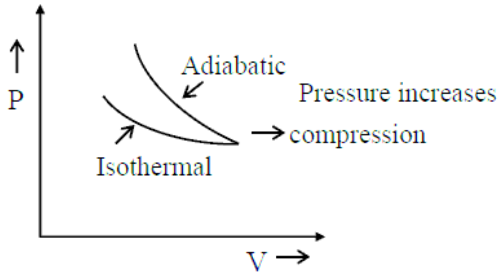
Physics
Section - A

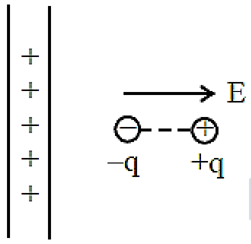
1. **(c)** See-back effect
Low thermal conductivity
High electrical conductivity

2. **(c)**



$$\left(\frac{dP}{dV}\right)_{Adiabatic} > \left(\frac{dP}{dV}\right)_{Isothermal}$$

3. **(c)**



Here $E = \frac{\sigma}{2\,\epsilon_0}, \vec{\tau} = \vec{P} \times \vec{E}$

$\vec{\tau} = 0$

$U = -\vec{P} \cdot \vec{E}\ U \rightarrow$ minimum

4. **(c)**

$$\frac{hC}{\lambda} = W + eV_S$$

$$\frac{hC}{\lambda} = W + (K_{max})$$

$$\therefore V_S = \frac{K_{max}}{e}$$

5. **(a)**

$$\phi = -2 \times 10^4 \frac{Nm^2}{C}$$

$r = 8.0$ cm

$$\phi = \frac{q}{\epsilon_0} \Rightarrow q = \epsilon_0 \phi$$

$$= (8.85 \times 10^{-12}) \times (-2 \times 10^4)$$

$$q = -17.7 \times 10^{-8} C$$

6. **(b)**

$\Delta Q_{AB} = 0$ adiabatic

$\Delta Q_{BC} = \Delta W_{BC}$

$$= nRT\ln\left(\frac{V_C}{V_B}\right) = 450R\ln\left(\frac{8 \times 10^{-6}}{6 \times 10^{-6}}\right)$$

$$= 450R\ln\left(\frac{4}{3}\right) = 450R(\ln 4 - \ell n3)$$

$$\therefore \Delta Q = \Delta Q_{AB} + \Delta Q_{BC}$$

$$\Delta Q = 450R(\ln 4 - \ell n3)$$

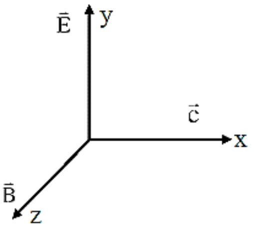
7. (d)

$f_{L_1} = f_{L_2} = f$

$f_{L_3} = f_{L_4} = 2f$

$\therefore f_{L_1} : f_{L_3} = 1:2$

8. (b)



Direction of propogation

$= \vec{E} \times \vec{B}$

9. (b) For B

$$\frac{\mu_2}{V} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

$$\frac{1.5}{V} + \frac{1}{\frac{R}{2}} = \frac{0.5}{-R}$$

$$\frac{1.5}{V} = -\frac{1}{2R} - \frac{2}{R}$$

$$\frac{1.5}{V} = \frac{-5}{2R} \Rightarrow V_B = -0.6R$$

For A

$$\frac{1.5}{V} + \frac{2}{3R} = \frac{0.5}{-R}$$

$$\frac{1.5}{V} = -\frac{1}{2R} - \frac{2}{3R}$$

$$\frac{1.5}{V} = -\frac{7}{6R}$$

$$V_A = -\frac{9}{7}R$$

Distance between images

$$= 2R - \left(0.6R + \frac{9}{7}R\right) = 0.114R$$

10. (a)

$V_1 = A_1\omega_1$

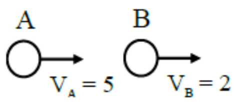
$V_2 = A_2\omega_2$

$A_1 = A_2$

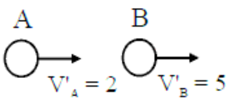
$$\frac{V_1}{V_2} = \frac{\omega_1}{\omega_2} = \frac{\sqrt{\frac{K_1}{m}}}{\sqrt{\frac{K_2}{m}}}$$

$$\frac{V_1}{V_2} = \sqrt{\frac{K_1}{K_2}}$$

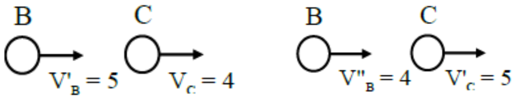
11. (d) In elastic collision for same mass, velocities interchange



Before collision



After collision



Before collision After collision

$V_A' = 2\ m/s\ V_B' = 4\ m/s\ V_C' = 5\ m/s$

12. (d)

$$\text{Power } = \vec{F} \cdot \vec{V}$$
$$F = \frac{dp}{dt}[p = mv]$$
$$F = \left(\frac{dm}{dt}\right)v = C(\sqrt{v})v$$
$$F = Cv^{\frac{3}{2}}$$
$$\text{Power } = C\left(v^{3/2}\right)v = Cv^{5/2}$$
$$p^2 \propto v^5$$

