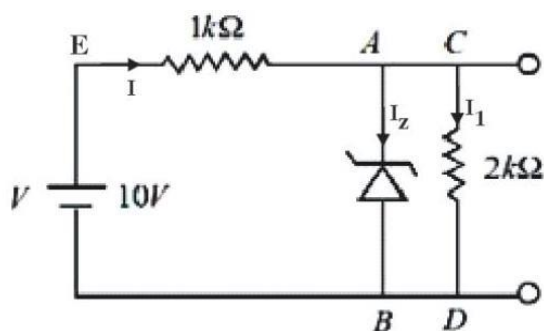


1. (b)



$$V_z = 3 \text{ V}$$

Let potential at B = 0 V

Potential at E (V_E) = 10 V

$$V_C = V_A = 3 \text{ V}$$

$$I_z + I_1 = I$$

$$I = \frac{10 - 3}{1000} = \frac{7}{1000} \text{ A}$$

$$I_1 = \frac{3}{2000} \text{ A}$$

$$\text{Therefore } I_z = \frac{7 - 1.5}{1000} = 5.5 \text{ mA}$$

2. (b) Given that

Current $I = I_0 + \beta t$

$$I_0 = 20 \text{ A}$$

$$\beta = 3 \text{ A/s}$$

$$I = 20 + 3t$$

$$\frac{dq}{dt} = 20 + 3t$$

$$\int_0^q dq = \int_0^{20} (20 + 3t) dt$$

$$q = \int_0^{20} 20 dt + \int_0^{20} 3t dt$$

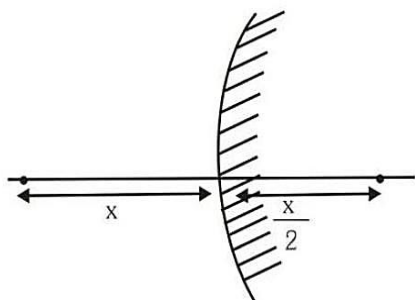
$$q = \left[20t + \frac{3t^2}{2} \right]_0^{20} = 1000 \text{ C}$$

3. (c) Surface tension will be less as temperature increases

$$h = \frac{2 T \cos \theta}{\rho g r}$$

Height of capillary rise will be smaller in hot water and larger in cold water.

4. (a)



Given $R = 30 \text{ cm}$

$f = R/2 = +15 \text{ cm}$

Magnification (m) = $\pm \frac{1}{2}$

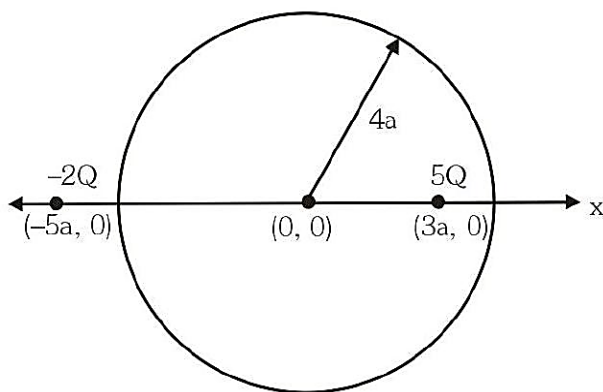
For convex mirror, virtual image is formed for real object.

Therefore, m is +ve

$$\frac{1}{2} = \frac{f}{f - u}$$

$u = -15 \text{ cm}$

5. (b)



$5Q$ charge is inside the spherical region

$$\text{flux through sphere} = \frac{5Q}{\epsilon_0}$$

6. (b) S_1 in first $(p - 1)$ sec

S_2 in first p sec

$$S_1 = \frac{1}{2} a(p - 1)^2$$

$$S_2 = \frac{1}{2} a(p)^2$$

$$S_1 + S_2 = \frac{1}{2}at^2$$

$$(p-1)^2 + p^2 = t^2$$

$$t = \sqrt{2p^2 + 1 - 2p}$$

7. (c) Given $U = 2x^2 + 3y^3 + 2z$

$$F_x = -\frac{\partial U}{\partial x} = -4x$$

At $x = 1$ magnitude of F_x is 4N

8. (a)

$$R = \frac{V}{I}$$

According to error analysis

$$\frac{dR}{R} = \frac{dV}{V} + \frac{dI}{I}$$

$$\frac{dR}{R} = \frac{5}{200} + \frac{0.2}{20}$$

$$\frac{dR}{R} = \frac{7}{200}$$

$$\% \text{ error } \frac{dR}{R} \times 100 = \frac{7}{200} \times 100 = 3.5\%$$

9. (c) Given $m = 100 \text{ kg}$

$$s = 10 \text{ m}$$

$$\mu = 0.4$$

$$\text{As } f = \mu mg = 0.4 \times 100 \times 10 = 400 \text{ N}$$

$$\text{Now } W = f.s = 400 \times 10 = 4000 \text{ J}$$

10. (c) Ampere - Maxwell law

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 i_c + \mu_0 \epsilon_0 \frac{d\phi_E}{dt}$$

$$\text{Faraday law } \oint \vec{E} \cdot d\vec{l} = \frac{d\phi_B}{dt}$$

$$\text{Gauss' law for electricity } \oint \vec{E} \cdot d\vec{A} = \frac{Q}{\epsilon_0}$$

$$\text{Gauss' law for magnetism } \oint \vec{B} \cdot d\vec{A} = 0$$

11. (a) Given $m_1 = m_2$

$$\text{and } \frac{r_1}{r_2} = \frac{3}{4}$$

$$\text{As centripetal force } F = \frac{mv^2}{r}$$

In order to have constant (same in this question) centripetal force

$$F_1 = F_2$$

$$\frac{m_1 v_1^2}{r_1} = \frac{m_2 v_2^2}{r_2}$$

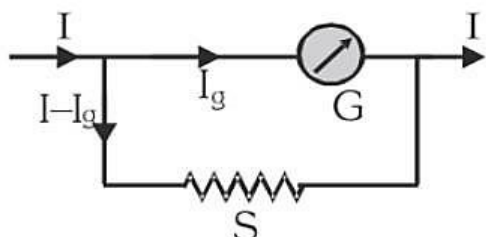
$$\Rightarrow \frac{v_1}{v_2} = \sqrt{\frac{r_1}{r_2}} = \frac{\sqrt{3}}{2}$$

