

PART-I PHYSICS

SECTION-A

1. Velocity (v) and acceleration (a) in two systems of units 1 and 2 are related as $v_2 = \frac{n}{m^2} v_1$ and $a_2 = \frac{a_1}{mn}$ respectively. Here m and n are constants. The relations for distance and time in two systems respectively are:

(a) $\frac{n^3}{m^3} L_1 = L_2$ and $\frac{n^2}{m} T_1 = T_2$

(b) $L_1 = \frac{n^4}{m^2} L_2$ and $T_1 = \frac{n^2}{m} T_2$

(c) $L_1 = \frac{n^2}{m} L_2$ and $T_1 = \frac{n^4}{m^2} T_2$

(d) $\frac{n^2}{m} L_1 = L_2$ and $\frac{n^4}{m^2} T_1 = T_2$

2. A ball is spun with angular acceleration

$\alpha = 6t^2 - 2t$ where t is in second and α is in rads^{-2} . At $t = 0$, the ball has angular velocity of 10 rads^{-1} and angular position of 4rad . The most appropriate expression for the angular position of the ball is:

(a) $\frac{3}{2}t^4 - t^2 + 10t$

(b) $\frac{t^4}{2} - \frac{t^3}{3} + 10t + 4$

(c) $\frac{2t^4}{3} - \frac{t^3}{6} + 10t + 12$

(d) $2t^4 - \frac{t^3}{2} + 5t + 4$

3. A block of mass 2 kg moving on a horizontal surface with speed of 4 ms^{-1} enters a rough surface ranging from $x = 0.5 \text{ m}$ to $x = 1.5 \text{ m}$. The retarding force in this range of rough surface is related to distance by $F = -kx$ where $k = 12 \text{ Nm}^{-1}$. The speed of the block as it just crosses the rough surface will be:

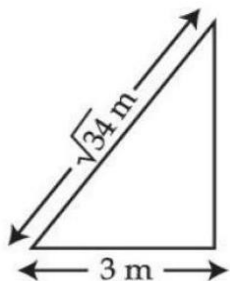
(a) Zero

(b) 1.5 ms^{-1}

(c) 2.0 ms^{-1}

(d) 2.5 ms^{-1}

4. A $\sqrt{34} \text{ m}$ long ladder weighing 10 kg leans on a frictionless wall. Its feet rest on the floor 3 m away from the wall as shown in the figure. If F_f and F_w are the reaction forces of the floor and the wall, then ratio of F_w/F_f will be: (Use $g = 10 \text{ m/s}^2$)



(a) $\frac{6}{\sqrt{110}}$

(b) $\frac{3}{\sqrt{113}}$

(c) $\frac{3}{\sqrt{109}}$

(d) $\frac{2}{\sqrt{109}}$

5. Water fall from a 40 m high dam at the rate of $9 \times 10^4 \text{ kg}$ per hour. Fifty percentage of gravitational potential energy can be converted into electrical energy. Using this hydroelectric energy number of 100 W lamps, that can be lit, is:

(Take $g = 10 \text{ ms}^{-2}$)

- (a) 25 (b) 50
(c) 100 (d) 18

6. Two objects of equal masses placed at certain distance from each other attracts each other with a force of F . If one-third mass of one object is transferred to the other object, then the new force will be:

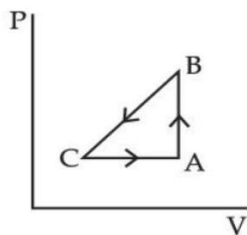
- (a) $\frac{2}{9} F$ (b) $\frac{16}{9} F$
(c) $\frac{8}{9} F$ (d) F

7. A water drop of radius $1 \mu\text{m}$ falls in a situation where the effect of buoyant force is negligible. Coefficient of viscosity of air is $1.8 \times 10^{-5} \text{ Nsm}^{-2}$ and its density is negligible as compared to that of water 10^6 gm^{-3} . Terminal velocity of the water drop is:

(Take acceleration due to gravity = 10 ms^{-2})

- (a) $145.4 \times 10^{-6} \text{ ms}^{-1}$ (b) $118.0 \times 10^{-6} \text{ ms}^{-1}$
(c) $132.6 \times 10^{-6} \text{ ms}^{-1}$ (d) $123.4 \times 10^{-6} \text{ ms}^{-1}$

8. A sample of an ideal gas is taken through the cyclic process ABCA as shown in figure. It absorbs, 40 J of heat during the part AB, no heat during BC and rejects 60 J of heat during CA. A work 50 J is done on the gas during the part BC. The internal energy of the gas at A is 1560 J. The work done by the gas during the part CA is:



- (a) 20 J (b) 30 J
(c) -30 J (d) -60 J

9. What will be the effect on the root mean square velocity of oxygen molecules if the temperature is doubled and oxygen molecule dissociates into atomic oxygen?

- (a) The velocity of atomic oxygen remains same
(b) The velocity of atomic oxygen doubles
(c) The velocity of atomic oxygen becomes half
(d) The velocity of atomic oxygen becomes four times

10. Two-point charges A and B of magnitude $+8 \times 10^{-6} \text{ C}$ and $-8 \times 10^{-6} \text{ C}$ respectively are placed at a distance d apart. The electric field at the middle point O between the charges is $6.4 \times 10^4 \text{ NC}^{-1}$. The distance 'd' between the point charges A and B is:

- (a) 2.0 m (b) 3.0 m
(c) 1.0 m (d) 4.0 m

11. Resistance of the wire is measured as 2Ω and 3Ω at 10°C and 30°C respectively. Temperature coefficient of resistance of the material of the wire is:

- (a) $0.033^\circ \text{C}^{-1}$ (b) $-0.033^\circ \text{C}^{-1}$
(c) $0.011^\circ \text{C}^{-1}$ (d) $0.055^\circ \text{C}^{-1}$

12. The space inside a straight current carrying solenoid is filled with a magnetic material having magnetic susceptibility equal to 1.2×10^{-5} . What is fractional increase in the magnetic field inside solenoid with respect to air as medium inside the solenoid?

