Physics
Section - A

1. **(c)**

2. **(c)**

$$H_{Max} = \frac{(u\sin\theta)^2}{2g}$$

$$\frac{(H_{max})_1}{(H_{max})_2} = \frac{u^2\sin^2(45-\alpha)}{u^2\sin^2(45+\alpha)}$$

$$= \frac{\left(\frac{1}{\sqrt{2}}\cos\alpha - \frac{1}{\sqrt{2}}\sin\alpha\right)^2}{\left(\frac{1}{\sqrt{2}}\cos\alpha + \frac{1}{\sqrt{2}}\sin\alpha\right)^2}$$

$$= \frac{1-\sin 2\alpha}{1+\sin 2\alpha}$$

3. **(d)** $I\omega 2\theta = q\ell E_0\theta$

$$2m\left(\frac{\ell}{2}\right)^2\omega^2 = q\ell E_0$$

$$\omega^2 = \frac{2qE_0}{m\ell}$$

$$T = 2\pi\sqrt{\frac{m\ell}{2qE_0}}$$

4. **(b)**

$[\tau] = [E]$

$[\sigma] \neq [I]$

$[L] = [h]$

$[P] = [Y]$

5. **(b)** As h increases, g decreases, T increases

$$T = 2\pi\sqrt{\frac{\ell}{g}}$$

$$g = \frac{g_0R^2}{(R+h)^2}$$

6. **(a)**

$$[LT^{-1}] = [A][T^2] = \frac{[B][T]}{[C]+[T]}$$

$[C] = [T]$

$[A] = [LT^{-3}]$

$[B] = [LT^{-1}]$

$[ABC] = [L^2\,T^{-3}]$

7. **(c)**

$$\phi_1 = L_1I_1 + M_{12}I_2$$

$$\varepsilon_1 = -\frac{d\phi_1}{dt} = -L_1\frac{dI_1}{dt} - M_{12}\frac{dI}{2}$$

8. **(d)**

9. **(b)** Maximum possible magnetic field is at the surface

$$B_{max} = \frac{\mu_0 I}{2\pi a}$$

$$\frac{B_{max}}{2} = \frac{\mu_0 I}{4\pi a}$$

It can be obtained inside as well as outside the wire For inside,

$$\frac{\mu_0 I}{4\pi a}=\frac{\mu_0 I r}{2\pi a^2}$$

$$\Rightarrow r=\frac{a}{2}$$

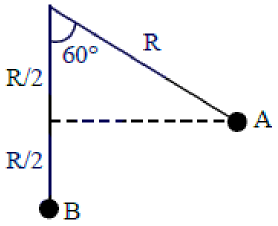
For outside

$$\frac{\mu_0 I}{4\pi a}=\frac{\mu_0 I}{2\pi r}$$

$$\Rightarrow r=2a$$

Correct answer $\left[\frac{a}{2}, 2a\right]$

10. (a)



Velocity of a just before hitting :

$$u=\sqrt{2\,g\frac{R}{2}}=\sqrt{gR}$$

Just after collision, let velocity of A and B are v_1 and v_2 respectively

∴ by COM:

$$mu=mv_1+\frac{m}{2}v_2$$

$$2v_1+v_2=2u \quad \ldots\ldots..(i)$$

$$e=1=\frac{v_2-v_1}{u}$$

$$\Rightarrow v_2-v_1=u \quad \ldots\ldots(ii)$$

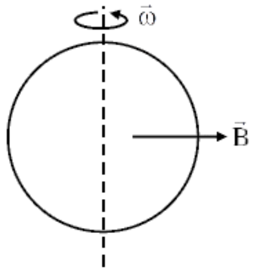
From (i) -(ii)

$$\Rightarrow 3v_1=u \Rightarrow v_1=\frac{u}{3}=\frac{1}{3}\sqrt{gR}$$

11. (d) (A) : True

(B) : True but not correct explanation

12. (b)



$$\phi = BAN\cdot\cos(\omega t)$$

$$\varepsilon = \frac{-d\phi}{dt}=BA\omega\, N\cdot\sin(\omega t)$$

When B is parallel to plane, $\underline{\underline{\omega t}}t=\frac{\pi}{2}$

$$\Rightarrow \phi=0, \varepsilon=BA\omega\, N$$

13. (d)

$$B = \frac{\rho g h}{\left(\frac{\Delta v}{v}\right)}$$

$$\frac{\Delta v}{v} \times 100 = \frac{\rho g h}{B} \times 100$$

$$\frac{1000 \times 10 \times 2.5 \times 10^3}{2 \times 10^9} \times 100\%$$

$$= 1.25\%$$

14. (b)

$$\lambda = \frac{h}{mv} = \frac{h}{\sqrt{2mK}}$$

$$\lambda^2 = \frac{h^2}{2\,m}\left(\frac{1}{k}\right)$$

$$Y = cx^2$$

Upward facing parabola passing through origin.

