1. (a)

$(N_0)_A = \frac{320}{16} = 20$ moles

$(N_0)_B = \frac{320}{32} = 10$ moles

$N_A = \frac{(N_0)_A}{2^{n_1}} = \frac{20}{4} = 5$

$N_B = \frac{(N_0)_B}{2^{n_2}} = \frac{10}{(2)^{\frac{2}{0.5}}} = \frac{10}{2^4} = 0.625$

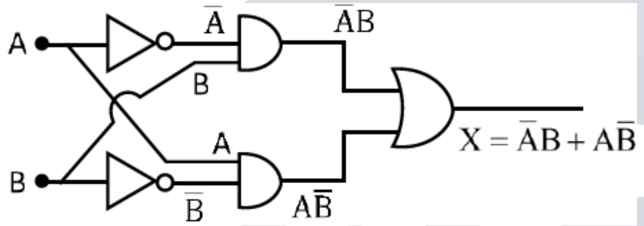
$(2)^{\frac{2}{0.5}}$

Total $N = 5.625$ moles

No. of atoms $= (N)(N_A)$

$= 5.625 \times 6.023 \times 10^{23} = (3.38 \times 10^{24})$

2. (c)



From Bodean Algebra:

$X = \bar{A}B + A\bar{B}$

The correct truth table will be

A	B	X
0	0	0
0	1	1
1	0	1
1	1	0

3. (c) Acceleration on the smooth inclined plane

$a_1 = g\sin\theta = \frac{g}{\sqrt{2}}$

Acceleration on the rough inclined plane

$a_2 = g\sin\theta - \mu g\cos\theta = \frac{g}{\sqrt{2}} - \frac{Kg}{\sqrt{2}} (K = \mu)$

Given that:

$t_2 = nt_1$ and $\frac{1}{2}a_1t_1^2 = \frac{1}{2}a_2t_2^2$

$a_1t_1^2 = a_2t_2^2$

$\frac{g}{\sqrt{2}}t_1^2 = \left(\frac{g}{\sqrt{2}} - \frac{Kg}{\sqrt{2}}\right)(n^2t_1^2)$

$\frac{g}{\sqrt{2}} = n^2\left(\frac{g}{\sqrt{2}} - \frac{Kg}{\sqrt{2}}\right)$

$K = 1 - \frac{1}{n^2}$

4. (a)

Heat energy given $= 184KJ = 184 \times 10^3$ J

Amount of heat required to raise the temperature

$\theta_1 = ms_{ice}\Delta T = 0.6 \times 2222.3 \times 12$

$= 16000.56$ J

Remaining heat $\theta_2 = 184000 - 16000.56 = 167999.44$ J

For melting at 0°C heat required $= mL_f$

$\therefore$ 100% ice is not melted

$= 0.6 \times 336000$
$= (201600)$J needed

Amount of ice melted

$167999.44 = m \times 336000$

$m =$ mass of water $= 0.4999$Kg

Mass of ice $= 0.1001$

Ratio $= \frac{0.1001}{0.4999} \approx 1:5$

5. (d) Work done by a man in lifting a bucket out of a well by means of a rope tied to the bucket is positive

→ Work done by friction on a body sliding down an inclined plane is negative

→ Work done by a applied force on a body moving on a rough horizontal plane with uniform velocity is positive

6. (a)

$$R.P = \frac{2\mu\sin\theta}{1.22\lambda}$$

$\mu \uparrow$, R.P $\uparrow$

$D \downarrow, \theta \downarrow$, R.P $\downarrow$

$\lambda \uparrow$, R.P $\downarrow$

R.P is independent of focal length of eye piece

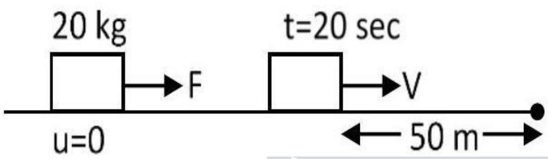
7. **(c)** If on displacing the jockey slightly from the null point position, the galvanometer shows a large deflection, than the potentiometer is said to be sensitive. The sensitivity of the potentiometer depends upon the potential gradient along the wire. The smaller potential gradient greater will be sensitivity.

Sensitivity $\uparrow$, potential gradient $\downarrow$, length $\uparrow$

Sensitivity $\propto$ length

Sensitivity $\propto \frac{1}{\text{Potential gradient}}$

8. **(b)**



$$50 = V \times 10$$
$$V = 5\ ms^{-1}$$
$$V = 0 + a \times 20$$
$$5 = a \times 20$$
$$a = \frac{1}{4}\ ms^{-2}$$
$$F = ma = 20 \times \frac{1}{4} = 5\ N$$

9. **(a)**

$$\mu = \text{Modulating index } = \frac{A_{max} - A_{min}}{A_{max} + A_{min}} = \frac{14-6}{14+6} = 0.4$$

10. **(a)**

Statement -I is correct as

EMW are neutral

Statement - II is wrong

$$E_0 = \sqrt{\frac{1}{\mu_0 \in_0}}\, B_0$$

11. **(a)**

$l = 5$ cm
$t = 1$sec
$V = \frac{0.05}{1} = 0.05\ ms^{-1}$
$I = \frac{40 \times 0.05 \times 0.05}{10} = \frac{BLV}{R} = 0.01$ A
$F = BIL = 40 \times 0.010.05 = 0.02$ N
$W = F\ell = 0.02 \times 0.05 = 1 \times 10^{-3}$ J

12. (b)

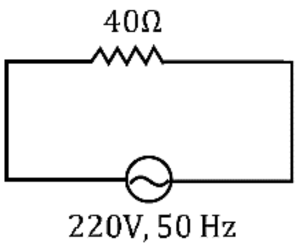


fig (a)

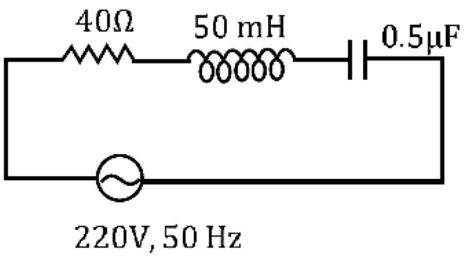


fig (a)

X_L is not equal to X_C, so rms current In (b) can never be large than (a)

13. (b)

$$P_2 A - P_1 A = 5.4 \times 10^5 \times g$$

$$P_2 - P_1 = \frac{5.4 \times 10^6}{500} = 10.8 \times 10^3$$

$$P_2 + 0 + \frac{1}{2}\rho v_2^2 = P_1 + 0 + \frac{1}{2}\rho v_1^2$$

$$P_2 - P_1 = \frac{1}{2}\rho(v_1^2 - v_2^2) = \frac{1}{2}\rho(v_1 + v_2)(v_1 - v_2)$$

$$10.8 \times 10^3 = \frac{1}{2} \times 1.2 \times (v_1 - v_2) \times 2 \times 3 \times 10^2$$

$$v_1 - v_2 = 30$$

$$\frac{v_1 - v_2}{v} \times 100 = \frac{30}{300} \times 100 = 10\%$$

14. (d)

$$\frac{\lambda_\alpha}{\lambda_p} = \frac{\frac{h}{\sqrt{2\,m_\alpha q_\alpha v}}}{\frac{h}{\sqrt{2\,m_p q_p v}}}$$

$$\frac{\lambda_\alpha}{\lambda_p} = \sqrt{\frac{1}{8}}$$

$M = 8$

15. (c) $T^2 \propto R^3$

$$\frac{T_1^2}{T_2^2} = \frac{R_1^3}{R_2^3} \Rightarrow \left(\frac{T_1}{T_2}\right)^2 = \left(\frac{R}{\frac{R}{4}}\right)^3$$

$$\frac{T_1^2}{T_2^2} = 64$$

$$T_2^2 = \frac{T_1^2}{64}$$

$$T_2 = \frac{T_1}{8} = \frac{24}{8} = 3$$

16. (b)

$$B = \frac{\mu_0 i}{2R} \times 4$$

$$B' = \frac{\mu_0 i}{2R'}$$

$$R' = 4R$$

$$B' = \frac{\mu_0 i}{8R}$$

$$\frac{B'}{B} = \frac{1}{16}$$

$$B' = 2T$$

17. (c)

