

PART-I PHYSICS

SECTION-A

- The dimensions of $\left(\frac{B^2}{\mu_0}\right)$ will be: (if μ_0 : permeability of free space and B : magnetic field)
 - $[ML^2 T^{-2}]$
 - $[MLT^{-2}]$
 - $[ML^{-1} T^{-2}]$
 - $[ML^2 T^{-2} A^{-1}]$
- A NCC parade is going at a uniform speed of 9 km/h under a mango tree on which a monkey is sitting at a height of 19.6 m. At any particular instant, the monkey drops a mango. A cadet will receive the mango whose distance from the tree at time of drop is: (Given $g = 9.8 \text{ m/s}^2$)
 - 5 m
 - 10 m
 - 19.8 m
 - 24.5 m
- In two different experiments, an object of mass 5 kg moving with a speed of 25 ms^{-1} hits two different walls and comes to rest within
 - 3 second, (ii) 5 seconds, respectively.

Choose the correct option out of the following:

- Impulse and average force acting on the object will be same for both the cases.
 - Impulse will be same for both the cases but the average force will be different.
 - Average force will be same for both the cases but the impulse will be different.
 - Average force and impulse will be different for both the cases.
- A balloon has mass of 10 g in air. The air escapes from the balloon at a uniform rate with velocity 4.5 cm/s. If the balloon shrinks in 5 s completely. Then, the average force acting on that balloon will be (in dyne).
 - 3
 - 9
 - 12
 - 18
 - If the radius of earth shrinks by 2% while its mass remains same. The acceleration due to gravity on the earth's surface will approximately:
 - decrease by 2%
 - decrease by 4%
 - increase by 2%
 - increase by 4%
 - The force required to stretch a wire of crosssection 1 cm^2 to double its length will be:
(Given Yong's modulus of the wire = $2 \times 10^{11} \text{ N/m}^2$)
 - $1 \times 10^7 \text{ N}$
 - $1.5 \times 10^7 \text{ N}$
 - $2 \times 10^7 \text{ N}$
 - $2.5 \times 10^7 \text{ N}$
 - A Carnot engine has efficiency of 50%. If the temperature of sink is reduced by 40°C , its efficiency increases by 30%. The temperature of the source will be:
 - 166.7 K
 - 255.1 K
 - 266.7 K
 - 367.7 K
 - Given below are two statements:

Statement I: The average momentum of a molecule in a sample of an ideal gas depends on temperature.

Statement II: The rms speed of oxygen molecules in a gas is v. If the temperature is doubled and the oxygen molecules dissociate into oxygen atoms, the rms speed will become 2v.

In the light of the above statements, choose the correct answer from the options given below:

- (a) Both Statement I and Statement II are true
 (b) Both Statement I and Statement II are false
 (c) Statement I is true but Statement II is false
 (d) Statement I is false but Statement II is true

9. In the wave equation

$$y = 0.5 \sin \frac{2\pi}{\lambda} (400t - x) \text{ m}$$

the velocity of the wave will be:

- (a) 200 m/s (b) $200\sqrt{2}$ m/s
 (c) 400 m/s (d) $400\sqrt{2}$ m/s

10. Two capacitors, each having capacitance $40\mu\text{F}$ are connected in series. The space between one of the capacitors is filled with dielectric material of dielectric constant K such that the equivalence capacitance of the system became $24\mu\text{F}$. The value of K will be:

- (a) 1.5 (b) 2.5
 (c) 1.2 (d) 3

11. A wire of resistance R_1 is drawn out so that its length is increased by twice of its original length. The ratio of new resistance to original resistance is:

- (a) 9:1 (b) 1:9
 (c) 4:1 (d) 3:1

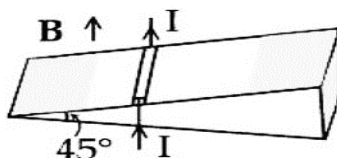
12. The current sensitivity of a galvanometer can be increased by:

- (A) decreasing the number of turns
 (B) increasing the magnetic field
 (C) decreasing the area of the coil
 (D) decreasing the torsional constant of the spring

Choose the most appropriate answer from the options given below:

- (a) (B) and (C) only (b) (C) and (D) only
 (c) (A) and (C) only (d) (B) and (D) only

13. As shown in the figure, a metallic rod of linear density 0.45 kg m^{-1} is lying horizontally on a smooth incline plane which makes an angle of 45° with the horizontal. The minimum current flowing in the rod required to keep it stationary, when 0.15 T magnetic field is acting on it in the vertical upward direction, will be: {Use $g = 10 \text{ m/s}^2$ }



- (a) 30 A (b) 15 A
 (c) 10 A (d) 3 A

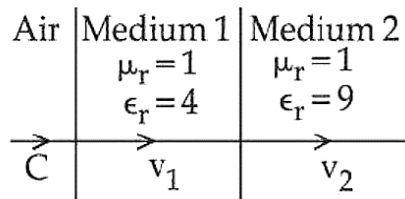
14. The equation of current in a purely inductive circuit is $5 \sin(49\pi t - 30^\circ)$. If the inductance is 30 mH then the equation for the voltage across the inductor, will be: (Let $\pi = \frac{22}{7}$)

