1. **(c)** Pressure and time

P: $\frac{\mathrm{N}}{\mathrm{m}^2}$, Time : Sec

$$\mathrm{Pt}=\frac{\mathrm{Nsec}}{\mathrm{m}^2}$$

$$\eta=\frac{\mathrm{F}}{6\pi\mathrm{rv}}:\frac{\mathrm{N}}{\mathrm{m}\cdot\mathrm{m/sec}}:\frac{\mathrm{Nsec}}{\mathrm{m}^2}$$

2. **(c)** $a=k^2\mathrm{rt}^2=\frac{\mathrm{V}^2}{\mathrm{r}}$

$\mathrm{V}=\mathrm{krt}$

$$\mathrm{a}_{\mathrm{t}}=\frac{\mathrm{dv}}{\mathrm{dt}}=\mathrm{kr}$$

$\mathrm{F}_{\mathrm{t}}=\mathrm{ma}_{\mathrm{t}}=\mathrm{mkr}$

$\mathrm{P}=\vec{\mathrm{F}}\cdot\vec{\mathrm{V}}$

$=\mathrm{F}\cos\theta\mathrm{V}=\mathrm{F}_{\mathrm{t}}\mathrm{V}=\mathrm{mkr}(\mathrm{krt})$

$\mathrm{P}=\mathrm{mk}^2\mathrm{r}^2\mathrm{t}$

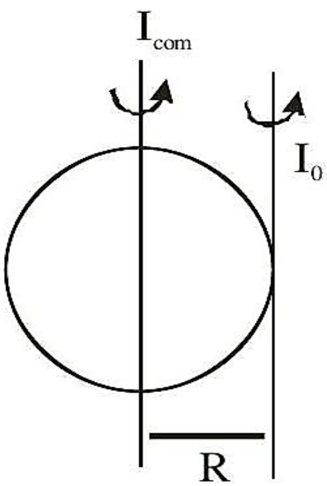
3. **(a)** $x=4\sin\left(\frac{\pi}{2}-\omega t\right)y=4\cos(\omega t)$

$x=4\cos(\omega t)\ y=4\sin(\omega t)$

Eliminate ' t ' to find relation between x and y $x^2+y^2=y^2\cos^2\omega t+y^2\sin^2\omega t=4^2$

$x^2+y^2=4^2$

4. **(a)** solid sphere



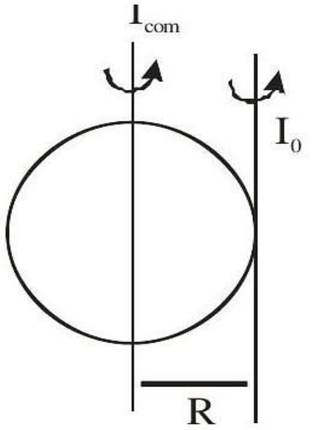
$\mathrm{I}_0=\mathrm{I}_{\mathrm{com}}+\mathrm{MR}^2$

(Parallel Axis theorem)

$I_0 = \frac{2}{5}MR^2 + MR^2$

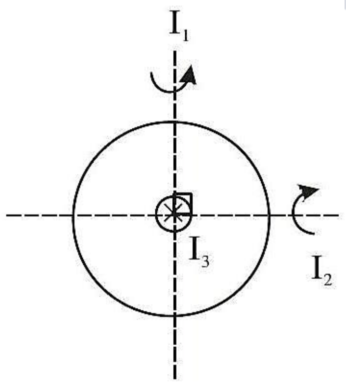
$I_0 = \frac{7}{5}MR^2$

Hollow sphere



$I_0 = I_{com} + MR^2$

$= \frac{2}{3}MR^2 + MR^2 = \frac{5}{3}MR^2$



$I_1 + I_2 + I_3$ (Perpendicular axis theorem)

By symmetry MOI

About 1" and 2" Axis are same i.e.

$I_1 = I_2$

$\therefore 2I_1 = I_3 = MR^2 (I_{com} = MR^2)$

$I_1 = \frac{MR^2}{2}$

Similarly in disc

$2I_1 = \frac{MR^2}{2}\left\{I_{com} = \frac{MR^2}{2}\right\}$

$I_1 = \frac{MR^2}{4}$

5. (c) $T = \frac{2\pi}{\sqrt{Gm_a}} r^{\frac{3}{2}}$

$T^2 \propto r^3$

$\left(\frac{T_A}{T_B}\right)^2 = \left(\frac{r_A}{r_B}\right)^3$

$\Rightarrow \left(\frac{2}{1}\right)^2 = \left(\frac{r_A}{r_B}\right)^3 \Rightarrow r_A^3 = 4r_B^3$

6. (a) $d = 2$ cm;

$r = 1$ cm;

$T = 0.075$

$\Delta SE = T\Delta A$

$= 0.075(A_f - A_1)$

$A_i = 4\pi r^2$

$A_f = 4\pi r_0^2 \times 64$

By volume conservation

$\frac{4}{3}\pi r^3 = 64 \cdot \frac{4}{3}\pi r_0^3$

$r_0 = \frac{r}{4}$

$A_f = 4\pi\left(\frac{r}{4}\right)^2 \cdot 64 = 16\pi r^2$

$\Delta SE = 0.075(16\pi r^2 - 4\pi r^2)$

$= 0.075(12\pi(0.01)^2)$

$= 2.8 \times 10^{-4}$ J

7. (a)

$W_{adiabatic} = \frac{NR(T_f - T_l)}{1-\gamma} \rightarrow$ statment 1

$Q = W + \Delta U$

$0 = W + \Delta U$

$\Delta U = -W$

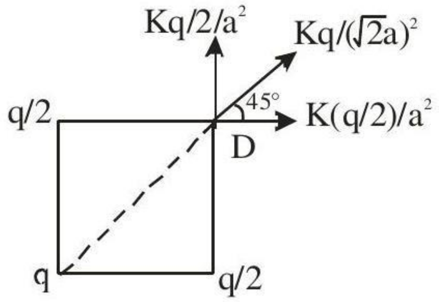
If work is done on the gas, i.e. work is negative

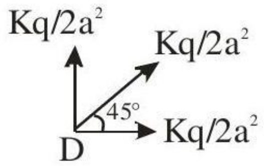
∴ ΔU is positive.

∴ Temperature will increase.

8. **(a)** If the electric field is in the positive direction and the positive charge is to the left of that point then the electric field will increase. But to the left of the positive charge the electric field would decrease. If the dipole is kept at the point where the electric field is maximum then the force on it will be zero.

