-\sin\frac{\pi}{7}}{2\sin\frac{\pi}{7}}$

$= -\frac{1}{2}$

79. (a)

$\sin^{-1}\left(\sin\frac{2\pi}{3}\right) + \cos^{-1}\left(\cos\frac{7\pi}{6}\right) + \tan^{-1}\tan\left(\frac{3\pi}{4}\right)$

$\sin^{-1}\sin\left(\frac{2\pi}{3}\right) = \pi - \frac{2\pi}{3} = \frac{\pi}{3}$

$\cos^{-1}\left(\cos\frac{2\pi}{6}\right) = 2\pi - \frac{7\pi}{6} = \frac{5\pi}{6}$

$\tan^{-1}\tan\left(\frac{3\pi}{4}\right) = \frac{3\pi}{4} - \pi = \frac{-\pi}{4}$

$\sin^{-1}\left(\sin\frac{2\pi}{3}\right) + \cos^{-1}\cos\frac{7\pi}{6} + \tan^{-1}\tan\frac{3\pi}{4}$

$= \frac{11\pi}{12}$

80. (d) $(\sim (p \wedge q)) \vee q$

$= (\sim p \vee \sim q) \vee q$

$= \sim p \vee \sim q \vee q$

$= \sim p \vee t$

= this statement is a tautology option D

$p \Rightarrow (p \vee q)$ is also a tautology.

OR

p	q	P^q	~ (p^q)	~ (p^q) ∨ q	p ∨ q	$p \rightarrow (p \vee q)$
T	T	T	F	T	T	T
T	F	F	T	T	T	T
F	T	F	T	T	T	T
F	F	F	T	T	F	T

81. (99)

$$f(x)+f(1-x)=\frac{2e^{2x}}{e^{2x}+e}+\frac{2e^{2-2x}}{e^{2-ex}+e}=\left[\frac{e^{2x}}{e^{2x}+e}+\frac{e^{2}}{e^{2}+e^{2x+1}}\right]$$

$$=2\left[\frac{e^{2x-1}}{e^{2x-1}+1}+\frac{1}{1+e^{2x-1}}\right]=2$$

$$f\left(\frac{1}{100}\right)+f\left(\frac{2}{100}\right)+f\left(\frac{3}{100}\right)+\cdots..+f\left(\frac{99}{100}\right)$$

$$=\left\{f\left(\frac{1}{100}\right)+f\left(\frac{99}{100}\right)\right\}+\left\{f\left(\frac{2}{100}\right)+f\left(\frac{98}{100}\right)\right\}+\cdots..+f\left\{\left(\frac{49}{100}\right)+f\left(\frac{51}{100}\right)\right\}+f\left(\frac{1}{2}\right)$$

$$=(2+2+2+----49\text{ times })+\frac{2e}{e+e}$$

$$=98+1=99$$

82. (45) $e^{2x}-11e^{x}-45e^{-x}+\frac{81}{2}=0\,]$

$$(e^{x})^{3}-11(e^{x})^{2}-45+\frac{81e^{x}}{2}=0]$$

$e^{x}=t$

$2t^{3}-22t^{2}+81t-90=0$

$t_1t_2t_3=45$

$e^{x_1}\cdot e^{x_2}\cdot e^{x_3}=45$

$e^{x_1+x_2+x_3}=45$

$\log_e e^{x_1+x_2+x_3} = \log_e 45$

$x_1 + x_2 + x_3 = \log_e 45$

$\log_e P = \log_e 45$

$\mathrm{P} = 45$

83. (14) $\operatorname{Adj}(\operatorname{Adj} A) = \begin{bmatrix} 14 & 18 & -14 \\ -14 & 14 & 28 \\ 28 & -14 & 14 \end{bmatrix}$

$$|\operatorname{Adj}(\operatorname{Adj} A)| = \begin{bmatrix} 14 & 28 & -14 \\ -14 & 14 & 28 \\ 28 & -14 & 14 \end{bmatrix} = 14 \times 14 \times 14 \begin{vmatrix} 1 & 2 & -1 \\ -1 & 1 & 2 \\ 2 & -1 & 1 \end{vmatrix}$$
$$= (14)^3[3 - 2(-5) - 1(-1)] = (14)^3[14] = (14)^4$$

$|A|^4 = (14)^4 \Rightarrow |A| = 14$

84. (56)