- (a) $1.47 \sin(49\pi t - 30^\circ)$
 (b) $1.47 \sin(49\pi t + 60^\circ)$

(c) $23.1\sin(49\pi t - 30^\circ)$

(d) $23.1\sin(49\pi t + 60^\circ)$

15. As shown in the figure, after passing through the medium 1. The speed of light v_2 in medium 2 will be:
(Given $c = 3 \times 10^8 \text{ ms}^{-1}$)



- (a) $1.0 \times 10^8 \text{ ms}^{-1}$ (b) $0.5 \times 10^8 \text{ ms}^{-1}$
(c) $1.5 \times 10^8 \text{ ms}^{-1}$ (d) $3.0 \times 10^8 \text{ ms}^{-1}$

16. In normal adjustment, for a refracting telescope, the distance between objective and eye piece is 30 cm. The focal length of the objective, when the angular magnification of the telescope is 2, will be:

- (a) 20 cm (b) 30 cm
(c) 10 cm (d) 15 cm

17. The equation $\lambda = \frac{1.227}{x} \text{ nm}$ can be used to find the de-Broglie wavelength of an electron. In this equation x stands for:

Where,

m = mass of electron

P = momentum of electron

K = Kinetic energy of electron

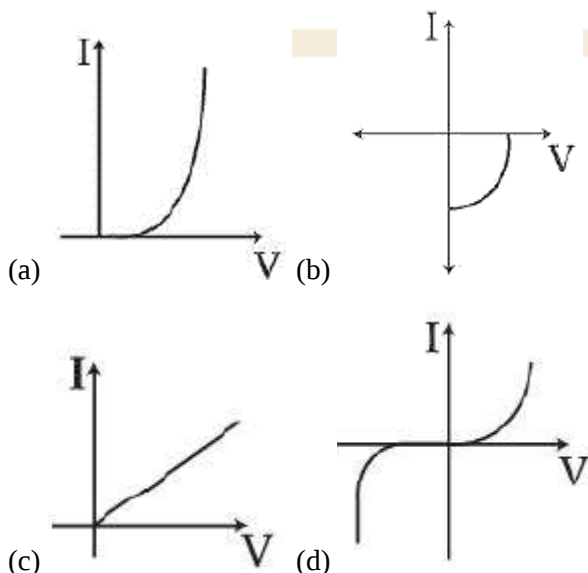
V = Accelerating potential in volts for electron

- (a) $\sqrt{mK}$ (b) $\sqrt{P}$
(c) $\sqrt{K}$ (d) $\sqrt{V}$

18. The half-life period of a radioactive substance is 60 days. The time taken for $\frac{7}{8}$ th of its original mass to disintegrate will be:

- (a) 120 days (b) 130 days
(c) 180 days (d) 20 days

19. Identify the solar cell characteristics from the following options:



20. In the case of amplitude modulation to avoid distortion the modulation index (μ) should be:

- (a) $\mu \leq 1$ (b) $\mu \geq 1$
(c) $\mu = 2$ (d) $\mu = 0$

SECTION-B

21. If the projection of $2\hat{i} + 4\hat{j} - 2\hat{k}$ on $\hat{i} + 2\hat{j} + \alpha\hat{k}$ is zero. Then, the value of α will be

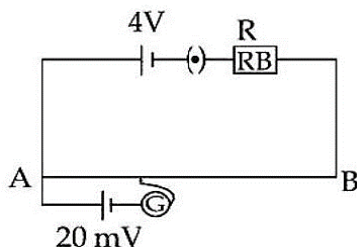
22. A freshly prepared radioactive source of half life

2 hours 30 minutes emits radiation which is 64 times the permissible safe level. The minimum time, after which it would be possible to work safely with source, will be _____ hours.

23. In a Young's double slit experiment, a laser light of 560 nm produces an interference pattern with consecutive bright fringes' separation of 7.2 mm. Now another light is used to produce an interference pattern with consecutive bright fringes' separation of 8.1 mm. The wavelength of second light is nm.

24. The frequencies at which the current amplitude in an LCR series circuit becomes $\frac{1}{\sqrt{2}}$ times its maximum value, are 212rads^{-1} and 232rads^{-1} . The value of resistance in the circuit is $R = 5\Omega$. The self-inductance in the circuit is mH.

25. As shown in the figure, a potentiometer wire of resistance 20Ω and length 300 cm is connected with resistance box (R.B.) and a standard cell of emf 4 V. For a resistance 'R' of resistance box introduced into the circuit, the null point for a cell of 20mV is found to be 60 cm. The value of 'R' is Ω .

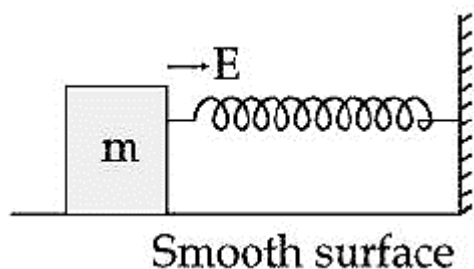


26. Two electric dipoles of dipole moments 1.2×10^{-30} cm and 2.4×10^{-30} cm are placed in two different uniform electric fields of strengths $5 \times 10^4 \text{NC}^{-1}$ and $15 \times 10^4 \text{NC}^{-1}$ respectively. The ratio of maximum torque experienced by the electric dipoles will be $\frac{1}{x}$. The value of x is

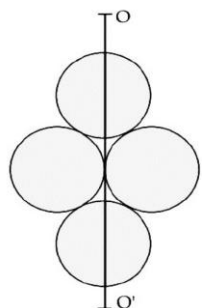
27. The frequency of echo will be Hz if the train blowing a whistle of frequency 320 Hz is moving with a velocity of 36 km/h towards a hill from which an echo is heard by the train driver. Velocity of sound in air is 330 m/s.

