Physics
Section - A

1. **(d)** potential difference,

$v \to$ same

$U = \frac{1}{2}cv^2$

as $q_1 < q_2$

$\therefore c_1 < c_2$

$\& U_1 < U_2$

2. **(b)** $V_T = \frac{2}{9}R^2\frac{g}{\eta}(d-\rho)$

3. **(b)**

4. **(c)** $v = -\int \vec{E}\cdot dr = \int \frac{2k\lambda}{r}dr = 2k\lambda \ln r + c$

Net potential due to all wire

$v = 2k\lambda \ln\sqrt{x^2+y^2} + 2k\lambda \ln\sqrt{y^2+z^2} + 2k\lambda \ln\sqrt{z^2+x^2} + c$

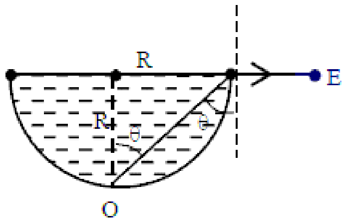
for $v = c$

$\sqrt{(x^2+y^2)(y^2+z^2)(z^2+x^2)} = c$

$\therefore (x^2+y^2)(y^2+z^2)(z^2+x^2) = c$

where c = constant

5. **(c)**



$\sin c = \frac{1}{\mu}$

for $\mu \to$ least, $c \to$ maximum

$\theta = c = 45$

$\mu = \frac{1}{\sin 45} = \sqrt{2}$

6. **(d)**

7. **(a)**

$\eta_{12} = 1 - \frac{273}{473} = \frac{200}{473} = 0.423$

$\eta_1 = 1 - \frac{373}{473} = \frac{100}{473} = 0.211$

$\eta_2 = 1 - \frac{273}{373} = \frac{100}{373} = 0.268$

8. **(d)** Solids have higher value of bulk modulus than gases.

9. **(a)**

$V_{rms} = \sqrt{\frac{3RT}{M}}$

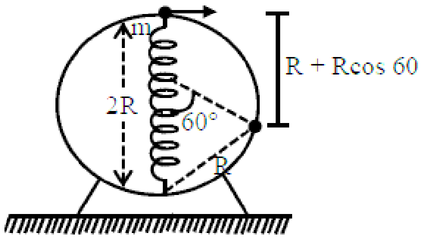
$V_{rms}^2 = 3RT/M$

Hence we can conclude that V_{rms}^2 is directly proportional to temperature

$y = mx$

$\Rightarrow$ Graph will be straight line

10. **(d)**



Work energy theorem

$$Mg(R+R\cos 60)+\frac{1}{2}k(R^2-0^2)=\frac{1}{2}mv^2$$

$$Mg\frac{3R}{2}+\frac{KR^2}{2}=\frac{1}{2}mv^2$$

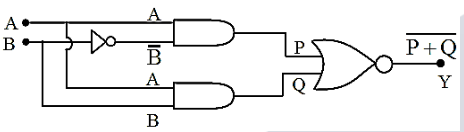
$$V=\sqrt{3gR+\frac{KR^2}{m}}$$

11. (b) Both statement are correct but Reason is not the correct explanation of Assertion.

12. (a) Theoretical equation for β^{-1} decay

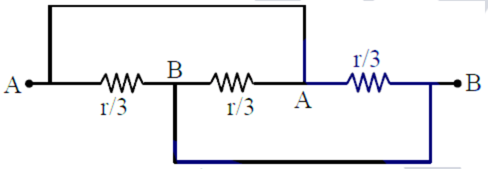
$n_0^1 \rightarrow p_1^1 + e_{-1}^{-0} + \bar{v}$

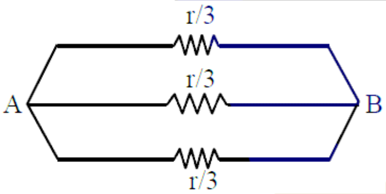
13. (a)



$$P = A \cdot \overline{B}$$
$$Q = A \cdot B$$
$$Y=\overline{P+Q}=\overline{A\cdot\overline{B}+A\cdot B}$$
$$=\overline{A\cdot(B+\overline{B})}=\overline{A\cdot I}$$
$$Y=\overline{A}$$

14. (c)





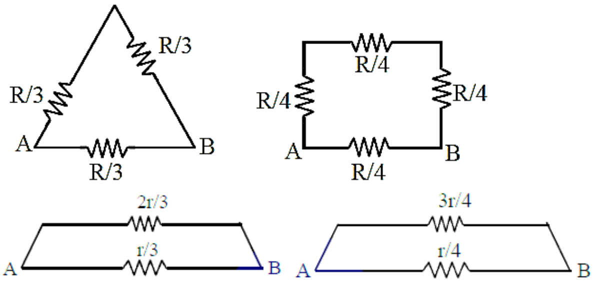
All are in parallel

$$R_{eq}=\frac{r/3}{3}=r/9$$

15. (d) $R=\frac{\rho\ell}{A}$

So, $R \propto \ell$

Side length of triangle is $1/3$ of total length.



$$\left(R_{eq}\right)_1=\frac{2r/3\times r/3}{2r/3+r/3}\quad\left(R_{eq}\right)_2=\frac{3r/4\times r/4}{3r/4+r/4}$$

$$\left(R_{eq}\right)_1=2r/9\quad\left(R_{eq}\right)_2=3r/16$$

$$\frac{\left(R_{eq}\right)_1}{\left(R_{eq}\right)_2}=\frac{2r/9}{3r/16}=\frac{32}{27}$$

16. (b)

$$\text{Energy density }=\frac{1}{2}\epsilon_0 E^2\times c$$

$$\text{Energy }=\frac{1}{2}\epsilon_0 E^2\times c\times\text{ volume}$$

$$(\text{Energy})_1=(\text{Energy})_2\ (\text{Given})$$

$$\frac{1}{2}\epsilon_0 E_1^2c\pi R_1^2\times L_1=\frac{1}{2}\epsilon_0 E_2^2c\pi R_2^2\times L_2$$

