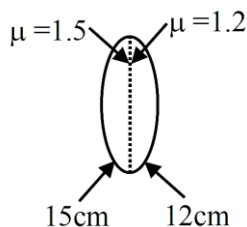
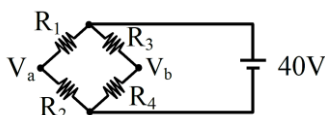


PHYSICS SECTION-A

1. A nucleus has mass number α and radius R_α . Another nucleus has mass number β and radius R_β . If $\beta = 8\alpha$ then R_α/R_β is :
 (a) 2 (b) 8
 (c) 1 (d) 0.5
2. A plane electromagnetic wave is moving in free space with velocity $c = 3 \times 10^8$ m/s and its electric field is given as $\vec{E} = 54\sin(kz - \omega t)\hat{j}$ V/m, where $\hat{j}$ is the unit vector along y -axis. The magnetic field vector $\vec{B}$ of the wave is :
 (a) $-1.8 \times 10^{-7}\sin(kz - \omega t)\hat{i}$ T
 (b) $1.4 \times 10^{-7}\sin(kz - \omega t)\hat{k}$ T
 (c) $1.4 \times 10^{-7}\sin(kz - \omega t)\hat{i}$ T
 (d) $+1.8 \times 10^{-7}\sin(kz - \omega t)\hat{i}$ T
3. A biconvex lens is formed by using two thin planoconvex lenses, as shown in the figure. The refractive index and radius of curved surfaces are also mentioned in figure. When an object is placed on the left side of lens at a distance of 30 cm from the biconvex lens, the magnification of the image will be :



- (a) -2 (b) +2
 (c) +2.5 (d) -2.5
4. The mean free path of a molecule of diameter 5×10^{-10} m at the temperature 41°C and pressure 1.38×10^5 Pa, is given as ____ m. (Given $k_B = 1.38 \times 10^{-23}$ J/K).
 (a) $2\sqrt{2} \times 10^{-10}$ (b) $10\sqrt{2} \times 10^{-8}$
 (c) $2\sqrt{2} \times 10^{-8}$ (d) 2×10^{-8}
5. Two p-n junction diodes D_1 and D_2 are connected as shown in figure. A and B are input signals and C is the output. The given circuit will function as a ____ .
 (a) OR Gate (b) NOR Gate
 (c) NAND Gate (d) AND Gate
6. A Wheatstone bridge is initially at room temperature and all arms of the bridge have same value of resistances ($R_1 = R_2 = R_3 = R_4$). When R_3 resistance is heated to some temperature, its resistance value has gone up by 10%. The potential difference ($V_b - V_a$) (after R_3 is heated) is ____ V.



- (a) 1.05 (b) 0
 (c) 0.95 (d) 2
7. In an experiment, a set of readings are obtained – 1.24 mm, 1.25 mm, 1.23 mm, 1.21 mm. The expected least count of the instrument used in recording these readings is ____ mm.
 (a) 0.01 (b) 0.001
 (c) 0.1 (d) 0.05
8. A particle starts moving from time $t = 0$ and its coordinate is given as $x(t) = 4t^3 - 3t$.
 (A) The particle returns to its original position (origin) 0.866 units later
 (B) The particle is 1 unit away from origin at its turning point.
 (C) Acceleration of the particle is non-negative.

- (D) The particle is 0.5 units away from origin at its turning point.
 (E) Particle never turns back as acceleration is non-negative.

Choose the correct answer from the options given below:

- (a) (A), (C), (D) only (b) (A), (B), (C) only
 (c) (C), (E) only (d) (A), (C) only

9. The speed of a longitudinal wave in a metallic bar is 400 m/s. If the density and Young's modulus of the bar material are increased by 0.5% and 1% respectively then the speed of the wave is changed approximately to ____ m/s.

- (a) 399 (b) 398
 (c) 402 (d) 401

10. Identify the correct statements:

- (A) Effective capacitance of a series combination of capacitors is always smaller than the smallest capacitance of the capacitor in the combination.
 (B) When a dielectric medium is placed between the charged plates of a capacitor, displacement of charges cannot occur due to insulation property of dielectric.
 (C) Increasing of area of capacitor plate or decreasing of thickness of dielectric is an alternate method to increase the capacitance.
 (D) For a point charge, concentric spherical shells centered at the location of the charge are equipotential surfaces.

Choose the correct answer from the options given below.

- (a) (A), (B) and (C) only
 (b) (C) and (D) only
 (c) (A), (C) and (D) only
 (d) (B) and (D) only

11. Number of photons of equal energy emitted per second by a 6 mW laser source operating at 663 nm is ____ .

(Given : $h = 6.63 \times 10^{-34}$ J.s and $c = 3 \times 10^8$ m/s)

- (a) 5×10^{16} (b) 5×10^{15}
 (c) 10×10^{15} (d) 2×10^{16}

12. When the position vector $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ changes sign as $-\vec{r}$, which one of the following vector will not flip under sign change ?

