

## Physics Section -A

1. The difference of temperature in a material can convert heat energy into electrical energy. To harvest the heat energy, the material should have

- low thermal conductivity and low electrical conductivity
- high thermal conductivity and high electrical conductivity
- low thermal conductivity and high electrical conductivity
- high thermal conductivity and low electrical conductivity

2. Given below are two statements. One is labelled as Assertion (A) and the other is labelled as Reason (R).

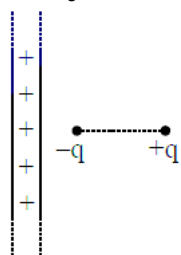
Assertion (A) : With the increase in the pressure of an ideal gas, the volume falls off more rapidly in an isothermal process in comparison to the adiabatic process.

Reason (R) : In isothermal process,  $PV = \text{constant}$ , while in adiabatic process  $PV^\gamma = \text{constant}$ . Here  $\gamma$  is the ratio of specific heats,  $P$  is the pressure and  $V$  is the volume of the ideal gas.

In the light of the above statements, choose the correct answer from the options given below :

- Both (A) and (R) are true but (R) is NOT the correct explanation of (A)
- (A) is true but (R) is false
- Both (A) and (R) are true and (R) is the correct explanation of (A).
- (A) is false but (R) is true

3. An electric dipole is placed at a distance of 2 cm from an infinite plane sheet having positive charge density  $\sigma_0$ . Choose the correct option from the following.



- Torque on dipole is zero and net force is directed away from the sheet.
- Torque on dipole is zero and net force acts towards the sheet.
- Potential energy of dipole is minimum and torque is zero.
- Potential energy and torque both are maximum

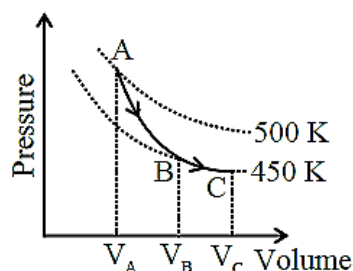
4. In an experiment with photoelectric effect, the stopping potential.

- increases with increase in the wavelength of the incident light
- increases with increase in the intensity of the incident light
- is  $\left(\frac{1}{e}\right)$  times the maximum kinetic energy of the emitted photoelectrons
- decreases with increase in the intensity of the incident light

5. A point charge causes an electric flux of  $-2 \times 10^4 \text{ Nm}^2\text{C}^{-1}$  to pass through a spherical Gaussian surface of 8.0 cm radius, centred on the charge. The value of the point charge is : (Given  $\epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}$ )

- $-17.7 \times 10^{-8} \text{ C}$
- $-15.7 \times 10^{-8} \text{ C}$
- $17.7 \times 10^{-8} \text{ C}$
- $15.7 \times 10^{-8} \text{ C}$

6.

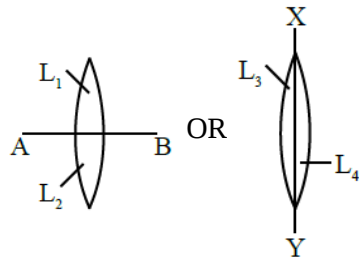


A poly-atomic molecule ( $C_V = 3R$ ,  $C_p = 4R$ , where  $R$  is gas constant) goes from phase space point A ( $P_A = 10^5 \text{ Pa}$ ,  $V_A = 4 \times 10^{-6} \text{ m}^3$ ) to point B ( $P_B = 5 \times 10^4 \text{ Pa}$ ,  $V_B = 6 \times 10^{-6} \text{ m}^3$ ) to point C ( $P_C = 10^4 \text{ Pa}$ ,  $V_C = 8 \times 10^{-6} \text{ m}^3$ ). A to B is an adiabatic path and B to C is an isothermal path.

The net heat absorbed per unit mole by the system is :

- (a)  $500R(\ln 3 + \ln 4)$  (b)  $450R(\ln 4 - \ln 3)$   
(c)  $500R\ln 2$  (d)  $400R\ln 4$

7. Two identical symmetric double convex lenses of focal length  $f$  are cut into two equal parts  $L_1, L_2$  by AB plane and  $L_3, L_4$  by XY plane as shown in figure respectively. The ratio of focal lengths of lenses  $L_1$  and  $L_3$  is

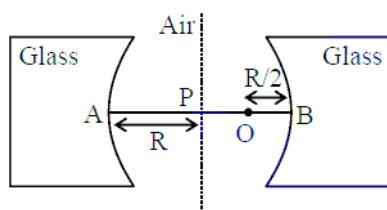


- (a) 1:4 (b) 1:1  
(c) 2:1 (d) 1:2

8. A plane electromagnetic wave propagates along the  $+x$  direction in free space. The components of the electric field,  $\vec{E}$  and magnetic field,  $\vec{B}$  vectors associated with the wave in Cartesian frame are :

- (a)  $E_y, B_x$  (b)  $E_y, B_z$   
(c)  $E_x, B_y$  (d)  $E_z, B_y$

9.