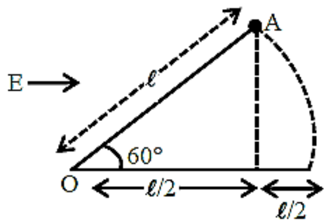
$$E_1^2R_1^2=E_2^2R_2^2$$

$$E_1R_1=E_2R_2$$

$$100\times R_1=E_2\times\frac{R_1}{2}$$

$$E_2=200\text{ N/C}$$

17. (c)



$$W_{all}=\Delta k$$

$$W_e=k_f-k_i$$

$$qE\frac{\ell}{2}=\frac{1}{2}mv^2-0$$

$$v=\sqrt{\frac{qE\ell}{m}}$$

18. (c) E is missing in the question but considering E as energy, the solution will be

$$E_{photen}=\frac{hc}{\lambda}=E;\ E_{proton}=\frac{1}{2}m_pv^2=E$$

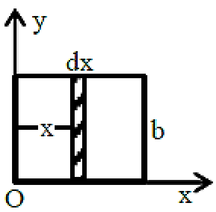
$$\frac{\lambda_{proton}}{\lambda_{photon}}=\frac{h/p}{hc/E}=\frac{h/\sqrt{2m_pE}}{hc/E}$$

$$=\frac{E}{c\sqrt{2m_pE}}$$

$$\frac{\lambda_{proton}}{\lambda_{photon}}=\frac{1}{c}\sqrt{\frac{E}{2m_p}}$$

19. (a) σ is constant in y-direction

So, $y_{cm}=b/2$



$$x_{cm}=\frac{\int_0^a x\,dm}{\int_0^a dm}$$

$$=\frac{\int_0^a x\sigma_x dA}{\int_0^a \sigma_x dA}$$

$$=\frac{\int_0^a x\frac{\sigma_0 x}{ab}bdx}{\int_0^a \frac{\sigma_0 x}{ab}bdx}$$

$$x_{cm}=\frac{\int_0^a x^2dx}{\int_0^a xdx}$$

$$=\frac{\left(\frac{x^3}{3}\right)_0^a}{\left(\frac{x^2}{2}\right)_0^a}=\frac{a^3/3}{a^2/2}$$

$$=\frac{2a}{3}$$

20. (b)

$\delta_{net}=0$

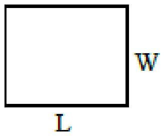
$(\mu_1-1)A_1-(\mu_2-1)A_2=0$

$(1.54-1)4-(1.72-1)A_2=0$

$A_2=3^\circ$

Section – B

21. (3)



Since least count of the instrument can be calculated as

$$\text{Least count }=\frac{\text{pitch length}}{\text{No. of division on circular scale}}$$

$$=\frac{0.75}{15}=0.05\text{ mm}$$

Here we are provided $L=5$ mm& $W=2.5$ mm

$L=5$ mm& $W=2.5$ mm

$\because$ We know that

$A=L\cdot W$

For calculating fractional error, we can write

$$\frac{dA}{A}=\frac{dL}{L}+\frac{dW}{W}$$

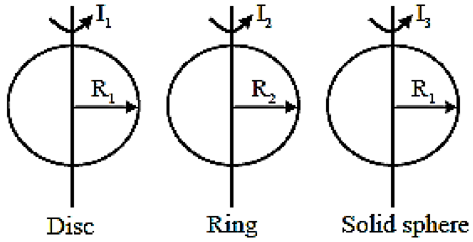
Here $dL=dW=0.05$ mm

$$\frac{dA}{A}=\frac{0.05}{5}+\frac{0.05}{2.5}$$

$$\Rightarrow\frac{dA}{A}=\frac{1}{100}+\frac{2}{100}=\frac{3}{100}$$

So, $x=3$

22. (4)



According to problem

$\frac{I_1}{I_2} = 2.5 \Rightarrow \frac{\frac{MR_1^2}{4}}{\frac{MR_2^2}{2}} = \frac{5}{2} \Rightarrow \frac{R_1^2}{R_2^2} = 5$ ...(1)

Now we are provided with information that

$$\frac{I_3}{I_2} = n$$

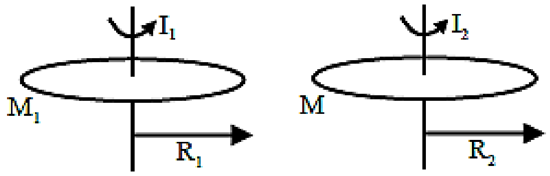
$$\Rightarrow \frac{\frac{2MR_1^2}{5}}{\frac{MR_2^2}{2}} = n \Rightarrow \frac{4R_1^2}{5R_2^2} = n \quad \ldots..(2)$$

From Eq', (1) and (2)

$\Rightarrow n = 4$

23. (0)

24. (16)



Given $R_2 = 2R_1$

$$M_1 = \sigma \times \pi R_1^2 = M_\circ$$
$$M_2 = \sigma \times \pi R_2^2 = M_\circ$$
$$M_2 = \sigma \times \pi R_2^2 = \sigma \times \pi [2R_1]^2 = \sigma \times 4\pi R_1^2 = 4M_\circ$$

$$\frac{I_1}{I_2} = \frac{\frac{M_1 R_1^2}{2}}{\frac{M_2 R_2^2}{2}} = \frac{M_1 R_1^2}{M_2 R_2^2} = \frac{1}{4} \times \frac{1}{4} = \frac{1}{16}$$

25. (11) In case of YDSE the distance of n^{th} maxima from central maxima is given by

$$Y = \frac{n\lambda D}{d}$$

Here n, D & d are same

So, $y \times \lambda$

$$\Rightarrow \frac{y_2}{y_1} = \frac{\lambda_2}{\lambda_1} \Rightarrow \frac{y_2}{10\ \text{mm}} = \frac{660\ \text{nm}}{600\ \text{nm}}$$

$$\Rightarrow y_2 = 11\ \text{mm}$$

Chemistry
Section - A

26. (c) Correct order of atomic radii : Be < Mg > Al > Si

27. (b) For the titration : Oxalic acid $v/sKMnO_4$

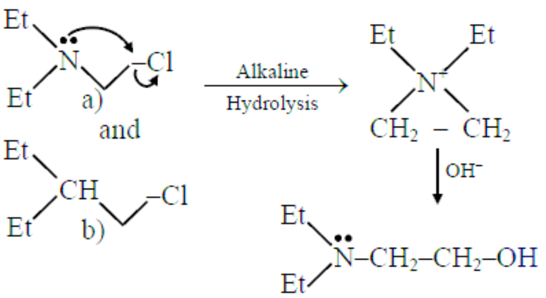
$$2MnO_4^- + 5(COO)_2^{2-} + 16H^+ \rightarrow$$
$$10CO_2 + 2Mn^{2+} + 8H_2O$$

