1. **(a)** $10 \times 3.36 \times 10^5 + 10 \times 2100 \times 10 + 10 \times 4200 \times (T - 0)$

$= 100 \times 4200 \times (25 - T)$

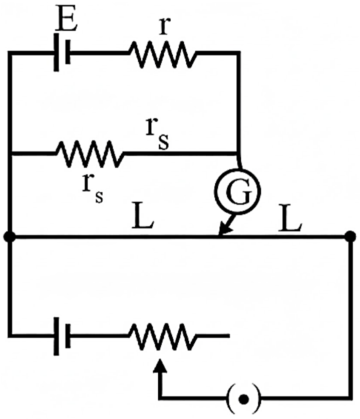
$\Rightarrow T = 15^\circ C$

$\Delta T = 25 - 15 = 10^\circ C$

2. **(d)** $i^2_{mms} = \dfrac{\int_0^T (i_0^2 t^2/T^2)dt}{\int_0^T dt} = \dfrac{i_0^2}{T^3} \cdot \dfrac{T^3}{3} = \dfrac{i_0^2}{3}$

$i_{rms} = \dfrac{i_0}{\sqrt{3}}$

3. **(b)** Let E is emf and r is internal resistance of cell.



$\dfrac{E \cdot 4}{r + 4} = 120K$

$\dfrac{E \cdot 12}{r + 12} = 180K$

$\Rightarrow \dfrac{1}{3}\dfrac{r + 12}{r + 4} = \dfrac{2}{3}$

$r + 12 = 2(r + 4)$

$\Rightarrow r = 4$

4. **(b)** $\alpha = \frac{\tau}{I}$

$I = \dfrac{1}{4}mR^2 + mR^2 + \dfrac{1}{4}mR^2 + m(2R)^2 + \dfrac{m(3R)^2}{12} + m\left(\dfrac{R}{2}\right)^2$

$= \left(\dfrac{3}{2} + 4 + 1\right)mR^2 = \dfrac{13}{2}mR^2 = \dfrac{13}{2} \times 600 \times 10^2 = 39 \times 10^4$

$\alpha = \dfrac{43 \times 10^5}{39 \times 10^4} rad/s^2 = \dfrac{430}{39} rad/s^2 \approx 11 rad/s^2$

5. **(c)** Time to reach ground $= \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 5}{10}} = 1sec$

Five drops per second

Time between each drop $= 0.2sec.$

Time of fall for 4^{th} drop is $1 - 0.6 = 0.4sec$

Height fall of 4^{th} drop is $= \frac{1}{2} \times 10 \times 0.4^2 = 0.8$ m

Height from ground $= 5 - 0.8 = 4.2$ m

6. **(c)** Surface area $x \propto A^{2/3}$

$X_i = 8^{2/3}K = 4K$

$X_f = (8 + 10 + 9)^{2/3}K = 9K$

% increase in surface area of nucleus

$x_1 = \dfrac{9K - 4K}{4K} \times 100 = 125\%$

7. **(c)** $\omega = 2 \times 10^{15} rad/s$

$k = 10^7\ m^{-1}$

$V = \frac{2\pi}{k} \cdot \frac{\omega}{2\pi} = \frac{\omega}{k} = \frac{2\times10^{15}}{10^7} = 2\times10^8 = \frac{C}{1.5}$

$\Rightarrow \mu = 1.5$

8. (b)

9. (b) $F_{net} = \frac{\mu_0}{2\pi} I_0 \left(\frac{I_1}{d_1} + \frac{I_2}{d_2}\right) \ell$

$F_{net} = 2\times10^{-7} \times 1 \left(\frac{3}{3} + \frac{2}{2}\right) \times \frac{15\times10^{-2}}{10^{-2}}$

$= 4 \times 15 \times 10^{-7}$

$F_{net} = 6 \times 10^{-6}$ N

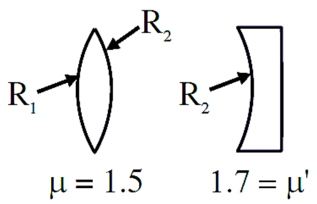
10. (d) $12 - 0.3\times10^3 I - 0.7 = 0 \frac{11.3}{0.3\times10^3} = I$

$37.66 \times 10^{-3}\ A = I$

Current through diode $D_1, I_1 = I/2$

$I_1 = 18.83$ mA

11. (a)



$|P_A| = |P_B|$

$0.5\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = \frac{0.7}{R_2}$

$\frac{5}{R_1} = \frac{2}{R_2}$

$\frac{R_1}{R_2} = \frac{5}{2}$

12. (d) w = Area of parabola

$= \frac{2}{3}$(Area of rectangle AC 31 A)

$= \frac{2}{3} P_0 (3-1) = \frac{4P_0}{3}$

When $V = 1$

$(1-2)^2 = 4aP_0$

$P_0 = \frac{1}{4a}$

$w = \frac{4}{3} P_0 = \frac{4}{3}\frac{1}{4a} = \frac{1}{3a}$

$w_{ga} = \frac{-1}{3a}$

13. (d) In series, $i_1 = \frac{2E}{6+2r}$

In parallel, $i_2 = \frac{E}{6+\frac{r}{2}}$

$i_1 = i_2 \Rightarrow \frac{2E}{6+2r} = \frac{E}{6+\frac{r}{2}}$

$12 + r = 6 + 2r$

$r = 6\Omega$

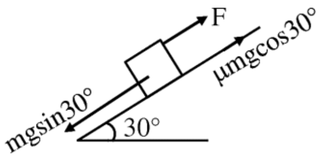
14. (d) $T = \frac{F/A}{\Delta\ell/\ell} \Rightarrow Y = \frac{F\ell}{A\Delta\ell}$

$\frac{Y_A}{Y_B} = \frac{\ell_A}{\ell_B}\left(\frac{A_B}{A_A}\right)$

$= \frac{6}{5.4}\left(\frac{4.5\times10^{-5}}{3\times10^{-5}}\right) = \frac{9}{5.4} = \frac{5}{3} \Rightarrow \frac{x}{3} = \frac{5}{3}$

$x = 5$

15. (b)



$\text{mgsin}30^\circ = \text{F} + \mu\text{mgcos}30^\circ$

$F = 5 \times 10 \times \frac{1}{2} - \frac{\sqrt{3}}{2} \times 5 \times 10 \times \frac{\sqrt{3}}{2}$

$F = 25 - \frac{75}{2} = 25 - 37.5$

$F = -12.5\ \text{N}$

∴ force will be downward on incline of magnitude 12.5 N

16. (d) Increasing the slit width ' a ' decreases the diffraction angle ($\theta = \lambda/\text{a}$) and reduces the spreading of the wave. A narrower slit produces a more pronounced spherical wave (high curvature) while a wider slit leads to a flatter, less curved wave.

