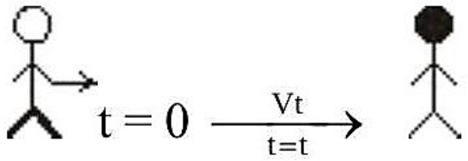
1. **(c)** $u = \frac{B^2}{2\mu_0}$

$u \rightarrow$ Energy per unit volume

$$\left[\frac{B^2}{\mu_0}\right] = [u] = \frac{[ML^2\ T^{-2}]}{[L^3]} = [ML^{-1}\ T^{-2}]$$

2. **(a)** Monkey



Time taken by mango $= \sqrt{\frac{2n}{g}}$

$$= \sqrt{\frac{2 \times 19.6}{9.8}} = 2 \text{ second}$$

Distance = vt

$$= 9 \times \frac{5}{18} \times 2 = 5 \text{ m}$$

3. **(b)** Impulse = change in momentum

$I = \Delta P$

$$F_{aug} = \frac{\Delta P}{\Delta t}$$

$\Delta t_1 = 3\ \Delta t_2 = 5$

$\Delta P_1 = \Delta P_2$

$I_1 = I_2$

F_{avg} in case (i) is more than (ii)

4. **(b)** $F = \frac{dm}{dt} v$

$$= \frac{10\text{ g}}{5\text{ s}}\left(4.5\frac{\text{cm}}{\text{s}}\right) = 9\frac{\text{gcm}}{\text{s}^2} = 9 \text{ dyne}$$

5. **(d)** $g = \frac{GM}{R^2}$

M = constant $g < \frac{1}{R^2}$

$$100\frac{\Delta g}{g} = -2\frac{\Delta R}{R}100$$

% change $= -2(-2)$

% change in g $= 4\%$

increase by 4%

6. **(c)** $F = \gamma A \frac{\Delta \ell}{\ell}$

$= 2 \times 10^{11} \times 10^{-4}\left(\frac{2\ell - \ell}{\ell}\right)$

$= 2 \times 10^{7}$ N

7. **(c)** $\eta = 1 - \frac{T_L}{T_H}$

$\frac{1}{2} = 1 - \frac{T_L}{T_H}$

$\frac{1}{2}(1 \cdot 3) = 1 - \left(\frac{T_L - 40}{T_H}\right)$

$\frac{1}{2}(1 \cdot 3) = \frac{1}{2} + \frac{40}{T_H}\ T_H = 266.7\ K$

8. **(d)** $\left[P_{arg} = 0\right]$ (due to random motion)

$v_{rms} = \sqrt{\frac{3RT}{M}}$

$T_{new} = 2\ T$

$M_{new} = \frac{M}{2}$

$\frac{v_{new}}{v} = \frac{\sqrt{\frac{2T}{M/2}}}{\sqrt{\frac{T}{M}}}$

$v_{new} = 2v$

9. **(c)** $y = 0.5 \sin\left(\frac{2\pi}{\lambda} 400t - \frac{2\pi}{\lambda} x\right)$

$\omega = \frac{2\pi}{\lambda} 400$

$K = \frac{2\pi}{\lambda}$

$v = \frac{\omega}{k}$

$[v = 400\ m/s]$

10. (a)

$$C_{eq} = \frac{C(KC)}{C + KC} = \frac{KC}{K + 1}$$

$$24 = \frac{K40}{K + 1}$$

$[K = 1 \cdot 5]$

11. (a) $R_1 = \rho \frac{L_1}{A_1}$

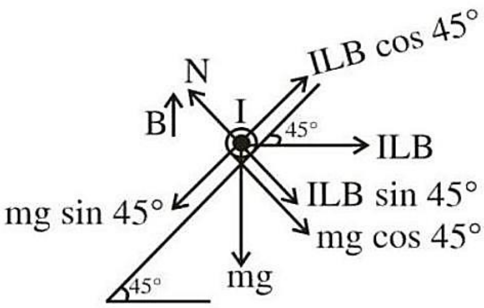
$$R_2 = \rho\left(\frac{3\,L_1}{A_1/3}\right) = 9\rho\frac{L_1}{A_1}$$

$$\therefore \frac{R_2}{R_1} = 9$$

12. (d) $i = \left(\frac{K}{NAB}\right)\theta$

$$\therefore \frac{d\theta}{di} = \frac{NAB}{K}$$

13. (a)



$$mg \sin 45^\circ = ILB\cos 45^\circ$$

$$\therefore I = \left(\frac{m}{L}\right)\frac{g}{B}$$

$$= \frac{(0.45)(10)}{0.15} = 30\text{ A}$$

14. (d) $v_0 = i_0 x_L$

$= i_0(wL)$

$= (5)(49\pi)(30 \times 10^{-3})$

$= 23.1$

Voltage will lead current by 90°.

$\therefore V = 23.1\sin(49\pi t + 60^\circ)$

15. (a) $\frac{\mu_2}{\mu_{air}} = \frac{C}{v_2}$

$\therefore \frac{\sqrt{\mu_{r_2}\varepsilon_{r_2}}}{(1)} = \frac{C}{v_2}$

$\therefore \sqrt{(1)(9)} = \frac{C}{V_2}$

$\therefore v_2 = \frac{C}{3}$

16. (a) $f_0 + f_e = 30$

$m = \frac{f_0}{f_e}$

$2 = \frac{f_0}{f_e} \Rightarrow f_0 = 2f_e$

So $f_0 + \frac{f_0}{2} = 30$

$f_0 = 20$ cm

17. (d) $\lambda = \frac{h}{mv}$ (de-Broglie's wavelength)

$\lambda \frac{h}{\sqrt{2\,m(K\cdot E)}}$

$h = \frac{h}{\sqrt{2mqV}}$

Putting the values of m; q

We get $\lambda = \frac{1\cdot 22}{\sqrt{V}}$ nm

18. (c) 7/8 disintegrates means 1/8 remains

Or $\left(\frac{1}{2}\right)^3$

$\therefore$ 3 half lives

$= 180$ days

19. (b)

20. (a) $\mu = \frac{A_m}{A_c}$

$\mu \leq 1$ to avoid distortion

because $\mu > 1$ will result in interference between career frequency & message frequency.

