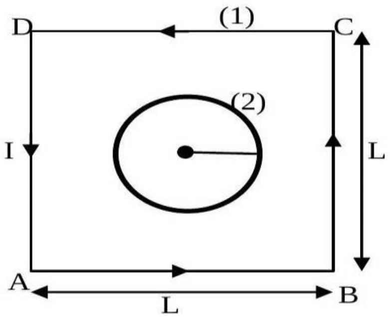
**1. (d)**




$\phi = MI$

$\phi_2 = MI_1$

$B_1 \; A_2 = MI_1$

$M = \frac{B_1 A_2}{I_1}$

$B_1 \rightarrow$ magnetic field due to square frame

$A_2 \rightarrow$ Area of circle

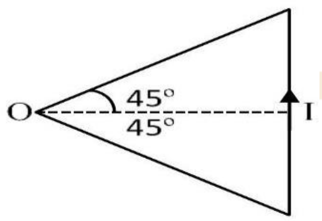
$I_1 \rightarrow$ current in square frame.

$B_1 \rightarrow$

$B_1 = 4. B_{AB}$

$= 4\left[\frac{\mu_0 I_1}{24\pi\frac{L}{2}}[\sin 45^\circ + \sin 45]\right]$

$B_1 = 2\frac{\mu_0 I_1}{\pi L}\left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right) = 2\sqrt{2}\frac{\mu_0 I_1}{\pi L}$

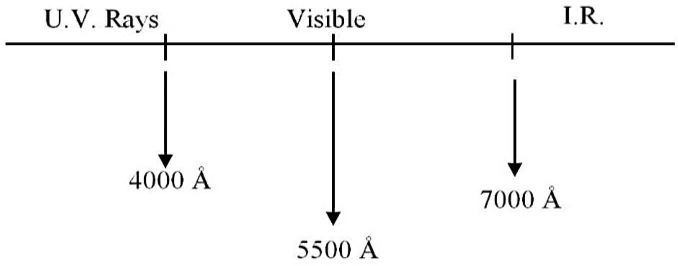



$M = \frac{B_1 \cdot A_2}{I_1}$

$M = \left(\frac{2\sqrt{2}\mu_0 I_1}{\pi L}\right) \times \frac{\pi R^2}{I_1} = \frac{2\sqrt{2}\mu_0 R^2}{L}$

**2. (d)** $\lambda_0 = 5500\text{Å} \rightarrow \phi_0 = \frac{12400}{5500} = 2.25\text{eV}$

$\phi = 3.6 \times 10^{-19}\ \text{J}$




P.E.E will occur if wavelength of incidence wave is less then threshold wavelength. So u. v. rays will be useful for emission.

So both U.V. rays lamps can be used.

**3. (c)**

(A) Pressure gradient $= \frac{\text{Pressure}}{\text{Length}} = \frac{\text{Force}}{\text{Area} \times \text{length}}$

$= \frac{MLT^{-2}}{L^2 \cdot L} = [ML^{-2}\ T^{-2}]$

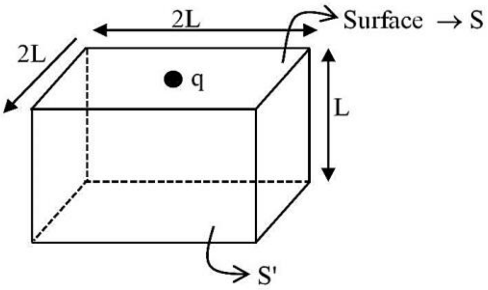
(B) Energy density $= \frac{\text{Energy}}{\text{Volume}} = \frac{ML^2\ T^{-2}}{L^3} = [ML^{-1}\ T^{-2}]$

(C) Electric field $= \frac{\text{Force}}{\text{Ch arge}} = \frac{MLT^{-2}}{AT} = [MLT^{-3}\ A^{-1}]$

(D) Latent heat $= \frac{\text{Heat}}{\text{Mass}} = \frac{M^2T^{-2}}{M} = [L^2T^{-2}]$

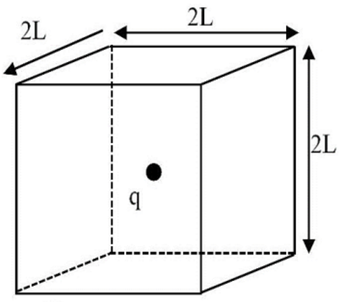
Ans: A-III, B-II, C-IV, D-I

**4. (b)**




When smaller box is considered on the given box then charge ' q ' will be at center.

So flux from surface $S' = \left(\frac{q}{\varepsilon_0}\right) \cdot \frac{1}{6} = \frac{q}{6\varepsilon_0}$




**5. (a)**

A → 30 m/s, Observer B → 30 m/s

$f_0 = 300$ Hz

$V = 330$ m/sec.

$$f_A = f_0\left[\frac{V}{V - V_A}\right] = 300\left[\frac{330}{330 - 30}\right] = 330 \text{ Hz}$$

$$f_B = f_0\left[\frac{V}{V + V_A}\right] = 300\left[\frac{330}{360}\right] = 275 \text{ Hz}$$

$$\Delta f = f_A - f_B = 330 - 275 = 55 \text{ Hz}$$

Ans.: (a)

**6.** **(a)** $f_k = \mu N$

$N = mg\cos\theta$

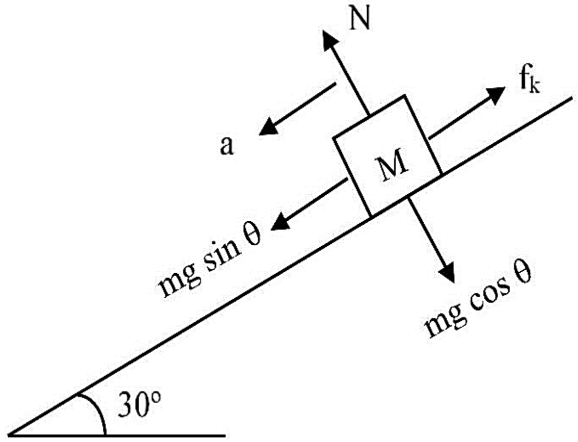
$f_k = \mu mg\cos\theta$

$$a = \frac{mg\sin\theta - \mu mg\cos\theta}{m}$$

$a = g\sin 30^\circ - \mu g\cos 30^\circ$

$$\frac{g}{4} = g\left[\frac{1}{2} - \frac{\sqrt{3}\mu}{2}\right]$$

$$\frac{1}{2} = 1 - \sqrt{3}\mu$$




$$\sqrt{3}\mu = \frac{1}{2}$$

$$\mu = \frac{1}{2\sqrt{3}}$$

**7.** **(d)** PV = nRT

n → const. V = const.

$P \alpha T$,

$P_1 = 270 \text{kpa}$,

$T_1 = 27^\circ C = 300 \text{ K}$

$P_2 = ?$,

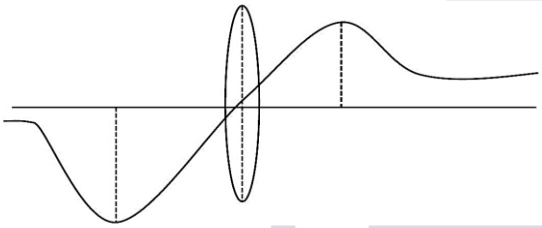
$T_2 = 36^\circ = 36 + 273 = 309 \text{ K}$

$$\frac{P_2}{P_1} = \frac{T_2}{T_1}$$

$$\frac{P_2}{270\text{KPa}} = \frac{309}{300}$$

$$P_2 = \frac{103}{100} \times 270\text{KPa} \simeq 278\text{KPa}$$

**8. (a)**



**9. (d)**

$T = 2.0 \times 10^{-2} \text{Nm}^{-1}$

$r_1 = 3.5 \text{ cm}, r_2 = 7 \text{ cm}$

$W = T\Delta A \times$ No. of air - liquid surface

$W = 2\, T. 4\pi(r_2^2 - r_1^2)$

$$W = 2 \times 2 \times 10^{-2} \times 4\pi \left[49 - \frac{49}{4}\right] \times 10^{-4}$$

$$W = 16\pi \times 10^{-6} \times 49 \times \frac{3}{4}$$

$W = 1847.26 \times 10^{-6}$

$W = 18.47 \times 10^{-4} \text{ J}$

**10. (a)** First law of thermodynamics is based on energy conservation

$dQ = dU + dW$

Here $dW \rightarrow$ work done on the system so volume decreases.

So $dW \rightarrow -ve$

$dQ = dU - dW$

**11. (a)**

$T = 30$ min.