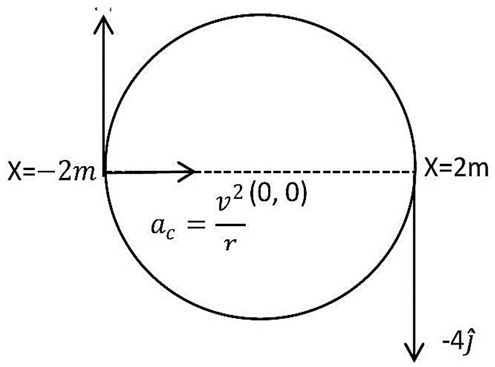
$$W_E = q\vec{E} \cdot \vec{S} = 2 \times 10^{-2} \times (-30)$$
$$= -0.6\text{ J} = -600\text{ mJ}$$

18. (c)

$$a_c = \frac{v^2}{r} = \frac{4^2}{2} = 8\text{ ms}^{-2}$$

$$\vec{v} = 4\hat{j}$$

$\vec{a_c} = 8i$



19. (a)

$$\sqrt{\frac{3RT}{M}} = \sqrt{\frac{\alpha+5}{\alpha}}\sqrt{\frac{8}{\pi}\frac{RT}{M}}$$

$$3 = \left(\frac{\alpha+5}{\alpha}\right)\left(\frac{8}{\pi}\right)$$

$$\alpha = 28$$

20. (a) $(x - At)^2 + \left(y - \frac{t}{B}\right)^2 = a^2$

$A = L^1\ T^1$

$\frac{t}{B}$ is in meter

$$\frac{t}{B} = L$$

$$\frac{T^{-1}}{B} = L$$

$$B = T^{-1}L^{-1}$$

21. (800)

$$\text{Use } \Delta L = \int_0^t \tau dt$$

$$L_0 = \int_0^2 (mg)(v_x t)dt$$

$$= (mgv_x)\frac{t^2}{2}$$

$$= (0.1)(10)(10)(\sqrt{2}) \times \frac{2^2}{2}$$

$$= 20\sqrt{2}$$

$$= \sqrt{800}$$

22. (7)

$$\mu_1 = \sqrt{2.8}$$

$\mu_2 = \sqrt{6.8}$

$\mu \sin i = \mu_2 \cos i$

$\tan i = \frac{\mu_2}{\mu_1} = \sqrt{\frac{6.8}{2.8}}$

$\tan i = \left(\frac{2.8 + 4}{2.8}\right)^{\frac{1}{2}}$

$i = \tan^{-1}\left(1 + \frac{10}{7}\right)^{\frac{1}{2}}$

$\theta = 7$

23. (40)

$F = ma$

$-25x = \frac{250}{100} a$

$a = -100x$

$\omega^2 = 100$

$\omega = 100$

$A\omega = 4$

$A = \frac{4}{10} = 0.4 \text{ m}$

$A = 40 \text{ cm}$

24. (40) $\frac{dv}{dt} = \frac{v^2}{R}$

$\frac{vdv}{dx} = \frac{v^2}{R}$

$\frac{dv}{dx} = \frac{v}{R}$

$\int_{15}^{v} \frac{dv}{v} = \int_{0}^{x} \frac{dx}{R}$

$\frac{v}{15} = \frac{x}{R}$

$\frac{v}{15} = e^{\frac{x}{R}}$

$v = 15e^{\frac{x}{R}}$

$$\frac{dx}{dt} = 15e^{\frac{x}{R}}$$

$$\int_0^{\frac{\pi R}{2}} e^{-\frac{x}{R}} dx = 15\int_0^{to} dt$$

$$t_0 = 40\left(1 - e^{-\frac{\pi}{2}}\right)s$$

$$t = 40$$

25. (30)

$$\frac{R_1 + R_2}{10} = \frac{60}{40}$$

$$R_1 + R_2 = 15$$

$$\frac{R_1R_2}{(R_1 + R_2) \times 3} = \frac{40}{60}$$

$$R_1R_2 = 30$$

26. (41)

$$\mu = \frac{h}{h^1} = \frac{\text{Real depth}}{\text{Apparent depth}}$$

Least Count = M.S.D. - V.S.D

$= \text{M.S.D.} - \frac{49}{50}\text{M.S.D}$

$= \left(\frac{50-49}{50}\right)\text{M.S.D}$

$= \frac{1}{50}\text{M.S.D}$

$$= \frac{1}{50} \times \frac{1}{20} \text{ cm}$$

$$= \frac{1}{1000} \text{ cm}$$

$$= \frac{10}{1000} \text{ mm} = 0.01 \text{ mm}$$

$$\ln \mu = \ln h - \ln h$$

$$\frac{d\mu}{\mu} = \frac{dh}{h} + \frac{dh}{h'}$$

$$d\mu = \mu\left[\frac{dh}{h} + \frac{dh'}{h'}\right]$$

$$d\mu = \mu\left[\frac{dh}{h} + \frac{dh'}{h}\right] = \frac{5.25}{5.00}\left[\frac{0.01}{5.25} + \frac{0.01}{5.00}\right]$$

$$= \frac{41}{10} \times 10^{-3}$$

27. (3872) For Maximum current is drawn

$$x_L = x_C$$

$$\omega L = \frac{1}{\omega C}$$

$$2\pi f L = \frac{1}{2\pi f c}$$

$$C = \frac{1}{4\pi^2 f^2 L} = \frac{1}{4 \times \pi^2 \times 49 \times 10^6 \times 2 \times 10^{-6}}$$

$$C = \frac{1}{3872}\ F$$

$$X = 3872$$

28. (5) Potential Gradient $= \frac{\Delta V}{L}$

$$E - Ir = \left(\frac{\Delta V}{L}\right) x$$

$$\frac{ER}{R + r} = \left(\frac{\Delta V}{L}\right) x$$

$$\frac{E \times 5}{5 + r} = \frac{\Delta V}{L} \times 200$$

$$\frac{E \times 15}{15 + r} = \frac{\Delta V}{L} \times 300$$

29. (12)

$$E = -\frac{dv}{dr} = -4ar$$

By the Gauss' theorem

$$\oint \vec{E} \cdot \overrightarrow{dA} = \frac{q_{inside}}{\varepsilon}$$

$$E \times 4\pi r^2 = \frac{\rho \times \frac{4}{3}\pi r^3}{\varepsilon}$$

$$E = \frac{\rho r}{3\varepsilon} = -4ar$$

$$\rho = -12a\varepsilon$$

30. (25) $|F| = \eta A \frac{\Delta v}{\Delta h}$

$$0.1 = 5 \times 10^{-3} \times 0.2 \times \frac{v}{0.25 \times 10^{-3}}$$

$v = 0.025\ ms^{-1}$

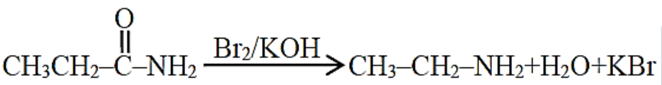
$v = 25 \times 10^{-3}\ ms^{-1}$

31. (d)

Molecules	Total No. of e^-	Bond order
O_2^{-2}	18	1
CO	14	3
NO^+	14	3

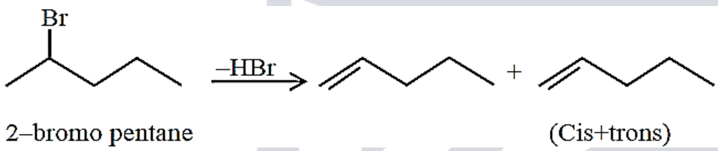
32. (c) Equanil is a tranquiliger, used for treatment of depression and hypertension.

33. (d)



Hoffmann's bromamide reaction

34. (d)

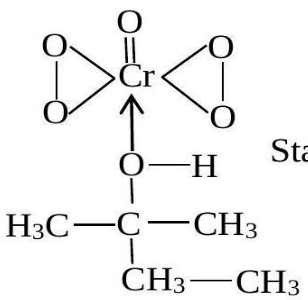


35. (b)

36. (b)

37. (d)

Blue

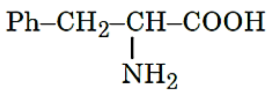


Stablize by L.P.