21. (5) $\vec{a} \cdot \vec{b} = 0$

$\therefore \vec{a} \cdot \vec{b} = 0$

$\therefore 2 \times 1 + 4 \times 2 - 2 \times \alpha = 0$

$\therefore \alpha = 5$

22. (15) $A = A_0 \times 2^{-t/T}$

$\frac{A_0}{64} = A_0 \times 2^{-t/T}$

$\therefore t = 6\,T = 6 \times 2 \cdot 5 = 15$ hours

23. (630)

$\beta \propto \lambda$

$\lambda_2 = \frac{9}{8}\lambda_1$

$\therefore \beta_2 = \frac{9}{8}\beta_1 = \frac{9}{8} \times 560 = 630$ nm

24. (250) Band width $= 232 - 212 = \frac{R}{L}$

$\therefore L = \frac{5}{20} = 250\mathrm{mH}$

25. (780)

$E = \frac{AC}{AB}(V_A - V_B)$

$\therefore 20 \times 10^{-3} = \frac{60}{300} \times \frac{4 \times 20}{R + 20}$

$\therefore R = 780\Lambda$

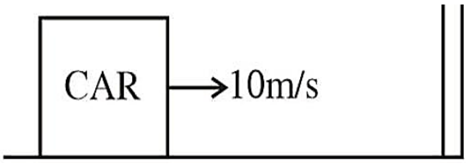
26. (6)

$|\tau|_{max} = PE$

$\frac{\tau_1}{\tau_2} = \frac{P_1E_1}{P_2E_2} = \frac{1 \cdot 2 \times 10^{-30} \times 5 \times 10^4}{2 \cdot 4 \times 10^{-30} \times 15 \times 10^4} = \frac{1}{6}$

Hence $x = 6$

27. (340) The hill will be a secondary source.



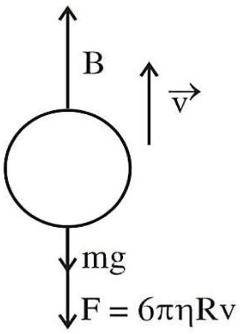
f_1 = frequency of the car w.r.t. the hill

$$f_1 = \left(\frac{v}{v - v_s}\right) f = \left(\frac{330}{320}\right) \times 320 = 330 \text{ Hz}$$

f_2 = Frequency of the sound reflected by hill w.r.t. the car (echo)

$$f_2 = \left(\frac{v + v_0}{v}\right) f_1 = \frac{(330 + 10)}{330} \times 330 = 340 \text{ Hz}$$

28. (11) As the bubble is rising steadily the net force acting on it will be zero



(Because of density of air the value of mg can be neglected)

So $B = F \Rightarrow \frac{4\pi}{3} R^3 \rho g = 6\pi\eta Rv$

Putting

$R = 1 \text{ mm} = 10^{-3} \text{ m}$

$\rho = 1 \cdot 75 \times 10^3 \text{ kg/m}^3$

$g = 10 \text{ m/s}^2$
$v = 0.35 \times 10^{-2} \text{ m/s}$

$\eta = \frac{10}{9} \simeq 1 \cdot 11$ SI unit $= 11$ poise (CGS)

29. (24) Using work - energy theorem

$W_{net} = (K_f - K_i)$

$$\Rightarrow -\frac{1}{2} Kx^2 = \frac{1}{2} m \left(\frac{v}{2}\right)^2 - \frac{1}{2} mv^2 = \frac{E}{4} - E$$

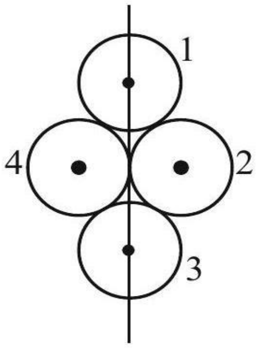
$$\Rightarrow \frac{1}{2} Kx^2 = \frac{3E}{4} \Rightarrow K = \frac{3E}{2x^2}$$

$$\Rightarrow K = \frac{3E}{2 \times \left(\frac{1}{4}\right)^2} = 24E$$

$n = 24$

30. (30) $I_1 = I_3 = \frac{M^2}{4}$

$I_2 = \frac{MR^2}{4} + MR^2 = \frac{5}{4}MR^2 = I_4$



So $I = I_1 + I_2 + I_3 + I_4$

$= \frac{MR^2}{2} + \frac{5}{2}MR^2$

$= 3MR^2$, Putting $R = \frac{a}{2}$

$I = \frac{3Ma^2}{4}$, So $x = 3$

31. (d) Atomic orbital is characterised by n, l, m.

32. (a) If $U + Pv$ (By definition)

$\Delta 14 = \Delta U + \Delta(Pr)$ at constant pressure

$\Delta H = \Delta U + P\Delta V$

33. (c)

(a) $Cd(s) + 2Ni(OH)_3(s) \rightarrow CdO(s) + 2Ni(OH)_2(s)$

$+H_2O(l)$

Discharge of secondary Battery

(b) $Zn(Hg) + HgO(s) \rightarrow ZnO(s) + Hg(l)$

(Primary Battery Mercury cell)

(c) $2PbSO_4(s) + 2H_2O(l) \rightarrow Pb(s) + PbO_2(s) + 2H_2SO_4$ (aq)

Charging of secondary Battery

(d) $2H_2(g) + O_2(g) \rightarrow 2H_2O(l)$ – Fuel cell

34. (c)

35. (c) Rb&Cs have nearly same electron gain enthalpy electron gain enthalpy $= -46$kj/ml

Ar & Kr have same ΔH_{eq}. Value is +96kj/ml

36. (c) $Al_2O_3 + 2NaOH + 3H_2O \rightarrow 2Na, [Al(OH)_4]$ Leaching

37. (b) Clark's Method Reaction

$$Ca(HCO_3)_2 + Ca(OH)_2 \rightarrow 2CaCO_3 + 2H_2O$$

$$Mg(HCO_3)_2 + 2Ca(OH)_2 \rightarrow 2CaCO_3 + Mg(OH)_2 + 2H_2O$$

38. (a) Low solubility of LiF in water is due to high lattice enthalpy

39. (c)

$$(NH_4)_2Cr_2O_7 \xrightarrow{\Delta} N_2 + Cr_2O_3 + 4H_2O$$

$$KMnO_4 + HCl \rightarrow MnCl_2 + KCl + Cl_2 + H_2O$$

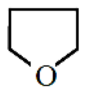
$$Al + NaOH + H_2O \rightarrow H_2 + Na[Al(OH)_4]$$

$$NaNO_3 \longrightarrow NaNO_2 + O_2$$

40. (a) Copper is least electropositive among the given metals and it lies below H in reactivity series

41. (d) Since eutrophication is result of excessive growth of weed in water bodies, which consume dissolved oxygen of water bodies. Eutrophication decreases amount of dissolved oxygen in water bodies. Polluted water has low value of dissolved oxygen, but high value of BOD (Biological oxygen demand), since chemical and organic matter requires dissolved oxygen to get decompose.