9. **(a)**



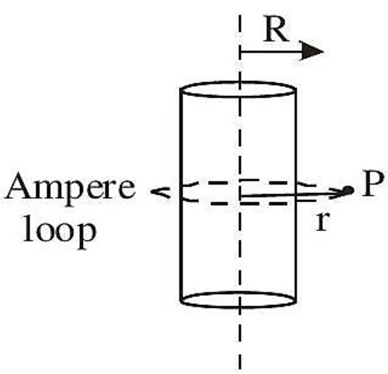


$$(E_{\text{net}})_D = \frac{kq}{2a^2} + \frac{\sqrt{2}\text{kq}}{2\text{a}^2}$$

$$(\text{E}_{\text{net}})_\text{D} = \frac{\text{kq}}{\text{a}^2}\left(\frac{1}{2} + \frac{1}{\sqrt{2}}\right)$$

$$(\text{E}_{\text{net}})_\text{D} = \frac{\text{q}}{4\pi \in_0 \text{a}^2}\left(\frac{1}{2} + \frac{1}{\sqrt{2}}\right)$$

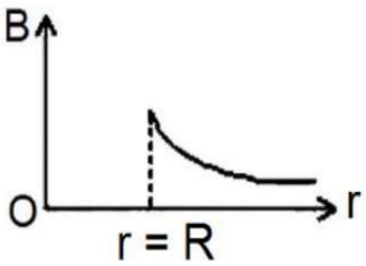
10. **(d)**



(1) $\text{r} < \text{R}, \text{B}_\text{p} = 0$

(i)$\text{r} \geq \text{R}, \text{B}_\text{p} = \frac{\mu_0 1}{2\pi\text{r}}$

$\text{B}_\text{p} \propto \frac{1}{\text{r}}$



11. (b) $C = \frac{E_0}{B_0} = \frac{2.25}{1.5\times10^{-8}} = 1.5\times10^{8}\ ms^{-1}$

$$t = \frac{6\times10^{3}}{1.5\times10^{8}} = 4\times10^{-5}\ s$$

12. (a)

$$\mu = \frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin\frac{A}{2}}$$

$$\mu = \cot\frac{A}{2}$$

$$\Rightarrow \sin\left(\frac{A+\delta_m}{2}\right) = \cos\frac{A}{2}$$

$$\delta_m = 180 - 2A$$

13. (c) $R.P = \frac{d}{1.22\lambda} = \frac{24.4\times10^{-2}}{1.22\times2440\times10^{-10}} = 8.2\times10^{5}$

14. (a) $\lambda_e = \frac{h}{\sqrt{2mk}}$

Also for photon $k = \frac{hc}{\lambda_p}$

$$\lambda_e = \frac{h\sqrt{\lambda_p}}{\sqrt{2mhc}}$$

$$\lambda_p \propto \lambda e^{2}$$

15. (d) $x + p \rightarrow \gamma + b$

$Q = k_\gamma + k_b - k_p$

$Q + k_p = k_\gamma + k_b$

$Q + k_p > 0$

16. (c) This is NAND gate

A	B	Y
0	0	1
1	0	1

0	1	1
1	1	0

17. (b) In forward bias diode conducts In revers bias it does not conducts.

18. (a)

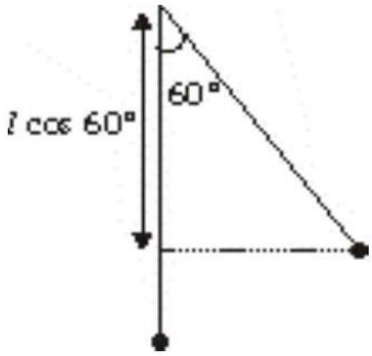
19. (c)

20. (d) $f_b = f_1 - f_2$

$$\frac{v}{4.08} - \frac{v}{4.16} = \frac{40}{12}$$

$\Rightarrow v = 707.2$

21. (5) $V_{max} = \sqrt{2gh}$



The speed will be highest at the lowest position.

$$h = (\ell - \ell \cos 60^\circ) = \frac{\ell}{2}$$

$$V_{max} = \sqrt{2 \times g \times \frac{\ell}{2}} = \sqrt{10 \times 2.5} = 5 \text{ m/s}$$

22. (19) Using the conditions of a balanced wheat stone bridge and adding the end correction.

$$\frac{15}{(43+2)} = \frac{R}{(102-45)} \Rightarrow R = \frac{57}{45} \times 15$$

$R = 19\Omega$

23. (10) Using the condition of a balanced wheat stone bridge,

$$\Rightarrow \frac{R}{3} = \frac{4}{6} \Rightarrow R = 2\Omega$$

So the effective resistance of the circuit is

$$R_{eq} = \frac{6 \times 9}{6+9} = \frac{18}{5}\Omega$$

$i = \frac{36}{R_{eq}} = 10\ A$

24. (15)

$|(X_L - X_C)| = |10 - 10^2| = 90\Omega$

$Z =$ Impendance

$= \sqrt{(X_L - X_C)^2 + R^2} = \sqrt{(90)^2 + (20)^2} = 150\Omega$

$i_{rms} = \frac{V_{rms}}{z} = \left(\frac{2}{15}\right) A$

Now $i_{rms}^2 R \Delta t = ms(\Delta T)$

$\Rightarrow \Delta t = 15\text{sec}$

25. (91) Using $\vec{L} = \vec{r} \times \vec{p} = \vec{r} \times m\vec{v}, m = 1kg$

$\vec{L} = (3\hat{i} - \hat{j}) \times (3\hat{j} + \hat{k}) = (9\hat{k} - 3\hat{j} - \hat{i}) N - s$
$\Rightarrow |\bar{L}| = \sqrt{91}\ N - s$

26. (6) Taking the system as man and trolley and using conservation of linear momentum.

$60 \times v = (60 + 120) \times 2$

$\Rightarrow v = 6\ m/s$

27. (6) Using $\vec{F}_{net} = \mu\vec{a}$, $\frac{T - Mg = Ma}{\Rightarrow a = \frac{g}{5}}$

$T = Mg + Ma = Mg + \frac{Mg}{5} = \frac{6}{5} Mg$

28. (7479) $U = nC_vT$

$= 2 \times \frac{3}{2} R \times 300$

$= 900R = 900 \times 8 \cdot 31 = 7479\ J$

29. (10) $R = \frac{mv}{qB} = \frac{\sqrt{2mK}}{qB}$

30. (100) For minimum impedance

$X_L = X_C$
$\Rightarrow \omega L = \frac{1}{\omega C} \Rightarrow L = \frac{1}{\omega^2 C} = 10^{-1} H = 100\text{mH}$

31. (d) Due to vacancy defect density of the substance will decrease.