- (a) Linear momentum (b) Velocity
 (c) Acceleration (d) Angular momentum

13. Which one of the following is not a measurable quantity?

- (a) Voltage difference (b) Resistance
 (c) Voltage (d) Displacement current

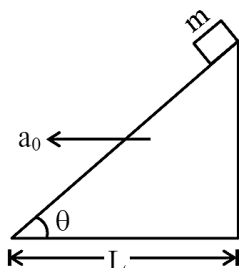
14. A long cylindrical conductor with large cross section carries an electric current distributed uniformly over its cross-section. Magnetic field due to this current is :

- (A) maximum at either ends of the conductor and minimum at the midpoint
 (B) maximum at the axis of the conductor
 (C) minimum at the surface of the conductor
 (D) minimum at the axis of the conductor
 (E) same at all points in the cross-section of the conductor

Choose the correct answer from the options given below :

- (a) (D) Only (b) (A), (D) Only
 (c) (B), (C) Only (d) (E) Only

15. A small block of mass m slides down from the top of a frictionless inclined surface, while the inclined plane is moving towards left with constant acceleration a_0 . The angle between the inclined plane and ground is θ and its base length is L . Assuming that initially the small block is at the top of the inclined plane, the time it takes to reach the lowest point of the inclined plane is ____ .



- (a) $\sqrt{\frac{2L}{g \sin 2\theta - a_0(1 + \cos 2\theta)}}$ (b) $\sqrt{\frac{4L}{g \sin 2\theta - a_0(1 + \cos 2\theta)}}$
(c) $\sqrt{\frac{4L}{g \cos^2 \theta - a_0 \sin \theta \cos \theta}}$ (d) $\sqrt{\frac{2L}{g \sin \theta - a_0 \cos \theta}}$

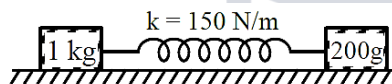
16. Identify the correct statements :

- (A) Electrostatic field lines form closed loops.
(B) The electric field lines point radially outward when charge is greater than zero.
(C) The Gauss-Law is valid only for inverse-square force.
(D) The work done in moving a charged particle in a static electric field around a closed path is zero.
(E) The motion of a particle under Coulomb's force must take place in a plane.

Choose the correct answer from the options given below:

- (a) (A), (B), (D), (E) Only (b) (A), (B), (C), (D) Only
(c) (B), (C), (D), (E) Only (d) (A), (C), (E) Only

17. As shown in the figure, a spring is kept in a stretched position with some extension by holding the masses 1 kg and 0.2 kg with a separation more than spring natural length and are released. Assuming the horizontal surface to be frictionless, the angular frequency (in SI unit) of the system is :



- (a) 30 (b) 27
(c) 20 (d) 5

18. For a transparent prism, if the angle of minimum deviation is equal to its refracting angle, the refractive index n of the prism satisfies.

- (a) $\sqrt{2} < n < 2\sqrt{2}$ (b) $1 < n < 2$
(c) $n \geq 2$ (d) $\sqrt{2} < n < 2$

19. The time period of a simple harmonic oscillator is $T = 2\pi \sqrt{\frac{k}{m}}$. The measured value of mass (m) of the object is 10 g with an accuracy of 10 mg, and time for 50 oscillations of the spring is found to be 60 s using a watch of 2 s resolution. Percentage error in determination of spring constant (k) is ____ %.

- (a) 3.43 (b) 3.35
(c) 7.60 (d) 6.76

20. Match List-I with List-II.

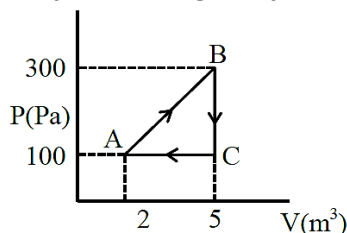
List-I		List-II	
(A)	Coefficient of viscosity	(I)	$[ML^{-1} T^{-2}]$
(B)	Surface tension	(II)	$[ML^2 T^{-2}]$
(C)	Pressure	(III)	$[ML^0 T^{-2}]$
(D)	Surface energy	(IV)	$[ML^{-1} T^{-1}]$

Choose the correct answer from the options given below:

- (a) (A)-(I), (B)-(II), (C)-(IV), (D)-(III)
 (b) (A)-(IV), (B)-(III), (C)-(I), (D)-(II)
 (c) (A)-(I), (B)-(III), (C)-(II), (D)-(IV)
 (d) (A)-(IV), (B)-(I), (C)-(II), (D)-(III)

SECTION-B

21. Two tuning forks A and B are sounded together giving rise to 8 beats in 2 s. When fork A is loaded with wax, the beat frequency is reduced to 4 beats in 2 s. If the original frequency of tuning fork B is 380 Hz, then the original frequency of tuning fork A is ____ Hz.
22. A thermodynamic system is taken through the cyclic process ABC as shown in the figure. The total work done by the system during the cycle ABC is ____ J.