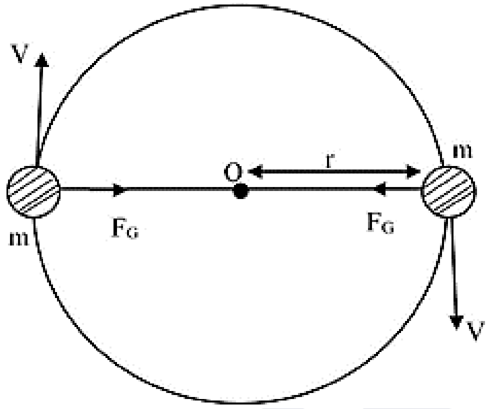
$t = 90$ min

$$n = \frac{t}{T} = \frac{90 \text{ min}}{30 \text{ min}} = 3$$

$$N \text{ (active)} = \frac{N_0}{2^n} = \frac{N_0}{2^3} = \frac{N_0}{8}$$

$$\frac{N}{N_0} = \frac{1}{8}$$

**12. (b)**

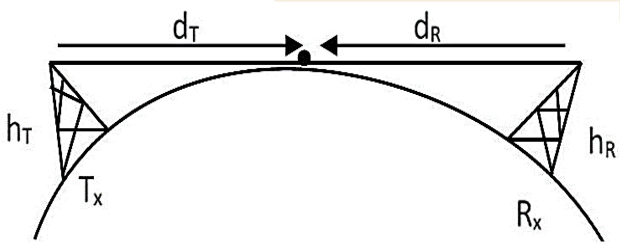



$$\frac{mv^2}{r} = \frac{Gm \cdot m}{(2r)^2}$$

$$\frac{v^2}{r} = \frac{Gm}{4r^2}$$

$$V = \sqrt{\frac{Gm}{4r}}$$

**13. (d)** $h_T = h_R = h = 80$ m




$d_T = \sqrt{2R.h}$ and $d_R = \sqrt{2R.h}$

Maximum line of sight $= d_T + d_R$

$= \sqrt{2Rh} + \sqrt{2Rh}$

$= 2\sqrt{2Rh} = 2\sqrt{2 \times 6.4 \times 10^6 \times 80}$

$= 2\sqrt{64 \times 16 \times 10^6}$

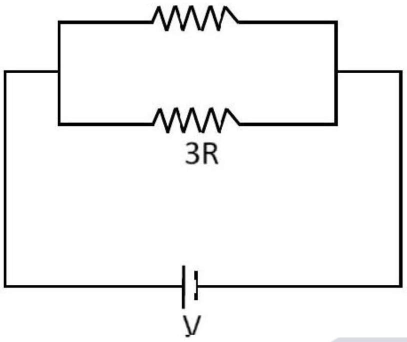
$= 2 \times 8 \times 4 \times 10^3$

$= 64 \times 10^3 = 64 \text{ km}$

**14. (b)** $\mu = 0.34, R = 50 \text{ m}$

$V = \sqrt{\mu R g} = \sqrt{0.34 \times 50 \times 10} = \sqrt{34 \times 5} = \sqrt{170} \simeq 13$

**15. (d)**



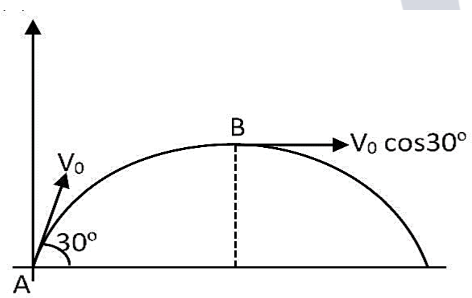

$H = I^2 R t = \frac{V^2}{R} . t$

$V = \text{const.}$

So, $H \alpha \frac{1}{R}$

$\frac{H_1}{H_2} = \frac{3R}{R} = \frac{3}{1}$

**16. (d)**




$K_A = \frac{1}{2} m V_A^2$

$K_A = \frac{1}{2} m V_0^2$

$K_B = \frac{1}{2} m(V_0 \cos 30°)^2$

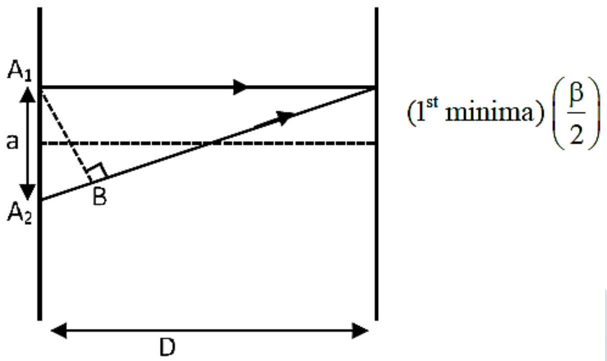
$K_B = \frac{m}{2} \cdot V_0^2 \frac{3}{4} = \frac{3}{8} m V_0^2$

$$\frac{K_A}{K_B} = \frac{\left(\frac{mV_0^2}{2}\right)}{\left(\frac{3mVV_0^2}{8}\right)}$$

$$\frac{K_A}{K_B} = \frac{4}{3}$$

**17. (a)** velocity of light depends on Refractive index of medium and independent of intensity and source.

**18. (d)**




$$\frac{\beta}{2} = \frac{a}{2}$$
$$\beta = a$$
$$\frac{\lambda D}{a} = a$$
$$\lambda D = a^2$$
$$a^2 = 800 \times 10^{-9} \times 5 \times 10^{-2}$$
$$a^2 = 4000 \times 10^{-11}$$
$$a = 2 \times 10^{-4}$$
$$a = 0.2 \text{ mm}$$

**19. (c)** Light emitting diode only used in forward bias

**20. (a)** Magnetic field due to "AB" and "ED" will be zero

magnetic field due to "BC" and "ET" will be equal in amount and direction.

'B'due BC $= \frac{\mu_0 I}{4\pi r} = \frac{\mu_0 I}{4\pi\frac{a}{2}} = \frac{\mu_0 I}{2\pi a} \odot$

'B'due to TE $= \frac{\mu_0 I}{2\pi a}$

$B_{net}$ at point 'O' $= \left(\frac{\mu_0 I}{2\pi a} + \frac{\mu_0 I}{2\pi a}\right) = \frac{\mu_0 I}{\pi a} \odot$ outward

**21. (24)**

Intensity of source $I_0 = 256 \frac{W}{m^2}$

intensity after passing $P_1$ is $I_1 = \frac{I_0}{2} = 128 \frac{W}{m^2}$

intensity after passing $P_2$ is $I_2 = I_1 \cos^2\theta$

$= (128) \cdot \cos^2 60°$

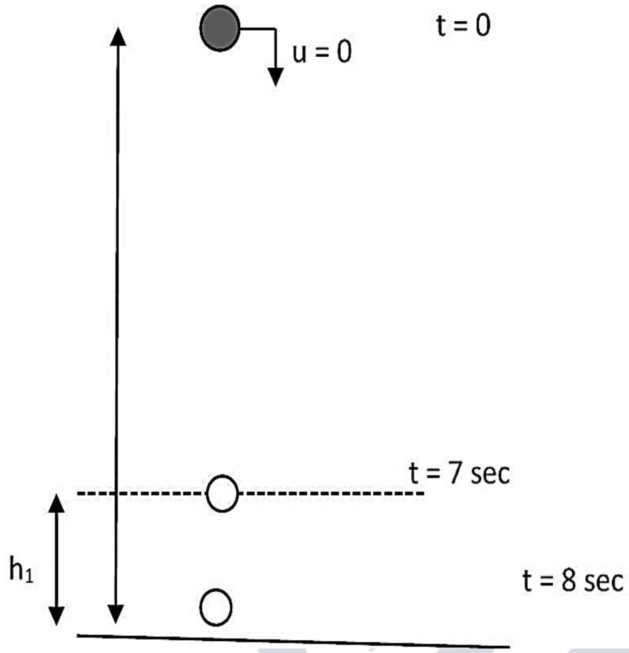
$128 \times \frac{1}{4} = 32\frac{\mathrm{W}}{\mathrm{m}^2}$

intensity after passing $P_3$ is $I_3 = I_2\cos^2\theta$

angle b/w$p_2$ and $p_3 = 30°$

So, $I_3 = 32\cos^2 30° = 32 \times \frac{3}{4} = 24\frac{\mathrm{W}}{\mathrm{m}^2}$

**22.** (**300 J**)




$S = ut + \frac{1}{2}at^2$

$h = 0 + \frac{1}{2}\cdot g(8)^2 = \frac{10}{2} \times 8 \times 8 = 320\ \mathrm{m}$

Distance covered in last second

$h_1 = u + \frac{a}{2}(2n - 1)$

$= 0 + \frac{10}{2}[2(8) - 1]$

$h_i = 5[15] = 75\ \mathrm{m}$

$\Delta U_{\text{loss}} = mg\Delta h$

$\Delta U_{\text{loss}} = 0.4 \times 10 \times 75 = 300\ \mathrm{J}$

Ans → 300 J

**23.** (**120**)

$A_1 = A\ A_2 = A\ A_{eq} = A$

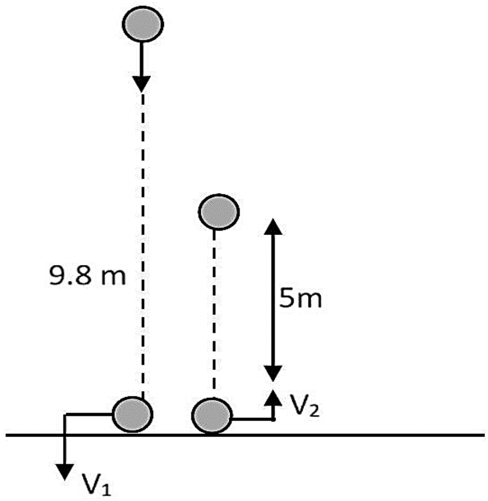
$A_1^2 + A_2^2 + 2A_1A_2\cos\phi = A_{eq}^2$