12. (c) Given $G = 10\Omega$

$$I_g = 3 \text{ mA}$$

$$I = 8 \text{ A}$$

In case of conversion of galvanometer into ammeter.



$$\text{We have } I_g G = (I - I_g)S$$

$$S = \frac{I_g G}{I - I_g}$$

$$S = \frac{(3 \times 10^{-3})10}{8 - 0.003} = 3.75 \times 10^{-3}\Omega$$

13. (b) For photon

$$E_p = \frac{hc}{\lambda_p} \Rightarrow \lambda_p = \frac{hc}{E_p}$$

For electron

$$\lambda_e = \frac{h}{m_e v_e} = \frac{h v_e}{2K_e}$$

$$\text{Given } v_e = 0.25c$$

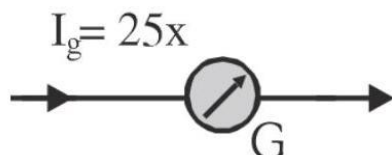
$$\lambda_e = \frac{h \times 0.25c}{2 K_e} = \frac{hc}{8 K_e}$$

Also $\lambda_p = \lambda_e$

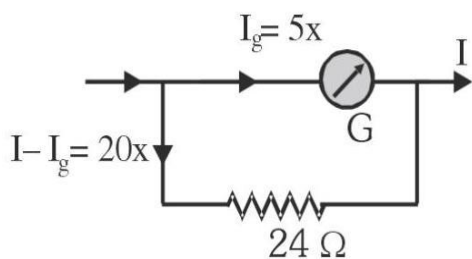
$$\frac{hc}{E_p} = \frac{hc}{8 K_e}$$

$$\frac{K_e}{E_p} = \frac{1}{8}$$

14. (b) Let x = current/division



After applying shunt



Now $5x \times G = 20x \times 24$

$$G = 4 \times 24$$

$$G = 96 \Omega$$

15. (b) $\mu_1 = 1.5$

$$\mu_m = 1.6$$

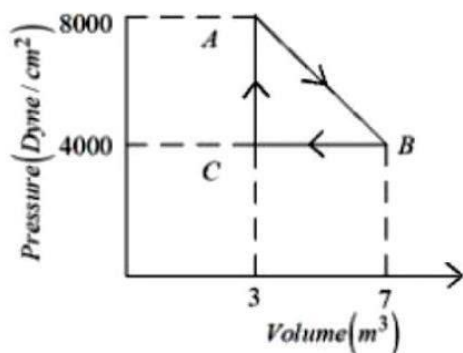
$$f_a = 20 \text{ cm}$$

$$\text{As } \frac{f_m}{f_a} = \frac{(\mu_1 - 1)\mu_m}{(\mu_1 - \mu_m)}$$

$$\frac{f_m}{20} = \frac{(1.5 - 1)1.6}{(1.5 - 1.6)}$$

$$f_m = -160 \text{ cm}$$

16. (c)



$$\text{Work done AB} = \frac{1}{2}(8000 + 6000) \text{ Dyne /cm}^2 \times$$

$$4 \text{ m}^3 = (6000 \text{ Dyne /cm}^2) \times 4 \text{ m}^3$$

$$\text{Work done BC} = -(4000 \text{ Dyne /cm}^2) \times 4 \text{ m}^3$$

$$\text{Total work done} = 2000 \text{ Dyne /cm}^2 \times 4 \text{ m}^3$$

$$\begin{aligned} &= 2 \times 10^3 \times \frac{1}{10^5} \frac{\text{N}}{\text{cm}^2} \times 4 \text{ m}^3 \\ &= 2 \times 10^{-2} \times \frac{\text{N}}{10^{-4} \text{ m}^2} \times 4 \text{ m}^3 \\ &= 2 \times 10^2 \times 4 \text{ Nm} = 800 \text{ J} \end{aligned}$$

17. (d)

18. (b) By energy conservation

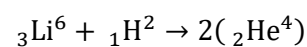
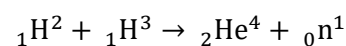
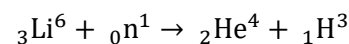
$$\frac{1}{2} CV^2 = \frac{1}{2} LI_{\text{max}}^2$$

$$I_{\text{max}} = \sqrt{\frac{C}{L}} V$$

$$= \sqrt{\frac{100 \times 10^{-6}}{6.4 \times 10^{-3}}} \times 12$$

$$= \frac{12}{8} = \frac{3}{2} = 1.5 \text{ A}$$

19. (d)



Energy released in process

$$Q = \Delta mc^2$$

$$Q = [M(\text{Li}) + M({}_1\text{H}^2) - 2 \times M({}_2\text{He}^4)] \times 931.5 \text{ MeV}$$

$$Q = [6.01690 + 2.01471 - 2 \times 4.00388] \times 931.5 \text{ MeV}$$

$$Q = 22.216 \text{ MeV}$$

$$Q = 22.22 \text{ MeV}$$

20. (a)

$$\frac{P_A V_A}{P_B V_B} = \frac{n_A R T_A}{n_B R T_B}$$

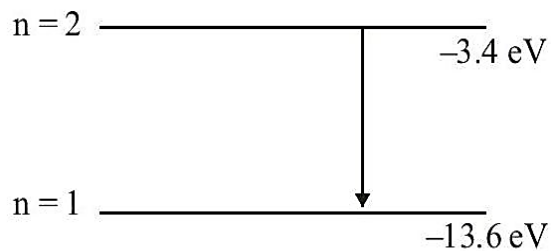
$$\text{Given } V_A = V_B$$

$$\text{And } T_A = T_B$$

$$\frac{P_A}{P_B} = \frac{n_A}{n_B}$$

$$\frac{P_A}{P_B} = \frac{1/2}{1/32} = 16$$

21. (17)



