Physics

Section - A

1. (c)

$B = 0.4\ T$

$r = 20\ cm$

$\omega = 10\pi rad/s$

$E = \frac{1}{2}\ B\omega R^2$

$= 0.2512\ V$

2. (a)

$C = 1\mu\ F$

$V = 20\ V$

$d = 1\mu\ m$

Energy density $== \frac{1}{2} \in_0\ E^2$

$E = \frac{V}{d} = 20 \times 10^6 v/m$

$U = 1.77 \times 10^3\ J/m^3$

3. (a)

Angular impulse $= [ML^2\ T^{-1}]$

Latent Heat $= [M^0\ L^2\ T^{-2}]$

Electrical resistivity $= [ML^3\ T^{-3}\ A^{-2}]$

Electromotive force $= [ML^2\ T^{-3}\ A^{-1}]$

4. (c) $\frac{\rho_1}{\rho_2} = \frac{4}{25}$

Ratio of rms velocities $= \sqrt{\frac{\rho_2}{\rho_1}} = \frac{5}{2}$

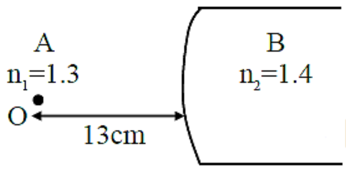
5. (b)

$(KE)_{\text{Trandational}} = \left[\frac{3}{2}KT\right] \times$ no. of molecule

No. of molecule $= \left[\frac{50}{44} \times 6.023 \times 10^{23}\right]$

$(KE)_{\text{Trandational}} = 4108.644\ J$

6. (c)



$\frac{n_2}{v} - \frac{n_1}{u} \frac{n_2 - n_1}{R}$

$\frac{1.4}{v} - \frac{1.3}{-13} = \frac{0.1}{R}$

$\frac{1.4}{v} = \frac{1 - R}{10R}$

$\frac{1.4}{v} = \frac{1 - R}{10R}$

$m = \frac{v/n_2}{u/n_1}$

$-2 \times \frac{(-13)}{1.3} = \frac{10R}{1 - R}$

$R = \frac{2}{3}\ cm$

7. **(b)** Frequency of revolution $\propto \frac{1}{n^3}$

8. **(d)** Comptan effect is based on particle nature of light.

9. **(a)** Distance = Area under v v/s t graph

Distance $= \frac{1}{2} \times 2 \times 10 + 2 \times 10 = 30$ m

10. **(b, / c)**

Method-1 :

Without considering uncentainity in ℓ.

$$B = \frac{\mu_0}{4\pi} \frac{m}{r^3}$$

$$B \propto \frac{1}{r^3}$$

$$\frac{\Delta B}{B} = 3 \times \left(\frac{\Delta r}{r}\right)$$

% uncertainity in B = 3%

Method-2 :

With considering uncentainity in ℓ.

$$B \propto \frac{1}{r^3}$$

$$\frac{\Delta B}{B} = \frac{\Delta \ell}{\ell} + 3 \times \left(\frac{\Delta r}{r}\right) = 1 + 3 \times 1 = 4\%$$

% uncertainity in B = 4%

11. **(b)**

$$V_{\text{escape}} = \sqrt{\frac{2GM}{R}}$$

$$\frac{(V_{\text{escape}})_{\text{Planet}}}{(V_{\text{escape}})_{\text{Earth}}} = \sqrt{\left(\frac{M_P}{M_E}\right) \times \left(\frac{R_E}{R_P}\right)} = \frac{1}{2}$$

$$(V_{\text{escape}})_{\text{Planet}} = \frac{1}{2} (V_{\text{escape}})_{\text{Earth}} = 5.6 \text{ km/s}$$

12. **(d)**

$$x = A\sin(\omega t + \phi)$$

$$x_0 = A\sin\phi \quad \ldots\ldots (1)$$

$$p = mA\omega\cos(\omega t + \phi)$$

$$p_0 = mA\omega\cos\phi \quad \ldots\ldots (2)$$

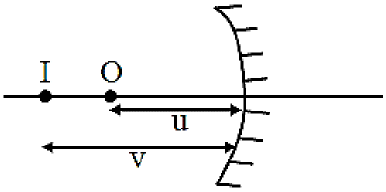
$$(2)/(1) \Rightarrow \tan\phi = \left(\frac{x_0}{p_0}\right) m\omega$$

$$\sin\phi = \frac{x_0\, m\omega}{\sqrt{(m\omega x_0)^2 + p_0^2}}$$

From (1), $A = \frac{x_0}{\sin\phi} = \frac{\sqrt{(m\omega x_0)^2 + p_0^2}}{m\omega}$

This means we can explain assertion with the given reason.

