

PART-I PHYSICS**SECTION-A**

1. Given below are two statements: One is labelled as Assertion A and the other is labelled as Reason R.

Assertion A : Product of Pressure (P) and time (t) has the same dimension as that of coefficient of viscosity.

Reason R: Coefficient of viscosity = $\frac{\text{Force}}{\text{Velocity gradient}}$

Question: Choose the correct answer from the options given below:

(a) Both A and R true, and R is correct explanation of A.

(b) Both A and R are true but R is NOT the correct explanation of A.

(c) A is true but R is false.

(d) A is false but R is true.

2. A particle of mass m is moving in a circular path of constant radius r such that its centripetal acceleration (a) is varying with time t as $a = k^2 r^2$, where k is a constant. The power delivered to the particle by the force acting on it is given as

(a) zero (b) $mk^2 r^2 t^2$

(c) $mk^2 r^2 t$ (d) $mk^2 r t$

3. Motion of a particle in $x - y$ plane is described by a set of following equations $x = 4\sin\left(\frac{\pi}{2} - \omega t\right)m$ and $y = 4\sin(\omega t)m$. The path of particle will be -

(a) circular (b) helical

(c) parabolic (d) elliptical

4. Match List-I with List-II

	List-I		List-II
A	Moment of inertia of solid sphere of radius R about any tangent	I	$\frac{5}{3}MR^2$
B	Moment of inertia of hollow sphere of radius (R) about any tangent	II	$\frac{7}{5}MR^2$
C	Moment of inertia of circular ring of radius (R) about its diameter.	III	$\frac{1}{4}MR^2$
D	Moment of inertia of circular disc of radius (R) about any diameter.	IV	$\frac{1}{2}MR^2$

Question: Choose the correct answer from the options given below

- (a) A-II, B-II, C-IV, D-III (b) A-I, B-II, C-IV, D-III
(c) A-II, B-I, C-III, D-IV (d) A-I, B-II, C-III, D-IV

5. Two planets A and B of equal mass are having their period of revolutions T_A and T_B such that $T_A = 2 T_B$. These planets are revolving in the circular orbits of radii r_A and r_B respectively. Which out of the following would be the correct relationship of their orbits?

- (a) $2r_A^2 = r_B^2$ (b) $r_A^3 = 2r_B^3$
(c) $r_A^3 = 3r_B^3$ (d) $T_A^2 - T_B^2 = \frac{\pi^2}{GM} (r_B^3 - 4r_A^3)$

6. A water drop of diameter cm is broken into 64 equal droplets. The surface tension of water is 0.075 N/m. In this process the gain in surface energy will be:

- (a) 2.8×10^{-4} J (b) 1.5×10^{-3} J
(c) 1.9×10^{-4} J (d) 9.4×10^{-5} J

7. Given below are two statement:

Statement - I : What μ amount of an ideal gas undergoes adiabatic change from state (P_1, V_1, T_1) to state (P_2, V_2, T_2) , the work done is $W = \frac{1R(T_2-T_1)}{1-\gamma}$, where $\gamma = \frac{C_p}{C_v}$ and R = universal gas constant,

Statement - II: In the above case. when work is done on the gas. the temperature of the gas would rise.

Choose the correct answer from the options given below:

- (a) Both statement-I and statement-II are true.
(b) Both statement-I and statement-II are false.
(c) Statement-I is true but statement-II is false.
(d) Statement-I is false but statement-II is true.

8. Given below are two statements:

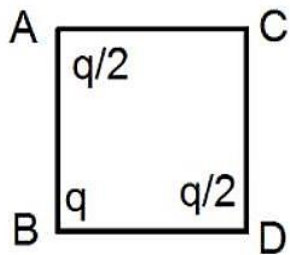
Statement-I: A point charge is brought in an electric field. The value of electric field at a point near to the charge may increase if the charge is positive.

Statement-II: An electric dipole is placed in a non-uniform electric field. The net electric force on the dipole will not be zero.

Choose the correct answer from the options given below:

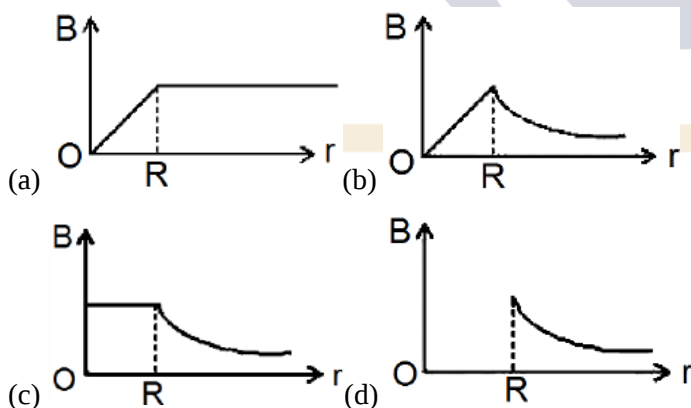
- (a) Both statement-I and statement-II are true.
- (b) Both statement-I and statement-I are false.
- (c) Statement-I is true but statement-II is false.
- (d) Statement-I is false but statement-II is true.