$$\Delta E = 10.2 \text{ eV}$$

$$\text{Recoil speed (v)} = \frac{\Delta E}{mc}$$

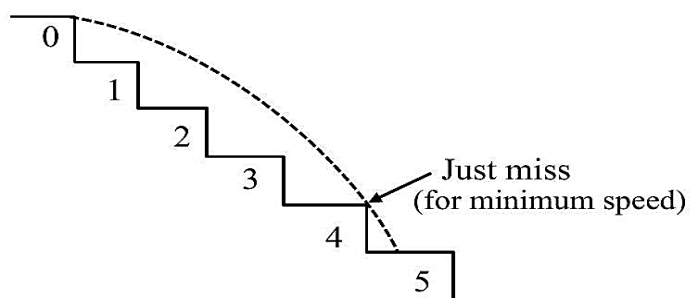
$$= \frac{10.2 \text{ eV}}{1.6 \times 10^{-27} \times 3 \times 10^8}$$

$$= \frac{10.2 \times 1.6 \times 10^{-19}}{1.6 \times 10^{-27} \times 3 \times 10^8}$$

$$v = 3.4 \text{ m/s} = \frac{17}{5} \text{ m/s}$$

$$\text{Therefore, } x = 17$$

22. (2)



The ball needs to just cross 4 steps to just hit 5th step

Therefore, horizontal range (R) = 0.4 m

$$R = u \cdot t$$

Similarly, in vertical direction

$$h = \frac{1}{2}gt^2$$

$$0.4 = \frac{1}{2}gt^2$$

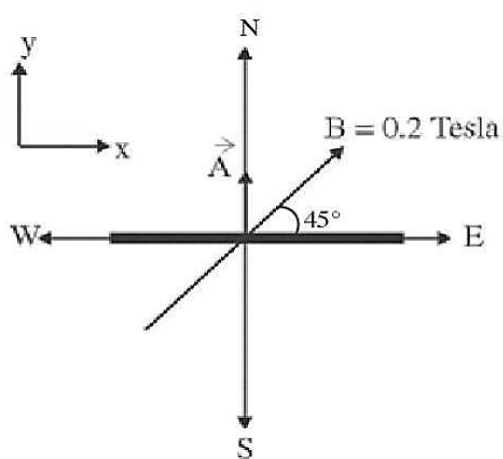
$$0.4 = \frac{1}{2}g\left(\frac{0.4}{u}\right)^2$$

$$u^2 = 2$$

$$u = \sqrt{2} \text{ m/s}$$

Therefore, $x = 2$

23. (2)



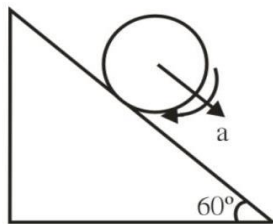
$$\vec{A} = (0.1)^2 \hat{j}$$

$$\vec{B} = \frac{0.2}{\sqrt{2}} \hat{i} + \frac{0.2}{\sqrt{2}} \hat{j}$$

Magnitude of induced emf

$$e = \frac{\Delta\phi}{\Delta t} = \frac{\vec{B} \cdot \vec{A} - 0}{1} = \sqrt{2} \times 10^{-3} \text{ V}$$

24. (10)



For rolling motion, $a = \frac{g \sin \theta}{1 + \frac{I_{cm}}{MR^2}}$

$$a = \frac{g \sin \theta}{1 + \frac{1}{2}}$$

$$= \frac{2 \times 10 \times \frac{\sqrt{3}}{2}}{3}$$

$$= \frac{10}{\sqrt{3}}$$

Therefore $x = 10$

25. (6) $V = \frac{\mu_0 M}{4\pi r^2}$

$$\Rightarrow 1.5 \times 10^{-5} = 10^{-7} \times \frac{M}{(20 \times 10^{-2})^2}$$

$$\Rightarrow M = \frac{1.5 \times 10^{-5} \times 20 \times 20 \times 10^{-4}}{10^{-7}}$$

$$M = 1.5 \times 4 = 6$$

26. (0.20)

Path difference for minima at P

$$2\sqrt{D^2 + d^2} - 2D = \frac{\lambda}{2}$$

$$\therefore \sqrt{D^2 + d^2} - D = \frac{\lambda}{4}$$

$$\therefore \sqrt{D^2 + d^2} = \frac{\lambda}{4} + D$$

$$\Rightarrow D^2 + d^2 = D^2 + \frac{\lambda^2}{16} + \frac{D\lambda}{2}$$

$$\Rightarrow d^2 = \frac{D\lambda}{2} + \frac{\lambda^2}{16}$$

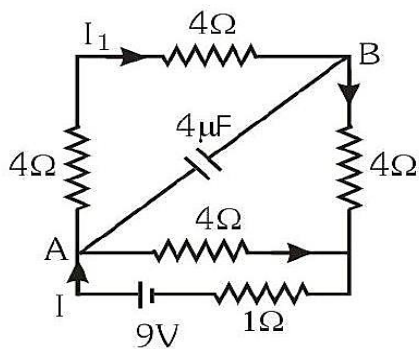
$$\Rightarrow d^2 = \frac{0.2 \times 400 \times 10^{-9}}{2} + \frac{4 \times 10^{-14}}{4}$$

$$\Rightarrow d^2 \approx 400 \times 10^{-10}$$

$$\therefore d = 20 \times 10^{-5}$$

$$\Rightarrow d = 0.20 \text{ mm}$$

27. (81)



$$I = \frac{V}{R_{eq}} \quad I = \frac{V}{R_{eq}} = \frac{9}{1 + \frac{12 \times 4}{12 + 4}} = \frac{9}{4}$$

$$I_1 = \frac{9}{4} \times \frac{4}{16} = \frac{9}{16}$$

$$V_A - V_B = I_1 \times 8 = \frac{9}{16} \times 8 = \frac{9}{2} \text{ V}$$

$$\therefore U = \frac{1}{2} \times 4 \times \frac{81}{4} \mu\text{J}$$

$$\therefore U = \frac{81}{2} \mu\text{J}$$

$$\therefore x = 81$$

28. (9) Let total energy = $E = \frac{1}{2} KA^2$

$$U = \frac{1}{2} K \left(\frac{A}{3} \right)^2 = \frac{KA^2}{2 \times 9} = \frac{E}{9}$$