13. **(d)**



$m = -3 = -\frac{v}{u}$ and $v - u = 20$ cm

$f = \frac{vu}{v+u} = \frac{(-30)(-10)}{-30-10}$

$\therefore R = +15$

14. (b)

$\langle p \rangle = \frac{(2\hat{i}+3\hat{j})\cdot(3\hat{i}+6\hat{j})}{4} = 6$

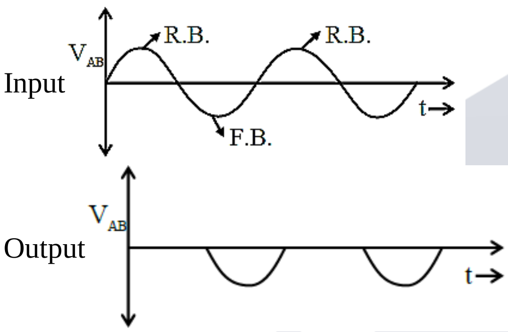
$\vec{a} = \left(\frac{\vec{F}}{m} = \frac{1}{2}\hat{i} + \frac{3}{4}\hat{j}\right)$

$\vec{v}$ at $t = 4\text{sec} = \left(\frac{1}{2}\hat{i} + \frac{3}{4}\hat{j}\right) \times 4 = (2\hat{i} + 3\hat{j})$

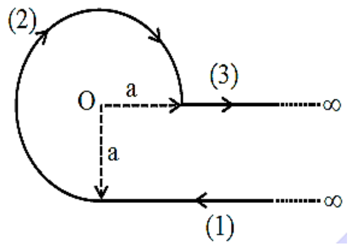
$P_{ins} = (2\hat{i} + 3)(2\hat{i} + 3\hat{j}) = 13$

$\frac{\langle P >}{P_{ins}} = \frac{6}{13}$

15. (d) $V = V_0 \sin \omega t$



16. (b)



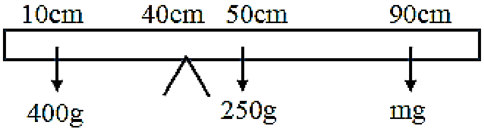
$B_1 = \frac{\mu_0 i}{4\pi a} \otimes$

$B_2 = \frac{\mu_0}{4\pi}\frac{i}{a}\left(\frac{3\pi}{2}\right) \otimes$

$B_3 = 0$

$B = \frac{\mu_0}{4\pi}\frac{i}{a}\left(1\frac{3\pi}{2}\right) \otimes$

17. (b)



$\tau_{Net} = 0 \Rightarrow (400\text{ g} \times 30) = (250\text{ g} \times 10)(\text{mg} \times 50)$

$m = \frac{12000 - 2500}{50} = \frac{9500}{50}$

$M = 190$ g

18. (d)

$Mg = F_B \Rightarrow (400 \times 10^{-3}) = 10^3 \times V_d$

$V_d = 400 \times 10^{-6}\ m^3$

$(\text{Vol.})_{\text{outside}} = (10 \times 10^{-2})^3 - 400 \times 10^{-6}$

$= 600 \times 10^{-6}\ m^2 = 600\ cm^3$

19. (b)

$$\vec{B} = \left(\frac{\sqrt{3}}{2}\hat{i} + \frac{1}{2}\hat{j}\right) 30\sin\left[\omega\left(t - \frac{z}{c}\right)\right]$$

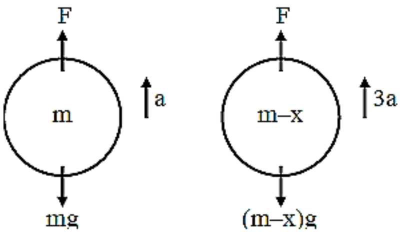
$\vec{E} = \vec{B} \times \vec{C}$ and $E = B_0 c$

Here $\vec{E}\left(\frac{\sqrt{3}}{2}(-\hat{j}) + \frac{1}{2}\hat{i}\right)$

$E_0 = 30c$

$$\vec{E} = \left(\frac{1}{2}\hat{i} - \frac{\sqrt{3}}{2}\hat{j}\right) 30c\sin\left[\omega\left(t - \frac{z}{c}\right)\right]$$

20. (c)



$F - mg = ma$

$F = ma + mg$

$F - (m - x)g = (m - x)3a$

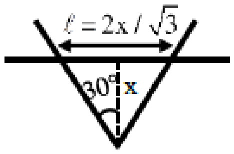
Put F

$Ma + mg - mg + xg = 3ma - 3xa$

$$x = \frac{2ma}{g + 3a}$$

Section - B

21. (1)



$E = \ell vB$

$E = \frac{2x}{\sqrt{3}} \times vB$ and $x = vt$

$E = \frac{2}{\sqrt{3}} v^2 Bt \quad E \propto t^1$

22. (12)

$p = 6 \times 10^{-6} Cm$

$E = 10^6 v/m$

$W = \Delta U = -pE(\cos\theta_f - \cos\theta_i)$

$W = 2pE = 12\ J$

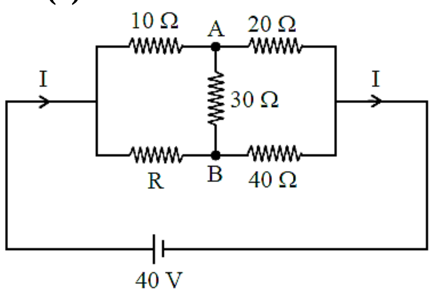
23. (50)

$$B = \frac{\Delta P}{\frac{\Delta V}{V}}$$

$$\Delta V = \frac{7 \times 10^6}{1.4 \times 10^{11}} \times (10 \times 10^{-2})^3$$

$\Delta V = 50\ mm^3$

24. (2)



$V_A = V_B \Rightarrow$ the bridge is balanced

$\Rightarrow \frac{10}{R} = \frac{20}{40}$

$R = 20\Omega$

$I = \frac{40}{20} = 2\text{ A}$

25. (54) Maxima condition

$2\mu t = n\lambda \Rightarrow t = \frac{n\lambda}{2\mu} \Rightarrow t = \frac{\lambda}{2\mu}, \frac{2\lambda}{2\mu}, \ldots\ldots$