9. The three charges $q/2$, q and $q/2$ are placed at the corners A, B and C of a square of side 'a' as shown in figure. The magnitude of electric field (E) at the corner D of the square, is:



- (a) $\frac{q}{4\pi\epsilon_0 a^2} \left(\frac{1}{\sqrt{2}} + \frac{1}{2} \right)$
- (b) $\frac{q}{4\pi\epsilon_0 a^2} \left(1 + \frac{1}{\sqrt{2}} \right)$
- (c) $\frac{q}{4\pi\epsilon_0 a^2} \left(1 - \frac{1}{\sqrt{2}} \right)$
- (d) $\frac{q}{4\pi\epsilon_0 a^2} \left(\frac{1}{\sqrt{2}} - \frac{1}{2} \right)$

10. An infinitely long hollow conducting cylinder with radius R carries a uniform current along its surface. Choose the correct representation of magnetic field (b) as a function of radial distance (r) from the axis of cylinder.



11. A radar sends an electromagnetic signal of electric field (E_0) = 2.25 V/m and magnetic field (B_0) = 1.5×10^{-8} T which strikes a target on line of sight at a distance of 3 km in a medium. After that, a part of signal (echo) reflects back towards the radar with the same velocity and by same path. If the signal was transmitted at time t_0 from radar, then after how much time echo will reach to the radar?

- (a) 2.0×10^{-5} s
- (b) 4.0×10^{-5} s

- (c) $1.0 \times 10^{-5} \text{ s}$ (d) $8.0 \times 10^{-5} \text{ s}$

12. The refracting angle of a prism is A and refractive index of the material of the prism is $\cot(A/2)$. Then the angle of minimum deviation will be -

- (a) $180 - 2A$ (b) $90 - A$
(c) $180 + 2A$ (d) $180 - 3A$

13. The aperture of the objective is 24.4 cm. The resolving power of this telescope. If a light of wavelength $2440\text{\AA}$ is used to see the object will be

- (a) 8.1×10^6 (b) 10.0×10^7
(c) 8.2×10^5 (d) 1.0×10^{-8}

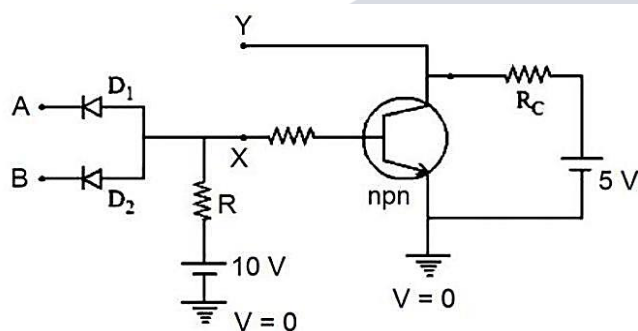
14. The de Brogue wavelengths for an electron and a photon are λ_e and λ_p respectively. For the same kinetic energy of electron and photon, which of the following presents the correct relation between the de Brogue wavelengths of two?

- (a) $\lambda_p \propto \lambda_e^2$ (b) $\lambda_p \propto \lambda_e$
(c) $\lambda_p \propto \sqrt{\lambda_e}$ (d) $\lambda_p \propto \sqrt{\frac{1}{\lambda_e}}$

15. The Q -value of a nuclear reaction and kinetic energy of the projectile particle, K_p are related as :

- (a) $Q = K_p$ (b) $(K_p + Q) < 0$
(c) $Q < K_p$ (d) $(K_p + Q) > 0$

16. In the following circuit, the correct relation between output (Y) and inputs A and B will be :



- (a) $Y = AB$ (b) $Y = A + B$
(c) $Y = \overline{AB}$ (d) $Y = \overline{A + B}$

17. For using a multimeter to identify diode from electrical components, choose the correct statement out of the following about the diode:

- (a) It is two terminal device which conducts current in both directions.
(b) It is two terminal device which conducts current in one direction only
(c) It does not conduct current gives an initial deflection which decays to zero.
(d) It is three terminal device which conducts current in ne direction only between central terminal and either of the remaining two terminals

18. Given below are two statements: One is labelled as Assertion A and the other is labelled as Reason R.

Assertion A: n-p-n transistor permits more current than a p-n-p transistor.

Reason R: Electrons have greater mobility as a charge carrier.

Choose the correct answer from the options given below:

- (a) Both A and R true, and R is correct explanation of A.

- (b) Both A and R are true but R is NOT the correct explanation of A.
- (c) A is true but R is false.
- (d) A is false but R is true.