23. An inductor stores 16 J of magnetic field energy and dissipates 32 W of thermal energy due to its resistance when an a.c. current of 2 A (rms) and frequency 50 Hz flows through it. The ratio of inductive reactance to its resistance is ____ ($\pi = 3.14$)
24. A beam of light consisting of wavelengths 650 nm and 550 nm illuminates the Young's double slits with separation of 2 mm such that the interference fringes are formed on a screen, placed at a distance of 1.2 m from the slits. The least distance of a point from the central maximum, where the bright fringes due to both the wavelengths coincide, is ____ $\times 10^{-5}$ m.
25. A fly wheel having mass 3 kg and radius 5 m is free to rotate about a horizontal axis. A string having negligible mass is wound around the wheel and the loose end of the string is connected to a 3 kg mass. The mass is kept at rest initially and released. Kinetic energy of the wheel when the mass descends by 3 m is ____ J. ($g = 10 \text{ m/s}^2$)

CHEMISTRY

SECTION-A

26. Identify the correct statements:

The presence of $-\text{NO}_2$ group in benzene ring

- (A) activates the ring towards electrophilic substitutions.
 (B) deactivates the ring towards electrophilic substitutions.
 (C) activates the ring towards nucleophilic substitutions.
 (D) deactivates the ring towards nucleophilic substitutions.

- (a) (B) and (D) Only (b) (C) and (A) Only
 (c) (A) and (D) Only (d) (B) and (C) Only

27. Given below are two statements :

Statement I : The increasing order of boiling point of hydrogen halides is $\text{HCl} < \text{HBr} < \text{HI} < \text{HF}$.

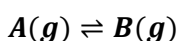
Statement II : The increasing order of melting point of hydrogen halides is $\text{HCl} < \text{HBr} < \text{HF} < \text{HI}$. In the light of the above statements, choose the correct answer from the options given below :

- (a) Both Statement I and Statement II are true
 (b) Statement I is true but Statement II is false
 (c) Both Statement I and Statement II are false
 (d) Statement I is false but Statement II is true

28. Consider the elements N, P, O, S, Cl and F. The number of valence electrons present in the elements with most and least metallic character from the above list is respectively.

- (a) 7 and 5 (b) 5 and 6
 (c) 5 and 7 (d) 6 and 7

29. Observe the following equilibrium in a 1 L flask.



At T(K), the equilibrium concentrations of A and B are 0.5 M and 0.375 M respectively. 0.1 moles of A are added into the flask and heated to T(K) to establish the equilibrium again. The new equilibrium concentrations (in M) of A and B are respectively.

- (a) 0.367, 0.275 (b) 0.53, 0.4
(c) 0.742, 0.557 (d) 0.557, 0.418

30. The plot of $\log_{10} K$ vs $\frac{1}{T}$ gives a straight line. The intercept and slope respectively are (where K is equilibrium constant).

- (a) $\frac{2.303R}{\Delta H^\circ}$, $\frac{2.303R}{\Delta S^\circ}$ (b) $\frac{\Delta S^\circ}{2.303R}$, $-\frac{\Delta H^\circ}{2.303R}$
(c) $-\frac{\Delta S^\circ R}{2.303}$, $\frac{\Delta H^\circ R}{2.303}$ (d) $-\frac{\Delta H^\circ}{2.303R}$, $\frac{\Delta S^\circ}{2.303R}$

31. The reactions which produce alcohol as the product are :

- (A) $\text{CH}_4 + \text{O}_2 \xrightarrow[\Delta]{\text{Mo}_2\text{O}_3}$
(B) $2\text{CH}_3\text{CH}_3 + 3\text{O}_2 \xrightarrow[\Delta]{(\text{CH}_3\text{COO})_2\text{Mm}}$
(C) $(\text{CH}_3)_3\text{CH} \xrightarrow{\text{KMnO}_4}$
(D) $2\text{CH}_4 + \text{O}_2 \xrightarrow{\text{KMnO}_4 \text{ Cu}/523 \text{ K}/100 \text{ zm.}}$
(E) $\text{CH}_3 - \text{CH} = \text{CH} - \text{CH}_3 \xrightarrow{\text{KMnO}_4/\text{H}^+}$

Choose the correct answer from the options given below:

- (a) (A) and (D) Only
(b) (A), (C) and (E) Only
(c) (C) and (D) Only
(d) (B), (D) and (E) Only

