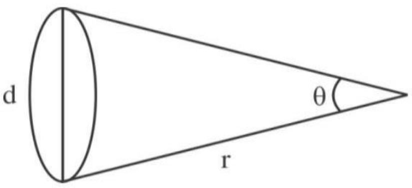
1. **(a)** Pascal second

$$\frac{F}{A}t = \frac{MLT^{-2}}{L^2}\ T = ML^{-1}\ T^{-1}$$

2. **(c)**



$$\theta = \frac{d}{r}$$

$$\frac{2000}{60\times 60}\times\frac{\pi}{180} = \frac{d}{1.5\times 10''}$$

$$\Rightarrow d = \frac{2000}{60\times 60}\times\frac{\pi}{180}\times 1.5\times 10''$$

$$= \frac{\pi\times 1.5}{3\times 6\times 18}\times 10'' = 1.45\times 10^9$$

3. **(b)**

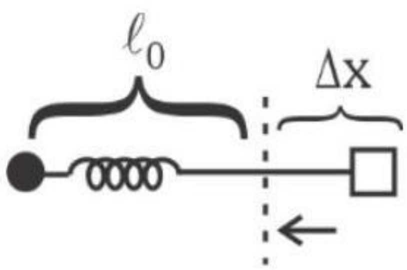
$$V^2 = 2\times 9.8\times 4.9$$

$V = 9.8\ m/s$

Depth = distance travelled in 3 seconds

$= 9.8\times 3 = 29.4\ m$

4. **(c)**



$$K\Delta x = m\left(\ell_0 + \Delta x\right)w^2$$

$$K\Delta x = m\ell_0 w^2 + mw^2\Delta x$$

$$\Delta x = \frac{m\ell_0 w^2}{k - mw^2}$$

5. **(b)**

$$v = \sqrt{u^2 - 2gL}$$

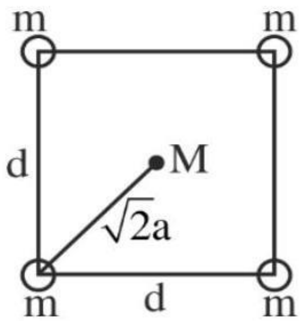
$\Delta v = \sqrt{u^2 + v^2}$

$\Delta v = \sqrt{u^2 + v^2 - 2gL}$

$\Delta v = \sqrt{2u^2 - 2gL}$

$\Delta v = \sqrt{2(u^2 - gL)}x = 2$

6. (a)



$-\frac{Gm^2}{d} \times 4 - \frac{Gm^2}{\sqrt{2}\,d} \times 2 - \frac{GMm}{d} \times 4\sqrt{2}$

$-\frac{Gm}{d}[(4+\sqrt{2})m + 4\sqrt{2}M]$

7. (a)

$PV = nRT$

$\frac{V}{T} = \frac{nR}{P}$

$\frac{nR}{P_1} < \frac{nR}{P_2}$

$P_2 < P_1$

8. (b)

9. (b) $m \times 125 \times 200 + m \times 2.5 \times 10^4 = \frac{1}{2}mv^2 \times \frac{40}{100}$

$V = 500 \text{ m/s}$

10. (157)

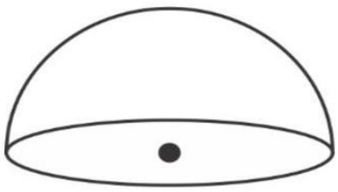
$x = \sin\pi\left(t + \frac{1}{3}\right)$

$x = \sin\left(\pi t + \frac{\pi}{3}\right)$

$V = \frac{dx}{dt} = \cos\left(\pi t + \frac{\pi}{3}\right)\pi$

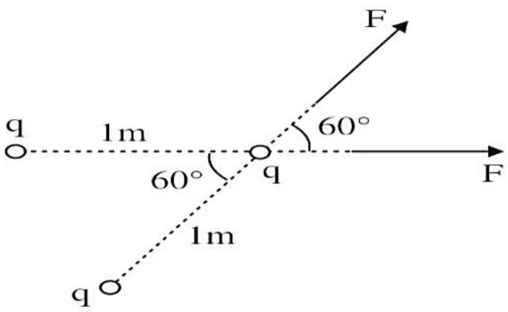
$= -\pi \times \frac{1}{2} = 157 \text{ cm/s}$

11. (b)



Total flux through complete spherical surface is $\frac{q}{\varepsilon_0}$. So the flux through curved surface will be $\frac{q}{2\varepsilon_0}$. The flux through flat surface will be zero. Remark: Electric flux through flat surface is zero but no option is given, option is available for electric flux passing through curved surface.

12. (d)



$$F=\frac{k(2)(2)}{(1)^2}$$

(F = Force between two charges).

F = 4k

$$F_{\text{net}}=2\,F\cos 30^{\circ}=2\cdot F\cdot\frac{\sqrt{3}}{2}=F\sqrt{3}$$

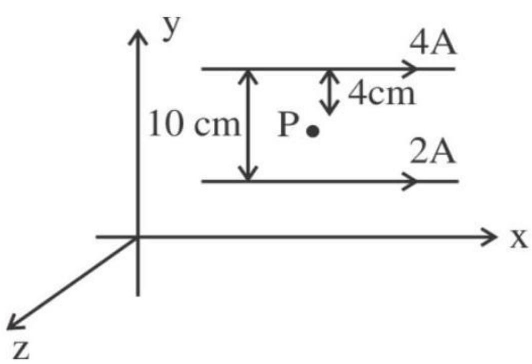
(F_{net} = Net electrostatic force on one charged ball)

$$\frac{F_{\text{net}}}{F}=\frac{\sqrt{3}\,F}{F}=(\sqrt{3})$$

Remark: Net force on any one of the ball is zero.

But no option given in options.

13. (c)



$$B_{net} = B_1 - B_2 = \frac{\mu_0 \times 4}{2\pi[.04]} - \frac{\mu_0 \times 2}{2\pi[.06]}$$

$$\vec{B}_{net} = \frac{\mu_0}{2\pi}\left[\frac{200}{3}\right](-\hat{k})$$

$$\vec{F} = q[\vec{v} \times \vec{B}]$$

$$= [3\pi]\left[(2\hat{i} + 3\hat{j}) \times \left(\frac{\mu_0}{2\pi}\right)\left(\frac{200}{3}\right) - \hat{k}\right] = 3\pi \times \frac{\mu_0}{2\pi}\left(\frac{200}{3}\right)[2 \times \hat{j} - 3(\hat{\imath})]$$

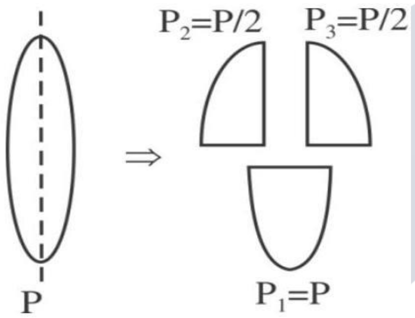
$$= (4\pi \times 10^{-7})(100)(-3\hat{\imath} + 2\hat{j})$$

$$= 4\pi \times 10^{-5} \times [-3\hat{i} + 2\hat{j}]$$

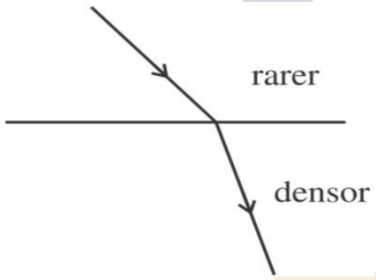
14. **(d)** $\left(\frac{L}{C}\right)$ does not have dimension of time. RC, $\frac{L}{R}$ are time constant while $\sqrt{LC}$ is reciprocal of angular frequency or having dimension of time.