28. The diameter of an air bubble which was initially 2 mm, rises steadily through a solution of density 1750kg m^{-3} at the rate of 0.35cms^{-1} . The coefficient of viscosity of the solution is poise (in nearest integer). (the density of air is negligible).

29. A block of mass 'm' (as shown in figure) moving with kinetic energy E compresses a spring through a distance 25 cm when, its speed is halved. The value of spring constant of used spring will be $n \text{ENm}^{-1}$ for $n =$ _____



30. Four identical discs each of mass 'M' and diameter 'a' are arranged in a small plane as shown in figure. If the moment of inertia of the system about OO' is $\frac{x}{4} Ma^2$. Then, the value of x will be



PART-II CHEMISTRY

SECTION-A

31. Identify the incorrect statement from the following.

- (a) A circular path around the nucleus in which an electron moves is proposed as Bohr's orbit.
- (b) An orbital is the one electron wave function (Ψ) in an atom.
- (c) The existence of Bohr's orbits is supported by hydrogen spectrum.
- (d) Atomic orbital is characterized by the quantum numbers n and l only

32. Which of the following relation is not correct?

- (a) $\Delta H = \Delta U - P\Delta V$ (b) $\Delta U = q + W$
- (c) $\Delta S_{\text{sys}} + \Delta S_{\text{sur}} \geq 0$ (d) $\Delta G = \Delta H - T\Delta S$

33. Match List-I with List-II.

	List-I		List-II
(A)	$\text{Cd(s)} + 2\text{Ni(OH)}_3(\text{s}) \rightarrow$ $\text{CdO(s)} + 2\text{Ni(OH)}_2(\text{s}) +$ $\text{H}_2\text{O(l)}$	(I)	Primary battery

(B)	$\text{Zn(Hg)} + \text{HgO(s)} \rightarrow \text{ZnO(s)} + \text{Hg(l)}$	(II)	Discharging of secondary battery
(C)	$2\text{PbSO}_4(\text{s}) + 2\text{H}_2\text{O(l)} \rightarrow \text{Pb(s)} + \text{PbO}_2(\text{s}) + 2\text{H}_2\text{SO}_4(\text{aq})$	(III)	Fuel cell
(D)	$2\text{H}_2(\text{g}) + \text{O}_2(\text{g}) \rightarrow 2\text{H}_2\text{O(l)}$	(IV)	Charging of secondary battery

Choose the correct answer from the options given below:

- (a) (A) - (I), (B) - (II), (C) - (III), (D) - (IV)
 (b) (A) - (IV), (B) - (I), (C) - (II), (D) - (III)
 (c) (A) - (II), (B) - (I), (C) - (IV), (D) - (III)
 (d) (A) - (II), (B) - (I), (C) - (III), (D) - (IV)

34. Match List-I with List-II.

	List-I Reaction		List-II Catalyst
(A)	$4\text{NH}_3(\text{g}) + 5\text{O}_2(\text{g}) \rightarrow 4\text{NO(g)} + 6\text{H}_2\text{O(g)}$	(I)	NO(g)
(B)	$\text{N}_2(\text{g}) + 3\text{H}_2(\text{g}) \rightarrow 2\text{NH}_3(\text{g})$	(II)	H ₂ SO ₄ (l)

(C)	$\text{C}_{12}\text{H}_{22}\text{O}_{11}(\text{aq}) + \text{H}_2\text{O}(\text{l}) \rightarrow \text{C}_6\text{H}_{12}\text{O}_6(\text{Glucose}) + \text{C}_6\text{H}_{12}\text{O}_6(\text{Fructose})$	(III)	Pt(s)
(D)	$2\text{SO}_2(\text{g}) + \text{O}_2(\text{g}) \rightarrow 2\text{SO}_3(\text{g})$	(IV)	Fe(s)

Choose the correct answer from the options given below:

- (a) (A) - (II), (B) - (III), (C) - (I), (D) - (IV)
 (b) (A) - (III), (B) - (II), (C) - (I), (D) - (IV)
 (c) (A) - (III), (B) - (IV), (C) - (II), (D) - (I)
 (d) (A) - (III), (B) - (II), (C) - (IV), (D) - (I)

35. In which of the following pairs, electron gain enthalpies of constituent elements are nearly the same or identical?

- (a) Rb and Cs
 (b) Na and K
 (c) Ar and Kr
 (d) I and At

Choose the correct answer from the options given below:

- (a) (A) and (B) only (b) (B) and (C) only
 (c) (A) and (C) only (d) (C) and (D) only

36. Which of the reaction is suitable for concentrating ore by leaching process?

- (a) $2\text{Cu}_2\text{S} + 3\text{O}_2 \rightarrow 2\text{Cu}_2\text{O} + 2\text{SO}_2$
 (b) $\text{Fe}_3\text{O}_4 + \text{CO} \rightarrow 3\text{FeO} + \text{CO}_2$
 (c) $\text{Al}_2\text{O}_3 + 2\text{NaOH} + 3\text{H}_2\text{O} \rightarrow 2\text{Na}[\text{Al}(\text{OH})_4]$
 (d) $\text{Al}_2\text{O}_3 + 6\text{Mg} \rightarrow 6\text{MgO} + 4\text{Al}$

37. The metal salts formed during softening of hardwater using Clark's method are:

- (a) $\text{Ca}(\text{OH})_2$ and $\text{Mg}(\text{OH})_2$ (b) CaCO_3 and $\text{Mg}(\text{OH})_2$
 (c) $\text{Ca}(\text{OH})_2$ and MgCO_3 (d) CaCO_3 and MgCO_3

38. Which of the following statement is incorrect?

- (a) Low solubility of LiF in water is due to its small hydration enthalpy.
 (b) KO_2 is paramagnetic.
 (c) Solution of sodium in liquid ammonia is conducting in nature.