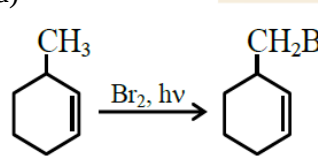
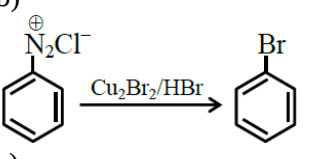
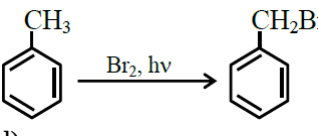
32. Consider the following statements about manganate and permanganate ions. Identify the correct statements :

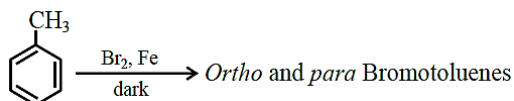
- (A) The geometry of both manganate and permanganate ions is tetrahedral.
(B) The oxidation states of Mn in manganate and permanganate are +7 and +6, respectively.
(C) Oxidation of Mn (II) salt by peroxodisulphate gives manganate ion as the final product.
(D) Manganate ion is paramagnetic and permanganate ions is diamagnetic.
(E) Acidified permanganate ion reduces oxalate, nitrite and iodide ions.

Choose the correct answer from the options given below.

- (a) (A), (C) and (D) Only
(b) (A), (B) and (C) Only
(c) (A), (D) and (E) Only
(d) (A) and (D) Only

33. Which of the following reaction is NOT correctly represented?

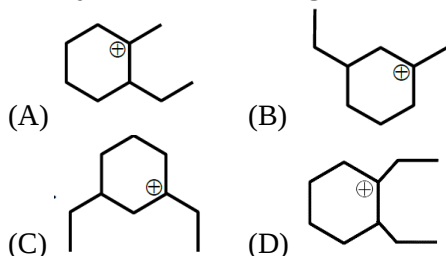
- (a) 
- (b) 
- (c) 
- (d)



34. The wavelength of photon 'A' is 400 nm. The frequency of photon 'B' is 10^{16} s^{-1} . The wave number of photon 'C' is 10^4 cm^{-1} . The correct order of energy of these photons is :

- (a) $C > B > A$ (b) $B > A > C$
(c) $A > B > C$ (d) $A > C > B$

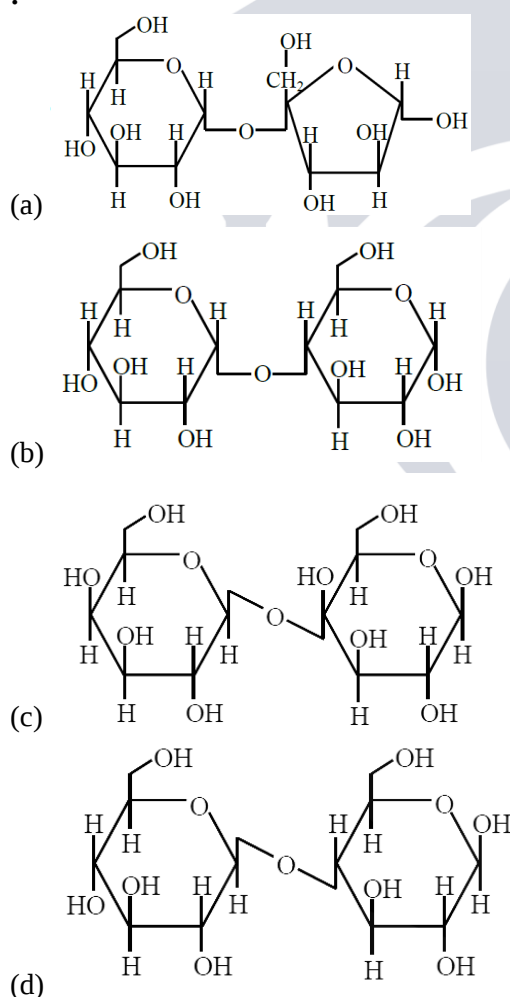
35. The cyclic cations having the same number of hyperconjugation are:



Choose the correct answer from the options given below:

- (a) (A) and (C) Only
(b) (B) and (C) Only
(c) (A) and (B) Only
(d) (A), (C) and (D) only

36. Structures of four disaccharides are given below. Among the given disaccharides, the non-reducing sugar is :



37. Match List-I with List-II according to shape.

List-I

(A) XeO_3

List-II

(I) BrF_5

- (B) XeF_2 (II) NH_3
 (C) $\text{XeO}_2 \cdot \text{F}_2$ (III) $[\text{I}_3]^-$
 (D) XeOF_4 (IV) SF_4

Choose the correct answer from the options given below:

- (a) (A)-(II), (B)-(I), (C)-(III), (D)-(IV)
 (b) (A)-(II), (B)-(III), (C)-(IV), (D)-(I)
 (c) (A)-(II), (B)-(III), (C)-(I), (D)-(IV)
 (d) (A)-(III), (B)-(II), (C)-(IV), (D)-(I)

38. A student performed analysis of aliphatic organic compound 'X' which on analysis gave C = 61.01%, H = 15.25%, N = 23.74%.