15. **(c)** The statement II is wrong as the velocity of εm wave in a medium is $\frac{1}{\sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{\mu_0\mu_r\varepsilon_0\varepsilon_r}}$.

16. **(a)**



17. **(c)**



No change in frequency but speed and wave-length decreases.

18. **(d)** When electron jump from lower to higher energy level, energy absorbed so statement-I incorrect. When electron jump from higher to lower energy level, energy of emitted photon

$$E = E_2 - E_1$$

$$hf = E_2 - E_1 \Rightarrow f = \frac{E_2 - E_1}{h}$$

so statement-II is correct.

19. **(d)** Transistor act as a switch in saturation and cut of region.

20. **(a)**

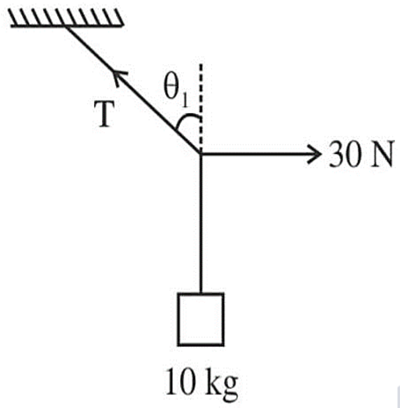
(a) For low frequency or high wavelength size of antenna required is high.

(b) E P R is low for longer wavelength.

(c) yes we want to avoid mixing up signals transmitted by different transmitter simultaneously.

(d) Low frequency signals sent to long distance by superimposing with high frequency.

21. (3)



$T\sin\theta = 30$

$T\cos\theta = 100$

$\Rightarrow \tan\theta = 0.3$

22. (3) Net loss in PE = Gain in KE

$$12gh - 3gh = \frac{1}{2}3v^2 + \frac{1}{2}12v^2 + \frac{1}{2}[12r^2]\left(\frac{v}{r}\right)^2$$

$$9gh = \frac{1}{2}[3 + 12 + 12]v^2$$

$$v^2 = \frac{2gh}{3} \Rightarrow v = \frac{1}{2}\sqrt{\frac{8}{3}gh}$$

$$x = \frac{8}{3} \simeq 3$$

23. (1400) $Q = nC_p\Delta T = \frac{n\nu}{\nu - 1}R\Delta T$

$$Q = \frac{\nu}{\nu - 1}\omega = \frac{1.4}{0.4} \times 400 = 1400\ J$$

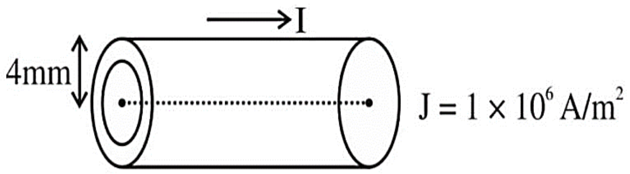
24. (1)

$$t = \frac{\Delta\phi}{\omega} = \frac{\pi/2 - \pi/6}{2\pi/6} = \frac{\pi/3}{\pi/3} = 1sec$$

25. (23) Parallel combination

$$c_{eq}=\varepsilon_0 A\left[\frac{1}{5b}+\frac{1}{3b}+\frac{1}{b}\right]=\frac{23}{15}\frac{\varepsilon_0 A}{b}$$

26. (12)



$$I=\int JdA$$

$$=\int 10^6\times 2\pi x dx$$

$$=10^6\times 2\pi\cdot x\frac{x^2}{2}\Big]_{\frac{r}{2}}^{r}$$

$$=\pi\times 10^6\left[r^2-\frac{r^2}{4}\right]=12\pi$$

$$x=12$$

27. (3)

$$R=\left(\frac{ma}{3}\right)+\left(\frac{a}{2m}\right)$$

$$\frac{dR}{dm}=\frac{a}{3}-\frac{a}{2m^2}=0$$

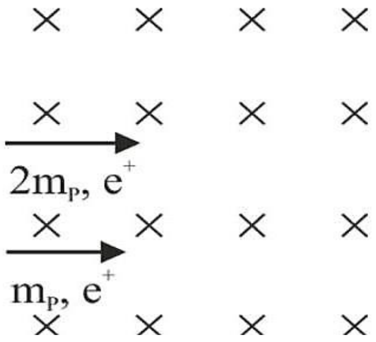
$$\frac{a}{3}=\frac{a}{2m^2}$$

$$m^2=\frac{3}{2}$$

$$m=\sqrt{\frac{3}{2}}$$

$$x=3$$

28. (2)



$R = \frac{mv}{q_B}$

$R_D = \frac{(2\ m_P)v_D}{eB}$

$R_P = \frac{(m_P)v_P}{eB}$

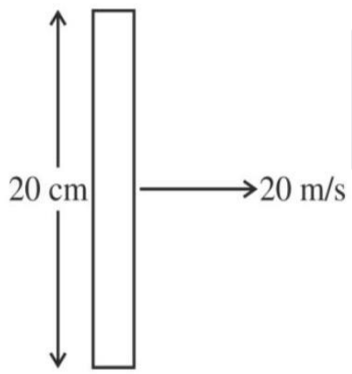
$\frac{R_D}{R_P} = \frac{2v_D}{v_P} = \frac{2v_D}{\sqrt{2}v_D} = \frac{\sqrt{2}}{1}$

$\frac{1}{2}(2mp)v_D^2 = \frac{1}{2}\ m_P \cdot v_P^2$

$\sqrt{2}v_D = v_P$

$x = 2$

29. (16)



$B_H = 4 \times 10^{-3}\ T$

$\theta \to 45°$

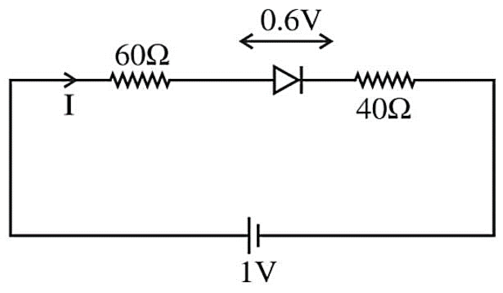
$B_V = B_H$

$\epsilon = (\vec{V} \times \vec{B}) \cdot \vec{\ell}$

$= \left((4 \times 10^{-3})(20)\right)\frac{20}{100}$

$= 16 \times 10^{-3}\ V = 16mV$

30. (4)