This reaction is slow at room temperature but becomes fast at 60°C. Manganese(II) ions catalyse the reaction; thus, the reaction is autocatalytic; once manganese(II) ions are formed, it becomes faster and faster. The titration of FAS $v/sKMnO_4$ do not require heating because at higher temperature the oxidation of Fe^{+2} to Fe^{+3} by atmospheric O_2 will be prominent.

28. (a) (A) Combustion of hydrocarbon

(B) Decomposition into gaseous product.
(C) Displacement of ' V ' by ' Ca ' atom.
(D) Disproportionation of $H_2O_2^{-1}$ into O^{-2} and O° oxidation states.

29. (c)



Rate of (a) is faster than rate of (b) because it is a intramolecular substitution.

30. (d)

$HA \rightleftharpoons H^{\oplus} + A^{\oplus}$

$t = 0 \quad a$

$t = t \; a(1-x) \; ax \; ax$

$$K_a = (ax)\frac{(x)}{1-x}; [H^{+}] = ax$$

$$-\log(K_a) = -\log(ax) - \log\left(\frac{x}{1-x}\right)$$

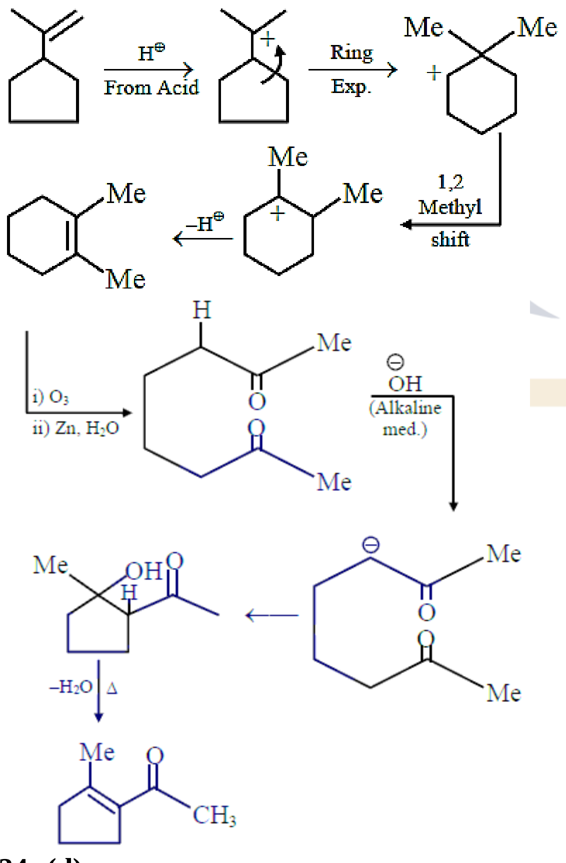
$$pKa = pH - \log\left(\frac{x}{1-x}\right)$$

$$pH - pKa = \log\left(\frac{x}{1-x}\right)$$

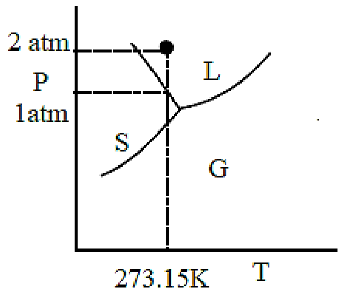
31. (b)

32. (c)

33. (b)



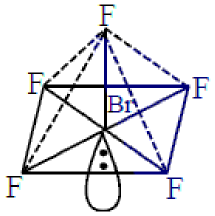
34. (d)



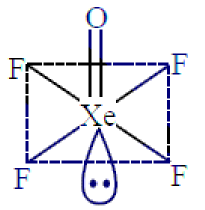
Phase diagram of H_2O

If pressure is made two time then mixture of ice and water will completely convert into water (liquid) form.

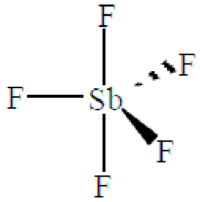
35. (a)



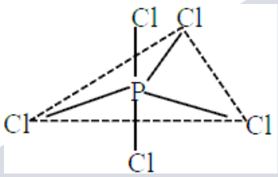
Square pyramidal



Square pyramidal



Trigonal Bipyramidal



Trigonal Bipyramidal

BrF_5 : Square pyramedal

$XeOF_4$: Square pyramedal

SbF_5 : Trigonal bipyramidal

PCl_5 : Trigonal bipyramidal

36. (b) Ni^{+2} gives violet coloured bead in non-luminous flame under hot conditions. Ni^{+2} has d^8 configuration which does not depend on nature of ligand present in octahedral complex.

