1. **(d)** $R_\alpha = R_0 \alpha^{13}$

$R_\beta = R_0 \beta^{1/3}$

$\frac{R_\alpha}{R_\beta} = \left(\frac{\alpha}{\beta}\right)^{1/3} = \frac{1}{2}$

2. **(a)** $\hat{B} = \hat{C} \times \hat{E} = \hat{k} \times \hat{j} = -\hat{i}$

$\therefore \vec{B} = \frac{54}{3 \times 10^8} \sin(kz - \omega t)(-\hat{i})$

$= -1.8 \times 10^{-7} \sin(kz - \omega t)\hat{i}$

3. **(a)** $\frac{1}{v} - \frac{1}{u} = \frac{1}{f_{net}} = \frac{1}{f_1} + \frac{1}{f_2}$

$\frac{1}{v} + \frac{1}{30} = (1.5 - 1)\left(\frac{1}{15} - \frac{1}{\infty}\right) + (1.2 - 1)\left(\frac{1}{\infty} + \frac{1}{12}\right)$

$\frac{1}{v} + \frac{1}{30} = \frac{1}{30} + \frac{1}{60}$

$v = 60$

$m = \frac{v}{u} = \frac{60}{-30} = -2$

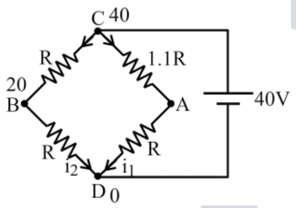
4. **(c)** $\lambda = \frac{k_B T}{\sqrt{2}\pi\sigma^2 P}$

$= \frac{1.38 \times 10^{-23} \times (273 + 41) \times 100}{\sqrt{2} \times 3.14 \times (5 \times 10^{-10})^2 \times 1.38 \times 10^5} = 2\sqrt{2} \times 10^{-8}$

5. **(d)** If either A or B is zero, in that case current flow and $v_e = 0$.

Hence the Gate will be AND Gate

6. **(c)**



$V_A = \frac{V}{2}$

$V_B = \frac{V}{2.1R} \times R = \frac{V}{2.1}$

$\therefore V_A - V_B = V\left[\frac{1}{2} - \frac{1}{2.1}\right]$

$V_A - V_B = \frac{0.1}{2 \times 2.1} \times 40$

$V_A - V_B = \frac{4}{4.2} = 0.95$

7. **(a)** Least count will be 0.01 mm .

8. **(b)** $x = 0 \Rightarrow t = 0, \frac{\sqrt{3}}{2}$

$v = 12t^2 - 3$

At turning point, $v = 0$

$t = \frac{1}{2} \Rightarrow x = \frac{4}{8} - \frac{3}{2} = -1$

$a = 24t$ (always positive)

9. **(d)** $V_{sound} = \sqrt{\frac{Y}{\rho}}$

$\frac{\Delta V}{V} \times 100 = \frac{1}{2}\left(\frac{\Delta Y}{Y} \times 100\right) - \frac{1}{2}\left(\frac{\Delta \rho}{\rho} \times 100\right)$

$= \frac{1}{2} \times 1 - \frac{1}{2} \times 0.5$

$\frac{\Delta V}{V} \times 100 = \frac{1}{4}$

$\Delta V = \frac{1}{4} \times \frac{V}{100}$

$\Delta V = 1 \text{ m/s}$

$V_{final} = 400 + 1 = 401 \text{ m/s}$

10. (c) For series combination

$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$

$\therefore C_{eq}$ is less than C_1 & C_2.

Note : In statement C, capacitor is assumed to be completely filled with dielectric then on decreasing thickness of dielectric capacitance will increase.

11. (d) $P = \frac{nhC}{\lambda}$

$6 \times 10^{-3} = \frac{n \times 6.63 \times 10^{-34} \times 3 \times 10^{8}}{663 \times 10^{-9}}$

$n = 2 \times 10^{16}$ photons

12. (d) $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$

$\vec{v} = \frac{d\vec{r}}{dt} = v_x\hat{\imath} + v_y\hat{\jmath} + v_z\hat{k}$

$\vec{p} = m\vec{v}$

$\vec{L} = m(\vec{r} \times \vec{v})$

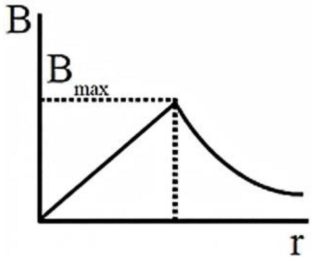
$= (x\hat{\imath} + y\hat{\jmath} + z\hat{k}) \times m(v_x\hat{\imath} + v_y\hat{\jmath} + v_z\hat{k})$

When sign of $\vec{r}$ changes, $\vec{L}$ remains same.

13. (c) Here from voltage, question refers to potential. We can measure potential difference between two points but not potential at any point.

Note : If the potential of reference point is known then we can measure potential as well.