$1 - I(60) - 0.6 - I(40) = 0$

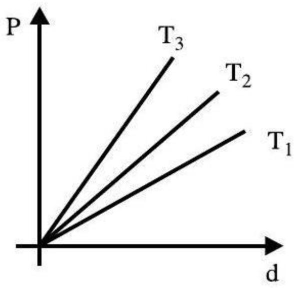
$\frac{0.4}{100} = I$

$I = 4\ mA$

31. (b)

P vs d :

$P = \left(\frac{RT}{M}\right) d$



$T_3 > T_2 > T_1$

32. (c) In PCl_5, axial bonds are weaker than equatorial.
33. (d) Statement-1: wrong, $Au^{+}$is correct, not Au^{+3}

Statement-2: correct

34. (c) Order of H-Bonding

$CH_4 < HCN < NH_3$

NCH ... NCH

H_2NH ... NH_3

35. (a) $N^{-3} > O^{-2} > F^{-} > Na^{+} > Mg^{+2}$ (Radii)

(Isoelectronic species)

36. (b)

$$NH_4Cl + NaNO_2 \rightarrow NH_4NO_2 + NaCl$$

$$\downarrow$$

$$N_2 + 2H_2O$$

37. (a) Both A and R are correct and R is the correct explanation of A.
38. (c) $Cr^{+2}/Cr \rightarrow -0.90\ V$

$Fe^{+2}/Fe \rightarrow -0.44\ V$

$Cu^{+2}/Cu \rightarrow +0.34\ V$

$Zn^{+2}/Zn \rightarrow -0.76$ V

So Ans. Cu^{+2}/Cu

39. (c)

$Tb \rightarrow 4f^{9}6\ s^{2}$

$Tb^{+4} \rightarrow 4f^{7}$

$Yb \rightarrow 4f^{14}6\ s^{2}$

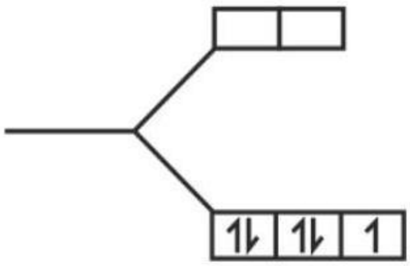
$Yb^{+2} \rightarrow 4f^{14}$

40. (b)

(a) $[FeF_6]^{3-}$

$Fe^{+3} \rightarrow 3\ d^{5}4\ s^{0}$
$n = 5$

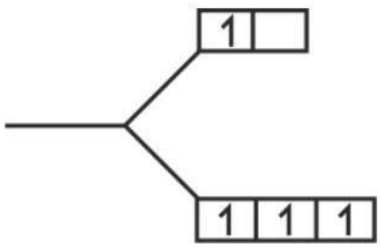
(b) $[Fe(CN)_6]^{3-}$



$Fe^{+3} \rightarrow 3\ d^{5}4\ s^{0}$

$n = 1$

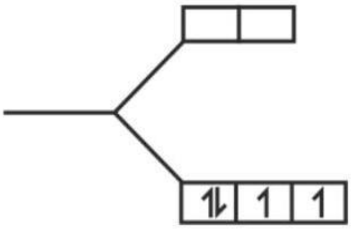
(c) $[MnCl_6]^{3-}$



$Mn^{+3} \rightarrow 3\ d^{4}4\ s^{0}$

$n = 4$

(d) $[Mn(CN)_6]^{3-}$



$Mn^{+3} \rightarrow 3\,d^4 4\,s^0$

$n = 2$

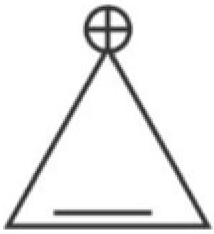
$\mu \Rightarrow A > C > D > B$

41. (a)

$$O_2N-O-Cl + H_2O \longrightarrow \underset{(B)}{O_2N-OH} + \underset{(A)}{HOCl}$$

$$O_2N-O-Cl + HCl \longrightarrow \underset{(B)}{O_2N-OH} + \underset{(C)}{Cl_2}$$

42. (a)

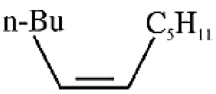


is most stable as it is aromatic.

43.(c)

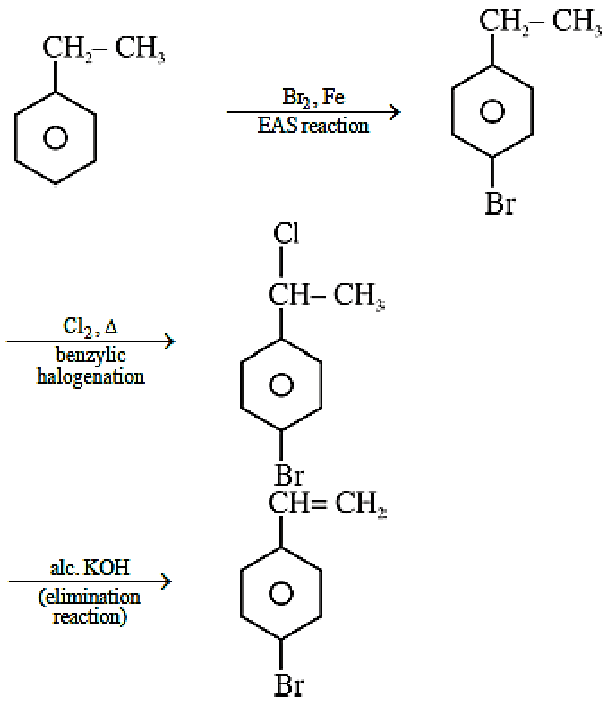
$$n-Bu-C\equiv CH \xrightarrow[\text{(acid base reaction)}]{nBuLi} n-Bu-C\equiv C^{-}Li^{+}$$

$$\downarrow n-C_5H_{11}Cl \text{ (SN reaction)}$$



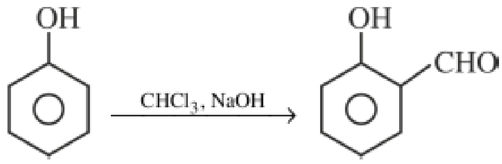
$$\xleftarrow[\text{Lindlar's Catalyst}]{H_2} n-Bu-C\equiv C-C_5H_{11}$$

44.(d)

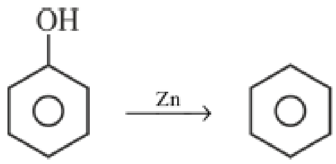


45. (a)

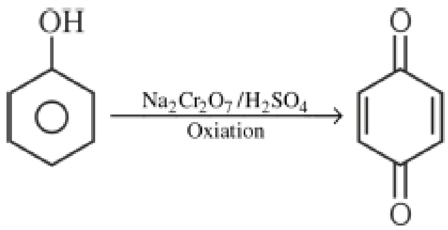
(A)



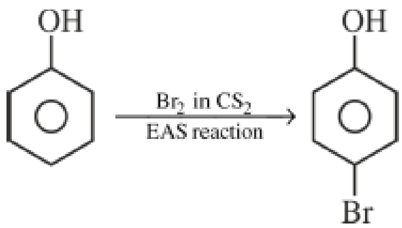
(B)