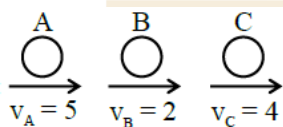
Two concave refracting surfaces of equal radii of curvature and refractive index 1.5 face each other in air as shown in figure. A point object O is placed midway, between P and B. The separation between the images of O, formed by each refracting surface is :

- (a)  $0.214 R$  (b)  $0.114 R$   
(c)  $0.411 R$  (d)  $0.124 R$

10. Two bodies A and B of equal mass are suspended from two massless springs of spring constant  $k_1$  and  $k_2$ , respectively. If the bodies oscillate vertically such that their amplitudes are equal, the ratio of the maximum velocity of A to the maximum velocity of B is

- (a)  $\sqrt{\frac{k_1}{k_2}}$  (b)  $\frac{k_1}{k_2}$  (c)  $\frac{k_2}{k_1}$  (d)  $\sqrt{\frac{k_2}{k_1}}$

11. Given below are two statements. One is labelled as Assertion (A) and the other is labelled as Reason (R).



Assertion (A)

Three identical spheres of same mass undergo one dimensional motion as shown in figure with initial velocities  $v_A = 5 \text{ m/s}$ ,  $v_B = 2 \text{ m/s}$ ,  $v_C = 4 \text{ m/s}$ . If we wait sufficiently long for elastic collision to happen, then  $v_A = 4 \text{ m/s}$ ,  $v_B = 2 \text{ m/s}$ ,  $v_C = 5 \text{ m/s}$  will be the final velocities.

Reason (R) : In an elastic collision between identical masses, two objects exchange their velocities.

In the light of the above statements, choose the correct answer from the options given below :

- (a) Both (A) and (R) are true but (R) is NOT the correct explanation of (A)  
(b) (A) is true but (R) is false  
(c) Both (A) and (R) are true and (R) is the correct explanation of (A).  
(d) (A) is false but (R) is true

12. A sand dropper drops sand of mass  $m(t)$  on a conveyer belt at a rate proportional to the square root of speed ( $v$ ) of the belt, i.e.  $\frac{dm}{dt} \propto \sqrt{v}$ . If  $P$  is the power delivered to run the belt at constant speed then which of

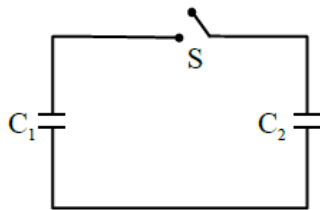
the following relationship is true?

- (a)  $P^2 \propto v^3$  (b)  $P \propto \sqrt{v}$   
(c)  $P \propto v$  (d)  $P^2 \propto v^5$

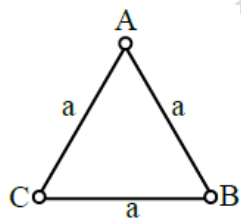
13. A convex lens made of glass (refractive index = 1.5) has focal length 24 cm in air. When it is totally immersed in water (refractive index = 1.33), its focal length changes to

- (a) 72 cm (b) 96 cm  
(c) 24 cm (d) 48 cm

14. A capacitor,  $C_1 = 6\mu\text{F}$  is charged to a potential difference of  $V_0 = 5\text{V}$  using a 5 V battery. The battery is removed and another capacitor,  $C_2 = 12\mu\text{F}$  is inserted in place of the battery. When the switch 'S' is closed, the charge flows between the capacitors for some time until equilibrium condition is reached. What are the charges ( $q_1$  and  $q_2$ ) on the capacitors  $C_1$  and  $C_2$  when equilibrium condition is reached.



- (a)  $q_1 = 15\mu\text{C}, q_2 = 30\mu\text{C}$   
(b)  $q_1 = 30\mu\text{C}, q_2 = 15\mu\text{C}$   
(c)  $q_1 = 10\mu\text{C}, q_2 = 20\mu\text{C}$   
(d)  $q_1 = 20\mu\text{C}, q_2 = 10\mu\text{C}$



15. Three equal masses  $m$  are kept at vertices (A, B, C) of an equilateral triangle of side  $a$  in free space. At  $t = 0$ , they are given an initial velocity  $\vec{V}_A = V_0\vec{AC}$ ,  $\vec{V}_B = V_0\vec{BA}$  and  $\vec{V}_C = V_0\vec{CB}$ . Here,  $\vec{AC}$ ,  $\vec{CB}$  and  $\vec{BA}$  are unit vectors along the edges of the triangle. If the three masses interact gravitationally, then the magnitude of the net angular momentum of the system at the point of collision is :