Minima condition $2\mu t = (2n-1)\lambda/2$

$\Rightarrow t = \frac{(2n-1)\lambda}{4\mu} \Rightarrow t = \frac{\lambda}{4\mu}, \frac{3\lambda}{4\mu}, \ldots..$

$\Delta t = \frac{2\lambda}{4\mu}$

Rate of evaporation $= \frac{A(\Delta t)}{\text{time}} = 54 \times 10^{-13}\ m^3/s$

Chemistry
Section - A

26. (b)

$R_1 = K[A]^1[B]^1$

$R_1 = K\left[\frac{n_A}{V}\right]^1\left[\frac{n_B}{V}\right]^1$

$R_2 = K\left[\frac{3n_A}{V}\right]^1\left[\frac{3n_B}{V}\right]^1$

$R_2 = 9R_1$

27. (c) $V_2O_5 +$ alkali $\rightarrow VO_4^{3-}$

In VO_4^{3-} ion, vanadium is in +5 oxidation state

28. (a)

(A) Sucrose $\rightarrow \alpha_1 - \beta_2$ Glycosidic linkage
(B) Maltose $\rightarrow \alpha$ 1-4 Glycosidic linkage
(C) Lactose $\rightarrow \beta$ 1-4 Glycosidic linkage
(D) Amylopectin $\rightarrow \alpha$ 1-4 and α 1-6 Glycosidic linkage

A-III, B-I, C-IV, D-II

29. (a)

30. (b) Based on the Ksp values and salt analysis cation identification, we can say that order of Ksp value is:

$HgS < PbS < AgBr < Ca(OH)_2$

Ksp values

$HgS \rightarrow 4 \times 10^{-53}$

$PbS \rightarrow 8 \times 10^{-28}$

$AgBr \rightarrow 5 \times 10^{-13}$

$Ca(OH)_2 \rightarrow 5.5 \times 10^{-6}$

31. (a)

32. (c) (A) Starch $\xrightarrow{H^+H_2O}$ Glucose

(B) Cane sugar $\xrightarrow{H^+H_2O}$ glucose + fructose
(Sucrose) 50% 50%

(C) Milk sugar $\xrightarrow{H^+H_2O}$ glucose + galactose
(Lactose)

(D) Amylopectin $\xrightarrow{H^+H_2O}$ Glucose

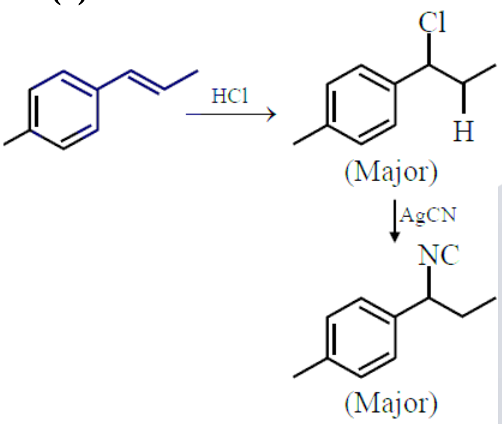
(E) Amylose $\xrightarrow{H^+H_2O}$ Glucose

So, correct options are B, C and E only

33. (d) As internal energy ' U ' is a state function, its cyclic integral must be zero in a cyclic process

$\therefore \Delta U$ case (I) $= \Delta U$ case (II) $= \Delta U$ case (III)

34. (d)



35. (c) %w/w of $HNO_3 = 75\%$

means 100 gm of solution containing 75 g of HNO_3

$\& \left(\frac{gm}{m_1}\right)_{solution} = 1.25 = \frac{100gm}{V}$

V_{ml} of 100 gm solution $= \frac{100}{1.25}$ ml

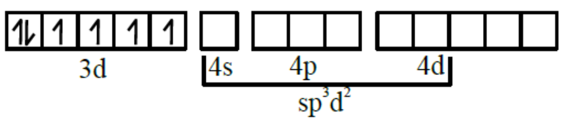
$\because$ 75gm of HNO_3 present in $\frac{100}{1.25}$ ml solution