15. (a)

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	1

$\Rightarrow$ OR Gate

16. (d)

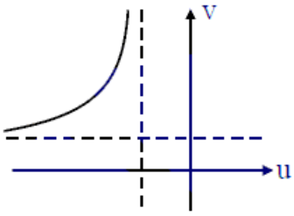
$$V_{Top} = \sqrt{n^2 gR}$$

$$V_{Bottom} = \sqrt{n^2 gR + 4gR}$$

$$\text{Ratio} = \frac{n^2 + 4}{n^2}$$

17. (b)

$$(u + f)(v - f) = f^2$$



18. (d) (A) $\rightarrow$ 0 (III)

(B) $\rightarrow \frac{\sigma}{2\varepsilon_0}$ (II)

(C) $\rightarrow \frac{\sigma R^2}{\varepsilon_0 r^2}$ (No row matching)

(D) $\rightarrow \frac{\sigma}{\varepsilon_0}$ (I)

19. (d) $\Delta W = -\Delta U = -nC_V\Delta T$

20. (b) Assertion is false because em waves have momentum.

Section - B

21. (2)

$$\vec{\tau} = \vec{r} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 1 & -1 & 1 \end{vmatrix}$$

$$\vec{\tau} = \hat{k}(-1-1) = -2\hat{k}$$

$$|\vec{\tau}| = 2\text{Nm}$$

22. (327)

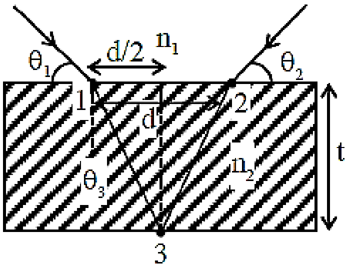
$$\frac{P_1}{T_1} = \frac{P_2}{T_2}$$

$$\frac{P}{300} = \frac{2P}{T_2}$$

$$T_2 = 600\text{ K}$$

$$T_2 = 327^\circ\text{C}$$

23. (6)



$$n_1 \sin(90-\theta_1) = n_2 \sin\theta_3$$

$$n_1 \cos\theta_1 = n_2 \sin\theta_3$$

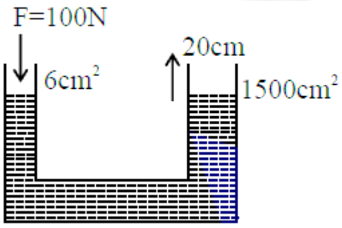
$$n_1 \frac{n_2}{2n_1} = n_2 \sin\theta_3$$

$$\frac{1}{2} = \sin\theta_3, \theta_3 = 30$$

$$\tan 30 = \frac{d}{2(t)}$$

$$t = \frac{d\sqrt{3}}{2} = \frac{4\sqrt{3}\times\sqrt{3}}{2}\text{ cm} = 6\text{ cm}$$

24. (5)



$$\frac{F_1}{A_1} = \frac{F_2}{A_2}, \frac{100}{6} = \frac{F}{1500}, F = \frac{50}{3}\times 1500$$

$$F = 50\times 500 = 25\times 10^3\text{ N}$$

$$\omega = \vec{F}\cdot\vec{S} = 25\times 10^3 \times \frac{20}{100}$$

$$= 5\times 10^3 = 5\text{ kJ}$$

25. (2000)

$\vec{v}_b = 9 + 27 = 36\text{ km/hr}$

$\vec{v}_b = 36 \times \frac{1000}{36000} = 10\text{ m/sec}$

Time of flight $= \frac{2\times 10}{10} = 2\text{sec}$

Range $= 10\times 2 = 20\text{ m} = 2000\text{ cm}$

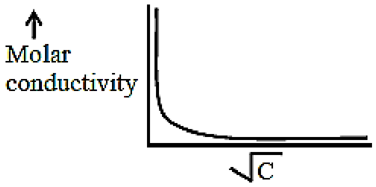
Chemistry
Section - A

26. (a) Total five nucleophiles are present

$NH_3, PhSH, (H_3C)_2S, CH_2=CH_2, \overset{\ominus}{O}H$

27. (d) Standard reduction potential value (+ve) increases oxidising capacity increases.

28. (d)



29. (c)

$$AB_{2(g)} \rightleftharpoons AB_{(g)} + \frac{1}{2}B_{2(g)}$$

$$t_{eq} \quad \frac{(1-x)}{1+\frac{x}{2}}P \quad \frac{xP}{1+\frac{x}{2}} \quad \frac{\left(\frac{x}{2}\right)P}{1+\frac{x}{2}}$$

$$\Rightarrow x \ll 1 \Rightarrow 1+\frac{x}{2} \simeq 1 \text{ and } 1-x \simeq 1$$

$$\Rightarrow k_P = \frac{(xp)\cdot\left(\frac{xp}{2}\right)^{\frac{1}{2}}}{P}$$

$$\Rightarrow k_P^2 = x^2 \cdot \frac{xP}{2}$$

$$x = \sqrt[3]{\frac{2k_P^2}{P}}$$

30. (3)

31. (d)

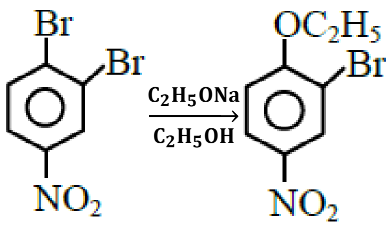
32. (c)

(I) $[Mn(CN)_6]^{3-}$ $\quad -1.6\Delta_\circ$
(II) $[Co(CN)_6]^{4-}$ $\quad -1.8\Delta_\circ$
(III) $[Fe(CN)_6]^{4-}$ $\quad -2.4\Delta_0$
(IV) $[Fe(CN)_6]^{3-}$ $\quad -2.0\Delta_\circ$

I < II < IV < III

33. (d)

34. (a) It is an example of nucleophillic Aromatic substitution reaction.