This compound, on treatment with $\text{HNO}_2/\text{H}_2\text{O}$ produced another compound 'Y' which did not contain any nitrogen atom. However, the compound 'Y' upon controlled oxidation produced another compound 'Z' that responded to iodoform test.

The structure of 'X' is:

- (a) $\text{CH}_3\text{CH}_2\text{CH}_2\text{NH}_2$ (b) $\begin{array}{c} \text{Ph}-\text{CH}-\text{NH}_2 \\ | \\ \text{CH}_3 \end{array}$
 (c) $\begin{array}{c} \text{CH}_3 \\ \diagup \\ \text{CH}-\text{NH}_2 \\ \diagdown \\ \text{CH}_3 \end{array}$ (d) $\begin{array}{c} \text{CH}_3-\text{CH}_2\text{CH}-\text{CH}_3 \\ | \\ \text{NH}_2 \end{array}$

39. Consider the following aqueous solutions.

- (I) 2.2 g Glucose in 125 mL of solution.
 (II) 1.9 g Calcium chloride in 250 mL of solution.
 (III) 9.0 g Urea in 500 mL of solution.
 (IV) 20.5 g Aluminum sulphate in 750 mL of solution.

The correct increasing order of boiling point of these solutions will be:

[Given: Molar mass in gmol^{-1} : H = 1, C = 12, N = 14, O = 16, Cl = 35.5, Ca = 40, Al = 27 and S = 32]

- (a) I < II < III < IV (b) III < I < II < IV
 (c) II < III < I < IV (d) II < III < IV < I

40. The correct order of acidic strength of the major products formed in the given reactions, is :

- (A) $\text{PhNH}_2 \xrightarrow[(2) \text{CuCN}, (3) \text{H}_3\text{O}^+/\Delta]{(1) \text{NaNO}_2 + \text{HCl} (< 5^\circ \text{C})} [\text{A}]$
 (B) $\text{CH}_3\text{CH}_2\text{CHO} \xrightarrow{[\text{Ag}(\text{NH}_3)_2]^+ \cdot \text{OH}^-} [\text{B}]$
 (C) $\text{CH}_4 + \text{O}_2 \xrightarrow[(ii) \text{Na}_2\text{Cr}_2\text{O}_7/\text{H}^+]{(i) \text{Mo}_2\text{O}_3, \Delta} [\text{C}]$
 (D) $\text{PhCH}_2\text{MgBr} + \text{CO}_2 \xrightarrow[\text{H}_3\text{O}^+]{\text{Dry ether}} [\text{D}]$

Choose the correct answer from the options given below:

- (a) C > B > A > D (b) A > D > C > B
 (c) A > D > B > C (d) C > A > D > B

41. Total number of alkali insoluble solid lipoamides obtained by reaction of given amines with Heinsberg's reagent is ____.

Aniline, N-Methylaniline, Methenamine,

N, N-Dimethylmethanamine,

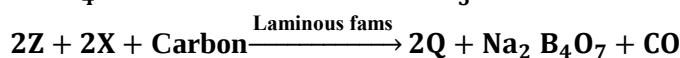
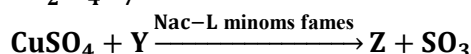
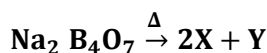
N-Methyl methanamine, Phenylmethanamine,

N-propylaniline, N-phenylaniline,

N, N-Dimethylaniline, Allyl amine, Isopropyl amine

- (a) 4 (b) 2
 (c) 8 (d) 5

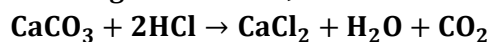
42. Consider the following reactions.



The oxidation states of Cu in Z and Q, respectively are:

- (a) +2 and +2 (b) +2 and +1
(c) +1 and +2 (d) +1 and +1

43. For the given reaction;



If 90 g CaCO_3 is added to 300 mL of HCl which contains 38.55% HCl by mass and has density 1.13 g mL^{-1} , then which of the following option is correct?

Given molar mass of H, Cl, Ca and O are 1, 35.5, 40 and 16 g mol^{-1} respectively.