$A^2 + A^2 + 2A^2\cos\phi = A^2$

$1 + 2\cos\phi = 0 \Rightarrow \cos\phi = -\frac{1}{2}$

$\phi = 120$

**24.** $\left(\mathbf{120\ m/sec^2}\right)$




$v_1 = \sqrt{2gh} = \sqrt{2 \times 10 \times 9.8} = \sqrt{196}$

$v_1 = 14\ m/sec$

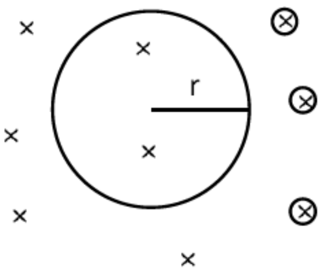
$v_2 = \sqrt{2gh}$

$v_2 = \sqrt{2 \times 10 \times 5} = 10\ m/sec.$

$a_{aug} = \frac{\Delta v}{\Delta t} = \frac{\vec{v}_2 - \vec{v}_1}{\Delta t} = \frac{10 - (-14)}{0.2}$

$a_{ay} = \frac{24}{0.2} = 120\ m/sec^2$

**25. (10)**



$B = 0.8T$
$\frac{dr}{dt} = 2\text{cms}^{-1}$
$\text{emf } = \frac{d\phi}{dt} = \frac{d(BA)}{dt}$
$\text{emf } = B\frac{d}{dt}\pi r^2 = \pi B(2r)\frac{dr}{dt}$
$\text{emf } = 2\pi Br \cdot (0.02)$
$= 2\pi(0.8)(0.1) \times 0.02$
$= 32\pi \times 10^{-4}$
$= 100.48 \times 10^{-4}$
$= 10.048 \times 10^{-3}$
$= 10.04\text{mV} \approx 10\text{mV}$

**26. (40)**

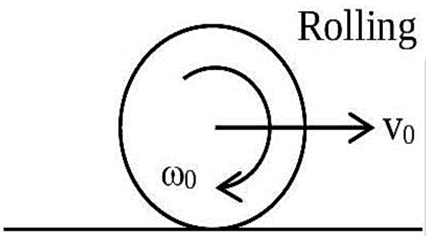
Mass = 2 kg

K. E = 2240 J

$\text{K.E} = \frac{1}{2}mv_0^2 + \frac{1}{2}I\omega_0^2$

$= \frac{1}{2}mv_0^2 + \frac{1}{2} \cdot \frac{2}{5}mR^2 \cdot \frac{v_0^2}{R^2}$

$= \frac{1}{2}mv_0^2 + \frac{mv_0^2}{5}$




$\text{K. E} = \frac{7}{10}mv_0^2$

$2240 = \frac{7}{10} \times 2 \times v_0^2$

$v_0^2 = \frac{22400}{14} = 1600$

$v_0 = 40 \text{ m/sec}$

**27. (28)**

$60°C \xrightarrow[6\text{ min}]{} 40°C \xrightarrow[6\text{ min}]{} T \qquad T_0 = 10°C$

$\frac{\Delta T}{\Delta t} = k(T - T_0)$

$\frac{(60-40)}{6\text{min}} = k[50 - 10]$

And $\frac{(40-T)}{6\text{ min}} = K\left[\frac{40+T}{2} - 10\right]$

(1)/(2)

$$\frac{20}{40-T} = \frac{40}{\left(\frac{40+T-20}{2}\right)}$$

$$\frac{20}{40-T} = \frac{40\times 2}{20+T}$$

$(20+T) = (40-T)4$

$20 + T = 160 - 4\,T \Rightarrow ST = 140$

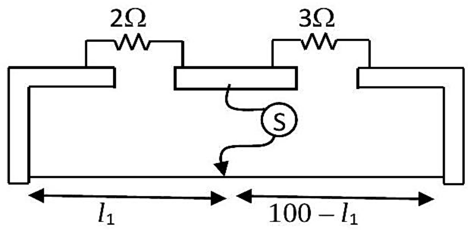
$$T = \frac{140}{5} = 28^\circ C$$

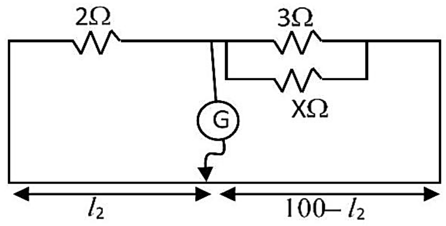
**28. (2)**

$x = 2$

$$\frac{2}{\ell_1} = \frac{3}{100-\ell_1}$$

$200 - 2\ell_1 = 3\ell_1$




$200 = 5\ell_1$

$\ell_1 = 40$ cm

Now $\ell_2 = \ell_1 + 22.5$

$\ell_2 = 40 + 22.5 = 62.5$ cm

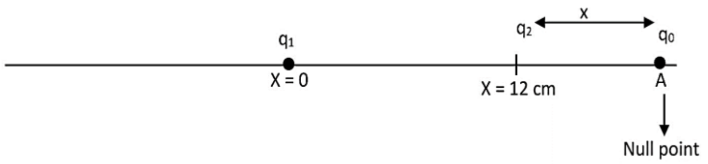
So, $\frac{2}{62.5} = \frac{\left(\frac{3\cdot x}{3+x}\right)}{37.5} \Rightarrow (37.5)\times 2 = \frac{(62.5)(3x)}{3+x}$

$$3 + x = \frac{(62.5)}{25}x$$

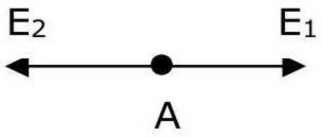
$3 + x = 2.5x$

$3 = 1.5x \Rightarrow x = 2$

**29. (24)** $q_1 = 4q_0$ and $q_2 = -q_0$




Electric field at point A will be zero.




$|\vec{E_1}| = |\vec{E_2}|$

$$\frac{kq_1 \cdot q_0}{(12+x)^2} = \frac{kq_2 \cdot q_0}{x^2}$$

$$\frac{4q_0}{(12+x)^2} = \frac{q_0}{x^2}$$

$4x^2 = (12+x)^2$

$\pm 2x = (12+x)$

$2x = 12 + x \quad -2x = 12 + x$

$x = 12 \quad -3x = 12$

$x = 12\text{ cm}\ x = x = -\frac{12}{3} = -4$

Position of proton from origin will be $\rightarrow 12 + 12$
$\rightarrow 24\text{ cm}$

**30. (87)**

$${}_{92}X^{242} \longrightarrow {}_{P}Y^{234} + 2\,{}_{2}\alpha^4 + {}_{-1}e^0 + 2\,{}_{+1}e^0$$

Using charge conservation:

$92 = P + 2(b) + (-1) + 2(a)$

$92 = P + 5$

$P = 87$ Ans.

**31. (d)**

**32. (d)** Photochemical smog is oxidizing smog. Its high concentration of oxidizing agent like ozone and $HNO_3$

**33. (c)** $V^{+2}$ and $Cr^{+2}$

The metal ion for which have less value of reduction potential can release $H_2$ on reaction with dilute acid.

**34. (b)** For lymen seriese → $\frac{1}{\lambda_{\min}} = R \times 1\left(\frac{1}{1^2} - \frac{1}{\infty^2}\right)$

For balmer seriese → $\frac{1}{\lambda_{\max}} = R \times 4\left(\frac{1}{2^2} - \frac{1}{3^2}\right)$

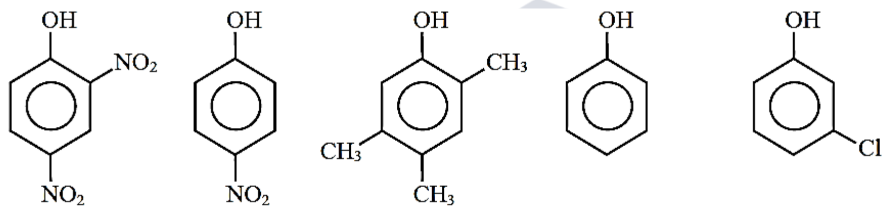
$$\frac{\frac{1}{\lambda_{\min}}}{\frac{1}{\lambda_{\max}}} = \frac{\lambda_{\max}}{\lambda_{\min}} = \frac{\lambda_{\max}}{\lambda} = \frac{9R}{5R}$$

$$\lambda_{\max} = \frac{9\lambda}{5}$$

**35. (d)** Order of B.D.E in halogen is

(E) $Cl - Cl > Br - Br > F - F > I - I$

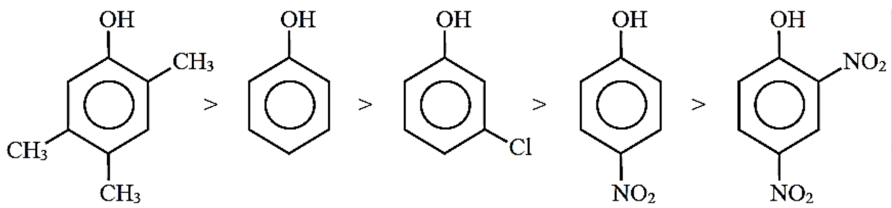
**36. (a)**



(A) (B) (C) (D) (E)

acedic strength $\propto K_a$

$\propto \frac{1}{PK_a}$



(C) (D) (E) (B) (A)

**37. (b)** At point A → low pressure, volume of gas very high

$\rightarrow V - b \approx V$

$$\left(p + \frac{a}{V^2}\right)(v - \underset{\text{neglect}}{b}) = RT$$

$$\left(p + \frac{a}{V^2}\right)v = RT$$

$$PV + \frac{a}{V} = RT$$

$$z + \frac{a}{RTV} = 1$$

$$z = 1 - \frac{a}{RTV}$$

**38. (d)**

Narrow Spectrum Antibiotic → Penicillin G (used in pathgens)

Antiseptic → Furacin

Disinfectants → Sulphur dioxide

Broad spectrum antibiotic → Chloramphenicol

**39. (b)**
**40. (b)**

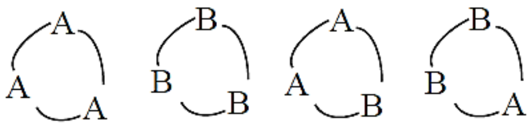
$FeCl_3 + NaOH \rightarrow \underset{\text{Negative colloidal particle}}{Fe(OH)_3/OH^-}$

Positive ion required for coagulation of sol.