$\therefore$ 30gm of HNO_3 present in

$\frac{100}{1.25 \times 75} \times 30 = 32$ml solution

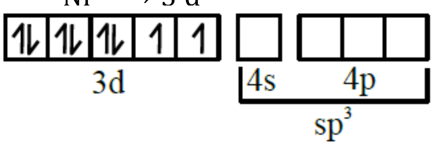
36. (b) (A) $[CoF_6]^{-3}$

$Co^{3+} \rightarrow 3\,d^6$



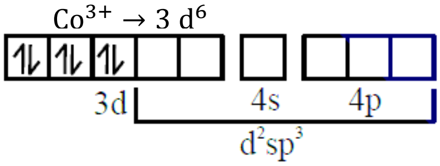
(B) $[NiCl_4]^{2-}$

$Ni^{2+} \rightarrow 3\,d^8$



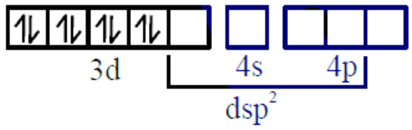
(C) $[Co(NH_3)_6]^{3+}$

$Co^{3+} \rightarrow 3\,d^6$

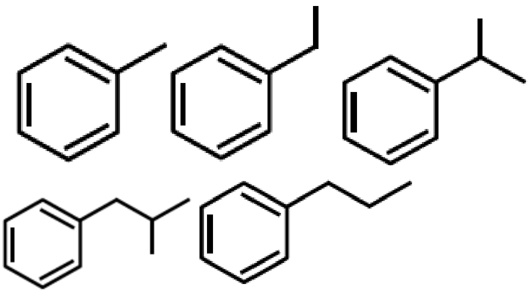


(D) $[Ni(CN)_4]^{2-}$

$Ni^{2+} \rightarrow 3\,d^8$

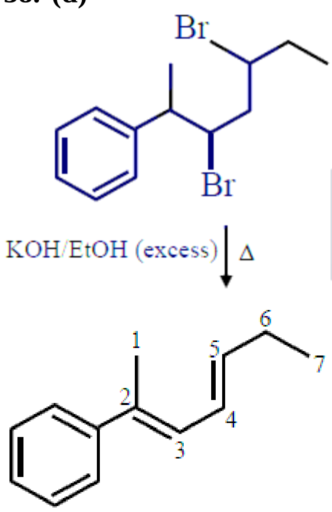


37. (d) Compounds having at least $1\alpha - H$ will react with $KMnO_4$ and give benzoic acid.



Total 5 compounds

38. (d)



2-Phenylhepta-2,4-diene

39. (d) Law of octaves was arranged in the increasing order of their atomic weight. Lothar Meyer plotted the physical properties such as atomic volume, melting point and boiling point against atomic weight.

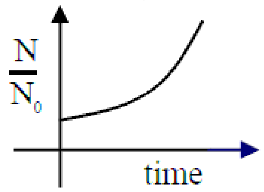
40. (d) Because no. of bacteria initial $= N_0$

and No. of bacteria at any time $t = N$

Since bacterial growth is given as

$N = N_0 e^{Kt}$

Where K = growth constant for bacterial growth



41. (c) For single electron species energy only depends on ' n ' (principal quantum number)

So energy of 2 s = 2p

and energy of 3 d < 4 s

42. (d) Living cell = 0.9gm in 100 gm of solution %w/w = 0.9

Solution is have equal moles of glucose and water = 0.5

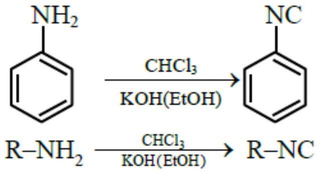
Weight of solution $= 0.5 \times 180 + 0.5 \times 18 = 99$gm %w/w $\simeq 90\%$

Concentrated solution

= Cell will shrink.

43. (a)

(A) $R-NH_2 \xrightarrow[HCl]{NaNO_2} R-N_2^{\oplus}Cl^{\ominus}$



(B)

(C) Only primary amine gives carbyl amine test

(D) $Ph-SO_2Cl \longrightarrow$ Hinsberg reagent

Benzene sulphonyl chloride

(E) Tertiary amine do not react with $Ph-SO_2Cl$

So correct options are (B) and (D) only

44. (b)

45. (b) $As_2\ S_3$ and $As_2\ S_5$ are yellow colour sulphides, $(NH_4)_2\ S$ is colourless, PbS is black, CuS is black in colour

Section - B

46. (5) Strongest oxidising power among the option is Mn_2O_3 because of E° value

$E^{\circ}_{Mn^{+3}/Mn^{+2}} = +1.57\ V$

$Mn^{+3} \rightarrow\ d^4$ configuration

$\mu = \sqrt{24}BM$

$= 4.89BM$

$\Rightarrow 5$

47. (48)

$\Delta H_f[CO_2(\ g)] = -393.5\ kJ/mole$

$\Delta H_f[H_2O(\ell)] = -286.0\ kJ/mole$

$\Delta H_c[C_6H_6] = -3267.0\ kJ/mole$

$\Delta H_f C_6H_6 = (?)$

$C_6H_6 + \frac{15}{2}O_2(\ g) \longrightarrow 6CO_2(\ g) + 3H_2O(\ell)$

$\Delta H_R = \Delta H_C = \Sigma\Delta H_f(P) - \Sigma\Delta H_f(R)$

$-3267 = 6 \times (-393.5) + 3(-286) - \Delta H_f(C_6H_6)$

$\Delta H_f(C_6H_6) = 48\ kJ/mole$

48. (2) Electrolysis of NaCl is

$Cl + H_2O(aq) \rightarrow NaOH(aq) + \frac{1}{2}Cl_2(\ g) + \frac{1}{2}H_2(\ g)$

Since during electrolysis pH changes to 12

So $[OH^{\ominus}] = 10^{-2}$ and $[H^+] = 10^{-12}$

So by Faraday law

Gram amount of substance deposited =

Amount of electricity passed

$10^{-2} \times \frac{600}{1000} \times 96500 = I \times t$

$\frac{10^{-2} \times 600}{1000} \times 96500 = I \times 5 \times 60$

$I = \frac{10^{-2} \times 600 \times 96500}{1000 \times 5 \times 60}$

$I = 1.93$ ampere

So, $I = 2$ ampere (nearest integer)

49. (15) Phosphorus belongs to 15^{th} group and forms $d\pi - d\pi$ bond with transition metal and PH_3 is strongest base among the other group members excepet NH_3.