- (a) 1.2×10^{-5} (b) 1.2×10^{-3}
(c) 1.8×10^{-3} (d) 2.4×10^{-5}

13. Two parallel, long wires are kept 0.20 m apart in vacuum, each carrying current of xA in the same direction. If the force of attraction per meter of each wire is 2×10^{-6} N, then the value of x is approximately:

- (a) 1 (b) 2.4
(c) 1.4 (d) 2

14. A coil is placed in a time varying magnetic field. If the number of turns in the coil were to be halved and the radius of wire doubled, the electrical power dissipated due to the current induced in the coil would be: (Assume the coil to be short circuited.)

- (a) Halved (b) Quadrupled
(c) The same (d) Doubled

15. An EM wave propagating in x-direction has a wavelength of 8 mm. The electric field vibrating ydirection has maximum magnitude of 60Vm^{-1} . Choose the correct equations for electric and magnetic fields if the EM wave is propagating in vacuum:

(a) $E_y = 60 \sin \left[\frac{\pi}{4} \times 10^3 (x - 3 \times 10^8 t) \right] \hat{j} \text{V}^{-1}$

$B_z = 2 \sin \left[\frac{\pi}{4} \times 10^3 (x - 3 \times 10^8 t) \right] \hat{k} \text{T}$

(b) $E_y = 60 \sin \left[\frac{\pi}{4} \times 10^3 (x - 3 \times 10^8 t) \right] \hat{j} \text{Vm}^{-1}$

$B_z = 2 \times 10^{-7} \sin \left[\frac{\pi}{4} \times 10^3 (x - 3 \times 10^8 t) \right] \hat{k} \text{T}$

(c) $E_y = 2 \times 10^{-7} \sin \left[\frac{\pi}{4} \times 10^3 (x - 3 \times 10^8 t) \right] \hat{j} \text{Vm}^{-1}$

$B_z = 60 \sin \left[\frac{\pi}{4} \times 10^3 (x - 3 \times 10^8 t) \right] \hat{k} \text{T}$

(d) $E_y = 2 \times 10^{-7} \sin \left[\frac{\pi}{4} \times 10^4 (x - 4 \times 10^8 t) \right] \hat{j} \text{Vm}^{-1}$

$B_z = 60 \sin \left[\frac{\pi}{4} \times 10^4 (x - 4 \times 10^8 t) \right] \hat{k} \text{T}$

16. In young's double slit experiment performed using a monochromatic light of wavelength λ , when a glass plate ($\mu = 1.5$) of thickness $x\lambda$ is introduced in the path of the one of the interfering beams, the intensity at the position where the central maximum occurred previously remains unchanged. The value of x will be:

- (a) 3 (b) 2
(c) 1.5 (d) 0.5

17. Let K_1 and K_2 be the maximum kinetic energies of photo-electrons emitted when two monochromatic beams of wavelength λ_1 and λ_2 , respectively are incident on a metallic surface. If $\lambda_1 = 3\lambda_2$ then:

- (a) $K_1 > \frac{K_2}{3}$ (b) $K_1 < \frac{K_2}{3}$
(c) $K_1 = \frac{K_2}{3}$ (d) $K_2 = \frac{K_1}{3}$

18. Following statements related to radioactivity are given below:

(A) Radioactivity is a random and spontaneous process and is dependent on physical and chemical conditions.

(B) The number of un-decayed nuclei in the radioactive sample decays exponentially with time.

(C) Slope of the graph of $\log_e$ (no. of undecayed nuclei) Vs. time represents the reciprocal of mean life time (τ).

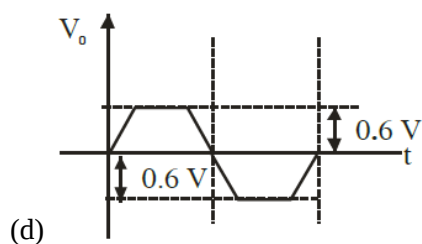
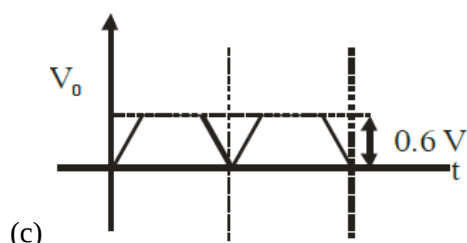
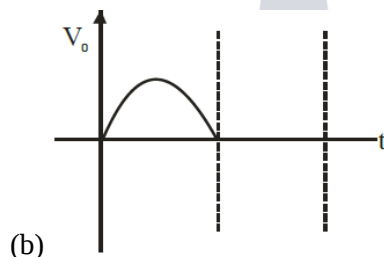
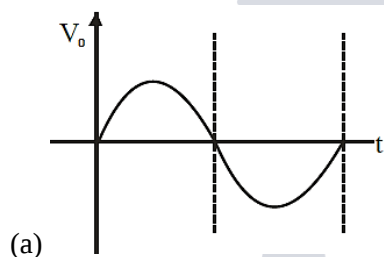
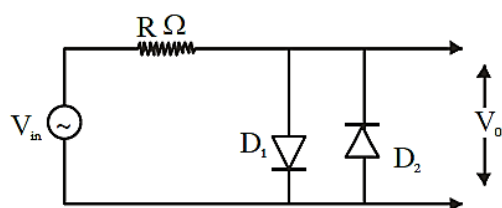
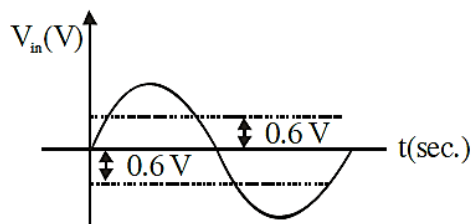
(D) Product of decay constant (λ) and half-life time ($T_{1/2}$) is not constant.

