1. (d) $[\alpha\beta] = [\eta] = [\sin\theta] = \text{Dimensionless}$

$[\eta^{-1}\sin\theta] = [\alpha\beta] = \text{D.L.}$

2. (b) $\vec{r} = 3t\hat{i} + 5t^3\hat{j} + 7k$

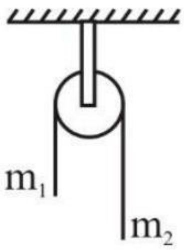
$$\frac{d^2\vec{r}}{dt^2} = 30t\hat{j}$$

At $t = 1 \Rightarrow \frac{d^2\vec{r}}{dt^2} = 30\hat{j}$

3. (d) $F = \rho a v^2 = 10^3 \times 10 \times 10^{-4} \times 20 \times 20$

$F = 400$

4. (d)



$$a = \frac{(m_2 - m_1)}{(m_2 + m_1)} g$$
$$\frac{g}{2} = \frac{(\lambda(L-\ell) - \lambda\ell)g}{\lambda L} \Rightarrow L = \frac{L}{4} = \frac{L}{x}$$
$$x = 4$$

5. (a) Using $mv = \sqrt{2mk}$

$$u = \frac{1}{0.2}\sqrt{2 \times 0.2 \times 90} = 30 \text{ m/s}$$

$$v = \frac{1}{0.2}\sqrt{2 \times 0.2 \times 40} = 20 \text{ m/s}$$

$$a = \frac{20 - 30}{1} = -10 \text{ m/s}^2$$

$$s = \frac{-u^2}{2a} = 45 \text{ m}$$

6. (c)

$t \propto \frac{1}{\sqrt{g}}$ and $g \propto \frac{1}{(R+h)^2}$

$$\frac{t_1}{t_2} = \sqrt{\frac{g'}{g}} = \sqrt{\frac{R^2}{(R+h)^2}}$$

$$\frac{t_1}{t_2} = \frac{4}{6} = \frac{R}{(R+h)} \Rightarrow h = 3200 \text{ km}$$

7. (c) Apply Bernoulli's theorem between Piston and hole $P_A + \rho g h = P_0 + \frac{1}{2}\rho v_e^2$

Assuming there is no atmospheric pressure on piston

$$\frac{5 \times 10^5}{\pi} + 10^3 \times 10 \times 10 = 1.01 \times 10^5 + \frac{1}{2} \times 10^3 \times v_e^2$$

$v_e = 17.8\ m/s$

8. (c)

$v_{rms} \propto \sqrt{T}$

$v_{rms} \propto \sqrt{300\ K}, v_{rms_f} = 2v_{rms_i}$

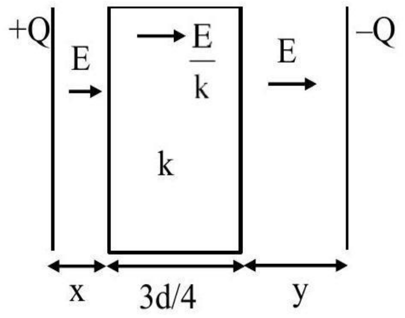
$v_{rms_f} \propto \sqrt{1200\ K}$

$$T_f = 1200\ K,\ T_i = 300\ K, n = \frac{14}{28} = \frac{1}{2}$$

$$Q = nC_v\Delta T = \frac{1}{2} \times \frac{5R}{2} \times 900$$

$Q = 9360\ J$

9. (a)



$$x + y + \frac{3d}{4} = d$$

$$x + y = \frac{d}{4}$$

$$\frac{A \in_0}{d} = C_0$$

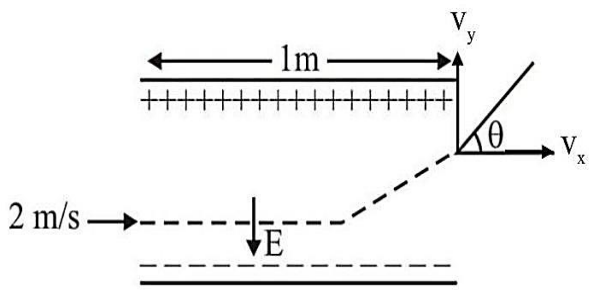
$$\Delta V = Ex + \frac{E}{k} \times \frac{3\ d}{4} + Ey$$

$$= \frac{3Ed}{4k} + E(x + y)$$

$$\Delta V = E\left[\frac{3\ d}{4k} + \frac{d}{4}\right] \Delta V = \frac{\sigma}{\epsilon_0}\left[\frac{3\ d + dk}{4k}\right] = \frac{Qd}{A\epsilon_0}\left[\frac{3+k}{4k}\right]$$

$$\frac{Q}{\Delta V} = C = \frac{A\in_0}{d}\left[\frac{4k}{3+k}\right] = \frac{4kC_0}{k+3}$$