38. (b)

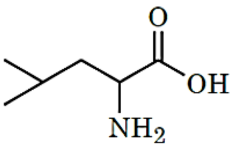
(i) Mn_2O_7
(ii) RuO_4 &OsO_4
(iii) $Sc(+4)$ oxidation state not possible in oxidizing nature
(iv) Cr show oxidizing nature in +6 oxidation state

39. (d) Hydrolysis of the given tetrapeptide will give the following:



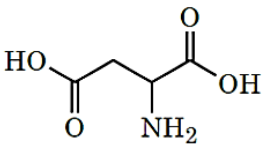
(F)

(Phenylalanine)



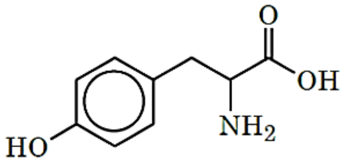
(L)

(Leucine)



(D)

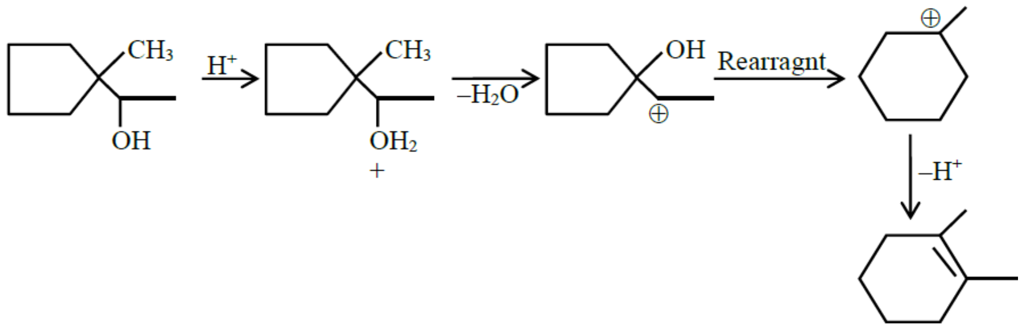
(Aspartic acid)



(Y)

(Tyrosine)

40. (d)



41. (c)

(A) van't Hoff factor, i

$$i = \frac{\text{Normal molar mass}}{\text{Abnormal molar mass}}$$

(B) k_f = Cryoscopic constant
(C) Solutions with same osmotic pressure are known as isotonic solutions.
(D) Solutions with same composition of vapour over them are called Azeotrope.'

42. (b)
43. (b)

Neoprene: Elastomer

Polyester Fibre

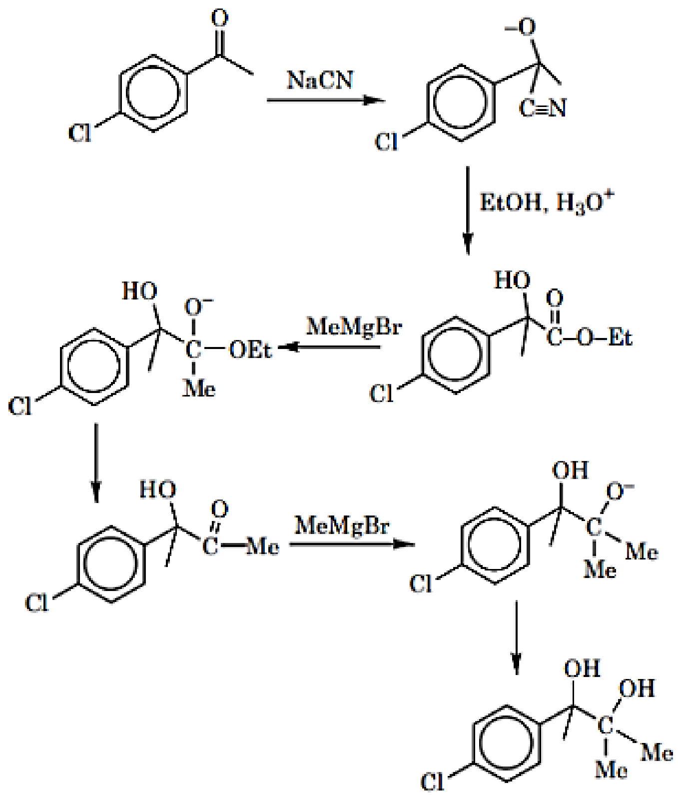
Polstyrene : THermolastic

Urea-Formaldhyde Resin: Thermosetting polymer

44. (b)

→ BOD value of water of water is in the range $-3-5$ (Less than 5)
→ dissolve oxygen in water for growth of wish → Less than (6)

45. (b)



46. (b) $C_xH_y + \left(x + \frac{y}{4}\right) O_2 \longrightarrow xCO_2 + \frac{y}{2} H_2O$

Number of equivalents of O_2 = Number of equivalents of H_2O

Number of equivalents of $H_2O = \frac{y}{2} = 3$

$y = 6$

Number of equivalents of $O_2 = x + \frac{y}{4} = 9.5$

$$x + \frac{6}{4} = 9.5$$

$x = 9.5 - 1.5 = 8$

$C_xH_y = C_8H_6$

47. (a)

48. (c) Only (B) and (C) are correct.

(B) $G = H - TS$

At constant T

$\Delta G = \Delta H - T\Delta S$

(A) First law is given by

$\Delta U = Q + W$

If we apply constant P and reversible work.

$\Delta U = Q - P\Delta V$

(C) By definition of entropy change

$$dS = \frac{q_{rev}}{T}$$

At constant T

$$\Delta S = \frac{q_{rev}}{T}$$

(D) $H = U + PV$
For ideal gas

$$H = U + nRT$$

At constant T

$$\Delta H = \Delta U + \Delta nRT$$

49. (a)
50. (b)

(i) Electro osmosis: When movement of colloidal particles is prevented by some suitable means (porous diaphragm or semi permeable membranes), it is observed that the D.M. begins to move in an electric field. This phenomenon is termed electrosmosis.

(ii) Solvent molecules pass through semi-permeable membrane towards solvent side is termed as reverse osmosis.

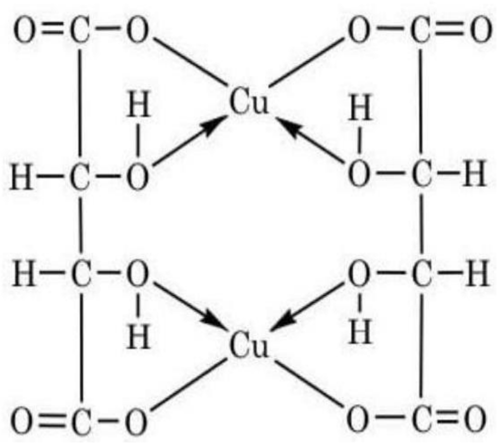
(iii) When an electric potential is applied across two platinum electrodes dipping in a colloidal solution, the colloidal particles move towards move towards one or the other electrode. The movement of colloidal particles under an applied electric potential is called electrophoresis.

(iv) Solvent molecules pass through semipermeable membrane towards the solution side is termed as osmosis.

51. (270)

$$\mathbf{r} \propto \frac{n^2}{Z}$$
$$r_{He^+} = r_H \times \frac{n^2}{Z}$$
$$r_{He^+} = 0.6 \times \frac{(3)^2}{2}$$
$$= 2.7\text{Å}$$
$$r_{He^+} = 270\text{pm}$$

52. (4) Acidic oxide $\rightarrow N_2O_3, NO_2, Cl_2O_7, SO_2$
53. (4)



Copper tartrate complex

Denticity = 2

54. (17) Given

$Zn(s) + Sn^{2+}(aq.) \rightleftharpoons Zn^{2+}(aq.) + Sn(s)$

$K_C = 1 \times 10^{20}$

$E^{\circ}_{Zn^{2+}/Zn} = -0.76\ V$

$$E_{cell} = E^{\circ}_{cell} - \frac{0.059}{n}\log_{10} K_c$$

$$0 = E^{\circ}_{cell} - \frac{0.059}{2} \times 20$$

$E^{\circ}_{cell} = 0.59$

$E^{\circ}_{cell} = \underset{(RP)}{E^{\circ}_{Cathode}} - \underset{(RP)}{E^{\circ}_{Anode}}$

$0.59 - E^{\circ}_{Sn^{2+}/sn} - E^{\circ}_{Zn^{2+}/Zn}$

$0.59 = E^{\circ}_{Sn^{2+}/Sn} - (-0.76)$

$E^{\circ}_{Sn^{2+}/Sn} = 0.17$

$E^{\circ}_{Sn/Sn^{2+}} = 17 \times 10^{-2}$

55. (15)

Mole of Na $= \frac{0.69}{23} = 3 \times 10^{-2}$

$$Na + H_2O \longrightarrow NaOH + \frac{1}{2}H_2$$

By using POAC

Moles of NaOH $= 3 \times 10^{-2}$

NaOH reacts with HCl

No. of equivalent of NaOH = No. of equivalent of HCl

$3 \times 10^{-2} \times 1 = \frac{73}{36.5} \times V(\text{ in L }) \times 1$

$V = 1.5 \times 10^{-2}$ L

Volume of HCl = 15ml.

