1. (d)

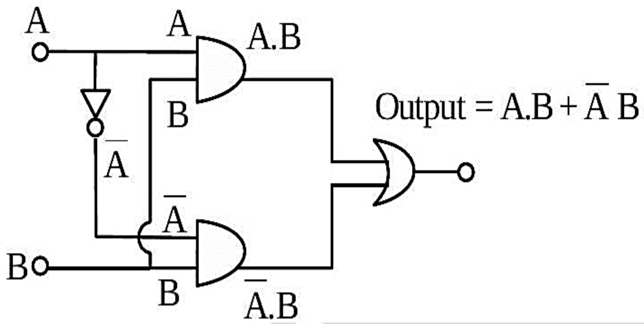
$n_1 \times \frac{hc}{\lambda_1} = 200$

$n_2 \times \frac{hc}{\lambda_2} = 200$

$\frac{n_1}{n_2} = \frac{\lambda_1}{\lambda_2} = \frac{300}{500}$

$\frac{n_1}{n_2} = \frac{3}{5}$

2. (b)



$Y = A \cdot B + \overline{A} \cdot B$
$= (A + \overline{A}) \cdot B$
$Y = 1.B$
$Y = B$

3. (c) $Q = \frac{a^4 b^3}{c^2}$

$\frac{\Delta Q}{Q} = 4\frac{\Delta a}{a} + 3\frac{\Delta b}{b} + 2\frac{\Delta c}{c}$

$\frac{\Delta Q}{Q} \times 100 = 4\left(\frac{\Delta a}{a} \times 100\right) + 3\left(\frac{\Delta b}{b} \times 100\right) + 2\left(\frac{\Delta c}{c} \times 100\right)$
% error in $Q = 4 \times 3\% + 3 \times 4\% + 2 \times 5\%$
$= 12\% + 12\% + 10\%$
$= 34\%$

4. (c) $P_{avg} = V_{rms} I_{rms} \cos(\Delta\phi)$

$= \frac{100}{\sqrt{2}} \times \frac{100 \times 10^{-3}}{\sqrt{2}} \times \cos\left(\frac{\pi}{3}\right)$

$= \frac{10^4}{2} \times \frac{1}{2} \times 10^{-3}$

$= \frac{10}{4} = 2.5$ W

5. (a) $PV = nRT$

$PV = \frac{N}{N_A} RT$

N = Total no. of molecules

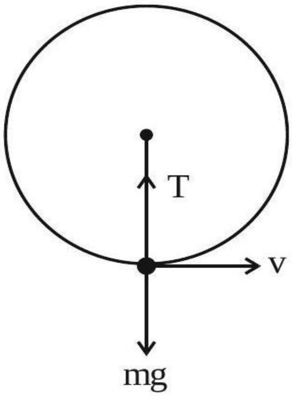
$P = \frac{N}{V} kT$

$1.38 \times 1.01 \times 10^5 = 2 \times 10^{25} \times 1.38 \times 10^{-23} \times T$

$1.01 \times 10^5 = 2 \times 10^2 \times T$

$T = \frac{1.01 \times 10^3}{2} \approx 500\ K$

6. **(b)** Given that



$m = 900 gm = \frac{900}{1000}\ kg = \frac{9}{10}\ kg$

$r = 1\ m$

$\omega = \frac{2\pi N}{60} = \frac{2\pi(10)}{60} = \frac{\pi}{3}\ rad/sec$

$T - mg = mr\omega^2$

$T = mg + mr\omega^2$

$= \frac{9}{10} \times 9.8 + \frac{9}{10} \times 1 \left(\frac{\pi}{3}\right)^2$

$= 8.82 + \frac{9}{10} \times \frac{\pi^2}{9}$

$= 8.82 + 0.98$

$= 9.80\ N$

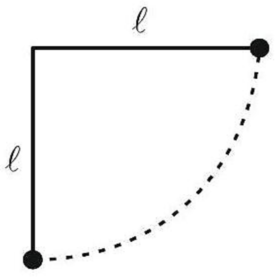
7. **(a)**

$\ell = 10\ m,$

Initial energy $= mg\ell$

So, $\frac{9}{10}mg\ell = \frac{1}{2}mv^2$

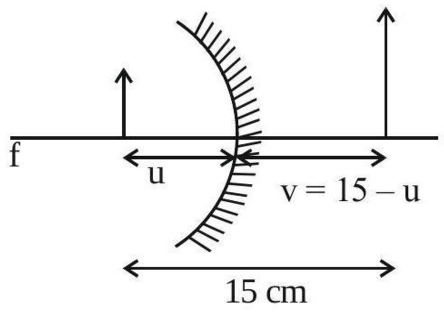
$\Rightarrow \frac{9}{10} \times 10 \times 10 = \frac{1}{2}v^2$



$v^2 = 180$

$v = \sqrt{180} = 6\sqrt{5}$ m/s

8. (c)



$m = 2 = \frac{-v}{u}$

$2 = \frac{-(15-u)}{-u}$

$2u = 15 - u$

$3u = 15 \Rightarrow u = 5$ cm

$v = 15 - u = 15 - 5 = 10$ cm

$\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$

$= \frac{1}{10} + \frac{1}{(-5)} = \frac{1-2}{10} = \frac{-1}{10}$

$f = -10$ cm

9. (b) $R = \frac{mv}{qB} = \frac{p}{qB} = \frac{\sqrt{2\,m(KE)}}{qB} = \frac{\sqrt{2mqV}}{qB}$

$R \propto \sqrt{m}$

$m \propto R^2$

$$\frac{m_1}{m_2}=\left(\frac{R_1}{R_2}\right)^2$$

10. (a) $\Delta x=\frac{7\lambda}{4}$

$$\phi=\frac{2\pi}{\lambda}\Delta x=\frac{2\pi}{\lambda}\times\frac{7\lambda}{4}=\frac{7\pi}{2}$$

$$I=I_{\max}\cos^2\left(\frac{\phi}{2}\right)$$

$$\frac{I}{I_{\max}}=\cos^2\left(\frac{\phi}{2}\right)=\cos^2\left(\frac{7\pi}{2\times 2}\right)=\cos^2\left(\frac{7\pi}{4}\right)$$

$$=\cos^2\left(2\pi-\frac{\pi}{4}\right)$$

$$=\cos^2\frac{\pi}{4}$$

$$=\frac{1}{2}$$

11. (a) Volume constant

$$\frac{4}{3}\pi R^3=27\times\frac{4}{3}\times\pi r^3$$

$$R^3=27r^3$$

$$R=3r$$

$$r=\frac{R}{3}$$

$$r^2=\frac{R^2}{9}$$

Work done $=T.\Delta A$

$$=27\ T(4\pi r^2)-T4\pi R^2$$

$$=27\ T4\pi\frac{R^2}{9}-4\pi R^2\ T$$

$$=8\pi R^2\ T$$

12. (b) Apply energy conservation between A & B

$$\frac{1}{2}mV_L^2=\frac{1}{2}mV_H^2+mg(2\ L)$$

$$\because V_L=\sqrt{5gL}$$

So, $V_H=\sqrt{gL}$

$$\frac{(K.E)_A}{(K.E)_B}=\frac{\frac{1}{2}\,m(\sqrt{5gL})^2}{\frac{1}{2}\,m(\sqrt{gL})^2}=\frac{5}{1}$$