32. (c) The potential difference between the fixed and diffused layer of charges in a colloidal particle is called zeta potential

33. (a) $E \Rightarrow [Ar]3d^{10}4s^2 4p^4$

Element above E ⇒ $[Ne]3\ s^2 3p^4$

34. **(b)** From Ellingham diagram given in NCERT, it can be seen that Mg, MgO line crosses Al, Al_2O_3 line after 1350°C hence assertion is true. Yes, Mg have lower MP and BP than aluminium but it does not explain the above fact.

35. **(b)** $CuO + H_2 \rightarrow Cu + H_2O$ (under hot conditions)

36. **(b)** $Ba(N_3)_2 \rightarrow Ba + 3\ N_2$

37. **(d)** As +ve oxidation state increases, EN of element increases hence acidic character increases. Down the group, non-metallic character decreases, acidic character decreases.

Acidic character: $E_2O_5 > E_2O_3$

Down the group, acidic character of E_2O_3 decreases

38. **(c)** $E^o_{M^{3+}/M^{2+}} \Rightarrow \begin{array}{cc} Eu & Yb \\ -0.35 & -1.05 \end{array}$

Hence, due to more reduction potential in Eu as compared to Yb, it can concluded that Eu^{2+} is more stable than Yb^{2+}.

39. **(b)**

$[Ni(CN)_4]^{2-}$: d^8 configuration, SFL, sq. planar splitting (dsp^2), diamagnetic.

$[Ni(CO)_4]$: d^{10} config (after excitation), SFL, tetrahedral splitting (sp^3), diamagnetic.

$[NiCl_4]^{2-}$: d^8 config, WFL, tetrahedral splitting (sp^3), paramagnetic (2 unpaired e^-).

40. **(d)**

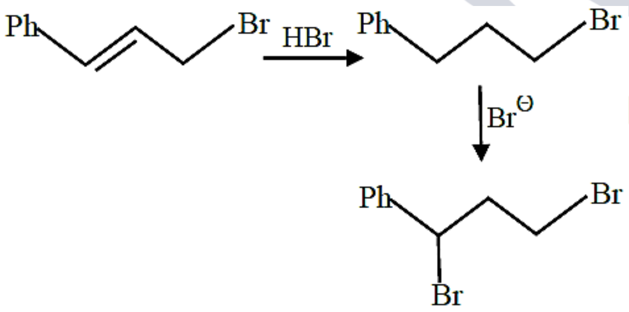
41. **(c)**

42. **(b)**

A, B aromatic

C, D is nonaromatic

43. **(c)**

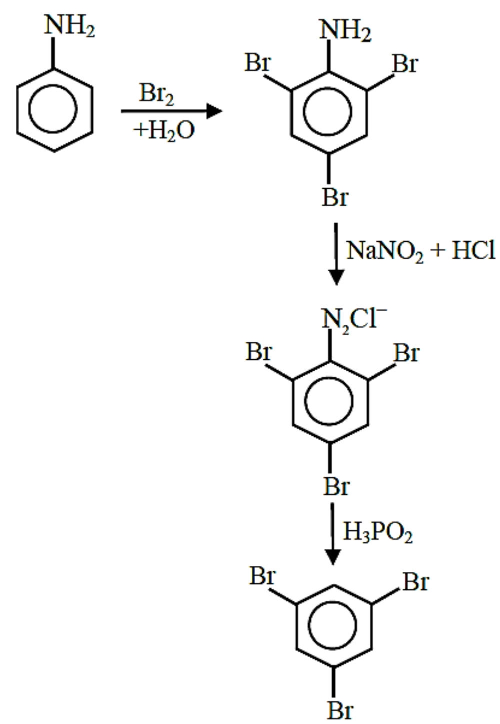


44. **(a)**

$$PhCH = O + PhCH = O \xrightarrow{OD/D_2O} PhCH_2OD + PhCO_2^-$$

45. **(c)** The carbocation fromed is very unstable. So it is inactive towards S_N1

46. **(c)**



47. (b)

$$R-NH_2 \xrightarrow[+HCl]{NaNO_2} R-N_2^+ \rightarrow R^+ \xrightarrow[H_2O]{} R-OH$$

48. (b) Buna-S, PHBr and Butadiene-styrene are copolymer. Only neoprene is namopolymer.

49. (b)

50. (d) $Na_2S + Na_2[Fe(CN)_5NO] \rightarrow Na_4[Fe(CN)_5NO_5]$

51. (13)

$MnO_2 + HCl \longrightarrow Cl_2 + Mn^{+2}$
6meq
6meq
$= 3$ m mol
$Cl_2 + KI \longrightarrow Cl^- + I_2$
6meq 6meq
$I_2 + Na_2\,S_2O_3 \longrightarrow I^- + Na_2\,S_4O_6$
6meq6 m mol
$= 6$meq

$$\%MnO_2 = \frac{3 \times 10^{-3} \times 87}{2} \times 100$$
$= 13.05\%$

52. (300) $\phi = 6.63 \times 10^{-19}\ \mathrm{J} = \frac{hc}{\lambda} = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{\lambda} \Rightarrow \lambda = 3 \times 10^{-7}\ \mathrm{m} = 300\ \mathrm{nm}$

53. (1) $PF_5 \Rightarrow sp^3$ d hybridisation

(5 sigma bonds, zero lone pair on central atom)

Value of $y = 1$

54. (0) For free expansion:

$P_{ext} = 0, w = 0$
$q = 0, \Delta U = 0$

55. (14)

$$\frac{y_B}{1-y_B} = \frac{P_B^o}{P_A^o}\left[\frac{X_B}{1-X_B}\right]$$

$$\Rightarrow \frac{y_B}{1-y_B} = \frac{100}{50}\left[\frac{0.7}{0.3}\right] = \frac{14}{3}$$

$$\Rightarrow y_B = \frac{14}{17}$$

56. (3)

$A_2X_{3(s)} \rightleftharpoons 2\,A^{+3}_{(aq)} + 3X^{-2}_{(aq)}$

solubility $= sM\ 2\,s\ 3\,s$

$(2\,s)^2(3\,s)^3 = 1.1 \times 10^{-23}$

$108\,s^5 = 1.1 \times 10^{-23}$

$$s \simeq 10^{-5}M = 10^{-5}\frac{mol}{L} = 0.01\frac{mol}{m^3}$$

Now $\Lambda_m \simeq \Lambda_m^\infty = \frac{k}{m} = \frac{k}{s}$

$$\Rightarrow \Lambda_m^\infty = \frac{3 \times 10^{-5}}{0.01} = 3 \times 10^{-3}\ S - m^2/mol$$