$$KE = E - \frac{E}{9} = \frac{8E}{9}$$

$$\text{Ratio } \frac{\text{Total}}{\text{KE}} = \frac{E}{\frac{8E}{9}} = \frac{9}{8}$$

$$x = 9$$

$$29. (8) \quad u = 1 \text{ m/s}; a = -\frac{\sigma e}{2\epsilon_0 m}$$

$$t = 1 \text{ s}$$

$$S = -1 \text{ m}$$

$$\text{Using } S = ut + \frac{1}{2}at^2$$

$$-1 = 1 \times 1 - \frac{1}{2} \times \frac{\sigma e}{2\epsilon_0 m} \times (1)^2$$

$$\therefore \sigma = 8 \frac{\epsilon_0 m}{e}$$

$$\therefore \alpha = 8$$

30. (810)

$$F = \frac{1}{2} \rho (v_1^2 - v_2^2) A$$

$$F = \frac{1}{2} \times 1.2 \times (70^2 - 65^2) \times 2$$

$$= 810 \text{ N}$$

31. (c) First ionisation energy increases along the period. Along the period Z increases which outweighs the shielding effect

32. (d)

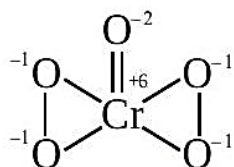
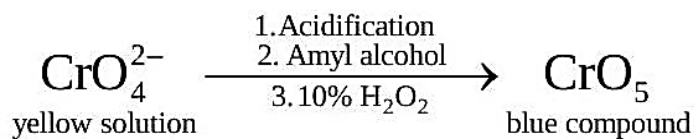
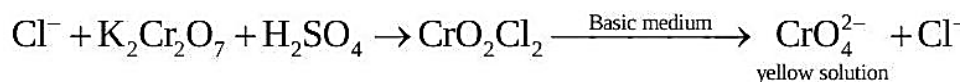
Ziegler catalyst → Titanium

Blood pigment → Iron

Wilkinson catalyst → Rhodium

Vitamin B₁₂ → Cobalt

33. (a)



34. (b) The difference in energy between the actual structure and the lowest energy resonance structure for the given compound is known as resonance energy.

35. (a)

Gr-14	EN
C	2.5
Si	1.8
Ge	1.8
Sn	1.8
Pb	1.9

The electronegativity values for elements from Si to Pb are almost same. So Statement I is false.

36. (a)

$$\text{Rb} = [\text{Kr}]5s^1$$

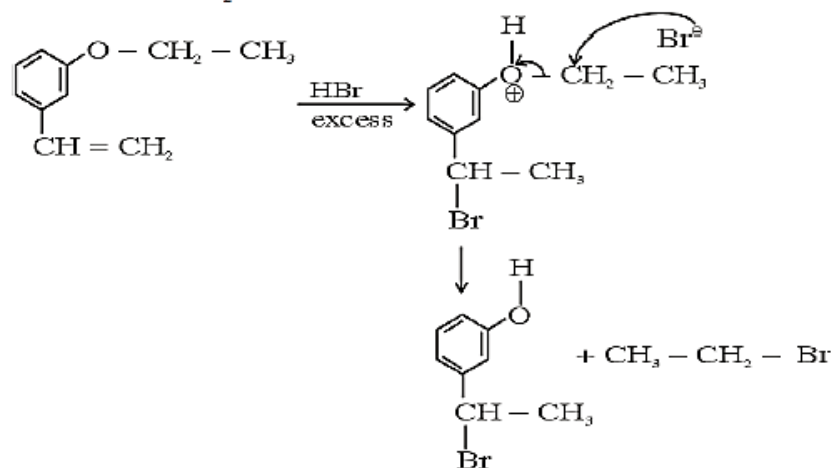
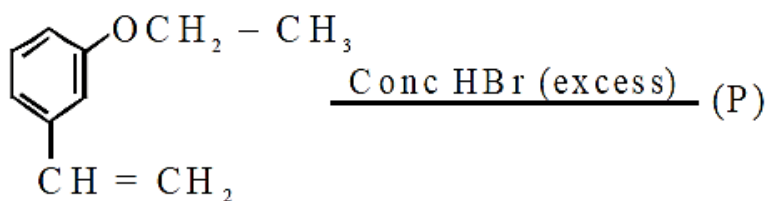
$$n = 5$$

$$l = 0$$

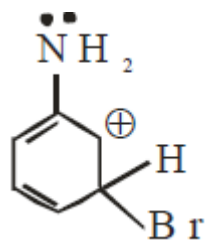
$$m = 0$$

$$s = +1/2 \text{ or } -1/2$$

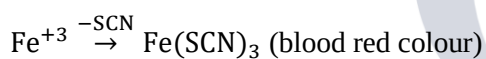
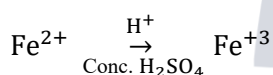
37. (d)



38. (c) Since $-\text{NH}_2$ group is o/p directing hence arenium ion will not be formed by attack at meta position i.e.



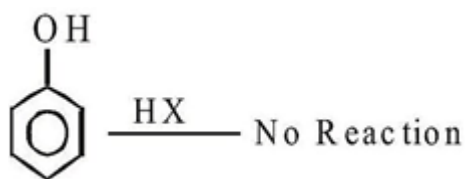
39. (c)



Appearance of blood red colour indicates presence of both nitrogen and sulphur.

40. (c) Assertion (A): Given statement is correct because in phenol hydroxyl group cannot be replaced by halogen atom.