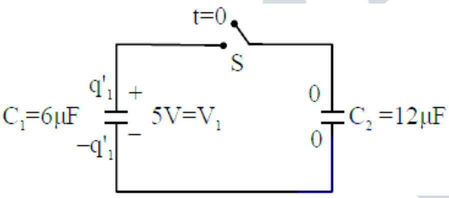
13. (b)

$$\frac{1}{8} = \left(\frac{\mu_\ell}{\mu_s} - 1\right)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$
$$\frac{1}{24} = (1.5 - 1)\left[\frac{2}{R}\right] \quad \ldots..(i)$$
$$\frac{1}{f'} = \left(\frac{1.5}{1.33} - 1\right)\left(\frac{2}{R}\right)$$
$$\frac{1}{f'} = \left(\frac{1.5 \times 3}{4} - 1\right)\frac{2}{R} \quad \ldots.(ii)$$

(i) divided by (ii)

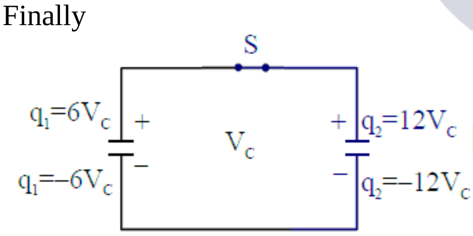
$$\frac{f'}{24} = 4$$
$$f' = 96\text{ cm}$$

14. (c)



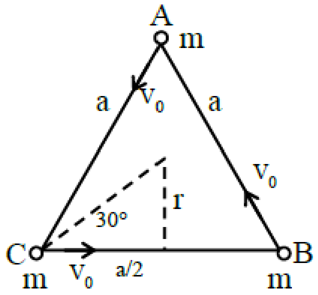
$q_1' = 6 \times 5 = 30\mu C$

Finally



$$6\ V_c + 12\ V_c = 30 + 0$$
$$18\ V_c = 30$$
$$V_C = \frac{30}{18} = \frac{5}{3}\text{Volt}$$
$$\Rightarrow q_1 = \frac{6 \times 5}{3} = 10\mu C$$
$$\Rightarrow q_2 = \frac{12 \times 5}{3} = 20\mu C$$

15. (c)



$$\tan 30^\circ = \frac{2r}{a} = \frac{1}{\sqrt{3}}$$

$$r = \frac{a}{2\sqrt{3}}$$

$$L = (mvr_\perp) \times 3$$

$$= mv_0 \frac{a}{2\sqrt{3}} \times 3$$

$$= \frac{\sqrt{3}}{2} mv_0 a$$

16. (d)

(A) $[Y] = \frac{F}{A\left(\frac{\Delta \ell}{\ell}\right)} \Rightarrow \frac{MLT^{-2}}{L^2} = ML^{-1}\,T^{-2}$(II)

(B) Torque $(\vec{\tau}) = \vec{r} \times \vec{F}$

$(\vec{\tau}) = L \times MLT^{-2} = ML^2\,T^{-2}$(IV)

(C) Coefficient of viscosity $\Rightarrow F = \eta A \frac{dV}{dt}$

$\eta \to Pa \cdot sec$

$[\eta] = \frac{MLT^{-2}}{L^2} \times T = ML^{-1}\,T^{-1}$ (I)

(D) Gravitational constant (G)

$$F = \frac{GM_1M_2}{r^2}$$

$$[G] = \frac{F \cdot r^2}{m_1\, m_2} = \frac{MLT^{-2} \times L^2}{M^2} = M^{-1}\,L^3\,T^{-2}\text{(III)}$$

17. (b) (A) Magnetic induction → Gauss (III)

(B) Magnetic intensity

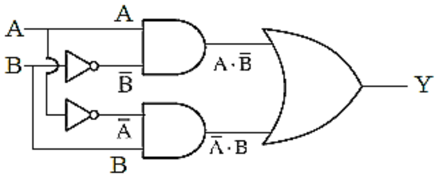
$\left(H = \frac{B}{\mu}\right) \to$ Ampere / meter (IV)

(C) Magnetic flux → Weber (Wb) (II)

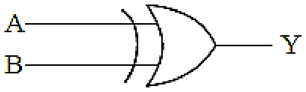
(D) Magnetic moment → Ampere-meter ^{2}

$(\vec{M} = i\,\vec{A})$

18. (a)



$Y = A \cdot \overline{B} + \overline{A} \cdot B$



XOR (Exclusive OR)

19. (a) By using average form of Newton's law of cooling

$$\frac{90-80}{t}=k\left(\frac{90+80}{2}-20\right) \quad \ldots\ldots..(i)$$

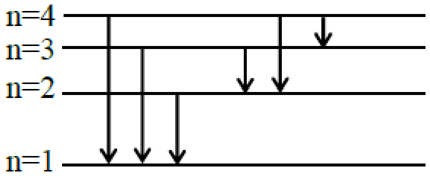
$$\frac{80-60}{t'}=k\left(\frac{80+60}{2}-20\right) \quad \ldots..(ii)$$