10. (b)



$$a_y = \frac{F_y}{m} = \frac{e(E)}{m} = \frac{e\left(\frac{8m}{e}\right)}{m} = 8\ m/s^2$$

$s_x = u_x t$

$1 = 2 \times t$

$t = \frac{1}{2}\sec$

$v_y = u_y + a_y t$

$v_y = 0 + 8 \times \frac{1}{2}$

$v_y = 4\ m/s$

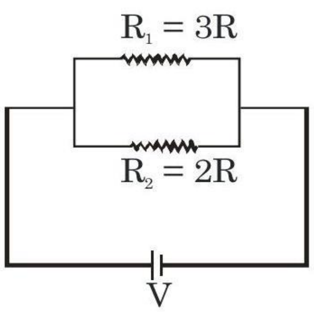
$\tan\theta = \frac{v_y}{v_x} = \frac{4}{2} = 2 \Rightarrow \theta = \tan^{-1}$

11. (c) Statement 1 $- R = 80\Omega$

$R_1 = R_2 = R_3 = R_4 = 20\Omega$

In parallel $R_{eq} = \frac{20}{4} = 5\Omega$

Statement 2 -



$$P_{th} = \frac{v^2}{R}$$

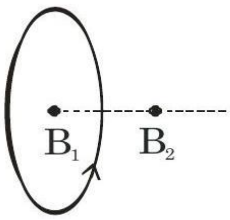
$\frac{P_1}{P_2} = \left(\frac{R_2}{R_1}\right) = \frac{2}{3}$ (where P is power)

12. (c) $F_M(CD) = BI\ell_{eff}$

$= 0.5 \times (10) \times (5 \sin 60 \times 10^{-2})$

$= 0.216\ N$

13. (c)



$$B_1 = \frac{\mu_0 I}{2R}$$

$$B_2 = \frac{\mu_0 I R^2}{2(R^2 + 3R^2)^{3/2}} = \frac{1}{8}\left(\frac{\mu_0 I}{2R}\right) = \frac{B_1}{8}$$

$$\frac{B_1}{B_2} = \frac{8}{1}$$

14. (c) $\frac{(8 \times 10^3)^2}{R_p} = 80 \times 10^3$

$R_P = 800\Omega$

$$\frac{(160)^2}{R_S} = 80 \times 10^3$$

$R_S = 0.32\Omega$

15. (d)

$\frac{I}{c} \times$ area $=$ force

$\frac{I}{C} \times 36 \times 10^{-4} = 7.2 \times 10^{-9}$

$$I = \frac{7.2 \times 10^{-9} \times 3 \times 10^8}{36 \times 10^{-9} \times 10}$$

$$= \frac{6 \times 10^{-1}}{10^{-3}}$$

$$I = 6 \times 10^2 \frac{W}{m^2}$$

$$= 0.06 \frac{W}{cm^2}$$

16. (b)

17. (c) $\frac{1}{2}mv_1^2 = 4\phi$

$$\frac{1}{2}mv_2^2 = 9\phi$$

$$\frac{v_1}{v_2} = \frac{2}{3}$$

18. (a) Remaining $= \frac{1}{8}$

$$3t_{1/2} = 15 \text{ min}$$

$$t_{1/2} = 5 \text{ min}$$

19. (b) $\frac{v_{out}}{v_{in}} = \beta\frac{R_{out}}{R_{in}}$

$$v_{out} = \frac{100 \times 10 \times 10^3}{10^3} \times 10^{-3}$$

$$= 1 \text{ V}$$

20. (d) Given

FM broadcast

Modulating frequency $= 20\text{kHz} = f$

Deviation ratio $= \frac{\text{frequency deviation}}{\text{modulating frequency}} = \frac{\Delta f}{f}$

$\Rightarrow$ frequency deviation $-\Delta f = f \times 10$

$$= 20\text{kHz} \times 10 = 200\text{kHz}$$

$\Rightarrow$ Bandwidth $= 2(f + \Delta f)$

$$= 2(20 + 200)\text{kHz}$$

$$= 440\text{kHz}$$

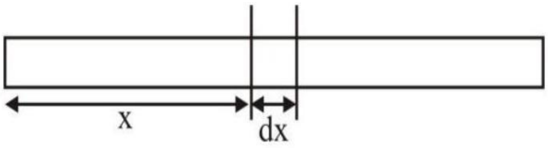
21. (392)

$$t_a = \frac{u}{g} = \frac{19.6}{9.8} = 2 \text{ s}$$

$$t_d = 6 - 2 \text{ s} = \sqrt{\frac{2\, h_{max}}{g}}$$

$$\Rightarrow h_{max} = \frac{16 \times 9.8}{2} = \frac{392}{5}$$

22. (8)