57. (6)

$Cr_2O_7^{-2} + 6e^- \longrightarrow 2Cr^{+3}$
1 mol 6 mol
$\Rightarrow$ number of faradays $=$ moles of electrons
$= 6$

58. (16)

$$t_{67\%} = \frac{1}{k}\ln\left(\frac{1}{1-0.67}\right) = \frac{t_{1/2}}{\ln 2} \times \ln\left(\frac{1}{1-\frac{2}{3}}\right)$$

$$t_{67\%} = \frac{t_{1/2}}{\log 2} \times \log 3 = \frac{t_{1/2} \times 0.4771}{0.301}$$

$\Rightarrow t_{67\%} = 1.585 \times t_{1/2}$

$X \times 10^{-1} = 1.585$

$\Rightarrow X = 15.85$

59. (3) Carbonyl complex compounds have tendency to show synergic bonding

60. (34)

O.C AgBr

0.5 g 0.4 g

mol of Br = mol of AgBr $= \frac{0.4}{188}$

$$\%Br = \%Br = \frac{\frac{0.4}{188}\times 80}{0.5}\times 100$$

$= 34.04\%$

61. (a)

$\sum_{R=1}^{31} {}^{31}C_R \cdot {}^{31}C_{R-1}$

$= {}^{31}C_1 \cdot {}^{31}C_0 + {}^{31}C_2 \cdot {}^{31}C_1 + \cdots . + {}^{31}C_{31} \cdot {}^{31}C_{30}$

$= {}^{31}C_0 \cdot {}^{31}C_{30} + {}^{31}C_1 \cdot {}^{31}C_{29} + \cdots . + {}^{31}C_{30} \cdot {}^{31}C_0$

$= {}^{62}C_{30}.$

Similarly

$\sum_{R=1}^{30} ({}^{30}C_R \cdot {}^{30}C_{R-1}) = {}^{60}C_{29}$

$${}^{62}C_{30} - {}^{60}C_{29} = \frac{62!}{30!\,32!} - \frac{60!}{29!\,31!}$$

$$= \frac{60!}{29!\,31!}\left\{\frac{62\cdot 61}{30\cdot 32} - 1\right\}$$

$$= \frac{60!}{30!\,31!}\left(\frac{2822}{32}\right)$$

$$\therefore 16\alpha = 16\times\frac{2822}{32} = 1411$$

62. (d) $f(x) = \begin{cases} 4R & ; \quad n = 2R \\ 4R-2 & ; \quad n = 4R-1 \\ 2R-1 & ; \quad n = 4R-3 \end{cases}$

$(R \in N)$

Note that for any element, it will fall into exactly. one of these sets.

$\{y: y = 4R; y \in N\}$

$\{y: y = 4R-2; y \in N\}$

$\{y: y = 2R-1; y \in N\}$

Corresponding to that y, we will get exactly one value of n.

Thus, f is one - one & onto.

63. (b)

$$\begin{vmatrix} 2 & 3 & -1 \\ 1 & 1 & 1 \\ 1 & -1 & |\lambda| \end{vmatrix} = 0$$

$\Rightarrow |\lambda| = 7 \Rightarrow \lambda = \pm 7$

System :

$2x + 3y - z = -2$

$\mathrm{x} + \mathrm{y} + \mathrm{z} = 4$

$x - y + |\lambda| z = 4\lambda - 4$

Eliminating y from equal (2) & (3) we get

$\mathrm{x} + 4\mathrm{z} = 14$

$(3) + (4) \Rightarrow x + \left(\frac{|\lambda| + 1}{2}\right) z = 2\lambda$

Clearly for $\lambda = -7$, system is inconsistent.

64. (a)

$|(\det(\mathrm{a}))\operatorname{adj}(5\operatorname{adj}(\mathrm{a}))|$

$= \left|2\operatorname{adj}\left(5\operatorname{adj}(\mathrm{A}^3)\right)\right|$

$= 2^3 \operatorname{ladj}(5\operatorname{adj}(\mathrm{A}^3)\ \|$

$= 2^3 \cdot |5\operatorname{adj}(\mathrm{A}^3)|^2$

$= 2^3\left(5^3 \cdot \operatorname{ladj}(\mathrm{A}^3)\right)^2$

$= 2^3 \cdot 5^6 \cdot |\operatorname{adj} A^3|^2$

$= 2^3 \cdot 5^6((|\ \mathrm{A}|^3)^2)^2$

$= 2^3 \cdot 5^6 \cdot 2^{12} = 2^{15} \times 5^6$

$= 2^9 \times 10^6$
$= 512 \times 10^6.$

65. (d) To make a no. divisible by 3 we can use the digits 1,2,5,6,7 or 1,2,3,5,7.

Using 1,2,5,6,7, number of even numbers is

$= 4 \times 3 \times 2 \times 1 \times 2 = 48$

Using 1,2,3,5,7, number of even numbers is

$= 4 \times 3 \times 2 \times 1 \times 1 = 24$

Required answer is 72.

66. (c) $A_1 \cdot A_3 \cdot A_5 \cdot A_7 = \frac{1}{1296}$

$(A_4)^4 = \frac{1}{1296}$

$A_4 = \frac{1}{6}$

$A_2 + A_4 = \frac{7}{36}$

$A_2 = \frac{1}{36}$

$A_6 = 1$

$A_8 = 6$

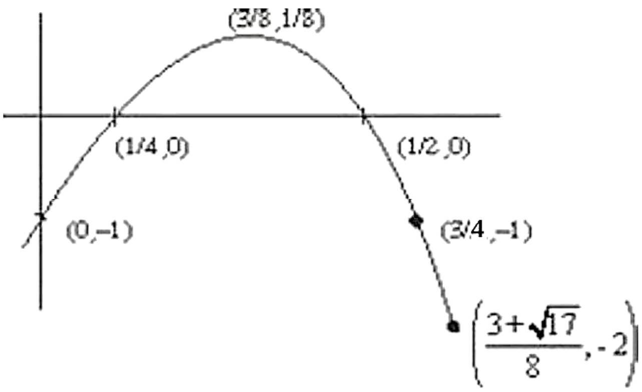
$A_{10} = 36$

$A_6 + A_8 + A_{10} = 43$

67. (c)

$\int_0^1 [-8x^2 + 6x - 1]dx$

$= \int_0^{1/4} -1dx + \int_{1/4}^{1/2} 0dx + \int_{1/2}^{3/4} -1dx$



$+ \int_{3/4}^{\frac{3+\sqrt{17}}{8}} -2dx + \int_{\frac{3+\sqrt{17}}{8}}^{1} -3dx$