Choose the most appropriate answer from the options given below:

(a) (A) and (B) only (b) (B) and (D) only

(c) (B) and (C) only (d) (C) and (D) only

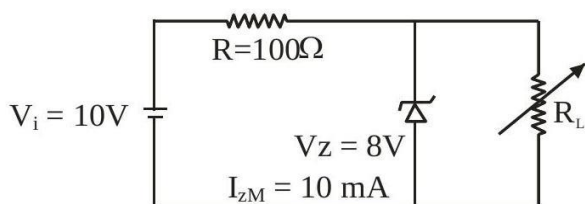
19. In the given circuit the input voltage V_{in} is shown in figure. The cut-in voltage of p – n junction diode (D_1 or D_2) is 0.6 V. Which of the following output voltage (V_o) waveform across the diode is correct?



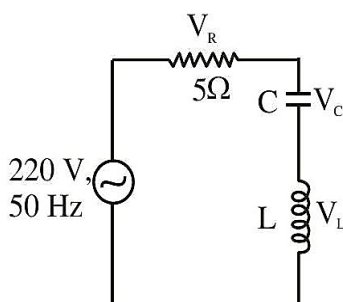
20. Amplitude modulated wave is represented by $V_{AM} = 10[1 + 0.4\cos(2\pi \times 10^4 t)]\cos(2\pi \times 10^7 t)$. The total bandwidth of the amplitude modulated wave is :
- (a) 10kHz (b) 20MHz
(c) 20kHz (d) 10MHz

SECTION-B

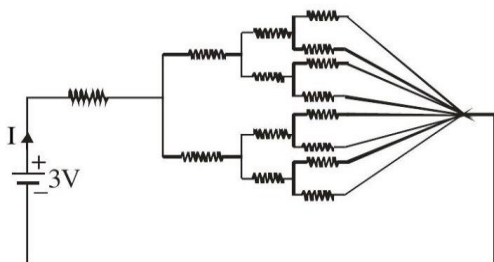
21. A student in the laboratory measures thickness of a wire using screw gauge. The readings are 1.22 mm, 1.23 mm, 1.19 mm and 1.20 mm. The percentage error is $\frac{x}{121}\%$. The value of x is
22. A Zener of breakdown voltage $V_Z = 8\text{ V}$ and maximum zener current, $I_{ZM} = 10\text{ mA}$ is subjected to an input voltage $V_i = 10\text{ V}$ with series resistance $R = 100\Omega$. In the given circuit R_L represents the variable load resistance. The ratio of maximum and minimum value of R_L is



23. In a Young's double slit experiment, an angular width of the fringe is 0.35° on a screen placed at 2 m away for particular wavelength of 450 nm. The angular width of the fringe, when whole system is immersed in a medium of refractive index $7/5$, is $\frac{1}{\alpha}$. The value of α is
24. In the given circuit, the magnitude of V_L and V_C are twice that of V_R . Given that $f = 50\text{ Hz}$, the inductance of the coil is $\frac{1}{K\pi}\text{ mH}$. The value of K is

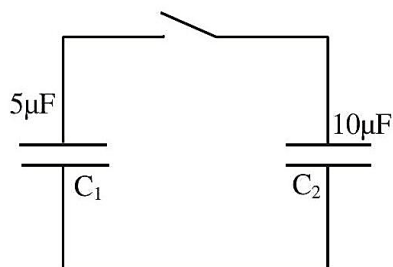


25. All resistances in figure are 1Ω each. The value of current 'I' is $\frac{a}{5}\text{ A}$. The value of a is

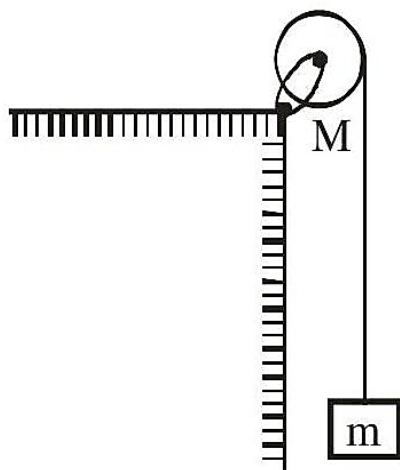


26. A capacitor C_1 of capacitance $5\mu\text{F}$ is charged to a potential of 30 V using a battery. The battery is then removed and the charged capacitor is connected to an uncharged capacitor C_2 of capacitance $10\mu\text{F}$ as

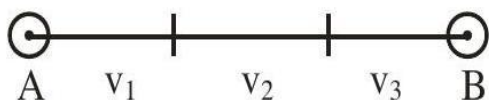
shown in figure. When the switch is closed charge flows between the capacitors. At equilibrium, the charge on the capacitor C_2 is μC .