35. (b) According to Nernst equation :-

$$E = E^\circ - \frac{RT}{nF}\ln Q.$$

Cell reaction :-

$$Mg_{(s)} + 2Ag^{+}_{(aq)} \rightleftharpoons 2Ag_{(s)} + Mg^{+2}_{(aq)}$$

$$\Rightarrow Q = \frac{[Mg^{+2}]}{[Ag^{+}]^{2}}$$

$$\Rightarrow E = E^{\circ}_{Cell} - \frac{RT}{2F}\ln\left[\frac{[Mg^{+2}]}{[Ag^{+}]^{2}}\right]$$

36. (c) M.P. $\Rightarrow$ Mn < Fe, Tc < Ru, Os < Re

37. (d)

For $AX_2: -\Delta T_b = K_b \times m \times i$

$$0.0156 = 0.52 \times \frac{0.01}{1} \times i_{AX_2}$$

$\Rightarrow i_{AX_2} = 3 \Rightarrow$ complete ionisation

For $AY_2: -\Delta T_b = K_b \times m \times i$

$0.026 = 0.52 \times 0.0508 \times i_{AY_2}$

$\Rightarrow i_{AY_2} \simeq 1 \therefore$ complete unionisation

38. (a)

$q_p = n \times c_p \times \Delta T$

$$\Rightarrow 500 = 0.5 \times \frac{5}{2} \times 8.3(T_f - 298)$$

$\Rightarrow T_f \simeq 346.2$ K

$$\frac{\Delta H}{\Delta U} = \frac{C_p}{C_v} = \left(\frac{5}{3}\right)$$

$$\Rightarrow \Delta U = \frac{3}{5} \times 500 = 300 \text{ J}$$

39. (a)

rate $= k_2[A][B_2]$ ...... (1)

$$\left(\frac{k_1}{k_{-1}}\right) = \left(\frac{[A]^2}{[A_2]}\right)$$

$$\Rightarrow [A] = \sqrt{\frac{k_1}{k_{-1}}} \cdot \sqrt{[A_2]}$$

Substituting in (1) ; we get

$$\text{Rate} = k_2\sqrt{\frac{k_1}{k_{-1}}} \cdot [A_2]^{\frac{1}{2}} \cdot [B_2]$$

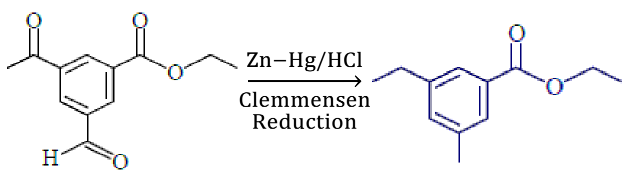
$$\therefore \text{ order } = \left(\frac{3}{2}\right) = 1.5$$

40. (b) $2\pi r_n = n\lambda$

$2\pi(4a_0) = n\lambda$

$$= \lambda = \frac{8\pi a_0}{n}$$

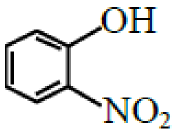
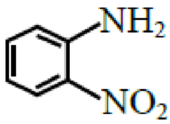
41. (c)



42. (a) Difference between I.E. & E.G.E increases, ionic character increases.

43. (b)

44. (c)

(A)  & (B) 

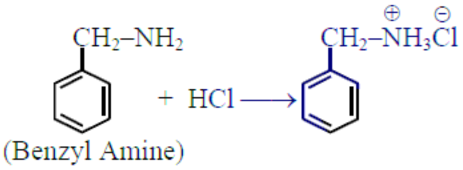
are steam volatile due to intramolecular hydrogen bonding.

45. (d) (i) For isoelectronic species -ve charge increases, radii increases.

(ii) Magnitude of E.G.E : Cl > F > Br > I

Section - B

46. (341) Benzyl Amine is most basic due to localised lone pair.