(d) Sodium metal has higher density than potassium metal

39. Match List-I with List-II, match the gas evolved during each reaction.

	List-I		List-II
(a)	$(\text{NH}_4)_2\text{Cr}_2\text{O}_7 \xrightarrow{\Delta}$	(I)	H_2
(b)	$\text{KMnO}_4 + \text{HCl} \rightarrow$	(II)	N_2
(c)	$\text{Al} + \text{NaOH} + \text{H}_2\text{O} \rightarrow$	(III)	O_2
(d)	$\text{NaNO}_3 \xrightarrow{\Delta}$	(IV)	Cl_2

Choose the correct answer from the options given below:

(a) (A) - (II), (B) - (III), (C) - (I), (D) - (IV)

(b) (A) - (III), (B) - (I), (C) - (IV), (D) - (II)

(c) (A) - (II), (B) - (IV), (C) - (I), (D) - (III)

(d) (A) - (III), (B) - (IV), (C) - (I), (D) - (II)

40. Which of the following has least tendency to liberate H_2 from mineral acids?

(a) Cu

(b) Mn

(c) Ni

(d) Zn

41. Given below are two statements:

Statement I: In polluted water values of both dissolved oxygen and BOD are very low.

Statement II: Eutrophication results in decrease in the amount of dissolved oxygen.

In the light of the above statements, choose the most appropriate answer from the options given below:

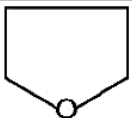
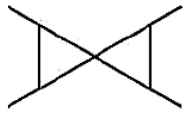
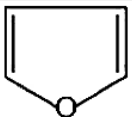
(a) Both Statement I and Statement II are true

(b) Both Statement I and Statement II are false

(c) Statement I is true but Statement II is false

(d) Statement I is false but Statement II is true

42. Match List-I with List-II.

	List-I		List-II
(A)		(I)	Spiro compound
(B)		(II)	Aromatic compound
(C)		(III)	Non-planar Heterocyclic compound
(D)		(IV)	Bicyclo compound

Choose the correct answer from the options given below:

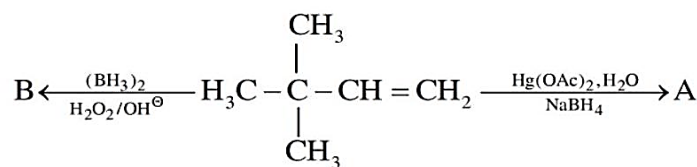
(a) (A) - (II), (B) - (I), (C) - (IV), (D) - (III)

(b) (A) - (IV), (B) - (III), (C) - (I), (D) - (II)

(c) (A) - (III), (B) - (IV), (C) - (I), (D) - (II)

(d) (A) - (IV), (B) - (III), (C) - (II), (D) - (I)

43. Choose the correct option for the following reactions.



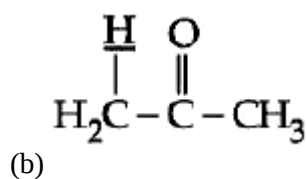
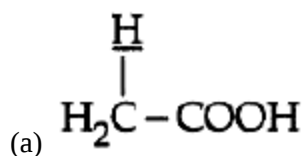
(a) 'A' and 'B' are both Markovnikov addition products.

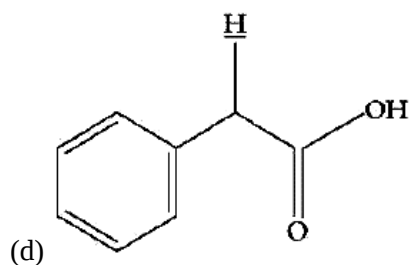
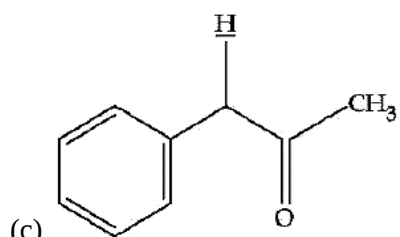
(b) 'A' is Markovnikov product and 'B' is antiMarkovnikov product.

(c) 'A' and 'B' are both anti-Markovnikov products.