27. A tuning fork of frequency 340 Hz resonates in the fundamental mode with an air column of length 125 cm in a cylindrical tube closed at one end. When water is slowly poured in it, the minimum height of water required for observing resonance once again is cm. (Velocity of sound in air is 340 ms^{-1})
28. A liquid of density 750 kg m^{-3} flows smoothly through a horizontal pipe that tapers in crosssectional area from $A_1 = 1.2 \times 10^{-2} \text{ m}^2$ to $A_2 = \frac{A_1}{2}$. The pressure difference between the wide and narrow sections of the pipe is 4500 Pa. The rate of flow of liquid is $\times 10^{-3} \text{ m}^3 \text{ s}^{-1}$.
29. A uniform disc with mass $M = 4 \text{ kg}$ and radius $R = 10 \text{ cm}$ is mounted on a fixed horizontal axle as shown in figure. A block with mass $m = 2 \text{ kg}$ hangs from a massless cord that is wrapped around the rim of the disc. During the fall of the block, the cord does not slip and there is no friction at the axle. The tension in the cord is N.
(Take $g = 10 \text{ ms}^{-2}$)



30. A car covers AB distance with first one-third at velocity $v_1 \text{ ms}^{-1}$, second one-third at $v_2 \text{ ms}^{-1}$ and last one-third at $v_3 \text{ ms}^{-1}$. If $v_3 = 3v_1$, $v_2 = 2v_1$ and $v_1 = 11 \text{ ms}^{-1}$ then the average velocity of the car is ms^{-1} .



SECTION-A

31. Compound A contains 8.7% Hydrogen, 74% Carbon and 17.3% Nitrogen. The molecular formula of the compound is,

Given: Atomic masses of C, H and N are 12, 1 and 14 amu respectively.

The molar mass of the compound A is 162 g mol^{-1} .

- (a) $\text{C}_4\text{H}_6\text{N}_2$ (b) $\text{C}_2\text{H}_3\text{N}$
(c) $\text{C}_5\text{H}_7\text{N}$ (d) $\text{C}_{10}\text{H}_{14}\text{N}_2$

32. Consider the following statements:

- (A) The principal quantum number 'n' is a positive integer with values of 'n' = 1, 2, 3,
(B) The azimuthal quantum number 'l' for a given 'n' (principal quantum number) can have values as 'l' = 0, 1, 2, ..., n
(C) Magnetic orbital quantum number 'm_l' for a particular 'l' (azimuthal quantum number) has (2l + 1) values.
(D) $\pm 1/2$ are the two possible orientations of electron spin.
(E) For l = 5, there will be a total of 9 orbital.

Which of the above statements are correct?

- (a) (A), (B) and (C) (b) (A), (C), (D) and (E)
(c) (A), (C) and (D) (d) (A), (B), (C) and (D)

33. In the structure of SF_4 , the lone pair of electrons on S is in.

- (a) equatorial position and there are two lone pair-bond pair repulsions at 90°
(b) equatorial position and there are three lone pair-bond pair repulsions at 90°
(c) axial position and there are three lone pair bond pair repulsion at 90° .
(d) axial position and there are two lone pair bond pair repulsion at 90° .

34. A student needs to prepare a buffer solution of propanoic acid and its sodium salt with pH 4. The ratio of $\frac{[\text{CH}_3\text{CH}_2\text{COO}^-]}{[\text{CH}_3\text{CH}_2\text{COOH}]}$ required to make buffer is

Given: $K_a(\text{CH}_3\text{CH}_2\text{COOH}) = 1.3 \times 10^{-5}$

- (a) 0.03 (b) 0.13
(c) 0.23 (d) 0.33

35. Match List-I with List-II.

List-I		List-II	
(A)	Negatively charged sol	(I)	$\text{Fe}_2\text{O}_3 \cdot x\text{H}_2\text{O}$

(B)	Macromolecular colloid	(II)	CdS sol
(C)	Positively charged sol	(III)	Starch
(D)	Cheese	(IV)	a gel

Choose the correct answer from the options given below:

- (a) (A) - (II), (B) - (III), (C) - (IV), (D) - (I)
- (b) (A) - (II), (B) - (I), (C) - (III), (D) - (IV)
- (c) (A) - (II), (B) - (III), (C) - (I), (D) - (IV)
- (d) (A) - (I), (B) - (III), (C) - (II), (D) - (IV)

36. Match List-I with List-II.

List-I (Oxide)		List-II (Nature)	
(A)	Cl_2O_7	(I)	Amphoteric
(B)	Na_2O	(II)	Basic
(C)	Al_2O_3	(III)	Neutral
(D)	N_2O	(IV)	Acidic

Choose the correct answer from the options given below:

- (a) (A) - (IV), (B) - (III), (C) - (I), (D) - (II)
- (b) (A) - (IV), (B) - (II), (C) - (I), (D) - (III)
- (c) (A) - (II), (B) - (IV), (C) - (III), (D) - (I)
- (d) (A) - (I), (B) - (II), (C) - (III), (D) - (IV)

37. In the metallurgical extraction of copper, following reaction is used:

$\text{FeO} + \text{SiO}_2 \rightarrow \text{FeSiO}_3$ FeO and FeSiO₃ respectively are.

- (a) gangue and flux (b) flux and slag
- (c) slag and flux (d) gangue and slag

38. Hydrogen has three isotopes: protium (^1H), deuterium (^2H or D) and tritium (^3H or T). They have nearly same chemical properties but different physical properties. They differ in

- (a) number of protons (b) atomic number

(c) electronic configuration (d) atomic mass

39. Among the following basic oxide is:

- (a) SO_3 (b) SiO_2
(c) CaO (d) Al_2O_3

40. Among the given oxides of nitrogen; N_2O , N_2O_3 , N_2O_4 and N_2O_5 , the number of compound(s) having N – N bond is :

- (a) 1 (b) 2
(c) 3 (d) 4

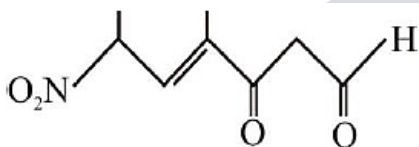
41. Which of the following oxoacids of sulphur contains " S " in two different oxidation states?