16 cubes $<$ 11 Blue, 5 Red

$$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 11$$
$$x_1, x_6 \geq 0,\ x_2, x_3, x_4, x_5 \geq 2$$
$$x_2 = t_1 + 2$$
$$x_3 = t_3 + 2$$
$$x_4 = t_4 + 2$$
$$x_5 = t_5 + 2$$
$$x_1, t_2, t_3, t_4, t_5, x_6 \geq 0$$

No. of solutions $= {}^{6+3-1}C_3 = {}^{8}C_3 = 56$

85. (5) $\left(\frac{\sqrt{x}}{5^{1/4}} + \frac{\sqrt{5}}{x^{1/3}}\right)^{60}$

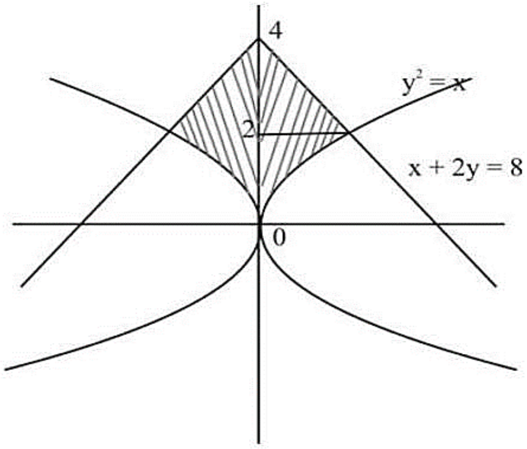
$$T_{r+1} = {}^{60}C_r \left(\frac{x^{1/2}}{5^{1/4}}\right)^{60-r} \left(\frac{5^{1/2}}{x^{1/3}}\right) r$$
$$= {}^{60}C_r 5\frac{3r-60}{4} \cdot x\frac{180-5r}{6}$$
$$\frac{180-5r}{6} = 10 \Rightarrow r = 24$$

Coeff. of $x^{10} = {}^{60}C_{24} 5^3 = \frac{60}{\underline{24}|36} 5^3$

Powers of 5 in $= {}^{60}C_{24}. 5^3 = \frac{5^{14}}{5^4 \times 5^8} \times 5^3 = 5^5$

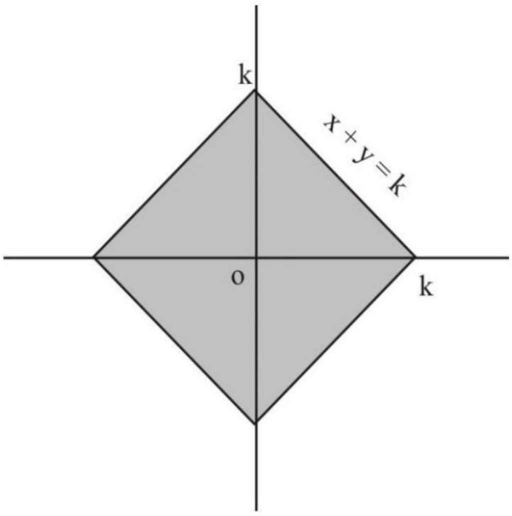
86. (6) $A_1 = \{(x, y): |x| \leq y^2, |x| + 2y \leq 8\}$ and

$A_2 = \{(x, y): |x| + |y| \leq k\}$.



$$\text{area}(A_1) = 2\left[\int_0^2 y^2 dy + \int_2^4 (8-2y)dy\right]$$

$$= 2\left[\left(\frac{y^3}{3}\right)_0^2 + (8y - y^2)_2^4\right]$$



$$\text{area}(A_1) = 2 \times \frac{20}{3} = \frac{40}{3}$$

$$\text{Area}(A_2) = 4 \times \frac{1}{2}k^2$$

$$\text{Area}(A_2) = 2k^2$$

Now

$$27\ (\text{Area } A_1) = 5(\text{ Area } A_2)$$

$$9 \times 4 = k^2$$

$$k = 6$$

87. (276) $\frac{1}{5} + \frac{2}{65} + \frac{3}{325} + \frac{4}{1025} + \frac{5}{2501} + \cdots \ldots$