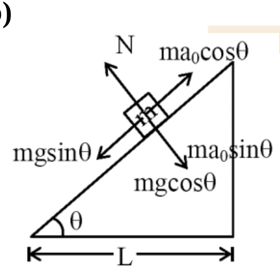
14. (a) Solid cylinder



B_{max} at surface

B_{min} at Axis

15. (b)



$mg\sin\theta - ma_0\cos\theta = ma$

$a = g\sin\theta - a_0\cos\theta$

Now using,

$S = ut + \frac{1}{2}a_{down}t^2$

$\frac{L}{\cos\theta} = \frac{1}{2}(g\sin\theta - a_0\cos\theta)t^2$

$t = \sqrt{\frac{2L}{g\sin\theta\cos\theta - a_0\cos^2\theta}}$

$t=\sqrt{\dfrac{4L}{g\sin 2\theta-a_0(1+\cos 2\theta)}}$

16. (c)

17. (a) $\mu=\frac{m_1\, m_2}{m_1+m_2}=\frac{1\times 0.2}{1.2}$

$\mu=\dfrac{1}{6}$

$\omega=\sqrt{\dfrac{k}{\mu}}=\sqrt{\dfrac{150}{1/6}}=30$

18. (d)$\delta_{\min}=2i-A\Rightarrow i=\delta_{\min}=A$

Also, $\mu=\frac{\sin\left(\frac{\delta_{\min}+A}{2}\right)}{\sin\left(\frac{A}{2}\right)}$

$\Rightarrow\mu=\dfrac{\sin A}{\sin\dfrac{A}{2}}=2\cos\left(\dfrac{A}{2}\right)$

$1<\mu<2$ ...(1)

$\delta_{\min}=2i-A$

$A=2i-A\Rightarrow i=A$

$i<90^{\circ}$ (grazing incidence)

$A<90^{\circ}$

$\mu=2\cos(A/2)$

& $A<90^{\circ}$

$\mu>\sqrt{2}$ ...(2)

from (i) & (2)

$\sqrt{2}<\mu<2$

19. (d) $\frac{\Delta K}{K}=\frac{2\Delta T}{T}+\frac{\Delta m}{m}$

$T=\dfrac{60}{50}=1.2\,\text{sec}$

$\Delta T=\dfrac{2}{50}$

$\therefore\dfrac{\Delta K}{K}=\dfrac{2\times 2}{50\times 1.2}+\dfrac{10\times 10^{-3}}{10}=0.0676$

$\therefore$ Error $=6.76\%$

20. (b) (A) $\eta=\frac{Fdr}{Adv}=\frac{[MLT^{-2}][L]}{[L^2][LT^{-1}]}=[ML^{-1}T^{-1}]$

(B) $S=\frac{F}{L}=\frac{[MLT^{-2}]}{[L]}=[MT^{-2}]$

(C) $P=\frac{F}{A}=\frac{[MLT^{-2}]}{[L^2]}=[ML^{-1}T^{-2}]$

(D) $E=S\times A=[MT^{-2}][L^2]=[ML^2T^{-2}]$

21. (384) $|f_A-f_B|=4$

$|f_A-380|=4$

So, $f_A=384$ Hz or 376 Hz

on loading with wax f_A decreases

So, $f_A=384$ Hz

22. (300) Work done = Area bounded by cycle

$=\dfrac{1}{2}\times 3\times 200=300$ J

23. (314) $\frac{1}{2}Li_{mas}^2=16\Rightarrow L=8$

$i^2R=32\Rightarrow R=8$

$x_L=\omega L\Rightarrow 2\times 3.14\times 50\times 8$

$\Rightarrow 800 \times 3.14$
$R = 8$
$\frac{x_L}{R} = 314$

24. (429) $y = n\frac{\lambda D}{d}$

$y_1 = y_2$

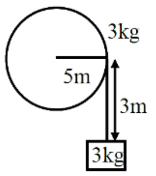
$n_1\lambda_1\frac{D}{d} = n_2\lambda_2\frac{D}{d}$

$\frac{n_1}{n_2} = \frac{\lambda_2}{\lambda_1} = \frac{550}{650} = \frac{11}{13}$

$y = 11 \times \frac{\lambda_1 D}{d} = \frac{11 \times 650 \times 10^{-9} \times 1.2}{2 \times 10^{-3}}$

$y = 429 \times 10^{-5}$

25. (30)



$\text{mg} \times 3 = \frac{1}{2} \cdot \frac{\text{mR}^2}{2}\omega^2 + \frac{1}{2}\text{mv}^2 \quad \ldots(i)$

$\& \text{v} = \omega \text{R} \quad \ldots(ii)$

From equation (i) & (ii)

$\text{g} \times 3 = \frac{3}{4} \cdot \text{v}^2$

K.E. of flywheel $= \frac{1}{2} \times \frac{\text{mR}^2}{2} \times \omega^2 = \frac{1}{4}\text{mv}^2$

$= \frac{1}{4} \times 3 \times 40 = 30$ Joule

26. (d) Presence of NO_2 group in Benzene ring deactivate ring towards electrophilic substitution reaction due to -M effective & activate ring towards nucleophilic substitution.

27. (a) Correct order of
(i) Boiling point: HF > HI > HBr > HCl
(ii) Melting point: HI > HF > HBr > HCl

28. (c) Least metallic = F, valence electrons = 7
Most metallic = P, valence electrons = 5

29. (d) $A \rightleftharpoons B$
0.5M 0.375 (At equilibrium)

$\text{K}_{\text{eq}} = \frac{[\text{B}]_{\text{eq}}}{[\text{A}]_{\text{eq}}} = \frac{0.375\text{M}}{0.5} = 0.75$

Now 0.1 mole of A is added so reaction will move in forward direction.