42.(c)



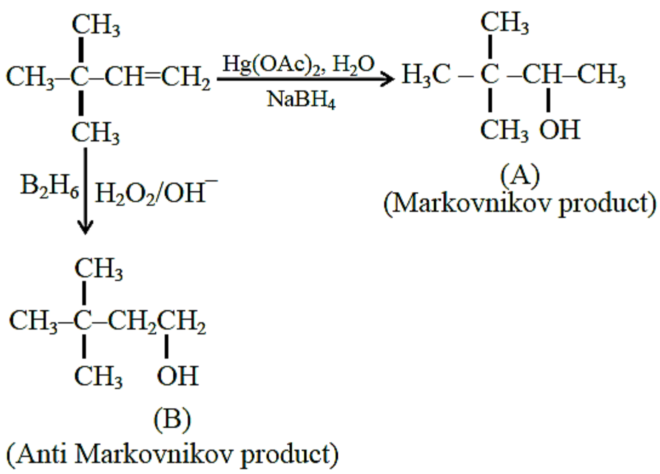
: Non-planar heterocyclic Compound

: Bicyclo Compound

: Spiro Compound

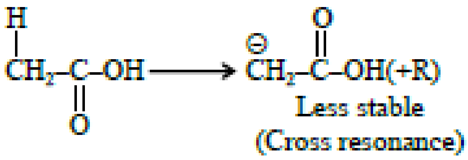
: Aromatic Compound

43.(b)

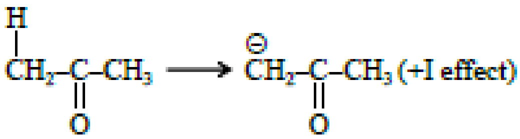


44. (c)

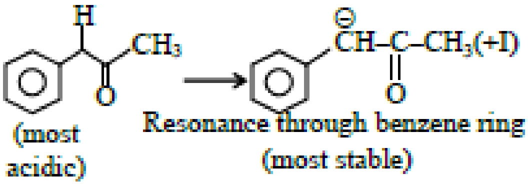
(A)



(B)

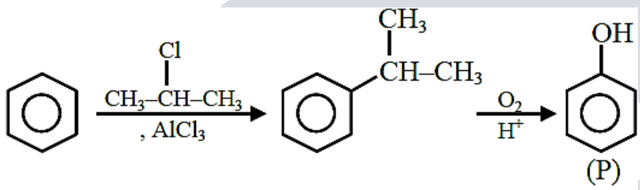


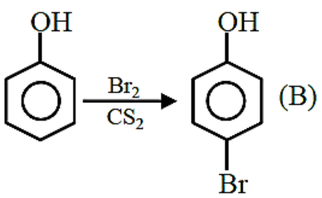
(C)

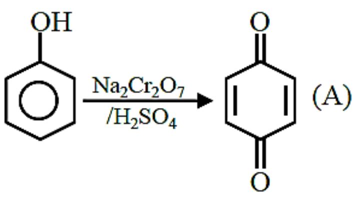


So it has least pK_a value.

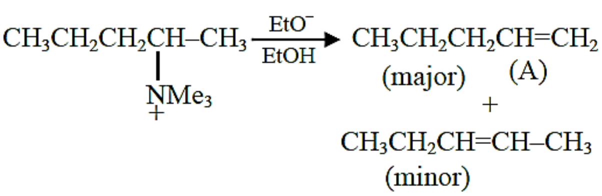
45. (b)



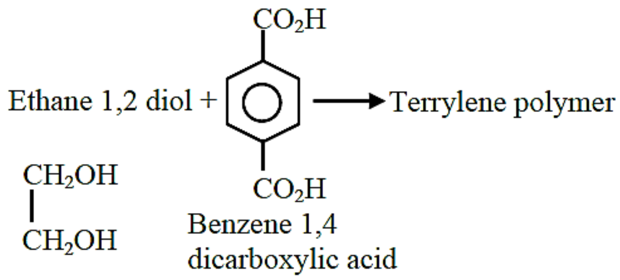




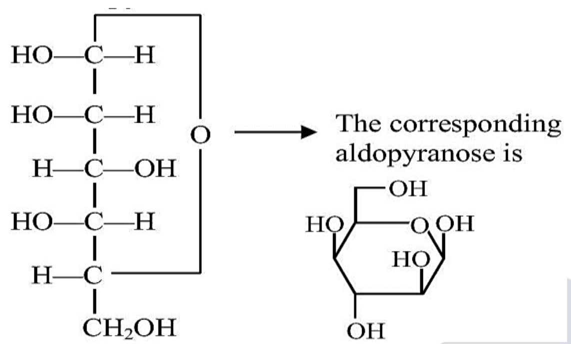
46. (c)



47. (c)



48. (d) Correct pyranose structure is



X (Hemiacetal)

49. (c) Enzyme inhibitors can be competitive inhibitors (inhibit the attachment of substrate on active site of enzyme) and non-competitive inhibitor (changes the active site of enzyme after binding at allosteric site.)

50. (a) The is recorded immediately after the blue colour appears.

$Na_2\ S_2O_3$ is kept in limited amount.

51. (2)

$$\underset{1\text{mol}}{X} + \underset{1\text{ mol}}{Y} + \underset{0.05\text{ mol}}{3Z} \rightleftharpoons \mathbf{XYZ_3}$$

Z is L.R.

$\frac{0.05}{3} = 1$ mole of XYZ_3

Mass of $XYZ_3 = \frac{0.05}{3} \times (10 + 20 + 30 \times 3)$

$= 2$ g

52. (22) M is body certred cubic, $\therefore Z = 2$

Let mass of 1 atom of M is A

Edge length $= 300$pm

Density $= 6\ \text{g/cm}^3$

$$\therefore 6\ \text{g/cm}^3 = \frac{Z \times A}{(300 \times 10^{-10})^3} = \frac{2 \times A}{27 \times 10^{-24}}$$

$A = 81 \times 10^{-24}$ g

$\therefore$ Atomic mass $= 48.6$ g

$\therefore$ Mole in 180 g $= \frac{180}{48.6} = 3.7$ moles

Atoms of M $= 3.7 \times 6 \times 10^{23}$

$= 22.22 \times 10^{23}$ atoms

53. (4)

54. (15)

$150\ gCH_3COOH$

10.2 g ascorbic acid $\Rightarrow$ 0.058 moles

$\Delta T_f = (x \times 10^{-1})\ ^\circ C$

$\Delta T_f = k_f \cdot$ molality

$$= 3.9 \times \frac{0.058}{150} \times 1000$$

$= 1.5^\circ C$

$= 15 \times 10^{-1\circ} C$

55. (27)

K_a of Butyric acid $\Rightarrow 2 \times 10^{-5} PKa = 4.7$

pH of 0.2M solution

$$pH = \frac{1}{2} pK_a - \frac{1}{2} \log C$$

$$= \frac{1}{2}(4 \cdot 7) \frac{1}{2} \log(0.2)$$

$= 2.35 + 0.35 = 2.7$

$pH = 27 \times 10^{-1}$

56. (100)

$A \rightarrow B\ t_{1/2} = 0.3010$ min

A_0/A_t at time 2 min $=?$

$$K = \frac{2.303}{t} \log \left[\frac{A_0}{A_t}\right]$$

$$\Rightarrow \frac{0 \cdot 693}{t_{\frac{1}{2}}} = \frac{2 \cdot 303}{2} \log \left(\frac{A_0}{A_t}\right)$$

Or $\frac{2.303 \times 0.3010}{0.3010} = \frac{2.303}{2} \log \frac{A_0}{A_t}$

$\log\frac{A_0}{A_t} = 2$

$\therefore \frac{A_0}{A_t} = 10^2 = 100$

57. (3) Square pyramidal structures are

BrF_5, IF_5 and ClF_5.