13. (d)

$$Y=\frac{\text{stress}}{\text{strain}}$$

$$Y=\frac{\frac{F}{\pi r^2}}{\frac{\ell}{L}}$$

$$F=Y\pi r^2\times\frac{\ell}{L}$$

$$Y=\frac{\frac{F/2}{\pi r^2/4}}{\frac{\Delta\ell}{L}}$$

$$F=Y\frac{\Delta\ell}{L}\times 2\times\frac{\pi r^2}{4}$$

From (i)

$$Y\pi r^2\frac{\ell}{L}=Y\frac{\Delta\ell}{L}\frac{\pi r^2}{2}$$

$$\Delta\ell=2\ell$$

14. (a) $T^2\propto r^3$

$$\frac{T_1^2}{r_1^3}=\frac{T_2^2}{r_2^3}$$

$$\frac{(200)^2}{r^3}=\frac{T_2^2}{\left(\frac{r}{4}\right)^3}$$

$$\frac{200\times 200}{4\times 4\times 4}=T_2^2$$

$$T_2=\frac{200}{4\times 2}$$

$$T_2=25\text{ days}$$

15. (a) $\frac{E}{B}=C$

$$\frac{E}{B}=3\times 10^8$$

$$B=\frac{E}{3\times 10^8}=\frac{9.6}{3\times 10^8}$$

$B = 3.2 \times 10^{-8}$ T

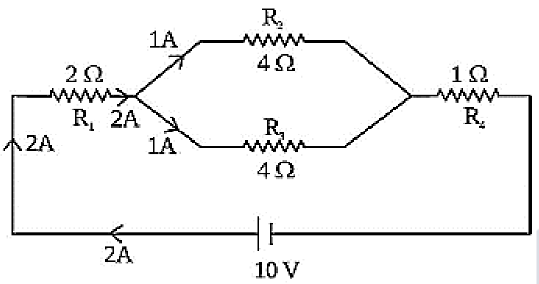
$\hat{B} = \hat{v} \times \hat{E}$

$= \hat{\imath} \times \hat{\jmath} = \hat{k}$

So,

$\vec{B} = 3.2 \times 10^{-8} \hat{k} T$

16. (a)



$R_{eq} = 2\Omega + 2\Omega + 1\Omega = 5\Omega$

$i = \frac{V}{R_{eq}} = \frac{10}{5} = 2$ A

Current in resistance $R_3 = 2 \times \left(\frac{4}{4+4}\right)$

$= 2 \times \frac{4}{8}$
$= 1$ A

17. (b) $x = t^3 - 6t^2 + 20t + 15$

$\frac{dx}{dt} = v = 3t^2 - 12t + 20$

$\frac{dv}{dt} = a = 6t - 12$

When $a = 0$

$6t - 12 = 0; t = 2sec$

At $t = 2sec$

$v = 3(2)^2 - 12(b) + 20$

$v = 8$ m/s

18. (c) $f_{eq} = \frac{n_1 f_1 + n_2 f_2}{n_1 + n_2}$

For diatomic gas $f_{eq} = 5$

$$5 = \frac{(N)(6) + (b)(c)}{N + 2}$$

$$5N + 10 = 6N + 6$$

$$N = 4$$

19. (c) According to Rutherford atomic model, most of mass of atom and all its positive charge is concentrated in tiny nucleus & electron revolve around it.

According to Thomson atomic model, atom is spherical cloud of positive charge with electron embedded in it.

Hence, Statement I is true but statement II false.

20. (c)

$$\vec{E} = 6\hat{i} + 5\hat{j} + 3\hat{k}$$

$$\vec{A} = 30\hat{i}$$

$$\phi = \vec{E}.\vec{A}$$

$$\phi = (6\hat{i} + 5\hat{j} + 3\hat{k}) \cdot (30\hat{i})$$

$$\phi = 6 \times 30 = 180$$

21. (16)

$$Y = \frac{\text{Stress}}{\text{Strain}} = \frac{F/A}{\Delta\ell/\ell} = \frac{F\ell}{A\Delta\ell}$$

$$\Delta\ell = \frac{F\ell}{AY}$$

$$V = A\ell \Rightarrow \ell = \frac{V}{A}$$

$$\Delta\ell = \frac{FV}{A^2Y}$$

Y& V is same for both the wires

$$\Delta\ell \propto \frac{F}{A^2}$$

$$\frac{\Delta\ell_1}{\Delta\ell_2} = \frac{F_1}{A_1^2} \times \frac{A_2^2}{F_2}$$

$$\Delta\ell_1 = \Delta\ell_2$$

$$F_1 A_2^2 = F_2 A_1^2$$

$$\frac{F_1}{F_2} = \frac{A_1^2}{A_2^2} = \left(\frac{4}{1}\right)^2 = 16$$

22. (3)

$B_H = 0.60 \times 10^{-4}\ Wb/m^2$

Induced emf $e = B_H v\ell$

$= 0.60 \times 10^{-4} \times 10 \times 5$
$= 3 \times 10^{-3}\ V$

23. (121) For minimum potential difference electron has to make transition from $n = 3$ to $n = 2$ state but first electron has to reach to $n = 3$ state from ground state. So, energy of bombarding electron should be equal to energy difference of $n = 3$ and $n = 1$ state.

$$\Delta E = 13.6\left[1 - \frac{1}{3^2}\right]e = eV$$

$$\frac{13.6 \times 8}{9} = V$$

$V = 12.09\ V \approx 12.1\ V$

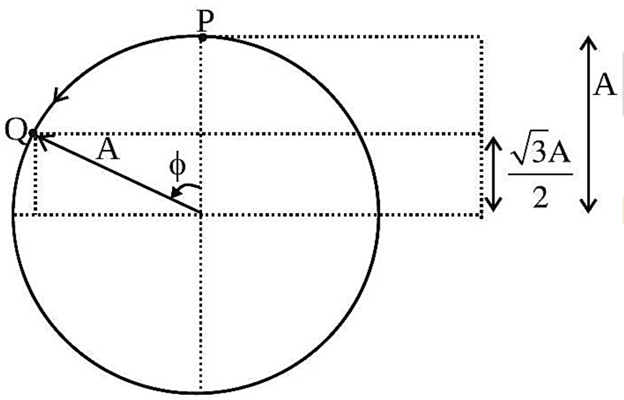
So, $\alpha = 121$

24. (32)