56. **(2)** As unit of rate constant is (conc.) $^{1-n}$ time $^{-1}$

Put n = 2

then $\text{Lmol}^{-1}\ \text{s}^{-1}$

So order of the reaction is 2.

57. **(3)** $4LiNO_3 \longrightarrow 2Li_2O + 4NO_2 + O_2$

58. **(200)**

Let M is the molar mass of the compound (g/mol)

mass of compound = 0.01Mgm

mass of carbon $= 0.01M \times \frac{60}{100}$

mass of carbon $= \frac{0.01M}{12} \times \frac{60}{100}$

moles of CO_2 from combustion $= \frac{4.4}{44}$ = moles of carbon

$$\frac{0.01M}{12} \times \frac{60}{100} = \frac{4.4}{44}$$

$$M = \frac{4.4}{44} \times \frac{100}{60} \times \frac{12}{0.01} = 200\text{gm/mol}$$

59. **(4)** Reverse equation (a) So $K_1^1 = \frac{1}{K_1}$

Add equation

(b) $K_1^1 = K_2$

Multiply equation (c) by (c) $K_3^1 = k_3^3$

Add (a), (b) & (c)

$$K_c^1 = \frac{K_2 \times K_3^3}{K_1} = \frac{1.6 \times 10^{12} \times 1 \times 10^{-39}}{4 \times 10^5}$$

$\Rightarrow 4 \times 10^{-33}$

60. **(36)** One-unit cell of hcp contains = 18 voids

No. of voids in 0.02 mol of hcp

$$\frac{18}{6} \times 6.02 \times 10^{23} \times 0.02$$
$$\approx 3.6 \times 10^{22}$$
$$\approx 36 \times 10^{21}$$

61. (a, c, d)

B ⇒ (~ A)VB

A	B	~ A	~ AVB	B ⇒ (~A) VB
T	T	F	T	T
T	F	F	F	T
F	T	T	T	T
F	F	T	T	T

A ⇒ B	~A ⇒ B	B ⇒ A ⇒ B	A ⇒ ((~A) ⇒ B)	B ⇒ ((~A) ⇒ B)
T	T	T	T	T
F	T	T	T	T
T	T	T	T	T
T	F	T	T	T

62. (d)

$$I = \int_1^2 \left(\frac{t^4+1}{t^6+1}\right) dt$$
$$\Rightarrow \int \frac{t^4+1-t^2+t^2}{(t^2+1)(t^4-t^2+1)} dt$$
$$\Rightarrow \int \frac{(t^4-t^2+1)+t^2}{(t^2+1)(t^4-t^2+1)} dt$$
$$\Rightarrow \int_1^2 \left[\frac{t^4-t^2+1}{(t^2+1)(t^4-t^2+1)} + \frac{t^2}{t^6+1}\right] dt$$
$$\Rightarrow \int_1^2 \frac{1}{t^2+1} dt + \frac{1}{3}\int_1^2 \frac{3t^2}{(t^3)^2+1} dt$$
$$\Rightarrow \left[\tan^{-1} t + \frac{1}{3}\tan^{-1}(t^3)\right]_1^2$$
$$\Rightarrow \tan^{-1} 2 + \frac{1}{3}\tan^{-1}(8) - \tan^{-1}(a) - \frac{1}{3}\tan^{-1}(a)$$
$$\Rightarrow \tan^{-1} 2 + \frac{1}{3}\tan^{-1} 8 - \frac{\pi}{4} - \frac{\pi}{4}\cdot\frac{1}{3}$$
$$\Rightarrow \tan^{-1} 2 + \frac{1}{3}\tan^{-1} 8 - \frac{3\pi+\pi}{12}$$
$$\Rightarrow \tan^{-1} 2 + \frac{1}{3}\tan^{-1} 8 - \frac{\pi}{3}$$

63. (c)

$$\cos^2 2x - 2\left(\frac{1-\cos 2x}{2}\right)^2 - (1+\cos 2x) = \lambda$$

$$\Rightarrow \cos^2 2x - 2\left(\frac{1-\cos^2 2x - 2\cos 2x}{4}\right) - 1 - \cos 2x = \lambda$$

Let $\cos 2x = t$

$\Rightarrow \quad 2t^2 - 1 - t^2 + 2t - 2 - 2t = 2\lambda$

$\Rightarrow \quad t^2 - 3 = 2\lambda$

$\Rightarrow \quad t^2 = 2\lambda + 3$

$0 \le 2\lambda + 3 \le 1$

$-3 \le 2\lambda \le -2$

$\frac{-3}{2} \le \lambda \le -1$

64. (a)

Reflexive

Let $a \in \mathbb{N}$

$aa \Rightarrow 2a + 3a$ is a multiple of 5

$\Rightarrow$ 5a which is a multiple of 5

$\Rightarrow$ R is reflexive

Symmetric

Let $a, b \in N$

$a\,Rb \Rightarrow 2a + 3b = 5\lambda_1 \quad \lambda_1 \in N$

$b\,Ra \Rightarrow 2b + 3a = 5\lambda_2 \quad \lambda_2 \in N$

On Adding

$(2a + 3\,b) + (2\,b + 3a) = 5(\lambda_1 + \lambda_2)$

$5a + 5\,b = 5(\lambda_1 + \lambda_2)$

$\Rightarrow$ Both sides are multiple of 5

$\Rightarrow$ R is symmetric

Transitive

Let $a, b, c \in N$

$aR\,b \Rightarrow 2a + 3\,b = 5\lambda_1 \ldots (1)$

$bc \Rightarrow 2a + 3c = 5\lambda_2 \ldots (2)$

$2a + 5\,b + 3c = 5(\lambda_1 + \lambda_2)$

$\Rightarrow (2a + 3c) = 5(\lambda_1 + \lambda_2 - b)$

$2a + 3c$ is divisible by 5

$\Rightarrow$ ac is true

$\Rightarrow$ R is transitive

R is Equivalence Relation

65. (a)

$f(a) + 2f(b) + 3f(c) + \cdots \ldots + xf(x) = x^2 f(x) + xf(x)$

$\Rightarrow f(a) + 2f(b) + 3f(c) + \cdots \ldots + (x-1)f(x-1) = x^2 f(x)$

$x = 2\ f(a) = 2^2 f(b) \Rightarrow f(b) = \frac{1}{4}$

$x = 3\ f(a) + 2f(b) = 3^2 f(c)$

$\Rightarrow f(c) = \frac{1}{9}\left(1 + \frac{2}{4}\right) = \frac{1}{9} \times \frac{3}{2} = \frac{1}{6}$

$x = 4\ f(a) + 2f(b) + 3f(c) = 4^2 f(d)$

$\Rightarrow f(d) = \left(1 + \frac{1}{2} + \frac{1}{2}\right) \cdot \frac{1}{16} \Rightarrow f(d) = \frac{1}{8}$

$x = 5\ f(a) + 2f(b) + 3f(c) + 4f(d) = 5^2 f(5)$

$f(5) = \left(1 + \frac{1}{2} + \frac{1}{2} + \frac{1}{2}\right)\frac{1}{25} = \frac{5}{2} \cdot \frac{1}{25} = \frac{1}{10}$