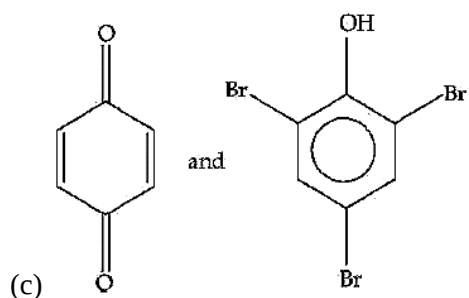
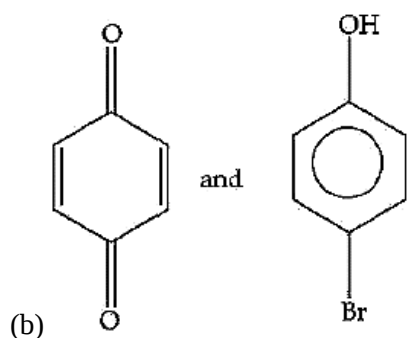
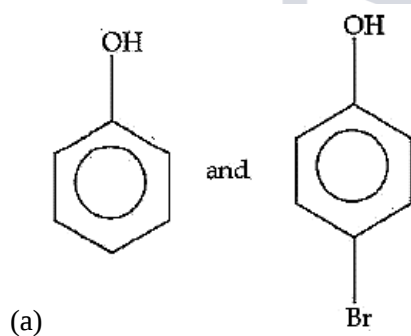
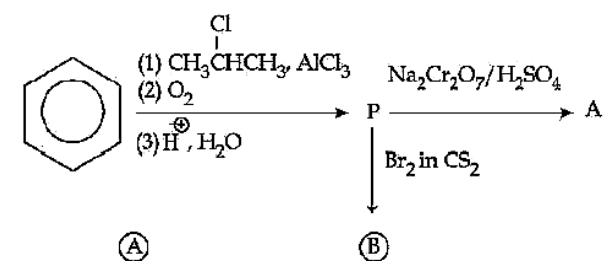
(d) 'B' is Markovnikov and 'A' is antiMarkovnikov product.

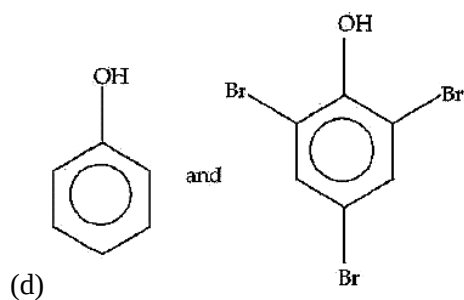
44. Among the following marked proton of which compound shows lowest pK_a value?



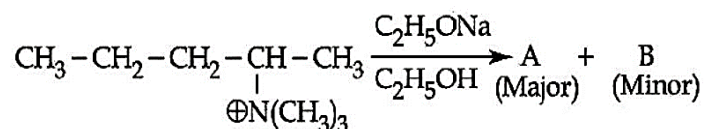


45. Identify the major product A and B for the below given reaction sequence.





46. Identify the correct statement for the below given transformation.



(a) A – $\text{CH}_3\text{CH}_2\text{CH} = \text{CH} - \text{CH}_3$,

B – $\text{CH}_3\text{CH}_2\text{CH}_2\text{CH} = \text{CH}_2$,

Saytzeff products

(b) A – $\text{CH}_3\text{CH}_2\text{CH} = \text{CH} - \text{CH}_3$,

B – $\text{CH}_3\text{CH}_2\text{CH}_2\text{CH} = \text{CH}_2$,

Hafmann products

(c) A – $\text{CH}_3\text{CH}_2\text{CH}_2\text{CH} = \text{CH}_2$,

B – $\text{CH}_3\text{CH}_2\text{CH} = \text{CHCH}_3$,

Hofmann products

(d) A – $\text{CH}_3\text{CH}_2\text{CH}_2\text{CH} = \text{CH}_2$,

B – $\text{CH}_3\text{CH}_2\text{CH} = \text{CHCH}_3$,

Saytzeff products

47. Terylene polymer is obtained by condensation of

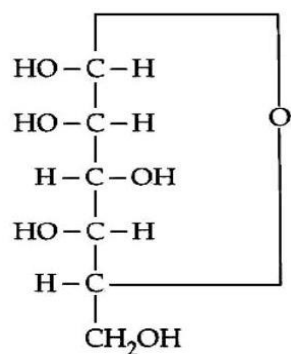
(a) Ethane-1, 2-diol and Benzene-1, 3 dicarboxylic acid

(b) Propane-1, 2-diol and Benzene-1, 4 dicarboxylic acid

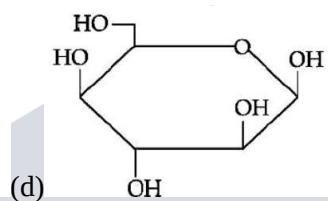
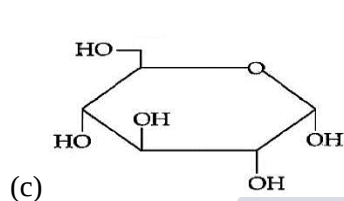
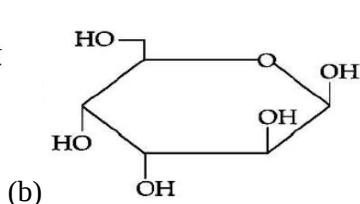
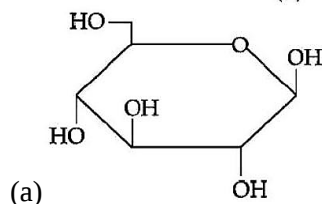
(c) Ethane-1, 2-diol and Benzene-1, 4 dicarboxylic acid

(d) Ethane-1, 2-diol and Benzene-1, 2 dicarboxylic acid

48. For the below given cyclic hemiacetal (X), the correct pyranose structure is :



(X)



49. Statements about Enzyme Inhibitor Drugs are given below:

(A) There are Competitive and Non-competitive inhibitor drugs.