$q = 4\mu C, \vec{v} = 4 \times 10^6 \hat{j}\, m/s$

$\vec{B} = 2\hat{k}r$
$\vec{F} = q(\vec{v} \times \vec{B})$
$= 4 \times 10^{-6}\left(4 \times 10^6 \hat{j} \times 2\hat{k}\right)$
$= 4 \times 10^{-6} \times 8 \times 10^6 \hat{i}$
$\vec{F} = 32\hat{i}N$
$x = 32$

25. (2)



From phasor diagram particle has to move from P to Q in a circle of radius equal to amplitude of SHM.

$$\cos\phi = \frac{\frac{\sqrt{3}A}{2}}{A} = \frac{\sqrt{3}}{2}$$

$$\phi = \frac{\pi}{6}$$

Now, $\frac{\pi}{6} = \omega t$

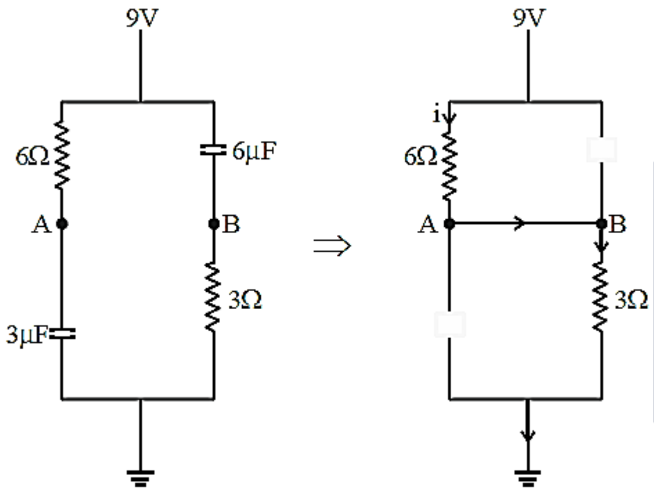
$$\frac{\pi}{6} = \frac{2\pi}{T} t$$

$$\frac{\pi}{6} = \frac{2\pi}{6\pi} t$$

$$t = \frac{\pi}{2}$$

So, $x = 2$

26. (36) At steady state, capacitor behaves as an open circuit and current flows in circuit as shown in the diagram.



$R_{eq} = 9\Omega$

$i = \frac{9\text{ V}}{9\Omega} = 1\text{ A}$

$\Delta V_{6\Omega} = 1 \times 6 = 6\text{ V}$

$V_A = 3\text{ V}$

So, potential difference across $6\mu F$ is 6 V.

Hence $Q = C\Delta V$

$= 6 \times 6 \times 10^{-6} C$

$= 36\mu C$

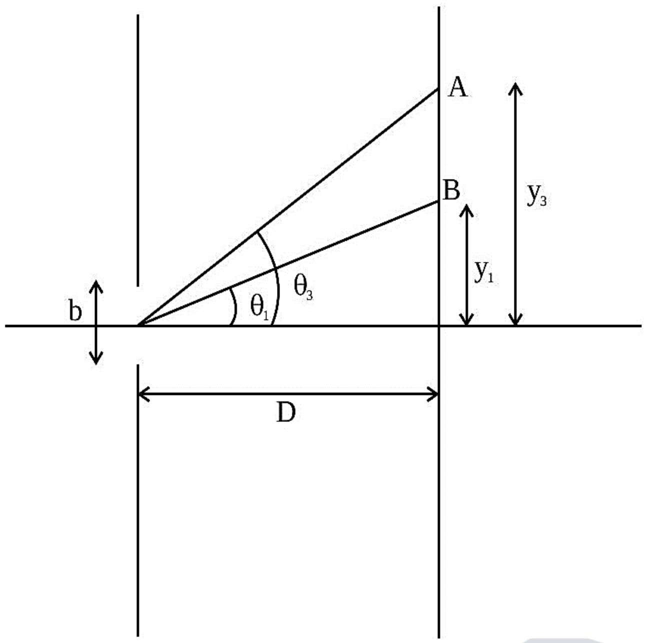
27. (2) For n^{th} minima

$b\sin\theta = n\lambda$

(λ is small so $\sin\theta$ is small, hence $\sin\theta \simeq \tan\theta$) $b\tan\theta = n\lambda$

$$b\frac{y}{D} = n\lambda$$

$\Rightarrow y_n = \frac{n\lambda D}{b}$ (Position of n^{th} minima)



B → 1^{st} minima, A → 3^{rd} minima

$y_3 = \frac{3\lambda D}{b}, y_1 = \frac{\lambda D}{b}$

$\Delta y = y_3 - y_1 = \frac{2\lambda D}{b}$

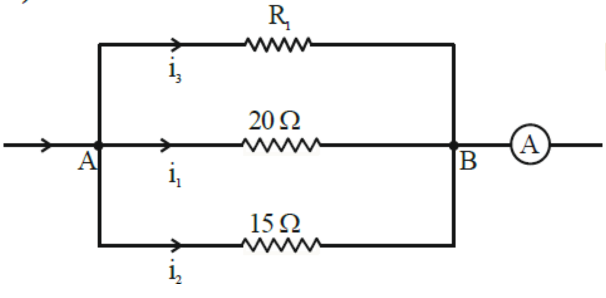
$3 \times 10^{-3} = \frac{2 \times 6000 \times 10^{-10} \times 0.5}{b}$

$b = \frac{2 \times 6000 \times 10^{-10} \times 0.5}{3 \times 10^{-3}}$

$b = 2 \times 10^{-4}$ m

$x = 2$

28. (30)



Given, $i_1 = 0.3$ A, $i_1 + i_2 + i_3 = 0.9$ A

So, $V_{AB} = i_1 \times 20\Omega = 20 \times 0.3\text{ V} = 6\text{ V}$

$i_2 = \frac{6\text{ V}}{15\Omega} = \frac{2}{5}$ A

$i_1 + i_2 + i_3 = \frac{9}{10}$ A

$\frac{3}{10}+\frac{2}{5}+i_3=\frac{9}{10}$

$\frac{7}{10}+i_3=\frac{9}{10}$

$i_3=0.2\ A$

So, $i_3\times R_1=6\ V$

$(0.2)R_1=6$

$R_1=\frac{6}{0.2}=30\Omega$

29. (8)

$|\vec{a}_C|=|\vec{a}_t|$

$\frac{v^2}{r}=\frac{dv}{dt}$

$\Rightarrow \int_4^v \frac{dv}{v^2}=\int_0^t \frac{dt}{r}$

$\Rightarrow \left[\frac{-1}{v}\right]_4^v=\frac{t}{r}$

$\Rightarrow \frac{-1}{v}+\frac{1}{4}=2t \Rightarrow v=\frac{4}{1-8t}=\frac{ds}{dt}$

$4\int_0^t \frac{dt}{1-8t}=\int_0^s ds$

$(r=0.5\ m$

$s=2\pi r=\pi)$

$4\times\frac{[\ln(1-8t)]_0^t}{-8}=\pi$

$\ell n(1-8t)=-2\pi$

$1-8t=e^{-2\pi}$

$t=(1-e^{-2\pi})\frac{1}{8}\ s$

So, $\alpha=8$

30. (60) $y-x-4=0$

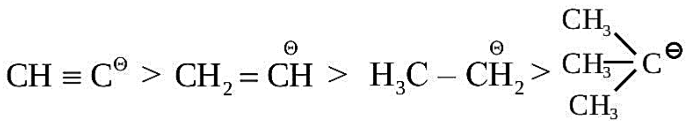
d_1 is perpendicular distance of given line from origin.