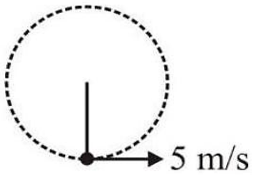


$$dm = \lambda \cdot dx = \lambda_0\left(1-\frac{x^2}{\ell^2}\right)$$

$$X_{cm} = \frac{\int xdm}{\int dm_\ell}$$

$$= \frac{\lambda_0 \int_0^\ell \left(x-\frac{x^3}{\ell^2}\right)dx}{\int_0^\ell \lambda_0\left(1-\frac{x^2}{\ell^2}\right)dx} = \frac{\frac{\ell^2}{2}-\frac{\ell^4}{4\ell^2}}{\ell-\frac{\ell^3}{3\ell^2}} = \frac{3\ell}{8}$$

23. (30)



Strain = F/AY

$$= \frac{mg+\frac{mv^2}{R}}{AY}$$

$$= \frac{20+\frac{2(5)^2}{0.5}}{3\times10^{-6}\times10^{11}} = 30\times10^{-5}$$

24. (750)

$$W = nR\Delta T = 150\ J$$

$$\mathbf{Q} = \left(\frac{f}{2}+1\right)nR\Delta T = \left(\frac{8}{2}+1\right)150 = 750\ J$$

25. (2) $U = 4(1-\cos 4x)$

$$F = -\frac{dU}{dx} = -4(+\sin 4x)4 = -16\sin(4x)$$

For small θ

$$\sin\theta \approx \theta$$

$$F = -64x$$

$$a = -64x/m = -16x$$

$$\omega^2 = 16$$

$T = \frac{2\pi}{\omega} = \frac{\pi}{2}$

26. (14) $R_1 = \frac{V^2}{P} = \frac{220^2}{100} = 484$

$R_2 = \frac{V^2}{P} = \frac{220^2}{60} = 484\left(\frac{10}{6}\right)$

$I = \frac{220}{484 + 484 \times \frac{10}{6}}$

$P_1 = I^2 R_1 = 14.06 \text{ W}$

27. (1) Just after closing the switch S, inductor behaves like an open circuit.

$I = \frac{6}{2+4} = 1 \text{ A}$

28. (400) After 10sec

$\mathbf{u} = -80 \text{ cm}$

$\mathbf{f} = -100 \text{ cm}$

$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

$v = 400 \text{ cm}$

29. (15) $vu = f^2$ (by Newton's formula)

$f^2 = 225$

$f = 15 \text{ cm}$

30. (5)

$T = 2\pi\sqrt{\frac{\ell}{g}}$

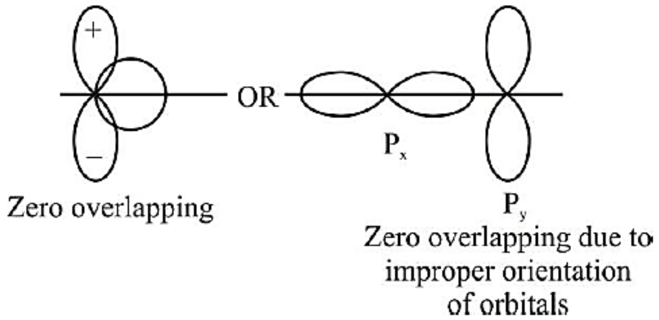
$g = \frac{1}{4\pi^2}\frac{T^2}{\ell}$

$\frac{\Delta g}{g} = \frac{2\Delta T}{T} + \frac{\Delta \ell}{\ell}$

$\frac{\Delta g}{g} = 2 \cdot \frac{1}{100 \times 0.5} + \frac{1 \text{ mm}}{10 \text{ cm}}$

$\frac{\Delta g}{g} = \frac{5}{100}$

31. (a)



32. (a) Across a period metallic character decreases

33. (a) $\Delta G = \Delta H - T\Delta S$

∵ Entropy of liquid is more than solid

∴ on melting the entropy increases and ΔG becomes more negative and hence it becomes easier to reduce metal

34. (c) In Clark's method lime water is used

$$Ca(HCO_3)_2 + 2Ca(OH)_2 \rightarrow 2CaCO_3 + 2H_2O$$
$$Mg(HCO_3)_2 + 2Ca(OH)_2 \rightarrow 2CaCO_3 + Mg(OH)_2 + 2H_2O$$

35. (b) Alloy of Li and Mg is used to make armour plates and not aircraft plates.

Calcium plays important roles in neuromuscular function, interneuronal transmission and cell membrane integrity

36. (b) $P_4 + 8SOCl_2 \rightarrow 4PCl_3 + 4SO_2 + 2\ S_2Cl_2$

37. (c) $I_2 + 10HNO_{3(\text{ conc })} \Rightarrow 2HIO_3 + 10NO_2 + 4H_2O$

38. (d)

$$\left.\begin{array}{l} Sm^{2+} \rightarrow \text{ electron } = 60 \\ Er^{3+} \rightarrow \text{ electron } = 65 \\ Tb^{2+} \rightarrow \text{ electron } = 63 \\ Tm^{4+} \rightarrow \text{ electron } = 65 \end{array}\right\} \text{(not isoelectronic)}$$