**41. (c)** To amine acid

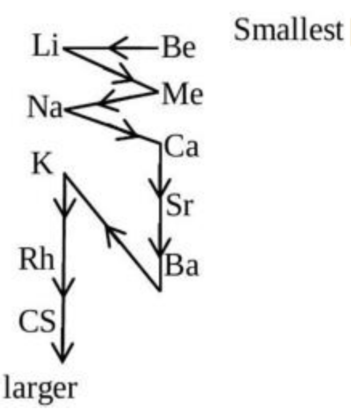
$$\underset{\text{(A)}}{H_2N-\underset{\underset{R_1}{|}}{CH}-COOH}$$

Tripeptide are formed →



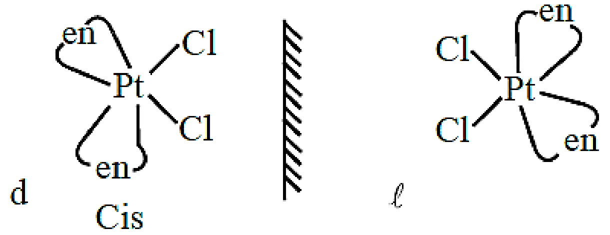

$$\underset{\text{(B)}}{H_2N-\underset{\underset{R_2}{|}}{CH}-COOH}$$

**42. (d)** Order of hydration enthalpy is size order




$Mg^{2+} > Ca^{2+} > K^+ > Rb^+ > CS^+$

**43. (a)**




**44. (d)**

(i) B.P. $\propto$ Molecular mass

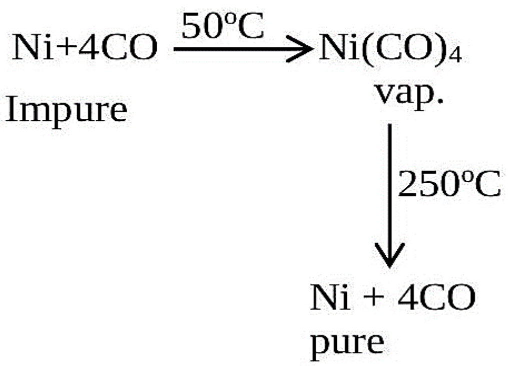
(ii) B.P. $\propto$ polarity $\uparrow$

(iii) B.P. $\propto \frac{1}{\text{No. of Branches}}$

**45. (d)**

| | | |
|---|---|---|
| $Li_2O$ | $O^{--}$ | Diamagnetic |
| $Na_2O_2$ | $O_2^{--}$ | Diamagnetic |
| $KO_2$ | $O_2^{-}$ | paramagnetic |

**46. (d)**




**47. (b)** Ni can adsorb 800 times more hydrogen then its own volume

**48. (a)**

Hoffmann degradation → $Br_2$, NaOH

Clemenson reduction → Zn − Hg/HCl

Cannizaro reaction → Conc. KOH, $\Delta$

Reimer-Tiemann reaction → $CuCl_3, NaOH/H_3O^{\oplus}$

**49. (d)**

**50. (b)** Lassaigne test for both N and X is given by the compound which have C, N as well X atom in compound.

**51. (5)**

$pH = 12, pOH = 2 [OH^-] = 10^{-2}$ N

Molarity of $Ca(OH)_2 = \frac{N}{2} = \frac{10^{-2}}{2} = 0.005\ N$

$$0.005 = \frac{\text{milimoles}}{100}$$

$$= \frac{5}{1000} = \frac{\text{mili moles}}{100}$$

$= 5 \times 10^{-1}$ milimoles

**52. (2)** $H_2O(g) \rightarrow H_2(g) + 1/2O_2$

$1-\propto \quad \propto \quad \propto/2$

$$k_p = \frac{\propto (\propto/2)^{1/2}}{1-\propto} = 2 \times 10^{-3}$$

$$2 \times 10^{-3} = \frac{\propto^{3/2}}{\sqrt{2}(1-\propto)}$$

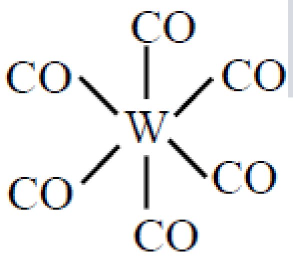
$\propto \ll 1$

$2^{3/2} \times (10^{-2})^{3/2} = \propto^{3/2}$

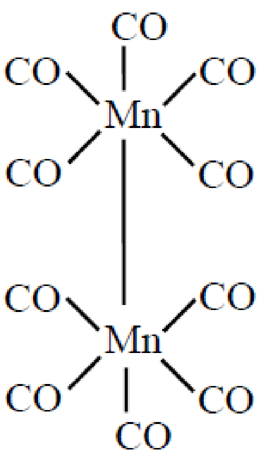
$\propto = 2 \times 10^{-2}$

**53. (0)**

$W(CO)_6 \rightarrow 0$ Bridge CO



$Mn_2(CO)_{10} \rightarrow 0$



**54. (13)**

**55. (3)** Moles of hydrocarbon $= \frac{17\times10^{-3}}{136} = 1.25\times10^{-4}$

$$nH_2 = 1\times\frac{8.4}{1000} = n\times0.0821\times273$$

$$\Rightarrow n = 3.75\times10^{-4}$$

Hydrogen molecule used for 1 molecule of hydrogen is 3

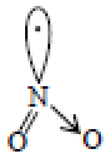
$$= \frac{3.75\times10^{-4}}{1.25\times10^{-4}} = 3$$

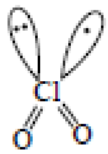
**56. (6)**

$$K_{eq} = \frac{K_f}{K_b} = \frac{10^3}{10^2} = 10$$
$$\Delta G^\circ = -RT\ln K_{eq}$$
$$= -8.3\times300\ln 10$$
$$= -8.3\times300\times2.3$$
$$= -5.72\times10^{+3}\text{ J}$$
$$= 5.72\text{KJ}$$

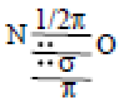
**57. (3)**

odd $e^-$

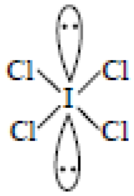


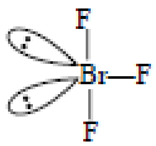


$ICl_4^-$, $BrF_3$ and $NO_2^+$ do not have odd number of electron.



Odd $e^-$ absent





**58. (6)**

$$R_f = \frac{\text{Distance moved by the substance from base line}}{\text{Distance moved by the solvent from base line}}$$

$$= \frac{3.0 \text{ cm}}{5.0 \text{ cm}} = 0.6 \text{ or } 6 \times 10^{-1}$$

**59. (2)**

**60. (2)**

**61. (a)**

$A^2 = 3A + \alpha I$

and $A^4 = 21A + \beta I$

Now $A^4 = A^2 \cdot A^2$

$A^4 = (3A + \alpha I) \cdot (3A + \alpha I)$ { from (a)}

$A^4 = 9A^2 + 6\alpha A + \alpha^2 I$ ……… (c)

From (b) and (c)

$9A^2 + 6\alpha A + \alpha^2 I = 21A + \beta I$

putting value of $A^2$ from (a)

$9(3\,A+\alpha I)+6\alpha A+\alpha^2 I=21\,A+\beta I$

$(27+6\alpha)A+(9\alpha+\alpha^2)I=21A+\beta I$

by comparison

$27+6\alpha=21$ and $9\alpha+\alpha^2=\beta$

$\Rightarrow 6\alpha=-6$ putting $\alpha=-1$

$\Rightarrow \alpha=-1\ \therefore \beta=-8$

**62. (a)**

$$f(x)=\begin{cases}\dfrac{1-\cos(x^2-4px+q^2+8q+16)}{(x-2p)^4} & ,x\neq 2p\\ 0 & ,x=2p\end{cases}$$