Mole of benzyl Amine $\Rightarrow \frac{1}{107} = 0.00934$ mole

1 Mole of Benzyl amine consumed 1 mole of HCl

So, Mole of HCl consumed $\rightarrow 0.00934$ mole

Mass of HCl consumed $\rightarrow 0.00934 \times$ molar mass of HCl

$= 0.00934 \times 36.5$

$= 0.341\text{gm}$

$= 341\text{mg}$

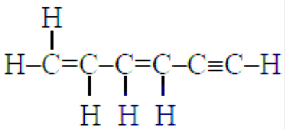
47. (160)

$4FeCr_2O_4 + 8Na_2CO_3 + 7O_2 \rightarrow 8Na_2CrO_4 + 2Fe_2O_3 + 8CO_2$

Fe_2O_3 is water insoluble, so its molar mass

$\Rightarrow [2 \times 56 + 3 \times 16] \Rightarrow 160$ g/mol

48. (15)



Number of σ bond = 11

Number of π bond = 4

σ + π = 11 + 4 = 15

49. (16)

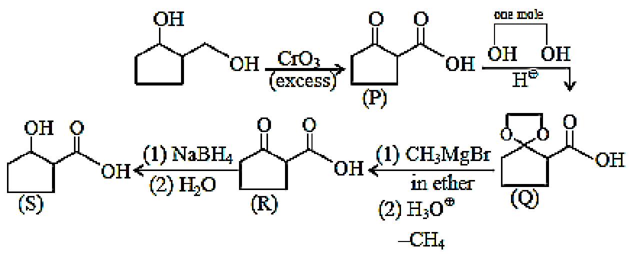
$A_2B \rightarrow 2A^+ + B^{-2}; y = 3$

$\alpha = 0.3$

$i = 1 + (y-1)\alpha$

$= 1 + (3-1)(0.3) = 1.6 = 16 \times 10^{-1}$

50. (13)

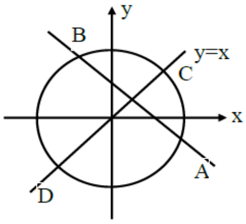


0.1 mole of compound (S) weight in gm = 0.1 × molar mass of compound (S) = 0.1 × 130 = 13gm

Mathematics

Section - A

51. (b)



By solving $x = y$ with circle
We get

$C(\sqrt{2}, \sqrt{2})$

$D(-\sqrt{2}, -\sqrt{2})$

By solving $x + y = 1$ with circle $x^2 + y^2 = 4$ we set

$A\left(\frac{1+\sqrt{7}}{2}, \frac{1-\sqrt{7}}{2}\right)$

& $B\left(\frac{1-\sqrt{7}}{2}, \frac{1+\sqrt{7}}{2}\right)$

$\therefore$ Area of Quadrilateral ACBD $= 2 \times$ Area of $\triangle$ BCD

$$= 2 \times \frac{1}{2}\begin{vmatrix} \sqrt{2} & \sqrt{2} & 1 \\ \frac{1-\sqrt{7}}{2} & \frac{1+\sqrt{7}}{2} & 1 \\ -\sqrt{2} & -\sqrt{2} & 1 \end{vmatrix}$$

$= 2\sqrt{14}$

52. (a)

$$\begin{vmatrix} 1+\sin^2 x & \cos^2 x & 4\sin 4x \\ \sin^2 x & 1+\cos^2 x & 4\sin 4x \\ \sin^2 x & \cos^2 x & 1+4\sin 4x \end{vmatrix}, x \in R$$

$R_2 \to R_2 - R_1 \& R_3 \to R_3 - R_1$

$$f(x)\begin{vmatrix} 1+\sin^2 x & \cos^2 x & 4\sin 4x \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{vmatrix}$$

Expand about R_1, we get

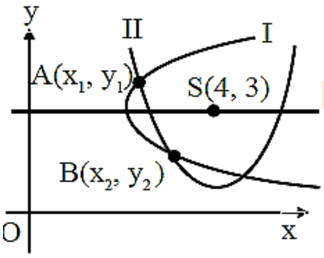
$f(x) = 2 + 4\sin 4x$

$\therefore M =$ max value of $f(x) = 6$

$m =$ min value of $f(x) = -2$

$\therefore M^4 - m^4 = 1280$

53. (a)



Let intersection points of these two parabolas are $A(x_1, y_1)$& $B(x_2, y_2)$

$\because$ equation of parabola I and II are given below

$\therefore (x-4)^2 + (y-3)^2 = x^2$ ……… (1)

$\&(x-4)^2 + (y-3)^2 = y^2$ …… (2)

Here $A(x_1, y_1)$& $B(x_2, y_2)$ will satisfy the equation Also from equations (1) & (2), we get $x = y$..(3)

Put $x = y$ in equation (1)

We get $x^2 - 14x + 25 = 0$

$x_1 + x_2 = 14$

$x_1 x_2 = 25$

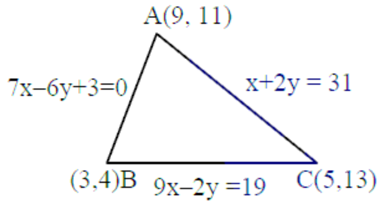
$\therefore AB^2 = (x_1 - x_2)^2 + (y_1 - y_2)^2$