(B) These can bind at the active sites and allosteric sites.

(C) Competitive Drugs are allosteric site blocking drugs.

(D) Non-competitive Drugs are active site blocking drugs.

Choose the correct answer from the options given below:

(a) (A), (D) only

(b) (A), (C) only

(c) (A), (B) only

(d) (A), (B), (C) only

50. For kinetic study of the reaction of iodide ion with H_2O_2 at room temperature:

(A) Always use freshly prepared starch solution.

(B) Always keep the concentration of sodium thiosulphate solution less than that of KI solution.

(C) Record the time immediately after the appearance of blue colour.

(D) Record the time immediately before the appearance of blue colour.

(E) Always keep the concentration of sodium thiosulphate solution more than that of KI solution. Choose the correct answer from the options given below:

(a) (A), (B), (C) only

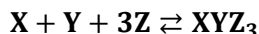
(b) (A), (D), (E) only

(c) (D), (E) only

(d) (A), (B), (E) only

SECTION-B

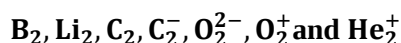
51. In the given reaction,



if one mole of each of X and Y with 0.05 mol of Z gives compound XYZ_3 . (Given: Atomic masses of X, Y and Z are 10, 20 and 30 amu, respectively). The yield of XYZ_3 is g. (Nearest integer)

52. An element M crystallises in a body centred cubic unit cell with a cell edge of 300 pm. The density of the element is 6.0 g cm^{-3} . The number of atoms present in 180 g of the element is $\times 10^{23}$ (Nearest integer)

53. The number of paramagnetic species among the following is



54. 150 g of acetic acid was contaminated with 10.2 g ascorbic acid ($C_6H_8O_6$) to lower down its freezing point by $(x \times 10^{-1})^\circ \text{C}$. The value of x is

(Nearest integer) [Given $K_f = 3.9 \text{ K kg mol}^{-1}$; Molar mass of ascorbic acid = 176 g mol^{-1}]

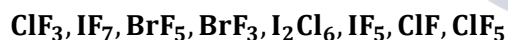
55. K_a for butyric acid (C_3H_7COOH) is 2×10^{-5} . The pH of 0.2 M solution of butyric acid is $\times 10^{-1}$. (Nearest integer) [Given $\log 2 = 0.30$]

56. For the given first order reaction



the half-life of the reaction is 0.3010 min. The ratio of the initial concentration of reactant to the concentration of reactant at time 2.0 min will be equal to . ____ (Nearest integer)

57. The number of interhalogens from the following having square pyramidal structure is:



58. The disproportionation of MnO_4^{2-} in acidic medium resulted in the formation of two manganese compounds A and B. If the oxidation state of Mn in B is smaller than that of A, then the spin-only magnetic moment (μ) value of B in BM is (Nearest integer)

59. Total number of relatively more stable isomer(s) possible for octahedral complex $[Cu(en)_2(SCN)_2]$ will be

60. On complete combustion of 0.492 g of an organic compound containing C, H and O, 0.7938 g of CO_2 and 0.4428 g of H_2O was produced. The % composition of oxygen in the compound is