$Ni^{+2}: t_{2g}^{\ 6} e_g^{\ 2}$

37. (b)

S.No.	Name of Reaction	Acetaldehyde $CH_3-\underset{\underset{O}{\parallel}}{C}-H$	Acetone $CH_3-\underset{\underset{O}{\parallel}}{C}-CH_3$
1	Iodoform reaction	⊕ ve	⊕ ve
2	Cannizaro	⊖ ve	⊖ ve
3	Aldol reaction	⊕ ve	⊕ ve
4	Tollen's test	⊕ ve	Θve
5	Clemmensen reduction	⊕ ve	⊕ ve

38. (d)

	Orbital
A: $n = 1, \ell = 0,\ m_\ell = 0$	1 s
B: $n = 2, \ell = 0,\ m_\ell = 0$	2 s

C: $n=3, \ell=1, m_\ell=1$	3 p
D: $n=3, \ell=2, m_\ell=1$	3 d
E: $n=3, \ell=2, m_\ell=0$	3 d

In absence of electric and magnetic fields, all orbitals of 3 d are degenerate

39. (d)

$$\underset{(A)}{CH_3-CH_2-CH_2-NO_2} \xleftrightharpoons{Ag-NO_2} CH_3-CH_2-CH_2-Br$$

$$\xrightarrow{AgCN} \underset{(B)}{CH_3-CH_2-CH_2-NC}$$

40. (a)

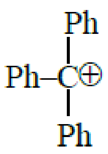
$\Delta T_f = K_b \cdot m$

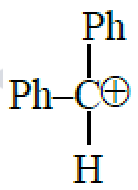
$$0.4 = K_b \frac{\frac{1}{256}}{50\times10^{-3}}$$

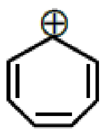
$K_b = 5.12\ \mathrm{K\ kg/mol}$

41. (b) Among Al, In, Tl, Ge, Sn, Pb, the metal having highest IE_1 is Ge and lowest IE_1 is In . Most stable oxidation state of Ge is +4 and In is +3 .

42. (d)

(A) 

(B) 

(C) 

(D)$H_3C-CH_2-\overset{\oplus}{C}H-CH_3$

C is aromatic due to ⊕ ve charge hence it is most stable
A have more resonance structure
B have less resonance structure
D have only hyper conjugation
Consider First Aromaticity > Resonance > Hyper conjugation

43. (c) A, C, D produce CO_2 with aqueous $NaHCO_3$ solution.
A, C, D acids are stronger acid than H_2CO_3 (Carbonic acid)

44. (c) $I^- \xrightarrow{H^+} I_2$ $\qquad I^- \xrightarrow{OH^-} IO_3^-$

$S^{-2} \xrightarrow{H^+} S$ $\qquad S_2O_3^{2-} \xrightarrow{OH^-} SO_4^{2-}$

$Fe^{+2} \rightarrow Fe^{+3}$

$S_2O_3^{2-} \xrightarrow{H^+} S\downarrow + SO_4^{2-}$

45. (b)

Section - B

46. (150)

$R = \rho \frac{\ell}{A}$

$\kappa = G \cdot G^* \; G = \frac{1}{R}; \kappa = \frac{1}{\rho}$

$G^* = \frac{\ell}{A}$

$R =$ Resistance

$\rho =$ Resistivity

$\frac{\ell}{A} =$ cell constant (G^*)

$\frac{\kappa_c}{\kappa_d} = \frac{R_d}{R_c}; \lambda_m = \frac{\kappa \times 1000}{C}$

$\frac{\kappa_c}{\kappa_d} = \frac{(\lambda_m \cdot C)}{(\lambda_m \cdot C)_d} = \frac{R_d}{R_c}$ {c = concentrated sol.}

$\frac{100 \cdot (0.15)^2}{150 \cdot (0.1)^2} = \frac{R_d}{100}$ {d = diluted solution}

$R_d = 150\Omega$

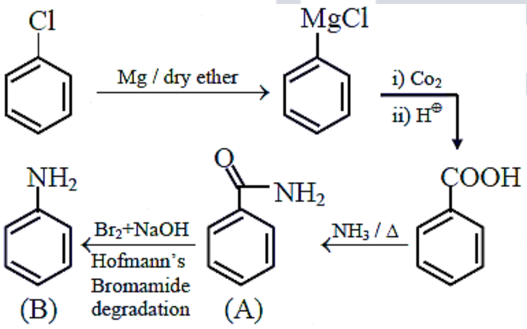
47. (1665)

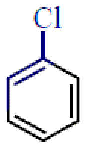
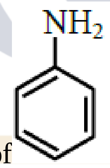
48. (125) Assuming 100 gm solution contain 70 gm solute.

Volume of 100 gm solution will be $\frac{100}{1.25}$ ml.

Molarity $= \frac{70/70}{100/1.25} \times 1000 = 12.5$ or 125×10^{-1}

49. (93)



No. of moles of  = No. of moles of 

$\frac{11.25 \times 10^{-3}}{112.5} = \frac{x \times 10^{-1} \times 10^{-3}}{93}$

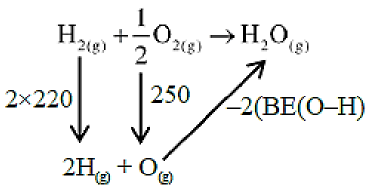
$x \times 10^{-1} = 93 \times 0.1$

$x = 93mg$

50. (466)

$\frac{1}{2}H_{2(g)} \rightarrow H(g);\ \Delta_f H\left(H_{(g)}\right) = 220KJ/mol$

$\frac{1}{2}O_{2(g)} \rightarrow O(g)\ ;\ \Delta_f H\left(O_{(g)}\right) = 250KJ/mol$



$\Delta_r H(H_2O_{(g)}) = -242 KJ/mol$

$\Delta H_f(H_2O_{(1)}) = -242 = 440 + 250 - 2(\text{ B.E.}(O - H))$

$BE(O - H) = 466 KJ/mol$

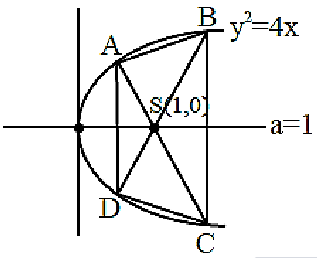
Mathematics
Section - A

51. (d)

Case – I	5 _ _ _	0
Case - II	5 _ _ _	1
	5	2
	5	3
	6	0
	6	1
	6	2
	7	0
Case IX	7_ _ _	1

$9 \times (8 \times 8 \times 8) = 4608$ but 50000 is not included, so total numbers $4608 - 1 = 4607$

52. (a)



$A(at_1^2, 2a_1) \& C\left(\frac{a}{t_1}, -\frac{2a}{t_1}\right)$

Length AC $= a\left(t_1 + \frac{1}{t_1}\right)^2 = \frac{25}{4}, t_1 + \frac{1}{t_1} = \pm\frac{5}{2}$