(i)/(ii)

$$\frac{10\times t'}{t\times 20}=\frac{65}{50}$$

$$t'=\frac{65}{50}\times 2t=\frac{65}{25}t=\frac{13}{5}t$$

20. (a)



Total possible transition = 6

Section - B

21. (8) Assuming long solenoid

$$B=\mu_0\left(\frac{N}{\ell}\right)i$$

$$\ell=\frac{\mu_0 Ni}{B}=\frac{(4\pi\times10^{-7})(200)(0.29)}{2.9\times10^{-4}}\ m$$

$$=8\pi\ cm$$

22. (1320)

$$V=\frac{Q}{C}=\frac{it}{\left(\frac{\epsilon_0 A}{d}\right)}=\frac{itd}{\epsilon_0(\pi r^2)}$$

$$\Rightarrow d=\frac{\epsilon_0(\pi r^2)}{i}\left(\frac{V}{t}\right)$$

$$=\frac{(9\times10^{-12})\left(\frac{22}{7}\right)(0.1)^2}{0.15}(7\times10^8)m$$

$$d=1320\mu\ m$$

23. (7700)

$$Q=\frac{ab^4}{cd}$$

$$\Rightarrow\frac{\Delta Q}{Q}\times100=\left[\frac{\Delta a}{a}+4\frac{\Delta b}{b}+\frac{\Delta c}{c}+\frac{\Delta d}{d}\right]\times100$$

$$\Rightarrow\frac{x}{1000}=\left[\frac{3}{60}+4\left(\frac{0.1}{20}\right)+\left(\frac{0.2}{40}\right)+\frac{0.1}{50}\right]\times100$$

$$\Rightarrow x=7700$$

24. (8)

$$L=mv_0R=m\sqrt{\frac{GM}{R}}R=m\sqrt{GMR}$$

here M is mass of star

$$\frac{L_B}{L_A} = \frac{m_B}{m_A}\sqrt{\frac{R_B}{R_A}}$$

$$= 4\sqrt{2}\sqrt{\frac{2}{1}}$$

$$\frac{L_B}{L_A} = 8$$

25. (3)

$a_p = kt$, k is constant

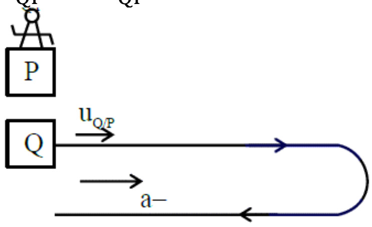
$a_Q = a$, a is constant

$a_{QP} = a_Q - a_p = a - kt$

as initial velocities are not mentioned in question, so will have to assume two cases.

Case-I

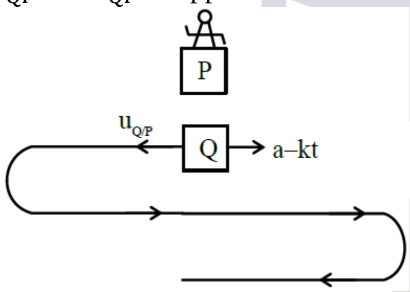
u_{QP} and a_{QP} in same direction



Total number of crossing = 2

Case-II

u_{QP} and a_{QP} in opposite direction



Total number of crossing = 3

Chemistry
Section - A

26. (b)

27. (b)

28. (b) Statement I is true.

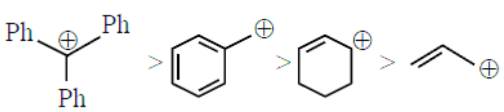
In partition chromatography, stationary phase is thin liquid film present in the inert support.

Statement II is false.

Because stationary phase in paper chromatography is water.

29. (a) Valine, Lysine and Threonine are essential amino acids.

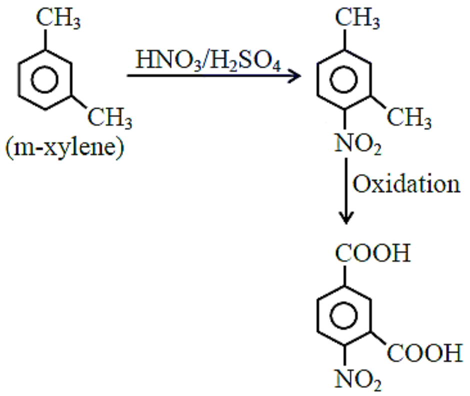
30. (d) Stability order of carbocation



31. (a)

32. (b)

33. (c) Statement-I



Statement-II $-CH_3$ group is o/p directing while $-NO_2$ group is meta directing.