- (a)  $\frac{1}{2}amV_0$  (b)  $3amV_0$   
(c)  $\frac{\sqrt{3}}{2}amV_0$  (d)  $\frac{3}{2}amV_0$

16. Match List-I with List-II.

|     | List-I                   |       | List-II                                  |
|-----|--------------------------|-------|------------------------------------------|
| (A) | Young's Modulus          | (I)   | $\text{ML}^{-1} \text{T}^{-1}$           |
| (B) | Torque                   | (II)  | $\text{ML}^{-1} \text{T}^{-2}$           |
| (C) | Coefficient of Viscosity | (III) | $\text{M}^{-1} \text{L}^3 \text{T}^{-2}$ |
| (D) | Gravitational Constant   | (IV)  | $\text{ML}^2 \text{T}^{-2}$              |

Choose the correct answer from the options given below :

- (a) (A)-(I), (B)-(III), (C)-(II), (D)-(IV)  
(b) (A)-(II), (B)-(I), (C)-(IV), (D)-(III)  
(c) (A)-(IV), (B)-(II), (C)-(III), (D)-(I)  
(d) (A)-(II), (B)-(IV), (C)-(I), (D)-(III)

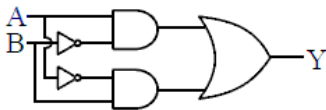
17. Match List-I with List-II.

|     | List-I             |       | List-II                   |
|-----|--------------------|-------|---------------------------|
| (A) | Magnetic induction | (I)   | Ampere meter <sup>2</sup> |
| (B) | Magnetic intensity | (II)  | Weber                     |
| (C) | Magnetic flux      | (III) | Gauss                     |
| (D) | Magnetic moment    | (IV)  | Ampere meter              |

Choose the correct answer from the options given below :

- (a) (A)-(III), (B)-(IV), (C)-(I), (D)-(II)  
 (b) (A)-(III), (B)-(IV), (C)-(II), (D)-(I)  
 (c) (A)-(I), (B)-(II), (C)-(III), (D)-(IV)  
 (d) (A)-(III), (B)-(II), (C)-(I), (D)-(IV)

18. The truth table for the circuit given below is :



| A | B | Y |  | A | B | Y |
|---|---|---|--|---|---|---|
| 0 | 0 | 0 |  | 0 | 0 | 0 |
| 0 | 1 | 1 |  | 1 | 0 | 0 |
| 1 | 0 | 1 |  | 1 | 1 | 0 |
| 1 | 1 | 0 |  | 0 | 1 | 1 |

| A | B | Y |  | A | B | Y |
|---|---|---|--|---|---|---|
| 0 | 0 | 0 |  | 0 | 0 | 0 |
| 1 | 0 | 1 |  | 1 | 1 | 1 |
| 0 | 1 | 0 |  | 1 | 0 | 1 |
| 1 | 1 | 0 |  | 0 | 1 | 1 |

19. A cup of coffee cools from 90°C to 80°C in t minutes when the room temperature is 20°C. The time taken by the similar cup of coffee to cool from 80°C to 60°C at the same room temperature is :

- (a)  $\frac{13}{5}t$  (b)  $\frac{10}{13}t$   
 (c)  $\frac{13}{10}t$  (d)  $\frac{5}{13}t$

20. The number of spectral lines emitted by atomic hydrogen that is in the 4<sup>th</sup> energy level, is

- (a) 6 (b) 0 (c) 3 (d) 1

### Section - B

21. The magnetic field inside a 200 turns solenoid of radius 10 cm is  $2.9 \times 10^{-4}$  Tesla. If the solenoid carries a current of 0.29 A, then the length of the solenoid is \_\_\_\_  $\pi$ cm.

22. A parallel plate capacitor consisting of two circular plates of radius 10 cm is being charged by a constant current of 0.15 A. If the rate of change of potential difference between the plates is  $7 \times 10^8$  V/s then the integer value of the distance between the parallel plates is -

(Take,  $\epsilon_0 = 9 \times 10^{-12} \frac{F}{m}$ ,  $\pi = \frac{22}{7}$ ) \_\_\_\_  $\mu$ m.

23. A physical quantity Q is related to four observables a, b, c, d as follows :  $Q = \frac{a^4}{cd}$  where,  $a = (60 \pm 3)$ Pa;  $b = (20 \pm 0.1)$ m;  $c = (40 \pm 0.2)$ Nsm<sup>-2</sup> and  $d = (50 \pm 0.1)$ m, then the percentage error in Q is  $\frac{x}{1000}$ , where x = \_\_\_\_.