$d_1 = \left|\frac{-4}{\sqrt{1^2+1^2}}\right| \Rightarrow 2\sqrt{2}\text{ m}$

So, $|\vec{L}| = mvd_1 = 5 \times 3\sqrt{2} \times 2\sqrt{2}\text{ kg m}^2/\text{s}$

$= 60\text{ kg m}^2/\text{s}$

31. (a)



Stability of conjugate base α acidic strength

$C < D < B < A$

32. (d) A-II, B-III, C-I, D-IV

Fact based.

33. (d)

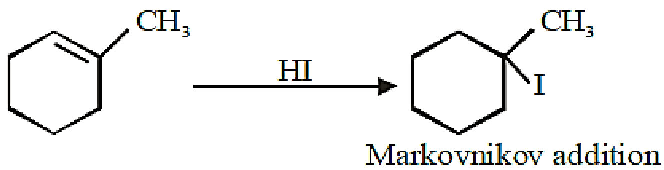
Ethanol → 15.9

Phenol → 10

M-Nitrophenol → 8.3

P-Nitrophenol → 7.1

34. (b)



35. (d) Cyclohex-2-en-1-ol

36. (b) K_2MnO_4

$2 + x - 8 = 0$

$\Rightarrow x = +6$

O.S. of Mn $= +6$

IUPAC Name =

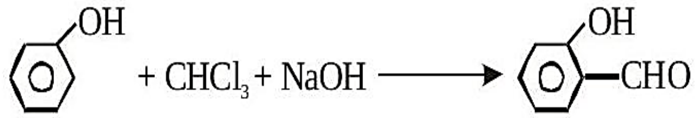
Potassium tetraoxidomanganate (VI)

37. (d)

$Ni^{2+} + 2dmg^- \rightarrow [Ni(dmg)_2]$

Rosy red/Bright Red precipitate

38. (d)



α-hydroxy-benzaldehyde

It is Reimer Tiemann Reaction

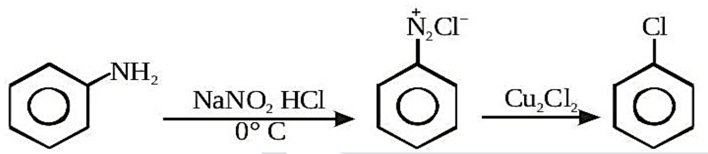
39. (c) A - II, B - IV, C - III, D - I

Fact based.

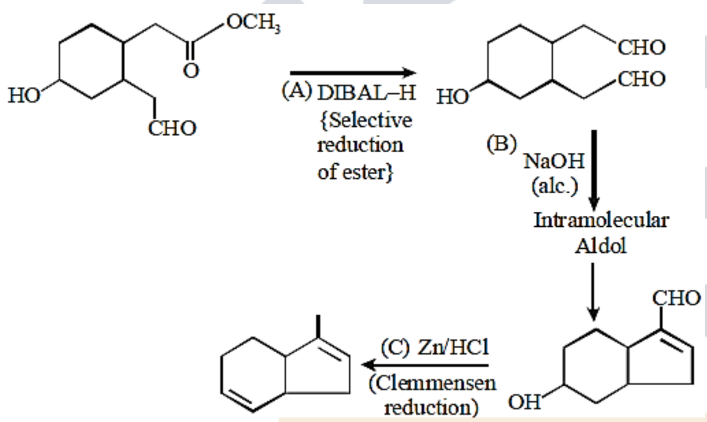
40. (c) Nessler's Reagent Reaction:

$$\underset{(\text{Nessler's Reagent})}{2K_2HgI_4} + NH_3 + 3KOH \rightarrow \underset{\left(\begin{array}{l}\text{Iodine of Millon's base}\\ \text{Brown precipitate}\end{array}\right)}{HgO.\,Hg(NH_2)I} + 7KI + 2H_2O$$

41. (c)



42. (d)



43. (c)

$Eu^{+2} \longrightarrow \quad Eu^{+3} + 1e^{-}$

$[Xe]4f^{7}6\,s^{0} \quad [Xe]4f^{6}6\,s^{0}$

44. (c)

45. (a)

(A) Zn, Cd, Hg exhibit lowest enthalpy of atomization in respective transition series.

(C) Compounds of Zn, Cd and Hg are diamagnetic in nature.

46. (c) $Al < Si < C < N; IE_1$ order.

47. (d) Covalent character of AgCN.

48. (c) Due to unsymmetrical.

49. (d) Statement-1 is false because chlorine has most negative electron gain enthalpy in its group.

50. (c)

51. (4) Antibonding molecular orbital from 2 s = 1 Antibonding molecular orbital from $2p$ = 3 Total = 4

52. (1)

$[Fe(H_2O)_5(NO)]^{2+}$,

Oxidation no. of Fe = +1

53. (417)

$$K_C = \frac{[NH_3]^2}{[N_2][H_2]^3}$$

$$K_C = \frac{(1.5 \times 10^{-2})^2}{(2 \times 10^{-2}) \times (3 \times 10^{-2})^3}$$

$$K_C = 417$$

54. (815)

$$m = \frac{M \times 1000}{d_{sol} \times 1000 - M \times \text{ Molar mass }_{\text{solute}}}$$
$$815 \times 10^{-3}\ m$$

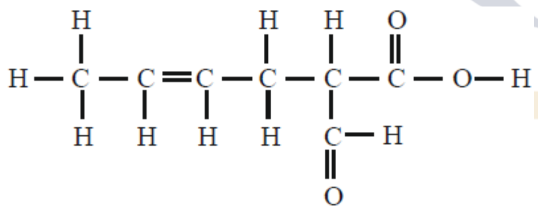
55. (4) Equivalent of Oxalic acid = Equivalents of NaOH

$$50 \times 0.5 \times 2 = 25 \times M \times 1$$

$$M_{NaOH} = 2M$$

W_{NaOH} in 50ml $= 2 \times 50 \times 40 \times 10^{-3}$ g = 4 g

56. (22)



