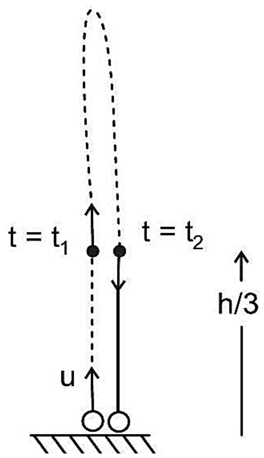
1. (d)

$$T = k\sqrt{\frac{\rho r^3}{s^{3/2}}}$$

Dimensions of RHS $= \frac{[M^{1/2}\,L^{-3/2}][L^{3/2}]}{[MT^{-2}]^{3/4}} = M^{1/8}\,L^0\,T^{3/2}$

Dimensions of L.H.S $\neq$ Dimensions of R.H.S

2. (b)



Max. Height $= h = \frac{u^2}{2\,g}$

$\Rightarrow u = \sqrt{2gh}$

$S = ut + \frac{1}{2}at^2$

$\frac{h}{3} = \sqrt{2gh}t + \frac{1}{2}(-g)t^2$

$\frac{gt^2}{2} - \sqrt{2gh}t + \frac{h}{3} = 0$ (Roots are $t_1 \& t_2$)

$$\frac{t_2}{t_1} = \frac{\sqrt{2gh} + \sqrt{2gh - 4 \times \frac{g}{2} \times \frac{h}{3}}}{\sqrt{2gh} - \sqrt{2gh - 4 \times \frac{g}{2} \times \frac{h}{3}}} = \frac{\sqrt{2gh} + \sqrt{\frac{4gh}{3}}}{\sqrt{2gh} - \sqrt{\frac{4gh}{3}}} = \frac{\sqrt{3} + \sqrt{2}}{\sqrt{3} - \sqrt{2}}$$

3. (b)

$t = \sqrt{x} + 4$

$\Rightarrow x = (t-4)^2 = t^2 - 8t + 16$

$\Rightarrow \frac{dx}{dt} = 2t - 8$

$\Rightarrow \left.\frac{dx}{dt}\right|_{t=4} = 2 \times 4 - 8 = 0$

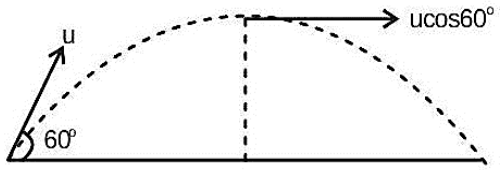
4. (a)

$$N = \frac{mv^2}{r}$$

Curve is parabola

$Y = kx^2$

5. (c)



$$E = \frac{1}{2}mu^2$$

At Highest point, Velocity $V = u\cos 60° = \frac{u}{2}$

$\therefore$ K.E at topmost point $= \frac{1}{2}\, m\left(\frac{u}{2}\right)^2 = \frac{E}{4}$

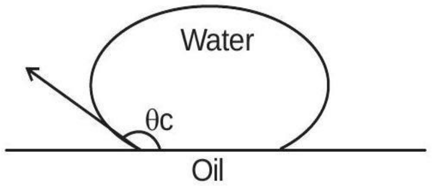
6. (a)

$$\overrightarrow{r_{com}} = \frac{m_1\vec{r}_1 + m_2\overrightarrow{r_2}}{m_1 + m_2} = \frac{1(\hat{\imath} + 2\hat{\jmath} + \hat{k}) + 3(-3\hat{\imath} - 2\hat{\jmath} + \hat{k})}{1 + 3}$$

$$= -2\hat{\imath} - \hat{\jmath} + \hat{k}$$

$$|2\hat{\imath} - \hat{\jmath} + \hat{k}| = \sqrt{(2)^2 + (1)^2 + (1)^2} = \sqrt{6}$$

7. (a)

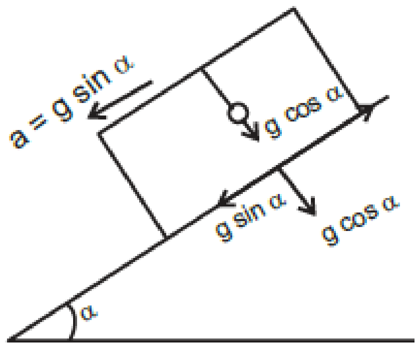


$\theta c > 90°$

For water oil interface

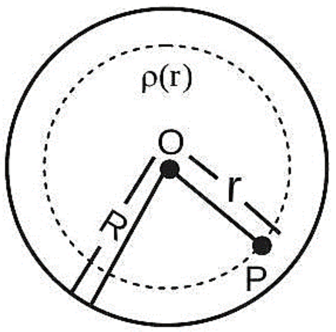
8. (a) Y depends on material of wire

9. (a) $g_{cff} = g\cos\alpha$



$$T = 2\pi\sqrt{\frac{L}{g\cos\alpha}}$$

10. (c) By Gauss law



$$\oint \vec{E}\cdot d\vec{s} = \frac{Q_{in}}{\varepsilon_0}$$

$$E.\ 4\pi r^2 = \frac{\int_0^r \rho_0\left(\frac{3}{4}-\frac{r}{R}\right)4\pi r^2 dr}{\varepsilon_0}$$

$$E4\pi r^2 = \frac{\rho_0 4\pi}{\varepsilon_0}\left(\frac{3}{4}\frac{r^3}{3}-\frac{r^4}{4R}\right)$$

$$Er^2 = \frac{\rho_0 r^3}{4\varepsilon_0}\left\{1-\frac{r}{R}\right\}$$

$$E = \frac{\rho_0 r}{4\varepsilon_0}\left\{1-\frac{r}{R}\right\}$$

11. (a) (Properties of conductor)

Statement - I, true as body of conductor acts as equipotential surface.