In general $f(x) = \frac{1}{2x}$

$\therefore \frac{1}{f(2022)} + \frac{1}{f(2028)}$

$\frac{1}{\frac{1}{2 \times 2022}} + \frac{1}{\frac{1}{2 \times 2028}}$

$\Rightarrow 2[2022 + 2028]$

$\Rightarrow 2 \times 4050$

$\Rightarrow 8100$

66. (d)

$\vec{r} \times \vec{b} + \vec{b} \times \vec{c} = 0$

$\Rightarrow \vec{r} \times \vec{b} - \vec{c} \times \vec{b} = 0$

$\Rightarrow (\vec{r} - \vec{c}) \times \vec{b} = 0$

$\vec{r} - \vec{c} \parallel \vec{b}$

$\vec{r} - \vec{c} = \lambda\vec{b}$

$\vec{r} = \lambda\vec{b} + \vec{c}$

$= \lambda(i + j + k) + (7i - 3j + 4k)$

$= i(\lambda + 7) + j(\lambda - 3) + k(\lambda + 4)$

$\vec{r} \cdot \vec{a} = 0$

$\Rightarrow (7 + \lambda) + 2(\lambda + 4) = 0$

$\Rightarrow 3\lambda = -15 \Rightarrow \lambda = -5$

$\therefore \vec{r} = 2i - 8j - k$

$\vec{r} \cdot \vec{c} = (2i - 8j - k) \cdot (7i - 3j + 4k)$

$= 14 + 24 - 4 = 34$

67. (d)

$L_1 = \frac{x-1}{2} = \frac{y+8}{-7} = \frac{z-4}{5} = \lambda$

$L_2 \frac{x-1}{2} = \frac{y-2}{1} = \frac{z-6}{-3} = \mu$

S.D. $= \left|\frac{(\vec{b}-\vec{a})\cdot(\vec{b}_1\times\vec{b}_2)}{|\vec{b}_1\times\vec{b}_2|}\right|$ $\quad \vec{a} = i - 8j + 4k$

$\vec{b} = i + 2j + 6k$

$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} i & j & k \\ 2 & -7 & 5 \\ 2 & 1 & -3 \end{vmatrix}$

$= i(21-5) - j(-6-10) + k(2+14)$

$= 16i + 16j + 16k$

$|\vec{b}_1 \times \vec{b}_2| = |16(i + j + k)|$

$= 16 \times \sqrt{3}$

$\vec{b} - \vec{a} = (10j + 2k)$

S.D. $= \left|\frac{(10j+2k)\cdot 16(i+j+k)}{16\sqrt{3}}\right|$

$= \left|\frac{16(10+2)}{16\sqrt{3}}\right| = \frac{12}{\sqrt{3}} \Rightarrow \frac{12}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$

$= \frac{12\sqrt{3}}{3} = 4\sqrt{3}$

68. (b)

69. (a)

Let $x = \frac{1}{t}$

$$dx = -\frac{1}{t^2}dt$$

$$I = \int_2^{1/2} \frac{\tan^{-1}\left(\frac{1}{t}\right)}{\frac{1}{t}} \times -\frac{1}{t^2}dt$$

$$\Rightarrow \int_{1/2}^{2} \frac{\cot^{-1}(t)}{t} dt$$

$$2I = \int_{1/2}^{2} \frac{\tan^{-1} x + \cot^{-1} x}{x} dx$$

$$\Rightarrow \int_{1/2}^{2} \frac{\pi/2}{x} dx$$

$$\Rightarrow \frac{\pi}{2}[\ell n x]_{1/2}^{2}$$

$$\Rightarrow \frac{\pi}{2}\left(\ell n 2 - \ell n \frac{1}{2}\right)$$

$$\Rightarrow \frac{\pi}{2}(\ell n 2 + \ell n 2)$$

$$2I = \pi \ell n 2$$

$$\Rightarrow I = \frac{\pi}{2} \ell n 2$$

70. (c)

The words start from G $\quad \lfloor 4$

The words start from H $\quad \lfloor 4 \quad 24\times 3$

The words start from O $\quad \lfloor 4$

T

Start form TG $\quad \lfloor 3$

TH $\quad \lfloor 3 \quad 6\times 2$

TO

TOG $\lfloor 2$ $\quad 2\times 2$

TOH $\lfloor 2$

TOU

TOUG $\lfloor 1$ $\quad 1$

89

71. (a)

$$|A| = \begin{vmatrix} e^t & e^{-t}(s-2c) & e^{-t}(-2s-c) \\ e^t & e^{-t}(2s+c) & e^{-t}(s-2c) \\ e^t & e^{-t}c & e^{-t}s \end{vmatrix}$$

$$\Rightarrow = e^t \cdot e^{-t} \cdot e^{-t} \begin{vmatrix} 1 & s-2c & -2s-c \\ 1 & 2s+c & s-2c \\ 1 & c & s \end{vmatrix}$$

$R_1 \to R_1 - R_2 \& R_2 \to R_2 - R_3$

$$= e^t \begin{vmatrix} 0 & -s-3c & -3s-c \\ 0 & 2s & -2c \\ 1 & c & s \end{vmatrix}$$

$$\Rightarrow e^{-t}[1(2sc + 6c^2 + 6s^2 + 2sc)]$$

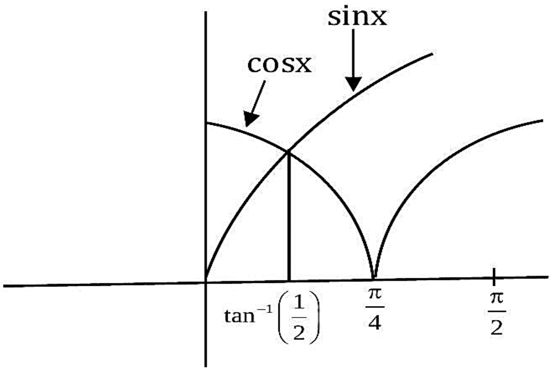
$$\Rightarrow e^{-t}[4sc + 6(c^2 + s^2)] = e^{-t}(6 + 2\sin 2t)$$

$\because 2\sin 2t \in [-2,2]$

$\therefore e^{-t}(6 + 2\sin 2t) \neq 0\ \forall t \in R$

72. (d)

$|\cos x - \sin x|$



A = area under the curve $y = \sin x$& above the curve $|\cos x - \sin x|$ $A = \int_0^{\pi/2}(\sin x - |\cos x - \sin x|)dx$

When 0 to $\frac{\pi}{4}$

$|\cos x - \sin x| = \cos x - \sin x$

$\sin x = \cos x - \sin x$

$2\sin x = \cos x$

$\tan x = \frac{1}{2}$

$x = \tan^{-1}\left(\frac{1}{2}\right)$

$A = \int_{\tan^{-1}(1/2)}^{\pi/4}\{\sin x - (\cos x - \sin x)\}dx + \int_{\pi/4}^{\pi/2}\{\sin x + (\cos x - \sin x)\}dx$ when $\frac{\pi}{4}$ to $\frac{\pi}{2}$

$|\cos x - \sin x| = \sin x - \cos x$

$\sin x = \sin x - \cos x$

$\cos x = 0$

$x = \frac{\pi}{2}$

$$\Rightarrow \int_{\tan^{-1}\left(\frac{1}{2}\right)}^{\pi/4}(2\sin x - \cos x)dx + \int_{\pi/4}^{\pi/2}\cos x dx$$

$$\Rightarrow (-2\cos x - \sin x)_{\tan^{-1}\left(\frac{1}{2}\right)}^{\pi/4} + (\sin x)_{\pi/4}^{\pi/2}$$

$$\Rightarrow \left(-2 \times \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}\right) - \left\{-2\cos\left(\tan^{-1}\frac{1}{2}\right) - \sin\left(\tan^{-1}\frac{1}{2}\right)\right\} + \left(1 - \frac{1}{\sqrt{2}}\right)$$

$$\Rightarrow -\frac{3}{\sqrt{2}} + 2 \times \frac{2}{\sqrt{5}} + \frac{1}{\sqrt{5}} + 1 - \frac{1}{\sqrt{2}} = \frac{-4}{\sqrt{2}} + \frac{5}{\sqrt{5}} + 1 = -2\sqrt{2} + \sqrt{5} + 1$$