$= -[x]_0^{1/4} + 0 - [x]_{1/2}^{3/4} + -2[x]_{3/4}^{\frac{3+\sqrt{17}}{8}} - 3[x]_{\frac{3+\sqrt{17}}{8}}^{1}$

$= -\left(\frac{1}{4} - 0\right) - \left(\frac{3}{4} - \frac{1}{2}\right) - 2\left(\frac{3+\sqrt{17}}{8} - \frac{3}{4}\right) - 3\left(1 - \frac{3+\sqrt{17}}{8}\right)$

$= -\frac{1}{4} - \frac{1}{4} + \frac{-6-2\sqrt{17}}{8} + \frac{3}{2} - 3 + \frac{9+3\sqrt{17}}{8}$

$= \frac{\sqrt{17}-13}{8}$

68. (c) f(x) is discontinuous at $x = 1$

For continuous at $x = 0; a = 1$

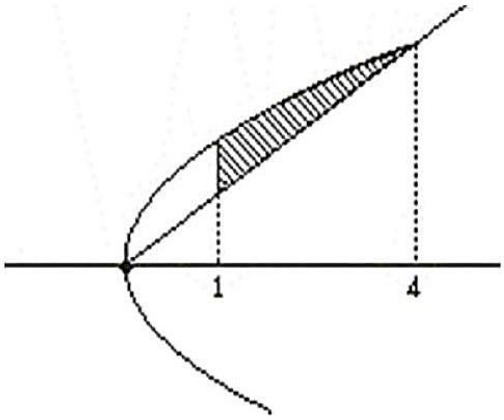
For continuous at $x = 2; b + c = 1$

$a + b + c = 2$

69. (b) $y^2 = 8x$

$y = \sqrt{2}x$

$y^2 = 2x^2$



$\Rightarrow 8x = 2x^2$

$\Rightarrow x = 0 \& 4$

Area : $= \int_1^4 2\sqrt{2}\sqrt{x} - \sqrt{2}x dx$

$= 2\sqrt{2}\left(\frac{x^{\frac{3}{2}}}{3/2}\right)_1^4 - \sqrt{2}\left(\frac{x^2}{2}\right)_1^4$

$= \frac{4\sqrt{2}}{3}(8-1) - \frac{\sqrt{2}}{3}(16-1)$

$= \frac{28\sqrt{2}}{3} - \frac{15\sqrt{2}}{2} = \frac{11\sqrt{2}}{6}$

70. (a) $\left(\frac{x}{\sqrt{x^2-y^2}} + e^{\frac{y}{x}}\right)x\frac{dy}{dx} = x + \left(\frac{x}{\sqrt{x^2y^2}} + e^{\frac{y}{x}}\right)y$

$\Rightarrow e^{\frac{y}{x}}(xdy - ydx) + \frac{x}{\sqrt{x^2-y^2}}(xdy - ydx) = xdx$

Dividing both side by x^2

$$\Rightarrow e^{\frac{y}{x}}\left(\frac{xdy-ydx}{x^2}\right)+\frac{1}{\sqrt{1-\left(\frac{y}{x}\right)^2}}\left(\frac{xdy-y}{x^2}\right.$$

$$\Rightarrow e^{\frac{y}{x}} \mid d\left(\frac{t}{x}\right)+\frac{1}{\sqrt{1-\left(\frac{y}{x}\right)^2}} d\left(\frac{y}{x}\right)=\frac{dy}{x}$$

Integrate both side.

$$\int e^{\frac{y}{x}} d\left(\frac{y}{x}\right)+\int \frac{1}{\sqrt{1-\left(\frac{y}{x}\right)^2}} d\left(\frac{y}{x}\right)=\int \frac{dx}{x}$$

$$\Rightarrow e^{\frac{y}{x}}+\sin^{-1}\left(\frac{y}{x}\right)=\ln x+c$$

It passes through $(1,0)$

$1+0=0+c \Rightarrow c=1$

It passes through $(2\alpha, \alpha)$

$$e^{\frac{1}{2}}+\sin^{-1}\frac{1}{2}=\ln 2\alpha+1$$

$$\Rightarrow \ln 2\alpha=\sqrt{e}+\frac{\pi}{6}-1$$

$$\Rightarrow 2\alpha=e^{\left(\sqrt{e+\frac{\pi}{6}-1}\right)}$$

$$\Rightarrow \alpha=\frac{1}{2}e^{\left(\frac{\pi}{6}+\sqrt{e}-1\right)}$$

71. (a) $x(1-x^2)\frac{dy}{dx}+(3x^2y-y-4x^3)=0$

$$x(1-x^2)\frac{dy}{dx}+(3x^2-1)y=4x^3$$

$$\frac{dy}{dx}+\frac{(3x^2-1)}{(x-x^3)}y=\frac{4x^3}{(x-x^3)}\ \frac{dy}{dx}+Py=Q$$

$$IF=e^{\int Pdx}=e^{\int\frac{3x^2-1}{x-x^3}dx}$$

$$x-x^3=t \Rightarrow IF=e^{\int\frac{-dt}{t}}$$

$$=e^{-\ell nt}=\frac{1}{t}$$

$$\therefore IF=\frac{1}{x-x^3}$$

$$y\times IF=\int Q\times IFdx$$

$$y\left(\frac{1}{x-x^3}\right)=\int\frac{4x^3}{x-x^3}\times\frac{1}{(x-x^3)}dx$$

$= \int \frac{4x^3}{(x-x^3)^2} dx$

$= \int \frac{4\mathrm{x}}{(1-\mathrm{x}^2)^2} \mathrm{dx}\ 1-\mathrm{x}^2 = \mathrm{K}$

$= 2\int \frac{-\mathrm{dK}}{\mathrm{K}^2}\ \ -2\mathrm{xdx} = \mathrm{dK}$

$= -2\left(-\frac{1}{\mathrm{K}}\right) + \mathrm{c}$

$\frac{\mathrm{y}}{\mathrm{x}-\mathrm{x}^3} = \frac{2}{\mathrm{K}} + \mathrm{c}$

$\frac{\mathrm{y}}{\mathrm{x}-\mathrm{x}^3} = \frac{2}{1-\mathrm{x}^2} + \mathrm{c}$