Statement - 2 True, as conductor is equipotential.

Tangential component of electric field should be zero. Therefore, electric field should be perpendicular to surface.

12. (b)

Let length of wire be 'ℓ'

Area of wire as 'A'

For equivalent wire length $= 2\ell$ & area will be A
Thermal resistance

$R_{eq} = R_1 + R_2$

$$\frac{2\ell}{\sigma_{eq} A} = \frac{\ell}{\sigma_1 A} + \frac{\ell}{\sigma_1 A}$$

$$\frac{2\ell}{\sigma_{eq}} = \frac{\ell}{\sigma_1} + \frac{\ell}{\sigma_2} \Rightarrow \sigma_{eq} = \frac{2\sigma_1\sigma_2}{\sigma_1 + \sigma_2}$$

13. (c) $E = 440 \operatorname{Sin} 100\pi t, L = -\frac{\sqrt{2}}{\pi} H$

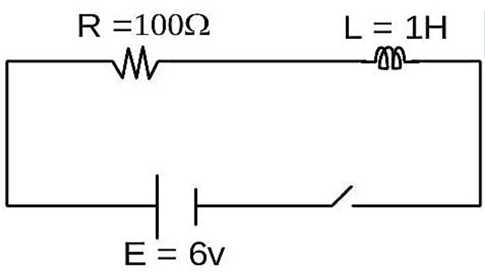
$$X_L = \omega L = 100\pi \frac{\sqrt{2}}{\pi} = 100\sqrt{2}\Omega$$

Peak current $I_0 = \frac{E_0}{X_L} = \frac{440}{100\sqrt{2}} = 2.2\sqrt{2}$ A

AC ammeter reads RMS value therefore reading will be I_{rms}

$$I_{rms} = \frac{I_0}{\sqrt{2}} = 2.2 \text{ A}$$

14. (c) Given circuit is $R - L$ growth circuit



$$i = \frac{E}{R}\left(1 - e^{-t/\tau}\right)$$

$$i = \frac{E}{2R} = \frac{E}{R}\left(1 - e^{-t/\tau)}\right)$$

Solving $t = \tau \ln 2$

$$t = \frac{1}{R}\ln 2 = \frac{1}{100} 0.693 = 0.00693$$

$= 7$ ms

$$i(15 \text{ ms}) = \frac{E}{R}\left(1 - e^{-\frac{15}{10}}\right)$$

$$i = \frac{6}{100}(1 - 1/4) = \frac{3}{4} \times \frac{6}{100}$$

$U = \frac{1}{2} LI^2$,

by solving we get $U = 1$ mJ.

15. (b)

(a) uv rays - used for water purification

(b) x-rays used for diagnosing fracture

(c) Microwaves are used for mobile and radar communication

(d) Infrared waves show less scattering therefore used in foggy days

(a-ii), (b-i), (c-iii), (d-iv)

16. (b) $E = \frac{hc}{\lambda} - \phi - (i)$

$2E = \frac{hc}{\lambda'} - \phi - (ii)$

(ii) - (i)

$$E = hc\left(\frac{1}{\lambda'} - \frac{1}{\lambda}\right)$$

$$\Rightarrow \lambda' = \frac{hc\lambda}{E\lambda + hc}$$

17. (a) $E_n = \frac{-13.6}{n^2} ev$

$$\Rightarrow \frac{E_2 - E_1}{E_\infty - E_1} = \frac{13.6\left(1 - \frac{1}{4}\right)}{13.6} = \frac{3}{4}$$

18. (a) Modulation index: $m = \frac{A_m}{A_c}$

Given $2\ A_m = 8$

$A_m + A_c = 9 \Rightarrow A_c = 5$

$$\therefore m = \frac{4}{5} = 0.8$$

19. (c) $1MSD = \frac{1}{20}$ cm

$$1VSD = \frac{24}{25} MSD = \frac{24}{25} \times \frac{1}{20} \text{ cm}$$

$\therefore$ Least count $= \frac{1}{20}\left(1 - \frac{24}{25}\right)$ cm

$$= \frac{1}{20} \times \frac{1}{25} = \frac{1}{500} \text{ cm}$$

$= 0.002$ cm

20. (c) MSR = 2.5 mm

$$CSR = 45 \times \frac{0.5}{50} \text{ mm}$$

$= 0.45$ mm

Diameter reading = MSR + CSR − zero error

$= 2.5 + 0.45 - (-0.03)$
$= 2.98$ mm

21. (15) $R_{max} = \frac{u^2 \sin 2(45°)}{g} = \frac{u^2}{g}$

$$\frac{R}{2} = \frac{u^2}{2\,g} = \frac{u^2 \sin 2\theta}{g}$$

$$\sin 2\theta = \frac{1}{2}$$

$2\theta = 30°, 150°$

$\theta = 15°, 75°$

Ans. 15, 75

22. (2)

$$g\left(1 - \frac{2h}{R}\right) = g\left(1 - \frac{d}{R}\right)$$

$$\frac{2h}{R} = \frac{d}{R}$$

$\alpha h = d$

$\alpha = 2$

23. (4)