50. (6)

$O_2 \rightarrow 2$ unpaired electrons according to MOT

$O_2{}^+ \rightarrow 1$ unpaired electrons according to MOT

$O_2^- \rightarrow 1$ unpaired electrons according to MOT
$NO \rightarrow$ odd electron species
$NO_2 \rightarrow$ odd electron species
$K_2[NiCl_4] \rightarrow Ni^{2+} \Rightarrow 3\ d^8$ weak Ligand, C.N. $= 4$
$\Rightarrow$ Tetrahedral, Paramagnetic with 2 unpaired electrons

Mathematics

Section - A

51. (b) E_1 : Bag B_1 is selected

B_1	B_2	B_3
6 W4 B	4 W6 B	5 W5 B

E_2: bag B_2 is selected
E_3: Bag B_3 is selected
A : Drawn ball is white
We have to find $P\left(\frac{E_2}{A}\right)$

$$P\left(\frac{E_2}{A}\right) = \frac{P(E_2)P\left(\frac{A}{E_2}\right)}{P(E_1)P\left(\frac{A}{E_1}\right) + P(E_2)P\left(\frac{A}{E_2}\right) + P(E_3)P\left(\frac{A}{E_3}\right)}$$

$$= \frac{\frac{1}{3} \times \frac{4}{10}}{\frac{1}{3} \times \frac{6}{10} + \frac{1}{3} \times \frac{4}{10} + \frac{1}{3} \times \frac{5}{10}}$$

$$= \frac{4}{15}$$

52. (a) Equation of angle bisector : $x - y = 0$

$$\left|\frac{a(1-a)}{\sqrt{2}}\right| = \frac{9}{\sqrt{2}} \Rightarrow a = 5 \text{ or } -4$$

Sum $= 5 + (-4) = 1$

53. (d) let

$\vec{a}_{11} =$ component of $\vec{a}$ along $\vec{b}$

$\vec{a}_1 =$ component of $\vec{a}$ perpendicular to $\vec{b}$

$$\vec{a}_{11} = \frac{16}{11}(3\hat{i} + \hat{j} - \hat{k})$$

$$\vec{a}_1 = \frac{1}{11}(-4\hat{i} - 5\hat{j} - 17\hat{k})$$

$$\because \vec{a} = \vec{a}_{11} + \vec{a}_1$$

$$\therefore \vec{a} = \frac{16}{11}(3\hat{i} + \hat{j} - \hat{k}) + \frac{1}{11}(-4\hat{i} - 5\hat{j} - 17\hat{k})$$

$$= \frac{44}{11}\hat{i} + \frac{11}{11}\hat{j} - \frac{33}{11}\hat{k}$$

$$\vec{a} = 4\hat{i} + \hat{j} - 3\hat{k}$$

$\alpha = 4\ \beta\ \beta = 1\ \gamma = -3$

$\alpha^2 + \beta^2 + \gamma^2 = 16 + 1 + 9 = 26$

54. (b)

$$x^2-(3-2i)x-(2i-2)=0$$

$$x=\frac{(3-2i)\pm\sqrt{(3-2i)^2-4(1)(-(2i-2))}}{2(1)}$$

$$==\frac{(3-2i)\pm\sqrt{9-4-12i+8i-8}}{2}$$

$$=\frac{3-2i\pm\sqrt{-3-4i}}{2}$$

$$=\frac{3-2i\pm\sqrt{(1)^2+(2i)^2-2(1)(2i)}}{2}$$

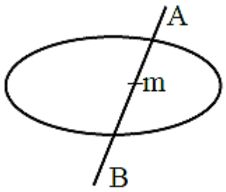
$$=\frac{3-2i\pm(1-2i)}{2}$$

$$\Rightarrow\frac{3-2i+1-2i}{2}\text{ or }\frac{3-2i-1+2i}{2}$$

$$\Rightarrow 2-2i\text{ or }1+0i$$

So $\alpha\gamma+\beta\delta=2(1)+(-2)(0)=2$

55. (b)



If m $\left(\sqrt{2},\frac{4}{3}\right)$ than equation of of AB is

$T=S_1$

$$\frac{x\sqrt{2}}{9}+\frac{y}{4}\left(\frac{4}{3}\right)=\frac{(\sqrt{2})^2}{9}+\frac{\left(\frac{4}{3}\right)^2}{4}$$

$$\frac{\sqrt{2}x}{9}+\frac{y}{3}=\frac{2}{9}+\frac{4}{9}$$

$\sqrt{2}x+3y=6\Rightarrow y=\frac{6-\sqrt{2}x}{3}$ put in ellipse

So, $\frac{x^2}{9}+\frac{(6-\sqrt{2}x)^2}{9\times 4}=1$

$$4x^2+36+2x^2-12\sqrt{2}x=36$$

$$6x^2-12\sqrt{2}x=0$$

$$6x(x-2\sqrt{2})=0$$

$$x=0\&x=2\sqrt{2}$$

So $y=2$ $y=\frac{2}{3}$

Length of chord $=\sqrt{(2\sqrt{2}-0)^2+\left(\frac{2}{3}-2\right)^2}$

$$=\sqrt{8+\frac{16}{9}}$$

$$=\sqrt{\frac{88}{9}}=\frac{2}{3}\sqrt{22}\text{ so }\alpha=22$$

56. (d)