PART-III MATHEMATICS

SECTION-A

61. Let the solution curve of the differential equation $xydy = (\sqrt{x^2 + y^2} + y) dx$, $x > 0$, intersect the line $x = 1$ at $y = 0$ and the line $x = 2$ at $y = \alpha$. Then the value of α is:
- (a) $\frac{1}{2}$ (b) $\frac{3}{2}$
(c) $-\frac{3}{2}$ (d) $\frac{5}{2}$
62. Considering only the principal values of the inverse trigonometric functions, the domain of the function $f(x) = \cos^{-1}\left(\frac{x^2 - 4x + 2}{x^2 + 3}\right)$ is :
- (a) $\left(-\infty, \frac{1}{4}\right]$ (b) $\left[-\frac{1}{4}, \infty\right)$
(c) $\left(-\frac{1}{3}, \infty\right)$ (d) $\left(-\infty, \frac{1}{3}\right]$
63. Let the vectors $\vec{a} = (1 + t)\hat{i} + (1 - t)\hat{j} + \hat{k}$, $\vec{b} = (1 - t)\hat{i} + (1 + t)\hat{j} + 2\hat{k}$ and $\vec{c} = t\hat{i} - t\hat{j} + \hat{k}$, $t \in \mathbb{R}$ be such that for $\alpha, \beta, \gamma \in \mathbb{R}$, $\alpha\vec{a} + \beta\vec{b} + \gamma\vec{c} = \vec{0} \Rightarrow \alpha = \beta = \gamma = 0$. Then, the set of all values of t is:
- (a) a non-empty finite set (b) equal to $\mathbb{N}$
(c) equal to $\mathbb{R} - \{0\}$ (d) equal to $\mathbb{R}$
64. Considering the principal values of the inverse trigonometric functions, the sum of all the solutions of the equation $\cos^{-1}(x) - 2\sin^{-1}(x) = \cos^{-1}(2x)$ is equal to:
- (a) 0 (b) 1
(c) $\frac{1}{2}$ (d) $-\frac{1}{2}$
65. Let the operations $*, \odot \in \{\wedge, \vee\}$. If $(p * q) \odot (p \odot \sim q)$ is a tautology, then the ordered pair $(*, \odot)$ is:
- (a) $(\vee, \wedge)$ (b) $(\vee, \vee)$
(c) $(\wedge, \wedge)$ (d) $(\wedge, \vee)$
66. Let a vector $\vec{a}$ has a magnitude 9. Let a vector $\vec{b}$ be such that for every $(x, y) \in \mathbb{R} \times \mathbb{R} - \{(0, 0)\}$, the vector $(x\vec{a} + y\vec{b})$ is perpendicular to the vector $(6y\vec{a} - 18x\vec{b})$. Then the value of $|\vec{a} \times \vec{b}|$ is equal to:
- (a) $9\sqrt{3}$ (b) $27\sqrt{3}$
(c) 9 (d) 81
67. For $t \in (0, 2\pi)$, if ABC is an equilateral triangle with vertices $A(\sin t, -\cos t)$, $B(\cos t, \sin t)$ and $C(a, b)$ such that its orthocentre lies on a circle with centre $\left(1, \frac{1}{3}\right)$, then $(a^2 - b^2)$ is equal to:
- (a) $\frac{8}{3}$ (b) 8
(c) $\frac{77}{9}$ (d) $\frac{80}{9}$
68. For $\alpha \in \mathbb{N}$, consider a relation R on $\mathbb{N}$ given by $R = \{(x, y) : 3x + \alpha y \text{ is a multiple of } 7\}$. The relation R is an equivalence relation if and only if:
- (a) $\alpha = 14$
(b) α is a multiple of 4
(c) 4 is the remainder when α is divided by 10
(d) 4 is the remainder when α is divided by 7
69. Out of 60% female and 40% male candidates appearing in an exam, 60% candidates qualify it. The number of females qualifying the exam is twice the number of males qualifying it. A candidate is randomly chosen from the qualified candidates. The probability, that the chosen candidate is a female, is:
- (a) $\frac{3}{4}$ (b) $\frac{11}{16}$
(c) $\frac{23}{32}$ (d) $\frac{13}{16}$
70. If $y = y(x)$, $x \in \left(0, \frac{\pi}{2}\right)$ be the solution curve of the differential equation

$$(\sin^2 2x) \frac{dy}{dx} + (8\sin^2 2x + 2\sin 4x)y = 2e^{-4x}(2\sin 2x + \cos 2x), \text{ with } y\left(\frac{\pi}{4}\right) = e^{-\pi},$$

then $y\left(\frac{\pi}{6}\right)$ is equal to:

- (a) $\frac{2}{\sqrt{3}}e^{-2\pi/3}$ (b) $\frac{2}{\sqrt{3}}e^{2\pi/3}$
(c) $\frac{1}{\sqrt{3}}e^{-2\pi/3}$ (d) $\frac{1}{\sqrt{3}}e^{2\pi/3}$

71. If the tangents drawn at the points P and Q on the parabola $y^2 = 2x - 3$ intersect at the point R(0, 1), then the orthocentre of the triangle PQR is:

- (a) (0,1) (b) (2, -1)
(c) (6,3) (d) (2,1)

72. Let C be the centre of the circle $x^2 + y^2 - x + 2y = \frac{11}{4}$ and P be a point on the circle. A line passes through the point C, makes an angle of $\frac{\pi}{4}$ with the line CP and intersects the circle at the points Q and R. Then the area of the triangle PQR (in unit²) is:

- (a) 2 (b) $2\sqrt{2}$
(c) $8\sin\left(\frac{\pi}{8}\right)$ (d) $8\cos\left(\frac{\pi}{8}\right)$

73. The remainder when $7^{2022} + 3^{2022}$ is divided by 5 is:

- (a) 0 (b) 2
(c) 3 (d) 4

74. Let the matrix $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$ and the matrix $B_0 = A^{49} + 2A^{98}$. If $B_n = \text{Adj}(B_{n-1})$ for all $n \geq 1$, then

$\det(B_4)$ is equal to:

- (a) 3^{28} (b) 3^{30}
(c) 3^{32} (d) 3^{36}

75. Let $S_1 = \{z_1 \in \mathbb{C} : |z_1 - 3| = \frac{1}{2}\}$, $S_2 = \{z_2 \in \mathbb{C} : |z_2 - |z_2 + 1|| = |z_2 + |z_2 - 1||\}$. Then, for $z_1 \in S_1$ and $z_2 \in S_2$, the least value of $|z_2 - z_1|$ is:

- (a) 0 (b) $\frac{1}{2}$
(c) $\frac{3}{2}$ (d) $\frac{5}{2}$

76. The foot of the perpendicular from a point on the circle $x^2 + y^2 = 1, z = 0$ to the plane $2x + 3y + z = 6$ lies on which one of the following curves?