39. (a) $2KMnO_4 + 16HCl \rightarrow 2MnCl_2 + 2KCl + 8H_2O + Cl_2$ HCl gets oxidised by $KMnO_4$ into Cl_2

40. (b)

$Ni(CO)_4$ Hybridisation sp^3

$[Ni(CN)_4]^{2-}$ Hybridisation dsp^2

$[Co(CN)_6]^{3-}$ Hybridisation d^2sp^3

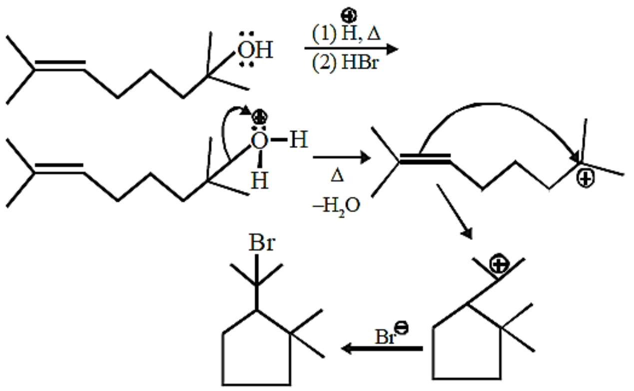
$[Co(F)_6]^{3-}$ Hybridisation $sp^3\ d^2$

41. (d)

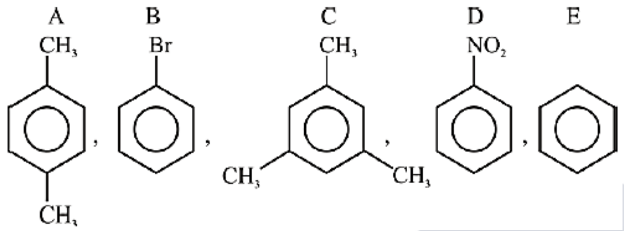
$$N_2 + O_2 \xrightleftharpoons{(1483\text{–}2000\ K)} 2NO$$

(Endothermic and feasible at high temperature)

42. (c)



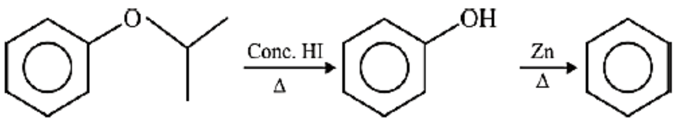
43. (b)



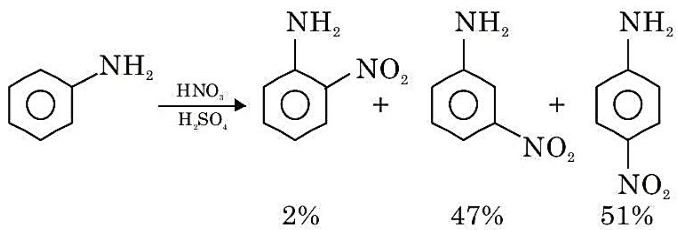
$-NO_2$ is strongly deactivating
$-Br$ – deactivating
$-CH_3$-activating group

D < B < E < A < C

44.(d)



45.(a)



Due to formation of anilinium ion in acidic medium meta product is also obtained in significant amount

46. (b)

Neoprene is elastomer

Nylon-6, 6 is fiber

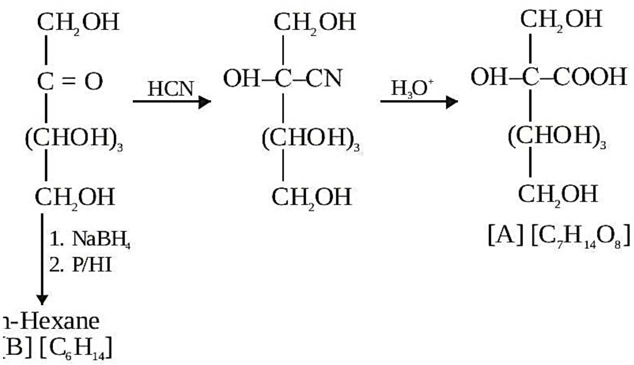
PVC is thermoplastic

Novolac is thermosetting

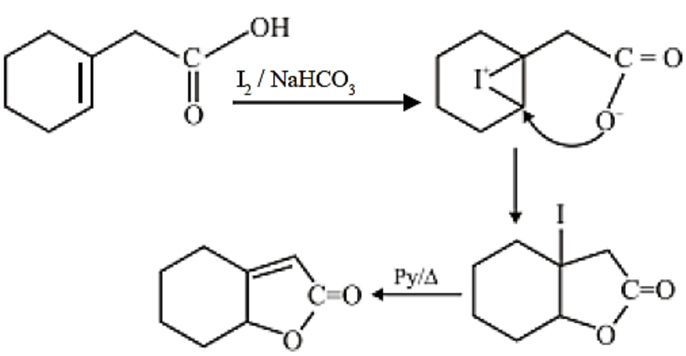
47. (d) Some drugs do not bind to active sites. These bind to different site of enzyme called allosteric sites.