$PV^\gamma = \text{const}$

$$d = \frac{m}{v}$$

$$p\left(\frac{m}{d}\right)^\gamma = \text{const}$$

$\frac{p}{d^\gamma} = \text{const} \quad \frac{d_2}{d_1} = 32$

$$\frac{p_1}{p_2} = \left(\frac{d_1}{d_2}\right)^\gamma = \left(\frac{1}{32}\right)^{7/5} = \frac{1}{128}$$

$$\frac{T_1}{T_2} = \frac{P_1\,V_1}{P_2\,V_2} = \frac{1}{128} 32 = \frac{1}{4}$$

24. (3)

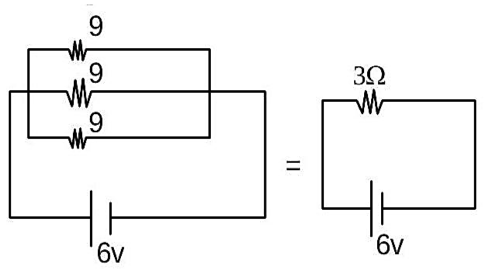
$$C_V/mix = \frac{n_1 Cv_1 + n_2 Cv_2}{n_1 + n_2}$$

$$= \frac{1 \cdot \frac{3R}{2} + 3 \cdot \frac{5R}{2}}{1+3}$$

$$= \frac{9R}{4} = \frac{\alpha^2}{4} R$$

$\alpha = 3$

25. (2) Equivalent circuit



$I = 6/3 = 2\text{ A}$

26. (3)

$$B_{centre} = \frac{N\mu_0 I}{2R}$$

$$37.68 \times 10^{-4} = \frac{100 \times 4\pi \times 10^{-7} \times I}{2 \times 5 \times 10^{-2}}$$

$I = 3\text{ A}$

27. (24)

$$I_{net} = I_1 + I_2 + 2\sqrt{I_1}\sqrt{I_2}\cos\phi$$

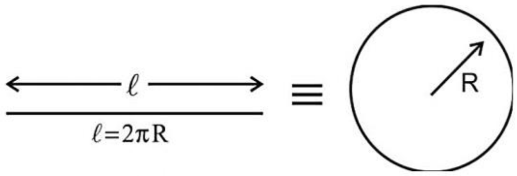
I_{max} for $\phi = 0$ & I_{min} for $\phi = \pi$

$$I_{max} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2 = (\sqrt{9I} + \sqrt{4I})^2 = 25I$$

$$I_{min} = \left(\sqrt{I_1} - \sqrt{I_2}\right)^2 = (\sqrt{9I} - \sqrt{4I})^2 = I$$

$$I_{max} - I_{min} = 25I - I = 24I$$

28. (11)



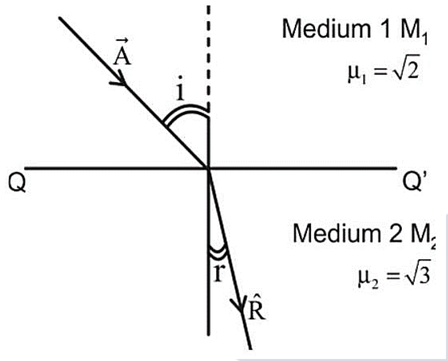
$\frac{314}{100} = 2\pi R\ R = 0.5\ m$

Magnetic Moment $= IA$

$= 14 \times \pi R^2$
$= 14 \times (3.14) \times \frac{1}{4}$
$= 10.99 \approx 11.00$

29. (15)

$\vec{A} = 4\sqrt{3}\hat{i} - 3\sqrt{3}\hat{j} - 5\hat{k}$



$\mu_1 \sin i = \mu_2 \sin r$
As incident vector A makes i angle with normal z-axis & refracted vector R makes r angle with normal z - axis with help of direction cosine

$$i = \cos^{-1}\left(\frac{A_z}{A}\right) = \cos^{-1}\left(\frac{5}{\sqrt{(4\sqrt{3})^2 + (3\sqrt{3})^2 + 5^2}}\right)$$

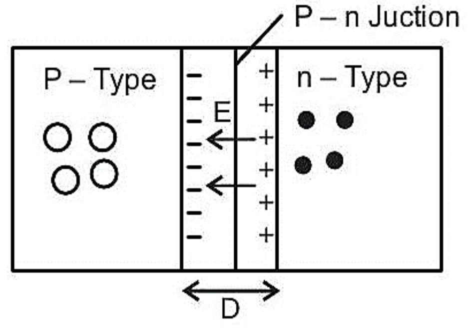
$= \cos^{-1}\left(\frac{5}{10}\right) \Rightarrow i = 60°$

$\sqrt{2}\sin 60 = \sqrt{3} \times \sin r$

$r = 45°$

Difference between i&r $= 60 - 45 = 15$

30. (1)



$E=\frac{V}{d}=\frac{\text{Potential barrier Across Junction}}{\text{width of Depletion layer}}$

$=\frac{0.6\ V}{6\times10^{-6}\ m}=1\times10^{5}\ V/m$

$=1\times10^{5}\ N/C$

31. (b)
32. (c)

$N_2(g)+3H_2(g)\rightleftharpoons 2NH_3(g)$
$W_2=20\ g\ 5\ g.$
$n=\frac{20}{28}\ \frac{5}{2}$

Stoichiometric Amount:

$N_2\to\frac{20/28}{1}=\frac{20}{28}\ H_2\to\frac{5/2}{3}=\frac{5}{6}$

$\therefore$ N_2 is the Limiting Reagent.