19. Match List-I with List-II

	List-I		List-II
A	Television signal	I	03KHz
B	Radio signal	II	20KHz
C	High Quality Music	III	02MHz
D	Human speech	IV	06MHz

Choose the correct answer from the options given below:

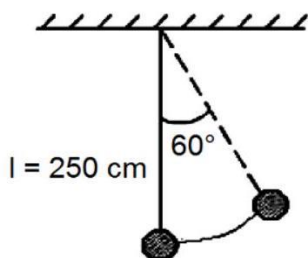
- (a) A-I, B-II, C-III, D-IV (b) A-IV, B-III, C-I, D-II
- (c) A-IV, B-III, C-II, D-I (d) A-I, B-II, C-IV, D-III

20. The velocity of sound in a gas, in which two wavelengths 4.08 m and 4.16 m produce 40 beats in 12 s, will be :
- (a) $2.82.8 \text{ ms}^{-1}$ (b) 175.5 ms^{-1}
- (c) 353.6 ms^{-1} (d) 707.2 ms^{-1}

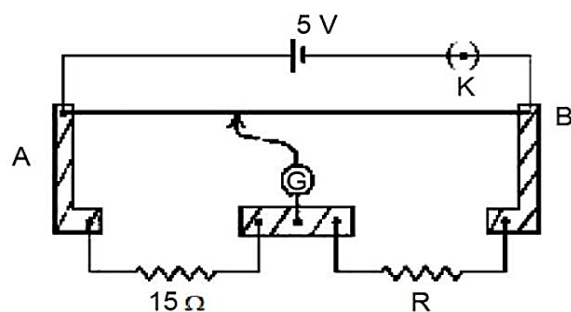
SECTION - B

21. A pendulum is suspended by a string of length 250 cm. The mass of the bob of the pendulum is 200 g. The bob is pulled aside until the string is at 60° with vertical as shown in the figure. After releasing the bob, the maximum velocity attained by the bob will be ms^{-1} .

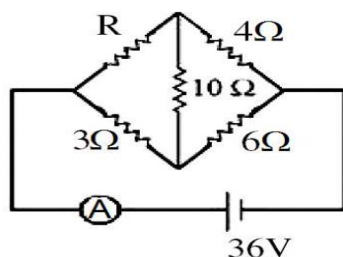
(if $g = 10 \text{ m/s}^2$)



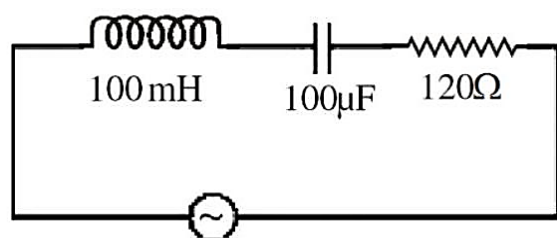
22. A meter bridge setup is shown in the figure. It is used to determine an unknown resistance R using a given resistor of 15Ω . The galvanometer (G) shows null deflection when tapping key is at 43 cm mark from end A. If the end correction for end A is 2 cm, then the determined value of R will be Ω .



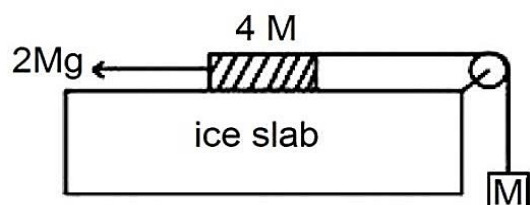
23. Current measured by the ammeter (a) in the reported circuit when no current flows through 10Ω resistance. will be A.



24. An AC source is connected to an inductance of 100 mH , a capacitance of $100\mu\text{F}$ and a resistance of 120Ω as shown in figure. The time in which the resistance having a thermal capacity $2\text{ J}^\circ\text{C}$ will get heated by 16°C is S.



25. The position vector of 1 kg object is $\vec{r} = (3\hat{i} - \hat{j})\text{m}$ and its velocity $\vec{v} = (3\hat{j} + k)\text{ms}^{-1}$. The magnitude of its angular momentum is $\sqrt{x}\text{Nm}$ where x is
26. A man of 60 kg is running on the road and suddenly jumps into a stationary trolley car of mass 120 kg . Then, the trolley car starts moving with velocity 2 ms^{-1} . The velocity of the running man was ms^{-1} , when he jumps into the car.
27. A hanging mass M is connected to a four times bigger mass by using a string-pulley arrangement, as shown in the figure. The bigger mass is placed on a horizontal ice-slab and being pulled by $2Mg$ force. In this situation, tension in the string is $\frac{x}{5}Mg$ for $x =$ Neglect mass of the string and friction of the block (bigger mass) with ice slab. (Given $g =$ acceleration due to gravity)



28. The total internal energy of two mole monoatomic ideal gas at temperature $T = 300 \text{ K}$ will be J. (Given $R = 8.31 \text{ J/mol.K}$)
29. A singly ionized magnesium atom (A_{24}) ion is accelerated to kinetic energy 5 keV and is projected perpendicularly into a magnetic field B of the magnitude 0.5 T . The radius of path formed will be cm.
30. A telegraph line of length 100 km has a capacity of $\frac{0.01 \mu\text{F}}{\text{km}}$ and it carries an alternating current at 0.5 kHz cycle per second. If minimum impedance is required, then the value of the inductance that needs to be introduced in series is mH. (if $\pi = \sqrt{10}$)

PART-II CHEMISTRY

SECTION-A

31. The incorrect statement about the imperfections in solids is:

- (a) Schottky defect decreases the density of the substance.
- (b) Interstitial defect increases the density of the substance.
- (c) Frenkel defect does not alter the density of the substance.
- (d) Vacancy defect increases the density of the substance.