24. Two planets, A and B are orbiting a common star in circular orbits of radii  $R_A$  and  $R_B$ , respectively, with  $R_B = 2R_A$ . The planet B is  $4\sqrt{2}$  times more massive than planet A. The ratio  $\left(\frac{L_B}{L_A}\right)$  of angular momentum ( $L_B$ ) of planet B to that of planet A ( $L_A$ ) is closest to integer \_\_\_\_
25. Two cars P and Q are moving on a road in the same direction. Acceleration of car P increases linearly with time whereas car Q moves with a constant acceleration. Both cars cross each other at time  $t = 0$ , for the first time. The maximum possible number of crossing(s) (including the crossing at  $t = 0$ ) is \_\_\_\_

### Chemistry

#### Section - A

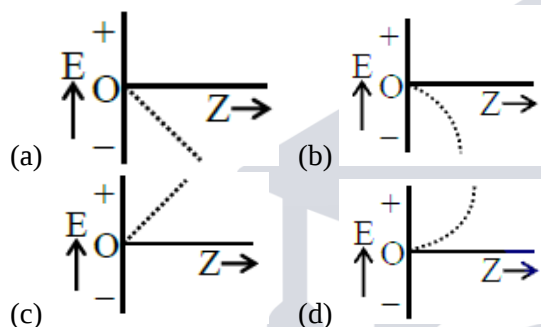
26. The calculated spin-only magnetic moments of  $K_3[Fe(OH)_6]$  and  $K_4[Fe(OH)_6]$  respectively are :

- (a) 4.90 and 4.90 B.M.  
 (b) 5.92 and 4.90 B.M.  
 (c) 3.87 and 4.90 B.M.  
 (d) 4.90 and 5.92 B.M.

27. For hydrogen like species, which of the following graphs provides the most appropriate representation of  $E$  vs  $Z$  plot for a constant  $n$  ?

[ $E$  : Energy of the stationary state,

$Z$  : atomic number,  $n$  = principal quantum number]



28. Given below are two statements :

Statement (I) : In partition chromatography, stationary phase is thin film of liquid present in the inert support.

Statement (II) : In paper chromatography, the material of paper acts as a stationary phase.

In the light of the above statements, choose the correct answer from the options given below :

- (a) Both Statement I and Statement II are false  
 (b) Statement I is true but Statement II is false  
 (c) Both Statement I and Statement II are true  
 (d) Statement I is false but Statement II is true

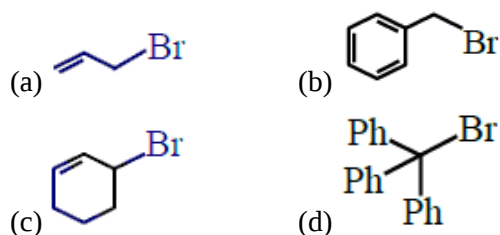
29. Identify the essential amino acids from below :

- (a) Valine (b) Proline  
 (c) Lysine (d) Threonine  
 (e) Tyrosine

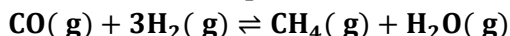
Choose the correct answer from the options given below :

- (a) (A), (C) and (D) only  
 (b) (A), (C) and (E) only  
 (c) (B), (C) and (E) only  
 (d) (C), (D) and (E) only

30. Which among the following halides will generate the most stable carbocation in Nucleophilic substitution reaction?



31. Consider the equilibrium



If the pressure applied over the system increases by two fold at constant temperature then

- (A) Concentration of reactants and products increases.  
 (B) Equilibrium will shift in forward direction.  
 (C) Equilibrium constant increases since concentration of products increases.  
 (D) Equilibrium constant remains unchanged as concentration of reactants and products remain same.

Choose the correct answer from the options given below :

- (a) (A) and (B) only      (b) (A), (B) and (D) only  
 (c) (B) and (C) only      (d) (A), (B) and (C) only

32. Given below are two statements :

Statement (I): NaCl is added to the ice at 0°C, present in the ice cream box to prevent the melting of ice cream.

Statement (II) : On addition of NaCl to ice at 0°C, there is a depression in freezing point.

In the light of the above statements, choose the correct answer from the options given below :

- (a) Statement I is false but Statement II is true  
 (b) Both Statement I and Statement II are true  
 (c) Both Statement I and Statement II are false  
 (d) Statement I is true but Statement II is false

33. Given below are two statements :

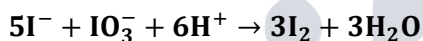
Statement (I) : On nitration of m-xylene with  $\text{HNO}_3$ ,  $\text{H}_2\text{SO}_4$  followed by oxidation, 4-nitrobenzene-1, 3-dicarboxylic acid is obtained as the major product.

Statement (II) :  $\text{CH}_3$  group is o/p-directing while  $\text{NO}_2$  group is m-directing group.