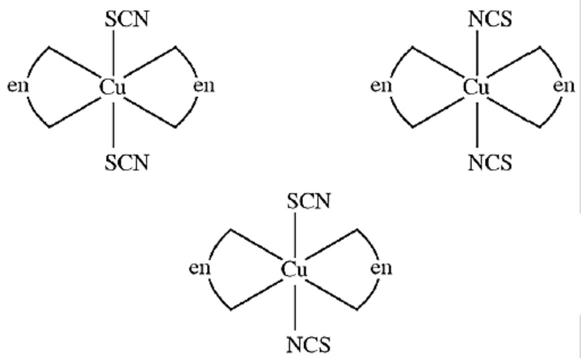
58. (4) $MnO_4^{2-} \xrightarrow{H^+} MnO_4^- + MnO_2$

No. of unpaired $\bar{e} = 3$

$\therefore \mu = \sqrt{15} = 3.877$

Nearest Integer = 4

59.(3) $[Cu(en)_2(SCN)_2]$



60. (46)

0.492 g of $C_xH_yO_z$

Gives $0.7938\ gCO_2 = 0.018$ moles

$0.4428\ gH_2O = 0.0246$ moles

So moles of C = 0.018 ⇒ 0.216 g

Moles of H = 0.049 ⇒ 0.049 g

∴ wt. of Oxygen = 0.492 − 0.216 − 0.049

= 0.227 g

% of Oxygen $= \frac{0.227}{0.492} \times 100$ 46 (approx.)

61. (b) $xdy = \left(\sqrt{x^2+y^2}+y\right)dx$

$xdy - ydx = \sqrt{x^2+y^2}dx$

$$\frac{xdy - ydx}{x^2} = \sqrt{1 + \frac{y^2}{x^2}} \cdot \frac{dx}{x}$$

$$\frac{d(y/x)}{\sqrt{1 + \left(\frac{y}{x}\right)^2}} = \frac{dx}{x}$$

$$\ln\left(\frac{y}{x} + \sqrt{\left(\frac{y}{x}\right)^2 + 1}\right) = \ln x + R$$

$$\frac{y + \sqrt{y^2 + x^2}}{x} = cx$$

$$y + \sqrt{y^2 + x^2} = cx^2$$

$x = 1, y = 0 \Rightarrow 0 + 1 = C \Rightarrow C = 1$

Curve is $y + \sqrt{x^2 + y^2} = x^2$

$x = 2, y = \alpha$

$2 + \sqrt{4 + \alpha^2} = 4$

$4 + \alpha^2 = 16 + \alpha^2 = 8\alpha$

$\alpha = \frac{3}{2}$

62. (b)

$$\left|\frac{x^2 + 4x + 2}{x^2 + 3}\right| \leq 1$$
$\Leftrightarrow (x^2 - 4x + 2)^2 \leq (x^2 + 3)^2$
$\Leftrightarrow (x^2 - 4x + 2)^2 - (x^2 + 3)^2 \leq 0$
$\Leftrightarrow (2x^2 - 4x + 5)(-4x - 1) \leq 0$
$\Leftrightarrow -4x - 1 \leq 0 \rightarrow x \geq -\frac{1}{4}$

63. (c) By its given condition

$: \vec{a}, \vec{b}, \vec{c}$ are linearly independent vectors

$\Rightarrow [\bar{a}\bar{b}\bar{c}] \neq 0$

Now, $[\bar{a}\bar{b}\bar{c}]$

$$= \begin{vmatrix} 1+t & 1-t & 1 \\ 1-t & 1+t & 2 \\ t & -t & 1 \end{vmatrix}$$

$C_2 \rightarrow C_1 + C_2$

$$\begin{vmatrix} 1+t & 2 & 1 \\ 1-t & 2 & 2 \\ t & 0 & 1 \end{vmatrix}$$

$$= 2\begin{vmatrix} 1+t & 1 & 1 \\ 1-t & 1 & 2 \\ t & 0 & 1 \end{vmatrix}$$

$= 2[(1+t) - (1-t) + t]$

$= 2[3t] = 6t$

$[\bar{a}\bar{b}\bar{c}] \neq 0 \Rightarrow t \neq 0$

64. (a) $\cos^{-1} x = 2\sin^{-1} x = \cos^{-1} 2x$

$\cos^{-1} x - 2\left(\frac{\pi}{2} - \cos^{-1} x\right) = \cos^{-1} 2x$

$\cos^{-1} x - \pi + 2\cos^{-1} x = \cos^{-1} 2x$

$3\cos^2 x = \pi + \cos^{-1} 2x$

$\cos(3\cos^{-1} x) = \cos(\pi + \cos^{-1} 2x)$

$4x^3 - 3x = -2x$

$4x^3 = x \Rightarrow x = 0, \pm\frac{1}{2}$

All satisfy the original equation

sum $= -\frac{1}{2}$ to $+\frac{1}{2} = 0$

65. (b) Well check each option

For A $\pi = \text{v of } 0 = \Lambda$

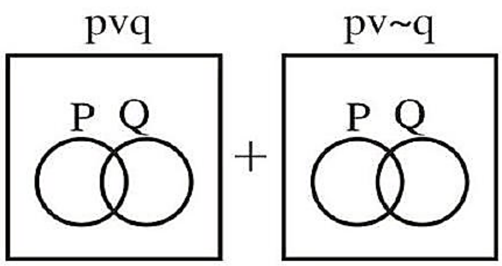
$(p \vee q) \wedge (p \vee \sim q)$

$\equiv p v(q \wedge \sim q)$

$\equiv p v(c) \equiv p$

For B : $* = v, O = v$

$(pvq) \vee (p \vee \sim q) \equiv t$ using Venn Diagrams



66. (b)

$|\vec{a}|=9\&(x\vec{a}+y\vec{b})\cdot(6y\vec{a}-18x\vec{b})=0$

$\Rightarrow 6xy|\bar{a}|^2-18x^2(\bar{a}\cdot\bar{b})+6y^2(\bar{a}\cdot\bar{b})-18xy|\bar{b}|^2=0$