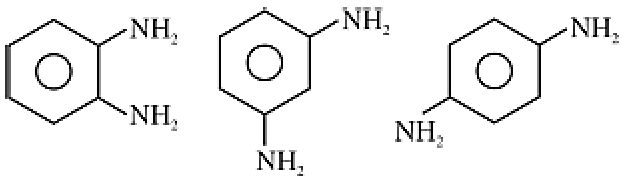
(C)



(D)



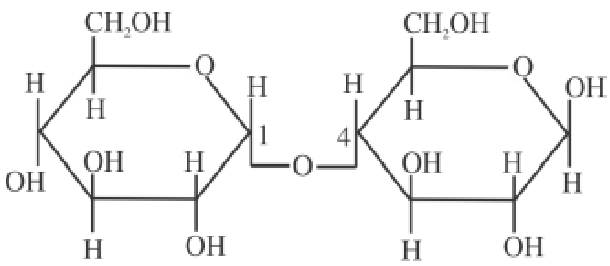
46. (d)



M.P. 142°C

47. (c) It is copolymerization of 1, 3-butadiene and acrylonitrile.

48. (c)



Maltose

49. (a)

A. Antipyretic	Reduces fever
B. Analgesic	Reduces pain
C. Tranquilizer	Reduces stress
D. Antacid	Reduces acidity (Stomach)

50. (d)

$CO_3{}^{2-}$ will give $CO_2(g)$ which will turns lime water milky.
S^{2-} will give H_2S (g), will turns lead acetate paper black
$SO_3{}^{2-}$ will give $SO_2(g)$, which will turns acidified potassium dichromate solution green.
NO_2^- will give brown $NO_2(g)$ will turn KI solution blue.

51. (2)

$$\%H = \frac{7.5}{116} \times 100 = 6.5$$

$$\%O = \frac{60}{116} \times 100 = 51.7$$

$$\%C = \frac{48.5}{116} \times 100 = 41.8$$

Relative atomicities $=$ $H \Rightarrow 6.5$

$$O \Rightarrow \frac{51.7}{16} = 3.25$$

$$C \Rightarrow \frac{41.8}{12} = 3.5$$

Emperically formula is approx.. CH_2O
(a) $C_2H_4O_2$
(b) CH_2O relate to this formula.

52. (2) Quantum no. of set (b) and (c) can be correct.

(a) and (d) are wrong as $n = \ell$ is not possible.

53. (2)

$$BeO + HF + NH_3 \rightarrow \underset{[A]}{(NH_4)_2[BeF_4]} \xrightarrow{\Delta} NH_4F + \underset{[B]}{BeF_2}$$

Oxidation state of Be in A is (+2)

54. (8630) $n = 5$ mol

$T = 300$ K

$V_1 = 10$ L

$V_2 = 20$ L

$$w = -nRT\ell n\frac{V_2}{V_1}$$

$$= -5 \times 8.3 \times 300 \times \ell n\frac{20}{10}$$

$$= -8630.38 \text{ J}$$

55. (45) $w = 2.5$ g

$W_{solvent} = 75$ g

$T_B' = 373.535$ K

$T_B^o = 373.15$ K

$\Delta T_B = 0.385 = K_b$ molality

$$0.385 = 0.52 \times \left(\frac{2.5}{M} \times \frac{1000}{75}\right)$$

$M = 45 \text{ g mol}^{-1}$

56. (11) $0.001 M NaOH$

$[OH^-] = 10^{-3}$

$pOH = 3$

$pH = 11$

57. (51) $\frac{1}{2}H_2 + Ag^+ \rightarrow H^+ + Ag$

$\Delta G^\circ = -nE^\circ F$

$= -1 \times 0.5332 \times 96500 \text{ J}$

$= -51.35 \text{ kJ}$

($n = 2$ for $H_2 + 2Ag^+ \rightarrow 2H^+ + 2Ag$)

58. (59)

$$\log_{10}\frac{K_2}{K_1} = \frac{E_a}{2.303R}\left(\frac{1}{300} - \frac{1}{309}\right)$$

$$0.3 = \frac{E_a}{2.303 \times 8.3}\left(\frac{9}{300 \times 309}\right)$$

$$E_a = \frac{0.3 \times 2.303 \times 8.3 \times 300 \times 309}{9}$$

$= 59065.04 \text{ J}$

$E_a = 59.06 \text{ kJ}$

59. (12) $\frac{x}{m} = kP^{\frac{1}{n}}$

$$\log\frac{x}{m} = \log k + \frac{1}{n}\log P$$

From graph

Slope $= \frac{1}{n} = 1 \Rightarrow n = 1$

Intercept $= \log k = 0.602$

$k = 4$

$$\frac{x}{m} = 4 \times (0.03)^{\frac{1}{1}}$$

$$\frac{x}{m} = 12 \times 10^{-2}$$

60. (40) wt. of organic compound $= 0.25$ g

mass of Cl $= \frac{35.5}{143.5} \times 0.4$ g

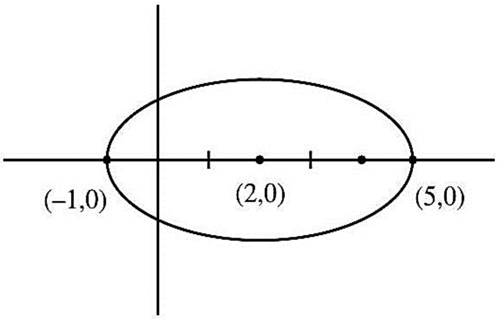
mass % of Cl in the organic compound

$$= \frac{35.5 \times 0.4}{143.5 \times 0.25} \times 100$$

$= 39.58\%$

61. (c) $C: (x-4)^2 + (y-3)^2 = 4$

$E: \frac{(x-2)^2}{9} + \frac{y^2}{5} = 1$



Lower Extremity of vertical diameter of

circle $\rightarrow (4,1)$

Put in ellipse $\Rightarrow \frac{(4-2)^2}{9} + \frac{1}{5} - 1$

$= \frac{4}{9} + \frac{1}{5} - 1$

$= \frac{29}{45} - 1 < 0$

Two Solutions

62. (c) $f(x) = \begin{vmatrix} a & -1 & 0 \\ ax & a & -1 \\ a^2 & ax & a \end{vmatrix}$