At $x = 2, y = -2$

$\frac{-2}{2-8} = \frac{2}{1-4} + \mathrm{c}$

$\frac{1}{3} = \frac{-2}{3} + c$

$\therefore \mathrm{C} = 1$

$\frac{\mathrm{y}}{\mathrm{x}-\mathrm{x}^3} = \frac{2}{1-\mathrm{x}^2} + 1$

Put $x = 3$

$\frac{y}{3-27} = \frac{2}{1-9} + 1$

$\frac{y}{-24} = -\frac{1}{4} + 1$

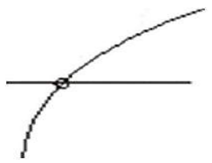
$\frac{y}{-24} = \frac{3}{4}$

$y = \frac{3}{4}(-24) = -18$

72. (b) $\mathrm{f(x)} = \mathrm{x}^7 + 5\mathrm{x}^3 + 3\mathrm{x} + 1$

$\mathrm{f}'(\mathrm{x}) = 7\mathrm{x}^6 + 15\mathrm{x}^2 + 3 > 0$

$\therefore$ f(x) is strictly increasing function



$\mathrm{x} \to -\infty, \mathrm{y} \to -\infty$

$x \to \infty, y \to \infty$

$\therefore$ no. of real solution $= 1$

73. (20) $y = mx \pm \sqrt{a^2 m^2 - b^2}$

$m = 2, c^2 = a^2 m^2 - b^2$

$c^2 = 4a^2 - b^2$

$e^2 = 1 + \frac{b^2}{a^2} \frac{5}{2} = 1 + \frac{b^2}{a^2}$

$\frac{3}{2} = \frac{b^2}{a^2} \Rightarrow b^2 = \frac{3a^2}{2}$

$\frac{2b^2}{a} = 6\sqrt{2}$

$\frac{2}{a} \times \frac{3a^2}{2} = 6\sqrt{2}$

$3a = 6\sqrt{2}$

$a = 2\sqrt{2}$

$b^2 = \frac{3}{2} \times 8 = 12$

$b = 2\sqrt{3}$

$\therefore c^2 = 4 \times 8 - 12$

$c^2 = 20$

74. (c) Tangent at O

$-(x + 0) - 2(y + 0) = 0$

$\Rightarrow x + 2y = 0$

Tangent at P

$x(1 + \sqrt{5}) + y \cdot 2 - (x + 1 + \sqrt{5}) - 2(y + 2 = 0)$

Put $x = -2y$

$-2y(1 + \sqrt{5}) + 2y + 2y - 1 - \sqrt{5} - 2y - 4 = 0$

$-2\sqrt{5}y = 5 + \sqrt{5} \Rightarrow y = \left(\frac{\sqrt{5} + 1}{2}\right)$

$Q\left(\sqrt{5} + 1, -\frac{\sqrt{5} + 1}{2}\right)$

Length of tangent $OQ = \frac{5+\sqrt{5}}{2}$

Area $= \frac{RL^3}{R^2+L^2}$

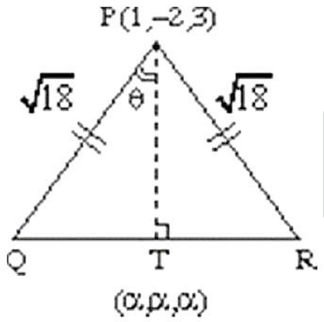
$R = \sqrt{5}$

$$= \frac{\sqrt{5} \times \left(\frac{5+\sqrt{5}}{2}\right)^3}{5 + \left(\frac{5+\sqrt{5}}{2}\right)^2}$$

$$= \frac{\sqrt{5}}{2} \times \frac{4 \times (125 + 75 + 75\sqrt{5} + 5\sqrt{5})}{(20 + 25 + 10\sqrt{5} + 5)}$$

$$= \frac{5 + 3\sqrt{5}}{2}$$

75. (b)



$-x + 2y - z = 0$

$3x - 5y + 2z = 0$ $\vec{n} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ -1 & 2 & -1 \\ 3 & -5 & 2 \end{vmatrix}$

$= \hat{\imath}(-1) - \hat{\jmath}(1) + \hat{k}(-1)$

$\vec{n} = -\hat{\imath} - \hat{\jmath} - \hat{k}$

Equation of LOI is $\frac{x}{1} = \frac{y}{1} = \frac{z}{1}$

DR: of PT $\rightarrow \alpha - 1, \alpha + 2, \alpha - 3$

DR: of QR $\rightarrow$ 1, 1,1

$\Rightarrow (\alpha - 1) \times 1 + (\alpha + 2) \times 1 + (\alpha - 3) \times 1 = 0$

$3\alpha = 2$

$\alpha = \frac{2}{3}$

$PT^2 = \frac{1}{9} + \frac{64}{9} + \frac{49}{9}$

$$PT^2 = \frac{114}{9}$$

$$PT = \frac{\sqrt{114}}{3}$$

$$\cos\theta = \frac{\sqrt{114}}{3} \times \frac{1}{3\sqrt{2}} = \frac{\sqrt{57}}{9} = \frac{\sqrt{19 \times 3}}{3 \times 3}$$

$$= \frac{\sqrt{19}}{3\sqrt{3}}$$

$$\cos 2\theta = \frac{2 \times 19}{27} - 1 = \frac{11}{27}$$

$$\sin 2\theta = \sqrt{1 - \left(\frac{11}{27}\right)^2} = \frac{\sqrt{38}\sqrt{16}}{27}$$

$$= \frac{4}{27}\sqrt{38}$$

$$\text{Area} = \frac{1}{2} \times \sqrt{18}\sqrt{18} \times \frac{4}{27}\sqrt{38}$$

$$= \frac{18}{2} \times \frac{4}{27}\sqrt{38} = \frac{36}{27}\sqrt{38} = \frac{4}{3}\sqrt{38}$$

76. **(a)** Equation of plane passing through the intersection of planes $5x + 8y + 13z - 29 = 0$ and $8x - 7y + z - 20 = 0$ is

$5x + 8y + 3z - 29 + \lambda(8x - 7y + z - 20) = 0$ and

if it is passing through $(2,1,3)$ then $\lambda = \frac{7}{2}$

P_1 : Equation of plane through intersection of $5x + 8y + 13z - 29 = 0$ and $8x - 7y + z - 20 = 0$ and the point $(2,1,3)$ is

$$5x + 8y + 3z - 29 + \frac{7}{2}(8x - 7y + z - 20) = 0$$

$\Rightarrow 2x - y + z = 6$

Similarly, P_2 : Equation of plane through intersection of

$5x + 8y + 13z - 29 = 0$ and $8x - 7y + z - 20 = 0$

and the point $(0,1,2)$ is

$\Rightarrow x + y + 2z = 5$

Angle between planes $= \theta = \cos^{-1}\left(\frac{3}{\sqrt{6}\sqrt{6}}\right) = \frac{\pi}{3}$