22 bonds

57. (63) Half-life of bromine −82 = 36 hours

$$t_{1/2} = \frac{0.693}{K}$$
$$K = \frac{0.693}{36} = 0.01925 hr^{-1}$$

1st order rxn kinetic equation

$t = \frac{2.303}{K}\log\frac{a}{a-x}$

$\log\frac{a}{a-x} = \frac{t \times K}{2.303}$ (t = 1 day = 24hr)

$\log\frac{a}{a-x} = \frac{24hr \times 0.01925hr^{-1}}{2.303}$

$\log\frac{a}{a-x} = 0.2006$

$\frac{a}{a-x} = \text{antilog}(0.2006)$

$\frac{a}{a-x} = 1.587$

If a = 1

$\frac{1}{1-x} = 1.587 \Rightarrow 1 - x = 0.6301 =$ Fraction remain after one day

58. (56)

$\Delta H^0_{vap}\ CCl_4 = 30.5\ kJ/mol$

Mass of $CCl_4 = 284gm$

Molar mass of $CCl_4 = 154\ g/mol$

Moles of $CCl_4 = \frac{284}{154} = 1.844$ mol

$\Delta H_{vap}{}^\circ$ for 1 mole = 30.5 kJ/mol

$\Delta H_{vap}{}^\circ$ for 1.844 mol $= 30.5 \times 1.844$

$= 56.242$ kJ

59. (2)

$\frac{W}{E} = \frac{\text{ch arge}}{1\ F}$

$\frac{1.314}{\frac{197}{3}} = \frac{Q}{1\ F}$

$Q = 2 \times 10^{-2}$ F

60. (3) Molecules with zero dipole moment $= CO_2, CH_4, BF_3$

61. (a) $|P^{-1}AP - 2I| = |P^{-1}AP - 2P^{-1}P|$

$= |P^{-1}(A - 2I)P|$

$= |P^{-1}||A - 2I||P|$

$= |A - 2I|$

$= \begin{vmatrix} 0 & 1 & 2 \\ 6 & 0 & 11 \\ 3 & 3 & 0 \end{vmatrix} = 69$

So, Prime factor of 69 is 3&23

So, sum = 26

62. (d)

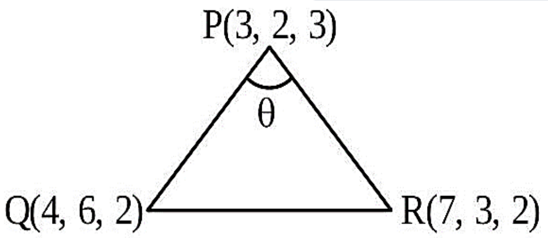
3 Shelf empty : $(8,0,0,0) \rightarrow 1$ way

2 shelf empty : $\left.\begin{matrix}(7,1,0,0)\\(6,2,0,0)\\(5,3,0,0)\\(4,4,0,0)\end{matrix}\right] \rightarrow 4$ ways

$\left.\begin{matrix}\text{1 shelf empty : } & (6,1,1,0) & (3,3,2,0)\\ & (5,2,1,0) & (4,2,2,0)\\ & (4,3,1,0) & \\ & (1,2,3,2) & (5,1,1,1)\\ & (2,2,2,2) & \\ & & \text{0 Shelf empty : } 3,3,1,1)\\ & & (4,2,1,1)\end{matrix}\right] \rightarrow 5$ ways

Total = 15 ways

63. (d)



Direction ratio of PR $= (4,1,-1)$

Direction ratio of PQ $= (1,4,-1)$

Now, $\cos\theta = \left|\frac{4+4+1}{\sqrt{18}\cdot\sqrt{18}}\right|$

$\theta = \frac{\pi}{3}$

64. (c)

$\overline{X} = \frac{24}{5}; \sigma^2 = \frac{194}{25}$

Let first four observation be x_1, x_2, x_3, x_4

Here, $\frac{x_1+x_2+x_3+x_4+x_5}{5} = \frac{24}{5}$

Also, $\frac{x_1+x_2+x_3+x_4}{4} = \frac{7}{2}$

$\Rightarrow x_1 + x_2 + x_3 + x_4 = 14$

Now from eqn -1

$x_5 = 10$

Now, $\sigma^2 = \frac{194}{25}$

$$\frac{x_1^2 + x_2^2 + x_3^2 + x_4^2 + x_5^2}{5} - \frac{576}{25} = \frac{194}{25}$$

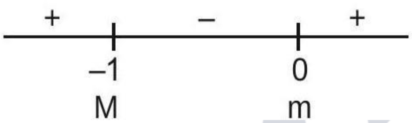
$\Rightarrow x_1^2 + x_2^2 + x_3^2 + x_4^2 = 54$

Now, variance of first 4 observations

$$\text{Var} = \frac{\sum_{i=1}^{4} x_i^2}{4} - \left(\frac{\sum_{i=1}^{4} x_i}{4}\right)^2$$
$$= \frac{54}{4} - \frac{49}{4} = \frac{5}{4}$$

65. (c) $f(x) = 2x + 3(x)^{\frac{2}{3}}$

$$f'(x) = 2 + 2x^{\frac{-1}{3}}$$
$$= 2\left(1 + \frac{1}{x^{\frac{1}{3}}}\right)$$
$$= 2\left(\frac{x^{\frac{1}{3}} + 1}{x^{\frac{1}{3}}}\right)$$



So, maxima (M) at $x = -1$ & minima (m) at $x = 0$

66. (a) $z = 2 - i\left(2\tan\frac{5\pi}{8}\right) = x + iy$ (let)

$$r = \sqrt{x^2 + y^2} \ \& \ \theta = \tan^{-1}\frac{y}{x}$$

$$r = \sqrt{(2)^2 + \left(2\tan\frac{5\pi}{8}\right)^2}$$

$$= \left|2\sec\frac{5\pi}{8}\right| = \left|2\sec\left(\pi - \frac{3\pi}{8}\right)\right|$$

$$= 2\sec\frac{3\pi}{8}$$

$$\& \theta = \tan^{-1}\left(\frac{-2\tan\frac{5\pi}{8}}{2}\right)$$

$$= \tan^{-1}\left(\tan\left(\pi - \frac{5\pi}{8}\right)\right)$$

$=\frac{3\pi}{8}$

67. (c)

$$\frac{3\cos 2x+\cos^3 2x}{\cos^6 x-\sin^6 x}=x^3-x^2+6$$

$$\Rightarrow \frac{\cos 2x(3+\cos^2 2x)}{\cos 2x(1-\sin^2 x\cos^2 x)}=x^3-x^2+6$$