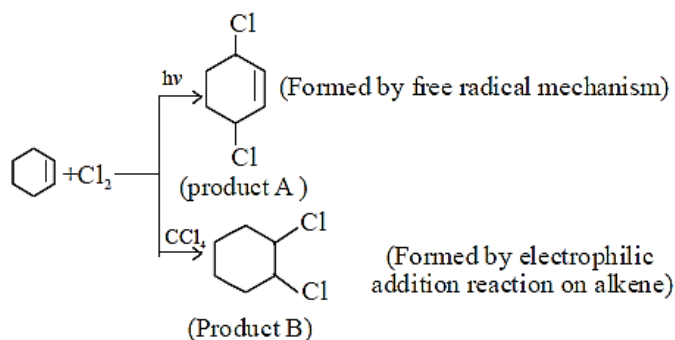
Reason (R):



Given reason is false

Hence Assertion (A) is correct but Reason (R) is false

41. (d)

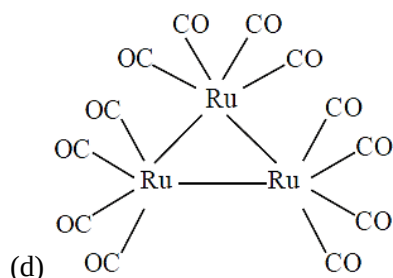
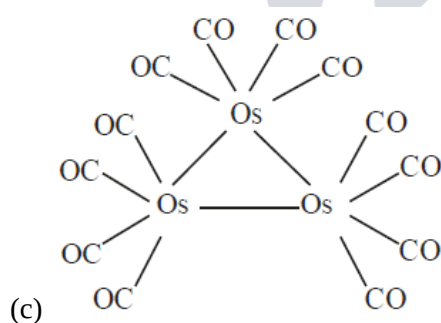
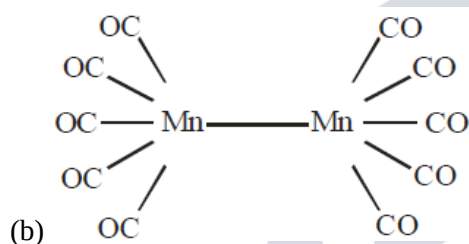
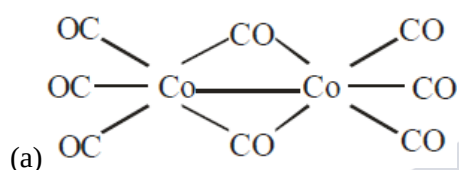


42. (a) Fluorspar is CaF_2

43. (d) It is a type of conjugation responsible for resonance.

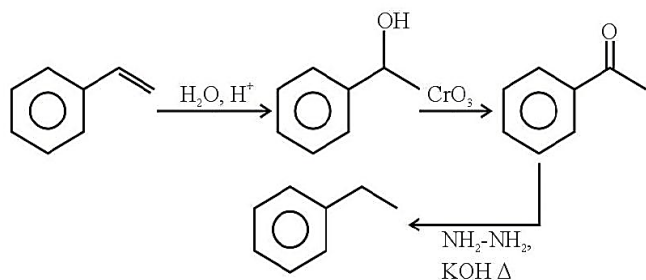
44. (d) $\text{KMnO}_4 \xrightarrow{\Delta} \text{K}_2\text{MnO}_4 + \text{MnO}_2 + \text{O}_2$

45. (a)



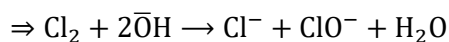
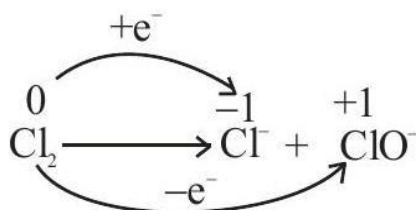
46. (b) Proteins are natural polymers composed of α -amino acids which are connected by peptide linkages. Hence proteins upon acidic hydrolysis produce α -amino acids.

47. (a)

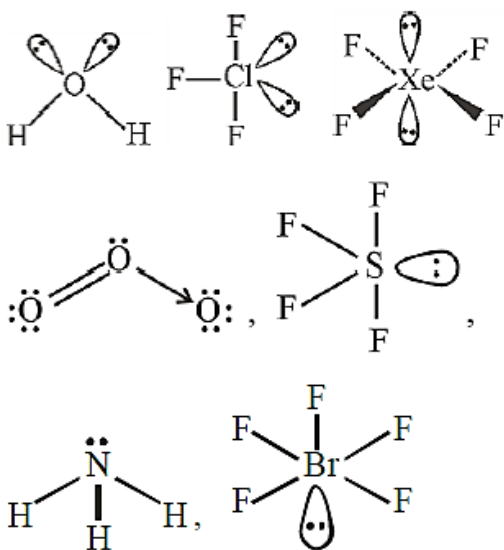


48. (b) $(\Delta G)_{P,T} = (+)$ ve for non-spontaneous process

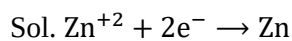
49. (a)



51. (4)



52. (45.75) or (46)



$$W = Z \times i \times t$$

$$= \frac{65.4}{2 \times 96500} \times 0.015 \times 15 \times 60$$

$$= 45.75 \times 10^{-4} \text{ gm}$$

53. (30)

$$K = \frac{K_1 \cdot K_2}{K_3} = \frac{A_1 \cdot A_2}{A_3} \cdot e^{-\frac{(E_{a1} + E_{a2} - E_{a3})}{RT}}$$

$$A \cdot e^{-E_a/RT} = \frac{A_1 A_2}{A_3} \cdot e^{-\frac{(E_{a1} + E_{a2} - E_{a3})}{RT}}$$

$$E_a = E_{a1} + E_{a2} - E_{a3} = 40 + 50 - 60 = 30 \text{ kJ/mole.}$$

54. (2)

$$K_P = K_C \cdot (RT)^{\Delta n_g}$$

$$\Delta n_g = 1$$

$$\Rightarrow K_c = \frac{K_P}{RT} = \frac{0.492}{0.082 \times 300} = 2 \times 10^{-2}$$

55. (4)

$$M = \frac{n_{\text{solute}}}{V} \times 1000$$

$$= \frac{\left(\frac{31.4}{98}\right)}{\left(\frac{100}{1.25}\right)} \times 1000$$

$$= 4.005 \approx 4$$

56. (72.56) or (73)