73. (b) Nos div. by 3

102,105,108 999

A.P. $a = 102 \quad d = 3 \; \ell = 999$

$$n = \frac{\ell - a}{d} + 1 = \frac{999 - 102}{3} + 1 = 300$$

Numbers div. by 4 100,104,108

A.P. $a = 100 \quad d = 4 \; \ell = 996$

$$n = \frac{996 - 100}{4} + 1 = \frac{896}{4} + 1 = 224 + 1 = 225$$

Numbers div. by 12

108, 120, 996

A.P. $a = 108 \quad d = 12 \; \ell = 996$

$$n = \frac{996 - 108}{12} + 1 = \frac{888}{12} + 1 = 74 + 1 = 75$$

Numbers div. by 48

144, 192, 960

A.P. $a = 144 \quad d = 48 \; \ell = 960$

$$n = \frac{996 - 144}{12} + 1 = \frac{816}{48} + 1 = 17 + 1 = 18$$

$\therefore$ No. Div. by 354 but not by 48

$300 + 225 - 75 - 18$

$= 450 - 18 = 432$

74. (a)

Let $\frac{x-1}{1} = \frac{y-2}{2} = \frac{z+3}{1} = \lambda$

$P(\lambda + 1, 2\lambda + 2, \lambda - 3)$

$\& \frac{x - a}{2} = \frac{y + 2}{3} = \frac{z - 3}{1} = \mu$

$P(2\mu + a, 3\mu - 2, \mu + 3)$

$\lambda + 1 = 2\mu + a$	$2\lambda + 2 = 3\mu - 2$	$\lambda - 3 = \mu + 3$
$22 + 1 = 2 \times 16 + a$	$2(\mu + 6) + 2 = 3\mu - 2$	$\lambda = \mu + 6$
$a = 23 - 32$	$2\mu + 12 = 3\mu - 4$	
$a = -9$		$\mu = 16$
		$\therefore \lambda = 22$

$\therefore P(23,46,19)$

Plane is $z = a$

$z = -9$

The distance of p from $z = -9$ is $19 - (-9) = 28$

75. (b)

D.E. $\frac{dy}{dx} + \frac{1}{x\log x} y = x$

Linear diff. eq $q^{dy}\frac{dy}{dx} + py = Q$

$$\text{If } = e^{\int Pt}$$
$$= e^{\int \frac{1}{x\log x} d} \log x = t$$
$$\frac{1}{x} dx = dt$$
$$= e^{\int \frac{1}{t} t} = e^{\ln t} = t$$
$$\text{I.F. } = \ell n x$$

Solution of DE.

$$y \cdot \text{ If } = \int q(\text{If}) dx + c$$
$$y \cdot \ell n x = \int x \cdot (\ell n x) dx + c$$
$$= \ln x \cdot \frac{x^2}{2} \ln x - \frac{x^2}{4} + C$$

At $x = 2, y = 2$

$$2\ell n 2 = \frac{4}{2} \ln 2 - \frac{4}{4} + C \Rightarrow C = 1$$
$$\therefore y \ell \ln x = \frac{x^2}{2} \ell n x - \frac{x^2}{4} + 1$$

At $x = e$

$$y(e) \ln e = \frac{e^2}{2} \ell n e - \frac{e^2}{4} + 1$$
$$y(e) = \frac{e^2}{4} + 1$$

76. (c)
77. (b)
78. (b) Let $\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$

$\vec{C} \cdot (i + j + k) = 4$

$\vec{c} \cdot \vec{a} \times \vec{b}$

$C_1 + C_2 + C_3 = 4$

$$\begin{vmatrix} C_1 & C_2 & C_3 \\ 4 & 3 & 0 \\ 3 & -4 & 5 \end{vmatrix} = -25$$

$\Rightarrow c_1(15-0) - c_2(20-0) + c_3(-16-9) = -25$

$\Rightarrow 15c_1 - 20c_2 - 25c_3 = -25$

$\Rightarrow 3c_1 - 4c_2 - 5c_3 = -5$

Proj. of $\vec{c}$ on $\vec{a} = \frac{\vec{a}\cdot\vec{c}}{|\vec{a}|} = 1$

$$\Rightarrow \frac{(4\hat{i}+3\hat{j})(c_1\hat{i}+c_2\hat{j}+c_3\hat{k})}{\sqrt{16+9}} = 1$$

$\Rightarrow 4c_1 + 3c_2 = 5$

$\Rightarrow 4c_1 = 5 - 3c_2$

$$\Rightarrow c_1 = \frac{5-3c_2}{4}$$

$Eq^n \cdot (1)\&(3)$

$$\frac{5-3c_2}{4} + C_2 + C_3 = 4$$

$Eq^n \cdot (2)\&(3)$

$5 - 3c_2 + 4c_2 + 4c_3 = 16$

$$3\left(\frac{5-3c_2}{4}\right) - 4c_2 - 5c_3 = -5$$

$C_2 + 4c_3 = 11$

$15 - 9c2 - 16c2 - 20c_3 = -20$

$Eq^n \cdot (4)\&(5)$

$C_2 = 11 - 4c_3$

$C_2 = 11 - 4 \times 3$

$-25c_2 - 20c_3 = -35$

$-25c_2 - 20c_3 = -35$

$= 11 - 12$

$-25(11 - 4c_3) - 20c_3 = -35$

$C_2 = -1$

$5(11-4c_3)+4c_3=7$

$55-20c_3+4c_3=7$

$c_1=\frac{5-3c_2}{4}$

$-16c_3=-48$

$C_3=3$

$=\frac{5-3(-1)}{4}$

$\vec{C}=2\hat{\imath}-\hat{\jmath}+3\hat{k}$

$C_1=2$

Projection of $\vec{c}$ on $\vec{b}=\left|\frac{\vec{c}\cdot\vec{b}}{|\vec{b}|}\right|$

$\Rightarrow\left|\frac{(2\hat{\imath}-\hat{\jmath}+3\hat{k})\cdot(3\hat{\imath}-4\hat{\jmath}+5\hat{k})}{\sqrt{9+16+25}}\right|$

$\Rightarrow\left|\frac{6+4+15}{5\sqrt{2}}\right|=\frac{25}{5\sqrt{2}}=\frac{5}{\sqrt{2}}$

79. (a)
80. (b)

$K=\frac{(1+1)^{99}}{2}=2^{98}$

$a={}^{200}C_{100}2^{100}\times\frac{1}{(\sqrt{2})^{100}}$

$a={}^{200}C_{100}2^{50}$

$\frac{{}^{200}C_{99}\cdot K}{a}=\frac{{}^{200}C_{99}\cdot 2^{98}}{{}^{200}C_{100}\cdot 2^{50}}$

$\because\frac{{}^{200}C_{99}}{{}^{200}C_{100}}=\frac{1200}{99\mid 101}\times\frac{100\mid 100}{\underline{200}}$

$=\frac{100}{101}$

$\therefore\frac{{}^{200}C_{99}\,K}{a}=\frac{100}{101}\times 2^{48}$

$=\frac{25\times 2^{50}}{101}$

$\ell=50, n=101$

$(\ell, n)=(50,101)$

81. (3000)
82. (461)

$$T=\frac{1}{2^0}+\frac{2}{2^1}+\frac{3}{2^2}+\frac{4}{2^3}+\cdots\ldots\ldots\ldots\frac{8}{2^7}$$

$$S=\frac{b_1}{2}+\frac{b_2}{2^2}+\frac{b_3}{2^3}+\frac{b_4}{2^4}+\cdots\ldots\ldots\ldots.\frac{b_{10}}{2^{10}}$$

$$\frac{S}{2}=\frac{b_1}{2^2}+\frac{b_2}{2^3}+\frac{b_3}{2^4}+\cdots\ldots\ldots\ldots.\frac{b_9}{2^{10}}+\frac{b_{10}}{2^{11}}\text{ (Subtract)}$$

$$\frac{\bar{S}}{2}=\frac{b_1}{2}+\left(\frac{b_2-b_1}{2^2}\right)+\left(\frac{b_3-b_2}{2^3}\right)+\left(\frac{b_4-b_3}{2^4}\right)+\cdots\ldots+\left(\frac{b_{10}-b_9}{2^{10}}\right)-\frac{b_{10}}{2^{11}}$$