$A \rightleftharpoons B$
$0.6 - x$ $0.375 + x$

$K_{eq} = 0.75 = \frac{0.375 + x}{0.6 - x}$

$0.45 - 0.75x = 0.375 + x$

$1.75x = 0.075$

$X = \frac{0.075}{1.75} = \frac{3}{70} = 0.043$

Moles of $A = 0.043 = 0.557$

Moles of B = 0.418

30. **(b)** $\log_{10} K = -\frac{\Delta H^0}{2.303RT} + \frac{\Delta S^0}{2.303R}$

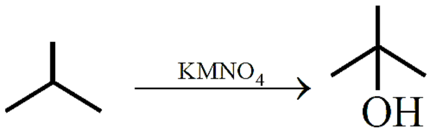
y -intercept $= \frac{\Delta S^\circ}{2.303R}$

Slope $= -\frac{\Delta H^\circ}{2.303R}$

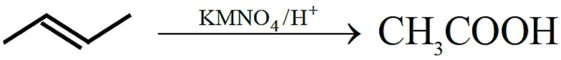
31. **(c)**Reaction given Alcohol

(A) $CH_4 + O_2 \xrightarrow[\Delta]{Mo_2O_3} HCHO$

(B) $2CH_3CH_3 + 3O_2 \xrightarrow[\Delta]{(CH_3COO)_2Mm} CH_3COOH$

(C) 

(D) $2CH_4 + O_2 \xrightarrow[\Delta 523\text{ K},100\text{am.}]{Cam} CH_3 - OH$

(E) 

32. **(d)** Manganate ion $\rightarrow MnO_4^{-2}$

Permanganate ion $\rightarrow MnO_4^-$

(A) Both are tetrahedral (d's Hybridization)

(B) MnO_4^- (+7 oxidation state)

MnO_4^{-2} (+6 oxidation state)

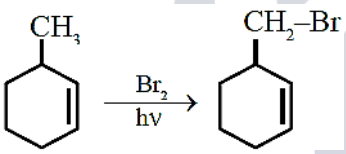
(C) $Mn^{2+} + S_2O_8^{2-} \rightarrow MnO_4^-$(Permanganate ion)

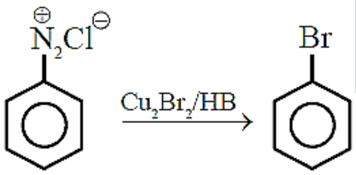
(D) $MnO_4^- \rightarrow$ Diamagnetic

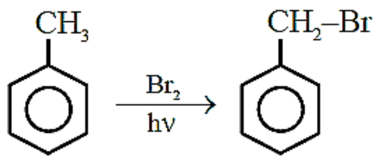
$MnO_4{}^{-2} \rightarrow$ Paramagnetic

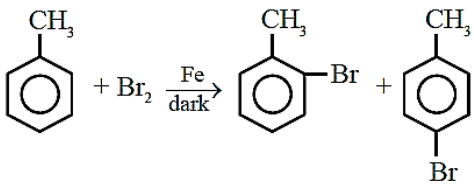
(E) It is oxidising agent

33. **(a)**

(a) 

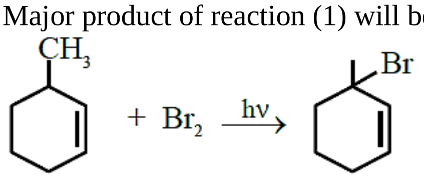
(b) 

(c) 



(d)

Major product of reaction (1) will be



As 3° radical more stable

34. (b) (a) Wavelength of $A = 400$ nm.

(b) Wavelength of $\mathrm{B}(\lambda) = \frac{\mathrm{C}}{\mathrm{v}} = \frac{3\times10^{8}}{10^{16}} = 3\times10^{-8} = 30\times10^{-9} = 30$ nm.

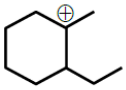
(c) Wavelength of $\mathrm{C}(\lambda) = \frac{1}{\bar{\nu}} = \frac{1}{10^{4}} = 10^{-4}$ cm $= 10^{-6}$ m $= 1000$ nm

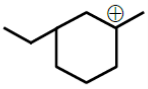
Here $\lambda_C > \lambda_A > \lambda_B$

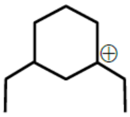
Energy (E) $\propto \frac{1}{\lambda}$

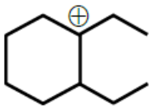
So $\mathrm{E_B} > \mathrm{E_A} > \mathrm{E_C}$

35. (a) $6\alpha\mathrm{H}5\alpha\mathrm{H}$

(A)  $6\alpha\mathrm{H}$

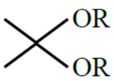
(B)  $7\alpha\mathrm{H}$

(C)  $6\alpha\mathrm{H}$

(D)  $5\alpha\mathrm{H}$

36. (a) Structure (a) given is of sucrose which is non reducing.

For non-reducing sugar compound should have acetal linkage not hemi acetal linkage.



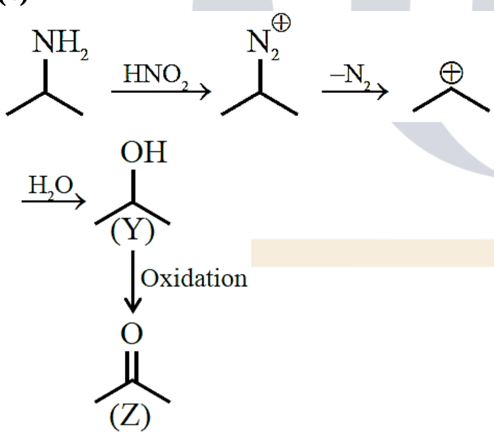
37. (b) $\mathrm{XeF_2}$&$\mathrm{I_3^-}$: 2 bond pair 3 lone pair ; Linear

$\mathrm{XeOF_4}$&$\mathrm{BrF_5}$: 5 bond pair 1 lone pair ; Square pyramidal

$\mathrm{XeO_2\ F_2}$&$\mathrm{SF_4}$: 4 bond pair 1 lone pair ; See saw

$\mathrm{XeO_3}$&$\mathrm{NH_3}$: 3 bond pair 1 lone pair ; Pyramidal

38.(c)



Give iodoform test

39. (a) $\Delta\mathrm{T_b} = \mathrm{ik_b}\cdot\mathrm{m}$

For dilute solution (M = m)

Molarity	**i × m**
(I) $\mathrm{M_{glucose}} = \frac{2.2}{180}\times\frac{1000}{125} = 0.098$	0.098×1