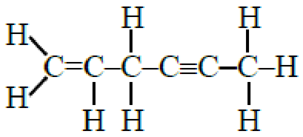
34. (a)

35. (a)

36. (a)

37. (a)

38. (a)



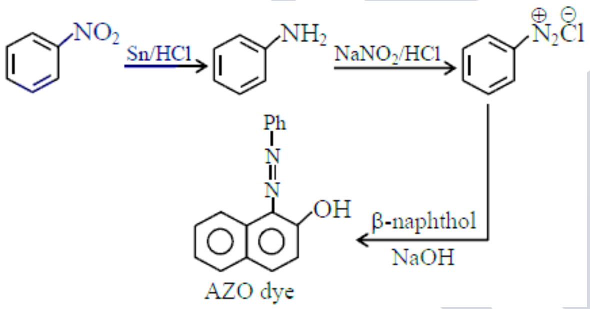
σ bonds = 13

π bonds = 3

39. (d)

40. (b)

41. (a)

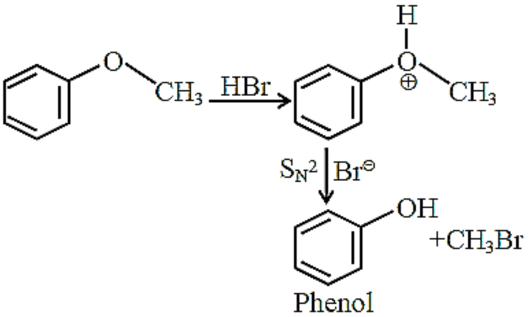


42. (d)

43. (a)

44. (a)

45. (b)



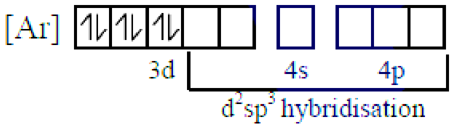
Section - B

46. (0) Yellow coloured & low spin complex are

$K_3[Co(NO_2)_6]$ & $K_4[Fe(CN)_6]$

(I) $K_3[Co(NO_2)_6]$
$Co^{+3} \Rightarrow [Ar]3\,d^6$

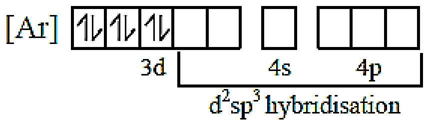
In presence of ligand field



Magnetic moment $= \sqrt{n(n+2)}\,BM$
$= 0\,BM$

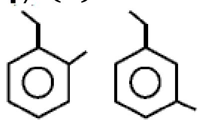
(II) $K_4[Fe(CN)_6]$
$Fe^{+2} \Rightarrow [Ar]3\,d^6$

In presence of ligand field



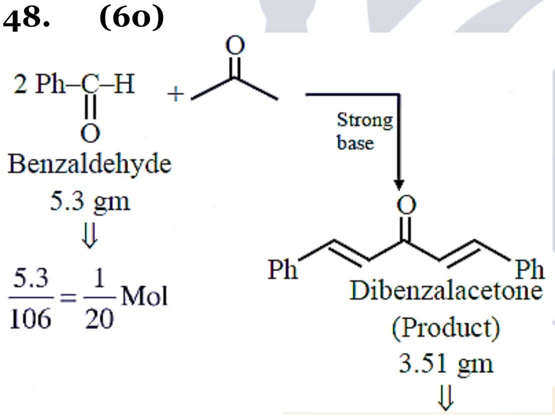
Magnetic moment $= \sqrt{n(n+2)}\,BM$
$= 0\,BM$

47. (b)



Above two isomers of C_9H_{12} have four different sites for aromatic electrophilic substitution reaction.

48. (60)



$\frac{3.51}{234} = 0.015\,Mol$

(Actual)

Theoretical $= \frac{1}{40}\,Mol$

% yield $= \frac{0.015}{1/40} \times 100 \Rightarrow 60\%$

49. (275)
50. (12)

Mathematics
Section - A

51. (c)

$(a-5)^2 - 8(15-3a) < 0$
$a^2 + 14a + 25 - 120 < 0$
$a^2 + 14a - 95 < 0$
$(a+19)(a-5) < 0$
$a \in (-19, 5)$
$\therefore -19 < x < 5$
$\therefore \sum_{x \in X} x^2 = (1^2 + 2^2 + \cdots. + 4^2) + (1^2 + 2^2 + \cdots + 18^2)$
$= \frac{4 \times 5 \times 9}{6} + \frac{18 \times 19 \times 37}{6}$
$= 30 + 2109$
$= 2139$

52. (c)

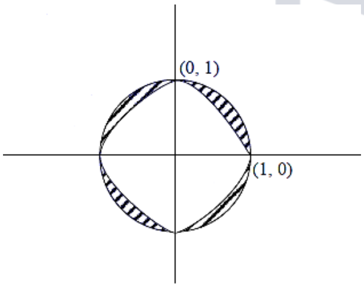
$\sin x + \sin^2 x = 1$
$\Rightarrow \sin x = \cos^2 x \Rightarrow \tan x = \cos x$

$\therefore$ Given expression

$= 2\cos^{12} x + 6[\cos^{10} x + \cos^8 x] + 2\cos^6 x$
$= 2[\sin^6 x + 3\sin^5 x + 3\sin^4 x + \sin^3 x]$
$= 2\sin^3 x[(\sin x + 1)^3]$
$= 2[\sin^2 x + \sin x]^3$
$= 2$

53. (d)

$C_1 : |y| = 1 - x^2$
$C_2 : x^2 + y^2 = 1$



$\therefore$ Required Area
$= \alpha = 4[\text{Area of circle in } 1^{st} \text{ quad.} - \int_0^1 (1 - x^2)dx]$