$= 2(x_1 - x_2)^2$

$= 2[(x_1 + x_2)^2 - 4x_1x_2]$

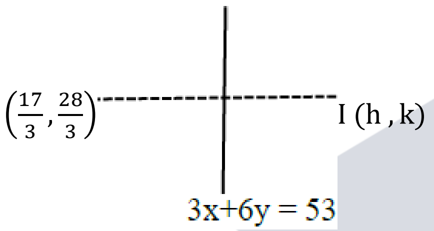
$= 192$

54. (a)



$$\therefore \text{ centroid of } \triangle ABC = \left(\frac{9+3+5}{3}, \frac{11+4+13}{3}\right)$$

$$= \left(\frac{17}{3}, \frac{28}{3}\right)$$



Let image of centroid with respect to line mirror is (h, k)

$$\therefore \left(\frac{k - \frac{28}{3}}{h - \frac{17}{3}}\right)\left(-\frac{1}{2}\right) = -1$$

$$\& 3\left(\frac{h + \frac{17}{3}}{2}\right) + 6\left(\frac{k + \frac{28}{3}}{2}\right) = 53$$

Solving (1) & (2) we get $h = 3, k = 4$

$\therefore h^2 + k^2 + hk = 37$

55. (c)

$\vec{a} = 2\hat{i} - \hat{j} + 3\hat{k}$
$\vec{b} = 3\hat{i} - 5\hat{j} + \hat{k}$
$\vec{a} \times \vec{c} = \vec{c} \times \vec{b}$
$\vec{a} \times \vec{c} + \vec{b} \times \vec{c} = 0$
$(\vec{a} + \vec{b}) \times \vec{c} = 0$
$\Rightarrow \vec{c} = \lambda(\vec{a} + \vec{b})$
$\vec{c} = \lambda(5\hat{i} - 6\hat{j} + 4\hat{k}) \ldots . (1)$
$|\vec{c}|^2 = \lambda^2(25 + 36 + 16)$
$|\vec{c}|^2 = 77\lambda^2$
$(\vec{a} + \vec{c}) \cdot (\vec{b} + \vec{c}) = 168$
$\vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{c} + \vec{c} \cdot \vec{b} + |\vec{c}|^2 = 168$
$14 + \vec{c} \cdot (\vec{a} + \vec{b}) + 77\lambda^2 = 168$
using equation (1)
$\lambda|5\hat{i} - 6\hat{j} + 4\hat{k}|^2 + 77\lambda^2 = 154$
$77\lambda + 77\lambda^2 - 154 = 0$
$\lambda^2 + \lambda - 2 = 0$
$\lambda = -2, 1$
$\therefore$ Maximum value of $|\vec{c}|^2$ occurs when $\lambda = -2$
$|\vec{c}|^2 = 77\lambda^2$
$= 77 \times 4$
$= 308$

56. (d) (i) number of numbers created using

$1111133 = \dfrac{7!}{5!\,2!} \Rightarrow 21$

(ii) number of numbers created using

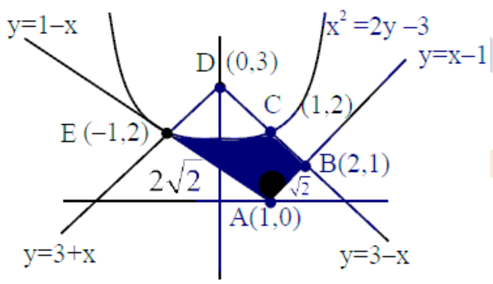
$1111223 = \dfrac{7!}{4!\,2!} \Rightarrow 105$

(iii) number of numbers created using

$1112222 = \dfrac{7!}{4!\,3!} \Rightarrow 35$

Total $= 161$

57. (d)



A $\Rightarrow$ Rectangle ABDE - Area of region EDC

$$A \Rightarrow 4 - 2\int_0^1 (3 - x) - \left(\frac{x^2 + 3}{2}\right) dx$$

$$A \Rightarrow 4 - 2\left\{3x - \frac{x^2}{2} - \frac{x^3}{6} - \frac{3}{2}x\right\}_0^1$$

$$A \Rightarrow 4 - 2\left\{3 - \frac{1}{2} - \frac{1}{6} - \frac{3}{2}\right\} = \frac{7}{3}$$

So $6A = 14$

58. (a) General term $= {}^nC_r\left(7^{1/3}\right)^{n-r}\left(11^{1/12}\right)^r$

$= {}^{n}C_r(7)^{\frac{n-r}{3}}(11)^{r/12}$

For integral terms, r must be multiple of 12

$\therefore r = 12k, k \in W$

Total values of $r = 183$

Hence $\max r = 12(182)$

$= 2184$

Min value of $n = 2184$

59. (b)

Consider $\frac{1}{\sqrt{x}} = \alpha \qquad (x > 0)$

$(9\alpha^2 - 9\alpha + 2)(2\alpha^2 - 7\alpha + 3) = 0$

$(3\alpha - 2)(3\alpha - 1)(\alpha - 3)(2\alpha - 1) = 0$

$\alpha = \frac{1}{3}, \frac{1}{2}, \frac{2}{3}, 3$

$x = 9, 4, \frac{9}{4}, \frac{1}{9}$

So, no. of solutions $= 4$

60. (d)

61. (a)

$\sec^2 x - \tan^2 x = 1$ (on replacing y with x)