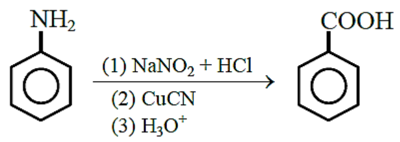
(II) $M_{CaCl_2} = \frac{1.9}{111} \times \frac{1000}{250} = 0.068$	0.068×3
(III) $M_{urea} = \frac{9}{60} \times \frac{1000}{500} = 0.3$	0.3×1
(IV) $M_{Al_2(SO_4)_2} = \frac{20.5}{342} \times \frac{1000}{750} = 0.08$	0.08×5

Order of $\Delta T_b = Al_2(SO_4)_3 > \text{Urea} > CaCl_2 > \text{Glucose}$

So order of $BP = Al_2(SO_4)_3 > \text{Urea} > CaCl_2 > \text{Glucose}$

So Answer will be $I < II < III < IV$

40. (d) Correct order of acidic strength of major product formed in the given reaction is

(A) 

(B) $CH_3CH_2 - CHO \xrightarrow[\Delta]{Ag_2(NH_3)_2OH} CH_3 - CH_2 - COOH$

(C) $CH_4 + O_2 \xrightarrow[Na_2Cr_2O_3/H^+]{MO_2O_3} HCOOH$

(D) $Ph - CH_2MgBr + CO_2 \xrightarrow[H_3O^\circ]{\text{Dryother}} Ph\_CH_2 - COOH$

41. (a) 2° Amine are insoluble with hinsberg reagent.

$Ph - NH - CH_3, Me - NH - Me,$

$Ph - NH - CH_2 - CH_2 - CH_3, Ph - NH - Ph$

42. (b) $Na_2\ B_4O_7 \xrightarrow{\Delta} (2NaBO_2) + B_2O_3$

(x) $CuSo_4 + B_2O_3 \xrightarrow[Non-Luminous\ Flame]{\Delta} \underset{(Blue\ bead)\ (Z)}{Cu(BO_{20})_2} + SO_3$

$2Cu(BO_2)_2 + 2NaBO_2 + C \xrightarrow[\text{Laminoss Flane}]{\Delta}$

$2 \underset{\substack{\text{Colorless} \\ (Q)}}{\overset{+1}{Cu}} BO_2 + Na_2\ B_4O_7 + CO$

43. (a) Density of HCl solution $(d) = 1.13\ g/ml$

$V = 300ml$

Wt. of HCl solution $= 339\ g$

Wt. of HCl $= 339 \times \frac{38.55}{100} = 130.68\ g$

(LR)

$CaCO_3 + 2HCl \rightarrow CaCl_2 + H_2O + CO_2$

$\frac{90}{100}\ \ \frac{130.68}{36.5}$

$= 0.90$ mole $= 3.58$ mole

Moles of HCl remained $= 1.78$ mole.

Mass of HCl remained $= 64.97$ g.

44. (d)$Mn^{2+} 3\ d^5 n = 5$

$Cu^{2+} 3\ d^9 t_{2g}^{2,2,2} e_g^{2,1} n = 1$

$Ni^{2+} 3\ d^3$ square planar $n = 0$

$Ni^{2+} 3\ d^8$ tetrahedral $e^{2,2} t_2^{2,1,1} n = 2$

45. (c) $\%S = \frac{32}{233} \times \frac{0.4813}{0.314} \times 100$

$= 21.052\%$

46. **(2)** $\lambda_d = \frac{h}{\sqrt{2mKE.}}$

Here KE is same i.e. 200 k eV

So $\lambda_d \propto \frac{1}{\sqrt{m}}$

$\frac{(\lambda_d)_{m_1}}{(\lambda_d)_{m_2}} = \sqrt{\frac{m_2}{m_1}} = \sqrt{4} = 2$

$(\lambda_d)_{m_1} = 2(\lambda_d)_{m_2}$

So $x = 2$.

47. **(5)** For $A \xrightarrow{K_1} B$

$ln(2) = \frac{E_{a_1}}{R}\left[\frac{1}{300} - \frac{1}{500}\right]$

$E_{a_1} = \frac{ln\, 2 \times R \times 1500}{2}$

$E_{a_2} = \frac{E_{a_1}}{2} = \frac{ln\, 2 \times R \times 1500}{4}$

$(K_1)_{21500\ K} = \frac{\ln 2}{2}$

$(K_2)_{21500\ K} = \ln 2$

Now for $C \xrightarrow{K_2} D$

$ln\left[\frac{(K_2)_{at\ 500\ K}}{(K_2)_{at\ 300\ K}}\right] = \left(\frac{ln\, 2 \times R \times 1500}{4}\right) \times \frac{1}{R} \times \left[\frac{1}{300} - \frac{1}{500}\right]$

$(K_2)_{at\ 300\ K} = \frac{\ln 2}{\sqrt{2}} = 0.49$

$(K_2)_{at\ 300\ K} = 4.9 \times 10^{-1}$

48. **(4)** Using equation : $\Lambda_m = \Lambda_m^0 - A\sqrt{c}$

$96.1 = \Lambda_m^0 - A\sqrt{0.04}$

$96.1 = \Lambda_m^0 - A \times 0.2 \quad \ldots(1)$

$95.7 = \Lambda_m^0 - A \times \sqrt{0.09}$

$95.7 = \Lambda_m^0 - A \times 0.3 \quad \ldots(2)$

From eq. (1) and eq. (2)

$A = 4$

49. **(78)** We have considered

$E^\circ_{[Sn(OH)_6]^{2-}/HSnO_2^-} = -0.9\ V$

$Pt|HSnO_2^-(aq), [Sn(OH)_6]^{2-}(aq), OH^-(aq)|Bi_2O_3(s)|Bi(s)|$

0.5M 0.05M

$E^\circ_{cell} = +0.9 - 0.44 = 0.46\ V$

Oxidation Half :