$\because x=2$ is a root of equation $x^2+px+q=0$

$\therefore 4+2p+q=0$

$\Rightarrow 2p=-q-4$

$\Rightarrow 4p^2=(q+4)^2=q^2+8q+16$

Now $\lim_{x\to 2p^+} f(x)=\lim_{x\to 2p^+}\frac{1-\cos(x^2-4px+4p^2)}{(x-2p)^4}$ (from (a))

$$=\lim_{x\to 2p^+}\left[\frac{1-\cos(x-2p)^2}{\{(x-2p)^2\}^2}\right]$$

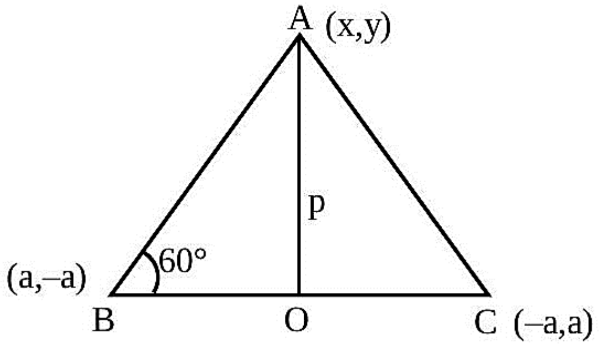
$$=\frac{1}{2}\left\{\because \lim_{x\to 0}\frac{1-\cos\theta}{\theta^2}=\frac{1}{2}\right\}$$

$$\therefore \lim_{x\to p^+}[f(x)]=\left[\frac{1}{2}\right]=0$$

**63. (d)** Since, A lies on perpendicular bisector of BC, whose equation is $y=x$

Now, $A$ is the point of intersection of $y=x$ and $y-2x=2$

$\therefore$ point A, after solving is $A(-2,-2)$



In $\triangle$ AOC $\tan 60^\circ = \frac{p}{OC} \Rightarrow OC = \frac{p}{\sqrt{3}} \{\because OA = p\}$

$\therefore BC = 2 \times OC = \dfrac{2p}{\sqrt{3}}$

Now, Area of $\triangle ABC = \frac{1}{2} \times BC \times OA$

$= \dfrac{1}{2} \times \dfrac{2p}{\sqrt{3}} \times p = \dfrac{p^2}{\sqrt{3}}$ sq. unit

and $p = OA = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$

So, Area of $\triangle ABC = \frac{(2\sqrt{2})^2}{\sqrt{3}} = \frac{8}{\sqrt{3}}$ sq. unit

**64. (d)**

$\because D = \begin{vmatrix} \alpha & 2 & 1 \\ 2\alpha & 3 & 1 \\ 3 & \alpha & 2 \end{vmatrix}$

$D = \alpha(6-\alpha) + 2(3-4\alpha) + 1(2\alpha^2 - 9)$
$= 6\alpha - \alpha^2 + 6 - 8\alpha + 2\alpha^2 - 9$

$D = \alpha^2 - 2\alpha - 3$

for no solution, $D = 0$

$\Rightarrow \ \alpha^2 - 2\alpha - 3 = 0$

$(\alpha+1)(\alpha-3) = 0$

$\Rightarrow \ \alpha = -1, \alpha = 3$

Now,

$D_1 = \begin{vmatrix} 1 & 2 & 1 \\ 1 & 3 & 1 \\ \beta & \alpha & 2 \end{vmatrix}, D_2 = \begin{vmatrix} \alpha & 1 & 1 \\ 2\alpha & 1 & 1 \\ 3 & \beta & 2 \end{vmatrix}$ and $D_3 = \begin{vmatrix} \alpha & 2 & 1 \\ 2\alpha & 3 & 1 \\ 3 & \alpha & \beta \end{vmatrix}$

if $\alpha = -1$ then

$D_1 = \begin{vmatrix} 1 & 2 & 1 \\ 1 & 3 & 1 \\ \beta & -1 & 2 \end{vmatrix}, D_2 = \begin{vmatrix} -1 & 1 & 1 \\ -2 & 1 & 1 \\ 3 & \beta & 2 \end{vmatrix}, D_3 = \begin{vmatrix} -1 & 2 & 1 \\ -2 & 3 & 1 \\ 3 & -1 & \beta \end{vmatrix}$

$\Rightarrow$ only for $\beta = 2, D_1 = 0, D_2 = 0, D_3 = 0$

$\therefore$ It has no solution if $\alpha = -1$ and $\beta \neq 2$

if $\alpha = 3$

$D_1 = \begin{vmatrix} 1 & 2 & 1 \\ 1 & 3 & 1 \\ \beta & 3 & 2 \end{vmatrix}, D_2 = \begin{vmatrix} 3 & 1 & 1 \\ 6 & 1 & 1 \\ 3 & \beta & 2 \end{vmatrix}, D_3 = \begin{vmatrix} 3 & 2 & 1 \\ 6 & 3 & 1 \\ 3 & 3 & \beta \end{vmatrix}$

$\Rightarrow$ Only for $\beta = 2, D_1 = D_2 = D_3 = 0$

$\Rightarrow$ It has no solution for $\beta \neq 2$

$\therefore$ It has no solution for $\alpha = 3$ and for all $\beta \neq 2$

**65. (c)**

$$y(x+1)dx - x^2 dy = 0,\ y(a) = e$$
$$\Rightarrow \frac{dy}{dx} = \frac{y(x+1)}{x^2}$$
$$\Rightarrow \int \frac{dy}{y} = \int \frac{(x+1)dx}{x^2}$$
$$\ell ny = \ell nx - \frac{1}{x} + c$$
$$\because y(a) = e$$
$$\therefore 1 = 0 - 1 + C \Rightarrow C = 2$$

Now, $\ell\, ny = \ell nx - \frac{1}{x} + 2$

$$\Rightarrow \ell n\left(\frac{y}{x}\right) = 2 - \frac{1}{x}$$

$$\Rightarrow \frac{y}{x} = e^{2-\frac{1}{x}}$$

$$\Rightarrow y = x, e^{2-\frac{1}{x}}$$

So, $\lim_{x\to 0^+} y = \lim_{x\to 0^+} xe^{2-\frac{1}{x}} = 0$

**66. (c)**

$$f(x) = \frac{\log_{(x+1)}(x-2)}{e^{2\ln x} - (2x+3)}$$

case (i) $x - 2 > 0 \Rightarrow x > 2$

$x \in (2, \infty)$

case (ii) $x + 1 > 0$ and $x + 1 \neq 1$

$\therefore x_t(-1,0) \cup (0, \infty)$

case (iii) $x > 0 \Rightarrow x_t(0, \infty)$

case (iv) $e^{2\text{ in } x} - (2x+3) \neq 0$

$\Rightarrow\ x^2 - 2x + 3 \neq 0$

$(x-3)(x+1) \neq 0$

$\Rightarrow\ x \neq 3, x \neq -1$

$\therefore$ from (i) n (ii) n (iii)n (iv)

$X_t(2, \infty) - \{3\}$

**67. (b)**

Required probability $= 1 - \frac{D_{(15)} + {}^{15}C_1 D_{(14)} + {}^{15}C_2 D_{(c)}}{15!}$

Taking $D_{(15)}$ as $\frac{15!}{e}$

$D_{(14)}$ as $\frac{14!}{e}$

$D_{(13)}$ as $\frac{13!}{e}$

We get $1 - \left(\frac{\frac{15!}{e} + 15\frac{14!}{e} + \frac{15\times14}{2\times1}\times\frac{13!}{e}}{15!}\right)$

$= 1 - \left(\frac{1}{e} + \frac{1}{e} + \frac{1}{2e}\right) = 1 - \frac{5}{2e} \simeq 0.08$

**68. (a)** $f(x) = \text{Max.}\{x^2, 1 + [x]\}$

Now, $f(x) = \begin{cases} 1 + [x] & 0 \le x \le \sqrt{2} \\ x^2 & \sqrt{2} < x \le 2 \end{cases}$

$\int_0^2 f(x)dx = \int_0^{\sqrt{2}} (1 + [x])dx + \int_{\sqrt{2}}^2 x^2 dx$

$= \int_0^1 1dx + \int_1^{\sqrt{2}} 2dx + \int_{\sqrt{2}}^2 x^2 dx$

$= (x)_0^1 + 2(x)_1^{\sqrt{2}} + \frac{1}{3}(x^3)_{\sqrt{2}}^2$

$= 1 + 2(\sqrt{2} - 1) + \frac{1}{3}(8 - 2\sqrt{2})$

$= \frac{4\sqrt{2} + 5}{3}$

**69. (c)**

$\text{Re}(z_1z_2) = 0$ and $\text{Re}(z_1 + z_2) = 0$
Let $z_1 = a_1 + ib_1$ and $z_2 = a_2 + ib_2$

$z_1z_2 = (a_1a_2 - b_1 b_2) + i(a_1 b_2 + b_1a_2)$
$\because \text{Re}(z_1z_2) = a_1a_2 - b_1 b_2 = 0$
$\therefore a_1a_2 = b_1 b_2 \ldots\ldots (1)$

and $\text{Re}(z_1 + z_2) = 0 \Rightarrow a_1 + a_2 = 0$

$\Rightarrow a_2 = -a_1$

from (1) and (2)

$b_1 b_2 = -a_1^2 < 0$

Product of $b_1 b_2$ is Negative.
$\therefore \operatorname{Im}(z_1)$ and $\operatorname{Im}(z_2)$ are also of opposite sign.