$\therefore n(NH_3)=2\times n(N_2)=2\times\frac{20}{28}$

$=1.42$

33. (a) Standard method for the preparation of lyophilic sol. (Discussed in lab Manual)
34. (c) I. E: $Na<Al<Mg<Si$

$\therefore\ 496<IE(Al)<737$

Option (c), matches the condition.

i.e $IE(Al)=577 kJmol^{-1}$

35. (a) Earthy and undesired materials present in the ore, other then the desired metal, is known as gangue.
36. (d)

Zinc dissolves in excess of aqueous alkali

$Zn+2OH^-+2H_2O\to[Zn(OH)_4]^{2-}+H_2\uparrow$

Tetrahydroxozincate(II) ion

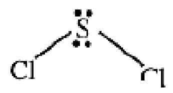
However, this reaction in NCERT is given as

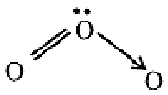
$Zn+2NaOH\to Na_2ZnO_2+H_2\uparrow$

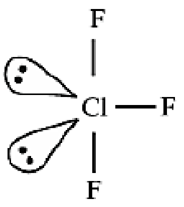
ZnO_2^{2-} is anhydrous form of $[Zn(OH)_4]^{2-}$

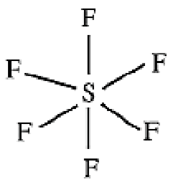
So in aqueous medium best answer of this question is $[Zn(OH)_4]^{2-}$

37. (c)
38.(b)


(2 ℓ .p.)


(1 ℓ .p.)


(2 ℓ .p.)

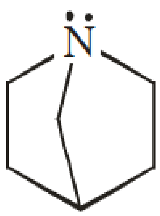

(0 ℓ .p.)

39. (a)
40. (d)

$$8\overset{+7}{Mn}O_4^{\,-} + 3\,S_2O_3^{2-} + H_2O \rightarrow 8\overset{+4}{Mn}O_2 + 6SO_4^{2-} + 2OH$$

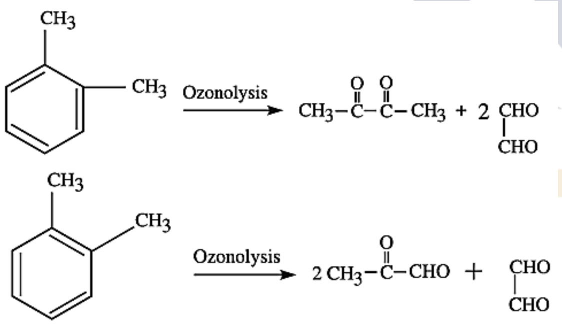
Change in oxidation state of Mn is from +7 to +4 which is 3.

41. (d) Both sodium chlorate and sodium arsenite behave as herbicide.
42. (d)

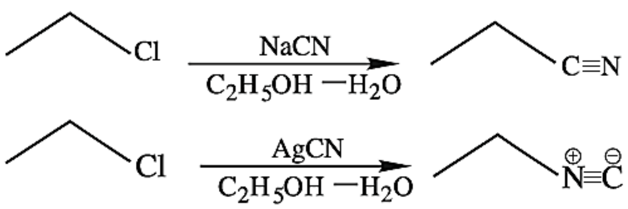


It is most basic because there is no amine inversion.

43.(c)



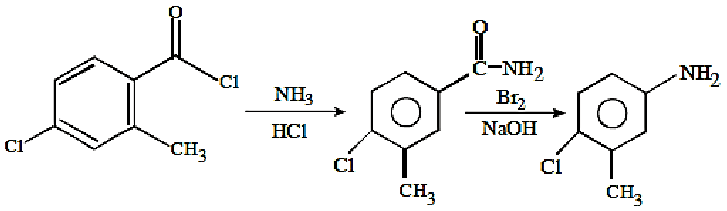
44. (c)



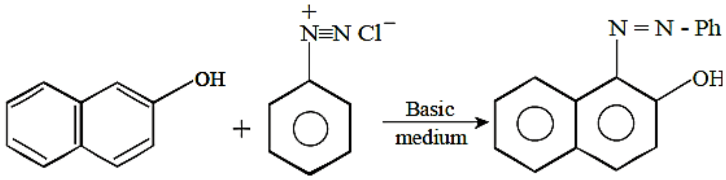
In NaCN; carbon is more nucleophilic atom. Whereas in AgCN; Ag – C has covalent bond.

45. (d)

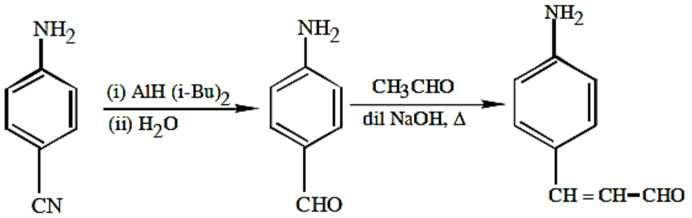
46. (c)



47. (a)

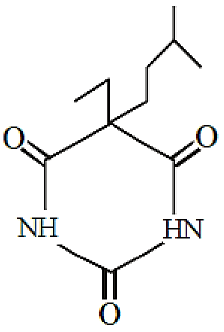


48. (b)



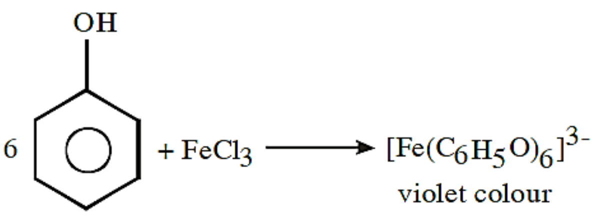
Cross aldol condensation

49. (b) Amytal is hypnotic drug used to treat sleeping disorder.