$HSnO_2^- + H_2O + 3OH^- \rightarrow [Sn(OH)_6]^{2-} + 2e^-$

Reduction Half :

$Bi_2O_3 + 3H_2O + 6e^- \rightarrow 2Bi + 6OH^-$

$3HSnO_2^-(aq) + Bi_2O_3(s) + 6H_2O + 3OH^-(aq)$

$\rightarrow 3[Sn(OH)^2]^2(aq) + 2Bi(s)$

$E_{cell} = E^\circ_{cell} - \frac{0.059}{6}\log\frac{(0.5)^3}{(0.05)^3 \times [OH^-]^3}$

$0.2353 = 0.46 - \frac{0.059}{6} \times 3log\left[\frac{10}{[OH^-]}\right]$

$\log\left[\frac{10}{OH^-}\right] = \frac{2 \times 0.2247}{0.059} = 7.6$

$1 + pOH = 7.6$

$pOH = 6.6$

$pH = 14 - 6.6 = 7.4$

$pH = pK_{a_1} + \log\frac{[HCO_3^-]}{[H_2CO_3]}$

$7.4 = 6.11 + \log\frac{5x}{20}$

$1.29 = \log\frac{x}{4}$

$\frac{x}{4} = 19.5$

$x = 78$

50. (6)

Sc^{+3}	18
Cr^{+2}	22
Mn^{+3}	22
Co^{+3}	24
Fe^{+3}	23

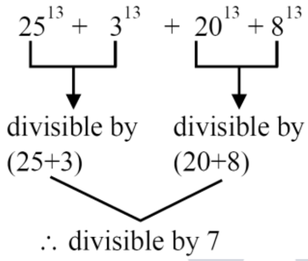
Cr^{2+} and Mn^{3+} are isoelectronic

$n = 2$

Complex is : $[Co(en)_2NH_3Cl]Cl_2$

$\Rightarrow Co^{3+}\ 3d^6\ t_{2g}^{2,2,2}\ e_g^{0,0}$

51. (b) Statement I :



Statement II: $R = (7 + 4\sqrt{3})^{25} = I + f$

$R' = (7 - 4\sqrt{3})^{25} = f'$

$\therefore R + R' = 2\left[{}^{25}C_0 7^{25} + {}^{25}C_2 7^{23}(4\sqrt{3})^2 + \cdots\right]$

$I + f + f = $ even integer

$I = $ odd integer

$\because 0 < f + f' < 2 \Rightarrow f + f' = 1$

$\Rightarrow$ Both the statements are correct

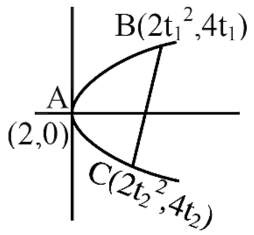
52. (b) $S = (1+x)^{1000} + x(1+x)^{999} + x^2(1+x)^{998} + \cdots. + x^{1000}$

$= (1+x)^{1000}\frac{\left(1-\left(\frac{x}{1+x}\right)^{1001}\right)}{1-\frac{x}{1+x}}$

$= (1+x)^{1001} - x^{1001}$

Required sum $= {}^{1001}C_{499} + {}^{1001}C_{500} = {}^{1002}C_{500}$

53. (b)



Coordinates of centroid of triangle ABC are

$\frac{2}{3}(t_1^2 + t_2^2 + 1) = \frac{7}{3} \Rightarrow t_1^2 + t_2^2 = \frac{5}{2}$

$\frac{4}{3}(t_1 + t_2) = \frac{4}{3} \Rightarrow t_1 + t_2 = 1$

$(t_1 + t_2)^2 = t_1^2 + t_2^2 + 2t_1t_2 \Rightarrow t_1t_2 = \frac{-3}{4}$

$(t_1 - t_2)^2 = (t_1 + t_2)^2 - 4t_1t_2 = 4$
$(BC)^2 = 4(t_1^2 - t_2^2)^2 + 16(t_1 - t_2)^2$
$\Rightarrow (BC)^2 = 8q0$

54. (c) $E(X) = \sum X_i P(X_i) = \frac{526k}{15\times7} = \frac{263}{15} \Rightarrow k = \frac{7}{2}$

X	14	15	16	17	18	19	20	21
P(X)	$\frac{2}{15}$	$\frac{1}{15}$	$\frac{2}{15}$	$\frac{1}{5}$	$\frac{1}{15}$	$\frac{2}{15}$	$\frac{1}{5}$	$\frac{1}{15}$

$P(X < 20) = \sum_{X=14}^{19} P(X) = \frac{11}{15}$

55. (a) $f(x) = \begin{cases} \cos \pi x & x \to 1^- \\ \frac{-\sin(x-1)}{(x-1)} & x \to 1^+ \end{cases}$

$\text{RHL} = \lim_{x\to1} \frac{-\sin(x-1)}{(x-1)} = -1$

$\text{LHL} = \lim_{x\to1} \cos \pi x = -1, f(1) = -1$

f(x) is continuous at $x = 1$

$f(x) = \begin{cases} \frac{-\sin(x-1)}{-(x-1)} & x \to -1^- \\ \cos \pi x & x \to -1^+ \end{cases}$

$\text{RHL} = \lim_{x\to-1} \cos \pi x = -1$

$\text{LHL} = \lim_{x\to-1} \frac{-\sin(x-1)}{(x-1)} = \frac{\sin 2}{-2}$

f(x) is discontinuous at $x = -1$

56. (b) Let $\sin^{-1}\frac{2}{\sqrt{13}} = \theta$ & $\cos^{-1}\frac{3}{\sqrt{10}} = \phi$