$f(x) = a\begin{vmatrix} 1 & -1 & 0 \\ x & a & -1 \\ x^2 & ax & a \end{vmatrix}$

$= a[1(a^2 + ax) + 1(ax + x^2)]$

$\Rightarrow f(x) = a(x + a)^2$

so, $f'(x) = 2a(x + a)$

as, $2f'(10) - f'(5) + 100 = 0$

$\Rightarrow 2 \times 2a(10 + a) - 2a(5 + a) + 100 = 0$

$\Rightarrow 40a + 4a^2 - 10a - 2a^2 + 100 = 0$

$2a^2 + 30a + 100 = 0$

$\Rightarrow a^2 + 15a + 50 = 0$

$(a + 10)(a + 5) = 0$

$a=-10$ or $a=-5$

Required $=(-10)^2+(-5)^2=125$

63. (b) $a=\alpha-i\beta; \alpha \in R; \beta \in R$

$4ix+(1+i)y=0$ and

$8\left(\cos\frac{2\pi}{3}+i\sin\frac{2\pi}{3}\right)x+\bar{a}y=0$

$\begin{vmatrix} 4i & 1+i \\ 8e^{i2\pi/3} & \bar{a} \end{vmatrix}=0$

$\Rightarrow 4i\bar{a}-(1+i)8e^{i2\pi/3}=0$

$\Rightarrow 4i(\alpha+i\beta)-8(1+i)\left(\frac{-1+i\sqrt{3}}{2}\right)=0$

$\Rightarrow i\alpha-\beta+1+\sqrt{3}+i(1-\sqrt{3})=0$

$\Rightarrow \beta=\sqrt{3}+1$

$\alpha=\sqrt{3}-1$

So, $\frac{\alpha}{\beta}=\frac{\sqrt{3}-1}{\sqrt{3}+1}=2-\sqrt{3}$

64. (c) $AB=i$

$|\operatorname{adj}(B\operatorname{adj}(2A)|=|B\operatorname{adj}(2A)|^2$

$=|B|^2|\operatorname{adj}(2A)|^2$

$=|B|^2(|2A|^2)^2=|B|^2(2^6|A|^2)^2$

$|A|=\frac{1}{8}$ and $|AB|=1\Rightarrow |A||B|=1$

$\Rightarrow \frac{1}{8}|B|=1$

$\Rightarrow |B|=8$

required value $=64$

65. (c) $S=2+\frac{6}{7}+\frac{12}{7^2}+\frac{20}{7^3}+\frac{30}{7^4}+\cdots\ldots$

Considering infinite sequence,

$S=2+\frac{6}{7}+\frac{12}{7^2}+\frac{20}{7^3}+\frac{30}{7^4}+$

$\frac{S}{7}=\frac{2}{7}+\frac{6}{7^2}+\frac{12}{7^3}+\frac{20}{7^4}+$

$$\Rightarrow \frac{6S}{7} = 2 + \frac{4}{7} + \frac{6}{7^2} + \frac{8}{7^3} + \frac{10}{7^4} + \cdots \ldots$$

$$\Rightarrow \frac{6S}{7^2} = \frac{2}{7} + \frac{4}{7^2} + \frac{6}{7^3} + \frac{8}{7^4} +$$

$$\frac{6S}{7}\left(1 - \frac{1}{7}\right) = 2 + \frac{2}{7} + \frac{2}{7^2} + \frac{2}{7^3} +$$

$$\Rightarrow \frac{6^2 S}{7^2} = \frac{2}{1 - \frac{1}{7}} = \frac{2}{6} \times 7$$

$$\Rightarrow S = \frac{2 \times 7^3}{6^3} \Rightarrow 4S = \frac{7^3}{3^3} = \left(\frac{7}{3}\right)^3$$

66. (d) $a_1, a_2, a_3 \ldots$ A.P. ; $a_1 = 2; a_{10} = 3; d_1 = \frac{1}{9}$

$b_1, b_2, b_3, \ldots$ A.P. ; $b_1 = \frac{1}{2}; b_{10} = \frac{1}{3}; d_2 = \frac{-1}{54}$

[Using $a_1 b_1 = 1 = a_{10} b_{10}$; $d_1 \& d_2$ are common differences respectively]

$$a_4 \cdot b_4 = (2 + 3d_1)\left(\frac{1}{2} + 3d_2\right)$$
$$= \left(2 + \frac{1}{3}\right)\left(\frac{1}{2} - \frac{1}{18}\right)$$
$$= \left(\frac{7}{3}\right)\left(\frac{8}{18}\right) = \frac{28}{27}$$

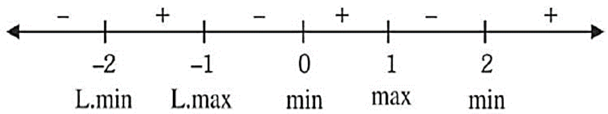
67. (b) $m = L \cdot \max$

$N = L \cdot \min$

$$f(x) = \int_0^{x^2} \frac{t^2 - 5t + 4}{2 + e^t} dt$$

$$f'(x) = \frac{(x^4 - 5x^2 + 4)2x}{2 + e^{x^2}} = \frac{2x(x^2 - 1)(x^2 - 4)}{2 + e^{x^2}}$$

$$= \frac{2x(x-1)(x+1)(x-2)(x+2)}{2 + e^{x^2}}$$



so, $m = 2$ and $n = 3$

68. (b) At right hand vicinity of $x = 0$ given equation does not satisfy

$\because$ LHS $= \int_{1^-}^{1} t^2 f(t) dt = 0$, RHS $= \lim_{x \to 0^+} (\sin^3 x + \cos x) = 1$

LHS $\neq$ RHS hence data given in question is wrong hence BONUS

Correct data should have been

$\int_{\cos x}^{1} t^2 f(t) dt = \sin^3 x + \cos x - 1$

Calculation for option

differentiating both sides

$-\cos^2 x f(\cos x) \cdot (-\sin x) = 3\sin^2 x \cdot \cos x - \sin x$

$\Rightarrow f(\cos x) = 3\tan x - \sec^2 x$

$\Rightarrow f'(\cos x)(-\sin x) = 3\sec^2 x - 2\sec^2 x \tan x$

$\Rightarrow f'(\cos x)\cos x = \frac{2}{\cos^2 x} - \frac{3}{\sin x \cdot \cos x}$

When $\cos x = \frac{1}{\sqrt{3}}; \sin x = \frac{\sqrt{2}}{\sqrt{3}} \therefore f'\left(\frac{1}{\sqrt{3}}\right)\frac{1}{\sqrt{3}} = 6 - \frac{9}{\sqrt{2}}$.

69. (a)

$\int_0^1 \frac{1}{7^{\left[\frac{1}{x}\right]}} dx = -\int_1^0 \frac{1}{7^{\left[\frac{1}{x}\right]}} dx$

$= (-1)\left[\int_1^{1/2} \frac{1}{7} dx + \int_{1/2}^{1/3} \frac{1}{7^2} dx + \int_{1/3}^{1/4} \frac{1}{7^3} dx + \cdots \ldots \infty\right]$

$= \left(\frac{1}{7} + \frac{1}{2.7^2} + \frac{1}{3 \cdot 7^3} + \cdots \infty\right) - \left(\frac{1}{7 \cdot 2} + \frac{1}{7^2 \cdot 3} + \frac{1}{7^2 \cdot 4} \ldots \infty\right)$

$= -\ln\left(1 - \frac{1}{7}\right) - 7\left(\frac{1}{7^2 \cdot 2} + \frac{1}{7^3 \cdot 3} + \frac{1}{7^4 \cdot 4} + \cdots \ldots \infty\right)$

(as $\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} \ldots \infty]$

(as $\ln(1-x) = -\left(x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} \ldots \infty\right)]$

$= -\ln\frac{6}{7} - 7\left(-\ln\left(1 - \frac{1}{7}\right) - \frac{1}{7}\right)$

$= 6\ln\frac{6}{7} + 1$

70. (b) $\frac{dx}{dy} + \frac{x}{1+y^2} = \frac{\tan^{-1} y}{1+y^2}$

I.f $= e^{\int \frac{1}{1+y^2} dy} = e^{\tan^{-1} y}$

$x e^{\tan^{-1} y} = \int \frac{\tan^{-1} y}{1+y^2} e^{\tan^{-1} y} dy$

$x \cdot e^{\tan^{-1} y} = (\tan^{-1} y - 1)e^{\tan^{-1} y} + c$

$\because$ (1,0) lies exit $c = 2$.