$\Rightarrow t_1 = 2$ or $\frac{1}{2}, A\left(\frac{1}{2}, 1\right), D\left(\frac{1}{4}, -1\right), B(4,4), C(4, -4)$

So, area of trapezium $= \frac{1}{2}(8 + 2)\left(4 - \frac{1}{4}\right) = \frac{75}{4}$

53. (c) $i^{k_1} + i^{k_2} \neq 0$ $i^{k_1} \rightarrow 4$ option for $i, -1, -i, 1$

Total cases $\Rightarrow 4 \times 4 = 16$

Unfovourble cases $\Rightarrow i^{k_1} + i^{k_2} = 0$

$$\begin{Bmatrix} 1, -1 \\ -1, 1 \\ i, -i \\ -i, i \end{Bmatrix}$$

4 Cases $\Rightarrow$ Probability $= \frac{16-4}{16} = \frac{3}{4}$

54. (b) $f(x) = \frac{2^x}{2^x + \sqrt{2}}$

$$f(x)+f(1-x)=\frac{2^x}{2^x+\sqrt{2}}+\frac{2^{1-x}}{2^{1-x}+\sqrt{2}}$$
$$=\frac{2^x}{2^x+\sqrt{2}}+\frac{2}{2+\sqrt{2}2^x}=\frac{2^x+\sqrt{2}}{2^x+\sqrt{2}}=1$$
$$\text{Now, }\sum_{k=1}^{81} f\left(\frac{k}{82}\right)=f\left(\frac{1}{82}\right)+f\left(\frac{2}{82}\right)+\cdots\ldots+f\left(\frac{81}{82}\right)$$
$$=f\left(\frac{1}{82}\right)+f\left(\frac{2}{82}\right)+\cdots\ldots+f\left(1-\frac{2}{82}\right)+f\left(1-\frac{1}{82}\right)$$
$$=\left[f\left(\frac{1}{82}\right)+f\left(1-\frac{1}{82}\right)\right]+\left[f\left(\frac{2}{82}\right)+f\left(1-\frac{2}{82}\right)\right]+\cdots..40\text{ cases }+f\left(\frac{41}{82}\right)$$
$$=(1+1+\cdots 40\text{ times })+\frac{2^{1/2}}{2^{1/2}+2^{1/2}}$$
$$40+\frac{1}{2}=\frac{81}{2}$$

55. (d)

$$f(x)=(3a+2)x^2+\left(\frac{a+2}{a-1}\right)x+b$$
$$f\left(x+\frac{1}{2}\right)=f(x)+f(y)+1-\frac{2}{7}xy\quad \ldots\ldots(1)$$

In (1) Put $x=y=0 \Rightarrow f(0)=2f(0)+1 \Rightarrow f(0)=-1$

So, $f(0)=0+0+b=-1 \Rightarrow b=-1$

In (1) Put $y=-x \Rightarrow f(0)=f(x)+f(-x)+1+\frac{2}{7}x^2$

$$-1=2(3a+2)x^2+2b+1+\frac{2}{7}x^2$$
$$-1=\left(2(3a+2)+\frac{2}{7}\right)x^2+1-2$$
$$\Rightarrow 6a+4+\frac{2}{7}=0$$
$$a=-\frac{5}{7}$$

So $f(x)=-\frac{1}{7}x^2-\frac{3}{4}x-1$

$$\Rightarrow |f(x)|=\frac{1}{28}|4x^2+21x+28|$$

Now, $28\sum_{i=1}^{5}|f(i)|=28(|f(1)|+|f(2)|+\cdots+|f(5)|)$

$$28\cdot\frac{1}{28}\cdot 675=675$$

56. (b)

$A(x,y,z)$ Let $P(0,3,2),Q(2,0,3),R(0,0,1)$

$AP=AQ=AR$

$x^2+(y-3)^2+(z-2)^2=(x-2)^2+y^2+(z-3)^2=x^2+y^2+(z-1)^2$

In xy plane $z=0$

So, $x^2-4x+4+y^2+9=x^2+y^2+1$

$\Rightarrow y=2$

$x=3$

$9+y^2-6y+9+4=x^2+y^2+1$

So, $A(3,2,0)$ also $B(1,4,-1)\&C(2,0,-2)$

Now $AB=\sqrt{4+4+1}=3$

$AC=\sqrt{1+4+4}=3$

$BC=\sqrt{1+16+1}=\sqrt{18}$

$AB=AC$

isosceles $\Delta \& AB^2 + AC^2 = BC^2$ right angle Δ

Area of $\triangle$ ABC $= \frac{1}{2} \times$ base.height

$$\frac{1}{2} \times 3 \times 3 = \frac{9}{2}$$

So only S_1 is true

57. (c) $R = \{(x, y): x, y \in z \text{ and } x + y \text{ is even }\}$

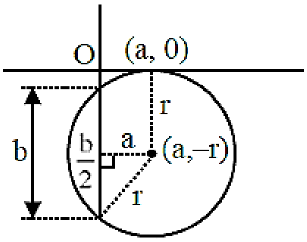
reflexive $x + x = 2x$ even

symmetric of $x + y$ is even, then $(y + x)$ is also even

transitive of $x + y$ is even $\& y + z$ is even then $x + z$ is also even

So, relation is an equivalence relation.