$\sin\theta = \frac{2}{\sqrt{13}}$ & $\cos\phi = \frac{3}{\sqrt{10}}$

$\tan(2\theta - 2\phi) = \frac{\tan 2\theta - \tan 2\phi}{1 + \tan 2\theta \tan 2\phi}$

$\left(\because \tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}\right)$

$= \frac{\frac{12}{5} - \frac{3}{4}}{1 + \frac{12}{5}\cdot\frac{3}{4}}$

$= \frac{33}{56}$

57. (a) $a = 4 - d, \alpha = 4 + d, b = 4 + 2d$
$\Rightarrow (4 + d)x^2 - (4 - d)x + 2(4 + d - 8 - 4d) = 0$
$\Rightarrow (4 + d)x^2 - (4 - d)x + 2(-4 - 3d) = 0$

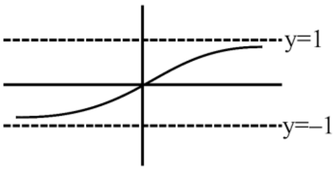
Also $\frac{\frac{1}{a}+\frac{1}{b}}{2} = \frac{5}{16}$

$\Rightarrow \frac{\frac{1}{4-d}+\frac{1}{4+2d}}{2} = \frac{5}{16}$

$\Rightarrow d = 2$
Equation becomes
$6x^2 - 2x - 20 = 0$
$3x^2 - x - 10 = 0$
$x = 2, \frac{-5}{3}$

58. (b) Statement 1: $f(x) = \frac{x}{1+|x|}$

$f(x) = \begin{cases} \frac{x}{1+x} & x \geq 0 \\ \frac{x}{1-x} & x < 0 \end{cases}$



f(x) is one-one

Statement 2: $f(x) = \frac{x^2+4x-30}{x^2-8x+18}, f(0) = \frac{-30}{18} = \frac{-5}{3}$

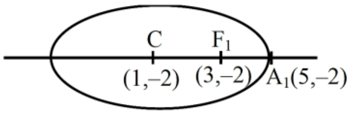
$\frac{-5}{3} = \frac{x^2+4x-30}{x^2-8x+18}$

On solving $x = 0, -1$

$\Rightarrow f(0) = f(-1) = \frac{-5}{3}$

$\therefore$ f(x) is many-one

59. (b)



$CA_1 = a = 4$

$CF_1 = ae = 2$

$e = \frac{1}{2}$

$LR = 2e\left(\frac{a}{e} - ae\right)$

$= 2 \times \frac{1}{2} \times \left(\frac{4}{1/2} - 2\right)$

$= 6$

60. (a) Equation of hyperbola: $\frac{y^2}{\lambda^2} - \frac{x^2}{16} = 1$

Equation of ellipse : $\frac{x^2}{144} + \frac{y^2}{169} = 1$

$e' = \sqrt{1 - \frac{144}{169}} = \frac{5}{13}$

focus $\Rightarrow (0,5)$

$\Rightarrow \lambda\sqrt{1 + \frac{16}{\lambda^2}} = 5$

$\Rightarrow \lambda^2 + 16 = 25$

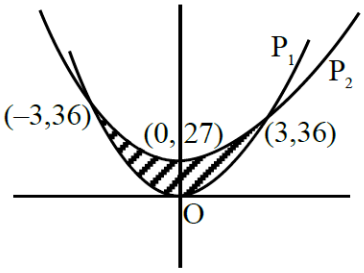
$\lambda = 3$

Eccentricity of hyperbola $= \sqrt{1 + \frac{16}{\lambda^2}} = \frac{5}{3}$

Length of latus rectum of hyperbola $= \frac{2(16)}{3} = \frac{32}{3}$

$24(e + \ell) = 24\left[\frac{5}{3} + \frac{32}{3}\right] = 8 \times 37 = 296$

61. (c)



Area bounded between P_1 & P_2 is

$\int_{-3}^{3} \left((x^2 + 27) - (4x^2)\right) dx$

(P.O.I. of P_1&P_2 is $x = \pm 3$)

$= 2\int_0^3 (27-3x^2)dx = 2[27x - x^3]_0^3$

$\therefore$ Area bounded between P_1 & L is 18 sq. units (Area between $x^2 = 4ay$ & line $x = my$) is $\frac{8a^2}{3m^3}$ $\therefore$ Area between $x^2 = \frac{y}{4}$ &$x = \frac{y}{\alpha}$ is

$\frac{8.\left(\frac{1}{16}\right)^2}{3.\left(\frac{1}{\alpha}\right)^3} = 18$

$\Rightarrow \frac{\frac{8}{16.16}}{\frac{3}{\alpha^3}} = 18 \Rightarrow \alpha^3 = 2^6.3^3$

$\Rightarrow \alpha = 12$

62. (a) Let point of intersection $R(h, k)$

$m_{BR} = m_{BP} \Rightarrow \frac{k}{h+2} = \frac{2\sin\alpha}{2\cos\alpha+2} \Rightarrow \frac{k}{h+2} = \tan\frac{\alpha}{2}$

$m_{AR} = m_{AQ} \Rightarrow \frac{k}{h-2} = \frac{2\sin\beta}{2\cos\beta-2} = \frac{\sin\beta}{\cos\beta-1} = -\cot\frac{\beta}{2}$

$\frac{\alpha}{2} - \frac{\beta}{2} = \frac{\pi}{4}$

$\tan\left(\frac{\alpha}{2} - \frac{\beta}{2}\right) = \tan\frac{\pi}{4} = 1$

$\frac{\tan\frac{\alpha}{2} - \tan\frac{\beta}{2}}{1+\tan\frac{\alpha}{2}\tan\frac{\beta}{2}} = 1$