Amytal

50. (b)



violet colour

51. (1000)

$$\frac{\ell}{A} = 129 \text{ m}^{-1}$$

KCl solution 1:

$74.5 ppm, R_1 = 100\Omega$

KCl solution 2:

$149 ppm, R_2 = 50\Omega$

$149 ppm, R_2 = 50\Omega$

Here, $\frac{ppm_1}{ppm_2} = \frac{M_1}{M_2}\left(= \frac{W_1/M_0}{V} \times \frac{V}{W_2/M_0}\right)$

$$\frac{\Lambda_1}{\Lambda_2} = \frac{\kappa_1 \times \frac{1000}{M_1}}{\kappa_2 \times \frac{1000}{M_2}}$$

$$= \frac{\kappa_1}{\kappa_2} \times \frac{M_2}{M_1}$$

$$= \frac{50}{100} \times 2$$

$$= \frac{\Lambda_1}{\Lambda_2} = 1{,}000 \times 10^{-3}$$

52. (512)

$a = 2(r_+ + r_-)$

$a = 2(102 + 181)$

$a = 2(283)$

$a = 566 pm$

53. (548) Heisenberg's uncertainty principle

$$\Delta x \times \Delta p_x \geq \frac{h}{4\pi}$$

$\Rightarrow 2a_0 \times m\Delta v_x = \frac{h}{4\pi}$ (minimum)

$$\Rightarrow \Delta v_x = \frac{h}{4\pi} \times \frac{1}{2a_0} \times \frac{1}{m}$$

$$= \frac{6.63 \times 10^{-34}}{4 \times 3.14 \times 2 \times 52.9 \times 10^{-12} \times 9.1 \times 10^{-31}}$$

$= 548273\ ms^{-1}$

$= 548.273\ km\ s^{-1}$

$= 548\ km\ s^{-1}$

54. (54)
55. (1)

$p = K_H \times x$

$0.920 = 46.82 \times 10^3 \text{ bar} \times \frac{\text{mol of } O_2}{\text{mol of } H_2O}$

$0.920 = 46.82 \times 10^6 \times \frac{\text{molof}_2}{1000/18}$

$0.920 = 46.82 \times n_{o_2}$

$p = \frac{0.920}{46.82 \times 18} = n_{0_2}$

$\Rightarrow 1.09 \times 10^{-3} = n_{0_2}$

$\Rightarrow$ m mol of $O_2 = 1$

56. (282) $K_{sp} = S^2$

$\mathbf{S} = \sqrt{K_{sp}} = \sqrt{8 \times 10^{-28}} = 2\sqrt{2} \times 10^{-14}$

$= 2.82 \times 10^{-14}$
$= 282 \times 10^{-16}$

Ans. $= 282$

57. (40)

$r = k[x][y]^0 = k[x]$

Using I & II

$\frac{4 \times 10^{-3}}{2 \times 10^{-3}} = \left(\frac{L}{0.1}\right) \Rightarrow L = 0.2$

Using I & III

$\frac{M \times 10^{-3}}{2 \times 10^{-3}} = \frac{0.4}{0.1} \Rightarrow M = 8$

$\frac{M}{L} = \frac{8}{0.2} = 40$

Ans. 40

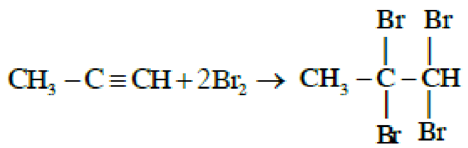
58. (1)

In Tetrapeptide,

No. of Amino Acids = 4

No. of Peptide bonds = 3

59. (3)



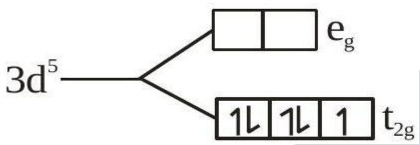
$$= \frac{1}{160} \times \frac{1}{2} \times 360 \times 0.27$$
$$= 0.30375$$
$$= 3.0375 \times 10^{-1}$$

60. (2)

$[Fe(CN)_6]^{3-}$

$CN^{-}$is strong field ligand

$Fe^{+3} 3\ d^5 (t_{2\ g}^5 e_g^0)$



$CFSE = 5(-0.4\Delta_0) = -2.0\Delta_0$

(2)

61. (b) Number of possible values of $a = 60$, for $b = pq$, If $p = 3, q = 3,5,7,11,13,17,19$

If $p = 5 q = 5,7,11$

If $p = 7\ q = 7$

Total cases $= 60 \times 11 = 660$

62. (a)