$= 4\left[\frac{\pi}{4} - \left[x - \frac{x^3}{3}\right]_0^1\right]$

$\alpha = \pi - \frac{8}{3}$

$\therefore 3\alpha = 3\pi - 8$
$\therefore 9\alpha = 9\pi - 24$
$\therefore \beta = 9, \gamma = -24$
$\therefore |\beta - \gamma| = 33$

54. (c)

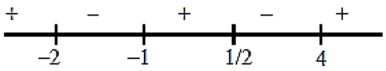
$f_1(x) = \log_5(18x - x^2 - 77)$
$\therefore\ 18x - x^2 - 77 > 0$
$x^2 - 18x + 77 < 0$
$x \in (7,11)\alpha = 7, \beta = 11$

$f_2(x) = \log_{(x-1)}\left(\frac{2x^2 + 3x - 2}{x^2 - 3x - 4}\right)$

$\therefore\ x - 1 > 0, x - 1 \neq 1, \frac{2x^2 + 3x - 2}{x^2 - 3x - 4} > 0$

$x > 1, x \neq 2, \frac{(2x-1)(x+2)}{(x-4)(x+1)} > 0$

$x > 1, x \neq 2,$



$\therefore\quad x \in (4, \infty)$
$\therefore\quad \gamma = 4$
$\therefore\quad \alpha^2 + \beta^2 + \gamma^2 = 49 + 121 + 16$
$= 186$

55. (a)

56. (c) Vector parallel to ' L '

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -2 \\ 1 & 3 & 4 \end{vmatrix} = 10\hat{i} - 10\hat{j} + 5\hat{k}$$

$= 5(2\hat{i} - 2\hat{j} + \hat{k})$

Equation of 'L'

$\frac{x-2}{2} = \frac{y+1}{-2} = \frac{z-3}{1} = \lambda(\text{ say })$

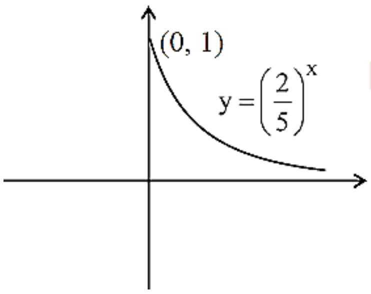
Let $Q(2\lambda + 2, -2\lambda - 1, \lambda + 3)$
$\Rightarrow 2\lambda + 2 = 0 \Rightarrow \lambda = -1$
$\Rightarrow Q(0,1,2)$
$d(P, Q) = 3$

57. (b)

$S = \{0,1,2,3 \ldots \ldots\}$

$\log_e y = \log_e\left(\frac{2}{5}\right)$

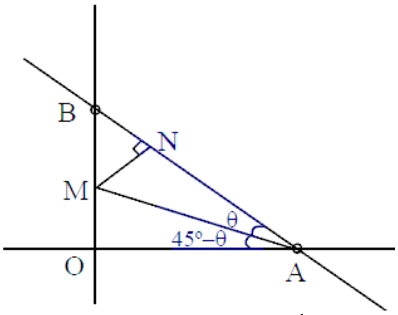
$\Rightarrow y = \left(\frac{2}{5}\right)^x$



Required

$\text{Sum} = 1 + \left(\frac{2}{5}\right)^1 + \left(\frac{2}{5}\right)^2 + \left(\frac{2}{5}\right)^3 + \cdots .. -= \frac{1}{1 - \frac{2}{5}} = \frac{5}{3}$

58. (d)



Area of $\triangle AOB = \frac{1}{2}$

Area of $\triangle AMN = \frac{4}{9} \times \frac{1}{2} = \frac{2}{9}$

Equation of AB is $x + y = 1$

$OA = 1, AM = \sec(45^\circ - \theta)$

$AN = \sec(45^\circ - \theta)\cos\theta$

$MN = \sec(45^\circ - \theta)\sin\theta$

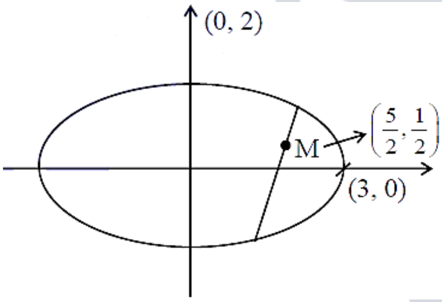
$Ar(\triangle AMN) = \frac{1}{2} \times \sec^2(45^\circ - \theta)\sin\theta \cdot \cos\theta = \frac{2}{9}$

$\Rightarrow \tan\theta = 2, \frac{1}{2}$

$\tan\theta = 2$ is rejected

$\frac{AN}{NB} = \frac{\lambda}{1} = \cot\theta = 2$

59. (c)



Equation of chord $T = S_1$

$\frac{5}{2}\left(\frac{x}{9}\right) + \frac{1}{2}\left(\frac{y}{4}\right) = \frac{25}{36} + \frac{1}{16}$

$\Rightarrow \frac{5x}{18} + \frac{y}{8} = \frac{100 + 9}{144} = \frac{109}{144}$