**70.** **(b)** Vector $\vec{a} = \lambda\hat{i} + \mu\hat{j} + 4\hat{k}, \vec{b} = -2\hat{i} + 4\hat{j} - 2\hat{k}$ and $\vec{c} = 2\hat{i} + 3\hat{j} + \hat{k}$

are coplanar then

$$[\vec{a}\vec{b}\vec{c}] = 0 \Rightarrow \begin{vmatrix} \lambda & \mu & 4 \\ -2 & 4 & -2 \\ 2 & 3 & 1 \end{vmatrix} = 0$$
$$\Rightarrow 10\lambda - 2\mu - 56 = 0$$
$$\Rightarrow 5\lambda - \mu = 28$$

also projection of $\vec{a}$ on the $\vec{b}$ is $\sqrt{54}$ units. then

$$\vec{a} \cdot \vec{b} = \sqrt{54}$$

$$\Rightarrow \frac{-2\lambda + 4\mu - 8}{\sqrt{24}} = \sqrt{54}$$

$$\Rightarrow -2\lambda + 4\mu - 8 = 36$$

$$\Rightarrow -2\lambda + 4\mu = 44$$

from (a) and (b)

$\lambda = \frac{26}{3}$ and $\mu = \frac{46}{3}$

$$\Rightarrow \lambda + \mu = \frac{26 + 46}{3} = \frac{72}{3} = 24$$

**71.** **(b)**

$$f(\theta) = 3\left(\sin^4\left(\frac{3\pi}{2} - \theta\right) + \sin^4(3\pi + \theta)\right) - 2(1 - \sin^2 2\theta)$$
$$= 3(\cos^4\theta + \sin^4\theta) - 2\cos^2 2\theta$$
$$= 3\left(1 - \frac{\sin^2 2\theta}{2}\right) - 2\cos^2 2\theta$$

$$= 3\left(\frac{2 - \sin^2 2\theta}{2}\right) - 2\cos^2 2\theta$$

$$= 3\left(\frac{1 + \cos^2 2\theta}{2}\right) - 2\cos^2 2\theta$$

$$f(\theta) = \frac{3 - \cos^2 2\theta}{2}$$

$$f^1(\theta) = \frac{2}{2}\cos 2\theta \sin 2\theta \times 2$$

$$f^1(\theta) = \sin 4\theta = \frac{-\sqrt{3}}{2}$$

$\theta \in [0, \pi]$

$4\theta \in [0, 4\pi]$

$\sin 4\theta = \dfrac{-\sqrt{3}}{2}$

$4\theta = \dfrac{4\pi}{3}, \dfrac{5\pi}{3}, \dfrac{10\pi}{3}, \dfrac{11\pi}{3}$

$\theta = \dfrac{\pi}{3}, \dfrac{5\pi}{12}, \dfrac{5\pi}{6}, \dfrac{11\pi}{12}$

$4\beta = \sum_{\theta \in s}\theta = \dfrac{\pi}{3} + \dfrac{5\pi}{12} + \dfrac{5\pi}{6} + \dfrac{11\pi}{12} = \dfrac{4\pi + 5\pi + 10\pi + 11\pi}{12} = \dfrac{30\pi}{12} = \dfrac{5\pi}{2}$

$\beta = \dfrac{5\pi}{8}$

$f(\beta) = f\left(\dfrac{5\pi}{8}\right) = \dfrac{3 - \cos^2\left(\frac{5\pi}{4}\right)}{2} = \dfrac{3 - \frac{1}{2}}{2} = \dfrac{5}{4}$

**72. (c)**

$(p \vee q) \vee (\sim p) \vee r) \rightarrow ((\sim q) \vee r)$
$T \rightarrow F \equiv F$
$\therefore (p \vee q) \wedge ((\sim p) \vee r) \equiv T$ …….
$(\sim q) \vee r \equiv F$
$\Rightarrow \sim q = F, r = F$
$\Rightarrow q = T$

From (a) $p \vee q \equiv T$

$\sim p \vee r \equiv T$

$\therefore r = F$

$\Rightarrow \sim p = T$

$\Rightarrow p = F$
$\therefore p = F, q = T, r = F$

**73. (b)** Area of Required Region

$\Delta = 2\left[\int_1^3 2\sqrt{x}dx + \int_3^{\sqrt{21}} \sqrt{21 - x^2}dx\right]$

$= 2\left[2\dfrac{\left(x^{3/2}\right)_1^3}{(3/2)} + \left\{\dfrac{(21)}{2}\sin^{-1}\left(\dfrac{x}{\sqrt{21}}\right) + \dfrac{x}{2}\sqrt{21 - x^2}\right\}_3^{\sqrt{21}}\right]$

$= 2\left[4\sqrt{3} - \dfrac{4}{3}\right] + (21\sin^{-1} 1 + 0) - \left(21\sin^{-1}\left(\dfrac{3}{\sqrt{21}}\right) + 3\sqrt{12}\right)$

$$\Delta = 8\sqrt{3} - \frac{8}{3} + \frac{21\pi}{2} - 6\sqrt{3} - 21\sin^{-1}\sqrt{\frac{3}{7}}$$

$$\Delta = 2\sqrt{3} + \frac{21\pi}{2} - \frac{8}{3} - 21\sin^{-1}\left(\sqrt{\frac{3}{7}}\right)$$

Now,

$$\frac{1}{2}\left(\Delta_1 - 21\sin^{-1}\frac{2}{\sqrt{7}}\right) = \frac{1}{2}\left[2\sqrt{3} + \frac{21}{2}\pi - \frac{8}{3} - 21\sin^{-1}\left(\sqrt{\frac{3}{7}}\right) - 21\sin^{-1}\left(\frac{2}{\sqrt{7}}\right)\right]$$

$$= \frac{1}{2}\left[2\sqrt{3} + \frac{21}{2}\pi - \frac{8}{3} - 21\sin^{-1}1\right]$$

$$\left\{\text{using } \sin^{-1}x + \sin^{-1}y = \sin^{-1}\left\{x\sqrt{1-y^2} + y\sqrt{1-x^2}\right\}\right\}$$

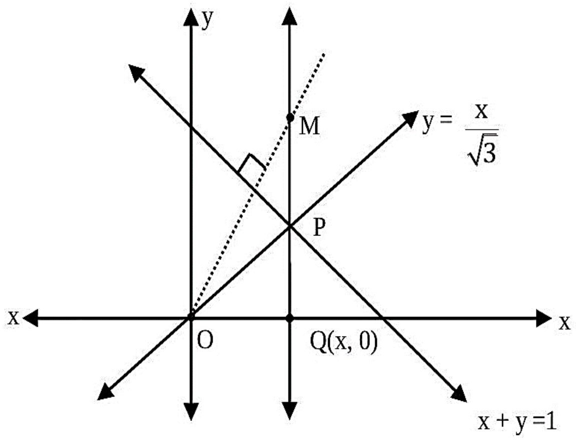
$$= \frac{1}{2}\left[2\sqrt{3} - \frac{8}{3}\right] = \sqrt{3} - \frac{4}{3}$$

**74. (b)** Equation of ray is

$$y = \frac{1}{\sqrt{3}}x$$

Image of $0(0,0)$ in the line $x + y = 1$ is lies on reflected ray.

$$\frac{x-0}{1} = \frac{y-0}{1} = -2\frac{(0+0-1)}{2}$$
$$\Rightarrow M(1,1)$$




$\therefore$ Point of Intersection of lines $y = \frac{x}{\sqrt{3}}$ and $x + y = 1$ is $p(x, y)$

$$\therefore p\left(\frac{3-\sqrt{3}}{2}, \frac{\sqrt{3}-1}{2}\right)$$

Now Reflected Ray is same as line passing through PM.

$$\therefore \text{Slope of PM} = \frac{\frac{\sqrt{3}-1}{2} - 1}{\frac{3-\sqrt{3}}{2} - 1} = \frac{\sqrt{3}-3}{1-\sqrt{3}} = \sqrt{3}$$

Equation of PM whose slope is $\sqrt{3}$ and passing through $M(1,1)$.

$y - 1 = \sqrt{3}(x - 1)$

$y = \sqrt{3}x + (-\sqrt{3} + 1)$

$\because$ ray, Intersects x-axis at $\alpha(x, 0)$

$\therefore y = 0$

$\Rightarrow \sqrt{3}x = -1(-\sqrt{3} + 1) \Rightarrow \sqrt{3}x = \sqrt{3} - 1$

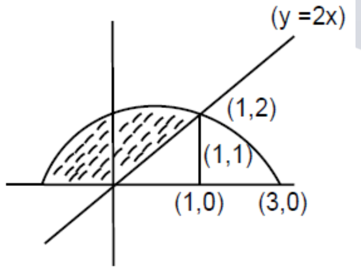
$\Rightarrow x = 1 - \frac{1}{\sqrt{3}}$

$x = \frac{\sqrt{3} - 1}{\sqrt{3}} \times \frac{\sqrt{3} + 1}{(\sqrt{3} + 1)} = \frac{2}{3 + \sqrt{3}}$

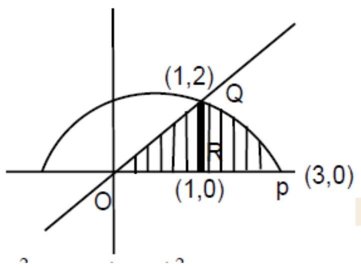
$\therefore$ abscissa of $\alpha$ is $\frac{2}{3+\sqrt{3}}$