17. (b) Reading = MSR + (VSR × LC) − (zero Error)

$= 15\ \text{mm} + (5 \times 0.1\ \text{mm}) - (4 \times 0.1\ \text{mm})$

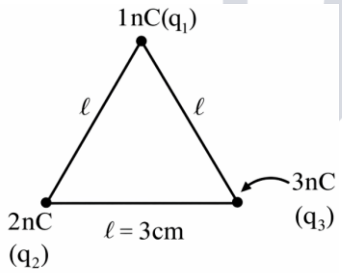
Reading $= 15.1$ mm

$\therefore \ell = 15.1\ \text{mm}$

18. (c) $\frac{\mu_0 \text{I}}{2\text{R}} = 16\mu\ \text{T}$

$$\frac{\mu_0 \text{IR}^2}{2(\text{x}^2 + \text{R}^2)^{3/2}} = \frac{\mu_0 \text{IR}^2}{2 \times 8\text{R}^3} = 2\mu\ \text{T}$$

19. (a)



$W = \left(\frac{\text{kq}_1}{\ell} + \frac{\text{kq}_2}{\ell}\right)\text{q}_3$

$= \frac{9 \times 10^9}{3 \times 10^{-2}}(3 \times 10^{-9}) \times 3 \times 10^{-9}$

$= 27 \times 10^{-7}\ \text{J} = 2.7\mu\ \text{J}$

20. (b) $m \cdot \frac{\text{dv}}{\text{dt}} = mg - kv$

$$\int_0^{\text{v}} \frac{\text{dv}}{\text{mg} - \text{kv}} = \int_0^{\text{t}} \frac{\text{dt}}{\text{m}}$$

$$\frac{-1}{\text{k}} \ell\text{n}\left(\frac{\text{mg} - \text{kv}}{\text{mg}}\right) = \frac{\text{t}}{\text{m}}$$

$$\text{v} = \frac{\text{mg}}{\text{k}}\left(1 - \text{e}^{-\text{kt}/\text{m}}\right)$$

21. (2) Potential energy is maximum at extreme position the particle starting at mean position reaches extreme position in time $\frac{\text{T}}{4}$.

22. (2) $\lambda = \frac{\text{h}}{\text{mv}} = \frac{\text{h}}{\sqrt{2\ \text{m} \cdot \text{KE}}}$

$$\frac{\lambda_{\text{d}}}{\lambda_\alpha} = \sqrt{\frac{\text{m}_\alpha \cdot \text{KE}_\text{a}}{\text{m}_\text{d} \cdot \text{KE}_\text{d}}} = \sqrt{\frac{4\ \text{m} \cdot 2\text{E}}{2\ \text{m} \cdot \text{E}}} = 2:1$$

23. (265) Let radius of gyration is k

$\Rightarrow mk^2 = \frac{2}{3}mR^2 + md^2$

$k^2 = \frac{2}{3} \times 10^2 + 15^2 = 265$

$(\sqrt{n})^2 = 265 \Rightarrow n = 265$

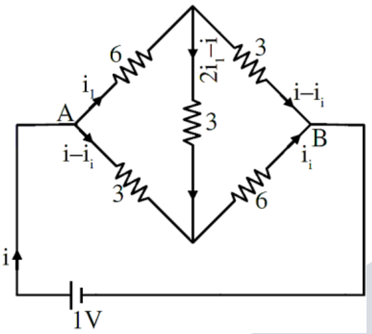
24. (3) $\frac{1}{f_{Air}} = \left(\frac{1.5-1}{1}\right)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$

$\frac{1}{f_{witer}} = \left(\frac{1.5 - 4/3}{4/3}\right)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$

$\Rightarrow \frac{f_{waer}}{f_{uar}} = \frac{0.5}{0.5/4} = 4$

$\Rightarrow f_{waxer} - f_{uir} = 3f$

25. (21)



$6i_1 + 3(2i_1 - i) = 3(i - i_1)$

$\Rightarrow 15i_1 = 6i \Rightarrow i_1 = \frac{2}{5}I$ ...(i)

$3(i - i_1) + 6i_1 = 1$

$3i + 3i_1 = 1$

$\left(3 + \frac{6}{5}\right)i = 1$

$\Rightarrow i = \frac{5}{21}\ A = \frac{1\ V}{R_{eq}} \Rightarrow R_{eq} = \frac{21}{5}\Omega$

26. (a) For isothermal reversible process $\Delta U = \Delta H = 0$

$w_{iso} = -p_1v_1 ln\frac{P_1}{P_2}$

$w_{iso} = -0.5 \times 10^6 \times 20 \times 10^{-3} ln\frac{0.5}{0.2}$

$w_{iso} = -0.5 \times 10^6 \times 20 \times 10^{-3} \times 2.303 \times (.6989 - .3010)$

$w = -9.1$ kJ

$q = -w = 9.1$ kJ

27. (c) $CH_2 = \overline{C}H\ CH_3 = CH_2\ CH \equiv \overline{C}$

Stability $\propto$ % S

Order of stability

$CH \equiv \overline{C} > CH_2 = C\overline{H} > CH_3 - CH_2$

28. (d)

$$\underbrace{\text{2 moles of A + 3 moles of B}}_{\Downarrow}$$

$X_A = 2/5, X_B = 3/5$

$P_S = X_AP_A^\circ + X_BP_B^\circ$

$320 = P_A^\circ\left(\frac{2}{5}\right) + P_B^\circ\left(\frac{3}{5}\right)$

$2P_A^\circ + 3P_B^\circ = 1600$ ... ... (I)