32. The Zeta potential is related to which property of colloids"

- (a) Colour
- (b) Tyndall effect
- (c) Charge on the surface of colloidal particles
- (d) Brownian movement

33. Element "E" belongs to the period 4 and group 16 of the periodic table. The valence shell electron configuration of the element, which is just above 'E' in the group is

- (a) $3s^2 \cdot 3p^4$
- (b) $3d^{10} \cdot 4s^2, 4p^4$
- (c) $4d^{10} \cdot 5s^2, 5p^4$
- (d) $2s^2, 2p^4$

34. Given are two statements one is labelled as Assertion A and other is labelled as Reason R. Assertion A : Magnesium can reduce Al_2O_3 at a temperature below 1350°C , while above 1350°C aluminium can reduce MgO . Reason R : The melting and boiling points of magnesium are lower than those of aluminium. In light of the above statements. choose most appropriate answer from the options given below:

- (a) Both A and R are correct. and R is correct explanation of A.
- (b) Both A and R are correct. but R is NOT the correct explanation of A.
- (c) A is correct R is not correct.
- (d) A is not correct. R is correct.

35. Dihydrogen reacts with CuO to give

- (a) CuH_2
- (b) Cu

- (c) Cu_2O (d) $\text{Cu}(\text{OH})_2$

36. Nitrogen gas is obtained by thermal decomposition of

- (a) $\text{Ba}(\text{NO}_3)_2$ (b) $\text{Ba}(\text{N}_3)_2$
(c) NaNO_2 (d) NaNO_3

37. Given below are two statements:

Statement -I: The pentavalent oxide of group- 15 element. E_2O_5 . is less acidic than trivalent oxide. E_2O_3 . of the same element.

Statement -II: The acidic character of trivalent oxide of group 15 elements. E_2O_3 . decreases down the group.

In light of the above statements. choose most appropriate answer from the options given below:

- (a) Both Statement I and Statement II are true.
(b) Both Statement I and Statement II are false.
(c) Statement I true. but statement II is false.
(d) Statement I is false but statement II is true.

38. Which one of the lanthanoids given below is the most stable in divalent form?

- (a) Ce (Atomic Number 58) (b) Sm (Atomic Number 62)
(c) Eu (Atomic Number 63) (d) Yb (Atomic Number 70)

39. Given below are two statements:

Statement I: $[\text{Ni}(\text{CN})_4]^{2-}$ is square planar and diamagnetic complex. with dsp^2 hybridization for Ni but $[\text{Ni}(\text{CO})_4]$ is tetrahedral. paramagnetic and with sp^3 -hybridization for Ni.

Statement II: $[\text{NiCl}_4]^{2-}$ and $[\text{Ni}(\text{CO})_4]$ both have same d-electron configuration have same geometry and are paramagnetic.

In light the above statements. choose the correct answer form the options given below:

- (a) Both Statement I and Statement II are true.
(b) Both Statement I and Statement II are false.
(c) Statement I is correct but statement II is false.
(d) Statement I is incorrect but statement II is true.

40. Which amongst the following is not a pesticide?

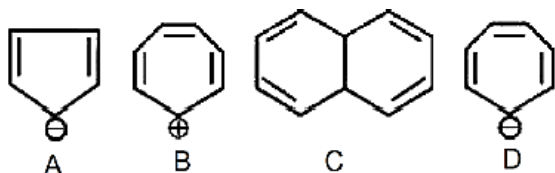
- (a) DDT (b) Organophosphates
(c) Dieldrin (d) Sodium arsenite

41. Which one of the following techniques is not used to spot components of a mixture separated on thin layer chromatographic plate?

- (a) I_2 (Solid)
(b) U.V. Light
(c) Visualisation agent as a component of mobile phase

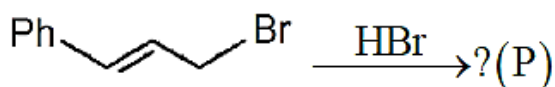
(d) Spraying of an appropriate reagent

42. Which of the following structures are aromatic in nature?

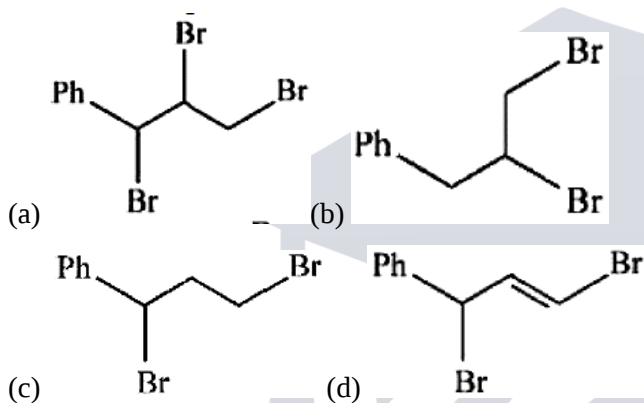


- (a) A, B, C and D (b) Only A and B
(c) Only A and C (d) Only B, C and D

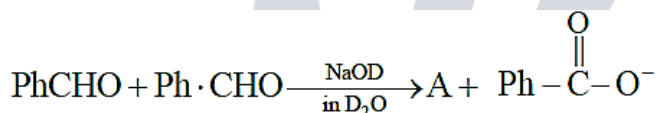
43. The major product (P) in the reaction