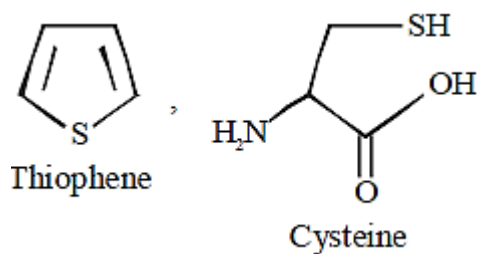
$$\pi = CRT$$

$$\Rightarrow \frac{\pi_1}{\pi_2} = \frac{T_1}{T_2}$$

$$\Rightarrow \pi_2 = \frac{\pi_1 T_2}{T_1} = \frac{7 \times 10^5 \times 283}{273}$$

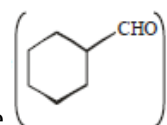
$$= 72.56 \times 10^4 \text{ Nm}^{-2}$$

57. (2)



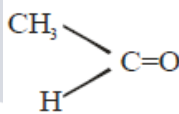
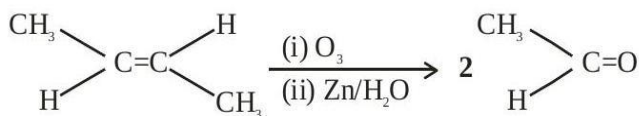
58. (1)

Magnetic behaviour		Bond order
H ₂	Diamagnetic	1
He ₂ ⁺	Paramagnetic	0.5
O ₂ ⁺	Paramagnetic	2.5
N ₂ ²⁻	Paramagnetic	2
O ₂ ²⁻	Diamagnetic	1
F ₂	Diamagnetic	1
Ne ₂ ⁺	Paramagnetic	0.5
B ₂	Paramagnetic	1



59. (3) Acetaldehyde (CH₃CHO), Methanal (HCHO), and cyclohexane carbaldehyde

60. (1)



Hence total number of oxygen atom present per molecule

is 1.

61. (d) $a + ar + ar^2 + ar^3 + \dots + ar^{63}$

$$= 7(a + ar^2 + ar^4 + \dots + ar^{62})$$

$$\Rightarrow \frac{a(1 - r^{64})}{1 - r} = \frac{7a(1 - r^{64})}{1 - r^2}$$

$$r = 6$$

62. (b) $a_6 = 2 \Rightarrow a + 5d = 2$

$$a_1 a_4 a_5 = a(a + 3d)(a + 4d)$$

$$= (2 - 5d)(2 - 2d)(2 - d)$$

$$f(d) = 8 - 32d + 34d^2 - 20d + 30d^2 - 10d^3$$

$$f'(d) = -2(5d - 8)(3d - 2)$$

$$\frac{2}{3} + \frac{8}{5}$$

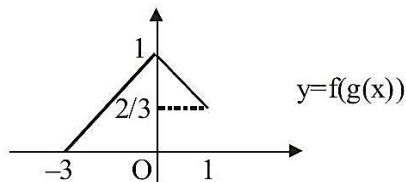
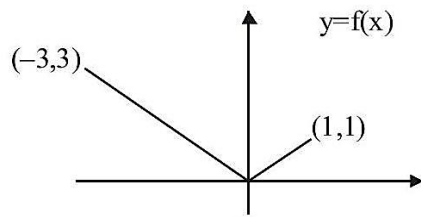
$$d = \frac{8}{5}$$

63. (c)

$$f(g(x)) = \begin{cases} 2 + 2g(x) & , -1 \leq g(x) < 0 \\ 1 - \frac{g(x)}{3} & , 0 \leq g(x) \leq 3 \end{cases}$$

By (a) $x \in \phi$

And by (b) $x \in [-3, 0]$ and $x \in [0, 1]$



$$y = f(g(x))$$

Range of $f(g(x))$ is $[0, 1]$

64. (c) Required probability =

$$\begin{aligned} & \frac{5}{6} \times \frac{1}{6} + \left(\frac{5}{6}\right)^3 \times \frac{1}{6} + \left(\frac{5}{6}\right)^5 \times \frac{1}{6} + \dots \\ &= \frac{1}{6} \times \frac{\frac{5}{6}}{1 - \frac{25}{36}} = \frac{5}{11} \end{aligned}$$

65. (b) $z = \frac{1}{2} - 2i$

$$|z + 1| = \alpha z + \beta(1 + i)$$

$$\left| \frac{3}{2} - 2i \right| = \frac{\alpha}{2} - 2\alpha i + \beta + \beta i$$

$$\left| \frac{3}{2} - 2i \right| = \left(\frac{\alpha}{2} + \beta \right) + (\beta - 2\alpha)i$$

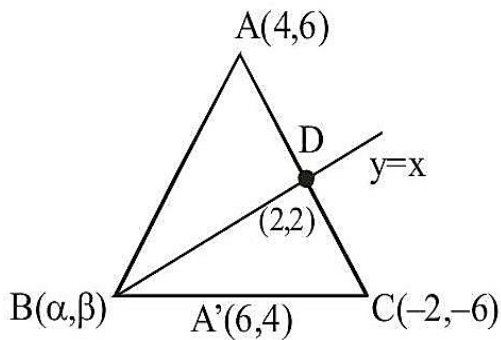
$$\beta = 2\alpha \text{ and } \frac{\alpha}{2} + \beta = \sqrt{\frac{9}{4} + 4}$$

$$\alpha + \beta = 3$$

66. (c) Using L'hospital rule

$$\begin{aligned} &= \lim_{x \rightarrow \frac{\pi}{2}} \frac{0 - \cos x \times 3x^2}{2 \left(x - \frac{\pi}{2} \right)} \\ &= \lim_{x \rightarrow \frac{\pi}{2}} \frac{\sin \left(x - \frac{\pi}{2} \right)}{2 \left(x - \frac{\pi}{2} \right)} \times \frac{3\pi^2}{4} \\ &= \frac{3\pi^2}{8} \end{aligned}$$