77. **(b)** Equation of plane passing through line of intersection of planes $P_1 : \vec{r}((\hat{\imath} + 3\hat{\jmath} - \hat{k}) = 6$ and $P_2 : \vec{r} \cdot (-6\hat{\imath} + 5\hat{\jmath} - \hat{k}) = 7$ is

$P_1 + \lambda P_2 = 0$

$(\bar{r} \cdot (\hat{\imath} + 3\hat{\jmath} - \hat{k}) - 6) + \lambda(\bar{r} \cdot (-6\hat{\imath} + 5\hat{\jmath} - \hat{k}) - 7) = 0$ and it passes through point $\left(2,3,\frac{1}{2}\right)$

$$\Rightarrow \left(2 + 9 - \frac{1}{2} - 6\right) + \lambda\left(-12 + 15 - \frac{1}{2} - 7\right) = 0$$

$\Rightarrow \lambda = 1$

Equation of plane is $\bar{r} \cdot (-5\hat{\imath} + 8\hat{\jmath} - 2\hat{k}) = 13$

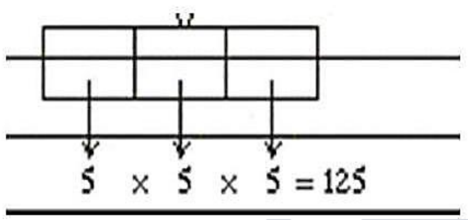
$|\vec{a}|^2 = 25 + 64 + 4 = 93; d = 13$

Value of $\frac{|13\bar{a}|^2}{d^2} = 93$

78. (a) At-least two digits are odd

= exactly two digits are odd + exactly there 3 digits are odd

For exactly three digits are odd



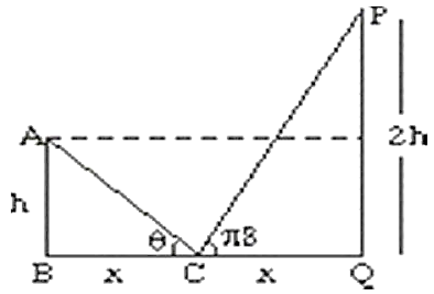
For exactly two digits odd:

If 0 is used then: $2 \times 5 \times 5 = 50$

If 0 is not used then: ${}^3C_1 \times 4 \times 5 \times 5 = 300$

Required Probability $= \frac{475}{900} = \frac{19}{36}$

79. (c)



Let $BC = CQ = x \& AB = h$ and $PQ = 2h$

$$\tan\theta = \frac{h}{x}, \tan\frac{\pi}{8} = \frac{2h}{x}$$

$\frac{\tan\theta}{\tan\left(\frac{\pi}{8}\right)} = \frac{1}{2} \tan\theta = \frac{1}{2}\tan\left(\frac{\pi}{8}\right) = \frac{1}{2}(\sqrt{2} - 1)$

$\tan^2\theta = \frac{1}{4}(3 - 2\sqrt{2})$

80. $(\boldsymbol{c}) s_1 : (\sim p \vee q) \vee (\sim p \vee r)$

$\equiv \sim \mathrm{p} \vee (\mathrm{q} \vee \mathrm{r})$

$\mathrm{s}_2 : \mathrm{p} \rightarrow (\mathrm{q} \vee \mathrm{r})$

$\equiv \sim \mathrm{p} \vee (\mathrm{q} \vee \mathrm{r}) \rightarrow$ By conditional law

$\mathrm{S}_1 \equiv \mathrm{S}_2$

81. (28) Here, $p, p^n \in \{1, 2, \ldots 50\}$

Now p can take values 2,3,5,7,11,13,17,23,29,31,37,41,43 and 47.

$\therefore$ we can calculate no. of elements in R, as

$(2, 2^\circ), (2, 2^1) .. (2, 2^5)$

$(3, 3^\circ), \ldots (3, 3^3)$

$(5, 5^\circ), \ldots (5, 5^2)$

$(7, 7^\circ), \ldots (7, 7^2)$

$(11, 11^\circ), \ldots (11, 11^1)$

And rest for all other two elements each

$\therefore\ \mathrm{n}(\mathrm{R}_1) = 6 + 4 + 3 + 3 + (2 \times 10) = 36$

Similarly, for R_2

$(2, 2^\circ), (2, 2^1)$

$(47, 47^\circ), (47, 47^1)$

$\therefore\ \mathrm{n}(\mathrm{R}_2) = 2 \times 14 = 28$

$\therefore\ \mathrm{n}(\mathrm{R}_1) - \mathrm{n}(\mathrm{R}_2) = 36 - 28 = 8$

82. (2) $\mathrm{e}^{4\mathrm{x}} + 4\mathrm{e}^{3\mathrm{x}} - 58\mathrm{e}^{2\mathrm{x}} + 4\mathrm{e}^{\mathrm{x}} + 1 = 0$

Let $\mathrm{f}(\mathrm{x}) = \mathrm{e}^{2\mathrm{x}}\left(\mathrm{e}^{2\mathrm{x}} + \frac{1}{\mathrm{e}^{2\mathrm{x}}} + 4\left(\mathrm{e}^{\mathrm{x}} + \frac{1}{\mathrm{e}^{\mathrm{x}}}\right) - 58\right)$

$\mathrm{e}^{\mathrm{x}} + \frac{1}{\mathrm{e}^{\mathrm{x}}}$

Let $\mathrm{h}(\mathrm{t}) = \mathrm{t}^2 + 4\mathrm{t} - 58 = 0$

$\mathrm{t} = \frac{-4 \pm \sqrt{16 + 4.58}}{2}$

$\frac{-4 \pm 2\sqrt{62}}{2}$

$t_1 = -2 + 2\sqrt{62}$

$t_2 = -2 - 2\sqrt{62}$ (not possible)

$t \geq 2$

$e^x + \frac{1}{e^x} = -2 + 2\sqrt{62}$

$e^{2x} - (-2 + 2\sqrt{62})e^x + 1 = 0$

$(-2 + 2\sqrt{62}) - 4$

$4 + 4.62 - 8\sqrt{62} - 4$

$248 - 8\sqrt{62} > 0$

$\frac{-b}{2a} > 0$

both roots are positive

2 real roots

83. (17)

We have Variance $= \frac{\sum_{r=1}^{15} x_r^2}{15} - \left(\frac{\sum_{r=1}^{15} x_r}{15}\right)^2$