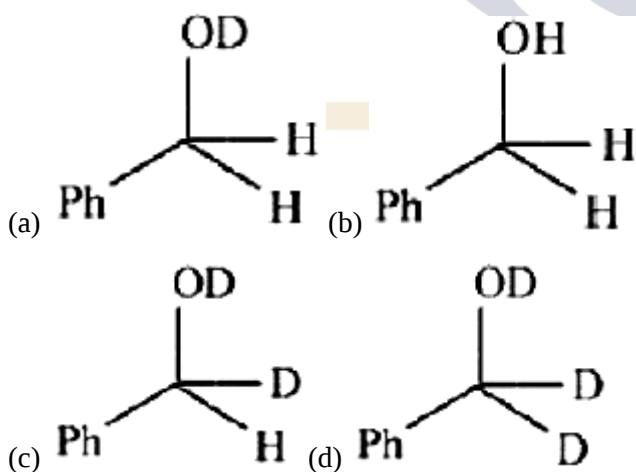
[Ph is C_6H_5] is



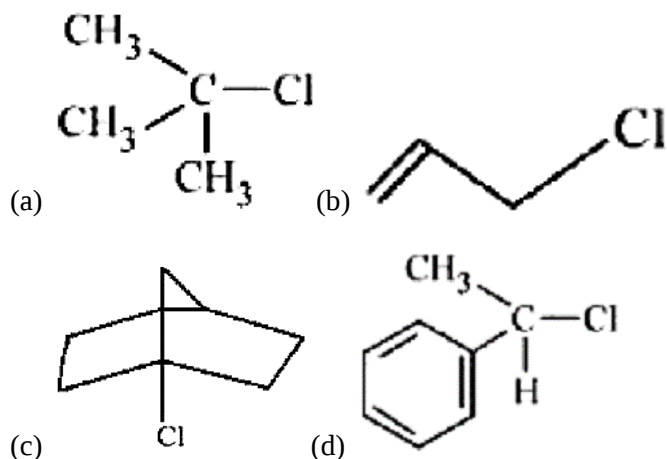
44. The correct structure of product 'A' formed in the following reaction.



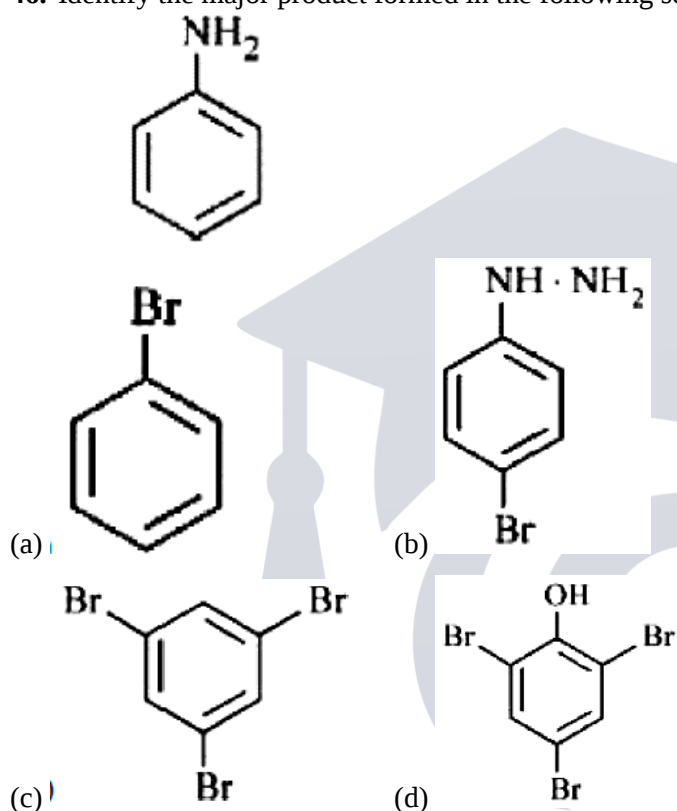
(Ph is C_6H_5)



45. Which one of the following compounds is inactive towards $\text{S}_\text{N}1$ reaction?



46. Identify the major product formed in the following sequence of reactions:



47. A primary aliphatic amine on reaction with nitrous acid in cold (273 K) and there after raising temperature of reaction mixture to room temperature (298 K). Gives a/an