67. (a)



$$AD:DC = 1:2$$

$$\frac{4-\alpha}{6-\alpha} = \frac{10}{8}$$

$$\alpha = \beta$$

$$\alpha = 14 \text{ and } \beta = 14$$

68. (a)

$$\vec{a} + 5\vec{b} = \lambda\vec{c}$$

$$\vec{b} + 6\vec{c} = \mu\vec{a}$$

Eliminating $\vec{a}$

$$\lambda\vec{c} - 5\vec{b} = \frac{6}{\mu}\vec{c} + \frac{1}{\mu}\vec{b}$$

$$\therefore \mu = \frac{-1}{5}, \lambda = -30$$

$$\alpha = 5, \beta = 30$$

69. (b) $A(a, -2), B(a, 6), C\left(\frac{a}{4}, -2\right), O\left(5, \frac{a}{4}\right)$

$$AO = BO$$

$$(a-5)^2 + \left(\frac{a}{4} + 2\right)^2 = (a-5)^2 + \left(\frac{a}{4} - 6\right)^2$$

$$a = 8$$

$$AB = 8, AC = 6, BC = 10$$

$$\alpha = 5, \beta = 24, \gamma = 24$$

70. (d)

$$y(x) = \int \frac{(1 + \sin^2 x) \cos x}{1 + \sin^4 x} dx$$

Put $\sin x = t$

$$= \int \frac{1+t^2}{t^4+1} dt = \frac{1}{\sqrt{2}} \tan^{-1} \frac{(t-\frac{1}{t})}{\sqrt{2}} + C$$

$$x = \frac{\pi}{2}, t = 1 \therefore C = 0$$

$$y\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}} \tan^{-1} \left(-\frac{1}{2}\right)$$

71. (c)

$$4 + 5 \tan \theta = \sec \theta$$

$$\text{Squaring: } 24 \tan^2 \theta + 40 \tan \theta + 15 = 0$$

$$\tan \theta = \frac{-10 \pm \sqrt{10}}{12}$$

$$\text{and } \tan \theta = -\left(\frac{10+\sqrt{10}}{12}\right) \text{ is Rejected.}$$

(c) is correct.

$$72. (a) \frac{dy}{dx} - \left(\frac{\sin 2x}{1+\cos^2 x}\right) y = \sin x$$

$$\text{I.F.} = 1 + \cos^2 x$$

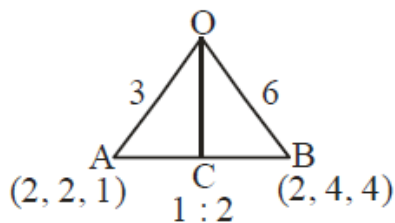
$$y \cdot (1 + \cos^2 x) = \int (\sin x) dx$$

$$= -\cos x + C$$

$$x = 0, C = 1$$

$$y\left(\frac{\pi}{2}\right) = 1$$

73. (b)



$$\text{length of OC} = \frac{\sqrt{136}}{3} = \frac{2\sqrt{34}}{3}$$

$$74. (d) f'(x) = 12\sqrt{2}x^2 - 3\sqrt{2} \geq 0 \text{ for } \left[\frac{1}{2}, 1\right]$$

$$f\left(\frac{1}{2}\right) < 0$$

$f(a) > 0 \Rightarrow (A)$ is correct.

$$f(x) = \sqrt{2}(4x^3 - 3x) - 1 = 0$$

Let $\cos \alpha = x$,

$$\cos 3\alpha = \cos \frac{\pi}{4} \Rightarrow \alpha = \frac{\pi}{12}$$

$$x = \cos \frac{\pi}{12}$$

(d) is correct.

75. (b) $|A| = \alpha^2 - \beta^2$

$$|2A|^3 = 2^{21} \Rightarrow |A| = 2^4$$

$$\alpha^2 - \beta^2 = 16$$

$$(\alpha + \beta)(\alpha - \beta) = 16 \Rightarrow \alpha = 4 \text{ or } 5$$

76. (c) Centroid G divides MR in 1: 2

$$G(1,2,2)$$

Point of intersection A of given lines is $(2, -6, 0)$

$$AG = \sqrt{69}$$

77. (a)

$$(a, b)R(a, b) \text{ as } ab - ab = 0$$

Therefore reflexive

$$\text{Let } (a, b)R(c, d) \Rightarrow ad - bc \text{ is divisible by } 5$$

$$\Rightarrow bc - ad \text{ is divisible by } 5 \Rightarrow (c, d)R(a, b)$$

Therefore symmetric

Relation not transitive as $(3,1)R(10,5)$ and $(10,5)R(1,1)$ but $(3,1)$ is not related to $(1,1)$

78. (a)

$$I = \int_{-\pi/2}^{\pi/2} \left(\frac{x^2 \cos x}{1 + \pi^x} + \frac{1 + \sin^2 x}{1 + e^{\sin x^{2023}}} \right) dx$$

$$I = \int_{-\pi/2}^{\pi/2} \left(\frac{x^2 \cos x}{1 + \pi^{-x}} + \frac{1 + \sin^2 x}{1 + e^{\sin(-x)^{2023}}} \right) dx$$

On Adding, we get

$$2I = \int_{-\pi/2}^{\pi/2} (x^2 \cos x + 1 + \sin^2 x) dx$$

On solving

$$I = \frac{\pi^2}{4} + \frac{3\pi}{4} - 2$$

$$a = 3$$

$$79. (c) f'(0) = \lim_{h \rightarrow 0} \frac{f(h) - f(0)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{(2^h + 2^{-h}) \tan h \sqrt{\tan^{-1}(h^2 - h + 1)} - 0}{(7h^2 + 3h + 1)^3 h}$$

$$= \sqrt{\pi}$$

80. (d)

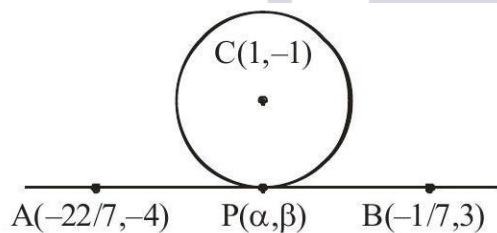
$$AA^T = I = A^T A$$

On solving given expression, we get

$$\frac{1}{2} A [A^2 + (A^T)^2 + 2AA^T + A^2 + (A^T)^2 - 2AA^T]$$

$$= A [A^2 + (A^T)^2] = A^3 + A^T$$

81. (2) Centre of circle is $(1, -1)$



Equation of AB is $7x - 3y + 10 = 0$

Equation of CP is $3x + 7y + 4 = 0$

Solving (i) and (ii)

$$\alpha = \frac{-41}{29}, \beta = \frac{1}{29} \therefore 17\beta - \alpha = 2$$