$\Rightarrow 40x + 18y = 109$

$\Rightarrow \alpha = 40, \beta = 18$

$\Rightarrow \alpha + \beta = 58$

60. (c)

61. (a)

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 4 & 10 \end{vmatrix} = 1(20 - 16) - 1(10 - 4) + 1(4 - 2)$$

$= 4 - 6 + 2 = 0$

For infinite solutions

$\Delta_x = \Delta_y = \Delta_z = 0$
$m^2 - 3x + 2 = 0$
$m = 1,2$
$\alpha = 1, \beta = 2$

$$\therefore \sum_{n=1}^{10} (n^{\alpha} + n^{\beta}) = \sum_{n=1}^{10} n^1 + \sum_{n=1}^{10} n^2$$

$$= \frac{10(11)}{2} + \frac{10(11)(21)}{6}$$

$= 55 + 385$
$= 440$

62. (d)

$$A = \begin{bmatrix} (\sqrt{2})^2 & (\sqrt{2})^3 & (\sqrt{2})^4 \\ (\sqrt{2})^3 & (\sqrt{2})^4 & (\sqrt{2})^5 \\ (\sqrt{2})^4 & (\sqrt{2})^5 & (\sqrt{2})^6 \end{bmatrix}$$

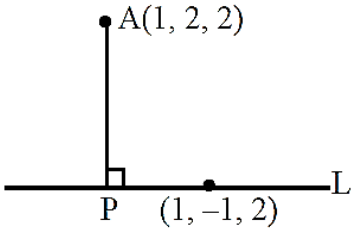
$$A = \begin{bmatrix} 2 & 2\sqrt{2} & 4 \\ 2\sqrt{2} & 4 & 4\sqrt{2} \\ 4 & 4\sqrt{2} & 8 \end{bmatrix}$$

$$A^2 = 2^2 \begin{bmatrix} 1 & \sqrt{2} & 2 \\ \sqrt{2} & 2 & 2\sqrt{2} \\ 2 & 2\sqrt{2} & 4 \end{bmatrix} \begin{bmatrix} 1 & \sqrt{2} & 2 \\ \sqrt{2} & 2 & 2\sqrt{2} \\ 2 & 2\sqrt{2} & 4 \end{bmatrix}$$

$$= 4 \begin{bmatrix} - & - & - \\ - & - & - \\ (2+4+8) & (2\sqrt{2} + 4\sqrt{2} + 8\sqrt{2}) & (4+8+16) \end{bmatrix}$$

Sum of elements of 3rd row $= 4(14 + 14\sqrt{2} + 28)$
$= 4(42 + 14\sqrt{2})$
$= 168 + 56\sqrt{2}$
$\alpha + \beta\sqrt{2}$
$\therefore \alpha \quad \alpha + \beta = 168 + 56 = 224$

63. (a)



$\longrightarrow (1, -1, 2) \equiv \vec{d}$

$$L: \frac{x-1}{1} = \frac{y+1}{-1} = \frac{z-2}{2} = \mu$$

$P(\mu + 1, -\mu - 1, 2\mu + 2)$

$\overrightarrow{AP} \cdot \vec{d} = 0 \Rightarrow (\mu, -\mu - 3, 2\mu) \cdot (1, -1, 2) = 0$

$$\Rightarrow \mu + \mu + 3 + 4\mu = 0 \Rightarrow \mu = -\frac{1}{2}$$

$$\therefore P\left(\frac{-1}{2} + 1, +\frac{1}{2} - 1, 2\left(\frac{-1}{2}\right) + 2\right)$$

$$P\left(\frac{1}{2}, \frac{-1}{2}, 1\right)$$

Now general pt. on L_2 is $Q(-1 + \lambda, 1 - \lambda, -2 + \lambda)$
Equate it with general pt of L

$\mu + 1 = -1 + \lambda \quad -\mu - 1 = 1 - \lambda \qquad 2\mu + 2 = -2 + \lambda$

$\mu = \lambda - 2 \qquad \mu = \lambda - 2 \qquad \downarrow$

$2(\lambda - 2) + 2 = -2 + \lambda$

$2\lambda - 4 + 2 = -2 + \lambda$

$\therefore\ \mu = -2, \lambda = 0$

$\therefore\ Q \equiv (-1, 1 - 2)$

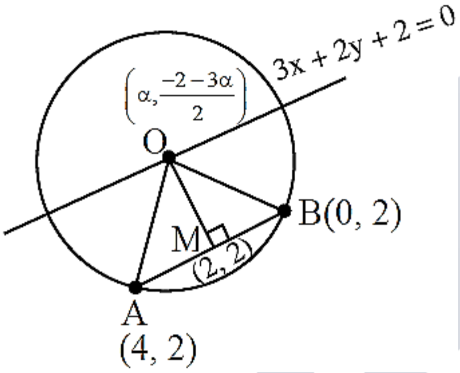
$P\left(\frac{1}{2}, \frac{-1}{2}, 1\right)$ and $Q(-1,1,-2)$

$$PQ = \sqrt{\left(\frac{1}{2}+1\right)^2 + \left(\frac{-1}{2}-1\right)^2 + (1+2)^2}$$

$$= \sqrt{\frac{9}{4} + \frac{9}{4} + 9} = \sqrt{\frac{54}{4}}$$

$$\therefore\ 2(PQ)^2 = 2\left(\frac{54}{4}\right) = 27$$

64. (b)