Now 1 mole of A & 1 mole of B is added

$X'_A = \frac{3}{7}, X'_B = \frac{4}{7}$

$P'_S = 328.6 = P^\circ_A\left(\frac{3}{7}\right) + P^\circ_B\left(\frac{4}{7}\right)$

$3P^\circ_A + 4P^\circ_B = 2300.2 \qquad \ldots(\text{II})$

Now eq (I) $\times 3$ − eq (II) $\times 2$

$6P^\circ_A + 9P^\circ_B = 4800$

$6P^\circ_A + 8P^\circ_B = 4600.4$

$P^\circ_B \simeq 200$ mm of Hg

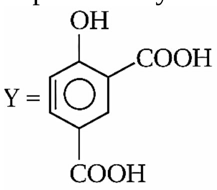
$P^\circ_A \simeq 500$ mm of Hg

29. (c) $(X) \xrightarrow[(2)H_3O^{\oplus}]{(1)CO_2/NaOH} Y_{Majec}$

76.6%C

6.38% H

Vapor Density 47

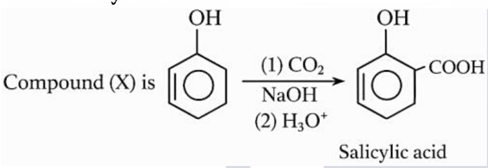


Gives colour with $FeCl_3$

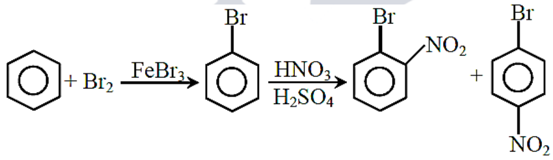
Soluble in $NaHCO_3$

Soluble in NaOH

Salicylic acid



30. (d)



B & C separate by

Fractional Distillation method

Due to their different boiling point.

31. (c)

$$\begin{array}{llll} & \boldsymbol{HCl} + & \boldsymbol{B} \rightarrow \boldsymbol{BH^+} + \boldsymbol{Cl^+} & \\ & 0.02\text{M} & 0.02\text{M} & \\ & \text{x ml} & \text{y ml} & \\ t_f & 0 & 0.02\text{y}-0.02\text{x} & 0.02\text{x} \end{array}$$

$\text{pOH} = \text{pK}_b + \log\left[\frac{\text{Salt}}{\text{Base}}\right]$

$5 = 5.699 + \log\left[\frac{\text{Salt}}{\text{Base}}\right]$

$\frac{x}{y-x} = \frac{1}{5}$

$6x = y$

$7x = 100$

$x = \frac{100}{7}$ ml

$\& \ y = \frac{600}{7}$ ml

32. (a) $t = \frac{1}{k}\ln\frac{A_0}{A_i}$

$$t_{1/2} = \frac{1}{k} ln \frac{A_0}{A_0/8} - \frac{1}{k} ln 8$$

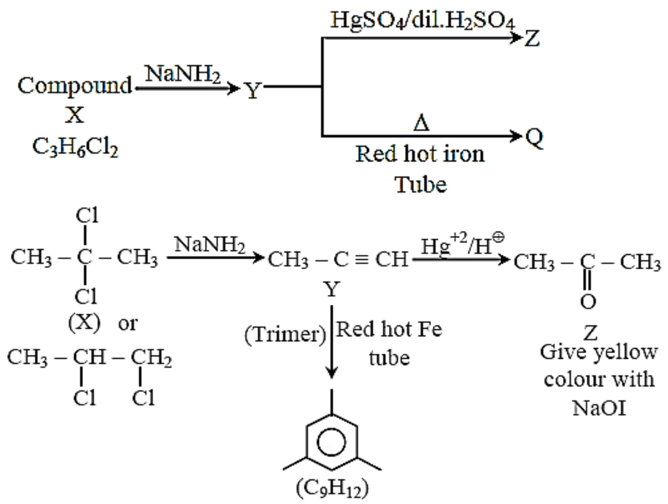
$$t_{1/10} = \frac{1}{k} ln \frac{A_0}{A_0/10} - \frac{1}{k} ln 10$$

$$\frac{t_{1/8}}{t_{1/10}} = \frac{ln 8}{ln 10} - \frac{log 8}{log 10}$$

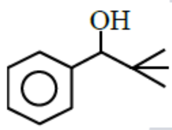
$$\frac{t_{1/8}}{t_{1/10}} = \log 8 - 3\log 2 - 0.9$$

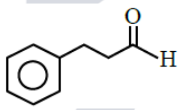
$$\frac{t_{1/k}}{t_{1/10}} \times 10 - 9$$

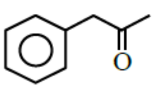
33. (b)

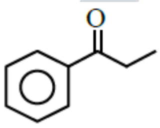


34. (a)

(P) 

(Q) 

(R) 

(S) 

(A) Q, R, S all three give 2,4 DNP test as they have Aldehyde/ketone group

(C) Q & R gives sooty flame

(E) Q gives Tollens reagent test

35. (a) In BF_4^-, SiF_4 and XeF_4 all bond lengths are identical

Molecules B.O.