$z^5 + (\bar{z})^5 = (2 + 3i)^5 + (2 - 3i)^5$

$= 2(\,{}^5C_0 2^5 + {}^5C_2 2^3 (3i)^2 + {}^5C_4 2^1 (3i)^4)$

$= 2(32 + 10 \times 8(-9) + 5 \times 2 \times 81) = 244$

63. (b) $AB = 0 \Rightarrow |AB| = 0$

$$|A|\ |B| = 0$$

$|A| = 0$ $\qquad$ $|B| = 0$

If $|A| \neq 0$, $B = 0$ (not possible)

If $|B| \neq 0$, $A = 0$ (not possible)

Hence $|A| = |B| = 0$

$\Rightarrow$ AX = 0 has infinitely many solutions

64. (c) By splitting

$$\frac{1}{20}\left[\left(\frac{1}{20-a}-\frac{1}{40-a}\right)+\left(\frac{1}{40-a}-\frac{1}{60-a}\right)\right.$$

$$\left.+\cdots+\left(\frac{1}{180-a}-\frac{1}{200-a}\right)\right]$$

$$\Rightarrow \frac{1}{20}\left(\frac{1}{20-a}-\frac{1}{200-a}\right)=\frac{1}{256}$$

$$(20-a)(200-a)=256\times 9$$

$$a^2-220a+1696=0$$

$$a=8,212$$

Hence maximum value of a is 212.

65. (c)

$$\lim_{x\to 0}\frac{\alpha\left(1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\cdots\right)+\beta\left(1-x+\frac{x^2}{2!}-\frac{x^3}{3!}+\cdots\right)+\gamma\left(x-\frac{x^3}{3!}+\cdots\right)}{x^3}$$

constant terms should be zero

$$\Rightarrow a+\beta=0$$

coeff of x should be zero

$$\Rightarrow \alpha-\beta+\gamma=0$$

coeff of x^2 should be zero

$$\lim_{x\to 0}\frac{x^3\left(\frac{\alpha}{3!}-\frac{\beta}{3!}-\frac{\gamma}{3!}\right)+x^4\left(\frac{\alpha}{3!}-\frac{\beta}{3!}-\frac{\gamma}{3!}\right)}{x^3}=\frac{-}{3}$$

$$\Rightarrow \frac{\alpha}{2}+\frac{\beta}{2}=0$$

$$\frac{\alpha}{6}-\frac{\beta}{6}-\frac{\gamma}{6}=2/3$$

$$\Rightarrow \alpha=1,\beta=-1,\gamma=-2$$

66. (b)

$$I=\int_0^{\frac{\pi}{2}}\frac{dx}{3+2\sin x+\cos x}=\int_0^{\frac{\pi}{2}}\frac{\sec^2\frac{x}{2}\cdot dx}{2\tan^2\frac{x}{2}+4\tan\frac{x}{2}+4}$$

Put $\tan\frac{x}{2}=t$, so

$I=\int_0^1 \frac{dt}{(t+1)^2+1}=\tan^{-1}(x+1)|_0^1=\tan^{-1}2-\frac{\pi}{4}$

67. (b) $\frac{dy}{dx}+y=\frac{1}{1+e^{2x}}$

So integrating factor is $e^{\int 1.dx}=e^x$

So solution is $y\cdot e^x=\tan^{-1}(e^x)+c$

Now as curve is passing through $\left(0,\frac{\pi}{2}\right)$ so

$\Rightarrow c=\frac{\pi}{4}$

$\Rightarrow \lim_{x\to\infty}(y\cdot e^x)=\lim_{x\to\infty}\left(\tan^{-1}(e^x)+\frac{\pi}{4}\right)=\frac{3\pi}{4}$

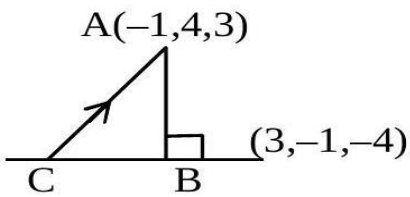
68. (b) Line is passing through intersection of $bx+10y-8=0$ and $2x-3y=0$ is $(bx+10y-8)+\lambda(2x-3y)=0$. As line is passing through $(1,1)$ so $\lambda=b+2$

Now line $(3b+4)x-(3b-4)y-8=0$ is tangent to circle $17(x^2+y^2)=16$

So $\frac{8}{\sqrt{(3b+4)^2+(3b-4)^2}}=\frac{4}{\sqrt{17}}$

$\Rightarrow b^2=2\Rightarrow e=\sqrt{\frac{3}{5}}$

69. (b)



Let B be foot of $\perp$ coordinates of $\mathrm{B}=\left(-2,\frac{7}{2},\frac{3}{2}\right)$ Direction ratio of line AB is $\langle 2,1,3\rangle$ so $\mathrm{m}=1,\mathrm{n}=3$

So equation of AC is $\frac{x+1}{3}=\frac{y-4}{-1}=\frac{z-3}{-4}=\lambda$

So point C is $(3\lambda-1,-\lambda+4,-4\lambda+3)$. But C lies on the plane, so

$6\lambda-2-\lambda+4-12\lambda+9=4$

$\Rightarrow\lambda=1\Rightarrow C(2,3,-1)$

$\Rightarrow AC=\sqrt{26}$

70. (a) $a=3\hat{i}+\hat{j},\vec{b}=\hat{i}+2\hat{j}+\hat{k}$

As $\vec{a}\times(\vec{b}\times\vec{c})=\vec{b}+\lambda\vec{c}$

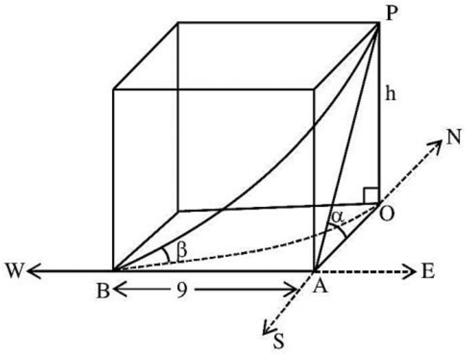
$\Rightarrow \vec{a}\cdot\vec{c}(\vec{b}) - (\vec{a}\cdot\vec{b})\vec{c} = \vec{b} + \lambda\vec{c}$