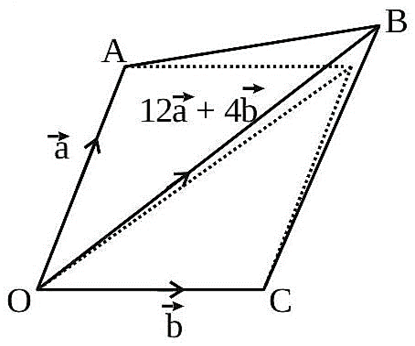
$$\Rightarrow \frac{4(3+\cos^2 2x)}{(4-\sin^2 2x)}=x^3-x^2+6$$

$$\Rightarrow \frac{4(3+\cos^2 2x)}{(3+\cos^2 2x)}=x^3-x^2+6$$

$x^3-x^2+2=0 \Rightarrow (x+1)(x^2-2x+2)=0$

so, sum of real solutions $=-1$

68. (d)



Area of parallelogram, $S=|\vec{a}\times\vec{b}|$

Area of quadrilateral $=$ Area(ΔOAB) + Area$(\triangle OBC)$

$=\frac{1}{2}\{|\vec{a}\times(12\vec{a}+4\vec{b})|+|\vec{b}\times(12\vec{a}+4\vec{b})|\}$

$=8|(\vec{a}\times\vec{b})|$

Ratio $=\frac{8|(\vec{a}\times\vec{b})|}{|(\vec{a}\times\vec{b})|}=8$

69. (a) $\log_e a, \log_e b, \log_e c$ are in A.P.

$\therefore b^2=ac$
Also

$\log_e\left(\frac{a}{2b}\right), \log_e\left(\frac{2b}{3c}\right), \log_e\left(\frac{3c}{a}\right)$ are in A.P.

$$\left(\frac{2b}{3c}\right)^2=\frac{a}{2b}\times\frac{3c}{a}$$

$\frac{b}{c}=\frac{3}{2}$

Putting in eq. (i) $b^2=a\times\frac{2b}{3}$

$\frac{a}{b}=\frac{3}{2}$

$a:b:c=9:6:4$

70. (d)

$$\int\frac{\sin^{\frac{3}{2}}x+\cos^{\frac{3}{2}}x}{\sqrt{\sin^3 x\cos^3 x\sin(x-\theta)}}dx$$

$$I=\int\frac{\sin^{\frac{3}{2}}x+\cos^{\frac{3}{2}}x}{\sqrt{\sin^3 x\cos^3 x(\sin x\cos\theta-\cos x\sin\theta)}}dx$$

$$=\int\frac{\sin^{\frac{3}{2}}x}{\sin^{\frac{3}{2}}x\cos^2 x\sqrt{\tan x\cos\theta-\sin\theta}}dx+\int\frac{\cos^{\frac{3}{2}}x}{\sin^2 x\cos^{\frac{3}{2}}x\sqrt{\cos\theta-\cot x\sin\theta}}dx=$$

$$\int\frac{\sec^2 x}{\sqrt{\tan x\cos\theta-\sin\theta}}dx+\int\frac{\operatorname{cosec}^2 x}{\sqrt{\cos\theta-\cot x\sin\theta}}dx$$

$I=I_1+I_2$ {Let}

For I_1, let $\tan x\cos\theta-\sin\theta=t^2$

$$\sec^2 x dx=\frac{2tdt}{\cos\theta}$$

For I_2, let $\cos\theta-\cot x\sin\theta=z^2$

$$\operatorname{cosec}^2 x dx=\frac{2zdz}{\sin\theta}$$

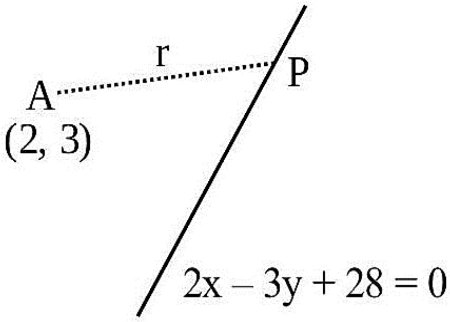
$$I=I_1+I_2$$
$$=\int\frac{2tdt}{\cos\theta t}+\int\frac{2zdz}{\sin\theta z}$$
$$=\frac{2t}{\cos\theta}+\frac{2z}{\sin\theta}$$

$$=2\sec\theta\sqrt{\tan x\cos\theta-\sin\theta}+2\operatorname{cosec}\theta\sqrt{\cos\theta-\cot x\sin\theta}$$

Comparing

$AB=8\operatorname{cosec}2\theta$

71. (d)



Writing P in terms of parametric co-ordinates $2 + r\cos\theta, 3 + r\sin\theta$ as $\tan\theta = \sqrt{3}$

$$P\left(2 + \frac{r}{2}, 3 + \frac{\sqrt{3}r}{2}\right)$$

P must satisfy $2x - 3y + 28 = 0$

So, $2\left(2 + \frac{r}{2}\right) - 3\left(3 + \frac{\sqrt{3}r}{2}\right) + 28 = 0$

We find $r = 4 + 6\sqrt{3}$

72. (a) Differential equation :-

$$x\cos\frac{y}{x}\frac{dy}{dx} = y\cos\frac{y}{x} + x$$
$$\cos\frac{y}{x}\left[x\frac{dy}{dx} - y\right] = x$$

Divide both sides by x^2

$$\cos\frac{y}{x}\left(\frac{x\frac{dy}{dx} - y}{x^2}\right) = \frac{1}{x}$$

Let $\frac{y}{x} = t$

$$\cos t\left(\frac{dt}{dx}\right) = \frac{1}{x}$$

$$\cos t\, dt = \frac{1}{x}dx$$

Integrating both sides

$$\sin t = \ln|x| + c$$
$$\sin\frac{y}{x} = \ln|x| + c$$

Using $y(a) = \frac{\pi}{3}$, we get $c = \frac{\sqrt{3}}{2}$

So, $\alpha = \sqrt{3} \Rightarrow \alpha^2 = 3$

73. (d) Let r' th term of the GP be ar^{n-1}. Given,

$$2a_r = a_{r+1} + a_{r+2}$$
$$2ar^{n-1} = ar^n + ar^{n+1}$$
$$\frac{2}{r} = 1 + r$$
$$r^2 + r - 2 = 0$$

Hence, we get, $r = -2(\text{ as } r \neq 1)$

So, $S_{20} - S_{18} =$ (Sum upto 20 terms) - (Sum upto 18 terms) $= T_{19} + T_{20}$

$T_{19} + T_{20} = ar^{18}(1 + r)$

Putting the values $a = \frac{1}{8}$ and $r = -2$;

we get $T_{19} + T_{20} = -2^{15}$

74. (d) Solving lines $L_1(3x + 2y = 14)$ and $L_2(5x - y = 6)$ to get $A(2,4)$ and solving lines $L_3(4x + 3y = 8)$ and $L_4(6x + y = 5)$ to get $B\left(\frac{1}{2}, 2\right)$.