$O_2^+ \rightarrow 2.5$

$O_2^- \rightarrow 1.5$

$O_2^{2-} \rightarrow 1$

$F_2 \rightarrow 1$

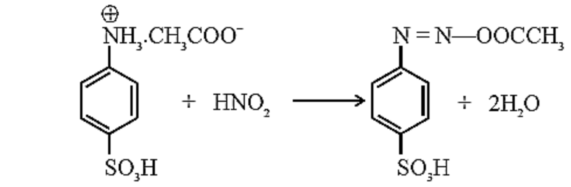
36. (a) Stability: $NH_3 > PH_3 > AsH_3 > SbH_3 > BiH_3$

Basicity: $NH_3 > PH_3 > AsH_3 > SbH_3 > BiH_3$

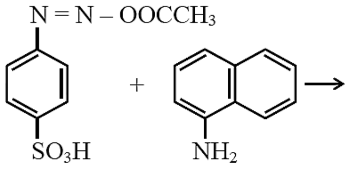
Reducing character : $NH_3 < PH_3 < AsH_3 < SbH_3 < BiH_3$

Boiling point: $PH_3 < AsH_3 < NH_3 < SbH_3 < BiH_3$

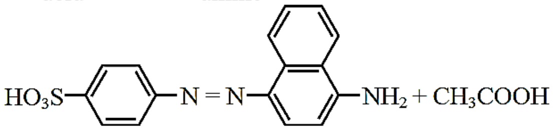
37. (b)$NO_2^- + CH_3COOH \rightarrow HNO_2 + CH_3COO^-$



(Sulphanilic acid solution)



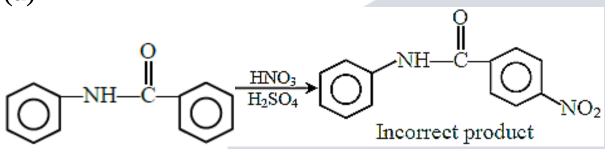
Diazotised acid   1-Naphthyl amine



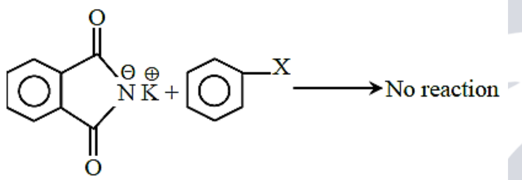
Red azo-dye

38. (d) $\xrightarrow{\Delta} H_3CCH_2NH_2 + 2KBr + K_2CO_3 + 2H_2O$

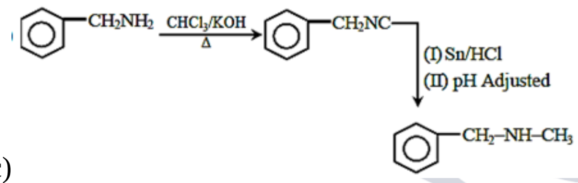
(a)



(b)



Aromatic halide does not give Gabriel phthalimide reaction



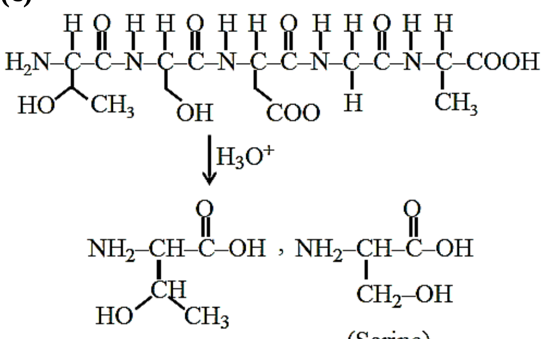
(c)

(d)Hoffmann bromamide degradation

$$CH_3-CH_2-\overset{\overset{O}{\|}}{C}-NH_2 \xrightarrow[4KOH]{Br_2} CH_3-CH_2-NH_2$$

$2KBr + K_2CO_3 + 2H_2O$

39. (c)



(Threonine)   (Serine)

$NH_2-CH(CH_2-COOH)-C(=O)-OH$ (Aspartic acid), $NH_2-CH_2-C(=O)-OH$ (Glycine)

$NH_2-CH(CH_3)-COOH$ (Alanine)

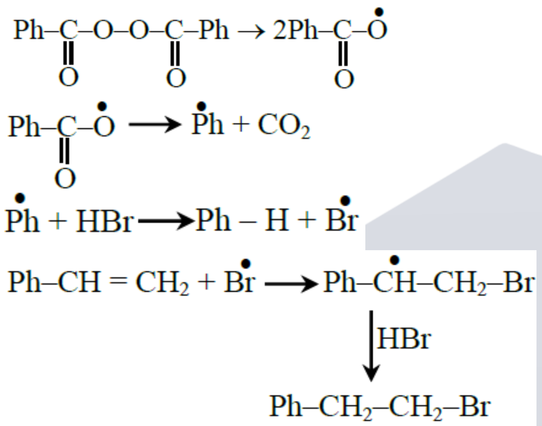
40. (d) $Ph-CH=CH_2 \xrightarrow[HBr]{(PhCOO)_2} Ph-CH_2-CH_2-Br$

Anti Markovnikov addition

• Reaction follow radical addition in presence of peroxide

• In absence of peroxide follow carbocation mechanism

• Benzene also formed



41. (a) Balmer series line $\Rightarrow \bar{\nu} = R_H Z^2\left[\frac{1}{2^2}-\frac{1}{n^2}\right]$

if $n=3 \Rightarrow \bar{\nu} = R(1)^2\left[\frac{1}{2^2}-\frac{1}{3^2}\right] = \frac{5R}{36}$

if $n=4 \Rightarrow \bar{\nu} = \frac{3R}{16}$

if $n=5 \Rightarrow \bar{\nu} = \frac{21R}{100}$

42. (a) At spherical node

$\psi_t = 0$

43. (c) Sc^{3+}, Ti^{4+} and Zn^{2+} are colourless Th^{4+} cannot act as a reducing agent.

44. (c) In a period moving from left to right atomic size decreases.

45. (c) $[Ni(CN)_4]^{2-} \to 3\,d^8 \to$ diamagnetic $\to dsp^2$

$[Ni(CO)_4] \to 3\,d^{10} \to$ diamagnetic $\to sp^3$

$[NiCl_4]^{2-} \to 3\,d^8 \to e^{2,2}t_2^{2,1,1} \to sp^3 \to$ paramagnetic.