$$T_n = \frac{n}{4n^4 + 1}$$

$$=\frac{n}{(2n^2+1)^2-(2n)^2}=\frac{n}{(2n^2+2n+1)(2n^2-2n+1)}$$

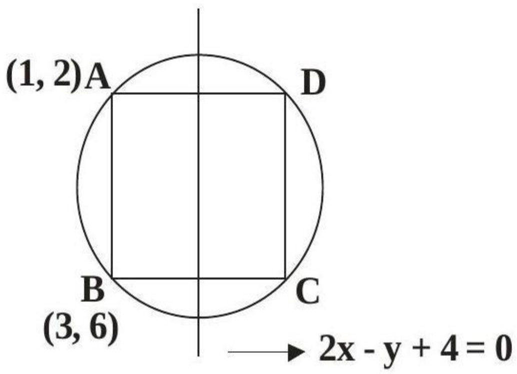
$$=\frac{1}{4}\left[\frac{1}{2n^2-2n+1}-\frac{1}{2n^2+2n+1}\right]$$

$$S_{10}=\sum_{n=1}^{10}T_n=\frac{1}{4}\left[\frac{1}{1}-\frac{1}{5}+\frac{1}{5}-\frac{1}{13}+\cdots\ldots\therefore\frac{1}{200+20+1}\right]$$

$$=\frac{1}{4}\left[1-\frac{1}{221}\right]=\frac{1}{4}\times\frac{220}{221}-\frac{55}{221}=\frac{m}{n}$$

$m+n=55+221=276$

88. (16)



Eq. of line AB

$y=2x$

Slope of AB $=2$

Slope of given diameter $=2$

So the diameter is parallel to AB

Distance between diameter and line AB

$$=\left(\frac{4}{\sqrt{2^2+12}}\right)=\frac{4}{\sqrt{5}}$$

Thus BC $=2\times\frac{4}{\sqrt{5}}=\frac{8}{\sqrt{5}}$

$AB=\sqrt{(1-3)^2+(2-6)^2}=\sqrt{20}=2\sqrt{5}$

Area $=$ AB $\times$ BC $=\frac{8}{\sqrt{5}}\times 2\sqrt{5}=16$

89. (63) Vertex and focus of parabola $y^2=2x$

are V(0,0) and $S\left(\frac{1}{2},0\right)$ resp.

Let equation of circle be

$(x-h)^2+(y-k)^2=4$

$\because$ Circle passes through (0,0)

$\Rightarrow h^2 + k^2 = 4$

$\because$ Circle passes through $\left(\frac{1}{2}, 0\right)$

$$\left(\frac{1}{2} - h\right)^2 + k^2 = 4$$

$$\Rightarrow h^2 + k^2 - h = \frac{15}{4}$$

On solving (1) and (2)

$$4 - h = \frac{15}{4}$$

$$h = 4 - \frac{15}{4} = \frac{1}{4}$$

$$k = +\frac{\sqrt{63}}{4}$$

$k = -\frac{\sqrt{63}}{4}$ is rejected as circle with centre $\left(\frac{1}{4}, -\frac{\sqrt{63}}{4}\right)$ can't touch given parabola.

Equation of circle is

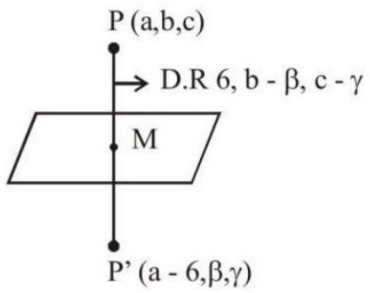
$$\left(x - \frac{1}{4}\right)^2 + \left(k - \frac{\sqrt{63}}{4}\right)^2 = 4$$

From figure $\alpha = 2 + \frac{\sqrt{63}}{4} = \frac{8+\sqrt{63}}{4}$

$$4\alpha - 8 = \sqrt{63}$$

$$(4\alpha - 8)^2 = 63$$

90. (137)



$$\mathrm{M} = \left(a - 3, \frac{\beta + b}{2}, \frac{\gamma + c}{2}\right)$$

Since M lies on $3x + 4y + 12z + 19 = 0$

$$\Rightarrow 6a - 4b + 12c - 4\beta + 12\gamma + 20 = 0$$

Since PP' is parallel to normal of the plane then $\frac{6}{3}=\frac{b-\beta}{-4}=\frac{c-\gamma}{12}$

$\Rightarrow \beta=b+8,\ \gamma=c-24$

$a+b+c=5 \Rightarrow a+\beta-8+\gamma+24=5$

$\Rightarrow a=-\beta-\gamma-11$

Now putting these values in (1) we get $6(-\beta-\gamma-11)-4(\beta-8)+12(\gamma+24)-4\beta+12\gamma+20=0$

$\Rightarrow 7\beta-9\gamma=170-33=137$