$\frac{\frac{k}{h+2} + \frac{h-2}{k}}{1+\left(\frac{k}{h+2}\right)\left(\frac{2-h}{k}\right)} = 1 \Rightarrow \frac{k^2+h^2-4}{\frac{k(h+2)}{4}} = 1$

$\frac{h^2 + k^2 - 4}{4k} = 1$

$x^2 + y^2 - 4y - 4 = 0$

63. (b) $I = \int_{\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{12(3+[x])dx}{3+[\sin x]+[\cos x]}$

$I = \int_{-\frac{\pi}{2}}^{-1} \frac{12(1)dx}{2} + \int_{-1}^{0} \frac{12(2)dx}{2} + \int_0^1 12(3)dx + \int_1^{\frac{\pi}{2}} \frac{12(4)dx}{3}$

$I = 6\left(\frac{\pi}{2} - 1\right) + 12(0+1) + 12(1-0) + 16\left(\frac{\pi}{2} - 1\right)$

$I = 3\pi - 6 + 12 + 12 + 8\pi - 16$

$I = 11\pi + 2$

64. (c) $xdy - ydx = x^2\cot x dx$

$x^2 d\left(\frac{y}{x}\right) = x^2 co\,t\,x\,dx$

$d\left(\frac{y}{x}\right) = co\,t\,x\,dx$

$\int d\left(\frac{y}{x}\right) = \int \cot x dx$

$\frac{y}{x} = \log_e \sin x + C$

given $y\left(\frac{\pi}{2}\right) = \frac{\pi}{2}$

$\Rightarrow c = 1$

$y = x(\log_e \sin x + 1)$

$y\left(\frac{\pi}{6}\right) = \frac{\pi}{6}[-\log_e 2 + 1]$

$y\left(\frac{\pi}{4}\right) = \frac{\pi}{4}\left[-\frac{1}{2}\log_e 2 + 1\right]$

$6y\left(\frac{\pi}{6}\right) - 8y\left(\frac{\pi}{4}\right)$

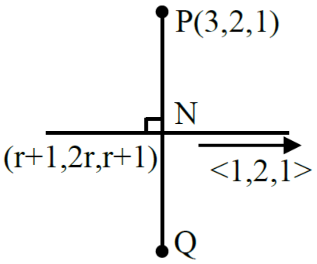
$= \pi\left[(-\log_e 2 + 1) + 2\left(\frac{1}{2}\log_e 2 - 1\right)\right]$

$= \pi[1-2] = -\pi$

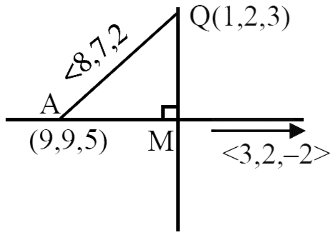
65. (b)$x \in (0, \pi/2) \Rightarrow y = 1+1+1+1 = 4$

$x \in (\pi/2, \pi) \Rightarrow y = 1 - 1 - 1 - 1 = -2$
$x \in (\pi, 3\pi/2) \Rightarrow y = -1 - 1 + 1 + 1 = 0$
$x \in (3\pi/2, 2\pi) \Rightarrow y = -1 + 1 - 1 - 1 = -2$
$\therefore$ Range of y is $\{-2, 0, 4\}$
Required sum $= -2 + 0 + 4 = 2$

66. (c)

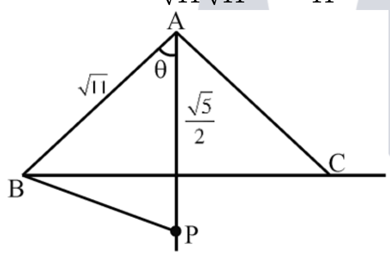


drs of PN $= \langle r-2, 2r-2, r \rangle$
$1.(r-2) + 2(2r-2) + 1.(r) = 0$
$6r = 6 \Rightarrow r = 1$
$\therefore$ N $\equiv (2,2,2)$
$\Rightarrow Q \equiv (1,2,3)$



$AQ = \sqrt{64+49+4} = \sqrt{117}$
$\text{AM} = \left|\frac{24+14-4}{\sqrt{9+4+4}}\right| = \frac{34}{\sqrt{17}} = 2\sqrt{17}$
$\therefore \text{QM} = \sqrt{117-68} = \sqrt{49} = 7$

67. (c) $\cos 2\theta = \frac{3-1-3}{\sqrt{11}\cdot\sqrt{11}} = -\frac{1}{11}$



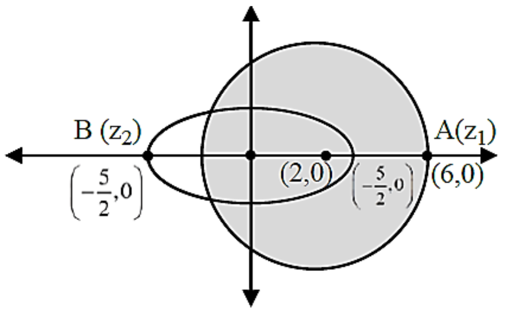
$1 - 2\sin^2\theta = -\frac{1}{11} \Rightarrow 2\sin^2\theta = \frac{12}{11} \Rightarrow \sin\theta = \sqrt{\frac{6}{11}}$

$\text{Area}(\triangle \text{APB}) = \frac{1}{2} \times \sqrt{11} \cdot \frac{\sqrt{5}}{2} \cdot \sqrt{\frac{6}{11}} = \frac{\sqrt{30}}{4}$