$\Rightarrow$ Reflexive

$\sec^2 x - \tan^2 y = 1$

$\Rightarrow 1 + \tan^2 x + 1 - \sec^2 y = 1$

$\Rightarrow \sec^2 y - \tan^2 x = 1$

$\Rightarrow$ symmetric

$\sec^2 x - \tan^2 y = 1$

$\sec^2 y - \tan^2 z = 1$

Adding both

$\Rightarrow \sec^2 x - \tan^2 y + \sec^2 y - \tan^2 z = 1 + 1$

$\sec^2 x + 1 - \tan^2 z = 2$

$\sec^2 x - \tan^2 z = 1$

$\Rightarrow$ Transitive

hence equivalence relation

62. (d)

$2ae = 4$

$2a\left(\frac{1}{\sqrt{3}}\right) = 4$

$\Rightarrow a = 2\sqrt{3}$

$\Rightarrow 1 - \frac{b^2}{12} = \frac{1}{3} \Rightarrow b^2 = 8$

Now $\frac{2b^2}{a} \cdot \frac{2A^2}{B} = \frac{32}{\sqrt{3}} \Rightarrow 2\left(\frac{8}{2\sqrt{3}}\right)\frac{2A^2}{B} = \frac{32}{\sqrt{3}}$

$\Rightarrow A^2 = 2B$

$1 - \frac{A^2}{B^2} = \frac{1}{3} \Rightarrow 1 - \frac{2B}{B^2} = \frac{1}{3} \Rightarrow B = 3$

$\Rightarrow A^2 = 6$

$\frac{x^2}{12} + \frac{y^2}{8} = 1 \ldots..(1)$

$\frac{x^2}{6} + \frac{y^2}{9} = 1 \ldots.(2)$

On solving (1) & (2) we get

$(x,y) \equiv \left(\frac{\sqrt{6}}{\sqrt{5}}, \frac{6}{\sqrt{5}}\right), \left(\frac{-\sqrt{6}}{\sqrt{5}}, \frac{6}{\sqrt{5}}\right), \left(\frac{\sqrt{6}}{\sqrt{5}}, \frac{-6}{\sqrt{5}}\right), \left(\frac{-\sqrt{6}}{\sqrt{5}}, \frac{-6}{\sqrt{5}}\right)$

he four points are vertices of rectangle and its area = $\frac{24\sqrt{6}}{5}$

63. (c)

$S_3 = 3a + 3d = 54$

$\Rightarrow a + d = 18$

$S_{20} = 10(2a + 19d)$

$\Rightarrow 10(36 + 17d)$

$\Rightarrow 1600 < 10(36 + 17d) < 1800$

$\Rightarrow 160 < 36 + 17d < 180$

$\Rightarrow 124 < 17d < 144$

$\Rightarrow 7\frac{5}{17} < d < 8\frac{8}{17}$

Common difference will be natural number

$\Rightarrow d = 8 \Rightarrow a = 10$

$\Rightarrow a_{11} = 10 + 10 \times 8 = 90$

64. (a)

$L_1: \vec{r} = (-\hat{i} + 2\hat{j} + \hat{k}) + \lambda(\hat{i} + 2\hat{j} + \hat{k})$

$\Rightarrow \vec{r} = (\lambda - 1)\hat{i} + 2(\lambda + 1)\hat{j} + (\lambda + 1)\hat{k}$

$L_2: \vec{r} = (\hat{j} + \hat{k}) + \mu(2\hat{i} + 7\hat{j} + 3\hat{k})$

$\Rightarrow \vec{r} = 2\mu\hat{i} + (1 + 7\mu)\hat{j} + (1 + 3\mu)\hat{k}$

For point of intersection equating respective components

$\Rightarrow \lambda - 1 = 2\mu \quad \ldots\ldots(1)$

$2(\lambda + 1) = 1 + 7\mu \quad \ldots\ldots(2)$

$\lambda + 1 = 1 + 3\mu \quad \ldots.(3)$

We get

$\Rightarrow \lambda = 3$ and $\mu = 1$

$\Rightarrow \vec{a} + \vec{b} = 3\hat{i} + 9\hat{j} + 4\hat{k}$

$L_3: \vec{r} = 2\hat{i} + 8\hat{j} + 4\hat{k} + \alpha(3\hat{i} + 9\hat{j} + 4\hat{k})$

For $\alpha = 2, \vec{r} = 8\hat{i} + 26\hat{j} + 12\hat{k}$

65. (d)

$$\lim_{n\to\infty} \sum_{k=1}^{n} \frac{k^3 + 6k^2 + 11k + 5}{(k+3)!}$$

$$= \lim_{n\to\infty} \sum_{k=1}^{n} \frac{k^3 + 6k^2 + 11k + 6 - 1}{(k+3)!}$$

$$= \lim_{n\to\infty} \sum_{k=1}^{n} \frac{(k+1)(k+2)(k+3) - 1}{(k+3)!}$$

$$= \lim_{n\to\infty} \sum_{k=1}^{n} \frac{(k+1)(k+2)(k+3)}{(k+3)!} - \frac{1}{(k+3)!}$$

$$= \lim_{n\to\infty} \sum_{k=1}^{n} \left(\frac{1}{k!} - \frac{1}{(k+3)!}\right)$$