$\Rightarrow 6xy\left(|\vec{a}|^2-3|\bar{b}|^2\right)+(\bar{a}\cdot\bar{b})(y^2-3x^2)=0$

This should hold $\forall x,y\in R\times R$

$\therefore |\bar{a}|^2=3|\bar{b}|^2\&(\bar{a}\cdot\bar{b})=0$

Now $|\bar{a}\times\bar{b}|^2=|\bar{a}|^2|\bar{b}|^2-(\bar{a}\cdot\bar{b})^2$

$=|\bar{a}|^2\cdot\dfrac{|\bar{a}|^2}{3}$

$\therefore |\bar{a}\times\bar{b}|=\dfrac{|\bar{a}|^2}{\sqrt{3}}=\dfrac{81}{\sqrt{3}}=27\sqrt{3}$

67. (b) $s\equiv\sin t, C\equiv\cos t$

Let ortho centre be (h,k)

Since it if an equilateral triangle hence orthocentre coincides with centroid.

$\therefore a+s+c=3h,\ b+s-c=3k$

$\therefore (3h-a)^2+(3k-b)^2=(s+c)^2+(s-c)^2=2(s^2+c^2)=2$

$\therefore \left(h-\frac{a}{3}\right)^2+\left(K-\frac{b}{3}\right)^2=\frac{2}{9},$

circle centre at $\left(\frac{a}{3},\frac{b}{3}\right)$

Gives, $\frac{a}{3}=1,\frac{b}{3}=\frac{1}{3}\Rightarrow a=3, b=1$

$\therefore a^2-b^2=8$

68. (d)

For R to be reflexive $\Rightarrow xRx$

$\Rightarrow 3x+\alpha x=7x\Rightarrow(3+\alpha)x=7K$

$\Rightarrow 3+\alpha=7\lambda\Rightarrow\alpha=7\lambda-3=7N+4, K,\lambda,N\in I$

$\therefore$ when α divided by 7 , remainder is 4 . R to be symmetric $xRy\Rightarrow yRx$

$3x+\alpha y=7N_1, 3y+\alpha x=7N_2$

$\Rightarrow(3+\alpha)(x+y)=7(N_1+N_2)=7N_3$

Which holds when $3+\alpha$ is multiple of 7

$\therefore \alpha=7N+4$ (as did earlier)

R to be transitive

$xRy \& yRz \Rightarrow xRz.$

$3x+\alpha y=7 N_1 \& 3y+\alpha z=7 N_2$

and

$3x+\alpha z=7 N_3$

$\therefore 3x+7 N_2-3y=7 N_3$

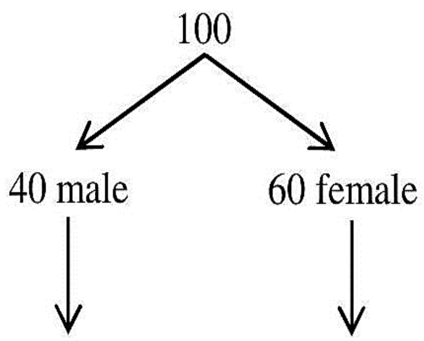
$\therefore 7 N_1-\alpha y+7 N_2-3y=7 N_3$

$\therefore 7(N_1+N_2)-(3+\alpha)y=7 N_3$

$\therefore (3+\alpha)y=7 N$

Which is true again when $3+\alpha$ divisible by 7 , i.e. when α divided by 7, remainder is 4 .

69. (a)



20 male qualified 40 female qualified

Probability that chosen candidate is female $=\frac{40}{60}=\frac{2}{3}$

70. (a) Given differential equation can be re-written as $\frac{dy}{dx}+(8+4\cot 2x)y=\frac{2e^{-4x}}{\sin^2 2x}(2\sin x+\cos 2x)$ which is a linear diff. equation.

$\text{I.f.}=e^{\int(8+4\cot 2x)dx}=e^{8x+2Cu(\sin 2x)}$

$=e^{8x}\cdot\sin^2 2x$

$\therefore$ solution is

$$y(e^{8x}\cdot\sin^2 2x)=\int 2e^{4x}(2\sin 2x+\cos 2x)dx+C$$
$$=e^{4x}\cdot\sin 2x+C$$

Given $y\left(\frac{\pi}{4}\right)=e^{-\pi}\Rightarrow C=0$

$$\therefore y=\frac{e^{-4x}}{\sin 2x}$$

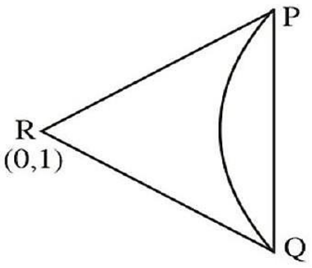
$$\therefore y\left(\frac{\pi}{6}\right)=\frac{e^{-4\cdot\frac{\pi}{6}}}{\sin\left(2\cdot\frac{\pi}{6}\right)}=\frac{2}{\sqrt{3}}e^{-\frac{2\pi}{3}}$$

71. **(b)** $y^2 = 2x - 3$

Equation of chord of contact

$PQ: r = 0$

$yx1 = (x + 0) - 3$
$y = x - 3$



from (1) and (2)