$$\frac{S}{2}=\frac{b_1}{2}+\frac{a_1}{2^2}+\frac{a_2}{2^3}+\frac{a_3}{2^4}+\cdots\ldots\ldots\ldots\ldots\ldots\ldots..\frac{a_9}{2^{10}}-\frac{b_{10}}{2^{11}}$$

$$\Rightarrow S=b_1+\frac{a_1}{2}+\frac{a_2}{2^2}+\frac{a_3}{2^3}+\cdots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\frac{a_9}{2^{10}}-\frac{b_{10}}{2^{10}}$$

$$S=\left(b_1-\frac{b_{10}}{2^{10}}\right)+\left(\frac{a_1}{2}+\frac{a_2}{2^2}+\cdots\ldots..\frac{a_9}{2^9}\right)$$

$$\Rightarrow\frac{S}{2}=\left(\frac{b_1}{2}-\frac{b_{10}}{2^{11}}\right)+\left(\frac{a_1}{2^2}+\frac{a_2}{2^3}+\cdots\ldots..\frac{a_8}{2^9}+\frac{a_9}{2^{10}}\right)\text{ (Subtract)}$$

$$\frac{S}{2}=\frac{b_1}{2}-\frac{b_{10}}{2^{11}}+\left(\frac{a_1}{2}-\frac{a_9}{2^{10}}\right)+\left(\frac{1}{2^2}+\frac{2}{2^3}+\cdots\ldots\ldots\ldots\ldots\frac{8}{2^9}\right)$$

$$=\frac{b_1}{2}+\frac{a_1}{2}-\left(\frac{b_{10}+2a_9}{2^{11}}\right)+\frac{T}{4}$$

from

Now $2s=2(a_1+b_1)-\left(\frac{b_{10}+2a_9}{2^9}\right)+T$

$$2S-T=2(a_1+b_1)-\left(\frac{b_{10}+2a_9}{2^9}\right)$$

$$2^7(2S-T)=2^8(a_1+b_1)-\frac{b_{10}+2a_9}{4}$$

$\because a_n-an_{-1}=n-1$

$a_2-a_1=1$

$a_3-a_2=2$

$a_4-a_3=3$

......

$a_9-a_8=8$

$a_9-a_1=1+2+3++8$

$a_9=36+1=37$

and $b_n=b_{n-1}=a_{n-1}$

$b_{10}-b_1=a_1+a_2+a_3+\cdots\ldots.+a_9$

$b_{10} - 1 = 1 + 2 + 4 + 7 + 11 + 16 + 22 + 29 + 37$

$b_{10} - 1 = 29$

$b_{10} = 130$

$2^7(2\,S - T) = 2^8 \times (1 + 1) - \dfrac{130 + 2 \times 37}{4}$
$= 2^9 - \dfrac{102}{2}$
$= 512 - 51 = 461$

83. (10) Tangent at $y^2 = 2x$

$T: 2y = 2\left(\dfrac{x+2}{2}\right)$

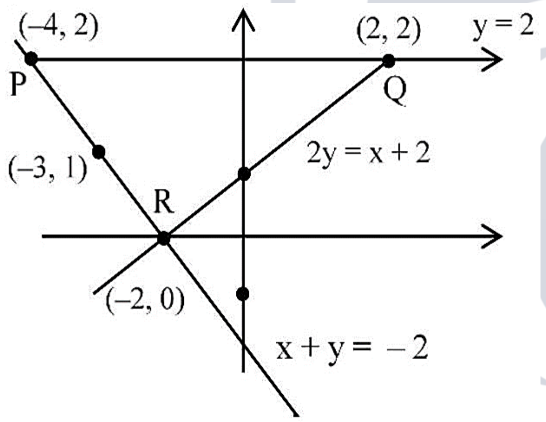
$2y = x + 2$

Tangent at $x^2 + y^2 = 4x$

$2x + 2y = \dfrac{4 \times (x+2)}{2}$

$2x + 2y = 2x + 4$

$y = 2$



$M_{PR} = -1$

Slope of $\perp^{r\ell}$ Bisector = 1

$y - 1 = 1(x + 3)$

$y = x + 3 + 1$

$y = x + 4$

$\perp^{r\ell}$ Bisector of PQ

$x = -1$

∴ Centre is

$y = -1 + 4 = 3$

$(-1,3)$

Radius: $r = \sqrt{(-1+4)^2 + (3-2)^2}$

$= \sqrt{9+1}$

$= \sqrt{10}$

$r^2 = 10$

84. (3) $\alpha_1, \alpha_2, \ldots \alpha_7$

$x^7 + 3x^5 - 13x^3 - 15x = 0$

$x(x^6 - 3x^4 - 13x^2 - 15) = 0$

$|\alpha_1| \geq |\alpha_2| \geq \cdots \geq |\alpha_7|$

$\alpha_1\alpha_2 - \alpha_3\alpha_4 + \alpha_3\alpha_6 = ?$

$\alpha_7 = 0$

$\Rightarrow x(x^6 + 3x^4 - 13x^2 - 15) = 0$

$x = 0x^6 + 3x^4 - 13x^2 - 15 = 0$

$\Rightarrow t^3 + 3t^2 - 13t - 15 = 0$

$\Rightarrow (t-3)(t^2 + 6t + 5) = 0$

$t = 3, t = -5, -1$

$x = 0, x = \pm\sqrt{3}, x = \pm\sqrt{5}i, x = \pm i$

$\alpha_1 = \sqrt{5}i$

$\alpha_2 = -\sqrt{5}i$

$\alpha_3 = \sqrt{3}$

$\alpha_4 = -\sqrt{3}$

$\alpha_5 = i$

$\alpha_6 = -i$

$\alpha_7 = 0$

$\alpha_1\alpha_2 = 5, \alpha_3\alpha_4 = 3, \alpha_5\alpha_6 = 1$

$\alpha_1\alpha_2 - \alpha_3\alpha_4 + \alpha_5\alpha_6$

$\Rightarrow 5 - 3 + 1 = 3$

85. (603)

$$\bar{x}=\frac{11+12+\cdots \ldots . .+41}{31}$$

$$=\frac{\frac{31}{2}(11+41)}{31}=\frac{52}{2}=26$$

$$\sigma^2=\frac{\sum x_i^2+\sum y_i^2}{31+31}-\bar{x}^2$$

$$=\frac{\left(\sum_{n=1}^{41} n^2-\sum_{n=1}^{10} n^2\right)+\left(\sum_{n=1}^{91} n^2-\sum_{n=1}^{60} n^2\right)}{62}-\left(\frac{31\times 26+76\times 31}{62}\right)^2$$

$$\Rightarrow \frac{\frac{41\times 42\times 83}{6}-\frac{10\times 11\times 21}{6}+\frac{91\times 92\times 183}{6}-\frac{60\times 61\times 121}{6}}{62}-(51)^2$$

$$\Rightarrow \frac{7(41\times 83-55)+61(91\times 46-1210)}{62}$$

$$\Rightarrow \frac{7(3403-55)+61(4186-1210)}{62}$$

$$\Rightarrow \frac{7\times 3348+61\times 2976}{62}-2601$$

$$\Rightarrow 3306-2601=705 \Rightarrow \sigma^2=705$$

$$\therefore |\bar{x}+\bar{y}-\sigma^2|=|26+76-705|=603$$

86. (-6)

$(1,-3)$ is on the curve

$$\therefore -3=\frac{1-a}{(1+b)(1-2)} \Rightarrow -3=\frac{1-a}{(-1)(1+b)}$$

$$\Rightarrow 3+3b=1-a \Rightarrow a+3b=-2$$

$$a=-2-3b$$

$$\ell ny=\ell n(x-a)-\ln(x+b)-\ln(x-2)$$

$$\frac{1}{y}\frac{dy}{dx}=\frac{1}{x-a}-\frac{1}{x+b}-\frac{1}{x-2}$$

$$\left.\frac{dy}{dx}\right|_{(1,-3)}=-3\left(\frac{1}{1-a}-\frac{1}{1+b}-\frac{1}{1-2}\right)=-4$$

$$\Rightarrow\left(\frac{1}{1+2+3b}-\frac{1}{1+b}+1\right)=\frac{4}{12}=\frac{1}{3}$$

$$\bar{y}=\frac{61+62+63+\cdots..+91}{31}$$

$$=\frac{\frac{31}{2}(61+91)}{31}=\frac{152}{2}=76$$

$$\Rightarrow \frac{1}{3(b+1)}-\frac{1}{b+1}=\frac{1}{3}-1$$

$$\Rightarrow \frac{1-3}{3(b+1)}=-\frac{2}{3}$$

$\Rightarrow b+1=3$

$b=2$

$a=-2-3\,b\,a+b$

$a=-2-3\times 2\ \Rightarrow -8+2$

$a=-2-6=-8\ \Rightarrow -6$

87. (5)