$M_{AB} = 0 \Rightarrow$ OM is vertical

$\Rightarrow \alpha = 2$

$\therefore$ Centre $(0) \equiv (2, -4)$

$r = OA = \sqrt{(2-4)^2 + (2+4)^2} = \sqrt{40}$

mid point of chord is $N \equiv (1,2)$ $\therefore ON = \sqrt{37}$

$\therefore$ length of chord $= 2\sqrt{r^2 - (ON)^2}$

$= 2\sqrt{40 - 37} = 2\sqrt{3}$

65. (b)

$|A| = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$

$= a_{11}a_{22} - a_{21}a_{12}$

$= \{-1, 0, 1\}$

x	P_i	P_iX_i	$P_1X_i^2$
-1	$\frac{3}{16}$	$-\frac{3}{16}$	$\frac{3}{16}$
0	$\frac{10}{16}$	0	0
1	$\frac{3}{16}$	$\frac{3}{16}$	$\frac{3}{16}$
		$\sum P_iX_i = 0$	$\sum P_iX_i^2 = \frac{3}{8}$

$\therefore \operatorname{var}(x) = \sum P_i X_i^2 - \left(\sum P_i X_i\right)^2$

$= \dfrac{3}{8} - 0 = \dfrac{3}{8}$

66. (d)

Bag 1 = {4 W, 5 B}

Bag 2 = {nW, 3B}

$P\left(\dfrac{W}{\text{Bag2}}\right) = \dfrac{29}{45}$

$\Rightarrow P\left(\dfrac{W}{B_1}\right) \times P\left(\dfrac{W}{B_2}\right) + P\left(\dfrac{B}{B_1}\right) \times P\left(\dfrac{W}{B_2}\right) = \dfrac{29}{45}$

$\dfrac{4}{9} \times \dfrac{n+1}{n+4} + \dfrac{5}{9} \times \dfrac{n}{n+4} = \dfrac{29}{45}$

$n = 6$

67. (a)

$7^{103} = 7(7^{102}) = 7(343)^{34} = 7(345-2)^{34}$

$7^{103} = 23\,K_1 + 7.2^{34}$

Now $7.2^{34} = 7.2^2 \cdot 2^{32}$

$= 28 \cdot (256)^4$

$= 28(253+3)^4$

$\therefore 28 \times 81 \Rightarrow (23+5)(69+12)$

$23\,K_2 + 60$

$\therefore$ Remainder $= 14$

68. (a) $f'(x) = x^3 - 9x^2 + 20x = x(x-4)(x-5)$

$$\begin{array}{ccccccc} - & & + & & - & & + \\ \hline & 0 & & 4 & & 5 & \end{array}$$

$\therefore f(x) = \dfrac{x^4}{4} - \dfrac{9x^3}{3} + \dfrac{20x^2}{2}$

$f(1) = \dfrac{1}{4} - 3 + 10 = \dfrac{29}{4} = \alpha$

$f(4) = \dfrac{256}{4} - 3(64) + 10(16) = 32 = \beta$

$4(\alpha + \beta) = 4\left(\dfrac{29}{4} + 32\right) = 157$

69. (c) Let $\vec{v} = \hat{i} + \hat{j} + \hat{k}$

$\vec{b} \times \vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 3 \\ 2 & 3 & -1 \end{vmatrix}$

$= -7\hat{i} + 7\hat{j} + 7\hat{k}$

$= -7(\hat{i} - \hat{j} - \hat{k})$

Now $\hat{a} = \frac{\hat{i}-\hat{j}-\hat{k}}{\sqrt{3}}$ or $\frac{-\hat{i}+\hat{j}+\hat{k}}{\sqrt{3}}$

$\downarrow \qquad\qquad \downarrow$

$\cos\theta = \dfrac{\hat{a}\cdot\vec{v}}{|\vec{v}|} = \dfrac{1-1-1}{\sqrt{3}\sqrt{3}} = \dfrac{-1}{3}$ $\quad \cos\theta = \dfrac{\hat{a}\cdot\vec{v}}{|\vec{v}|} = \dfrac{-1+1+1}{3} = \dfrac{1}{3}$

(rejected)

$\Rightarrow \hat{a} = \dfrac{\hat{i} - \hat{j} - \hat{k}}{\sqrt{3}}$

$$\text{Now } \cos\frac{\pi}{3} = \frac{\hat{a}\cdot(\hat{\imath}+\alpha\hat{\jmath}+\hat{k})}{\sqrt{1+\alpha^2+1}}$$

$$\Rightarrow \frac{1}{2} = \frac{1-\alpha-1}{\sqrt{3}\sqrt{\alpha^2+2}}$$

$$\Rightarrow \frac{\sqrt{3}}{2}\sqrt{\alpha^2+2} = -\alpha\ (\because \alpha<0)$$

$$3\alpha^2+6 = 4\alpha^2$$

$$\Rightarrow \alpha = -\sqrt{6}$$

70. (d)