46. (3) $MnO_4^- + I^- \to MnO_2 + I_2$

$I_2 + S_2O_3{}^{2-} \to S_4O_6{}^{2-} + I^-$

gram eq of $KMnO_4$ = gram eq of $Na_2\,S_2O_3$

$0.2 \times \frac{500}{1000} \times 3 = 0.1 \times V \times 1$

$V = 3$ L

47. (24) $3BH_4^- + 4ClO_3^- \longrightarrow 4Cl^- + 3H_2BO_3^- + 3H_2O$

n-factor = 8

moles = 3

$\therefore n = 3 \times 8 = 24$

48. (6) Here

$X = 3$ (Two cis + one trans isomers)

Y = 1 (trans isomer)

Z = 2 (Fac-mer isomer)

$X+Y+Z=3+1+2=6$

49. (4) $\underset{1\text{ g}}{C_xH_yBr_z} \rightarrow \underset{1.32\text{ g}}{CO_2}$

$$\%C=\frac{1.32\times 12}{44\times 1}\times 100=36\%$$

$\underset{0.53\text{ g}}{C_xH_yBr_z} \rightarrow \underset{0.75\text{ g}}{AgBr}$

$$\%Br=\frac{0.75\times 80}{188\times 0.53}\times 100=60.2\%$$

$\%H=100-(36+60.2)$

$\%H\simeq 4\%$

50. (3) $pH=\frac{1}{2}[pK_a-\log c]$

$$pH=\frac{1}{2}[4-\log 10^{-2}]$$

$pH=3$

51. (c) $g(2)=13$

$f(g(2))=f(13)$

Now $4g(f(x))=3x^2-32x+72$

$4[3f^2(x)+2f(x)-3]=3x^2-32x+72$

Let $f(x)=t$

$12t^2+8t-(3x^2-32x+84)=0$

$f(x)=\frac{-8\pm\sqrt{64+48(3x^2-32x+84)}}{24}$

$f(x)=\frac{-8\pm 4(3x-16)}{24}$

$\because f(0)=-3 \therefore$ we take + ve sign

$\therefore f(x)=\frac{-8+4(3x-16)}{24}$

$$\therefore f(13)=\frac{-8+4\times 23}{24}=\frac{84}{24}=\frac{7}{2}$$

52. (b) $T_k=(-1)^{k+1}\frac{k(k+1)}{\lfloor k}=(-1)^{k+1}\left(\frac{k(k-1)+2k}{\lfloor k}\right)$

$$\therefore \text{sum}=\sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{\lfloor k-2}+\sum_{k=1}^{\infty}\frac{2(-1)^{k+1}}{\lfloor k-1}$$

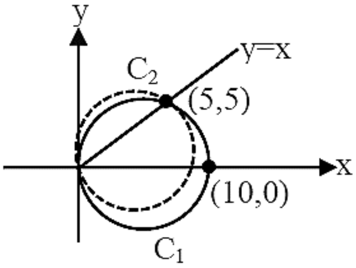
$$=\left(\frac{1}{\lfloor -1}-\frac{1}{\lfloor 0}+\frac{1}{\lfloor 1}-\frac{1}{\lfloor 2}+\frac{1}{\lfloor 3}\ldots\right)+\left(\frac{2}{\lfloor 0}-\frac{2}{\lfloor 1}+\frac{2}{\lfloor 2}-\frac{2}{\lfloor 3}\ldots\right)$$

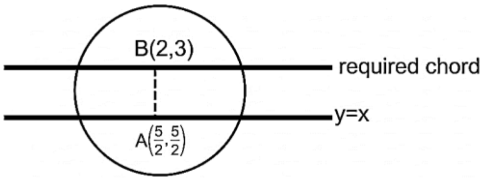
$$=\frac{1}{e}$$

53. (c) Equation of circle C_2 is

$x^2+y^2-5x-5y=0$

its center is $\left(\frac{5}{2},\frac{5}{2}\right)$





$m_{AB} = -1$

$\therefore$ Slope of required chord $= 1$

$\therefore$ equation of required chord is $x - y + 1 = 0$

$\therefore$ a $= -1$, b $= 2$

$\therefore$ a $-$ b $= -2$

54. **(a)** $\frac{\tan A - \tan B}{(1 + \tan A\ \tan B) \tan A} + \frac{1 + \cot^2\ A}{1 + \cot^2 C} = 1$

Put $\tan A = x, \tan B = y, \tan C = z$

$\therefore \frac{x-y}{(1+xy)x} + \frac{(x^2+1)\,z^2}{x^2(z^2+\ 1)} = 1$

$\therefore x(x-y)(z^2+1) + z^2(1+x^2)(1+xy)$

$= (1+xy)x^2(1+z^2)$

after solving we get

$z^2 = xy\ \ \because 1 + x^2 \neq 0$

$\therefore \tan^2 C = \tan A \cdot \tan B$

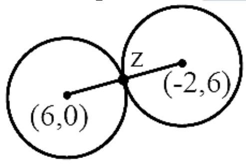
$\tan A, \tan C, \tan B$ are in G.P.

55. **(c)** Center of first circle $C_1(6,0), r_1 = 5$

Center of second circle $C_2(-2,6), r_2 = 5$

$C_1C_2 = r_1 + r_2$

common point Z is midpoint of C_1&C_2



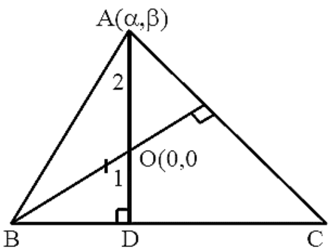
$z = 2 + 3i$

$z^2 = 4z - 13$

$z^3 = 3z - 52$

$z^3 + 3z^2 - 15z + 141 = 50$

56. **(d)**



$\because$ $m_{BC} \cdot\ m_{AD} = -1$

$\Rightarrow \left(-\frac{1}{2\sqrt{2}}\right)\left(\frac{\beta}{\alpha}\right) = -1$

$\Rightarrow \beta = 2\sqrt{2}\alpha$ .... (i)