48. (a) Thin layer chromatography (TLC) is another type of adsorption chromatography, which involve sepration of substance of a mixture ovel a thin layer of an adsorbent coated on glass plate. A thin layer (about 0.2 mm thick) of an adsorbent (silica gel) or (Alumina) in spread overa glass plate of suitable size. Hence Assertion (a) is correct and Reason (R) is correct explanation of (a)

49.(a)



50. (c)



51. (25) $H_2SO_4 + 2NaOH \rightarrow Na_2SO_4 + 2H_2O$

0.4 mol 0.2 mol -

0.3 mol $-$ 0.1 mol

Molarity of Na_2SO_4 is $\frac{0.1}{4} = 0.025M$

$= 25mM.$

52. (87)

$$a = 4 \times 10^{-8}\ \text{cm}$$
$$\mathbf{d} = 9.03\ \text{g/ml}$$
$$d = \frac{ZM}{N_A a^3}$$
$$M = \frac{9.03 \times 6.02 \times 10^{23} \times 64 \times 10^{-24}}{4} 86.97$$

53. (2)

$$\lambda = \frac{h}{\sqrt{2mK}}$$

$K=\frac{h^2}{2 m\lambda^2}$

$K=\frac{h^2}{2 m\lambda^2}=\frac{43.9\times10^{-68}}{2\times9.1\times10^{-31}\times10.89\times10^{-20}}$

$K=2.215\times10^{-18}$

$E_{abs}=E_{req}+K$

$\frac{E_{abs}}{E_{req}}=1+\frac{K}{E_{req}}=1+\frac{2.215\times10^{-18}}{13.6\times1.602\times10^{-19}}=2.0166$

54. (2) $Y_A=0.5\Rightarrow Y_B=0.5$

$\mathbf{P_A=P_B=0.4}\text{atm}$

$P_A=P_A^0X_A$

$P_A^0=2$

55. (2) $2NO+O_2\rightarrow2NO$

2 1 −

$2-2x$ $1-x$ $2x$

1.2 0.6 0.8

$K_p=\frac{\left(\frac{0.8}{2.6}\right)^2}{\left(\frac{1.2}{2.6}\right)^2\left(\frac{0.6}{2.6}\right)}=1.925$

56. (22)

$V=22.78\text{ml}, T=280\text{ K}$

$P_{total}=759\text{mmHg}$

$P_{N_2}=759-14.2=744.8\text{mmHg}$

$n_{N_2}=\frac{744.8\times22.78}{760\times1000\times0.082\times280}=0.00097$

$W_{Nitrogen}=0.02716$

$\%\text{ N}=\frac{0.02716}{0.125}\times1000=21.728$

57. (5) $IO_4^-+H_2O_2\rightarrow IO_3^-+O_2$

58. (8)

$K=Ae^{-Ea/RT}$

$\ln k = \frac{-Ea}{RT} + \ln A$

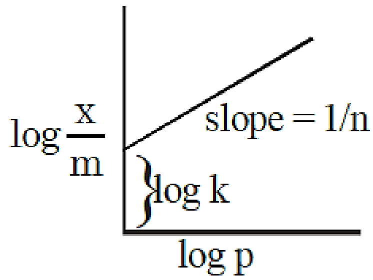
Slope $= \frac{Ea}{R} = \frac{20}{5}$

$E_a = 4R = 8\text{Cal/mol}$

59. (3)

$$\frac{X}{m} = KP^{\frac{1}{n}}$$

$$\log\frac{x}{m} = \frac{1}{n}\log p + \log k$$



60. (3) Internal energy, volume enthalpy are state variable

61. (d) $S \cap T = \{-5, -4, 3\}$

62. (b) $a = \frac{-1}{\alpha^2} - \frac{1}{\beta^2} - 2$

$$b = \frac{1}{\alpha^2} + \frac{1}{\beta^2} + 1 + \frac{1}{\alpha^2\beta^2}$$

$$a + b = \frac{1}{(\alpha\beta)^2} - 1 = \frac{1}{6} - 1 = -\frac{5}{6}$$

$$x^2 - \left(-\frac{5}{6} - 2\right)x + \left(2 - \frac{5}{6}\right) = 0$$

$$6x^2 + 17x + 7 = 0$$

$x = -\frac{7}{3}, x = -\frac{1}{2}$ are the roots

Both roots are real and negative.

63. (c) Given that $A^T = A, B^T = -B$

(a) $C = A^4 - B^4$

$C^T = (A^4 - B^4) = (A^4)^T - (B^4)^T = A^4 - B^4 = C$

(b) $C = AB - BA$

$C^T = (AB - BA)^T = (AB)^T - (BA)^T$

$= B^T A^T - A^T B^T = -BA + AB = C$

(c) $C = B^5 - A^5$

$C^T = (B^5 - A^5)^T = (B^5)^T - (A^5)^T = -B^5 - A^5$

(d) $C = AB + BA$

$C^T = (AB + BA)^T = (AB)^T + (BA)^T$

$= -BA - AB = -C$

$\therefore$ Option C is not true.