A, E,G R D N

Probabllity (P) $=\frac{\text{favourable case}}{\text{Total case}}$

(when A & E are in order)
Total case = 6 !
Favourable case = ${}^{6}C_2 . 4\,!$

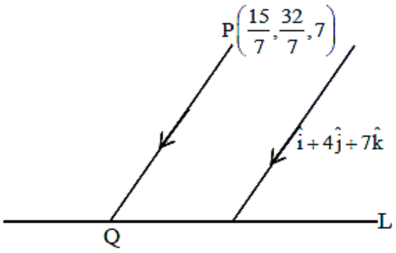
$P = \frac{(15)4!}{(30)4!}$

Probablity when not in order $= 1 - \frac{1}{2} = \frac{1}{2}$

57. (d)

$$g(x) = x2 + x^{\frac{7}{3}}$$
$$g'(x) = 2x + \frac{7}{3}x^{\frac{4}{3}}$$
$$f(x) = \frac{g'(x)}{x}$$
$$f(x) = 2 + \frac{7}{3}x^{\frac{1}{3}}$$
$$f(r^3) = 2 + \frac{7r}{3}$$
$$\sum_{r=1}^{15}\left(1 + \frac{7}{3}r\right) = 310$$

58. (c)



$$L = \frac{x+1}{3} = \frac{y+3}{5} = \frac{z+5}{7}$$
$$PQ = \frac{x - \frac{15}{7}}{1} = \frac{y - \frac{32}{7}}{4} = \frac{z-7}{7} = \lambda$$
$$\Rightarrow Q\left(\lambda + \frac{15}{7}, 4\lambda + \frac{32}{7}, 7\lambda + 7\right)$$

Since Q lies on -line L

$$\text{So, } \frac{\lambda + \frac{15}{7} + 1}{3} = \frac{7\lambda + 7 + 5}{7}$$
$$\Rightarrow 7\lambda + 22 = 21\lambda + 36$$
$$\Rightarrow \lambda = -1$$
$$\therefore \text{ Point } Q\left(\frac{8}{7}, \frac{4}{7}, 0\right)$$
$$PQ = \sqrt{\left(\frac{15}{7} - \frac{8}{7}\right)^2 + \left(\frac{32}{7} - \frac{4}{7}\right)^2 + (7-0)}$$
$$PQ = \sqrt{66}$$
$$\Rightarrow (PQ)^2 = 66$$

59. (c)
$$x(1 + y^2) = 1 \quad \ldots\ldots(1)$$
$$y^2 = 2x \quad \ldots\ldots(2)$$

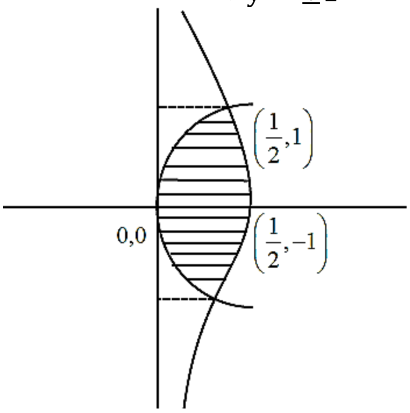
From equation (1) & (2)

$x(1+2x)=1 \quad \Rightarrow 2x^2+x-1=0$

$\Rightarrow x=\frac{1}{2}, x=-1$ (Reject)

$\Rightarrow y^2=2\left(\frac{1}{2}\right)$

$\Rightarrow y=\pm 1$



Area bounded $=\int_{-1}^{1}\left(\frac{1}{1+y^2}-\frac{y^2}{2}\right)dy$

$=\left(\tan^{-1}y-\frac{y^3}{6}\right)\Big|_{-1}^{1}$

$=\frac{\pi}{2}-\frac{1}{3}$

60. (a)

$P=\begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}$

$\because P^TP=I$

$B=PAPT$

Pre multiply by P^T (Given)

$P^T\,B=P^TPAPP^T=AP^T$

Now post multiply by P

$P^TBP=AP^TP=A$

So $A^2=\underbrace{P^TBPP^T}_{I}BP$

$A^2=P^T\,B^2P$

Similarly $A^{10}=P^TB^{10}P=C$

$A=\begin{bmatrix}\frac{1}{\sqrt{2}} & -2\\ 0 & 1\end{bmatrix}$ (Given)

$\Rightarrow A^2=\begin{bmatrix}\frac{1}{2} & -\sqrt{2}-2\\ 0 & 1\end{bmatrix}$

Similarly check A^3 and so on since $C=A^{10}$

$\Rightarrow$ Sum of diagonal elements of C is $\left(\frac{1}{\sqrt{2}}\right)^{10}+1$

$=\frac{1}{32}+1=\frac{33}{32}=\frac{m}{n}$

$g\,cd(m,n)=1$ (Given)

$\Rightarrow m+n=65$

61. (b)

let $x = t^4$

$dx = 4t^3 dt$

then $\int \frac{1}{x^{\frac{1}{4}}\left(1+x^{\frac{1}{4}}\right)} dx \Rightarrow \int \frac{4t^3 dt}{t(1+t)}$

$\Rightarrow \int \frac{4t}{1+t} dt \Rightarrow 4\int \frac{(t^2-1)+1}{1+t} dt$

$\Rightarrow 4\int (t-1) + \frac{1}{t+1} dt$

$\Rightarrow 4\left\{\frac{(t-1)^2}{2} + \ell n(t+1)\right\} + c$

hence $f(x) = 2\left(x^{\frac{1}{4}} - 1\right)^2 + 4\ell n\left(1 + x^{\frac{1}{4}}\right) + c$

$f(0) = -6 \Rightarrow 2 + 4\ell n + 6 = -6 \rightarrow C = -8$

now $f(1) = 4\ell n2 - 8$

$= 4(\ell \ln 2 - 2)$

62. (a)