- (a) $(6x + 5y - 12)^2 + 4(3x + 7y - 8)^2 = 1, z = 6 - 2x - 3y$
(b) $(5x + 6y - 12)^2 + 4(3x + 5y - 9)^2 = 1, z = 6 - 2x - 3y$
(c) $(6x + 5y - 14)^2 + 9(3x + 5y - 7)^2 = 1, z = 6 - 2x - 3y$
(d) $(5x + 6y - 14)^2 + 9(3x + 7y - 8)^2 = 1, z = 6 - 2x - 3y$

77. If the minimum value of $f(x) = \frac{5x^2}{2} + \frac{\alpha}{x^5}, x > 0$, is 14, then the value of α is equal to:

- (a) 32 (b) 64
(c) 128 (d) 256

78. Let α, β and γ be three positive real numbers. Let $f(x) = \alpha x^5 + \beta x^3 + \gamma x, x \in \mathbb{R}$ and $g: \mathbb{R} \rightarrow \mathbb{R}$ be such that $g(f(x)) = x$ for all $x \in \mathbb{R}$. If $a_1, a_2, a_3, \dots, a_n$ be in arithmetic progression with mean zero, then the value of

$f\left(g\left(\frac{1}{n} \sum_{i=1}^n f(a_i)\right)\right)$ is equal to:

- (a) 0 (b) 3
(c) 9 (d) 27

79. Consider the sequence $a_1, a_2, a_3, \dots$ such that $a_1 = 1, a_2 = 2$ and $a_{n+2} = \frac{2}{a_{n+1}} + a_n$ for $n = 1, 2, 3, \dots$. If

$$\left(\frac{a_1 + \frac{1}{a_2}}{a_3}\right) \cdot \left(\frac{a_2 + \frac{1}{a_3}}{a_4}\right) \cdot \left(\frac{a_3 + \frac{1}{a_4}}{a_5}\right) \cdots \left(\frac{a_{30} + \frac{1}{a_{31}}}{a_{32}}\right) = 2^\alpha ({}^{61}C_{31}), \text{ then } \alpha \text{ is equal to:}$$

- (a) -30 (b) -31
(c) -60 (d) -61

80. The minimum value of the twice differentiable function $f(x) = \int_0^x e^{x-t} f'(t) dt - (x^2 - x + 1)e^x, x \in \mathbb{R}$, is:

- (a) $-\frac{2}{\sqrt{e}}$ (b) $-2\sqrt{e}$
(c) $-\sqrt{e}$ (d) $\frac{2}{\sqrt{e}}$

SECTION-B

81. Let S be the set of all passwords which are six to eight characters long, where each character is either an alphabet from $\{A, B, C, D, E\}$ or a number from $\{1, 2, 3, 4, 5\}$ with the repetition of characters allowed. If the number of passwords in S whose at least one character is a number from $\{1, 2, 3, 4, 5\}$ is $\alpha \times 5^6$, then α is equal to _____

82. Let $P(-2, -1, 1)$ and $Q\left(\frac{56}{17}, \frac{43}{17}, \frac{111}{17}\right)$ be the vertices of the rhombus $PRQS$. If the direction ratios of the diagonal RS are $\alpha, -1, \beta$, where both α and β are integers of minimum absolute values, then $\alpha^2 + \beta^2$ is equal to _____

83. Let $f: [0, 1] \rightarrow \mathbb{R}$ be a twice differentiable function in $(0, 1)$ such that $f(0) = 3$ and $f(1) = 5$. If the line $y = 2x + 3$ intersects the graph of f at only two distinct points in $(0, 1)$, then the least number of points $x \in (0, 1)$, at which $f''(x) = 0$, is

84. If $\int_0^{\sqrt{3}} \frac{15x^3}{\sqrt{1+x^2} + \sqrt{(1+x^2)^3}} dx = \alpha\sqrt{2} + \beta\sqrt{3}$, where α, β are integers, then $\alpha + \beta$ is equal to

85. Let $A = \begin{bmatrix} 1 & -1 \\ 2 & \alpha \end{bmatrix}$ and $B = \begin{bmatrix} \beta & 1 \\ 1 & 0 \end{bmatrix}, \alpha, \beta \in \mathbb{R}$. Let α_1 be the value of α which satisfies $(A + B)^2 = A^2 + \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$ and α_2 be the value of α which satisfies $(A + B)^2 = B^2$. Then $|\alpha_1 - \alpha_2|$ is equal to

86. For $p, q \in \mathbb{R}$, consider the real valued function $f(x) = (x - p)^2 - q, x \in \mathbb{R}$ and $q > 0$. Let a_1, a_2, a_3 and a_4 be in an arithmetic progression with mean p and positive common difference. If $|f(a_i)| = 500$ for all $i = 1, 2, 3, 4$, then the absolute difference between the roots of $f(x) = 0$ is

87. For the hyperbola $H: x^2 - y^2 = 1$ and the ellipse $E: \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, a > b > 0$, let the

(1) eccentricity of E be reciprocal of the eccentricity of H , and

(2) the line $y = \frac{\sqrt{5}}{2}x + K$ be a common tangent of E and H .

Then $4(a^2 + b^2)$ is equal to

88. Let $x_1, x_2, x_3, \dots, x_{20}$ be in geometric progression with $x_1 = 3$ and the common ratio $\frac{1}{2}$. A new data is constructed replacing each x_i by $(x_i - i)^2$. If $\bar{x}$ is the mean of new data, then the greatest integer less than or equal to $\bar{x}$ is

89. $\lim_{x \rightarrow 0} \left(\frac{(x+2\cos x)^3 + 2(x+2\cos x)^2 + 3\sin(x+2\cos x)}{(x+2)^3 + 2(x+2)^2 + 3\sin(x+2)} \right)^{\frac{100}{x}}$ is equal to _____

90. The sum of all real values of x for which $\frac{3x^2-9x+17}{x^2+3x+10} = \frac{5x^2-7x+19}{3x^2+5x+12}$ is equal to_____