58. (d)



By pytogorus $r^2 = a^2 + \frac{b^2}{4} = P^2$

$$r = \sqrt{\frac{4a^2 + b^2}{4}}$$

Equation of circle is $(x - \alpha)^2 + (y - \beta)^2 = r^2$

$$x^2 + y^2 - 2ax - 2py + \alpha^2 + p^2 - r^2 = 0$$

comparision $x^2 + y^2 - \alpha x + \beta y + r = 0$

$-\alpha = -2a, \beta = -2p, r = a^2$

$$\Rightarrow 2a = \alpha, 4a^2 + b^2 = 4p^2$$

$$\alpha^2 + b^2 = 4p^2$$

$$\alpha^2 + b^2 = \beta^2$$

So, $(2a, b^2) = (\alpha, \beta^2 - 4r)$

59. (b)

$$a_0 = 0, a_1 = \frac{1}{2}$$

$$2a_{n+2} = 5a_{n+1} - 3a_n$$

$$2x^2 - 5x + 3 = 0 \Rightarrow x = 1, 3/2$$

$$\therefore a_n = A(1)^n + B\left(\frac{3}{2}\right)^n$$

$$n = 0\ 0 = A + B] A = -1$$

$$n = 1\ \frac{1}{2} = A + \frac{3}{2}\ B \Big] B = 1$$

$$\Rightarrow a_n = -1 + \left(\frac{3}{2}\right)^n$$

$$\sum_{k=1}^{100} a_k = \sum_{k=1}^{100} (-1) + \left(\frac{3}{2}\right)^k$$

$$= -100 + \frac{\left(\frac{3}{2}\right)\left(\left(\frac{3}{2}\right)^{100} - 1\right)}{\frac{3}{2} - 1}$$

$$= -100 + 3\left(\left(\frac{3}{2}\right)^{100} - 1\right)$$

$$= 3 \cdot (a_{100}) - 100$$

60. (b)

$$\cos\left(\sin^{-1}\frac{3}{5}+\sin^{-1}\frac{5}{13}+\sin^{-1}\frac{33}{65}\right)$$
$$\cos\left(\tan^{-1}\frac{3}{4}+\tan^{-1}\frac{5}{12}+\tan^{-1}\frac{33}{56}\right)$$
$$\cos\left(\tan^{-1}\left(\frac{\frac{3}{4}+\frac{5}{12}}{1+\frac{3}{4}\cdot\frac{5}{12}}\right)+\tan^{-1}\frac{33}{56}\right)$$
$$\cos\left(\tan^{-1}\frac{56}{33}+\cot^{-1}\frac{56}{33}\right)$$
$$\cos\left(\frac{\pi}{2}\right)=0$$

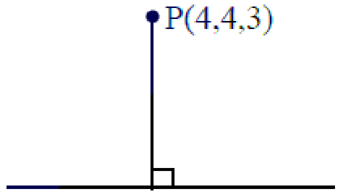
61. (b)

$$T_m=\frac{1}{25},\ T_{25}=\frac{1}{20},20\sum_{r=1}^{25} T_r=13$$
$$T_m=a+(m-1)d=\frac{1}{25}\ \ldots\ldots(1)$$
$$T_{25}=a+24\,d=\frac{1}{20}$$
$$20\cdot\frac{25}{2}\left[a+\frac{1}{20}\right]=13\Rightarrow a=\frac{1}{500}$$
also, $20\,S_{25}=20\cdot\frac{25}{2}[2a+24\,d]=13\Rightarrow\ d=\frac{1}{500}$

from (1) $\frac{1}{500}+\frac{m-1}{500}=\frac{1}{25}\Rightarrow\ m=20$

Now,
$$5\,m\sum_{r=m}^{2\,m} T_r=100\sum_{r=20}^{40} T_r=126$$

62. (a)



$$\frac{x-1}{2}=\frac{y-2}{1}=\frac{z-1}{3}$$

$Q(2\lambda+1,\lambda+2,3\lambda+11)$

$\overrightarrow{PQ}\perp(2\hat{i}+\hat{j}+3\hat{k})$

$\Rightarrow 2(2\lambda-3)+1(\lambda-2)+3(3\lambda-2)=0$

$\Rightarrow 14\lambda-14=0,\lambda=1$

So, $Q(3,3,4)$

Let image in $R(\alpha,\beta,\gamma)$
$$\frac{\alpha+4}{2}=3,\frac{\beta+4}{2}=3,\frac{\gamma+3}{2}=4$$
$(\alpha,\beta,\gamma)=(2,2,5)$

$\Rightarrow\alpha+\beta+\gamma=9$

63. (c) $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\frac{96x^2\cos^2 x}{(1+e^x)}dx$ (Apply King Property)

$$\int_0^{\frac{\pi}{2}} 96x^2\cos^2 x = 48\int_0^{\frac{\pi}{2}} x^2(1+\cos 2x)dx$$

$$48\left[\left(\frac{x^3}{3}\right)_0^{\pi/2} + \int_0^{\frac{\pi}{2}} x_I^2 \cos_{II} 2xdx\right]$$

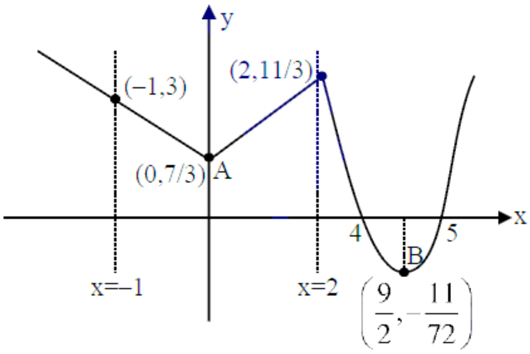
$\Rightarrow$ On solving $\pi(2\pi^2 - 12)$

$\Rightarrow \alpha = 2, \beta = -12$

$\Rightarrow (\alpha+\beta)^2 = 100$

64. (c)

$$f(x) = \begin{cases} 1-2x, & x < -1 \\ \frac{1}{3}(7-2x), & -1 \le x \le 2 \\ \frac{1}{3}(7+2x) & 0 \le x < 2 \\ \frac{11}{18}(x-4)(x-5), & x > 2 \end{cases}$$