A be a symmetric matrix such that $|A|=2$ and $\begin{bmatrix}2 & 1\\3 & 3\end{bmatrix}2\Big]\,A=\begin{bmatrix}1 & 2\\\alpha & \beta\end{bmatrix}$

$A=\begin{bmatrix}a & b\\b & d\end{bmatrix}$ $|A|=ad-b^2=2$

$$\begin{bmatrix}2 & 1\\3 & 3\end{bmatrix}\begin{bmatrix}a & b\\b & d\end{bmatrix}=\begin{bmatrix}1 & 2\\\alpha & \beta\end{bmatrix}$$

$$\begin{bmatrix}2a+b & 2b+d\\3a+3/2b & 3b+3/2d\end{bmatrix}=\begin{bmatrix}1 & 2\\\alpha & \beta\end{bmatrix}$$

$2a+b=1\ 2\,b+d=2$

$b=1-2a\ d=2-2\,b$

$=2-2(1-2a)$

$ad-b^2=2$

$=2-2+4a$
a. $4a-(1-2a)^2=2$

Now $\alpha=3a+\frac{3}{2}b$

$\Rightarrow 4a^2-1-4a^2+4a=2$

$$=\frac{9}{4}+\frac{3}{2}\cdot\left(\frac{-1}{2}\right)$$

$4a=3$

$= \frac{9-3}{4} = \frac{6}{4} = \frac{3}{2}$

$a = \frac{3}{4}$

$b = 1 - 2 \times \frac{3}{4}$

$\beta = 3b + \frac{3}{2}d$

$= \frac{-1}{2}$

$3 \times \left(\frac{-1}{2}\right) + \frac{3}{2} \times 3$

$d = 4 \times \frac{3}{4} = 3$

$\frac{-3+9}{2} = 3$

$A = \begin{bmatrix} 3/4 & -1/2 \\ -1/2 & 3 \end{bmatrix}$ $s = \frac{3}{4} + 3 = \frac{15}{4}$

$\frac{Bs}{\alpha^2} = \frac{3 \times \frac{15}{4}}{\frac{9}{4}} = 5$

88. (14)

$\alpha = 8 - 14iA = \left\{z \in c: \frac{\alpha z - \bar{\alpha}\bar{z}}{z^2 - (\bar{z})^2 - 112i} = 1\right\}$

$z^2 - \bar{z}^2$

$\Rightarrow (z + \bar{z})(z - \bar{z}) \Rightarrow 2x \cdot 2yi = 4xyi$

$\alpha z = (8 - 14i)(x + iy)$

$= 8x - 8iy + 14xi + 14y$

$= (8x + 14y) + i(8y - 14x)$

$\overline{\alpha z} = (8 + 14i)(x - iy)$

$= 8x - 8iy + 14ix + 14y$

$= (8x + 14y) + i(14x - 8y)$

$\frac{\alpha z - \overline{\alpha z}}{z^2 - \bar{z}^2 - 112i} = 1$

$\Rightarrow \frac{(16y - 28x)i}{4xyi - 112i} = 1$

$\Rightarrow 4y - 7x = xy - 28$

$\Rightarrow 4y - 7x - xy + 28 = 0$

$\Rightarrow y(4 - x) - 7(x - 4) = 0$

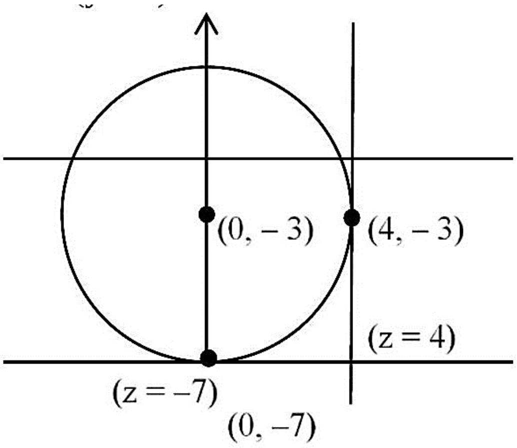
$(x - 4)(-y - 7) = 0$

$x = 4y = -7$

$z = 4$ or $z = -7i$

$B \Rightarrow |z + 3i| = 4$

$x^2 + (y - 3)^2 = 16$



$\sum Re(z) - Im(z)$
$= (0 + 4) - (-7 - 3)$
$= 4 + 10 = 14$

89. (11)

$$(x - 2)^2 + (y - 3)^2 = 16$$
$$x^2 + y^2 - 4x - 6y - 3 = 0$$

let the chord of contact w.r. to the point $S(\alpha, \beta)$ is

$T = 0$

$\alpha x + \beta y - 2(x + \alpha) - 3(y + \beta) - 3 = 0$

$x(\alpha - 2) + y(\beta - 3) - 2\alpha - 3\beta - 3 = 0$

Comparison with $x + y = 3$

$\frac{\alpha - 2}{1} = \frac{\beta - 3}{1} = \frac{-2\alpha - 3\beta - 3}{-3}$

$\alpha - 2 = \beta - 3 \quad -3\beta + 9 = -2\alpha - 3\beta - 3$

$\alpha - \beta = -1 \; 2\alpha = -3 - 9$

$\beta = \alpha + 1 \ \alpha = \frac{-12}{2}$

$= -6 + 1 = -5 \ \alpha = -6$

$4\alpha - 7\beta$

$4(-6) - 7(-5)$

$\Rightarrow -24 + 35 = 11$

90. (9)

$a_1 = 4$
GP $4, 4r_1, 4r_1^2$ - -
$b_1 = 4$
GP $4, 4r_2, 4r_2^2$ - -
$C_2 = a_2 + b_2$
$C_3 = a_3 + b_3$
$5 = 4r_1 + 4r_2$
$\frac{13}{4} = 4r_1^2 + 4r_2^2$
$\frac{5}{4} = r_1 + r_2 \ldots (1)$
$r_1^2 + r_2^2 = \frac{13}{16}.$

$\frac{25}{16} = r_1^2 + r_2^2 + 2r_1r_2$

$\frac{25}{16} = \frac{13}{16} + 2r_1r_2$

$\Rightarrow r_1r_2 = \frac{12}{16 \times 2} = \frac{3}{8}$

Now $r_1 + \frac{3}{8r_1} = \frac{5}{4}$

$8r_1^2 + 3 = 10r_1$
$\Rightarrow 8r_1^2 - 10r_1 + 3 = 0$

$r_1 = \frac{3}{4}, r_1 = \frac{1}{2}$

$r_2 = \frac{1}{2} r_2 = \frac{3}{4}$

$\because r_1 < r_2$

$r_1 = \frac{1}{2}$

$r_2 = \frac{3}{4}$

Now $C_k = a_k + b_k$

$\sum_{k=1}^{\infty} C_k = \dfrac{4}{1-r_1} + \dfrac{4}{1-r_2}$

$= \dfrac{4}{1-\frac{1}{2}} + \dfrac{4}{1-\frac{3}{4}}$

$= 8 + 16 = 24$

$\sum_{k=1}^{\infty} C_k - (12a_6 + 8\,b_4) \Rightarrow 24 - \left\{12 \times 4\left(\dfrac{1}{2}\right)^5 + 8 \times 4\left(\dfrac{3}{4}\right)^3\right\}$

$= 24 - \left(12 \times \dfrac{1}{8} + 8 \times \dfrac{27}{16}\right)$

$= 24 - \left\{\dfrac{3}{2} + \dfrac{27}{2}\right\}$

$= 24 - 15$

$= 9$