In the light of the above statements, choose the correct answer from the options given below :

- (a) Both Statement I and Statement II are false  
 (b) Statement I is false but Statement II is true  
 (c) Both Statement I and Statement II are true  
 (d) Statement I is true but Statement II is false

34. 0.1 M solution of KI reacts with excess of  $\text{H}_2\text{SO}_4$  and  $\text{KIO}_3$  solution. According to equation



Identify the correct statements :

- (A) 200 mL of KI solution reacts with 0.004 mol of  $\text{KIO}_3$   
 (B) 200 mL of KI solution reacts with 0.006 mol of  $\text{H}_2\text{SO}_4$   
 (3) 0.5 L of KI solution produced 0.005 mol of  $\text{I}_2$   
 (4) Equivalent weight of  $\text{KIO}_3$  is equal to  $\left(\frac{\text{Molecular weight}}{5}\right)$

Choose the correct answer from the options given below:

- (a) (A) and (D) only      (b) (B) and (C) only  
 (c) (A) and (B) only      (d) (C) and (D) only

35. Match List-I with List-II :

| List-I<br>Applications |                   | List-II<br>Batteries/Cell |                                              |
|------------------------|-------------------|---------------------------|----------------------------------------------|
| (A)                    | Transistors       | (I)                       | Anode -Zn/Hg;<br>Cathode -HgO + C            |
| (B)                    | Hearing aids      | (II)                      | Hydrogen fuel cell                           |
| (C)                    | Invertors         | (III)                     | Anode - Zn ;<br>Cathode - Carbon             |
| (D)                    | Apollo space ship | (IV)                      | Anode - Pb ;<br>Cathode -Pb PbO <sub>2</sub> |

Choose the correct answer from the options given below :

- (a) (A)-(III), (B)-(I), (C)-(IV), (D)-(II)  
 (b) (A)-(III), (B)-(II), (C)-(IV), (D)-(I)

- (c) (A)-(IV), (B)-(III), (C)-(II), (D)-(I)  
(d) (A)-(II), (B)-(III), (C)-(IV), (D)-(I)

36.  $O_2$  gas will be evolved as a product of electrolysis of :

- (a) an aqueous solution of  $AgNO_3$  using silver electrodes.  
(b) an aqueous solution of  $AgNO_3$  using platinum electrodes.  
(c) a dilute solution of  $H_2SO_4$  using platinum electrodes.  
(d) a high concentration solution of  $H_2SO_4$  using platinum electrodes.

Choose the correct answer from the options given below :

- (a) (B) and (C) only (b) (A) and (D) only  
(c) (B) and (D) only (d) (A) and (C) only

37. Identify the homoleptic complexes with odd number of d electrons in the central metal.

- (A)  $[FeO_4]^{2-}$  (B)  $[Fe(CN)_6]^{3-}$   
(C)  $[Fe(CN)_5NO]^{2-}$  (D)  $[CoCl_4]^{2-}$   
(E)  $[Co(H_2O)_3 F_3]$

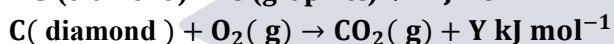
Choose the correct answer from the options given below :

- (a) (B) and (D) only  
(b) (C) and (E) only  
(c) (A), (B) and (D) only  
(d) (A), (C) and (E) only

38. Total number of sigma ( $\sigma$ ) \_\_\_\_ and pi ( $\pi$ ) \_\_\_\_ bonds respectively present in hex-1-en-4-yne are :

- (a) 13 and 3 (b) 11 and 3  
(c) 3 and 13 (d) 14 and 3

39. If  $C$  (diamond)  $\rightarrow C$  (graphite) +  $X kJ mol^{-1}$



At constant temperature. Then

- (a)  $X = Y + Z$  (b)  $-X = Y + Z$   
(c)  $X = -Y + Z$  (d)  $X = Y - Z$

40. Given below are two statements :

Statement (I): It is impossible to specify simultaneously with arbitrary precision, both the linear momentum and the position of a particle.

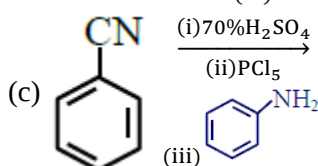
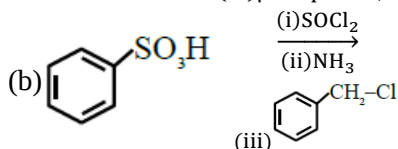
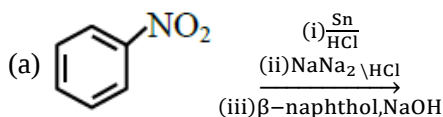
Statement (II) : If the uncertainty in the measurement of position and uncertainty in measurement of

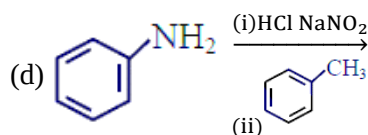
momentum are equal for an electron, then the uncertainty in the measurement of velocity is  $\geq \sqrt{\frac{h}{\pi}} \times \frac{1}{2m}$ .