Finding Eqn. of AB: $4x - 3y + 4 = 0$

Calculate distance PM

$$\Rightarrow \left|\frac{4(5) - 3(-2) + 4}{5}\right| = 6$$

75. (d) Assume $\sin^{-1} x = \theta$

$$\cos(2\theta) = \frac{1}{9}$$
$$\sin\theta = \pm\frac{2}{3}$$

as m and n are co-prime natural numbers,

$$x = \frac{2}{3}$$

i.e. $m = 2, n = 3$

So, the quadratic equation becomes $2x^2 - 3x + 1 = 0$ whose roots are $\alpha = 1, \beta = \frac{1}{2}$

$\left(1, \frac{1}{2}\right)$ lies on $5x + 8y = 9$

76. (b)

$$f(x) = \frac{x}{x^2 - 6x - 16}$$

Now,

$$f'(x) = \frac{-(x^2 + 16)}{(x^2 - 6x - 16)^2}$$

$f'(x) < 0$

Thus $f(x)$ is decreasing in

$(-\infty,-2) \cup (-2,8) \cup (8,\infty)$

77. (d) $y = \log_e\left(\frac{1-x^2}{1+x^2}\right)$

$$\frac{dy}{dx} = y' = \frac{-4x}{1-x^4}$$

Again,

$$\frac{d^2y}{dx^2} = y'' = \frac{-4(1+3x^4)}{(1-x^4)^2}$$

Again

$$y' - y'' = \frac{-4x}{1-x^4} + \frac{4(1+3x^4)}{(1-x^4)^2}$$

at $x = \frac{1}{2}$

$$y' - y'' = \frac{736}{225}$$

Thus $225(y' - y'') = 225 \times \frac{736}{225} = 736$

78. (a) Given set $\{1,2,3,4\}$

Minimum order pairs are

$(1,1), (2,2), (3,3), (4,4), (3,1), (2,1), (2,3), (3,2), (1,3), (1,2)$

Thus no. of elements $= 10$

79. (b)

Given set $= \{1,2,3,$

$P(A) =$ Probability that number is multiple of 4

$P(B) =$ Probability that number is multiple of 6

$P(C) =$ Probability that number is multiple of 7

Now,

$$P(A) = \frac{12}{50}, P(B) = \frac{8}{50}, P(C) = \frac{7}{50}$$

again

$$P(A \cap B) = \frac{4}{50}, P(B \cap C) = \frac{1}{50}, P(A \cap C) = \frac{1}{50}$$

$P(A \cap B \cap C) = 0$

Thus

$$P(A \cup B \cup C) = \frac{12}{50} + \frac{8}{50} + \frac{7}{50} - \frac{4}{50} - \frac{1}{50} - \frac{1}{50} + 0$$
$$= \frac{21}{50}$$

80. (b) Unit vector $\hat{u} = x\hat{i} + y\hat{j} + z\hat{k}$

$$\vec{p}_1 = \frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{k}, \vec{p}_2 = \frac{1}{\sqrt{2}}\hat{j} + \frac{1}{\sqrt{2}}\hat{k}$$

$$\vec{p}_3 = \frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{j}$$

Now angle between $\hat{u}$ and $\vec{p}_1 = \frac{\pi}{2}$

$$\hat{u} \cdot \vec{p}_1 = 0 \Rightarrow \frac{x}{\sqrt{2}} + \frac{z}{\sqrt{2}} = 0$$

$$\Rightarrow x + z = 0$$

Angle between $\hat{u}$ and $\vec{p}_2 = \frac{\pi}{3}$

$$\hat{u} \cdot \vec{p}_2 = |\hat{u}| \cdot |\vec{p}_2| \cos\frac{\pi}{3}$$

$$\Rightarrow \frac{y}{\sqrt{2}} + \frac{z}{\sqrt{2}} = \frac{1}{2} \Rightarrow y + z = \frac{1}{\sqrt{2}}$$

Angle between $\hat{u}$and $\vec{p}_3 = \frac{2\pi}{3}$

$$\hat{u} \cdot \vec{p}_3 = |\hat{u}| \cdot |\vec{p}_3| \cos\frac{2\pi}{3}$$

$$\Rightarrow \frac{x}{\sqrt{2}} + \frac{4}{\sqrt{2}} = \frac{-1}{2} \Rightarrow x + y = \frac{-1}{\sqrt{2}}$$

from equation (i), (ii) and (iii) we get

$$x = \frac{-1}{\sqrt{2}}\ y = 0\ z = \frac{1}{\sqrt{2}}$$

Thus $\hat{u} - \vec{v} = \frac{-1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{k} - \frac{1}{\sqrt{2}}\hat{i} - \frac{1}{\sqrt{2}}\hat{j} - \frac{1}{\sqrt{2}}\hat{k}$

$$\hat{u} - \vec{v} = \frac{-2}{\sqrt{2}}\hat{i} - \frac{1}{\sqrt{2}}\hat{j}$$

$$\therefore |\hat{u} - \vec{v}|^2 = \left(\sqrt{\frac{4}{2} + \frac{1}{2}}\right)^2 = \frac{5}{2}$$

81. (49)

$. x^2-\sqrt{6}x+6=0 \begin{smallmatrix}\nearrow\alpha \\ \searrow\beta\end{smallmatrix}$

$$x=\frac{\sqrt{6}\pm i\sqrt{6}}{2}=\frac{\sqrt{6}}{2}(1\pm i)$$

$$\alpha=\sqrt{3}\left(e^{i\frac{\pi}{4}}\right),\beta=\sqrt{3}\left(e^{-i\frac{\pi}{4}}\right)$$

$$\therefore \frac{\alpha^{99}}{\beta}+\alpha^{98}=\alpha^{98}\left(\frac{\alpha}{\beta}+1\right)$$

$$=\frac{\alpha^{98}(\alpha+\beta)}{\beta}=3^{49}\left(e^{i99\frac{\pi}{4}}\right)\times\sqrt{2}$$

$$=3^{49}(-1+i)$$

$$=3^{n}(a+ib)$$

$$\therefore n=49, a=-1, b=1$$

$$\therefore n+a+b=49-1+1=49$$

82. (113)

a, b, c and in A.P

$\Rightarrow 2b=a+c\Rightarrow a-2b+c=0$

$\therefore$ ax + by + c passes through fixed point $(1,-2)$

$\therefore P=(1,-2)$

For infinite solution,

$D=D_1=D_2=D_3=0$

$$D:\begin{vmatrix}1 & 1 & 1\\ 2 & 5 & \alpha\\ 1 & 2 & 3\end{vmatrix}=0$$

$\Rightarrow \alpha=8$

$$D_1:\begin{vmatrix}6 & 1 & 1\\ \beta & 5 & \alpha\\ 4 & 2 & 3\end{vmatrix}=0\Rightarrow\beta=6$$