- (a) $\text{H}_2\text{S}_2\text{O}_3$ (b) $\text{H}_2\text{S}_2\text{O}_6$
(c) $\text{H}_2\text{S}_2\text{O}_7$ (d) $\text{H}_2\text{S}_2\text{O}_8$

42. Correct statement about photo-chemical smog is:

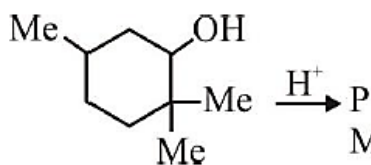
- (a) It occurs in humid climate.
(b) It is a mixture of smoke, fog and SO_2
(c) It is reducing smog.
(d) It results from reaction of unsaturated hydrocarbons.

43. The correct IUPAC name of the following compound is:

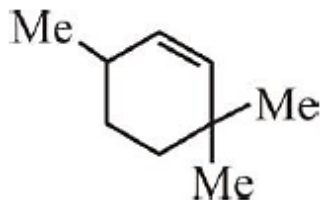


- (a) 4-methyl-2-nitro-5-oxohept-3-enal
(b) 4-methyl-5-oxo-2-nitrohept-3-enal
(c) 4-methyl-6-nitro-3-oxohept-4-enal
(d) 6-formyl-4-methyl-2-nitrohex-3-enal

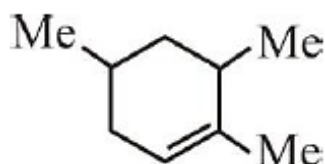
44. The major product (P) of the given reaction is (where, Me is $-\text{CH}_3$)



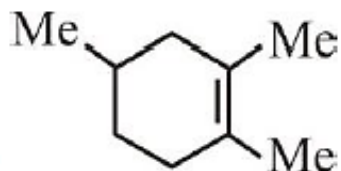
Major Product



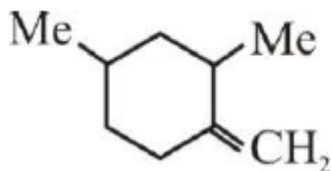
(a)



(b)



(c)



(d)



45.

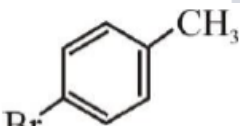
In the above reaction 'A' is



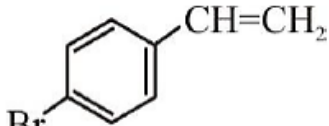
(a)



(b)

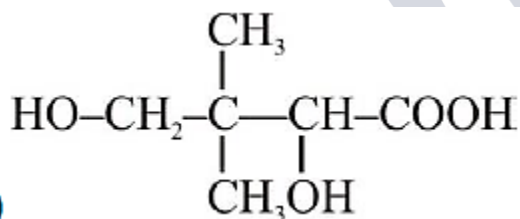


(c)

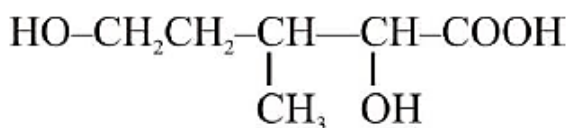


(d)

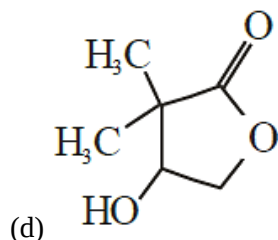
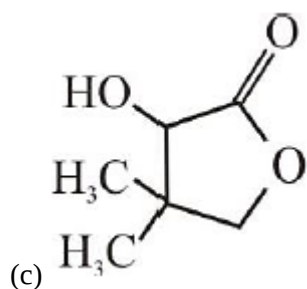
46. Iso butyraldehyde on reaction with formaldehyde and K_2CO_3 gives compound 'A'. Compound 'A' reacts with KCN and yields compound 'B', which on hydrolysis gives a stable compound 'C'. The compound 'C' is:



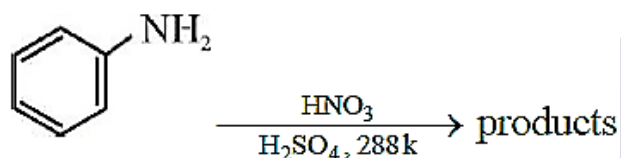
(a)



(b)



47. With respect to the following reaction, consider the given statements:



(A) o-Nitroaniline and p-nitroaniline are the predominant products

(B) p-Nitroaniline and m-nitroaniline are the predominant products

(C) HNO_3 acts as an acid

(D) H_2SO_4 acts as an acid

(a) (A) and (C) are correct statements.

(b) (A) and (D) are correct statements.

(c) (B) and (D) are correct statements.

(d) (B) and (C) are correct statements.

48. Given below are two statements, one is Assertion (a) and other is Reason (R).

Assertion (A): Natural rubber is a linear polymer of isoprene called cis-polyisoprene with elastic properties.

Reason (R): The cis-polyisoprene molecules consist of various chains held together by strong polar interactions with coiled structure.