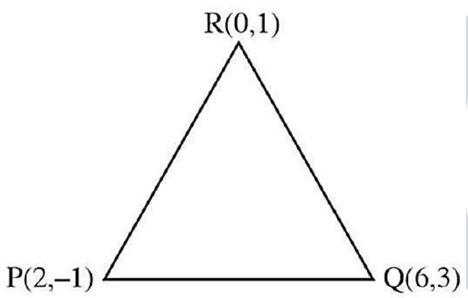
$(x \cdot 3)^2 = 2x - 3$

$x^2 - 8x + 12 = 0$

$(x - 2)(x - 6) = 0$

$x = 2$ or 6

$y = -1$ or 3



$MPQ = \frac{1}{4} = 1$

$MQR = \frac{2}{6} = \frac{1}{3}$

$MPR = \frac{2}{6} = \frac{1}{3}$

$MPR = \frac{2}{-2} = -1$

$MPQ \times MPR = - \Rightarrow PQ \perp PR$

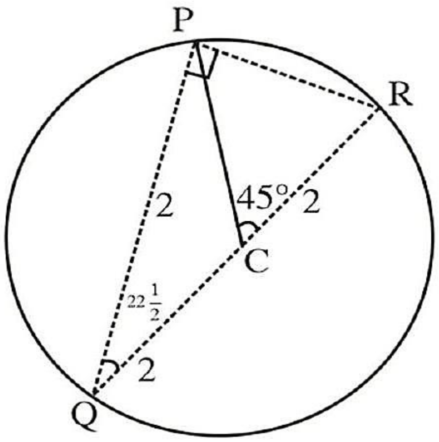
Orthocentre $= P(2, -1)$

72. **(b)** $x^2 + y^2 - x + 2y = \frac{11}{4}$

$\left(x-\frac{1}{2}\right)^2+(y+1)^2=(2)^2$

Or ΔPQR

$PR = QK\sin 2 \geq \frac{1}{3}$



$= 4\cdot 6\sin\frac{\pi}{8}$

$PQ = QR\cos 22\frac{1}{2}$

$\quad = 4\cos\frac{\pi}{8}$

As $\Delta PQR = \frac{1}{2}PR \times PQ$

$= \frac{1}{2}\left(4^2\sin\frac{\pi}{6}\right)\left(4\cos\frac{\pi}{8}\right)$

$= 4\sin\frac{\pi}{4} = \frac{4}{\sqrt{2}} = 2\sqrt{2}$

73. (c) $7^{2022}+3^{2022}$

$=(49)^{1011}+(9)^{1011}$

$=(50-1)^{1011}+(10-1)^{1011}$

$=5\lambda-1+5\text{ K}-1$

$=5\text{ m}-2$

Remainder $=5-2=3$

74. (c) $A^2=\begin{bmatrix}0&1&0\\0&0&1\\1&0&0\end{bmatrix}\begin{bmatrix}0&1&0\\0&0&1\\1&0&0\end{bmatrix}$

$=\begin{bmatrix}0&0&1\\1&0&0\\0&1&0\end{bmatrix}$

$a \leftrightarrow R_2$

$$-\begin{bmatrix}1 & 0 & 0\\0 & 0 & 1\\0 & 1 & 0\end{bmatrix}$$

$R_2 \leftrightarrow R_3$

$$\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix} = I$$

$B_0 = A^{49} + 2\,A^{98}$
$\quad = A + 2I$
$\quad B_n = \mathrm{Adj}(B_n - 1)$

$B_4 = \mathrm{Adj}\big(\mathrm{Adj}\big(\mathrm{Adj}(\mathrm{Adj}B_0)\big)\big)$
$= |B_0|^{\left((n-1)^4\right.}$
$= |B_0|^{16}$
$B_0 = \begin{bmatrix}0 & 1 & 0\\0 & 0 & 1\\1 & 0 & 0\end{bmatrix} + \begin{bmatrix}2 & 0 & 0\\0 & 2 & 0\\0 & 0 & 2\end{bmatrix}$
$= \begin{bmatrix}2 & 1 & 0\\0 & 2 & 1\\1 & 0 & 2\end{bmatrix}$
$= 2(4-0) - 1(0-1)$
$= 9$
$B_4(9)^{16} = (3)^{32}$

75. (c) $|z_2 + |z_2 - 1||^2 = |z_2 - |z_2 + 1||^2$

$$\Rightarrow |z_2 + |z_2 - 1||(\bar{z}_2 + |z_2 - 1|) = (z_2 - |z_2 + 1|)\big(\bar{z}_2 - (z_2 + 1)\big)$$

$\Rightarrow\ z_2|\bar{z}_2 + 12_2 - 1| - (\bar{z}_2 - |z_2 + 1|) + \bar{z}_2(|z_2 - 1| + |z_2 + 1|)$

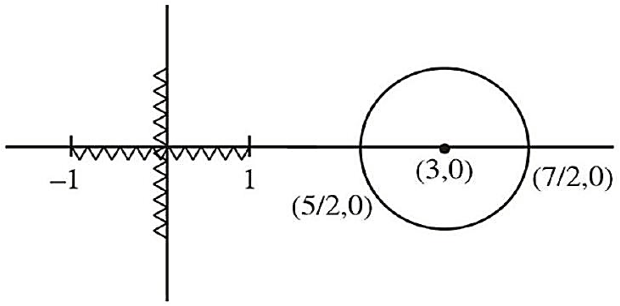
$$= |z_2 + 1|^2 = |z_2 - 1|^2$$

$\Rightarrow [z_2 + \bar{z}_2)(|z_2 - 1|) + (z_2 + 1\ |) = 2(z_2 + \bar{z}_2)$

$\Rightarrow (z_2 + \bar{z}_2)(|z_2 - 1| + |z_2 + 1| - 2) = 0$

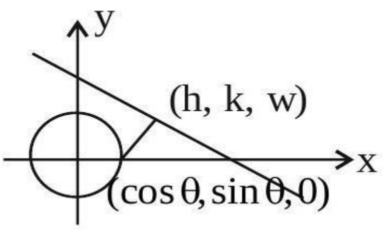
$\therefore z_2 + \bar{z}_2 = 0$ or $|z_2 - 1| + |z_2 + 1| - 2 = 0$

$\therefore z_2$ lie on imaginary axis. Or on real axis with in $[-1,1]$
Also $|z_1 - 3| = \frac{1}{2}$ lie on circle having centre 3 and radius $\frac{1}{2}$.



Clearly $|z_1 - z_2|\min = \frac{5}{2} - 1 = \frac{3}{2}$

76. (b)



$$\frac{h-\cos\theta}{2} = \frac{k-\sin\theta}{3} = \frac{w-0}{1}$$

$$= \frac{-1(2\cos\theta + 3\sin\theta - 6)}{14}$$

$$h = \cos\frac{-2(2\cos\theta + 3\sin\theta - 6)}{14}$$

$$= \frac{10\cos\theta - 6\sin\theta + 12}{14}$$

$$k = \sin\theta - \frac{3}{14}(2\cos\theta + 3\sin\theta - 6)$$

$$k = \frac{5\sin\theta - 6\cos\theta + 18}{14}$$

Elementary $\sin\theta$ and $\cos\theta$

$(5\,h + 6k - 12)^2 + 4(3\,h + 5k - 9)^2 = 1$

77. (c) $\frac{x^2}{2} + \frac{x^2}{2} + \frac{x^2}{2} + \frac{x^2}{2} + \frac{x^2}{2} + \frac{\alpha}{2x^5} + \frac{\alpha}{2x^5}$

$$\geq 7\left(\frac{\alpha^2}{2^7}\right)^{\frac{1}{7}}$$

$$\frac{7\cdot(\alpha)^{2/7}}{2} = 14$$

$(\alpha^2)^{1/7} = 2^2$

$\alpha = (2^2)^{7/2} = 2^7$

$\alpha = 128$

78. (0) Consider a case when $\alpha = \beta = 0$ then

$f(x) = yx$

$g(x) = \frac{x}{y}$

$$\frac{1}{n}\sum_{i=1}^{n} f(a_i) \Rightarrow \frac{y}{n}(a_1 + a_2 + \cdots .. + a_n)$$