- (a) nitrile (b) alcohol
(c) diazonium salt (d) secondary amine

48. Which one of the following is NOT a copolymer?

- (a) Buna-S (b) Neoprene
(c) PHBV (d) Butadiene-styrene

49. Stability of α - Helix structure of proteins depends upon

- (a) dipolar interaction (b) H-bonding interaction
(c) van der Waals forces (d) π -stacking interaction

50. The formula of the purple colour formed in Laissaigne's test for sulphur using sodium nitroprusside is

- (a) $\text{NaFe}[\text{Fe}(\text{CN})_6]$ (b) $\text{Na}[\text{Cr}(\text{NH}_3)_2(\text{NCS})_4]$
(c) $\text{Na}_2[\text{Fe}(\text{CN})_5(\text{NO})]$ (d) $\text{Na}_4[\text{Fe}(\text{CN})_5(\text{NOS})]$

SECTION-B

51. A 2.0 g sample containing MnO_2 is treated with HCl liberating Cl_2 . The Cl_2 gas is passed into a solution of KI and 60.0 mL of 0.1M $\text{Na}_2\text{S}_2\text{O}_3$ is required to titrate the liberated iodine. The percentage of MnO_2 in the sample is (Nearest integer) [Atomic masses (in u) $\text{Mn} = 55$; $\text{Cl} = 35.5$; $\text{O} = 16$, $\text{I} = 127$, $\text{Na} = 23$, $\text{K} = 39$, $\text{S} = 32$]
52. If the work function of a metal is $6.63 \times 10^{-19} \text{ J}$, the maximum wavelength of the photon required to remove a photoelectron from the metal is nm. (Nearest integer) [Given: $h = 6.63 \times 10^{-34} \text{ J s}$, and $c = 3 \times 10^8 \text{ m s}^{-1}$]
53. The hybridization of P exhibited in PF_5 is $\text{sp}^x \text{d}^y$. The value of y is
54. 4.0 L of an ideal gas is allowed to expand isothermally into vacuum until the total volume is 20 L. The amount of heat absorbed in this expansion is L atm.
55. The vapour pressures of two volatile liquids A and B at 25°C are 50 Torr and 100 Torr, respectively. If the liquid mixture contains 0.3 mole fraction of A , then the mole fraction of liquid B in the vapour phase is $\frac{x}{17}$. The value of x is
56. The solubility product of a sparingly soluble salt A_2X_3 is 1.1×10^{-23} . If specific conductance of the solution is $3 \times 10^{-5} \text{ S m}^{-1}$, the limiting molar conductivity of the solution is $x \times 10^{-3} \text{ S m}^2 \text{ mol}^{-1}$. The value of x is
57. The quantity of electricity in Faraday needed to reduce 1 mol of $\text{Cr}_2\text{O}_7^{2-}$ to Cr^{3+} is
58. For a first order reaction $\text{A} \rightarrow \text{B}$, the rate constant, $k = 5.5 \times 10^{-14} \text{ s}^{-1}$. The time required for 67% completion of reaction is $x \times 10^{-1}$ times the half life of reaction. The value of x is (Nearest integer) (Given: $\log 3 = 0.4771$)
59. Number of complexes which will exhibit synergic bonding amongst, $[\text{Cr}(\text{CO})_6]$, $[\text{Mn}(\text{CO})_5]$ and $[\text{Mn}_2(\text{CO})_{10}]$ is
60. In the estimation of bromine, 0.5 g of an organic compound gave 0.40 g of silver bromide. The percentage of bromine in the given compound is % (nearest integer) (Relative atomic masses of Ag and Br are 108u and 80u, respectively).

PART-III MATHEMATICS

SECTION-A

61. If

$$\sum_{k=1}^{31} ({}^{31}\text{C}_k) ({}^{31}\text{C}_{k-1}) - \sum_{k=1}^{30} ({}^{30}\text{C}_k) ({}^{30}\text{C}_{k-1}) = \frac{\alpha(60!)}{(30!)(31!)},$$

Where $\alpha \in \mathbb{R}$, then the value of 16α is equal to

- (a) 1411 (c) 1615
(b) 1320 (d) 1855

62. Let a function $f: \mathbb{N} \rightarrow \mathbb{N}$ be defined by

$$f(n) = \begin{cases} 2n, & n = 2, 4, 6, 8, \dots \\ n-1, & n = 3, 7, 11, 15, \dots \\ \frac{n+1}{2}, & n = 1, 5, 9, 13, \dots \end{cases}$$

then, f is

- (a) one-one but not onto (b) onto but not one-one
(c) neither one-one nor onto (d) one-one and onto

63. If the system of linear equations

$$2x + 3y - z = -2$$

$$x + y + z = 4$$

$$x - y + |\lambda|z = 4\lambda - 4$$

where $\lambda \in \mathbb{R}$, has no solution, then

- (a) $\lambda = 7$ (b) $\lambda = -7$
(c) $\lambda = 8$ (d) $\lambda^2 = 1$

64. Let A be a matrix of order 3×3 and $\det(a) = 2$. Then $\det(\det(a)\text{adj}(5\text{adj}(A^3)))$ is equal to

- (a) 512×10^6 (b) 256×10^6
(c) 1024×10^6 (d) 256×10^{11}

65. The total number of 5-digit numbers, formed by using the digits 1, 2, 3, 5, 6, 7 without repetition, which are multiple of 6, is