In the light of the above statements, choose the correct one from the options given below:

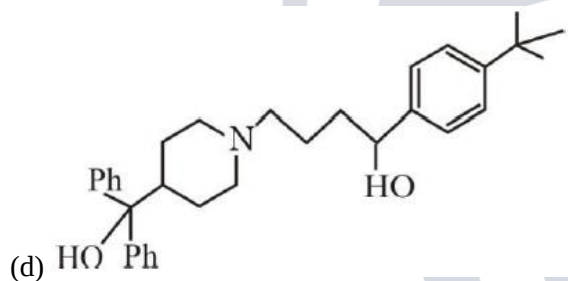
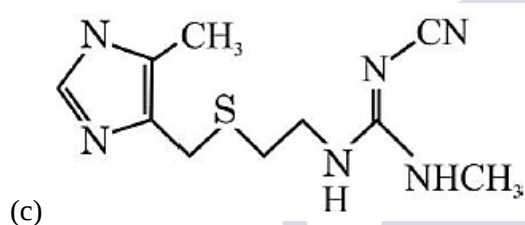
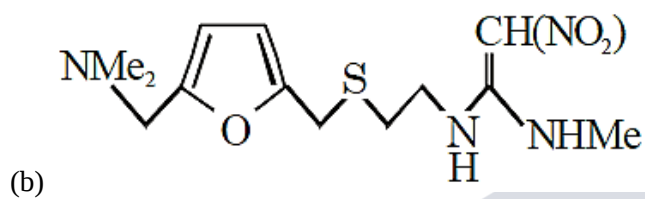
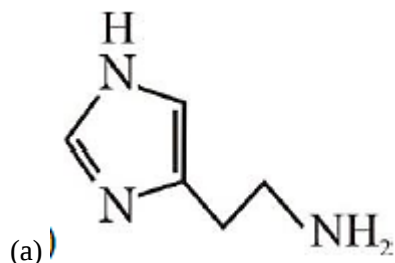
(a) Both (A) and (R) are true and (R) is the correct explanation of (A)

(b) Both (A) and (R) are true but (R) is not the correct explanation of (A).

(c) (A) is true but (R) is false.

(d) (A) is false but (R) is true.

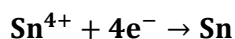
49. When sugar 'X' is boiled with dilute H_2SO_4 in alcoholic solution, two isomers 'A' and 'B' are formed. 'A' on oxidation with HNO_3 yields saccharic acid whereas 'B' is laevorotatory. The compound 'X' is:
- (a) Maltose (b) Sucrose
(c) Lactose (d) Strach
50. The drug tegamet is:



SECTION-B

51. 100 g of an ideal gas is kept in a cylinder of 416 L volume at 27°C under 1.5 bar pressure. The molar mass of the gas is gmol^{-1} . (Nearest integer) (Given: $R = 0.083 \text{ LbarK}^{-1} \text{ mol}^{-1}$)
52. For combustion of one mole of magnesium in an open container at 300 K and 1 bar pressure, $\Delta_c H^\ominus = -601.70 \text{ kJ mol}^{-1}$, the magnitude of change in internal energy for the reaction is kJ. (Nearest integer) (Given: $R = 8.3 \text{ J K}^{-1} \text{ mol}^{-1}$)
53. 2.5 g of protein containing only glycine ($\text{C}_2\text{H}_5\text{NO}_2$) is dissolved in water to make 500 mL of solution. The osmotic pressure of this solution at 300 K is found to be $5.03 \times 10^{-3} \text{ bar}$. The total number of glycine units present in the protein is (Given: $R = 0.083 \text{ LbarK}^{-1} \text{ mol}^{-1}$)
54. For the given reactions

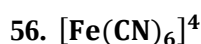




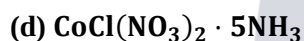
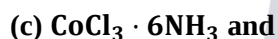
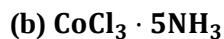
The electrode potentials are; $E_{\text{Sn}^{2+}/\text{Sn}}^{\circ} = -0.140 \text{ V}$ and $E_{\text{Sn}^{4+}/\text{Sn}}^{\circ} = 0.010 \text{ V}$. The magnitude of standard electrode potential for $\text{Sn}^{4+}/\text{Sn}^{2+}$ i.e. $E_{\text{Sn}^{4+}/\text{Sn}^{2+}}^{\circ}$ is $\times 10^{-2} \text{ V}$. (Nearest integer)

55. A radioactive element has a half-life of 200 days. The percentage of original activity remaining after 83 days is (Nearest integer)

(Given: $\text{antilog } 0.125 = 1.333$, $\text{antilog } 0.693 = 4.93$)



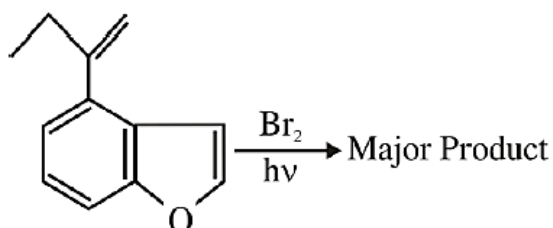
Among the given complexes, number of paramagnetic complexes is



Number of complex(es) which will exist in cis/trans is/are

58. The complete combustion of 0.492 g of an organic compound containing 'C', 'H' and 'O' gives 0.793 g of CO_2 and 0.442 g of H_2O . The percentage of oxygen composition in the organic compound is . (nearest integer)

59. The major product of the following reaction contains bromine atom(s).



60. 0.01M KMnO_4 solution was added to 20.0 mL of 0.05M Mohr's salt solution through a burette. The initial reading of 50 mL burette is zero. The volume of KMnO_4 solution left in the burette after the end point is mL. (nearest integer)

PART-III MATHEMATICS

SECTION-A

61. Let $R_1 = \{(a, b) \in \mathbb{N} \times \mathbb{N} : |a - b| \leq 13\}$ and

$R_2 = \{(a, b) \in \mathbb{N} \times \mathbb{N} : |a - b| \neq 13\}$. Then on $\mathbb{N}$:

- (a) Both R_1 and R_2 are equivalence relations
- (b) Neither R_1 nor R_2 is an equivalence relation
- (c) R_1 is an equivalence relation but R_2 is not
- (d) R_2 is an equivalence relation but R_1 is no

62. Let $f(x)$ be a quadratic polynomial such that $f(-2) + f(3) = 0$. If one of the roots of $f(x) = 0$ is -1 , then the sum of the roots of $f(x) = 0$ is equal to:

- (a) $\frac{11}{3}$
- (b) $\frac{7}{3}$
- (c) $\frac{13}{3}$
- (d) $\frac{14}{3}$

63. The number of ways to distribute 30 identical candies among four children C_1, C_2, C_3 and C_4 so that C_2 receives atleast 4 and atmost 7 candies, C_3 receives atleast 2 and atmost 6 candies, is equal to

- (a) 205
- (b) 615
- (c) 510
- (d) 430

64. The term independent of x in the expression of $(1 - x^2 + 3x^3) \left(\frac{5}{2}x^3 - \frac{1}{5x^2} \right)^{11}$, $x \neq 0$ is

- (a) $\frac{7}{40}$
- (b) $\frac{33}{200}$
- (c) $\frac{39}{200}$
- (d) $\frac{11}{50}$

65. If n arithmetic means are inserted between a and 100 such that the ratio of the first mean to the last mean is $1 : 7$ and $a + n = 33$, then the value of n is

- (a) 21
- (b) 22
- (c) 23
- (d) 24

66. Let $f, g: \mathbb{R} \rightarrow \mathbb{R}$ be functions defined by

$$f(x) = \begin{cases} [x], & x < 0 \\ |1 - x|, & x \geq 0 \end{cases} \text{ and}$$

$$g(x) = \begin{cases} e^x - x, & x < 0 \\ (x - 1)^2 - 1, & x \geq 0 \end{cases}$$

where $[x]$ denote the greatest integer less than or equal to x . Then, the function $f \circ g$ is discontinuous at exactly:

- (a) one point
- (b) two points
- (c) three points
- (d) four points

67. Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be a differentiable function such that $f\left(\frac{\pi}{4}\right) = \sqrt{2}$, $f\left(\frac{\pi}{2}\right) = 0$ and $f'\left(\frac{\pi}{2}\right) = 1$ and let $g(x) =$

$$\int_x^{\pi/4} (f'(t) \sec t + \tan t \sec t f(t)) dt \text{ for } x \in \left[\frac{\pi}{4}, \frac{\pi}{2}\right]. \text{ Then } \lim_{x \rightarrow (\frac{\pi}{2})^-} g(x) \text{ is equal to}$$

- (a) 2
- (b) 3
- (c) 4
- (d) -3

68. Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be continuous function satisfying $f(x) + f(x + k) = n$, for all $x \in \mathbb{R}$ where $k > 0$ and n is a positive integer. If $I_1 = \int_0^{4nk} f(x) dx$ and $I_2 = \int_{-k}^{3k} f(x) dx$, then

- (a) $I_1 + 2I_2 = 4nk$ (b) $I_1 + 2I_2 = 2nk$
(c) $I_1 + nI_2 = 4n^2k$ (d) $I_1 + nI_2 = 6n^2k$

69. The area of the bounded region enclosed by the curve $y = 3 - \left|x - \frac{1}{2}\right| - |x + 1|$ and the x-axis is

- (a) $\frac{9}{4}$ (b) $\frac{45}{16}$
(c) $\frac{27}{8}$ (d) $\frac{63}{16}$

70. Let $x = x(y)$ be the solution of the differential equation $2ye^{x/y^2} dx + (y^2 - 4xe^{x/y^2}) dy = 0$ such that $x(1) = 0$. Then, $x(e)$ is equal to

- (a) $\log_e(2)$ (b) $-\log_e(2)$
(c) $e^2 \log_e(2)$ (d) $-e^2 \log_e(2)$

71. Let the slope of the tangent to a curve $y = f(x)$ at (x, y) be given by $2 \tan x (\cos x - y)$. If the curve passes through the point $(\pi/4, 0)$, then the value of $\int_0^{\pi/2} y dx$ is equal to

- (a) $(2 - \sqrt{2}) + \frac{\pi}{\sqrt{2}}$ (b) $2 - \frac{\pi}{\sqrt{2}}$
(c) $(2 + \sqrt{2}) + \frac{\pi}{\sqrt{2}}$ (d) $2 + \frac{\pi}{\sqrt{2}}$

72. Let a triangle be bounded by the lines $L_1: 2x + 5y = 10$; $L_2: -4x + 3y = 12$ and the line L_3 , which passes through the point $P(2, 3)$, intersect L_2 at A and L_1 at B. If the point P divides the line-segment AB, internally in the ratio 1: 3, then the area of the triangle is equal to

- (a) $\frac{110}{13}$ (b) $\frac{132}{13}$
(c) $\frac{142}{13}$ (d) $\frac{151}{13}$

73. Let $a > 0$, $b > 0$. Let e and ℓ respectively be the eccentricity and length of the latus rectum of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. Let e' and ℓ' respectively the eccentricity and length of the latus rectum of its conjugate hyperbola. If $e^2 = \frac{11}{14} \ell$ and $(e')^2 = \frac{11}{8} \ell'$, then the value of $77a + 44b$ is equal to