$\therefore$ Local minimum values at A& B

$$\frac{7}{3} - \frac{11}{72}$$

$$\Rightarrow \frac{168-11}{72} \Rightarrow \frac{157}{72}$$

65. (c) $x^2 + |2x-3| - 4 = 0$

Case I: $x \ge \frac{3}{2}$

$x^2 + 2x - 3 - 4 = 0$

$x^2 + 2x - 7 = 0$

$x = 2\sqrt{2} - 1$

Case II : $x < \frac{3}{2}$

$x^2 + 3 - 2x - 4 = 0$

$x^2 - 2x - 1 = 0$

$x = 1 - \sqrt{2}$

Sum of squares $= (2\sqrt{2}-1)^2 + (1-\sqrt{2})^2$

$= 8 - 4\sqrt{2} + 1 + 1 - 2\sqrt{2} + 2$

$= 6(2-\sqrt{2}) \therefore (3)$

66. (a) $\int_0^x tf(t)dt = x^2 + (x), x > 0$

Diff. both side w.r. to x

$xf(x) = x^2f'(x) + 2xf(x)$

$-xf(x) = x^2f'(x)$

$$\int \frac{f'(x)}{f(x)}dx = \int \frac{-1}{x}dx$$

$\log d f(x) = -\log x + \log c$

$f(x) = \frac{c}{x}$

$f(2) = 3 \Rightarrow 3 = \frac{c}{2} \Rightarrow c = 6$

$f(x) = \frac{6}{x}$

$f(6) = 1 \quad (1)$

67. (a)

$$^{n}C_{r-1} = 28,\ ^{n}C_{r} = 56$$

$$\frac{^{n}C_{r-1}}{^{n}C_{r}} = \frac{28}{56}$$

$$\frac{\dfrac{n!}{(r-1)!\,(n-r+1)!}}{\dfrac{n!}{r!\,(n-r)!}} = \frac{1}{2}$$

$$\frac{r}{(n-r+1)} = \frac{1}{2}$$

$$3r = n + 1 \quad \ldots\ldots(i)$$

$$\frac{^{n}C_{r}}{^{n}C_{r+1}} = \frac{56}{70}$$

$$\frac{(r+1)}{(n-r)} = \frac{56}{70} \Rightarrow 9r = 4n - 5 \quad \ldots\ldots(ii)$$

By (i) & (ii)

$$(r = 3), (n = 8)$$

$$A(4\cos t, 4\sin t)\ B(2\sin t, -2\cos t) C(3r - n, r^2 - n - 1)$$

$$A(4\cos t, 4\sin t)\ B(2\sin t, -2\cos t) C(1,0)$$

$$(3x-1)^2 + (3y)^2 = (4\cos t + 2\sin t)^2 + (4\sin t - \cos t)^2$$

$$(3x-1)^2 + (3y)^2 = 20 \ \therefore \ \text{option (1)}$$

68. (d) $z_1 = \sqrt{3} + 2\sqrt{2}i \,\&\, \frac{|z_2|}{|z_1|} = \frac{1}{\sqrt{3}}$

given $\arg\left(\frac{z_2}{z_1}\right) = \frac{\pi}{6}$

$$z_2 = \frac{|z_2|}{|z_1|} \cdot z_1 e^{i\left(\frac{\pi}{6}\right)}$$

$$z_2 = \frac{1}{\sqrt{3}} \cdot \frac{(\sqrt{3} + 2\sqrt{2}i)(\sqrt{3} + i)}{2}$$

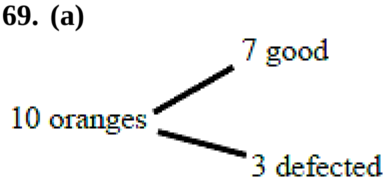
$$z_2 = \frac{(3 - 2\sqrt{2}) + i(2\sqrt{6} + \sqrt{3})}{2\sqrt{3}}$$

Now,

$$z_1 - z_2 = \frac{(3 + 2\sqrt{2}) + i(2\sqrt{6} - \sqrt{3})}{2\sqrt{3}}$$

$|z_1 - z_2| = |z_2| \Rightarrow \triangle$ ABO is isosceles with angles $\frac{\pi}{6}, \frac{\pi}{6} \,\&\, \frac{2\pi}{3}$

69. (a)



Probability distribution

x_i	p_i
$x = 0$	$\frac{^{7}C_2}{^{10}C_2} = \frac{42}{90}$

$x = 1$	$\frac{{}^7C_1 \times {}^3C_1}{{}^{10}C_2} = \frac{42}{90}$
$x = 2$	$\frac{{}^3C_2}{{}^{10}C_2} = \frac{6}{90}$

Now,

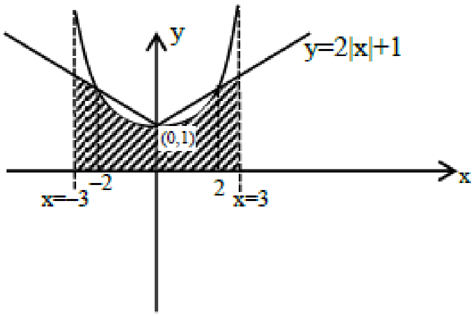
$$\mu = \sum x_i p_i = \frac{42}{90} + \frac{12}{90} = \frac{54}{90}$$

$$\sigma^2 = \sum p_i x_1^2 - \mu^2 = \frac{42}{90} + \frac{24}{90} - \left(\frac{54}{90}\right)^2$$

$$\Rightarrow \frac{66}{90} - \left(\frac{54}{90}\right)^2$$

$$\sigma^2 \Rightarrow \frac{28}{75} \therefore$$

70. (b)



$$\text{Area } = 2\left[\int_0^2 (x^2+1)dx + \int_2^3 (2x+1)dx\right]$$

$$\Rightarrow \frac{64}{3} \quad \therefore$$

Section - B

71. (1613) $\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$

No. of elements in $S_1 : A = A^T \Rightarrow 5^3 \times 5^3$

No. of elements in $S_2 : A = -A^T \Rightarrow 0$

since no zero in S_2

No. of elements in $S_3 \Rightarrow$

$$\left.\begin{array}{r} a_{11} + a_{22} + a_{33} = 0 \Rightarrow (1,2,-3) \Rightarrow 31 \\ \text{or} \\ (1,1,-2) \Rightarrow 3 \\ \text{or} \\ (-1,-1,2) \Rightarrow 3 \end{array}\right\} \Rightarrow 12 \times 5^6$$

$$n(S_1 \cap S_3) = 12 \times 5^3$$

$$n(S_1 \cup S_2 \cup S_3) = 5^6(1+12) - 12 \times 5^3$$

$$\Rightarrow 5^3 \times [13 \times 5^3 - 12] = 125\alpha$$

$$\alpha = 1613$$

72. (5)