In the light of the above statements, choose the correct answer from the options given below :

- (a) Statement I is true but Statement II is false.  
(b) Both Statement I and Statement II are true.  
(c) Statement I is false but Statement II is true.  
(d) Both Statement I and Statement II are false.

41. Which one of the following reaction sequences will give an azo dye?





42. Drug X becomes ineffective after 50% decomposition. The original concentration of drug in a bottle was 16mg/mL which becomes 4 mg/mL in 12 months. The expiry time of the drug in months is \_\_\_\_ -.

Assume that the decomposition of the drug follows first order kinetics.

- (a) 12 (b) 2 (c) 3 (d) 6

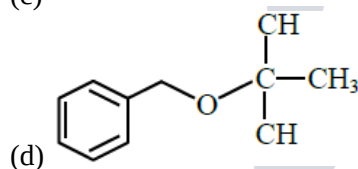
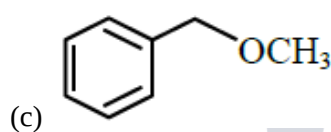
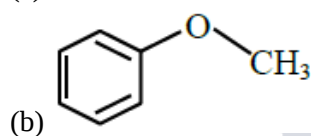
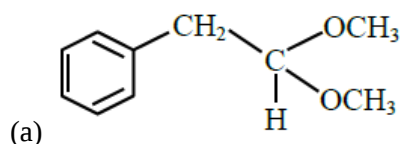
43. The type of oxide formed by the element among Li, Na, Be, Mg, B and Al that has the least atomic radius is :

- (a)  $A_2O_3$  (b)  $AO_2$   
(c)  $AO$  (d)  $A_2O$

44. First ionisation enthalpy values of first four group 15 elements are given below. Choose the correct value for the element that is a main component of apatite family :

- (a)  $1012 \text{ kJ mol}^{-1}$  (b)  $1402 \text{ kJ mol}^{-1}$   
(c)  $834 \text{ kJ mol}^{-1}$  (d)  $947 \text{ kJ mol}^{-1}$

45. Which one of the following, with HBr will give a phenol?



#### Section - B

46. Consider the following low-spin complexes  $K_3[Co(NO_2)_6]$ ,  $K_4[Fe(CN)_6]$ ,  $K_3[Fe(CN)_6]$ ,  $Cu_2[Fe(CN)_6]$  and  $Zn_2[Fe(CN)_6]$ . The sum of the spin-only magnetic moment values of complexes having yellow colour is \_\_\_\_ B.M. (answer is nearest integer)

47. Isomeric hydrocarbons  $\rightarrow$  negative Baeyer's test (Molecular formula  $C_9H_{12}$  ).

The total number of isomers from above with four different non-aliphatic substitution sites is -

48. In the Claisen-Schmidt reaction to prepare, dibenzalacetone from 5.3 g benzaldehyde, a total of 3.51 g of product was obtained. The percentage yield in this reaction was \_\_\_\_ %.

49. In the sulphur estimation, 0.20 g of a pure organic compound gave 0.40 g of barium sulphate. The percentage of sulphur in the compound is \_\_\_\_  $\times 10^{-1}$  %. (Molar mass :  $O = 16, S = 32, Ba = 137 \text{ in } g \text{ mol}^{-1}$  )

50. Total number of non-bonded electrons present in  $NO_2^-$  ion based on Lewis theory is \_\_\_\_ .

#### Mathematics

#### Section - A

51. If the set of all  $a \in R$ , for which the equation  $2x^2 + (a - 5)x + 15 = 3a$  has no real root, is the interval  $(\alpha, \beta)$ , and  $X = \{x \in Z: \alpha < x < \beta\}$ , then  $\sum_{x \in X} x^2$  is equal to