$\int_0^2 xF'(x)dx = 6$

$= xF(x)|_0^2 - \int_0^2 f(x)dx = 6$

$= 2\,F(2) - \int_0^2 xF(x)dx = 6[\therefore f(2) = 2\,F(2) = 2]$

$\int_0^2 xF(x)dx = -2 \quad \ldots..(1)$

$\Rightarrow \int_0^2 F(x)dx = -2 \quad \ldots..(2)$

Also

$\int_0^2 x^2F''(x)dx = x^2F'(x)|_0^2 - 2\int_0^2 xF'(x)dx = 40$

$= 4F'(2) - 2 \times 6 = 40$

$F'(2) = 13$

$\therefore F'(2) + \int_0^2 F(x) = 13 - 2 = 11$

63. (c)

$a_n \quad = \frac{n^2 + 5n + 6}{4}$

$S_n \quad = S_n = \sum_{k=1}^{n} \frac{1}{a_k} = \sum_1^n \frac{4}{k^2 + 5k + 6}$

$= 4\sum_{k=1}^{n} \frac{1}{(k+2)(k+3)}$

$= 4\sum_{k=1}^{n} \frac{1}{k+2} - \frac{1}{k+3}$

$= 4\left(\frac{1}{3} - \frac{1}{4}\right) + 4\left(\frac{1}{4} - \frac{1}{5}\right) + \cdots \ldots..$

$$4\left(\frac{1}{n+2}-\frac{1}{n+3}\right)$$
$$=4\left(\frac{1}{3}-\frac{1}{n+3}\right)$$
$$=\frac{4n}{3(n+3)}$$
$$507\ S_{2025}=\frac{(507)(4)(2025)}{3(2028)}$$
$$=675$$

64. (a)

f(x) is onto hence A is range of f(x)

as $\begin{aligned}\text{now } f'(x)&=6x^2-30x+36\\&=6(x-2)(x-3)\end{aligned}$

$f(2)=16-60+72+7=35$

$f(3)=54-135+108+7=34$

$f(0)=7$

hence range $\in[7,35]=A$

also for range of g(x)

$$g(x)=1-\frac{1}{x^{2025}+1}\in[0,1)=B$$

$s=\{0,7,8,\ldots..35\}$ hence $n(s)=30$

65. (b)

$2[x]+1\leq-1$ or $2[x]+1\geq1$

$\Rightarrow[x]\leq-1\cup[x]\geq0$

$\Rightarrow x\in(-\infty,0)\cup x\in[0,\infty)$

$\Rightarrow x\in(-\infty,\infty)$

66. (c)

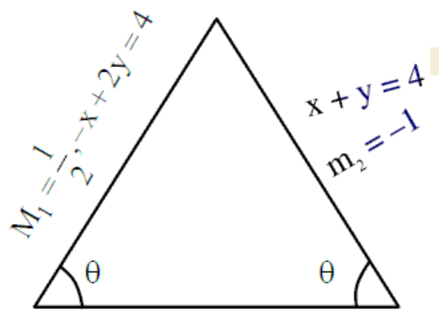
$$\frac{1}{\sin\frac{\pi}{6}}\sum_{r=1}^{13}\frac{\sin\left[\left(\frac{\pi}{4}+\frac{r\pi}{6}\right)-\left(\frac{\pi}{4}\right)-(r-1)\frac{\pi}{6}\right]}{\sin\left(\frac{\pi}{4}+(r-1)\frac{\pi}{6}\right)\sin\left(\frac{\pi}{4}+\frac{r\pi}{6}\right)}$$

$$\frac{1}{\sin\frac{\pi}{6}}\sum_{r=1}^{13}\left(\cot\left(\frac{\pi}{4}+(r-1)\frac{\pi}{6}\right)-\cot\left(\frac{\pi}{4}+\frac{r\pi}{6}\right)\right)$$

$=2\sqrt{3}-2=\alpha\sqrt{3}+b$

So $a^2+b^2=8$

67. (c)



$$\tan\theta=\frac{m-\frac{1}{2}}{1+\frac{1}{2}\cdot m}=\frac{-1-m}{1-m}=\frac{m+1}{m-1}$$

$$\frac{2m-1}{2+m}=\frac{m+1}{m-1}$$

$2m^2-3m+1=m^2+3m+2$

$m^2-6m-1=0$

sum of root = 6
sum is 6

68. (a)

$(a+b)^{\frac{1}{2}}$

$T_r\ T_{r+1},\ T_{r+2} \rightarrow GP$

So, $\frac{T_{r+1}}{T_r}=\frac{T_{r+2}}{T_{r+1}}$

$\frac{{}^{12}C_r}{{}^{12}C_{r-1}}=\frac{{}^{12}C_{r+1}}{{}^{12}C_r}$

$\frac{12-r+1}{r}=\frac{12-(r+1)+1}{r+1}$

$(13-r)(r+1)=(12-r)(r)$

$-r+12r+13=12r-r^2$

$13=0$

No value of r possible

So P = 0

$\left(3^{\frac{1}{4}}+4^{\frac{1}{3}}\right)^{12}=\sum\ {}^{12}C_r\left(3^{\frac{1}{4}}\right)^{12-r}\left(4^{\frac{1}{3}}\right)^r$

Exponent of $\left(3^{\frac{1}{4}}\right)$	exponent of $\left(4^{\frac{1}{3}}\right)$	term
12	0	27
0	12	256

$q=27+256=283$

$p+q=0+283=283$

69. (d)

$x^2+y^2-8x=0, \frac{x^2}{9}-\frac{y^2}{4}=1 \quad \ldots.(1)$

$4x^2-9y^2=36 \quad \ldots.(2)$

Solve (1)&(2)