**75. (c)**

A =




B =




$y^2 = 4 - (x - 1)^2$
$(x - 1)^2 + y^2 = 2^2$
$y = 2x$
$(x - 1)^2 + 4x^2 = 4$
$x^2 + 1 - 2x + 4x^2 = 4$
$5x^2 - 2x - 3 = 0$
$5x^2 - 5x + 3x - 3 = 0$
$5x(x - 1) + 3(x - 1) = 0$
$x = 1, -3/5$

For B : req. area $= ar(\triangle DRQ) + ar(RPQ)$

$$= \frac{1}{2} \times 1 \times 2 + \int_1^3 \sqrt{4-(x-1)^2}dx$$
$$= 1 + \left[\left(\frac{x-1}{2}\right)\sqrt{4-(x-1)^2} + \frac{4}{2}\sin^{-1}\left(\frac{x-1}{2}\right)\right]_1^3$$
$$= 1 + 2\sin^{-1} 1 = 1 + \pi$$

For A : req. area = area of semi circle - shaded area of B

$$= \frac{\pi r^2}{2} - (1+\pi)$$
$$= \frac{\pi \times 4}{2} - (1+\pi) \ \{\because r = 2\}$$
$$A = \pi - 1$$
$$\therefore \frac{A}{B} = \frac{\pi - 1}{\pi + 1}$$

**76. (a)**

$$14x^2 - 31x + 3\lambda = 0 \quad \alpha, \beta$$

and $35x^2 - 53x + 4\lambda = 0 \quad \alpha, \gamma$

Now, one root is common then

$$\therefore 14\alpha^2 - 31\alpha + 3\lambda = 0$$
$$35\alpha^2 - 53\alpha + 4\lambda = 0$$

$$\frac{\alpha^2}{-124\lambda + 159\lambda} = \frac{-\alpha}{56\lambda - 105\lambda} = \frac{1}{343}$$

$$\Rightarrow \frac{\alpha^2}{35\lambda} = \frac{\alpha}{49\lambda} = \frac{1}{343}$$

$\Rightarrow \alpha = \frac{\lambda}{7}$ {from (ii) and (iii) }

and $\alpha^2 = \frac{35\lambda}{343}$

$$\Rightarrow \frac{\lambda^2}{49} = \frac{35\lambda}{343}$$

$$\lambda^2 - 5\lambda = 0$$

$$\lambda(\lambda - 5) = 0$$

$$\lambda = 0, \lambda = 5 \ \Rightarrow \alpha = 5/7$$

not possible $\therefore$ only $\lambda = 5$ possible

Now, $\alpha + \beta = \frac{31}{14}, \alpha\beta = \frac{3\lambda}{14}, \alpha + \gamma = \frac{53}{35}, \alpha\gamma = \frac{4\lambda}{35}$

$\therefore \beta = \frac{3}{2}$ and $\gamma = \frac{4}{5}$

Now equation having roots $\left(\frac{3\alpha}{\beta}, \frac{4\alpha}{\gamma}\right) = \left(\frac{10}{7}, \frac{25}{7}\right)$ is

$$x^2 - \frac{35}{7}x + \frac{250}{49} = 0$$

$$\Rightarrow 49x^2 - 245x + 250 = 0$$

**77. (d)** Equation of line AB is

$$y + 5 = \left(\frac{-5+11}{8-4}\right)(x-8)$$

$$\Rightarrow y + 5 = \frac{3}{2}(x-8) \text{ P } 2y + 10 = 3x - 24$$

$$3x - 2y - 34 = 0$$

Let C be $(h, k)$ then equation of AB

$$hx + ky - \frac{3}{2}(x+h) + 5(y+k) - 15 = 0$$

$$x\left(h - \frac{3}{2}\right) + y(k+5) - \frac{3}{2}h + 5k - 15 = 0$$

Now, by comparing (i) and (ii)

$$\frac{h - \frac{3}{2}}{3} = \frac{k+5}{-2} = \frac{-\frac{3}{2}h + 5k - 15}{-34}$$

after solving centre C is

$$(h, k) = \left(8, \frac{-28}{3}\right)$$

and radius of circle is

$$r = \left|\frac{3(8) - 2\left(\frac{-28}{3}\right) - 34}{\sqrt{9+4}}\right| = \left|\frac{24 + 2\frac{56}{3} - 34}{\sqrt{13}}\right|$$

$$r = \left|\frac{26}{3\sqrt{13}}\right| = \frac{2\sqrt{13}}{3}$$

**78. (b)**

$$f(x) = x + \int_0^{\frac{\pi}{2}} (\sin x \cos y + \cos x \sin y) f(y)\, dy$$

$$f(x) = x + \int_0^{\frac{\pi}{2}} (\cos y f(y) dy) \sin x + (\sin y f(y) dy) \cos x\Big)$$

given : $f(x) = x + \frac{a}{\pi^2 - 4}\sin x + \frac{b}{\pi^2 - 4}\cos x$

by comparing (1) and (2)

$\Rightarrow \frac{a}{\pi^2-4} = \int_0^{\frac{\pi}{2}} \cos y f(y) dy$

and $\frac{b}{\pi^2-4} = \int_0^{\frac{\pi}{2}} \sin y f(y) dy$

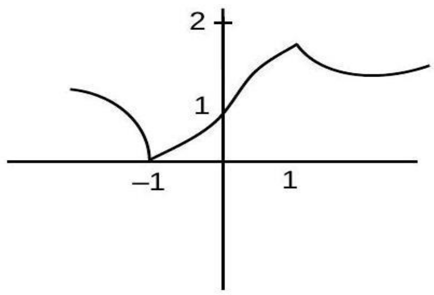
adding (3) and (4)

$$\frac{a+b}{\pi^2-4} = \int_0^{\frac{\pi}{2}} (\sin y + \cos y) f(y) dy$$
$$\frac{a+b}{\pi^2-4} = \int_0^{\frac{\pi}{2}} (\sin y + \cos y) f\left(\frac{\pi}{2} - y\right) dy$$

Additing (5) and (6)

$$\frac{2(a+b)}{\pi^2-4} = \int_0^{\frac{\pi}{2}} (\sin y + \cos y)\left(\frac{\pi}{2} + \frac{a+b}{\pi^2-4}(\sin y + \cos y)\right) dy$$
$$= \pi + \frac{a+b}{\pi^2-4}\left(\frac{\pi}{2} + 1\right)$$
$$\Rightarrow a + b = -2\pi(\pi + 2)$$

**79. (a)**




$$f(x) = \frac{(x+1)^2}{x^2+1} = 1 + \frac{2x}{x^2+1}$$

$$f(x) = 1 + \frac{2}{x + \frac{1}{x}}$$

Clearly, f(x) is one - one in $[1, \infty]$ but not in $(-\infty, \infty)$

**80. (d)**

Total Apple $= 10$, Rotten apple $= 3$, good apple $= 7$

Prob. of rotten apple $(p) = \frac{3}{10}$

Prob. of good apple $(q) = \frac{7}{10}$

$x \rightarrow$ Number of rotten apples

here $x = 0,1,2,3$

$$p(x=0) = {}^4C_0\left(\frac{3}{10}\right)^0 \times \frac{7}{10}\times\frac{6}{9}\times\frac{5}{8}\times\frac{4}{7} = \frac{1}{6}$$

$$p(x=1) = {}^4C_1\left(\frac{3}{10}\right)\times\frac{7}{9}\times\frac{6}{8}\times\frac{5}{7} = \frac{1}{2}$$

$$p(x=2) = {}^4C_2\left(\frac{3}{10}\times\frac{2}{9}\right)\times\frac{7}{8}\times\frac{6}{7} = \frac{3}{10}$$

$$p(x=3) = {}^4C_3\left(\frac{3}{10}\times\frac{2}{9}\times\frac{1}{8}\right)\times\frac{7}{7} = \frac{1}{30}$$

| $x_i$ | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| $p_i$ | $\frac{35}{210}$ | $\frac{105}{210}$ | $\frac{3}{10}$ | $\frac{1}{30}$ |

Now,

$$\mu = \sum p_i x_i = \frac{1}{6}\times 0 + \frac{1}{2}\times 1 + 2\times\frac{3}{10} + 3\times\frac{1}{30} = \frac{6}{5}$$

and $\sigma^2 = \sum p_i x_i^2 - \mu^2 = \frac{1}{2} + \frac{3}{10}\times 4 + \frac{1}{30}\times 9 - \frac{36}{25} = \frac{14}{25}$

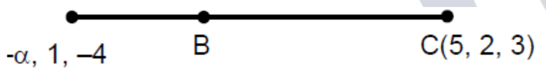
$$\therefore 10(\mu^2+\sigma^2) = 10\left(\frac{36}{25}+\frac{14}{25}\right)$$

$$= 10\times\left(\frac{50}{25}\right) = 10\times 2$$

$$= 20$$

**81. (9)**

$A(0,2,\alpha)$




$$\left|\frac{1}{2}\cdot 2\sqrt{21}\begin{vmatrix} i & j & k \\ \alpha & 1 & \alpha+4 \\ 5 & 2 & 3 \end{vmatrix}\frac{1}{\sqrt{25+4+9}}\right| = 21$$

$$\sqrt{(2\alpha+5)^2+(2\alpha+20)^2+(2\alpha-5)^2} = \sqrt{21}\sqrt{38}$$

$$12\alpha^2+80\alpha+450 = 798$$

$$12\alpha^2+80\alpha-398 = 0$$

$$\alpha = 3 \Rightarrow \alpha^2 = 9$$

**82. (3)**

Given $f(x+y) = f(x)+f(y)-1\forall x,y \in IR$ and $f'(0) = 2$

Partial differentiate w.r.t x

$\Rightarrow f'(x+y)f'(x)$

for $x = 0$

$f'(y) = f'(0) = 2$

on Integrating

$\Rightarrow f(y) = 2y + c$

for $y = 0$

$\Rightarrow f(0) = C$

Put $x = y = 0$ in (a)