- (a) 64.97 g of HCl remains unreacted
(b) 32.85 g CaCO_3 remains unreacted
(c) 97.30 g of HCl reacted
(d) 60.32 g of HCl remains unreacted

44. The correct increasing order of spin-only magnetic moment values of the complex ions

$[\text{MnBr}_4]^{2-}$ (A), $[\text{Cu}(\text{H}_2\text{O})_6]^{2+}$ (B), $[\text{Ni}(\text{CN})_4]^{2-}$ (C) and $[\text{Ni}(\text{H}_2\text{O})_6]^{2+}$ (D) is:

- (a) $A = B < C < D$ (b) $A = B < D < C$
(c) $C = D < B < A$ (d) $C < B < D < A$

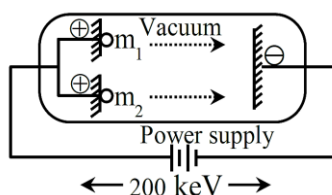
45. A student has been given 0.314 g of an organic compound and asked to estimate Sulphur. During the experiment, the student has obtained 0.4813 g of barium sulphate. The percentage of Sulphur present in the compound is ____.

(Given Molar mass in gmol^{-1} S: 32, BaSO_4 : 233)

- (a) 42.10% (b) 63.15%
(c) 21.05% (d) 48.24%

SECTION-B

46. Two positively charged particles m_1 and m_2 have been accelerated across the same potential difference of 200 keV as shown below.



[Given mass of $m_1 = 1\text{amu}$ and $m_2 = 4\text{amu}$]

The de Broglie wavelength of m_1 will be x times of m_2 . The value of x is ____ (nearest integer)

47. $A \rightarrow B$ (first reaction)

$C \rightarrow D$ (second reaction)

Consider the above two first-order reactions. The rate constant for first reaction at 500 K is double of the same at 300 K. At 500 K, 50% of the reaction becomes complete in 2 hour. The activation energy of the second reaction is half of that of first reaction. If the rate constant at 500 K of the second reaction becomes double of the rate constant of first reaction at the same temperature; then rate constant for the second reaction at 300 K is $\times 10^{-1} \text{ hour}^{-1}$ (nearest integer).

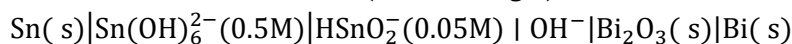
48. For strong electrolyte Λ_m increases slowly with dilution and can be represented by the equation $\Lambda_m = \Lambda_m^\circ - A c^{1/2}$

Molar conductivity values of the solutions of strong electrolyte AB at 18°C are given below :

$c[\text{molL}^{-1}]$	0.04	0.09	0.16	0.25
$\Lambda_m[\text{Scm}^2 \text{mol}^{-1}]$	96.1	95.7	95.3	94.9

The value of constant A based on the above data [in $\text{Scm}^2 \text{mol}^{-1}/(\text{mol/L})^{1/2}$] unit is ____.

49. A volume of x mL of 5MNaHCO_3 solution was mixed with 10 mL of $2\text{MH}_2\text{CO}_3$ solution to make an electrolytic buffer. If the same buffer was used in the following electrochemical cell to record a cell potential of 235.3 mV, then the value of $x =$ ____ mL (nearest integer).



Consider up to one place of decimal for intermediate calculations

$$\left[\begin{array}{l} \text{Given: } E_{\text{HSnO}_2^-/\text{Sn(OH)}_6^{2-}}^\circ = -0.9 \text{ V} \\ E_{\text{Bi}_2\text{O}_3/\text{Bi}}^\circ = -0.44 \text{ V} \\ \text{pK}_{\text{a}}(\text{H}_2\text{CO}_3) = 6.11 \\ \frac{2.303RT}{F} = 0.059 \text{ V} \\ \text{Anti log}(1.29) = 19.5 \end{array} \right]$$

50. The number of isoelectronic species among Sc^{3+} , Cr^{2+} , Mn^{3+} , Co^{3+} and Fe^{3+} is 'n'. If 'n' moles of AgCl is formed during the reaction of complex with formula $\text{CoCl}_3(\text{en})_2\text{NH}_3$ with excess of AgNO_3 solution, then the number of electrons present in the t_{2g} orbital of the complex is ____.

MATHEMATICS

SECTION-A

51. Given below two statements :

Statement I: $25^{13} + 20^{13} + 8^{13} + 3^{13}$ is divisible by 7.

Statement II: The integral part of $(7 + 4\sqrt{3})^{25}$ is an odd number.

In the light of the above statements, choose the correct answer from the options given below:

- (a) Both Statement I and Statement II are false.
(b) Both Statement I and Statement II are true.
(c) Statement I is false but Statement II is true.
(d) Statement I is true but Statement II is false.