Section -B

71. (12)

$$= \quad 24\int_0^{\frac{\pi}{48}} -\sin\left(4x-\frac{\pi}{12}\right) + \int_{\pi/48}^{\pi/4}\sin\left(4x-\frac{\pi}{12}\right)$$

$$+\int_0^{\frac{\pi}{6}}[0]dx + \int_{\pi/6}^{\pi/4}[2\underset{1}{\downarrow}\sin x]dx$$

$$= \quad 24\left[\frac{\left(1-\cos\frac{\pi}{12}\right)}{4} - \frac{\left(-\cos\frac{\pi}{12}-1\right)}{4}\right] + \frac{\pi}{4}-\frac{\pi}{6}$$

$$= \quad 24\left(\frac{1}{2}+\frac{\pi}{12}\right) = 2\pi+12$$

$\alpha = \quad 12$

72. (64)

1^{∞} form

$$\text{Now } L = e^{t\to 0}\,\frac{1}{t}\left(\frac{(3x+5)^{t+1}}{3(t+1)}\bigg|_0^1 - 1\right)$$

$$= e^{t\to 0}\,\frac{8^{t+1}-5^{t+1}-3t-3}{3t(t+1)}$$

$$= e\,\frac{8\ell\ln 8 - 5\ln 5 - 3}{3}$$

$$= \left(\frac{8}{5}\right)^{2/3}\left(\frac{64}{5}\right) = \frac{\alpha}{5e}\left(\frac{8}{5}\right)^{2/3}$$

On comparing

$\alpha = 64$

73. (11132)

$a_1 + a_5 + a_{10} + \cdots .. + a_{2020} + a_{2024} = 2233$

In an A.P. the sum of terms equidistant from ends is equal.

$$a_1 + a_{2024} = a_5 + a_{2020} = a_{10} + a_{2015} \ldots \ldots$$

$\Rightarrow 203$ pairs

$$\Rightarrow 203(a_1 + a_{2024}) = 2233$$

Hence,

$$S_{2024} = \frac{2024}{2}(a_1 + a_{2024})$$

$= \quad 1012 \times 11$

$= \quad 11132$

74. (10)

$a, b \in I, -3 \le a, b \le 3, a + b \ne 0$

$|z - a| = |z + b|$

$$\begin{vmatrix} z+1 & \omega & \omega^2 \\ \omega & z+\omega^2 & 1 \\ \omega^2 & 1 & z+\omega \end{vmatrix} = 1$$

$$\Rightarrow \begin{vmatrix} z & z & z \\ \omega & z+\omega^2 & 1 \\ \omega^2 & 1 & z+\omega \end{vmatrix} = 1$$

$$\Rightarrow \begin{vmatrix} 1 & 1 & 1 \\ \omega & z+\omega^2 & 1 \\ \omega^2 & 1 & z+\omega \end{vmatrix} = 1$$

$$\Rightarrow z\begin{vmatrix} 1 & 1 & 1 \\ \omega & z+\omega^2 & 1 \\ \omega^2 & 1 & z+\omega \end{vmatrix} = 1$$

$$\Rightarrow z\begin{vmatrix} 1 & 0 & 0 \\ \omega & z+\omega^2-\omega & 1-\omega \\ \omega^2 & 1-\omega^2 & z+\omega-\omega^2 \end{vmatrix} = 1$$

$\Rightarrow z^3 = 1$

$\Rightarrow z = \omega, \omega^2, 1$

Now

$|1 - a| = |1 + b|$

$\Rightarrow$ 10 pairs

75. (1328)

$y^2 = 12x$ $a = 3$ $SP \times SQ = \frac{147}{4}$

Let $P(3t^2, 6t)$ and $t_1 t_2 = -1$

(ends of focal chord)

So, $Q\left(\frac{3}{t^2}, \frac{-6}{t}\right)$

$S(3,0)$

$SP \times SQ = PM_1 \times QM_2$

(dist from directrix)

$$= (3 + 3t^2)\left(3 + \frac{3}{t^2}\right) = \frac{147}{4}$$

$$\Rightarrow \frac{(1+t^2)^2}{t^2} = \frac{49}{12}$$

$$t^2 = \frac{3}{4}, \frac{4}{3}$$

$$t = \pm\frac{\sqrt{3}}{2}, \pm\frac{2}{\sqrt{3}}$$

considering $t = \frac{-\sqrt{3}}{2}$

$P\left(\frac{9}{4}, -3\sqrt{3}\right)$ and $Q(4, 4\sqrt{3})$

Hence, diametric circle:

$$(x - 4)\left(x - \frac{9}{4}\right) + (y + 3\sqrt{3})(y - 4\sqrt{3}) = 0$$

$$\Rightarrow x^2 + y^2 - \frac{25}{4}x - \sqrt{3}y - 27 = 0$$

$\Rightarrow \alpha = 400, \beta = 1728$

$\beta - \alpha = 1328$