$\because OD = \left|\frac{-4}{\sqrt{1+8}}\right| = \frac{4}{3} \Rightarrow AO = \frac{8}{3}$

So $AD = \frac{8}{3} + \frac{4}{3} = 4$

$\Rightarrow \frac{|\alpha + 2\sqrt{2}\beta - 4|}{3} = 4 \Rightarrow \alpha = \frac{16}{9}$ or $-\frac{8}{9}$

$\because$ $A(\alpha, \beta)$&$(0,0)$ lies on same

side of given line

$\therefore (\alpha, \beta) = \left(\frac{16}{9}, \frac{32\sqrt{2}}{9}\right)$; (Rejected) so $(\alpha,\beta)=\left(-\frac{8}{9},\frac{-16\sqrt{2}}{9}\right)$

$=[|\alpha+\sqrt{2}\beta|]=\left[\left|\frac{-8-32}{9}\right|\right]=4$

57. (c) $S=1,2,3,\ldots\ldots,9$

$x={}^9C_1\cdot{}^8C_7\times\frac{9!}{2}=\frac{9\times 8\times 9!}{2}$

$y={}^9C_2\cdot{}^7C_5\times\frac{9!}{2!\times 2!}=\frac{9\times 8}{2}\times\frac{7\times 6}{2}\times\frac{9!}{2!\times 2!}$

$\Rightarrow\frac{x}{y}=\frac{4}{21}$

$21x=4y$

58. (d) $x^3+ax^2+bx+c=(x^2+2)\left(x+\frac{c}{2}\right)$

$x^2:a=\frac{c}{2}$

$x:b=2$

$b=2,a=\frac{c}{2}c\in\{2,4,\ldots,20\}$

Number of polynomials in ' S ' will be 10.

59. (d) Probability $=\frac{{}^1C_0\cdot{}^9C_3}{\sum_{k=0}^{10}{}^kC_0\cdot{}^{10-k}C_3}$

$=\frac{{}^9C_3}{{}^{10}C_3+{}^9C_3+{}^8C_3+\cdots.{}^3C_3}$

$=\frac{{}^9C_3}{{}^{11}C_4}=\frac{14}{55}$

60. (a) $\frac{\beta-\alpha}{\alpha\beta}=\frac{1}{3},\alpha+\beta=\frac{\lambda+3}{\lambda},\alpha\beta=\frac{3}{\lambda}$

$\beta-\alpha=\frac{\alpha\beta}{3}=\frac{1}{\lambda}$

on squaring

$\alpha^2+\beta^2-2\alpha\beta=\frac{1}{\lambda^2}$ ...(i)

$\alpha^2+\beta^2+2\alpha\beta=\frac{(\lambda+3)^2}{\lambda^2}$ ...(ii)

(ii) - (i)

$4\alpha\beta=\frac{(\lambda+3)^2-1}{\lambda^2}$

$\frac{12}{\lambda}=\frac{\lambda^2+6\lambda+8}{\lambda^2}$

$\Rightarrow\lambda^2-6\lambda^2+8\lambda=0$

$\Rightarrow\lambda=0,2,4$

Sum of possible values of λ is $=6$

61. (b) $\int\frac{dx}{\sin^5\cos^2x}-5\int\frac{dx}{\sin^5x}$

$\int\frac{\sec^2xdx}{\sin^5x}-5\int\frac{dx}{\sin^3x}$

By IBP

$=\frac{\tan x}{\sin^5x}-\int-\frac{5}{\sin^6x}\cdot\cos x\cdot\tan xdx-5\int\frac{dx}{\sin^5x}$

$=\frac{\tan x}{\sin^5x}+c$

$f(x)=\frac{ta\,n\,x}{\sin^5x}$

$f\left(\frac{\pi}{6}\right)-f\left(\frac{\pi}{4}\right)=\frac{2^5}{\sqrt{3}}-(\sqrt{2})^5=4\sqrt{2}-\frac{32}{\sqrt{3}}$

$= \frac{32}{\sqrt{3}} - 4\sqrt{2}$

$= \frac{4}{\sqrt{3}}(8-\sqrt{6})$

62. (b) $f(x^2+1) = x^4+5x^2+2$

$\{\text{put } x^2+1 = t\}$

$\Rightarrow f(t) = (t-1)^2+5(t-1)+2$

$\Rightarrow f(t) = t^2+3t-2$

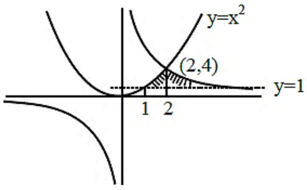
Now, $\int_0^3 f(t)dt = \int_0^3 (t^2+3t-2)dt$

$\left[\frac{t^3}{3}+\frac{3t^2}{2}-2t\right]_0^3$

$\left[\frac{27}{3}+\frac{27}{2}-6\right]$

$= \frac{33}{2}$

63. (c)



$$A = \int_1^2 (x^2-1)dx + \int_2^8 \left(\frac{8}{x}-1\right)dx$$

$$A = 8\log_e 4 - \frac{14}{3} = 16\log_e 2 - \frac{14}{3}$$

$$= \frac{2}{3}(24\log_e 2 - 7)$$

64. (c) $\Rightarrow \lim\limits_{x\to 0} \frac{\ln(\sec(ex))+\ln(\sec(e^2x))+\cdots..\ell n(\sec(e^{10}x))}{e^{2\cos x}\left(\frac{e^{2-2\cos x}-1}{2-2\cos x}\right)\times\frac{2-2\cos x}{x^2}\times x^2}$

$\Rightarrow \lim\limits_{x\to 10} \frac{\ell n(\sec(ex))+\ell n(\sec(e^2x))+\cdots..\ell n(\sec(e^{10}x))}{e^2x^2}$

Using L'H rule

$\Rightarrow \lim\limits_{x\to 10} \frac{e\tan ex+e^2\tan e^2x+\cdots..+e^{10}\tan e^{10}x}{2e^2x}$