52. The sum of the coefficients of x^{499} and x^{500} in $(1+x)^{1000} + x(1+x)^{999} + x^2(1+x)^{998} + \dots + x^{1000}$ is

- (a) $^{1001}C_{501}$ (b) $^{1002}C_{500}$
(c) $^{1002}C_{501}$ (d) $^{1000}C_{501}$

53. Let A be the focus of the parabola $y^2 = 8x$. Let the line $y = mx + c$ intersect the parabola at two distinct points B and C. If the centroid of the triangle ABC is $(\frac{7}{3}, \frac{4}{3})$, then $(BC)^2$ is equal to :

- (a) 41 (b) 80
(c) 89 (d) 32

54. The probability distribution of a random variable X is given below :

X	4k	$\frac{30}{7}k$	$\frac{32}{7}k$	$\frac{34}{7}k$	$\frac{36}{7}k$	$\frac{38}{7}k$	$\frac{40}{7}k$	6k
P(X)	$\frac{2}{15}$	$\frac{1}{15}$	$\frac{2}{15}$	$\frac{1}{5}$	$\frac{1}{15}$	$\frac{2}{15}$	$\frac{1}{5}$	$\frac{1}{15}$

If $E(X) = \frac{263}{15}$, then $P(X < 20)$ is equal to :

- (a) $\frac{3}{5}$ (b) $\frac{8}{15}$
(c) $\frac{11}{15}$ (d) $\frac{14}{15}$

55. Let $f(x) = \lim_{\theta \rightarrow 0} \left(\frac{\cos \pi x - x^{\left(\frac{2}{\theta}\right)} \sin(x-1)}{1 + x^{\left(\frac{2}{\theta}\right)} (x-1)} \right)$, $x \in R$.

Consider the following two statements :

(I) $f(x)$ is discontinuous at $x = 1$.

(II) $f(x)$ is continuous at $x = -1$.

Then,

- (a) Neither (I) nor (II) is True
(b) Both (I) and (II) are True

(c) Only (II) is True

(d) Only (I) is True

56. Considering the principal values of inverse trigonometric functions, the value of the expression

$\tan\left(2\sin^{-1}\left(\frac{2}{\sqrt{13}}\right) - 2\cos^{-1}\left(\frac{3}{\sqrt{10}}\right)\right)$ is equal to :

(a) $-\frac{33}{56}$

(b) $\frac{33}{56}$

(c) $\frac{16}{63}$

(d) $-\frac{16}{63}$

57. Let the arithmetic mean of $\frac{1}{a}$ and $\frac{1}{b}$ be $\frac{5}{16}$, $a > 2$. If α is such that $a, 4, \alpha, b$ are in A.P., then the equation

$\alpha x^2 - ax + 2(\alpha - 2b) = 0$ has :

(a) One root in $(1, 4)$ and another in $(-2, 0)$

(b) One root in $(0, 2)$ and another in $(-4, -2)$

(c) Complex roots of magnitude less than 2

(d) Both roots in the interval $(-2, 0)$

58. Given below are two statements :

Statement I : The function $f: R \rightarrow R$ defined by $f(x) = \frac{x}{1+|x|}$ is one-one.

Statement II : The function $f: R \rightarrow R$ defined by $f(x) = \frac{x^2+4x-30}{x^2-8x+18}$ is many-one.

In the light of the above statements, choose the correct answer from the options given below:

(a) Both Statement I and Statement II are false.

(b) Both Statement I and Statement II are true.

(c) Statement I is false but Statement II is true.

(d) Statement I is true but Statement II is false.

59. An ellipse has its center at $(1, -2)$, one focus at $(3, -2)$ and one vertex at $(5, -2)$. Then the length of its latus rectum is :

(a) $\frac{16}{\sqrt{3}}$

(b) 6

(c) $4\sqrt{3}$

(d) $6\sqrt{3}$

60. Let the ellipse $E: \frac{x^2}{144} + \frac{y^2}{169} = 1$ and the hyperbola $H: \frac{x^2}{16} - \frac{y^2}{\lambda^2} = -1$ have the same foci. If e and L respectively denote the eccentricity and the length of the latus rectum of H , then the value of $24(e + L)$ is :

(a) 296

(b) 126

(c) 148

(d) 67

61. Let $P_1: y = 4x^2$ and $P_2: y = x^2 + 27$ be two parabolas. If the area of the bounded region enclosed between P_1 and P_2 is six times the area of the bounded region enclosed between the line $y = \alpha x$, $\alpha > 0$ and P_1 , then α is equal to :

(a) 8

(b) 15

(c) 12

(d) 6

62. Let the circle $x^2 + y^2 = 4$ intersect x -axis at the points $A(a, 0)$, $a > 0$ and $B(b, 0)$. Let $P(2\cos \alpha, 2\sin \alpha)$, $0 < \alpha < \frac{\pi}{2}$ and $Q(2\cos \beta, 2\sin \beta)$ be two points such that $(\alpha - \beta) = \frac{\pi}{2}$. Then the point of intersection of AQ and BP lies on :

(a) $x^2 + y^2 - 4y - 4 = 0$

(b) $x^2 + y^2 - 4x - 4 = 0$

(c) $x^2 + y^2 - 4x - 4y = 0$

(d) $x^2 + y^2 - 4x - 4y - 4 = 0$

63. Let $[\cdot]$ denote the greatest integer function. Then $\int_{\frac{\pi}{2}}^{\pi} \left(\frac{12(3+[x])}{3+[\sin x] + [\cos x]} \right) dx$ is equal to:

(a) $15\pi + 4$

(b) $11\pi + 2$

(c) $13\pi + 1$

(d) $12\pi + 5$

64. Let $y = y(x)$ be the solution of the differential equation $x \frac{dy}{dx} - y = x^2 \cot x, x \in (0, \pi)$. If $y\left(\frac{\pi}{2}\right) = \frac{\pi}{2}$, then $6y\left(\frac{\pi}{6}\right) - 8y\left(\frac{\pi}{4}\right)$ is equal to :
- (a) 3π (b) -3π
(c) $-\pi$ (d) π
65. The sum of all the elements in the range of $f(x) = \text{Sgn}(\sin x) + \text{Sgn}(\cos x) + \text{Sgn}(\tan x) + \text{Sgn}(\cot x)$, $x \neq \frac{n\pi}{2}, n \in \mathbb{Z}$, where $\text{Sgn}(t) = \begin{cases} 1, & \text{if } t > 0 \\ -1 & \text{if } t < 0 \end{cases}$, is
- (a) 4 (b) 2
(c) -2 (d) 0
66. Let $Q(a, b, c)$ be the image of the point $P(3, 2, 1)$ in the line $\frac{x-1}{1} = \frac{y}{2} = \frac{z-1}{1}$. Then the distance of Q from the line $\frac{x-9}{3} = \frac{y-9}{2} = \frac{z-5}{-2}$ is
- (a) 6 (b) 8
(c) 7 (d) 5
67. Let P be a point in the plane of the vector $\overrightarrow{AB} = 3\hat{i} + \hat{j} - \hat{k}$ and $\overrightarrow{AC} = \hat{i} - \hat{j} + 3\hat{k}$ such that P is equidistant from the lines AB and AC . If $|\overrightarrow{AP}| = \frac{\sqrt{5}}{2}$, then the area of the triangle ABP is :
- (a) 2 (b) $\frac{3}{2}$
(c) $\frac{\sqrt{30}}{4}$ (d) $\frac{\sqrt{26}}{4}$
68. Let $A = \{z \in \mathbb{C} : |z - 2| \leq 4\}$ and $B = \{z \in \mathbb{C} : |z - 2| + |z + 2| = 5\}$. Then the max $\{|z_1 - z_2| : z_1 \in A \text{ and } z_2 \in B\}$ is
- (a) $\frac{15}{2}$ (b) 8
(c) $\frac{17}{2}$ (d) 9
69. Let $f(x) = \int \frac{dx}{x^{\frac{2}{3}} + 2x^{\frac{1}{2}}}$ be such that $f(0) = -26 + 24\log_c(2)$. If $f(1) = a + b\log_c(3)$, where $a, b \in \mathbb{Z}$, then $a + b$ is equal to:
- (a) -18 (b) -5
(c) -11 (d) -26
70. $\frac{6}{3^{26}} + \frac{10.1}{3^{25}} + \frac{10.2}{3^{24}} + \frac{10.2^2}{3^{23}} + \dots + \frac{10.2^{24}}{3}$ is equal to
- (a) 2^{25} (b) 2^{26}
(c) 3^{25} (d) 3^{26}

SECTION-B

71. If $\sum_{r=1}^{25} \left(\frac{r}{r^4 + r^2 + 1} \right) = \frac{p}{q}$, where p and q are positive integers such that $\gcd(p, q) = 1$, then $p + q$ is equal to
72. Three persons enter in a lift at the ground floor. The lift will go upto 10^{th} floor. The number of ways, in which the three persons can exit the lift at three different floors, if the lift does not stop at first, second and third floors, is equal to ____.
73. Let f be a differentiable function satisfying
- $$f(x) = 1 - 2x + \int_0^x e^{(x-t)} f(t) dt, x \in \mathbb{R} \text{ and let}$$
- $$g(x) = \int_0^x (f(t) + 2)^{15} (t - 4)^6 (t + 12)^{17} dt, x \in \mathbb{R}$$
- If p and q are respectively the points of local minima and local maxima of g , then the value of $|p + q|$ is equal to

74. If the distance of the point $P(4, 3, \alpha, \beta)$, $\beta < 0$, from the line $\vec{r} = 4\hat{i} - \hat{k} + \mu(2\hat{i} + 3\hat{k})$, $\mu \in \mathbb{R}$ along a line with direction ratios $3, -1, 0$ is $13\sqrt{10}$, then $\alpha^2 + \beta^2$ is equal to
75. Let $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$ and B be two matrices such that $A^{100} = 100B + I$. Then the sum of all the elements of B^{100} is ____.