$4x^2-9(8x-x^2)=36$

$13x^2-72x-36=0$

$(13x+6)(x=6)=0$

$x=\frac{-6}{13}, x=6$

$x=\frac{-6}{13}$ (rejected)

$y \rightarrow$ Imaginary

$n=6, \frac{36}{9}-\frac{y^2}{4}=1$

$y^2=12, y=I\sqrt{12}$

$A(6,\sqrt{12}), B(6,-\sqrt{12})$

$p\left(\alpha, \frac{2\alpha+4}{3}\right)$ P lies on

centroid (h,k) $\qquad 2x-3y+y=0$

$h = \frac{12+\alpha}{3}, \alpha = 3h - 12$

$k = \frac{\frac{2\alpha+4}{3}}{3} \Rightarrow 2\alpha + 4 = 9k$

$\alpha = \frac{9k-4}{2}$

$6h - 2y = 9k - 4$

$6x - 9y = 20$

70. (b) as f(x) is a polynomial of degree two let it be

$f(x) = ax^2 + bx + c\ (a \neq 0)$

on satisfying given conditions we get

$C = 1 \& a = \pm 1$

hence $f(x) = 1 \pm x^2$

also range $\in (-\infty, 1]$ hence

$f(x) = 1 - x^2$

now $f(k) = -2k$

$1 - k^2 = -2k \rightarrow k^2 - 2k - 1 = 0$

let roots of this equation be $\alpha \& \beta$

then $\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$

$= 4 - 2(-1) = 6$

Section - B

71. (64)

x	y	z

Let $x = 2 \Rightarrow y + z = 13$

$(4,9), (5,8), (6,7), (7,6), (8,5), (9,4), \rightarrow 6$

Let $x = 3 \rightarrow y + z = 12$

$(3,9), (4,8), \ldots \ldots .., (9,3) \rightarrow 7$

Let $x = 4 \rightarrow y + z = 11$

$(2,9), (3,8), \ldots \ldots \ldots (9,1) \rightarrow 9$

Let $x = 5 \rightarrow y + z = 10$

$(1,9), (2,8), \ldots \ldots (9,1) \rightarrow 10$

Let $x = 6 \rightarrow y + z = 9$

$(0,9), (1,8), \ldots \ldots .. (9,0) \rightarrow 9$

Let $x = 7 \rightarrow y + z = 8$

$(0,9), (1,7), \ldots \ldots \ldots, (8,0) \rightarrow 9$

Let $x = 8 \rightarrow y + z = 7$

$(0,7), (1,6), \ldots \ldots . (7,0) \rightarrow 8$

Let $x = 9 \rightarrow y + z = 6$

$(0,6), (1,5), \ldots \ldots .., (6,0) \rightarrow 7$

Total $= 6 = 7 + 8 + 9 + 10 + 9 + 8 + 7 = 64$

72. (1)

$f(x) = \lim_{n\to\infty} \sum_{r=0}^{n} \left(\tan\frac{x}{2^r} - \tan\frac{x}{2^{r+1}}\right) = \tan x$

$\lim_{x\to 0}\left(\frac{e^x - e^{\tan x}}{x - \tan x}\right) = \lim_{x\to 0} e^{\tan x}\frac{(e^{x-\tan x} - 1)}{(x - \tan x)}$

$= 1$

73. (20)

$\frac{n}{2}(2a + (n-1)6) = (n-2)\cdot 180°$

$an + 3n^2 - 3n = (n-2)\cdot 180° \quad \ldots \ldots (1)$

Now according to question

$a + (n-1)6^\circ = 219^\circ$

$\Rightarrow a = 225^\circ - 6n^\circ \quad \ldots\ldots\ldots (2)$

Putting value of a from equation (2) in (1)

We get

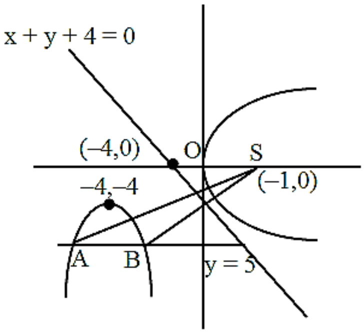
$(225n - 6n^2) + 3n^2 - 3n = 180n - 360$

$\Rightarrow 2n^2 - 42n - 360 = 0$

$\Rightarrow n2 - 14n - 120 = 0$

$n = 20, -6$ (rejected)

74. (14)



Area $= \frac{1}{2} \times 4 \times 5 = 10 = a$

$6 \quad = 4$

So $a + d = 14$

75. (4)

$$\frac{dy}{dx} + \frac{\left(\sin^{-1}\frac{x}{2}\right)}{\sqrt{4-x^2}} y = \frac{\left(\sin^{-3}\frac{x}{2}\right)^3}{\sqrt{4-x^2}}$$

$$ye^{\frac{\left(\sin^{-1}\frac{x}{2}\right)^2}{2}} = \int \frac{\left(\sin^{-3}\frac{x}{2}\right)^3}{4-x^2} e^{\frac{\left(\sin^{-1}\frac{x}{2}\right)^2}{2}} dx$$

$$y = \left(\sin^{-1}\frac{x}{2}\right)^2 - 2 + c \cdot e^{\frac{-\left(\sin^{-1}\frac{x}{2}\right)^2}{2}}$$

$$y(2) = \frac{\pi^2}{4} - 2 \Rightarrow c = 0$$

$$y(0) = -2$$