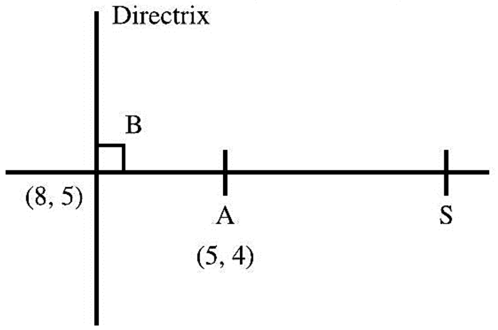
For $y = \tan 1 \Rightarrow x = \frac{2}{e}$

71. (d) Vertex (5,4)

Directrix: $3x + y - 29 = 0$

Co-ordinates of B (foot of directrix)

$$\frac{x-5}{3} = \frac{y-4}{1} = -\left(\frac{15+4-29}{10}\right) = 1$$



$x = 8, y = 5$

$S = (2,3)$ (focus)

Equation of parabola

$PS = PM$

so equation is

$x^2 + 9y^2 - 6xy + 134x - 2y - 711 = 0$

$a + b + c + d + k = 9 - 6 + 134 - 2 - 711 = -576$

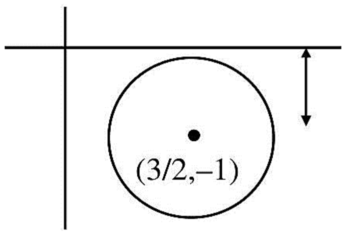
72. (d) $C: 4x^2 + 4y^2 - 12x + 8y + k = 0$

$\Rightarrow x^2 + y^2 - 3x + 2y + \left(\frac{k}{4}\right) = 0$

Centre $\left(\frac{3}{2}, -1\right)$; $r = \sqrt{\frac{13-k}{2}} \Rightarrow k \leq 13$ (i) Point $\left(1, \frac{-1}{3}\right)$ lies on or inside circle C

$\Rightarrow S_1 \leq 0 \Rightarrow k \leq \frac{92}{9}$

(ii) C lies in 4^{th} quadrant



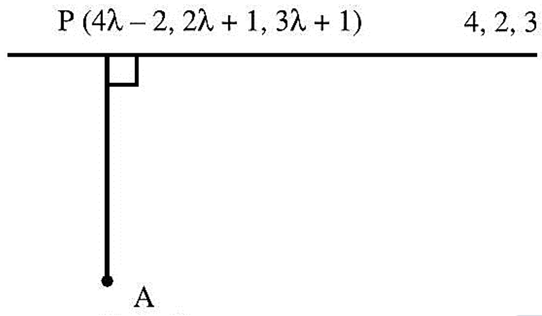
$r < 1$

$\Rightarrow \frac{\sqrt{13-\mathrm{k}}}{2} < 1$

$\Rightarrow \mathrm{k} < 9$

Hence $(1) \cap (2) \cap (3) \Rightarrow k \in \left(9, \frac{92}{9}\right]$

73. (a)



(1,2,4)

$\frac{\mathrm{x}+2}{4} = \frac{\mathrm{y}-1}{2} = \frac{\mathrm{z}+1}{3} = \lambda$

$(x, y, z) = (4\lambda - 2, 2\lambda + 1, 3\lambda - 1)$

$\overrightarrow{\mathrm{AP}} = (4\lambda - 3)\hat{\imath} + (2\lambda - 1)\hat{\jmath} + (3\lambda - 5)\hat{\mathrm{k}}$

$\vec{\mathrm{b}} = 4\hat{\imath} + 2\hat{\jmath} + 3\hat{\mathrm{k}}$

$\overrightarrow{\mathrm{AP}} \cdot \vec{\mathrm{b}} = 0$

$4(4\lambda - 3) + 2(2\lambda - 1) + 3(3\lambda - 5) = 0$

$29\lambda = 12 + 2 + 15 = 29$

$\lambda = 1$

$\mathrm{P} = (2,3,2)$

$3\mathrm{x} + 4\mathrm{y} + 12\mathrm{z} + 23 = 0$

$\mathrm{d} = \left|\frac{6 + 12 + 24 + 23}{\sqrt{9 + 16 + 144}}\right|$

$\mathrm{d} = \left|\frac{65}{13}\right| = 5$

74. (a) $\frac{x-3}{2} = \frac{y-2}{3} = \frac{z-1}{-1}$

$\frac{\mathrm{x}+3}{2} = \frac{\mathrm{y}-6}{1} = \frac{\mathrm{z}-5}{3}$

$\mathrm{A} = (3,2,1)\ \mathrm{B} = (-3,6,5)$

$\overrightarrow{n_1} = 2\hat{\imath} + 3\hat{\jmath} - \hat{k}$

$\overrightarrow{n_2} = 2\hat{\imath} + \hat{\jmath} - 3\hat{k}$

$\overrightarrow{BA} = 6\hat{\imath} - 4\hat{\jmath} - 4\hat{k}$

$$\text{SHORTEST DISTANCE} = \frac{\left[\overrightarrow{BA}\ \overrightarrow{n_1}\ \overrightarrow{n_2}\right]}{\left|\overrightarrow{n_1} \times \overrightarrow{n_2}\right|}$$

$$\overrightarrow{n_1} \times \overrightarrow{n_2} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2 & 3 & -1 \\ 2 & 1 & 3 \end{vmatrix}$$

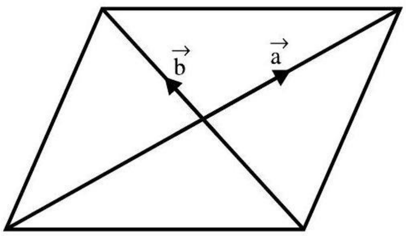
$= 10\hat{\imath} - 8\hat{\jmath} - 4\hat{k}$

$\left[\overrightarrow{BA} \quad \overrightarrow{n_1} \quad \overrightarrow{n_2}\right] = 60 + 32 + 16 = 108$

$|\overrightarrow{n_1} \times \overrightarrow{n_2}| = \sqrt{100 + 64 + 16} = \sqrt{180}$

$\text{S.D} = \frac{108}{\sqrt{180}} = \frac{108}{6\sqrt{5}} = \frac{18}{\sqrt{5}}$

75. (d)



Area $= \frac{1}{2}|\vec{a} \times \vec{b}| = 2\sqrt{2} \Rightarrow |\vec{a} \times \vec{b}| = 4\sqrt{2}$