68. (c) $|z-2| \le 4 \Rightarrow (x-2) + y^2 \le 16$

$|z-2| + |z+2| = 5 \Rightarrow \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$\Rightarrow \frac{4x^2}{25} + \frac{4y^2}{9} = 1$



Maximum value of $|z_1 - z_2| = 6 + \frac{5}{2} = \frac{17}{2}$

69. (c) $f(x) = \int \frac{dx}{x^{2/3}+2x^{1/2}}$

Put $x = t^6 \Rightarrow dx = 6t^5 dt$

$= \int \frac{6t^3 dt}{t^4 + 2t^3} = 6\int \frac{(t^2-4)+4}{t+2} dt$

$= 6\left[\int (t-2)dt + 4\int \frac{1}{t+2} dt\right]$

$= 6\left[\frac{t^2}{2} - 2t + 4\ell\text{n}(t+2)\right] + C$

$= 3x^{1/3} - 12x^{1/5} + 24\ell\text{n}(x^{16}+2) + C$

$f(0) = 24\ell\text{n}\,2 + C = -26 + 24\ell\text{n}\,2$ (given)

$\Rightarrow C = -26$

Now

$f(1) = -35 + 24\ell n3 = a + b\ell n3$ (as given in ques.)

$\Rightarrow \mathrm{a} = -35 \& \mathrm{~b} = 24$

$\Rightarrow \mathrm{a} + \mathrm{b} = -11$

70. (b) $\mathrm{S} = \frac{6}{3^{26}} + \frac{10}{3^{25}}\left[\frac{(6)^{25}-1}{6-1}\right]$

$\mathrm{S} = \frac{6}{3^{26}} + \frac{10}{3^{25}}\left[\frac{6^{25}-1}{5}\right]$

$\mathrm{S} = \frac{2}{3^{25}} + 2\left[2^{25} - \frac{1}{3^{25}}\right]$

$\mathrm{S} = 2^{26}$

71. (976) $S = \sum \frac{\mathrm{r}}{(\mathrm{r}^2+\mathrm{r}+1)(\mathrm{r}^2-\mathrm{r}+1)}$

$= \frac{1}{2}\sum_{\mathrm{r}=1}^{25}\left(\frac{1}{\mathrm{r}^2-\mathrm{r}+1} - \frac{1}{\mathrm{r}^2+\mathrm{r}+1}\right)$

$= \frac{1}{2}\left[\left(\frac{1}{1} - \frac{1}{3}\right) + \left(\frac{1}{3} - \frac{1}{7}\right) + \cdots + \left(\frac{1}{601} - \frac{1}{651}\right)\right]$

$= \frac{1}{2}\left[\frac{1}{1} - \frac{1}{651}\right]$

$= \frac{1}{2}\left[\frac{650}{651}\right] = \frac{325}{651}$

$\frac{\mathrm{p}}{\mathrm{q}} = \frac{325}{651} \Rightarrow \mathrm{p} + \mathrm{q} = 976$

72. (210) ${}^7\mathrm{C}_3 \times \underline{3}$

$= 210$

73. (9) $f(x) = 1 - 2x + e^x \int_0^x e^{-t} f(t) dt$

$e^{-x} f(x) = (1-2x)e^{-x} + \int_0^x e^{-t} f(t) dt$

$e^{-x} f(x) - e^{-x} f(x) = -2e^{-x} + (1-2x)e^{-x}(-1) + e^{-x} f(x)$

$f'(x) - 2f(x) = 2x - 3$

$\frac{dy}{dx} - 2y = 2x - 3$

$\Rightarrow y \cdot e^{-2x} = \int e^{-2x}(2x-3)dx$

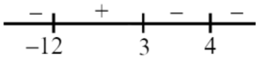
On solving we get

$f(x) = 1 - x$

$g(x) = \int_0^x (3-t)^{15}(t-4)^6(t+12)^{17}dt$

$g'(x) = (3-x)^{15}(x-4)^6(x+12)^{17}$

$= -(x-3)^{15}(x-4)^6(x+12)^{17}$



Local maxima $\Rightarrow q = 3$

Local minima $\Rightarrow p = -12 = |p+q| = 9$

74. (170) $\frac{x-43}{3} = \frac{y-\alpha}{-1} = \frac{z-\beta}{0} \Rightarrow P_1(43+3\lambda, \alpha-\lambda, \beta)$

$\frac{x-4}{2} = \frac{y}{0} = \frac{z+1}{3} \Rightarrow P_1(2\mu+4, 0, 3\mu-1)$

$\therefore \mu = \frac{3\lambda+39}{2}, \alpha = \lambda, \beta = \frac{9\lambda-115}{2}$

$P(43, \alpha, \beta), P_1(43+3\alpha, 0, \beta)$

$(PP_1)^2 = 1690 = 10\alpha^2, \therefore \alpha = 13, \beta = 1$

$\therefore \alpha^2 + \beta^2 = 170$

75. (0) $\mathrm{A} = \mathrm{I} + \begin{bmatrix} 2 & -4 \\ 1 & -2 \end{bmatrix}$, let $\mathrm{M} = \begin{bmatrix} 2 & -4 \\ 1 & -2 \end{bmatrix}$

$\mathrm{M}^2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = \mathrm{M}^3 = \mathrm{M}^4 = \cdots = \mathrm{M}^{100}$

$\mathrm{A}^{100} = (\mathrm{I}+\mathrm{M})^{100} = \sum_{r=0}^{100} \binom{100}{r} \mathrm{M}^{\mathrm{I}} \cdot \mathrm{I}$

$\mathrm{A}^{100} = \mathrm{I} + 100\mathrm{M} = \mathrm{I} + 100\,\mathrm{B}$

$\therefore \mathrm{M} = \mathrm{B} \Rightarrow \mathrm{M}^{100} = \mathrm{B}^{100} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$