$\Rightarrow \frac{1}{2e^2}[e^2+e^4+e^6+\cdots.+e^{20}]$

$\Rightarrow \frac{1}{2}\frac{e^2((e^2)^{10}-1)}{e^2(e^2-1)}$

$\Rightarrow \frac{1}{2}\frac{(e^{20}-1)}{(e^2-1)}$

65. (a) $\frac{2+3+5+10+11+13+15+21+a+b}{10} = 9$

$\frac{80+a+b}{10} = 9 \Rightarrow a+b = 10$ ...(i)

$\frac{\Sigma x_i^2}{10} - \left(\frac{\Sigma x_i}{10}\right)^2 = 34.2$

$\frac{2^2+3^2+5^2+10^2+11^2+13^2+15^2+21^2+a^2+b^2}{10} - (9)^2 = 34.2$

$1094+a^2+b^2-810 = 342$

$a^2+b^2 = 58$ ...(ii)

$a = 7, b = 3$

or $a = 3, b = 7$

Number $\rightarrow$ 2,3,5,7,10,11,13,15,21

Mean $= \frac{7+10}{2} = 8.5$

M.D $= \frac{6.5+5.5+5.5+3.5+1.5+1.5+2.5+4.5+6.5+12.5}{10}$

$= \frac{50}{5}$

66. (b) $B = (I + A)^{-1}, A + C = I$

$\Rightarrow B(I + A) = (I + A)B = I$

$\Rightarrow B + BA = B + AB$

$\Rightarrow B + B(I - C) = B + (I - C)B$

$\Rightarrow 2B - BC = 2B - CB$

$\Rightarrow BC = CB$

$\therefore CB\begin{bmatrix}x_1\\x_2\end{bmatrix} = \begin{bmatrix}1 & -5\\-1 & 2\end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix} = \begin{bmatrix}12\\-6\end{bmatrix}$

$\Rightarrow \begin{bmatrix}x_1\\x_2\end{bmatrix} = \begin{bmatrix}1 & -5\\-1 & 2\end{bmatrix}^{-1}\begin{bmatrix}12\\-6\end{bmatrix} = -\frac{1}{3}\begin{bmatrix}2 & 5\\1 & 1\end{bmatrix}\begin{bmatrix}32\\-6\end{bmatrix}$

$\Rightarrow \begin{bmatrix}x_1\\x_2\end{bmatrix} = \begin{bmatrix}2\\-2\end{bmatrix} \therefore x_1 + x_2 = 0$

67. (c) Let common difference of A.P.'s are d_1 & d_2

$\therefore d_1 = 13 + d_2$

$b_1 + 30d_2 = -277 \quad \ldots(i)$

$b_1 + 42d_2 = -385 \quad \ldots(ii)$

By (ii) - (i)

$12\,d_2 = -108$

$d_2 = -9$

$\therefore d_1 = 4$

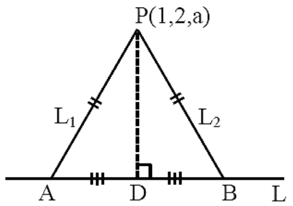
Now $a_{78} = 327$

$\Rightarrow a_1 + 77\,d_1 = 327$

$\Rightarrow a_1 + 308 = 327$

$a_1 = 19$

68. (a)



$L: \frac{x-1}{1} = \frac{y}{2} = \frac{z-1}{1}$

$L_1: \frac{x-1}{3} = \frac{y-2}{4} = \frac{z-a}{b} = \lambda$

$L_2: \frac{x-1}{1} = \frac{y-2}{4} = \frac{z-a}{c} = \mu$

Let $A(3\lambda + 1, 4\lambda + 2, b\lambda + a)$

It lies on L

$\therefore \frac{3\lambda}{1} = \frac{4\lambda+2}{2} = \frac{b\lambda + a - 1}{1}$

$\Rightarrow \lambda = 1$ and $a + b - 1 = 3$

$\Rightarrow A(4,6,4), a + b = 4 \quad \ldots(i)$

Let $B(\mu + 1, 4\mu + 2, c\mu + a)$

It also lies on L

$\frac{\mu}{1} = \frac{4\mu+2}{2} = \frac{c\mu + a - 1}{1}$

$\Rightarrow 2\mu = 4\mu + 2$

$\Rightarrow \mu = -1$

$a - c - 1 = -1$

$\Rightarrow a=c \quad ....(2)$ & $B(0,-2,0)$
also $PA=PB, P(1,2,a), A(4,6,4)$
$\Rightarrow 9+16+(a-4)^2=1+16+a^2$
$\Rightarrow 16+8=8a$
$a=3 \;\therefore c=3, b=1$
$\therefore a+b+c=7$

69. (d) $|\vec{a}-\vec{b}|^2+|\vec{b}-\vec{c}|^2+|\vec{c}-\vec{a}|^2=9$

$\Rightarrow \vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}-=-\frac{3}{2}$

$\Rightarrow \vec{a}+\vec{b}+\vec{c}=0 \Rightarrow \vec{b}+\vec{c}=-\vec{a}$
$|2\vec{a}+k(\vec{b}+\vec{c})|=3$
$|\vec{a}(2-k)|=3$
$K=5$ or -1
Positive value of k is 5

70. (b) $x\frac{dy}{dx}-\sin 2y=x^3(2-x^3)\cos^2 y$

$\sec^2 y\frac{dy}{dx}-2\tan y\cdot\frac{1}{x}=x^2(2-x^3)$

$\tan y=t \Rightarrow \sec^2 y\frac{dy}{dx}=\frac{dt}{dx}$

$\frac{dt}{dx}-\frac{2t}{x}=x^2(2-x^3)\ (LDE)$

$IF \cdot= e^{\int -\frac{2}{x}dx}=e^{-2\ell n\times}=\frac{1}{x^2}$

$\therefore \frac{t}{x^2}=\int \frac{1}{x^2}x^2(2-x^3)dx+C$

$\frac{\tan y}{x^2}=2x-\frac{x^4}{4}+C$

$y(2)=0 \Rightarrow 0=4-4+C \Rightarrow C=0$

$\tan y=2x^3-\frac{1}{4}x^6$

$x=1 \Rightarrow \tan y=2-\frac{1}{4}=\frac{7}{4} \Rightarrow (2)$

71. (90) Let first three terms of G.P. are $\frac{\mathrm{A}}{\mathrm{r}}, \mathrm{A}, \mathrm{Ar}$