$|\vec{a}| = 1$ and $|\vec{a} \cdot \vec{b}| = |\vec{a} \times \vec{b}|$

$\Rightarrow \cos\theta = \sin\theta$

$\Rightarrow \theta = \frac{\pi}{4}$

$\therefore |\vec{a} \times \vec{b}| = 4\sqrt{2} \Rightarrow |\vec{a}||\vec{b}|\sin\frac{\pi}{4} = 4\sqrt{2}$

$\Rightarrow |\vec{b}| = 8$

Now, $\vec{c} = 2\sqrt{2}(\vec{a} \times \vec{b}) - 2\vec{b}$

$|\vec{c}| = \sqrt{(2\sqrt{2})^2|\vec{a} \times \vec{b}|^2 + (2|\vec{b}|)^2} = 16\sqrt{2}$

Now, $\vec{b} \cdot \vec{c} = -2|\vec{b}|^2$

$\Rightarrow 8 \times 16\sqrt{2} \times \cos\alpha = -2.64$

$\Rightarrow \cos\alpha = -\frac{1}{\sqrt{2}} \Rightarrow \alpha = \frac{3\pi}{4}$

76. (b) mean $\bar{x} = \frac{4+5+6+6+7+8+x+y}{8} = 6$

$\Rightarrow x + y = 48 - 36 = 12$

Variance

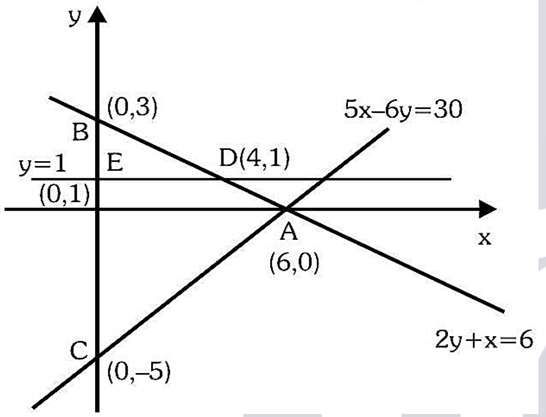
$= \frac{1}{8}(16 + 25 + 36 + 36 + 49 + 64 + x^2 + y^2) - 36 = \frac{9}{4}$

$\Rightarrow x^2 + y^2 = 80$

$\therefore x = 4; y = 8$

$x^4 + y^2 = 256 + 64 = 320$

77. (b) Required probability $= \frac{ar(ADEC)}{ar(ABC)}$



$= 1 - \frac{ar(BDE)}{ar(ABC)}$

$= 1 - \frac{\frac{1}{2} \times 2 \times 4}{\frac{1}{2} \times 8 \times 6} = 1 - \frac{1}{6} = \frac{5}{6}$

78. (a) $\tan^{-1}\frac{1}{1+n+n^2} = \tan^{-1}\left(\frac{(n+1)-n}{1+n(n+1)}\right)$

$= \tan^{-1}(n+1) - \tan^{-1} n$

so, $\sum_{n=1}^{50}(\tan^{-1}(n+1) - \tan^{-1} n)$

$= \tan^{-1} 51 - \tan^{-1} 1$

$\cot\left(\sum_{n=1}^{50} \tan^{-1}\left(\frac{1}{1+n+n^2}\right)\right) = \cot(\tan^{-1} 51 + \tan^{-1} 1)$

$= \frac{1}{\tan(\tan^{-1} 51 - \tan^{-1} 1)} = \frac{1 + 51 \times 1}{51 - 1} = \frac{52}{50} = \frac{26}{25}$

79. (c) $\cos 72^\circ = \frac{\sqrt{5}-1}{4}$

$\Rightarrow 1 - 2\sin^2 36^\circ = \dfrac{\sqrt{5}-1}{4}$

$\Rightarrow 4 - 8\alpha^2 = \sqrt{5} - 1$

$\Rightarrow 5 - 8\alpha^2 = \sqrt{5}$

$\Rightarrow (5 - 8\alpha^2)^2 = 5$

$\Rightarrow 25 + 64\alpha^4 - 80\alpha^2 = 5$

$\Rightarrow 64\alpha^4 - 80\alpha^2 + 20 = 0$

$\Rightarrow 16\alpha^4 - 20\alpha^2 + 5 = 0$

80. (c)

$(A)(\sim q \wedge p) \wedge q = (\sim q \wedge q) \wedge p = f$

(b) $(\sim q \wedge p) \wedge (p \wedge \sim p) = \sim q \wedge (p \wedge \sim p) = f$

(c) $(\sim q \wedge p) \vee (p \vee \sim p) = (\sim q \wedge p) \vee (t) = t$

(d) $(p \wedge q) \wedge (\sim (p \wedge q)) = f$

81. (190)

$$f^{-1}(n) = \begin{cases} \dfrac{n}{2} & ; n = 2,4,6,8,10 \\ \dfrac{n+11}{2} & ; n = 1,3,5,7,9 \end{cases}$$

$$f(g(n)) = \begin{cases} n+1 & ; \quad n \in \text{ odd} \\ n-1 & ; \quad n \in \text{ even} \end{cases}$$

$$\Rightarrow \; g(n) = \begin{cases} f^{-1}(n+1) & ; n \in \text{ odd} \\ f^{-1}(n-1) & ; n \in \text{ even} \end{cases}$$

$$\therefore \; g(n) = \begin{cases} \dfrac{n+1}{2} & ; \quad n \in \text{ odd} \\ \dfrac{n+10}{2} & ; \quad n \in \text{ even} \end{cases}$$

$g(10) \cdot [g(1) + g(2) + g(3) + g(4) + g(5)]$

$= 10 \cdot [1 + 6 + 2 + 7 + 3] = 190$

82. (98)

$x^2 - 4\lambda x + 5 = 0\langle {}^{\alpha}_{\beta}$

$x^2 - (3\sqrt{2} + 2\sqrt{3})x + (7 + 3\lambda\sqrt{3}) = 0\langle {}^{\alpha}_{\gamma}$

$\alpha+\beta=4\lambda$

$\alpha+\gamma=3\sqrt{2}+2\sqrt{3}$

$$\begin{aligned} & \beta+\lambda=3\sqrt{2} && \alpha\gamma=7+3\lambda\sqrt{3} \\ \therefore\quad & \alpha=2\lambda+\sqrt{3} && \alpha\beta=5 \\ & \beta=2\lambda-\sqrt{3} && 4\lambda^2=8\Rightarrow\lambda=\sqrt{2} \\ \therefore\quad & && (\alpha+2\beta+\lambda)^2=(4\alpha+3\sqrt{2})^2=(7\sqrt{2})^2=98 \end{aligned}$$

83. **(180)** Let $A=\begin{bmatrix} a & b \\ c & d \end{bmatrix}$; $a,b,c,d\in\{0,1,2,3,4,5\}$

$a+b+c+d=p,\ p\in\{3,5,7\}$

Case-(i)

$a+b+c+d=3;\ a,b,c,d\in\{0,1,2,3\}$

No. of ways $=\ {}^{3+4-1}C_{4-1}=\ {}^{6}C_3=56$

Case-(ii)