- (a) 36 (b) 48
(c) 60 (d) 72

66. Let A_1, A_2, A_3 , be an increasing geometric progression of positive real numbers. If $A_1 A_3 A_5 A_7 = \frac{1}{1296}$ and

$A_2 + A_4 = \frac{7}{36}$, then, the value of $A_6 + A_8 + A_{10}$ is equal to

- (a) 33 (b) 37
(c) 43 (d) 47

67. Let $[t]$ denote the greatest integer less than or equal to t . Then, the value of the integral

$$\int_0^1 [-8x^2 + 6x - 1] dx \text{ is equal to}$$

- (a) -1 (b) $-\frac{5}{4}$
(c) $\frac{\sqrt{17}-13}{8}$ (d) $\frac{\sqrt{17}-16}{8}$

68. Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be defined as

$$f(x) = \begin{cases} [e^x], & x < 0 \\ ae^x + [x-1], & 0 \leq x < 1 \\ b + [\sin(\pi x)], & 1 \leq x < 2 \\ [e^{-x}] - c, & x \geq 2 \end{cases}$$

where $a, b, c \in \mathbb{R}$ and $[t]$ denotes greatest integer less than or equal to t . Then, which of the following statements is true?

- (a) There exists $a, b, c \in \mathbb{R}$ such that f is continuous of $\mathbb{R}$
(b) If f is discontinuous at exactly one point, then $a + b + c = 1$.
(c) If f is discontinuous at exactly one point, then $a + b + c \neq 1$.

(d) f is discontinuous at at least two points, for any values of a, b and c .

69. The area of the region $S = \{(x, y) : y^2 \leq 8x, y \geq \sqrt{2}x, x \geq 1\}$ is

- (a) $\frac{13\sqrt{2}}{6}$ (b) $\frac{11\sqrt{2}}{6}$
(c) $\frac{5\sqrt{2}}{6}$ (d) $\frac{19\sqrt{2}}{6}$

70. Let the solution curve $y = y(x)$ of the differential equation,

$$\left[\frac{x}{\sqrt{x^2 - y^2}} + e^{\frac{y}{x}} \right] x \frac{dy}{dx} = x + \left[\frac{x}{\sqrt{x^2 - y^2}} + e^{\frac{y}{x}} \right] y$$

pass through the points $(1, 0)$ and $(2\alpha, \alpha)$, $\alpha > 0$.

Then α is equal to

- (a) $\frac{1}{2} \exp\left(\frac{\pi}{6} + \sqrt{e} - 1\right)$ (b) $\frac{1}{2} \exp\left(\frac{\pi}{3} + \sqrt{e} - 1\right)$
(c) $\exp\left(\frac{\pi}{6} + \sqrt{e} + 1\right)$ (d) $2 \exp\left(\frac{\pi}{3} + \sqrt{e} - 1\right)$

71. Let $y = y(x)$ be the solution of the differential equation $x(1 - x^2) \frac{dy}{dx} + (3x^2y - y - 4x^3) = 0$, $x > 1$, with $y(2) = -2$. Then $y(3)$ is equal to

- (a) -18 (b) -12
(c) -6 (d) -3

72. The number of real solutions of $x^7 + 5x^3 + 3x + 1 = 0$ is equal to

- (a) 0 (b) 1
(c) 3 (d) 5

73. Let the eccentricity of the hyperbola $H: \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ be $\sqrt{\frac{5}{2}}$ and length of its latus rectum be $6\sqrt{2}$. If $y = 2x + c$ is a tangent to the hyperbola H , then the value of c^2 is equal to

- (a) 18 (b) 20
(c) 24 (d) 32

74. If the tangents drawn at the point $O(0, 0)$ and $P(1 + \sqrt{5}, 2)$ on the circle $x^2 + y^2 - 2x - 4y = 0$ intersect at the point Q , then the area of the triangle OPQ is equal to

- (a) $\frac{3+\sqrt{5}}{2}$ (b) $\frac{4+2\sqrt{5}}{2}$
(c) $\frac{5+3\sqrt{5}}{2}$ (d) $\frac{7+3\sqrt{5}}{2}$

75. If two distinct point Q, R lie on the line of intersection of the planes $-x + 2y - z = 0$ and $3x - 5y + 2z = 0$ and $PQ = PR = \sqrt{18}$ where the point P is $(1, -2, 3)$, then the area of the triangle PQR is equal to

- (a) $\frac{2}{3}\sqrt{38}$ (b) $\frac{4}{3}\sqrt{38}$
(c) $\frac{8}{3}\sqrt{38}$ (d) $\sqrt{\frac{152}{3}}$