82. (553)

Words starting with E = 360

Words starting with GE = 60

Words starting with GN = 60

Words starting with GTE = 24

Words starting with GTN = 24

Words starting with GTT = 24

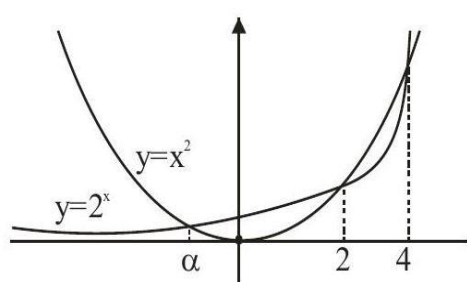
$$GTWENTY = 1$$

$$\text{Total} = 553$$

83. (13)

$$\begin{aligned} & \alpha^6 + \alpha^4 + \beta^4 - 5\alpha^2 \\ &= \alpha^4(\alpha - 2) + \alpha^4 - 5\alpha^2 + (\beta - 2)^2 \\ &= \alpha^5 - \alpha^4 - 5\alpha^2 + \beta^2 - 4\beta + 4 \\ &= \alpha^3(\alpha - 2) - \alpha^4 - 5\alpha^2 + \beta - 2 - 4\beta + 4 \\ &= -2\alpha^3 - 5\alpha^2 - 3\beta + 2 \\ &= -2\alpha(\alpha - 2) - 5\alpha^2 - 3\beta + 2 \\ &= -7\alpha^2 + 4\alpha - 3\beta + 2 \\ &= -7(\alpha - 2) + 4\alpha - 3\beta + 2 \\ &= -3\alpha - 3\beta + 16 = -3(a) + 16 = 13 \end{aligned}$$

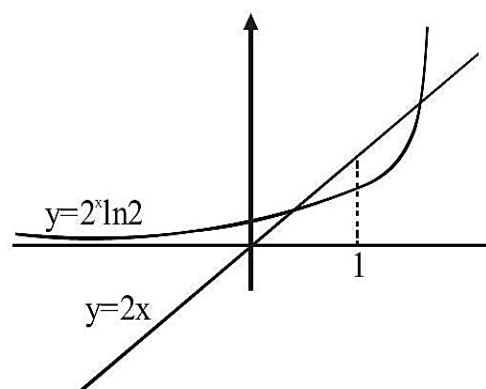
84. (5)



$$\therefore m = 3$$

$$f'(x) = 2^x \ln 2 - 2x = 0$$

$$2^x \ln 2 = 2x$$



$$\therefore n = 2$$

$$\Rightarrow m + n = 5$$

85. (432)

Putting $y^2 = 3x^2$ in both the conics

We get $x^2 = b$ and $\frac{b}{16} + \frac{3}{b} = 1$

$\Rightarrow b = 4, 12$ ($b = 4$ is rejected because curves coincide)

$\therefore b = 12$

Hence points of intersection are $(\pm\sqrt{12}, \pm 6) \Rightarrow$ area of rectangle = 432

86. (3)

$$\int \left(\frac{1}{x} + \frac{\ln x}{x} \right) dx + \int \frac{dy}{1+y^2} = 0$$

$$\ln x + \frac{(\ln x)^2}{2} + \tan^{-1} y = C$$

Put $x = y = 1$

$$\therefore C = \frac{\pi}{4}$$

$$\Rightarrow \ln x + \frac{(\ln x)^2}{2} + \tan^{-1} y = \frac{\pi}{4}$$

Put $x = e$

$$\Rightarrow y = \tan \left(\frac{\pi}{4} - \frac{3}{2} \right) = \frac{1 - \tan \frac{3}{2}}{1 + \tan \frac{3}{2}}$$

$$\therefore \alpha = 1, \beta = 1$$

$$\Rightarrow \alpha + 2\beta = 3$$

87. (6344)

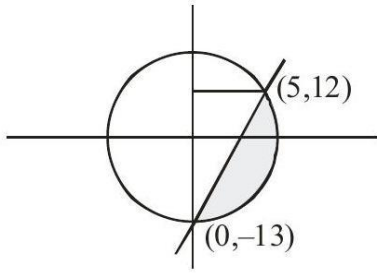
$$\bar{x} = 56$$

$$\sigma^2 = 66.2$$

$$\Rightarrow \frac{\alpha^2 + \beta^2 + 25678}{10} - (56)^2 = 66.2$$

$$\therefore \alpha^2 + \beta^2 = 6344$$

88. (171)



$$\text{Area} = \int_{-13}^{12} \sqrt{169 - y^2} dy - \frac{1}{2} \times 25 \times 5$$

$$= \frac{\pi}{2} \times \frac{169}{2} - \frac{65}{2} + \frac{169}{2} \sin^{-1} \frac{12}{13}$$

$$\therefore \alpha + \beta = 171$$

89. (2041)

$$\sum_{r=1}^9 \frac{{}^{11}C_r}{r+1}$$

$$= \frac{1}{12} \sum_{r=1}^9 {}^{12}C_{r+1}$$

$$= \frac{1}{12} [2^{12} - 26] = \frac{2035}{6}$$

$$\therefore m + n = 2041$$

90. (65)

Let $P(t, t-2, t)$ and $Q(2s-2, s, s)$

D.R's of PQ are 2, 1, 2

$$\frac{2s-2-t}{2} = \frac{s-t+2}{1} = \frac{s-t}{2}$$

$$\Rightarrow t = 6 \text{ and } s = 2$$

$$\Rightarrow P(6, 4, 6) \text{ and } Q(2, 2, 2)$$

$$PQ: \frac{x-2}{2} = \frac{y-2}{1} = \frac{z-2}{2} = \lambda$$

Let $F(2\lambda+2, \lambda+2, 2\lambda+2)$

$A(1, 2, 12)$

$$\overrightarrow{AF} \cdot \overrightarrow{PQ} = 0$$

$$\therefore \lambda = 2$$

So $F(6, 4, 6)$ and $AF = \sqrt{65}$