$$= \lim_{n\to\infty} \left(\frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} \ldots + \frac{1}{n!} - \frac{1}{4!} - \frac{1}{5!} - \frac{1}{6!} \ldots - \frac{1}{(n+3)!}\right)$$

$$= \frac{1}{1} + \frac{1}{2} + \frac{1}{6} = \frac{10}{6} = \frac{5}{3}$$

66. (c)

$$I = 80\int_0^{\frac{\pi}{4}}\left(\frac{\sin\theta+\cos\theta}{9+16(2\sin\theta\cdot\cos\theta)}\right)d\theta$$

$$= 80\int_0^{\frac{\pi}{4}}\frac{\sin\theta+\cos\theta}{9-16(1-2\sin\theta\cdot\cos\theta-1)}d\theta$$

$$= 80\int_0^{\frac{\pi}{4}}\frac{\sin\theta+\cos\theta}{9+16-16(\sin\theta-\cos\theta)^2}\ d\theta$$

Let $\sin\theta-\cos\theta = t$

$(\cos\theta+\sin\theta)d\theta = dt$

$$= 80\int_{-1}^{0}\frac{dt}{25-16t^2}$$

$$= \frac{80}{16}\int_{-1}^{0}\frac{dt}{\left(\frac{5}{4}\right)^2-t^2}$$

$$= \frac{5}{2\left(\frac{5}{4}\right)}\ln\left[\left|\frac{\frac{5}{4}+t}{\frac{5}{4}-t}\right|\right]_{-1}^{0}$$

$= 2\ln(1)+4\ln 3$

$= 4\ln 3$

67. (c)

$$\text{DR's of } L_3 = \vec{m}\times\vec{n} = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & -1 & 2\\ -1 & 2 & 1\end{vmatrix}$$

$= -5\hat{i}-3\hat{j}+\hat{k}$

$$L_3: \frac{x-\alpha}{-5} = \frac{y-\beta}{-3} = \frac{z-\gamma}{1} = \lambda$$

$A(\alpha-5\lambda, \beta-3\lambda, \gamma+\lambda)$

$$L_1: \frac{x-1}{1} = \frac{y-2}{-1} = \frac{z-1}{2} = k$$

$B(k+1, -k+2, 2k+1)$

Now

$\alpha-5\lambda = k+1 \Rightarrow \alpha = 5\lambda+k+1$

$\beta-3\lambda = -k+2 \Rightarrow \beta = 3\lambda-k+2$

$\gamma+\lambda = 2k+1 \Rightarrow \gamma = -\lambda+2k+1$

$|5\alpha-11\beta-8\gamma| = |-25|$

$= 25$

68. (a)

$$\sum_{i=1}^{10} x_i = 50, \ \therefore \ \text{mean} = 5$$

$$\text{Variance} = \frac{4}{5} = \frac{\sum x_i^2}{10} - \left(\frac{\sum x_i}{10}\right)^2$$

$$\frac{4}{5} = \frac{\sum x_i^2}{10} - 25$$

$$\Rightarrow \sum x_i^2 = 258 \quad \ldots.(1)$$

$$\text{Now} \sum_{i=1}^{10} (x_i - \beta)^2 = 98$$

$$\sum_{i=1}^{10} \left(x_i^2 - 2\beta \cdot x_i + \beta^2\right) = 98$$

$$258 - 2\beta(50) + 10\beta^2 = 98$$

$$(\beta - 8)(\beta - 2) = 0$$

$$\beta = \text{ or } \beta = 2 \text{ (as } \beta > 2)$$

$$\therefore \beta = 8 \quad \ldots..(2)$$

Now as per the question

$$2(x_1 - 1) + 4\beta, 2(x_2 - 1) + 4\beta, \ldots.2(x_{10} - 1) + 4\beta$$

can be simplified to

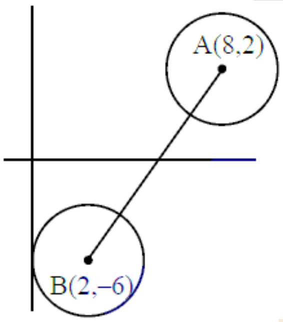
$$2x_1 + 30, 2x_2 + 30, \ldots..2x_{10} + 30 \text{ using eq. (2)}$$

$$\mu = 2(5) + 30 = 40 \quad \ldots..(3)$$

$$\sigma^2 = 2^2\left(\frac{4}{5}\right) = \frac{16}{5}$$

$$\because \frac{\beta\mu}{\sigma^2} = \frac{8 \times 40}{16/5} = 100$$

69. (b)



$$\because AB = \sqrt{100} = 10$$

$$\therefore |Z_1 - Z_2|_{min} = 10 - 2 - 1 = 7$$

70. (c)

$|A| = \frac{11}{2}$

$C_{11} = \sum_{k=1}^{2} a_{1k} \cdot A_{1k} = a_{11}\, A_{11} + a_{12}\, A_{12} = |A| = \frac{11}{2}$