- (a) 2109 (b) 2129  
(c) 2139 (d) 2119

52. If  $\sin x + \sin^2 x = 1$ ,  $x \in \left(0, \frac{\pi}{2}\right)$ , then  $(\cos^{12} x + \tan^{12} x) + 3(\cos^{10} x + \tan^{10} x + \cos^8 x + \tan^8 x) + (\cos^6 x + \tan^6 x)$  is equal to  
 (a) 4 (b) 3 (c) 2 (d) 1
53. Let the area enclosed between the curves  $|y| = 1 - x^2$  and  $x^2 + y^2 = 1$  be  $\alpha$ . If  $9\alpha = \beta\pi + \gamma$ ;  $\beta, \gamma$  are integers, then the value of  $|\beta - \gamma|$  equals  
 (a) 27 (b) 18 (c) 15 (d) 33
54. If the domain of the function  $\log_5(18x - x^2 - 77)$  is  $(\alpha, \beta)$  and the domain of the function  $\log_{(x-1)}\left(\frac{2x^2+3x-2}{x^2-3x-4}\right)$  is  $(\gamma, \delta)$ , then  $\alpha^2 + \beta^2 + \gamma^2$  is equal to :  
 (a) 195 (b) 174  
 (c) 186 (d) 179
55. Let the function  $f(x) = (x^2 - 1)|x^2 - ax + 2| + \cos |x|$  be not differentiable at the two points  $x = \alpha = 2$  and  $x = \beta$ . Then the distance of the point  $(\alpha, \beta)$  from the line  $12x + 5y + 10 = 0$  is equal to :  
 (a) 3 (b) 4 (c) 2 (d) 5
56. Let a straight line L pass through the point  $P(2, -1, 3)$  and be perpendicular to the lines  $\frac{x-1}{2} = \frac{y+1}{1} = \frac{z-3}{-2}$  and  $\frac{x-3}{1} = \frac{y-2}{3} = \frac{z+2}{4}$ . If the line L intersects the  $yz$ -plane at the point Q, then the distance between the points P and Q is :  
 (a) 2 (b)  $\sqrt{10}$   
 (c) 3 (d)  $2\sqrt{3}$
57. Let  $S = \mathbb{N} \cup \{0\}$ . Define a relation R from S to R by :  
 $R = \left\{ (x, y) : \log_e y = x \log_e \left(\frac{2}{5}\right), x \in S, y \in \mathbb{R} \right\}$ .  
 Then, the sum of all the elements in the range of R is equal to  
 (a)  $\frac{3}{2}$  (b)  $\frac{5}{3}$  (c)  $\frac{10}{9}$  (d)  $\frac{5}{2}$
58. Let the line  $x + y = 1$  meet the axes of  $x$  and  $y$  at A and B, respectively. A right angled triangle AMN is inscribed in the triangle OAB, where O is the origin and the points M and N lie on the lines OB and AB, respectively. If the area of the triangle AMN is  $\frac{4}{9}$  of the area of the triangle OAB and  $AN : NB = \lambda : 1$ , then the sum of all possible value(s) of  $\lambda$  is :  
 (a)  $\frac{1}{2}$  (b)  $\frac{13}{6}$  (c)  $\frac{5}{2}$  (d) 2
59. If  $\alpha x + \beta y = 109$  is the equation of the chord of the ellipse  $\frac{x^2}{9} + \frac{y^2}{4} = 1$ , whose mid point is  $\left(\frac{5}{2}, \frac{1}{2}\right)$ , then  $\alpha + \beta$  is equal to  
 (a) 37 (b) 46 (c) 58 (d) 72
60. If all the words with or without meaning made using all the letters of the word "KANPUR" are arranged as in a dictionary, then the word at 440<sup>th</sup> position in this arrangement, is :  
 (a) PRNAKU (b) PRKANU  
 (c) PRKAUN (d) PRNAUK
61. Let  $\alpha, \beta (\alpha \neq \beta)$  be the values of  $m$ , for which the equations  $x + y + z = 1$ ;  $x + 2y + 4z = m$  and  $x + 4y + 10z = m^2$  have infinitely many solutions. Then the value of  $\sum_{n=1}^{10} (n^\alpha + n^\beta)$  is equal to :  
 (a) 440 (b) 3080  
 (c) 3410 (d) 560
62. Let  $A = [a_{ij}]$  be a matrix of order  $3 \times 3$ , with  $a_{ij} = (\sqrt{2})^{i+j}$ . If the sum of all the elements in the third row of  $A^2$  is  $\alpha + \beta\sqrt{2}$ ,  $\alpha, \beta \in \mathbb{Z}$ , then  $\alpha + \beta$  is equal to  
 (a) 280 (b) 168  
 (c) 210 (d) 224