$\therefore Q=(8,6)$

$\therefore PQ^2=113$

83. (192) Parabola is $x^2=8y$

Chord with mid-point (x_1,y_1) is $T=S_1$

$\therefore xx_1-4(y+y_1)=x_1{}^2-8y_1$

$\therefore (x_1, y_1) = \left(1, \frac{5}{4}\right)$

$\Rightarrow x - 4\left(y + \frac{5}{4}\right) = 1 - 8 \times \frac{5}{4} = -9$

$\therefore x - 4y + 4 = 0$

(α, β) lies on (i) & also on $y^2 = 4x$

$\therefore \alpha - 4\beta + 4 = 0$

$\& \beta^2 = 4\alpha$

Solving (ii) & (iii)

$\beta^2 = 4(4\beta - 4) \Rightarrow \beta^2 - 16\beta + 16 = 0$

$\therefore \beta = 8 \pm 4\sqrt{3}$ and $\alpha = 4\beta - 4 = 28 \pm 16\sqrt{3}$

$\therefore (\alpha, \beta) = (28 + 16\sqrt{3}, 8 + 4\sqrt{3})$ &

$(28 - 16\sqrt{3}, 8 - 4\sqrt{3})$

$\therefore (\alpha - 28)(\beta - 8) = (\pm 16\sqrt{3})(\pm 4\sqrt{3})$

$= 192$

84. (6)

$= \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \sqrt{1 - \sin 2x} dx$

$= \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} |\sin x - \cos x| dx$

$= \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} (\cos x - \sin x) dx + \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} (\sin x - \cos x) dx$

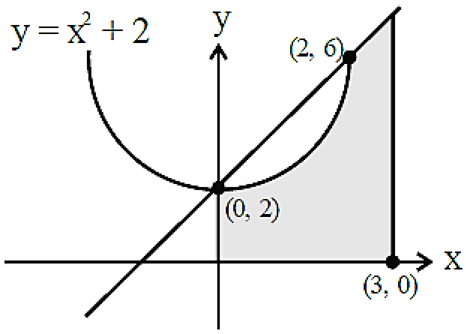
$= -1 + 2\sqrt{2} - \sqrt{3}$

$= \alpha + \beta\sqrt{2} + \gamma\sqrt{3}$

$\alpha = -1, \beta = 2, \gamma = -1$

$3\alpha + 4\beta - \gamma = 6$

85. (164)



$y = 2x + 2$

$A = \int_0^2 (x^2 + 2)dx + \int_2^3 (2x + 2)dx$

$A = \frac{41}{3}$

$12\text{ A} = 41 \times 4 = 164$

86. (9)

$L_1: \frac{x-5}{4} = \frac{y-4}{1} = \frac{z-5}{3} = \lambda \text{ drs}(4,1,3) = b_1 \; M(4\lambda + 5, \lambda + 4, 3\lambda + 5)$

$L_2: \frac{x+8}{12} = \frac{y+2}{5} = \frac{z+11}{9} = \mu$

$N(12\mu - 8, 5\mu - 2, 9\mu - 11)$

$\overrightarrow{MN} = (4\lambda - 12\mu + 13, \lambda - 5\mu + 6, 3\lambda - 9\mu + 16)$

Now

$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & 1 & 3 \\ 12 & 5 & 9 \end{vmatrix} = -6\hat{i} + 8\hat{k}$

Equation (a) and (b)

$\therefore \frac{4\lambda - 12\mu + 13}{-6} = \frac{\lambda - 5\mu + 6}{0} = \frac{3\lambda - 9\mu + 16}{8}$

I and II

$\lambda - 5\mu + 6 = 0$

I and III

$\lambda - 3\mu + 4 = 0$

Solve (c) and (d) we get

$\lambda = -1, \mu = 1$

$\therefore M(1,3,2)$

$N(4,3,-2)$

$\therefore \overrightarrow{OM} \cdot \overrightarrow{ON} = 4 + 9 - 4 = 9$

87. (2) $f(a) = 1, f(a) = 0$

$$f^2(x) = \lim_{r\to x}\left(\frac{2r^2(f^2(r) - f(x)f(r))}{r^2 - x^2} - r^3 e^{\frac{f(r)}{r}}\right)$$
$$= \lim_{r\to x}\left(\frac{2r^2 f(r)}{r+x}\frac{(f(r) - f(x))}{r-x} - r^3 e^{\frac{f(r)}{r}}\right)$$

$$f^2(x) = \frac{2x^2 f(x)}{2x} f'(x) - x^3 e^{\frac{f(x)}{x}}$$

$$y^2 = xy\frac{dy}{dx} - x^3 e^{\frac{y}{x}}$$

$$\frac{y}{x} = \frac{dy}{dx} - \frac{x^2}{y} e^{\frac{y}{x}}$$

Put $y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

$$v = v + x\frac{dv}{dx} - \frac{x}{v}e^v$$

$$\frac{dv}{dx} = \frac{e^v}{v} \Rightarrow e^{-v} v dv = dx$$

Integrating both side

$e^v(x + c) + 1 + v = 0$

$f(a) = 1 \Rightarrow x = 1, y = 1 \Rightarrow c = -1 - \frac{2}{e}$

$$e^v\left(-1 - \frac{2}{e} + x\right) + 1 + v = 0$$

$$e^{\frac{y}{x}}\left(-1 - \frac{2}{e} + x\right) + 1 + \frac{y}{x} = 0$$

$$x = a, y = 0 \Rightarrow a = \frac{2}{e}$$

$ae = 2$

88. (1)

Let $32^{32} = t$

$64^{33^{32}} = 64^t = 8^{2t} = (9 - 1)^{2t}$

$= 9k + 1$

Hence remainder $= 1$

89. (46)

$x^2 - 2^y = 2023$

$\Rightarrow x = 45, y = 1$

$\sum_{(x,y)\in C}(x + y) = 46$

90. (12) According to the question,

$$27r_1 + \frac{9r_2}{2} = -9$$

$$\lim_{x\to 3}\frac{\int_3^x 8t^2dt}{\frac{3r_2x}{2} - r_2x^2 - r_1x^3 - 3x}$$

$$= \lim_{x\to 3}\frac{8x^2}{\frac{3r_2^2}{2} - 2r_2x - 3r_1x^2 - 3}$$ (using LH' Rule)

$$= \frac{72}{\frac{3r_2}{2} - 6r_2 - 27r_1 - 3}$$

$$= \frac{72}{-\frac{9r_2}{2} - 27r_1 - 3}$$

$$= \frac{72}{9 - 3} = 12$$