$\Rightarrow f(0) = f(0) + f(0) - 1$

$\Rightarrow f(0) = 1$

from (c) & (d)

$c = 1$

$\Rightarrow f(y) = 2y + 1$

$\Rightarrow f(-2) = -4 + 1 = -3$

$\therefore |f(-2)| = 3$

**83. (10)** $f(x + y) = f(x) + f(y) \forall x, y \in N$ and $f(a) = \frac{1}{5}$

for $x = y = 1$

$f(b) = f(a) + f(a) = 2f(a)$

$f(c) = f(2 + 1) = f(b) + f(a) = 3f(a)$

In General

$f(n) = nf(a) = \frac{n}{5}$

$\sum_{n=1}^{m} \frac{f(n)}{n(n+1)(n+2)} = \frac{1}{12}$

$\Rightarrow \sum_{n=1}^{m} \frac{n}{5n(n+1)(n+2)} = \frac{1}{12}$

$\Rightarrow \frac{1}{5}\sum_{n=1}^{m} \frac{1}{(n+1)(n+2)} = \frac{1}{12}$

$\Rightarrow \sum_{n=1}^{m} \left(\frac{1}{n+1} - \frac{1}{n+2}\right) = \frac{5}{12}$

$\Rightarrow \left(\frac{1}{2} - \frac{1}{3}\right) + \left(\frac{1}{3} - \frac{1}{4}\right) + \cdots \ldots + \left(\frac{1}{m+1} - \frac{1}{m+2}\right) = \frac{5}{12}$

$\Rightarrow \frac{1}{2} - \frac{1}{m+2} = \frac{5}{12}$

$\Rightarrow \frac{1}{m+2} = \frac{1}{2} - \frac{5}{12} = \frac{1}{12}$

$\Rightarrow m = 10$

**84. (1120)**

Let $r+1, r+2$ and $r+3$ be three consecutive terms

$$\frac{{}^{n}C_{r}2^{r}}{{}^{n}C_{r+1}2^{r+1}} = \frac{2}{5}$$

$$\Rightarrow \frac{r+1}{n-r} = \frac{4}{5}$$

Also,

$$\frac{{}^{n}C_{r+1}2^{r+1}}{{}^{4}C_{r+2}2^{r+2}} = \frac{5}{8}$$

$$\Rightarrow \frac{r+2}{n-r-1} = \frac{5}{4}$$

on solving (a) & (b), we get

$n = 8, r = 3$

Here $n = 8$ (even)

middle term $= r+2 = 3+2 = 5$

coefficient of $T_5 = {}^{8}C_{4}2^{4} = 70(16) = 1120$

**85. (60)** Let first term of G.P be a with common ratio r

Given: $a_4 \cdot a_6 = 9$

$a_5 + a_7 = 24$
$a_4 = ar^3, a_5 = ar^4, a_6 = ar^5, a_7 = ar^6$
$a_4 \cdot a_6 = a^2r^8 = 9$

$\Rightarrow \quad ar^4 = 3$
$\quad a_5 = 3$
$\therefore \quad a_7 = 24 - 3 = 21$
$\Rightarrow \quad \frac{a_7}{a_5} = r^2 = 7$
$\Rightarrow \quad r = \sqrt{7}, a = \frac{3}{49}$

$a_1a_9 + a_2a_4a_9 + a_5 + a_7 = a_1a_9 + (ar_2)(ar^3)a_9 + 24$

$= a_1a_9 + a_1(ar^4)a_9 + 24$

$= a_1a_9(1 + a_5) + 24 = (ar^4)^2(d) + 24$

$= 36 + 24 = 60$

**86. (355)** Given equation of plane is

$ax + by + 3z = 2(a + b)$

It containing the line

$$\frac{x-(-10)}{1} = \frac{y-8}{-2} = \frac{z-0}{1}$$

$\therefore$ plane (1) must passes through $(-10,8,0)$ and parallel to $1,-2,1$
Hence,

$$a(-10) + 8b = 2a + 2b$$
$$\Rightarrow \quad 12a - 6b = 0$$
$$\text{and} \quad a - 2b + 3 = 0$$

on solving (2) and (3), we get

$b = 2, a = 1$

$\therefore$ equation of the plane is

$x + 2y + 3z = 6$

c is perpendicular distance from $(1,27,7)$ to the plane (4)

$$\Rightarrow c = \left|\frac{1 + 2 \times 27 + 3 \times 7 - 6}{\sqrt{1^2 + 2^2 + 3^2}}\right| = \left|\frac{70}{\sqrt{14}}\right| = \frac{10\sqrt{7}}{\sqrt{2}}$$

Now, $a^2 + b^2 + c^2 = 1 + 4 + \frac{700}{2} = \frac{710}{2} = 355$

**87. (1)** For $\left(\alpha x^3 + \frac{1}{\beta x}\right)^{11}$

$$T_{r+1} = {}^{11}C_r(\alpha x^3)^{11-r}\left(\frac{1}{\beta x}\right)^r$$
$$= {}^{11}C_r \alpha^{11-r}\beta^{-r}x^{33-4r}$$

Coefficient of $x^9 = {}^{11}C_6 \alpha^{11-6}\beta^{-6} = {}^{11}C_6\alpha^5\beta^{-6}$
For $\left(\alpha x - \frac{1}{\beta x^8}\right)^{11}$

$$T_{r+1} = {}^{11}C_r(\alpha x)^{11-r}\left(\frac{-1}{\beta x^3}\right)^r$$
$$= (-1)^{r}\,{}^{11}C_r\alpha^{11-r}\beta^{-r}x^{11-4r}$$

coefficient of $x^{-9} = -\,{}^{11}C_5\alpha^6\beta^{-5}$

$$\Rightarrow {}^{11}C_6\alpha^5\beta^{-6} = {}^{11}C_5\alpha^6\beta^{-5}$$
$$\Rightarrow \alpha\beta = -\frac{{}^{11}C_6}{{}^{11}C_5} = -1$$
$$\therefore (\alpha\beta)^2 = 1$$

**88. (2)**

$\overrightarrow{AB} = (\lambda\vec{a} - 3\vec{b} + 4\vec{c}) - (\vec{a} - \vec{b} + \vec{c})$
$\quad = (\lambda - 1)\vec{a} - 2\vec{b} + 3\vec{c}$
$\overrightarrow{AC} = (-\vec{a} + 2\vec{b} - 3\vec{c}) - (\vec{a} - \vec{b} + \vec{c})$
$\quad = -2\vec{a} + 3\vec{b} - 4\vec{c}$
$\overrightarrow{AD} = (2\vec{a} - 4\vec{b} + 6\vec{c}) - (\vec{a} - \vec{b} + \vec{c})$
$\quad = \vec{a} - 3\vec{b} + 5\vec{c}$

For coplanar vectors

$$\begin{vmatrix} \lambda - 1 & -2 & 3 \\ -2 & 3 & -4 \\ 1 & -3 & 5 \end{vmatrix} = 0$$

$\Rightarrow 3\lambda - 6 = 0$

$\therefore \lambda = 2$

**89. (1436)**

Number starting with 7 = $7\underset{5}{\underset{\uparrow}{\underline{\ \ }}}\ \underset{5}{\underset{\uparrow}{\underline{\ \ }}}\ \underset{5}{\underset{\uparrow}{\underline{\ \ }}}\ \underset{5}{\underset{\uparrow}{\underline{\ \ }}} = 625$

Number starting with 5 = $5\underset{5}{\underset{\uparrow}{\underline{\ \ }}}\ \underset{5}{\underset{\uparrow}{\underline{\ \ }}}\ \underset{5}{\underset{\uparrow}{\underline{\ \ }}}\ \underset{5}{\underset{\uparrow}{\underline{\ \ }}} = 625$

Number starting with 37 = $37\underset{5}{\underset{\uparrow}{\underline{\ \ }}}\ \underset{5}{\underset{\uparrow}{\underline{\ \ }}}\ \underset{5}{\underset{\uparrow}{\underline{\ \ }}} = 125$

Number starting with 357 = $357\ \bar{\uparrow}_5\ {}_5\bar{5}_5 = 25$

Number starting with 355 = $355\_\ ^{-} = 25$

Number starting with 3537 = $3537\ \_ = 5$

Number starting with 3535 = $3535 - = 5$

Number starting with $\underline{35337\ = 1}$

Total $= 1436$

Therefore, the serial number of 35337 is 1436

**90. (32)**

Number of six-digit number starting with 1 is 1

$= {}^{8}C_{5} = 56$

As remaining five digits can be selected from 8 digits that are greater than (i.e., 2, 3, 4, 5, 6, 7, 8)

Number of six-digit number starting with 23 = ${}^{6}C_{4} = 15$

Total $= 56 + 15 = 71$

Now, $72^{\text{nd}}$ number $= 245678$

$\therefore$ sum of the digits $= 2 + 4 + 5 + 6 + 7 + 8 = 32$