$= 0$

$\Rightarrow f(g(0)) \Rightarrow f(0)$

$\Rightarrow 0$

79. (c) $a_{n+2}a_{n+1} - a_{n+1} \cdot a_n = 2$

Series will satisfy

$a_1a_2, a_2a_3, a_3a_4, a_4a_5,$

1.2 2.2 2.3 2.4

$$\frac{a_n + \frac{1}{a_{n+1}}}{a_{n+2}} = \frac{a_{n+2} - \frac{1}{a_{n+1}}}{a_{n+2}}$$

$$= 1 - \frac{1}{a_{n+1}a_{n+2}}$$

$$= 1 - \frac{1}{2(r+1)}$$

$$= \frac{2r+1}{2(r+1)}$$

Now proof is given by

$$= \prod_{r=1}^{30} \frac{(2r+1)}{2(r+1)}$$

$$= \frac{(1 \cdot 3 \cdot 5 \cdot \ldots \ldots \ldots \cdot 61)}{2^{30} \cdot (2 \cdot 3 \cdot \ldots \ldots . \cdot 31)}$$

$$\Rightarrow \frac{(1 \cdot 3 \cdot 5 \cdot \ldots \ldots . .61)}{\underline{31} \cdot 2^{30}} \times \frac{2^{30} \times \underline{30}}{2^{30} \times \underline{30}}$$

$$= \frac{\underline{61}}{2^{60}\underline{31} \cdot \underline{30}}$$

$\alpha = -60$

80. (a) $f(x) = e^x \cdot \int_0^x \frac{f'(t)}{e^t} dt$

$$f'(x) = e^x \cdot \int_0^x \frac{f'(t)}{e^t} dt + e^x \cdot \frac{f'(x)}{e^x}$$

$-[(2x-1) \cdot e^x + (x^2 - x + 1) \cdot e^x]$

$\int_0^x \frac{f'(t)}{e^t} dt = x^2 + x$

$\frac{f'(x)}{e^x} = 2x + 1$

$f'(x) = (2x+1) \cdot e^x$

$f'(x) = 0 \Rightarrow x = -\frac{1}{2}$

$f(x) = (2x+1) \cdot e^x - 2e^x + C$

$f(0) = -1$

$-1 = 1 - 2 + C$

$C = 0$

$f(x) = e^x(2x-1)$

$f\left(-\frac{1}{2}\right) = \frac{-2}{\sqrt{e}}$

81. (7073)

Required no. = Total - no character from {1,2,3,4,5}

$= (10^6 - 5^6) + (10^7 - 5^7) + (10^8 - 5^8)$

$= 10^6(1 + 10 + 100) - 5^6(1 + 5 + 25)$

$= 10^6 \times 111 - 5^6 \times 31$

$= 2^6 \times 5^6 \times 111 - 5^6 \times 31$

$= 5^6(2^6 \times 111 - 31)$

$= 5^6 \times \underbrace{7073}_{\alpha}$

$\therefore \alpha = 7073$

82. (450)

$RS \equiv (\alpha, -1, \beta)$

DR of PQ $\equiv \left(\frac{56}{17} + 2, \frac{43}{17} + 1, \frac{111}{17} - 1\right)$

$\equiv \left(\frac{90}{17}, \frac{60}{17}, \frac{94}{17}\right)$

$\frac{90}{17}\alpha + \frac{60}{17}(-1) + \frac{94}{17}\beta = 0$

$90\alpha + 94\beta = 60$

$\beta = \frac{60 - 90\alpha}{94}$

$\beta = \frac{30(2 - 3\alpha)}{94}$

$\beta = -30\frac{(3\alpha - 2)}{94}$

$\beta = \frac{-15}{47}(3\alpha - 2)$

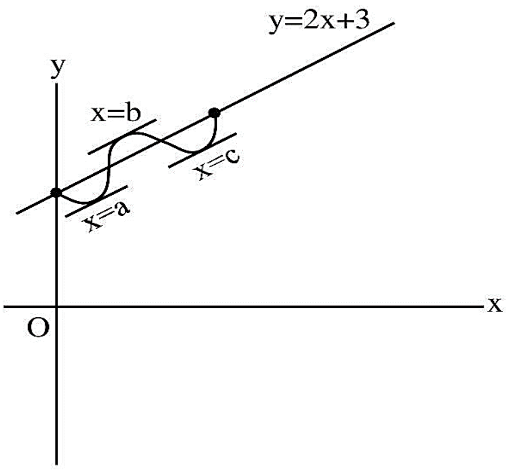
$\Rightarrow \frac{\beta}{-15} = \frac{3\alpha - 2}{47}$

$\Rightarrow \beta = -15, \alpha = -15$

$\alpha^2 + \beta^2 = 225 + 225$

$= 450$

83. (2)



$f'(a) = f'(b) = f'(c) = 2$

$\Rightarrow f''(x)$ is zero

for at-least $x_1 \in (a, b) \& x_2 \in (b, c)$

84. (10) Put $1 + x^2 = t^2$

$2xdx = 2tdt$

$Xdx = tdt$

$\therefore \int_1^2 \frac{15(t^2 - 1)tdt}{\sqrt{t^2 + t^3}}$

$15\int_1^2 \frac{t(t^2 - 1)}{t\sqrt{1 + t}}dt$

Put $1 + t = u^2$

$dt = 2udu$

$15\int_{\sqrt{2}}^{\sqrt{3}} \frac{(u^2-1)^2-1}{u} \times 2udu$

$30\int_{\sqrt{2}}^{\sqrt{3}} (u^4-2u^2)du$

$30\left(\frac{u^5}{5}-\frac{2u^3}{3}\right)_{\sqrt{2}}^{\sqrt{3}} 30\left[\frac{1}{5}\left(\sqrt{3}^5-\sqrt{2}^5\right)-\frac{2}{3}\left(\sqrt{3}^3-\sqrt{2}^3\right)\right]$

$30\left[\frac{1}{5}(9\sqrt{3}-4\sqrt{2})-\frac{2}{3}(3\sqrt{3}-2\sqrt{2})\right]$