$\Rightarrow \vec{a}\cdot\vec{c} = 1, \vec{a}\cdot\vec{b} = -\lambda$

$\Rightarrow (3\hat{i}+\hat{j})\cdot(\hat{i}+2\hat{j}+\hat{k}) = -\lambda$

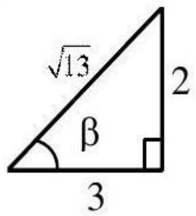
$\Rightarrow \lambda = -5$

71. (a)

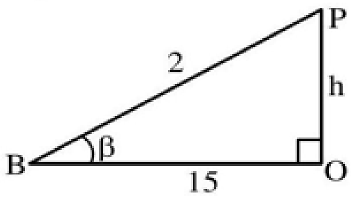


given $OB = 15$

$\cos\beta = \frac{3}{\sqrt{13}}$



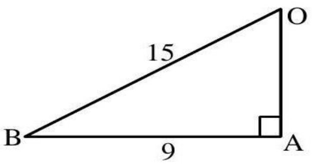
$\tan\beta = \frac{2}{3}$



$\tan\beta = \frac{h}{15}$

$\frac{2}{3} = \frac{h}{15}$

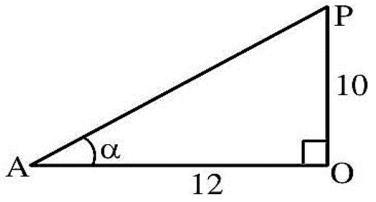
$10 = h$



$OA^2 + AB^2 = 225$

$OA^2 + 81 = 225$

$OA = 12$



$\tan\alpha = \frac{10}{12}$

$\cot\alpha = \frac{12}{10} = \frac{6}{5}$

72. (d)

$(p \wedge q) \Rightarrow (p \wedge r)$

$\sim (p \wedge q) \vee (p \wedge r)$

$(\sim p \vee \sim q) \vee (p \wedge r)$

$(\sim p \vee (p \wedge r)) \vee \sim q$

$(\sim p \vee p) \wedge (\sim p \vee r) \vee \sim q$

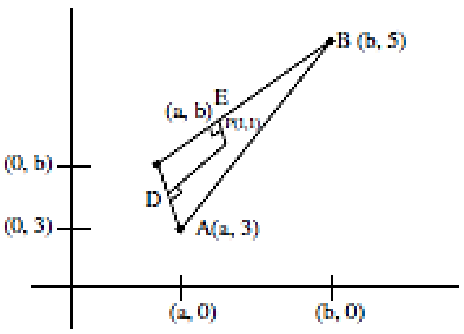
$(\sim p \vee r) \vee \sim q$

$(\sim p \vee \sim q) \vee r$

$\sim (p \wedge q) \vee r$

$(p \wedge q) \Rightarrow r$

73. (b)



$m_{AC} \longrightarrow \infty$
$m_{rv} = 0$
$D\left(\frac{a+a}{2}, \frac{b+3}{2}\right)$
$D\left(a, \frac{b+3}{2}\right)$
$m_n = 0$
$\frac{b+3}{2} - 1 = 0$
$b + 3 - 2 = 0$
$b = -1$
$E\left(\frac{b+a}{2}, \frac{5+b}{2}\right) = \left(\frac{af}{2}, 2\right)$
$m_{cal} \cdot m_{Hr} = -1$
$\left(\frac{5-b}{b-a}\right) = \left(\frac{2-1}{\frac{a-1}{2} - 1}\right) = -1$
$\left(\frac{6}{-1-a}\right) = \left(\frac{2}{a-3}\right) = -1$
$12 = (1+a)(a-3)$
$12 = a^2 - 3a + a - 3$
$a^2 - 2a - 15 = 0$
$(a-5)(a+3) = 0$

$a = 5$ or $a = -3$

Given $ab > 0$

$a(-1) > 0$
$-a > 0$
$a < 0$

$a = -3$ Accept

AP line $A(-3,3) P(1,1)$

$y - 1 = \left(\frac{3-1}{-3-1}\right)(x-1)$

$\Rightarrow \; -2y + 2 = x - 1$

Appling
Line BC $B(-1,5)$

$C(-3,-1)$
$(y-5) = \frac{6}{2}(x+1)$
$y - 5 = 3x + 3$
$y = 3x + 8$

Solving (1) & (2)

$$x+2(3x+8)=3$$

7 $\quad 7x+16=3$

$7x \quad =-13$

$$x=-\frac{13}{7}$$

$$y=3\left(-\frac{13}{7}\right)+8$$

$$=\frac{-39+56}{7}$$

$$y=\frac{17}{7}$$

$$x+y=\frac{-13+17}{7}=\frac{4}{7}$$

74. (a)