$a+b+c+d=5;\ a,b,c,d\in\{0,1,2,3,4,5\}$

No. of ways $=\ {}^{5+4-1}C_{4-1}=\ {}^{8}C_3=56$

Case-(iii)

$a+b+c+d=7$

No. of ways = total ways when $a,b,c,d\in\{0,1,2,3,4,5,6,7\}$ - total ways when $a,b,c,d\notin\{6,7\}$

No of ways $=\ {}^{7+4-1}C_{4-1}=\left(\frac{4}{\lfloor 3}+\frac{4}{\lfloor 2}\right)$

$=\ {}^{10}C_3-16=104$

Hence total no. of ways $=180$

84. **(57)** coefficients and there cumulative sum are :

Coefficient	Commulative sum
$x^{7n}\rightarrow {}^7C_0$	1
$x^{6n-5}\rightarrow 2\cdot {}^7C_1$	$1+14$
$x^{5n-10}\rightarrow 2^2\cdot {}^7C_2$	$1+14+84$
$x^{4n-15}\rightarrow 2^3\cdot {}^7C_3$	$1+14+84+280$
$x^{3n-20}\rightarrow 2^4\cdot {}^7C_4$	$1+4+84+280+560=939$

$x^{2n-25} \to 2^5 \cdot {}^7C_5$

$3n - 20 \geq 0 \cap 2n - 25 < 0 \cap n \in I$

$\therefore\ 7 \leq n \leq 12$

$\text{Sum} = 7 + 8 + 9 + 10 + 11 + 12 = 57$

85. (3) $f(x) = [1 + x] + \frac{\alpha^{2[x]+\{x\}}+[x]-1}{2[x]+\{x\}}$

$\lim_{x\to 0^-} f(x) = \alpha - \frac{4}{3} \Rightarrow 0 + \frac{\alpha^{-1} - 2}{-1} = \alpha - \frac{4}{3}$

$\Rightarrow 2 - \frac{1}{\alpha} = \alpha - \frac{4}{3}$

$\Rightarrow \alpha + \frac{1}{\alpha} = \frac{10}{3}$

$\Rightarrow \alpha = 3; \alpha \in I$

86. (16) $y = (x) = (x^x)^x$

$\ln y(x) = x^2 \cdot \ln x$

$\frac{1}{y(x)} \cdot y'(x) = \frac{x^2}{x} + 2x \cdot \ln x$

$y'(x) = y(x)[x + 2x\ln x]$

$y(1) = 1; y'(1) = 1$

$y''(x) = y'(x)[x + 2x \cdot \ln(x)]\ y''(1) = 1[1 + 0] + 1(1 + 2) = 4$

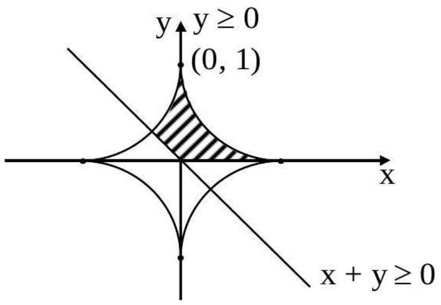
$\frac{d^2y}{dx^2} = -\left(\frac{dy}{dx}\right)^3 \cdot \frac{d^2x}{dy^2}$

$\Rightarrow 4 = -\frac{d^2x}{dy^2}$

$\frac{d^2x}{dy^2} = -4$

Ans. $-4 + 20 = 16$

87. (36)



$A=\frac{3}{2}\int_0^1\left(1-x^{2/3}\right)^{3/2}\mathrm{dx}$

Let $\mathrm{x}=\sin^3\theta$

$\mathrm{A}=\frac{3}{2}\int_0^{\pi/2}(1-\sin^2\theta)^{3/2}\cdot 3\sin^2\theta\cos\theta\mathrm{d}\theta$

$=\frac{3}{2}\int_0^{\pi/2}3\sin^2\theta\cos^4\theta\mathrm{d}\theta$

$=\frac{9}{2}\int_0^{\pi/2}\sin^2\theta\cos^4\theta\mathrm{d}\theta$

$\mathrm{A}=\frac{9}{2}\times\frac{1.3\cdot 1}{(2+4)(4)(2)}\cdot\frac{\pi}{2}$

$\Rightarrow \mathrm{A}=\frac{9\pi}{64}\Rightarrow\frac{64\mathrm{~A}}{\pi}=9$

$\Rightarrow\frac{256\mathrm{~A}}{\pi}=36$ Ans.

88. (320)

$(1-x^2)\frac{dy}{dx}=xy+(x^3+2)\sqrt{1-x^2}$

$\Rightarrow\frac{dy}{dx}+\left(\frac{-x}{1-x^2}\right)y=\frac{x^3+2}{\sqrt{1-x^2}}$

$\mathrm{IF}=\mathrm{e}^{\int\frac{-\mathrm{x}}{1-\mathrm{x}^2}\mathrm{dx}}=\sqrt{1-\mathrm{x}^2}$

$y(x)\cdot\sqrt{1-x^2}=\frac{x^4}{4}+2x+c$

$\mathrm{y}(0)=0\Rightarrow \mathrm{c}=0$

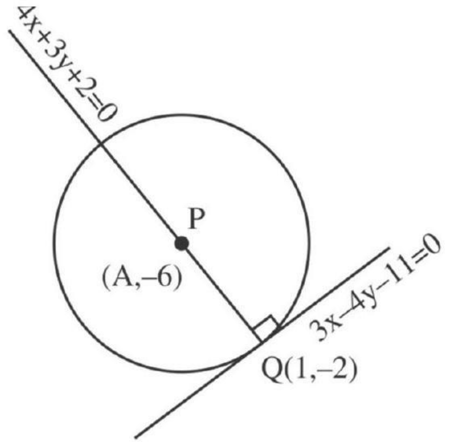
$\sqrt{1-x^2}y(x)=\frac{x^4}{4}+2x$

required value $=\int_{-1/2}^{1/2}\left(\frac{x^4}{4}+2x\right)dx-\frac{1}{4}\cdot 2\int_0^{1/2}x^4dx$

$=\frac{1}{10}(\mathrm{x}^5)_0^{1/2}=\frac{1}{320}$

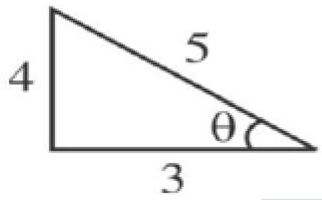
$\mathrm{k}^{-1}=320$

89. (11)



$4x + 3y + 2 = 0$

$3x - 4y - 11 = 0$



$$\frac{x}{-25} = \frac{y}{50} = \frac{1}{-25}$$

$$\frac{x-1}{\cos\theta} = \frac{y+2}{\sin\theta} = \pm 5$$

$$y = -2 + 5\left(-\frac{4}{5}\right) = -6$$

$$x = 1 + 5\left(\frac{3}{5}\right) = 4$$

Req. distance

$$\left|\frac{5(4) - 12(-6) + 51}{13}\right|$$

$$= \left|\frac{20 + 72 + 51}{13}\right|$$

$$= \frac{143}{13} = 11$$

90. (19)