$C_{12} = \sum_{k=1}^{2} a_{1k} \cdot A_{2k} = 0$

$C_{21} = \sum_{k=1}^{2} a_{2k} \cdot A_{1k} = 0$

$C_{22} = \sum_{k=1}^{2} a_{2k} \cdot A_{2k} = |A| = \frac{11}{2}$

$C = \begin{bmatrix} 11/2 & 0 \\ 0 & 11/2 \end{bmatrix}$

$|C| = \frac{121}{4}$

$8|C| = 242$

Section - B

71. (112)

$\int_0^1 f(\lambda x) d\lambda = af(x)$

$\lambda x = t$

$d\lambda = \frac{1}{x} dt$

$\frac{1}{x} \int_0^x f(t) dt = af(x)$

$\int_0^x f(t) dt = axf(x)$

$f(x) = a(xf'(x) + f(x))$

$(1 - a)f(x) = a \cdot xf'(x)$

$\frac{f'(x)}{f(x)} = \frac{(1 - a)}{a} \frac{1}{x}$

$\ell nf(x) = \frac{1 - a}{a} \ell nx + c$

$x = 1, f(1) = 1 \Rightarrow c = 0$

$x = 16, f(16) = \frac{1}{8}$

$\frac{1}{8} = (16)^{\frac{1-a}{a}} \Rightarrow -3 = \frac{4 - 4a}{a} \Rightarrow a = 4$

$f(x) = x^{-\frac{3}{4}}$

$f'(x) = -\frac{3}{4} x^{-\frac{7}{4}}$

$\therefore\ 16 - f'\left(\frac{1}{16}\right)$

$= 16 - \left(-\frac{3}{4}(2^{-4})^{-7/4}\right)$

$= 16 + 96 = 112$

72. (2)

$A = \begin{bmatrix} 2 & -1 \\ 1 & 0 \end{bmatrix}$

$A^2 = \begin{bmatrix} 3 & -2 \\ 2 & -1 \end{bmatrix}, A^3 = \begin{bmatrix} 4 & -3 \\ 3 & -2 \end{bmatrix}, A^4 = \begin{bmatrix} 5 & -4 \\ 4 & -3 \end{bmatrix}$

and so on

$$A^6 = \begin{bmatrix} 7 & -6 \\ 6 & -5 \end{bmatrix}$$

$$A^m = \begin{bmatrix} m+1 & -m \\ m & -m+1 \end{bmatrix},$$

$$A^{m^2} = \begin{bmatrix} m^2+1 & -m^2 \\ m^2 & -(m^2-1) \end{bmatrix}$$

$$A^{m^2} + A^m = 3I - A^{-6}$$

$$\begin{bmatrix} m^2+1 & -m^2 \\ m^2 & -(m^2-1) \end{bmatrix} + \begin{bmatrix} m+1 & -m \\ m & -m+1 \end{bmatrix}$$

$$= 3\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} -5 & 6 \\ -6 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & -6 \\ 6 & -4 \end{bmatrix}$$

$$= m^2 + 1 + m + 1 = 8$$

$$= m^2 + m - 6 = 0 \Rightarrow \ m = -3,2$$

$$n(\ s) = 2$$

73. (24)

$$\cdot \lim_{x\to 0^+}\left(x\left(\left[\frac{1}{x}\right]+\left[\frac{2}{x}\right]+\cdots.+\left[\frac{p}{x}\right]\right)-x^2\left(\left[\frac{1}{x^2}\right]+\left[\frac{2^2}{x^2}\right]+\left[\frac{9^2}{x^2}\right]\right)\right)\geq 1$$

$$(1+2+\cdots\ldots.+p)-(1^2+2^2+\cdots.9^2)\geq 1$$

$$\frac{p(p+1)}{2}-\frac{9.10.19}{6}\geq 1$$

$$p(p+1)\geq 572$$

Least natural value of p is 24

74. (1405)

(i) Single letter is used , then no. of words $= 5$

(ii) Two distinct letters are used, then no. of words

$${}^5C_2 \times \left(\frac{6!}{2!\,4!}\times 2+\frac{6!}{3!\,3!}\right) = 10(30+20) = 500$$

(iii) Three distinct letters are used, then no. of words

$${}^5C_3 \times \frac{6!}{2!\,2!\,2!} = 900$$

Total no. of words $= 1405$

75. (5)

$$\cos^{-1}x = \pi + \sin^{-1}x + \sin^{-1}(2x+1)$$

$$2\cos^{-1}x - \sin^{-1}(2x+1) = \frac{3\pi}{2}$$

$2\alpha - \beta = \frac{3\pi}{2}$ where $\cos^{-1}x = \alpha, \sin^{-1}(2x+1) = \beta$

$$2\alpha = \frac{3\pi}{2} + \beta$$

$$\cos 2\alpha = \sin\beta$$

$$2\cos^2\alpha - 1 = \sin\beta$$

$$2x^2 - 1 = 2x + 1$$

$$x^2 - x - 1 = 0$$

$$\Rightarrow x = \frac{1-\sqrt{5}}{2}, \left\{x = \frac{1+\sqrt{5}}{2} \text{ rejected}\right\}$$

$$\therefore 4x^2 - 4x = 4$$

$$(2x-1)^2 = 5$$