$\frac{\mathrm{A}}{\mathrm{r}}\cdot\ \mathrm{A}\cdot\mathrm{Ar}=27$
$\mathrm{A}=3$
$3\left(\frac{1}{\mathrm{r}}+1+\mathrm{r}\right)=3+3\left(\mathrm{r}+\frac{1}{\mathrm{r}}\right)$
We know, $r+\frac{1}{\mathrm{r}}\geq 2$ or $r+\frac{1}{\mathrm{r}}\leq -2$
$S\in R-(-3,9)$
$a^2+b^2=9+81=90$

72. (8) $\frac{x^2}{8}-\frac{y^2}{8\cos^2\theta}=1, e_1=\sqrt{1+\frac{8\cos^2\theta}{8}}$

$\ell_1=\frac{2b^2}{a}=\frac{2\cdot(8\cos^2\theta)}{2\sqrt{2}}$

$\frac{x^6}{6}+\frac{y^2}{6\cos^2\theta}=1; e_2=\sqrt{1-\frac{6\cos^2\theta}{6}}=\sin\theta$

$\ell_2=\frac{2b^2}{a}=\frac{2\cdot 6\cos^2\theta}{\sqrt{6}}$

$e_1^2=e_2^2(1+\sec^2\theta)$

$1+\cos^2\theta=\sin^2\theta\left(1+\frac{1}{\cos^2\theta}\right)$

$1+\cos^2\theta=\sin^2\theta+\tan^2\theta$

Solving we get $\theta=\frac{\pi}{4}$

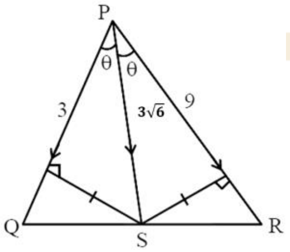
$\ell_1=2\sqrt{2}$

$e_1 = \sqrt{\frac{3}{2}}$
$\ell_2 = \sqrt{6}$
$e_2 = \dfrac{1}{\sqrt{2}}$
$\left(\dfrac{\ell_1 \ell_2}{e_1 e_2}\right) \tan^2 \theta = 8$ (By putting values)

73. (1) Let $\theta = \frac{1}{2}\sin^{-1}\frac{2}{3}$, then $\frac{1}{2}\cos^{-1}\frac{1}{3} = \left(\frac{\pi}{4} - \theta\right)$
$k = \tan\theta + \cot\theta = \dfrac{1}{\sin\theta\cos\theta} = \dfrac{2}{\sin 2\theta}$
$k = \dfrac{2}{\frac{2}{3}} = 3$
$\sin^{-1}(3x-1) = \sin^{-1}x - \cos^{-1}x$
$\sin^{-1}(3x-1) = \dfrac{\pi}{2} - 2\cos^{-1}x$
$3x - 1 = \sin\left(\dfrac{\pi}{2} - 2\cos^{-1}x\right)$
$3x - 1 = 2x^2 - 1 \Rightarrow x = 0, \dfrac{3}{2}$ (rejected)
No. of solution $= 1$

74. (210) Let $I_r = \int_0^r x|\sin \pi x|dx$ …(i)
Apply King Property
$= \int_0^r (r-x)|\sin\pi x|dx$ …(ii)
By (i) + (ii)
$2I_r = \int_0^r r|\sin\pi x|dx \Rightarrow I_r = \frac{r}{2}\int_0^r |\sin\pi x|dx$
$I_1 = \frac{1}{2}\int_0^1 |\sin\pi x|dx = \frac{1}{2\pi}\int_0^\pi |\sin t|dt = \frac{1}{2\pi}(2)$
$I_2 = \frac{2}{2}\int_0^2 |\sin\pi x|dx = \frac{2}{2\pi}\int_0^{2\pi} |\sin t|dt = \frac{2}{2\pi}(4)$
$S = \sqrt{\pi \cdot \frac{1}{2\pi} \cdot 2} + \sqrt{\pi \cdot \frac{2}{2\pi} \cdot 4} + \sqrt{\pi \cdot \frac{3}{2\pi} \cdot 6} + \cdots + \sqrt{\pi \cdot \frac{20}{2\pi}(2 \cdot 20)}$
$= 1 + 2 + 3 + \cdots + 20$
$= \dfrac{20 \times 21}{2} = 210$

75. (37) $\overrightarrow{PQ} = -2\hat{\imath} - \hat{\jmath} + 2\hat{k}$
$\overrightarrow{PR} = a\hat{\imath} + b\hat{\jmath} - 4\hat{k}$ $(a, b \in Z)$
$\overrightarrow{PS} = \hat{\imath} - 7\hat{\jmath} + 2\hat{k}$



$|\overrightarrow{PR}| = 9$
$a^2 + b^2 = 16 = 81$
$a^2 + b^2 = 65$ …(1)
$\cos\theta = \frac{\overrightarrow{PQ} \cdot \overrightarrow{PS}}{|\overrightarrow{PQ}||\overrightarrow{PS}|}$
$= \frac{-2+7+4}{3 \cdot 3\sqrt{6}} = \frac{9}{3 \cdot 3\sqrt{6}}$
$= \dfrac{1}{\sqrt{6}}$

$\frac{1}{\sqrt{6}} = \frac{\overrightarrow{PS} \cdot \overrightarrow{PR}}{|\overrightarrow{PS}||\overrightarrow{PR}|} = \frac{a-7b-8}{3\sqrt{6} \cdot 9}$

$a - 7b = 35 \quad \ldots(2)$

From (1) & (2)

$\Rightarrow a = 7, b = -4$

$\therefore 3a - 4b = 21 + 16 = 37$