64. (d) $f(0) + 3 + \lambda + 4 = 14$

$\therefore f(0) = 7 - \lambda = c$

$f(1) = a + b + c = 3$

$f(3) = 9a + 3\,b + c = 4$

$f(-2) = 4a - 2\,b + c = \lambda$

(ii) - (iii)

$a + b = \frac{4-\lambda}{5}$ put in equation (i)

$$\frac{4-\lambda}{5} + 7 - \lambda = 3$$

$6\lambda = 24;\ \lambda = 4$

65. (b) n should be given as a natural number.

$$f\left(x = \begin{cases} \frac{-\sin(x-1)}{x-1} & x < -1 \\ -(\sin 2 + 1) & x = -1 \\ \cos 2\pi x & -1 < x < 1 \\ 1 & x = 1 \\ \frac{-\sin(x-1)}{x-1} & x > 1 \end{cases}\right.$$

$f(x)$ is discontinuous at $x = -1$ and $x = 1$

66. (a) $f(x) = xe^{x(1-x)}$

$f'(x) = -e^{x(1-x)}(2x + 1)(x - 1)$

$f(x)$ is increasing in $\left(-\frac{1}{2}, 1\right)$

67. (c)

68. (d) $x = 2\sqrt{2}\cos t\sqrt{\sin 2t}$

$$\frac{dx}{dt} = \frac{2\sqrt{2}\cos 3t}{\sqrt{\sin 2t}}$$

$y(t)=2\sqrt{2}\sin t\sqrt{\sin 2t}$

$$\frac{dy}{dt}=\frac{2\sqrt{2}\sin 3t}{\sqrt{\sin 2t}}$$

$$\frac{dy}{dx}=\tan 3t$$

$\frac{dy}{dx}=-1$ at $t=\frac{\pi}{4}$

$\frac{d^2y}{dx^2}=\frac{3}{2\sqrt{2}}\sec^3 3t\cdot\sqrt{\sin 2t}=-3$ at $t=\frac{\pi}{4}$

$$\therefore \frac{1+\left(\frac{dy}{dx}\right)^2}{\frac{d^2y}{dx^2}}=\frac{1+1}{-3}=-\frac{2}{3}$$

69. (a) $I_n(x)=\int_0^x\frac{dt}{(t^2+5)^n}$

Applying integral by parts

$$I_n(x)=\left[\frac{t}{(t^2+5)^n}\right]_0^x-\int_0^x n(t^2+5)^{-n-1}\cdot 2t^2$$

$$I_n(x)=\frac{x}{(x^2+5)^n}+2n\int_0^x\frac{t^2}{(t^2+5)^{n+1}}dt$$

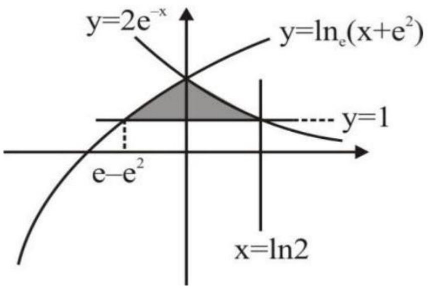
$$I_n(x)=\frac{x}{(x^2+5)^n}+2n\int_0^x\frac{(t^2+5)-5}{(t^2+5)^{n+1}}dt$$

$$I_n(x)=\frac{x}{(x^2+5)^n}+2nI_n(x)-10nI_{n+1}(x)$$

$10n\,I_{n+1}(x)+(1-2n)I_n(x)=\frac{x}{(x^2+5)^n}$

Put $n=5$

70. (b)



Required area is

$=\int_{e-e^2}^{0}\ln(x+e^2)-1dx+\int_0^{\ln 2}2e^{-x}-1dx=1+e-\ln 2$

71. (d) $\frac{dy}{dx}+\frac{1}{x^2-1}y=\left(\frac{x-1}{x+1}\right)^{\frac{1}{2}}$,

$$\frac{dy}{dx} + Py = Q$$

$$\text{I.F.} = e^{\int P dx} = \left(\frac{x-1}{x+1}\right)^{\frac{1}{2}}$$

$$y\left(\frac{x-1}{x+1}\right)^{\frac{1}{2}} = \int \left(\frac{x-1}{x+1}\right)^{1} dx$$

$$= x - 2\log_e|x+1| + C$$

Curve passes through $\left(2, \frac{1}{\sqrt{3}}\right)$

$$\Rightarrow C = 2\log_e 3 - \frac{5}{3}$$

at $x = 8$,

$$\sqrt{7}y(8) = 19 - 6\log_e 3$$

72. (a) Equation of circle passing through $(0, -2)$ and $(0,2)$ is

$$x^2 + (y^2 - 4) + \lambda x = 0, (\lambda \in R)$$

Divided by x we get

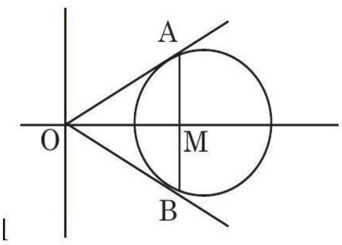
$$\frac{x^2 + (y^2 - 4)}{x} + \lambda = 0$$