76. The acute angle between the planes P_1 and P_2 , when P_1 and P_2 are the planes passing through the intersection of the planes $5x + 8y + 13z - 29 = 0$ and $8x - 7y + z - 20 = 0$ and the points $(2, 1, 3)$ and $(0, 1, 2)$, respectively, is

- (a) $\frac{\pi}{3}$ (b) $\frac{\pi}{4}$
(c) $\frac{\pi}{6}$ (d) $\frac{\pi}{12}$

77. Let the plane $P: \vec{r} \cdot \vec{a} = d$ contain the line of intersection of two planes $\vec{r} \cdot (\hat{i} + 3\hat{j} - \hat{k}) = 6$ and $\vec{r} \cdot (-6\hat{i} + 5\hat{j} - \hat{k}) = 7$. If the plane P passes through the point $(2, 3, \frac{1}{2})$, then the value of $\frac{|13\vec{a}|^2}{d^2}$ is equal to

- (a) 90 (b) 93
(c) 95 (d) 97

78. The probability, that in a randomly selected 3-digit number at least two digits are odd, is

- (a) $\frac{19}{36}$ (b) $\frac{15}{36}$
(c) $\frac{13}{36}$ (d) $\frac{23}{36}$

79. Let AB and PQ be two vertical poles, 160 m apart from each other. Let C be the middle point of B and Q, which are feet of these two poles. Let $\frac{\pi}{8}$ and θ be the angles of elevation from C to P and A, respectively. If the height of pole PQ is twice the height of pole AB, then $\tan^2 \theta$ is equal to

- (a) $\frac{3-2\sqrt{2}}{2}$ (b) $\frac{3+\sqrt{2}}{2}$
(c) $\frac{3-2\sqrt{2}}{4}$ (d) $\frac{3-\sqrt{2}}{4}$

80. Let p, q, r be three logical statements. Consider the compound statements

$S_1: ((\sim p) \vee q) \vee ((\sim p) \vee r)$ and

$S_2: p \rightarrow (q \vee r)$

Then, which of the following is NOT true?

- (a) If S_2 is True, then S_1 is True
(b) If S_2 is False, then S_1 is False
(c) If S_2 is False, then S_1 is True
(d) If S_1 is False, then S_2 is False

SECTION-B

81. Let R_1 and R_2 be relations on the set $\{1, 2, \dots, 50\}$ such that $R_1 = \{(p, p^n): p \text{ is a prime and } n \geq 0 \text{ is an integer}\}$ and $R_2 = \{(p, p^n): p \text{ is a prime and } n = 0 \text{ or } 1\}$. Then, the number of elements in $R_1 - R_2$ is

82. The number of real solutions of the equation $e^{4x} + 4e^{3x} - 58e^{2x} + 4e^x + 1 = 0$ is

83. The mean and standard deviation of 15 observations are found to be 8 and 3 respectively. On rechecking it was found that, in the observations, 20 was misread as 5. Then, the correct variance is equal to

84. If $\vec{a} = 2\hat{i} + \hat{j} + 3\hat{k}$, $\vec{b} = 3\hat{i} + 3\hat{j} + \hat{k}$ and $\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$ are coplanar vectors and $\vec{a} \cdot \vec{c} = 5$, $\vec{b} \perp \vec{c}$, then $122(c_1 + c_2 + c_3)$ is equal to

85. A ray of light passing through the point P(2, 3) reflects on the X-axis at point A and the reflected ray passes through the point Q(5, 4). Let R be the point that divides the line segment AQ internally into the ratio 2: 1. Let the co-ordinates of the foot of the perpendicular M from R on the bisector of the angle PAQ be (α, β) . Then, the value of $7\alpha + 3\beta$ is equal to

86. Let ℓ be a line which is normal to the curve $y = 2x^2 + x + 2$ at a point P on the curve. If the point Q(6, 4) lies on the line ℓ and O is origin, then the area of the triangle OPQ is equal to

87. Let $A = \{1, a_1, a_2, \dots, a_{18}, 77\}$ be a set of integers with $1 < a_1 < a_2 < \dots < a_{18} < 77$. Let the set $A + A = \{x + y: x, y \in A\}$ contain exactly 39 elements. Then, the value of $a_1 + a_2 + \dots + a_{18}$ is equal to

88. The number of positive integers k such that the constant term in the binomial expansion of

$$\left(2x^3 + \frac{3}{x^k}\right)^{12}, x \neq 0 \text{ is } 2^8 \cdot \ell, \text{ where } \ell \text{ is an odd integer, is}$$

89. The number of elements in the set

$\{z = a + ib \in \mathbb{C}: a, b \in \mathbb{Z} \text{ and } 1 < |z - 3 + 2i| < 4\}$ is

90. Let the lines $y + 2x = \sqrt{11} + 7\sqrt{7}$ and $2y + x = 2\sqrt{11} + 6\sqrt{7}$ be normal to a circle $C: (x - h)^2 + (y - k)^2 = r^2$. If the line $\sqrt{11}y - 3x = \frac{5\sqrt{77}}{3} + 11$ is tangent to the circle C , then the value of $(5h - 8k)^2 + 5r^2$ is equal to