$$\alpha \quad = 1 + \sum_{r=1}^{6} (-1)^{r-1}\, {}^{12}C_{2r-1} 3^{r-1}$$

$$\alpha \quad = 1 + \sum_{r=1}^{6} {}^{12}C_{2r-1} \frac{(\sqrt{3}i)^{2t-1}}{\sqrt{3}i} \quad i = \text{ iota, let } \sqrt{3}i = x$$

$$\alpha \quad = 1 + \frac{1}{\sqrt{3}i}\left({}^{12}C_1 x + {}^{12}C_3 x^3 + \cdots .\, {}^{12}C_{11} x^{11}\right)$$

$$= 1 + \frac{1}{\sqrt{3}i}\left(\frac{(1+\sqrt{3}i)^{12} - (1-\sqrt{3}i)^{12}}{2}\right)$$

$$= 1 + \frac{1}{\sqrt{3}i}\left(\frac{(-2\omega^2)^{12} - (2\omega)^{12}}{2}\right) = 1$$

so distance of $(12, \sqrt{3})$ from $x - \sqrt{3}y + 1 = 0$ is $\frac{12-3+1}{2} = 5$

73. (6)

$\vec{a} = \hat{i} + \hat{j} + \hat{k}$

$\vec{b} = 2\hat{i} + 2\hat{j} + \hat{k}$

$\vec{d} = \vec{a} \times \vec{b}$

$= -\hat{i} + \hat{j}$

$|\vec{c} - 2\vec{a}|^2 = 8$

$|c|^2 + 4|a|^2 - 4(a \cdot c) = 8$

$c^2 + 12 - 4c = 8$

$c^2 - 4c + 4 = 0$

$|c| = 2$

$\overrightarrow{d} = \vec{a} \times \vec{b}$

$\vec{d} \times \vec{c} = (\vec{a} \times \vec{b})^2 \times \vec{c}$

$\left(|d||c|\sin\frac{\pi}{4}\right)^2 = ((a \cdot c) \cdot b - (b \cdot c) \cdot a)^2$

$4 = 4\, b^2 + (b \cdot c)^2 2(a)^2 - 2(\, b \cdot c)(a \cdot b)$

Let $b.c = x$

$4 = 36 + 3x^2 - 20x$

$3x^2 - 20x + 32 = 0$

$3x^2 - 12x - 8x + 32 = 0$

$x = \frac{8}{3}, 4$

$b.c = \frac{8}{3}, 4$

$b.c = \frac{8}{3}$

Now $|10 - 3\, b.c| + |d \times c|^2$

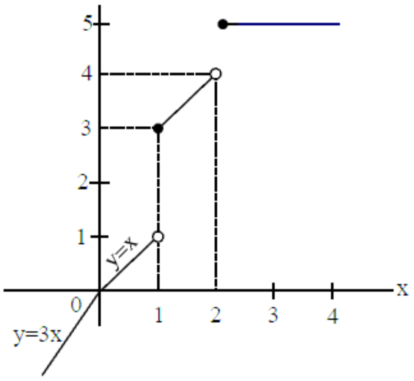
$|10 - 8| + (2)^2$

$\Rightarrow 6$ Ans.

74. (5)

$$f(x) = \begin{cases} 3x & ; \quad x < 0 \\ \min\{1 + x, x\} & ; \quad 0 \le x < 1 \\ \min\{2 + x, x + 2\} & ; \quad 1 \le x < 2 \\ 5 & ; \quad x > 2 \end{cases}$$

$$f(x) = \begin{cases} 3x & x \quad x < 0 \\ x & ; \quad 0 \le x < 1 \\ x + 2 & ; \quad 1 \le x < 2 \\ 5 & ; \quad x > 2 \end{cases}$$

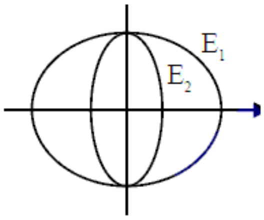


Not continuous at $x \in \{1,2\} \Rightarrow \alpha = 2$

Not diff. at $x \in \{0,1,2\} \Rightarrow \beta = 3$

$\alpha + \beta = 5$

75. (54)



$$E_1 = \frac{x^2}{9} + \frac{y^2}{4} \Rightarrow e = \sqrt{1 - \frac{4}{9}} = \frac{\sqrt{5}}{3}$$

$$E_2 : \frac{x^2}{a^2} + \frac{y^2}{4} = 1$$

$$e = \frac{\sqrt{5}}{3} = \sqrt{1 - \frac{a^2}{4}} \Rightarrow \frac{5}{9} = 1 - \frac{a^2}{4}$$

$$a^2 = \frac{16}{9}$$

$$E_2 : \frac{x^2}{\frac{16}{9}} + \frac{y^2}{4} = 1$$

$$E_3 : \frac{x^2}{\frac{16}{9}} + \frac{y^2}{b^2} = 1$$

$$e = \frac{\sqrt{5}}{3} = \sqrt{1 - \frac{b^2}{\frac{16}{9}}} \Rightarrow b^2 = \frac{64}{81}$$

$$E_3 = \frac{x^2}{\frac{16}{9}} + \frac{y^2}{\frac{64}{81}} = 1$$

$$A_1 = \pi \times 3 \times 2 \Rightarrow 6\pi$$

$$A_2 = \pi \times \frac{4}{3} \times 2 = \frac{8\pi}{3}$$

$$A_3 = \pi \times \frac{4}{3} \times \frac{8}{9} = \frac{32\pi}{81}$$

$$\sum_{i=1}^{\infty} A_i = 6\pi + \frac{8\pi}{3} + \frac{32\pi}{81} + \cdots \infty \Rightarrow \frac{6\pi}{1 - \frac{4}{9}} \Rightarrow \frac{54\pi}{5}$$

$$\therefore \frac{5}{\pi} \sum_{i=1}^{\infty} A_i \Rightarrow \frac{5}{\pi} \times \frac{54\pi}{5} = 54$$