Differentiating with respect to x

$$\frac{x\left[2x + 2y \cdot \frac{dy}{dx}\right] - [x^2 + y^2 - 4] \cdot 1}{x^2} = 0$$

$$\Rightarrow 2xy \cdot \frac{dy}{dx} + (x^2 - y^2 + 4) = 0$$

73. (b) $C: (x-2)^2 + y^2 = 1$



Equation of chord AB: $2x = 3$

$$OA = OB = \sqrt{3}$$

$$AM = \frac{\sqrt{3}}{2}$$

Area of triangle OAB $= \frac{1}{2}(2AM)(OM)$

$= \frac{3\sqrt{3}}{4}$ sq. units

74. (b) $H: \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

Foci : $S(ae, 0), S'(-ae, 0)$

Foot of directrix of parabola is $(-ae, 0)$

Focus of parabola is (ae, 0)

Now, semi latus rectum of parabola $= |SS'| = 2ae$

Given, $4ae = e\left(\frac{2b^2}{a}\right)$

$\Rightarrow b^2 = 2a^2$

Given, $(2\sqrt{2}, -2\sqrt{2})$ lies on H

$\Rightarrow \frac{1}{a^2} - \frac{1}{b^2} = \frac{1}{8}$

From (1) and (2)

$a^2 = 4,\ b^2 = 8$

$\because b^2 = a^2(e^2 - 1)$

$\therefore e = \sqrt{3}$

$\Rightarrow$ Equation of parabola is $y^2 = 8\sqrt{3}x$

75. (d) Given, $L_1: \frac{x-1}{\lambda} = \frac{y-2}{1} = \frac{z-3}{2}$

and $L_2: \frac{x+26}{-2} = \frac{y+18}{3} = \frac{z+28}{\lambda}$

are coplanar

$$\Rightarrow \begin{vmatrix} 27 & 20 & 31 \\ \lambda & 1 & 2 \\ -2 & 3 & \lambda \end{vmatrix} = 0$$

$\Rightarrow \lambda = 3$

Now, normal of plane P, which contains L_1 and L_2

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & 2 \\ -2 & 3 & 3 \end{vmatrix}$$

$= -3\hat{i} - 13\hat{j} + 11\hat{k}$

$\Rightarrow$ Equation of required plane P :

$3x + 13y - 11z + 4 = 0$

$(0,4,5)$ does not lie on plane P.

76. (b) Normal of plane P :

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -2 & 1 & -3 \\ -1 & 2 & -2 \end{vmatrix} = 4\hat{i} - \hat{j} - 3\hat{k}$$

Equation of plane P which passes through $(2,2,-2)$

is $4x - y - 3z - 12 = 0$

Now, A $(3,0,0), B(0,-12,0), C(0,0,-4)$

$\Rightarrow \alpha = 3, \beta = -12, \gamma = -4$

$\Rightarrow p = \alpha + \beta + \gamma = -13$

Now, volume of tetrahedron OABC

$$V = \left|\frac{1}{6}\overrightarrow{OA} \cdot (\overrightarrow{OB} \times \overrightarrow{OC})\right| = 24$$

$(V, p) = (24, -13)$

77. (c) For angle to be acute

$\vec{u} \cdot \vec{v} > 0$

$\Rightarrow a(\log_e b)^2 - 12 + 6a(\log_e b) > 0$

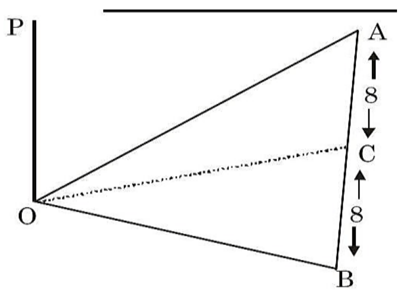
$\forall b > 1$

let $\log_e b = t \Rightarrow t > 0$ as $b > 1$

$y = at^2 + 6at - 12 \& y > 0, \forall t > 0$

$\Rightarrow a \in \phi$

78. (b)



$$\frac{OP}{OA} = \tan 15^\circ$$

$\Rightarrow OA = OP\cot 15^\circ$

$\frac{OP}{OC} = \tan 45^\circ \Rightarrow OP = OC$

Now, $OP = \sqrt{OA^2 - 8^2}$

$\Rightarrow OP^2 = (OP)^2 \cot^2 15^\circ - 64$

$\Rightarrow OP^2 = \frac{32}{\sqrt{3}}(2 - \sqrt{3})$

79. (a) $P(A \mid B) = \frac{1}{7} \Rightarrow \frac{P(A \cap B)}{P(b)} = \frac{1}{7}$