Now, as per information given in equation $\frac{\sum x_r^2}{15} - 8^2 = 3^2 \Rightarrow \sum x_r^2 = \log 5$

Now, the new $\sum x_r^2 = \log 5 - 5^2 + 20^2 = 1470$

And, new $\sum x_r = (15 \times 8) - 5 + (20) = 135$

$\therefore$ Variance $= \frac{1470}{15} - \left(\frac{135}{15}\right)^2 = 98 - 81 = 17$

84. (150)

$\bar{a} \cdot \bar{c} = 5 \Rightarrow 2c_1 + c_2 + 3c_3 = 5$

$\bar{b} \cdot \bar{c} = 0 \Rightarrow 3c_1 + 3c_2 + c_3 = 0$

And $[\bar{a}\bar{b}\bar{c}] = 0 \Rightarrow \begin{vmatrix} c_1 & c_2 & c_3 \\ 2 & 1 & 3 \\ 3 & 3 & 1 \end{vmatrix} = 0$

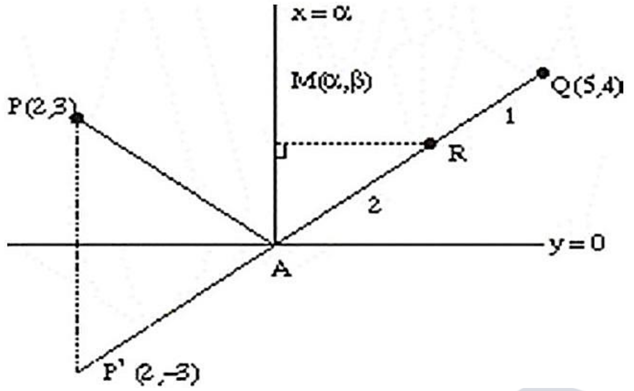
$\Rightarrow 8c_1 - 7c_2 - 3c_3 = 0$

By solving (1), (2), (3) we get

$C_1 = \frac{10}{122}, c_2 = \frac{-85}{122}, c_3 = \frac{225}{122}$

$\therefore 122(C_1 + C_2 + C_3) = 150$

85. (31)



By observation we see that $A(\alpha, 0)$.

And $\beta =$ y-cordinate of R

$= \frac{2 \times 4 + 1 \times 0}{2 + 1} = \frac{8}{3}$

Now P' is image of P in $y = 0$ which will be $P'(2, -3)$

$\therefore$ Equation of $P'Q$ is $(y + 3) = \frac{4+3}{5-2}(x - 2)$

i.e. $3y + 9 = 7x - 14$

$A \equiv \left(\frac{23}{7}, 0\right)$ by solving with $y = 0$

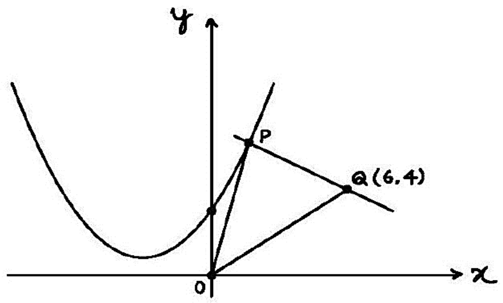
$\therefore \alpha = \frac{23}{7}$

By (1), (2)

$7\alpha + 3\beta = 23 + 8 = 31$

86. (13)

$y = 2x^2 + x + 2$



$$\frac{dy}{dx} = 4x + 1$$

Let P be (h, k), then normal at P is

$$y - k = -\frac{1}{4h+1}(x - h)$$

This passes through Q (6,4)

$$\therefore 4 - k = -\frac{1}{4h+1}(6 - h)$$
$$\Rightarrow (4h+1)(4-k) + 6 - h = 0$$

Also $k = 2h^2 + h + 2$

$$\therefore (4h+1)(4 - 2h^2 - h - 2) + 6 + h = 0$$
$$\Rightarrow 4h^3 - 3h^2 + 3h - 8 = 0$$
$$\Rightarrow h = 1, k = 5$$

Now area of $\triangle$ OPQ will be $= \frac{1}{2}\begin{vmatrix} 1 & 0 & 0 \\ 1 & 1 & 5 \\ 1 & 6 & 4 \end{vmatrix} = 13$

87. (702) $a_1, a_2, a_3, \ldots, a_{18}, 77$

are in AP i.e. 1, 5, 9, 13, ..., 77.

Hence $a_1 + a_2 + a_3 + \cdots + a_{18} = 5 + 9 + 13 + \cdots 18 \text{ terms} = 702$

88. (2) $\left(2x^3 + \frac{3}{x^k}\right)^{12}$

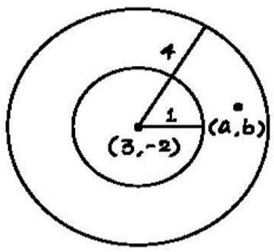
$$t_{r+1} = {}^{12}C_r (2x^3)^r \left(\frac{3}{x^k}\right)^{12-r}$$
$$x^{3r-(12-r)k} \rightarrow \text{ constant}$$
$$\therefore 3r - 12k + rk = 0$$
$$\Rightarrow k = \frac{3r}{12 - r}$$

$\therefore$ possible values of r are 3,6,8,9,10 and corresponding values of k are 1,3,6,9,15

Now ${}^{12}C_r = 220,924,495,220,66$

$\therefore$ possible values of k for which we will get 2^8 are 3,6

89. (40) $1 < |Z - 3 + 2i| < 4$



$$1 < (a-3)^2 + (b+2)^2 < 16$$

$(0, \pm 2), (\pm 2, 0), (\pm 1, \pm 2), (\pm 2, \pm 1)$

$(\pm 2, \pm 3), (3\pm, \pm 2), (\pm 1, \pm 1), (2\pm, \pm 2)$

$(\pm 3, 0), (0, \pm 3), (\pm 3 \pm 1), (\pm 1, \pm 3)$

Total 40 points

90. (816) Normal are

$$y + 2x = \sqrt{11} + 7\sqrt{7}$$
$$2y + x = 2\sqrt{11} + 6\sqrt{7}$$

Center of the circle is point of intersection of normals i.e.

$$\left(\frac{8\sqrt{7}}{3}, \sqrt{11} + \frac{5\sqrt{7}}{3}\right)$$

Tangent is $\sqrt{11}y - 3x = \frac{5\sqrt{77}}{3} + 11$

Radius will be $\perp$ distance of tangent from center i.e. $4\sqrt{\frac{7}{5}}$

Now $(5h - 8k)^2 + 5r^2 = 816$