- (a) 100 (b) 110
(c) 120 (d) 130

74. Let $\vec{a} = \alpha \hat{i} + 2\hat{j} - \hat{k}$ and $\vec{b} = -2\hat{i} + \alpha \hat{j} + \hat{k}$, where $\alpha \in \mathbb{R}$. If the area of the parallelogram whose adjacent sides are represented by the vectors $\vec{a}$ and $\vec{b}$ is $\sqrt{15(\alpha^2 + 4)}$, then the value of $2|\vec{a}|^2 + (\vec{a} \cdot \vec{b})|\vec{b}|^2$ is equal to

- (a) 10 (b) 7
(c) 9 (d) 14

75. If vertex of a parabola is $(2, -1)$ and the equation of its directrix is $4x - 3y = 21$, then the length of its latus rectum is

- (a) 2 (b) 8
(c) 12 (d) 16

76. Let the plane $ax + by + cz = d$ pass through $(2, 3, -5)$ and is perpendicular to the planes $2x + y - 5z = 10$ and $3x + 5y - 7z = 12$.

If a, b, c, d are integers $d > 0$ and $\gcd(|a|, |b|, |c|, d) = 1$, then the value of $a + 7b + c + 20d$ is equal to

- (a) 18 (b) 20
(c) 24 (d) 22

77. The probability that a randomly chosen one-one function from the set $\{a, b, c, d\}$ to the set $\{1, 2, 3, 4, 5\}$ satisfies $f(a) + 2f(b) - f(c) = f(d)$ is :

- (a) $\frac{1}{24}$ (b) $\frac{1}{40}$
(c) $\frac{1}{30}$ (d) $\frac{1}{20}$

78. The value of $\lim_{n \rightarrow \infty} 6 \tan \left\{ \sum_{r=1}^n \tan^{-1} \left(\frac{1}{r^2 + 3r + 3} \right) \right\}$ is equal to

- (a) 1 (b) 2
(c) 3 (d) 6

79. Let $\vec{a}$ be a vector which is perpendicular to the vector $3\hat{i} + \frac{1}{2}\hat{j} + 2\hat{k}$. If $\vec{a} \times (2\hat{i} + \hat{k}) = 2\hat{i} - 13\hat{j} - 4\hat{k}$, then the projection of the vector $\vec{a}$ on the vector $2\hat{i} + 2\hat{j} + \hat{k}$ is

- (a) $\frac{1}{3}$ (b) 1
(c) $\frac{5}{3}$ (d) $\frac{7}{3}$

80. If $\cot \alpha = 1$ and $\sec \beta = -\frac{5}{3}$, where $\pi < \alpha < \frac{3\pi}{2}$ and $\frac{\pi}{2} < \beta < \pi$, then the value of $\tan(\alpha + \beta)$ and the quadrant in which $\alpha + \beta$ lies, respectively are

- (a) $-\frac{1}{7}$ and IVth quadrant (b) 7 and Ist quadrant
(c) -7 and IVth quadrant (d) $\frac{1}{7}$ and Ist quadrant

SECTION-B

81. Let the image of the point P(1, 2, 3) in the line L: $\frac{x-6}{3} = \frac{y-1}{2} = \frac{z-2}{3}$ be Q. Let R(α, β, γ) be a point that divides internally the line segment PQ in the ratio 1:3. Then the value of $22(\alpha + \beta + \gamma)$ is equal to

82. Suppose a class has 7 students. The average marks of these students in the mathematics examination is 62, and their variance is 20. A student fails in the examination if he/she gets less than 50 marks, then in worst case, the number of students can fail is

83. If one of the diameters of the circle $x^2 + y^2 - 2\sqrt{2}x - 6\sqrt{2}y + 14 = 0$ is a chord of the circle $(x - 2\sqrt{2})^2 + (y - 2\sqrt{2})^2 = r^2$, then the value of r^2 is equal to

84. If $\lim_{x \rightarrow 1} \frac{\sin(3x^2 - 4x + 1) - x^2 + 1}{2x^3 - 7x^2 + ax + b} = -2$, then the value of $(a - b)$ is equal to

85. Let for $n = 1, 2, 50$, S_n be the sum of the infinite geometric progression whose first term is n^2 and whose common ratio is $\frac{1}{(n+1)^2}$. Then the value of $\frac{1}{26} + \sum_{n=1}^{50} \left(S_n + \frac{2}{n+1} - n - 1 \right)$ is equal to

86. If the system of linear equations

$$2x - 3y = \gamma + 5,$$

$\alpha x + 5y = \beta + 1$, where $\alpha, \beta, \gamma \in \mathbb{R}$ has infinitely many solutions, then the value of $|9\alpha + 3\beta + 5\gamma|$ is equal to

87. Let $A = \begin{pmatrix} 1+i & 1 \\ -i & 0 \end{pmatrix}$ where $i = \sqrt{-1}$. Then, the number of elements in the set $\{n \in \{1, 2, \dots, 100\} : A^n = A\}$ is

88. Sum of squares of modulus of all the complex numbers z satisfying $\bar{z} = iz^2 + z^2 - z$ is equal to

89. Let $S = \{1, 2, 3, 4\}$. Then the number of elements in the set $\{f: S \times S \rightarrow S: f \text{ is onto and } f(a, b) = f(b, a) \geq a \forall (a, b) \in S \times S\}$ is

90. The maximum number of compound propositions, out of $p \vee r \vee s, p \vee r \vee \sim s, p \vee \sim q \vee s,$

$$\sim p \vee \sim r \vee s, \sim p \vee \sim r \vee \sim s, \sim p \vee q \vee \sim s,$$

$$q \vee r \vee \sim s, q \vee \sim r \vee \sim s, \sim p \vee \sim q \vee \sim s$$

that can be made simultaneously true by an assignment of the truth values to p, q, r and s , is equal to