$\Rightarrow P(b) = \frac{7}{9}$

$P(B \mid A) = \frac{2}{5} \Rightarrow \frac{P(A \cap B)}{P(a)} = \frac{2}{5} \Rightarrow P(a) = \frac{5}{18}$

Now, $P(A' \cup B) = 1 - P(A \cup B) + P(b)$

$= 1 - P(a) + P(A \cap B) = \frac{5}{6}$

$P(A' \cap B') = 1 - P(A \cup B)$

$= 1 - P(a) - P(b) + P(A \cap B) = \frac{1}{18}$

$\Rightarrow$ Both (S1) and (S2) are true.

80. (d) $p \equiv$ Ramesh listens to music

$\sim q \equiv$ He is in village.

$r \vee s \equiv$ Saturday or sunday

$p \Rightarrow ((\sim q) \wedge (r \vee s))$

81. (57)

${}^4C_2 \times \frac{\beta^2}{6}, -6\beta, -{}^6C_3 \times \frac{\beta^3}{8}$ are in A.P

$\beta^2 - \frac{5}{2}\beta^3 = -12\beta$

$\beta = \frac{12}{5}$ or $\beta = -2 \therefore \beta = \frac{12}{5}$

$d = -\frac{72}{5} - \frac{144}{25} = -\frac{504}{25}$

$\therefore 50 - \frac{2d}{\beta^2} = 57$

82. (17)

$^{b}C_3 \times {}^{g}C_2 = 168$

$b(b-1)(b-2)(g)(g-1) = 8 \times 7 \times 6 \times 3 \times 2$

$b + 3\,g = 17$

83. (13)

Ellipse is

$\frac{x^2}{2} + \frac{y^2}{4} = 1; e = \frac{1}{\sqrt{2}}; S \equiv (0, -\sqrt{2})$

Chord of contact is

$$\frac{x}{\sqrt{2}} + \frac{(2\sqrt{2}-2)y}{4} = 1$$
$$\Rightarrow \frac{x}{\sqrt{2}} = 1 - \frac{(\sqrt{2}-1)y}{2} \text{ solving with ellipse}$$
$$\Rightarrow y = 0, \sqrt{2} \therefore x = \sqrt{2}, 1$$
$$P \equiv (1, \sqrt{2}) Q \equiv (\sqrt{2}, 0)$$
$$\therefore (SP)^2 + (SQ)^2 = 13$$

84. (99)

$1 + (1 + 2^{49})(2^{49} - 1) = 2^{98}$

$m = 1, n = 98$

$m + n = 99$

85. (9) $y^2 = -\frac{x}{2}$

$y = mx - \frac{1}{8m}$

this tangent pass through (2,0)

$m = \pm\frac{1}{4}$ i.e., one tangent is $x - 4y - 2 = 0$

$17r = 9$

86. (12)

$\frac{6}{3^{12}} + 10\left(\frac{1}{3^{11}} + \frac{2}{3^{10}} + \frac{2^2}{3^9} + \frac{2^3}{3^8} + \cdots .. + \frac{2^{10}}{3}\right)$

$\frac{6}{3^{12}} + \frac{10}{3^{11}}\left(\frac{6^{11}-1}{6-1}\right)$

$= 2^{12} \cdot 1; m \cdot n = 12$

87. (5) $\tan\theta + \sqrt{5}\tan 2\theta \tan\theta = \sqrt{5} - \tan 2\theta$

$\tan 3\theta = \sqrt{5}$

$\theta = \frac{n\pi}{3} + \frac{\alpha}{3};\ \tan\alpha = \sqrt{5}$

Five solution

88. (6)

$|z^2| = |\bar{z}| \cdot 2^{1-|z|} \Rightarrow |z| = 1$

$z^2 = \bar{z} \Rightarrow z^3 = 1 \therefore z = \omega \text{ or } \omega^2$

$\omega^n = (1+\omega)^n = (-\omega^2)^n$

Least natural value of n is 6

89. (56)

X	0	1	2	3
P(X)	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{3}{10}$	$\frac{1}{30}$

$\sigma^2 = \sum X^2 P(X) - (\sum XP(X))^2 = \frac{56}{100}$

$100\sigma^2 = 56$

90. (104)

$$I = 60\int_0^{\pi/2} \left(\frac{\sin 6x - \sin 4x}{\sin x} + \frac{\sin 4x - \sin 2x}{\sin x} + \frac{\sin 2x}{\sin x}\right) dx$$

$$I = 60\int_0^{\pi/2} (2\cos 5x + 2\cos 3x + 2\cos x) dx$$

$$I = 60\left(\frac{2}{5}\sin 5x + \frac{2}{3}\sin 3x + 2\sin x\right)\Bigg|_0^{\pi/2} = 104$$