63. Let P be the foot of the perpendicular from the point  $(1, 2, 2)$  on the line  $L: \frac{x-1}{1} = \frac{y+1}{-1} = \frac{z-2}{2}$ . Let the line  $\vec{r} = (-\hat{i} + \hat{j} - 2\hat{k}) + \lambda(\hat{i} - \hat{j} + \hat{k}), \lambda \in \mathbb{R}$ , intersect the line L at Q. Then  $2(PQ)^2$  is equal to:  
 (a) 27 (b) 25 (c) 29 (d) 19
64. Let a circle C pass through the points  $(4, 2)$  and  $(0, 2)$ , and its centre lie on  $3x + 2y + 2 = 0$ . Then the length of the chord, of the circle C, whose midpoint is  $(1, 2)$ , is:  
 (a)  $\sqrt{3}$  (b)  $2\sqrt{3}$   
 (c)  $4\sqrt{2}$  (d)  $2\sqrt{2}$
65. Let  $A = [a_{ij}]$  be a  $2 \times 2$  matrix such that  $a_{ij} \in \{0, 1\}$  for all i and j. Let the random variable X denote the possible values of the determinant of the matrix A. Then, the variance of X is:  
 (a)  $\frac{1}{4}$  (b)  $\frac{3}{8}$  (c)  $\frac{5}{8}$  (d)  $\frac{3}{4}$
66. Bag 1 contains 4 white balls and 5 black balls, and Bag 2 contains n white balls and 3 black balls. One ball is drawn randomly from Bag 1 and transferred to Bag 2. A ball is then drawn randomly from Bag 2. If the probability, that the ball drawn is white, is  $\frac{29}{45}$ , then n is equal to:  
 (a) 3 (b) 4 (c) 5 (d) 6
67. The remainder, when  $7^{103}$  is divided by 23, is equal to:  
 (a) 14 (b) 9 (c) 17 (d) 6
68. Let  $f(x) = \int_0^x t(t^2 - 9t + 20)dt, 1 \leq x \leq 5$ . If the range of f is  $[\alpha, \beta]$ , then  $4(\alpha + \beta)$  equals:  
 (a) 157 (b) 253  
 (c) 125 (d) 154
69. Let  $\hat{a}$  be a unit vector perpendicular to the vectors  $\vec{b} = \hat{i} - 2\hat{j} + 3\hat{k}$  and  $\vec{c} = 2\hat{i} + 3\hat{j} - \hat{k}$ , and makes an angle of  $\cos^{-1}\left(-\frac{1}{3}\right)$  with the vector  $\hat{i} + \hat{j} + \hat{k}$ . If  $\hat{a}$  makes an angle of  $\frac{\pi}{3}$  with the vector  $\hat{i} + \alpha\hat{j} + \hat{k}$ , then the value of  $\alpha$  is :  
 (a)  $-\sqrt{3}$  (b)  $\sqrt{6}$   
 (c)  $-\sqrt{6}$  (d)  $\sqrt{3}$
70. If for the solution curve  $y = f(x)$  of the differential equation  $\frac{dy}{dx} + (\tan x)y = \frac{2+\sec x}{(1+2\sec x)^2}$ ,  $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right), f\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{10}$ , then  $f\left(\frac{\pi}{4}\right)$  is equal to:  
 (a)  $\frac{9\sqrt{3}+3}{10(4+\sqrt{3})}$  (b)  $\frac{\sqrt{3}+1}{10(4+\sqrt{3})}$   
 (c)  $\frac{5-\sqrt{3}}{2\sqrt{2}}$  (d)  $\frac{4-\sqrt{2}}{14}$

### Section -B

71. If  $24 \int_0^{\frac{\pi}{4}} \left( \sin \left| 4x - \frac{\pi}{12} \right| + [2\sin x] \right) dx = 2\pi + \alpha$ , where  $[.]$  denotes the greatest integer function, then  $\alpha$  is equal to \_\_\_\_.
72. If  $\lim_{t \rightarrow 0} \left( \int_0^1 (3x+5)^t dx \right)^{\frac{1}{t}} = \frac{\alpha}{5e} \left( \frac{8}{5} \right)^{\frac{2}{3}}$ , then  $\alpha$  is equal to \_\_\_\_.
73. Let  $a_1, a_2, \dots, a_{2024}$  be an Arithmetic Progression such that  $a_1 + (a_5 + a_{10} + a_{15} + \dots + a_{2020}) + a_{2024} = 2233$ . Then  $a_1 + a_2 + a_3 + \dots + a_{2024}$  is equal to \_\_\_\_.
74. Let integers  $a, b \in [-3, 3]$  be such that  $a + b \neq 0$ . Then the number of all possible ordered pairs  $(a, b)$ , for which  $\left| \frac{z-a}{z+b} \right| = 1$  and  $\begin{vmatrix} z+1 & \omega & \omega^2 \\ \omega & z+\omega^2 & 1 \\ \omega^2 & 1 & z+\omega \end{vmatrix} = 1, z \in \mathbb{C}$ , where  $\omega$  and  $\omega^2$  are the roots of  $x^2 + x + 1 = 0$ , is equal to \_\_\_\_.
75. Let  $y^2 = 12x$  the parabola and S be its focus. Let PQ be a focal chord of the parabola such that  $(SP)(SQ) = \frac{147}{4}$ . Let C be the circle described taking PQ as a diameter. If the equation of a circle C is  $64x^2 + 64y^2 - \alpha x - 64\sqrt{3}y = \beta$ , then  $\beta - \alpha$  is equal to \_\_\_\_.