$30\left[-\frac{1}{5}\times\sqrt{3}+\frac{8}{15}\sqrt{2}\right]$

$-6\sqrt{3}+16\sqrt{2}=\alpha\sqrt{2}+\beta\sqrt{3}$

$\alpha = 16, \beta = -6$

$\therefore \alpha+\beta = 10$

85. (2) $A+B=\begin{bmatrix}\beta+1 & 0\\ 3 & \alpha\end{bmatrix}$

$$(A+B)^2=\begin{bmatrix}\beta+1 & 0\\ 3 & \alpha\end{bmatrix}\begin{bmatrix}\beta+1 & 0\\ 3 & \alpha\end{bmatrix}$$
$$=\begin{bmatrix}(\beta+1)^2 & 0\\ 3(\beta+1)+3\alpha & \alpha^2\end{bmatrix}$$

$$A^2=\begin{bmatrix}1 & -1\\ 2 & \alpha\end{bmatrix}\begin{bmatrix}1 & -1\\ 2 & \alpha\end{bmatrix}$$
$$=\begin{bmatrix}-1 & -1-\alpha\\ 2+2\alpha & \alpha^2-2\end{bmatrix}$$

$$\therefore \begin{bmatrix}1 & -\alpha+1\\ 2\alpha+4 & \alpha^2\end{bmatrix}=\begin{bmatrix}(\beta+1)^2 & 0\\ 3(\alpha+\beta+1) & \alpha^2\end{bmatrix}$$

$\alpha = 1 = \alpha_1$

$B^2=\begin{bmatrix}\beta & 1\\ 1 & 0\end{bmatrix}\begin{bmatrix}\beta & 1\\ 1 & 0\end{bmatrix}$

$=\begin{bmatrix}\beta^2+1 & \beta\\ \beta & 1\end{bmatrix}=\begin{bmatrix}(\beta+1)^2 & 0\\ 3(\beta+1)+3\alpha & \alpha^2\end{bmatrix}$

$\therefore \beta=0, \alpha=-1=\alpha_2$

$|\alpha_1-\alpha_2|=|1-(-1)|=2$

86. (50) $f(x)=0 \Rightarrow (x-p)^2-q=0.$

Roots are $p+\sqrt{q}, p-\sqrt{q}$ absolute difference between roots $2\sqrt{q}$.

Now, $|f(a_i)| = 500$

Let a_1, a_2, a_3, a_4 are $a_1 a + d, a + 2\,d, a + 3\,d$

$|f(a_4)| = 500$

$|(a_1 - p)^2 - q| = 500$

$\Rightarrow (a_1 - p)^2 - q = 500$

$\Rightarrow \frac{9}{4}\,d^2 - q = 500$

and $|f(a_1)|^2 = |f(a_2)|^2$

$((a_1 - p)^2 - q)^2 = ((a_2 - p)^2 - q)^2$

$\Rightarrow ((a_1 - p)^2 - (a_2 - p)^2)((a_1 - p)^2 - q + (a_2 - p)^2 - q) = 0$

$\Rightarrow \frac{9}{4}\,d^2 - q + \frac{d^2}{4} - q = 0$

$2q = \frac{10\,d^2}{4} \Rightarrow q = \frac{5\,d^2}{4}$

$\Rightarrow d^2 = \frac{4q}{5}$

From equation (1) $\frac{9}{4} \cdot \frac{4.q}{5} - q = 500$

$\frac{4q}{5} = 500$
$\frac{4q}{5} = 500$

and $2\sqrt{q} = 2 \times \frac{50}{2} = 50$

87. (3)

$e_E = \sqrt{1 - \frac{b^2}{a^2}}, e_H = \sqrt{2}$

If $\Rightarrow e_E = \frac{1}{e_H}$

$\Rightarrow \frac{a^2 - b^2}{a^2} = \frac{1}{2}$

$2a^{2-2\,b}2 = a^2$

$a^2 = 2b^2$

and $y = \sqrt{\frac{5}{2}}x + k$ is tangent to ellipse then

$K^2 = a^2 \times \frac{5}{2} + b^2 = \frac{3}{2}$

$6b^2 = \frac{3}{2} \Rightarrow b^2 = \frac{1}{4}$ and $a^2 = \frac{1}{2}$

$\therefore 4 \cdot (a^2 + b^2) = 3$

88. (142)

$$\sum x_0^1 = \frac{3\left(1-\left(\frac{1}{2}\right)\right)^{20}}{1\frac{-1}{2}} = 6\left(1-\frac{1}{2^{20}}\right)$$

$$= \sum_{i=1}^{20} (x_{i-i})^2$$

$$= \sum_{i=1}^{20} (x_i)^2 + (i)^2 - 2x_i i$$

$$\text{Now } = \sum_{i=1}^{20} (x_i)^2 = \frac{9\left(1-\left(\frac{1}{4}\right)\right)^{20}}{1-\frac{1}{4}} = 12\left(1-\frac{1}{2^{40}}\right)$$

$$\sum_{i=1}^{20} i^2 = \frac{1}{6} \times 20 \times 21 \times 41 = 2870$$

$$\sum_{i=1}^{20} x_i i = s = 3 + 2.3\frac{1}{2} + 3.3\frac{1}{2^2} + 4.3\frac{1}{2^3} + \cdots \ldots \text{ AGP}$$

$$= 6\left(2-\frac{22}{2^{20}}\right)$$

$$\bar{x} = \frac{12 - \frac{12}{2^{40}} + 2870 - 12\left(2-\frac{22}{2^{20}}\right)}{20}$$

$$\bar{x} = \frac{2858}{20} + \left(\frac{-12}{2^{40}} + \frac{22}{2^{20}}\right) \times \frac{1}{20}$$

$$[\bar{x}] = 142$$

89. (1)

$$= e^{\lim\limits_{x\to 0}\left[\left(\frac{(x+2\cos x)^3+2(x+2\cos x)^2+3\sin(x+2\cos x)}{(x+2)^3+2(x\div 2)^2+3\sin(x+2)}\right)-1\right]\times\frac{100}{x}}$$

$$= e^{\lim\limits_{x\to 0}\left[\frac{100}{x}\left(\frac{(x+2\cos x)^3\div 2(x+2\cos x)^2+3\sin(x+2\cos x)-\left((x+2)^3+2(x\div 2)^2+3\sin(x\div 2)\right)}{(x+2)^3\div 2(x+2)^2\div 3\sin(x+2)}\right)\right]}$$

$$= e^{\lim\limits_{x\to 0}\frac{100}{x}\left[\left(\frac{(x+2\cos x)^3+(x\div 2)^3+2(x+2\cos x)^2-2(x+2)^2\div 3\sin(x+2\cos x)-3\sin(x+2)}{8\div 8+3\sin^2}\right)\right]}$$

$$= e^{\frac{100}{16\div 3\sin^2}\lim\limits_{x\to 0}\frac{3(x+2\cos x)^2 x(1+2\sin x)-3(x+2)^2-4(x\div 2\cos x)}{x(1-2\sin x)-4(x+2)+3\cos(x\div 2\cos x)\times(1-2\sin x)-3\cos(x+2)}}$$

$$= e^{\frac{100}{16\div 3\sin 2}}\left(\frac{12-3(4)+8\times 1-8+3\cos 2-3\cos 2}{1}\right)$$

Using L’H rule.

$$= e^{\circ} = 1$$

90. (6)