$\hat{a}\wedge\hat{b}=\frac{\pi}{4}=\phi$

$$\hat{a}\cdot\hat{b}=|\hat{a}||\hat{b}|\cos\phi$$

$$\hat{a}\cdot\hat{b}=\cos\phi=\frac{1}{\sqrt{2}}$$

$$\cos\theta=\frac{(\hat{a}+\hat{b})\cdot(\hat{a}+2\hat{b}+2(\hat{a}\times\hat{b}))}{|\hat{a}+\hat{b}||\hat{a}+2\hat{b}+2(\hat{a}\times\hat{b})|}$$

$$|\hat{a}+\hat{b}|^2=(\hat{a}+\hat{b})\cdot(\hat{a}+\hat{b})$$

$$|\hat{a}+\hat{b}|^2=2+2\hat{a}\cdot\hat{b}$$

$$=2+\sqrt{2}$$

$$\hat{a}\times\hat{b}=|\hat{a}||\hat{b}|\sin\phi\hat{n}$$

$\hat{a}\times\hat{b}=\frac{\hat{n}}{\sqrt{2}}$ when $\hat{n}$ is vector $\perp$ $\hat{a}$ and $\hat{b}$

let $\vec{c}=\hat{a}\times\hat{b}$

We know.

$$\vec{c}\cdot\vec{a}=0$$

$$\vec{c}\cdot\vec{b}=0$$

$$|\hat{a}+2\hat{b}+2\vec{c}|^2$$

$$=1+4+\frac{(4)}{2}+4\hat{a}\cdot\hat{b}+8\hat{b}\cdot\vec{c}+4\vec{c}\cdot\hat{a}$$

$$=7+\frac{4}{\sqrt{2}}=7+2\sqrt{2}$$

Now

$$(\hat{a}+\hat{b})\cdot(\hat{a}+2\hat{b}+2\vec{c})$$
$$=|\hat{a}|^2+2\hat{a}\cdot\hat{b}+0+\hat{b}\cdot\hat{a}+2|\hat{b}|^2+0$$
$$=1+\frac{2}{\sqrt{2}}+\frac{1}{\sqrt{2}}+2$$
$$=3+\frac{3}{\sqrt{2}}$$
$$\cos\theta=\frac{3+\frac{3}{\sqrt{2}}}{\sqrt{2+\sqrt{2}}\sqrt{7+2\sqrt{2}}}$$
$$\cos^2\theta=\frac{9(\sqrt{2}+1)^2}{2(2+\sqrt{2})(7+2\sqrt{2})}$$
$$\cos^2\theta=\left(\frac{9}{2\sqrt{2}}\right)\frac{(\sqrt{2}+1)}{(7+2\sqrt{2})}$$
$$164\cos^2\theta=\frac{(82)(9)}{\sqrt{2}}\frac{(\sqrt{2}+1)}{(7+2\sqrt{2})}\frac{(7-2\sqrt{2})}{(7-2\sqrt{2})}$$
$$=\frac{(82)}{\sqrt{2}}\frac{(9)[7\sqrt{2}-4+7-2\sqrt{2}]}{(41)}$$
$$=(9\sqrt{2})[5\sqrt{2}+3]$$
$$=90+27\sqrt{2}$$

75. (d)

$$f(e^3)=\int_1^{a^3}\frac{\ell nt}{\ln 10(1+t)}dt\ \ldots$$
$$f(\alpha)=\int_1^{\alpha}\frac{\ell nt}{(\ln 10)(1+t)}dt$$
$$t=\frac{1}{x}\Rightarrow x=\frac{1}{t}$$
$$dt=\frac{-1}{x^2}dx$$
$$=\int_1^{\frac{1}{a}}\frac{-\ln x}{(\ln 10)\left(1+\frac{1}{x}\right)}\left(-\frac{1}{x^2}\right)dx$$
$$f(\alpha)=\frac{1}{\ln 10}\int_1^{\frac{1}{\alpha}}\frac{\ln x}{x(x+1)}dx$$
$$f(e^{-3})=\frac{1}{\ell n10}\int_1^3\frac{\ell nt}{t(t+1)}dt$$

Add (1) & (2)

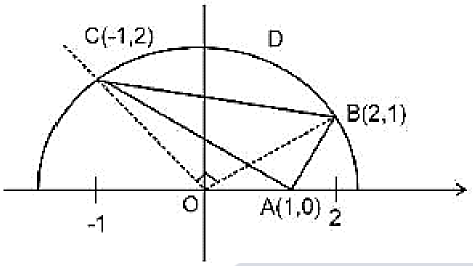
$f(e^3)+f(e^{-3})$

$$=\left(\frac{1}{\ln 10}\right)\int_1^{e^3}\frac{\ell nt}{(1+t)}\left[1+\frac{1}{t}\right]dt$$
$$=\left(\frac{1}{\ln 10}\right)\int_1^{3}\frac{\ln t}{t}dt$$

$\ell nt = r$

$$\frac{dt}{t} = dr$$
$$= \frac{1}{\ln 10}\int_0^3 r dr$$
$$= \left(\frac{1}{\ln 10}\right)\left(\frac{r^2}{2}\right)\Bigg|_0^3$$
$$= \left(\frac{1}{\log 10}\right)\left(\frac{9}{2}\right)$$
$$= \frac{9}{2\log_e 10}$$

76. (d)



$|x-1| < y < \sqrt{5-x^2}$

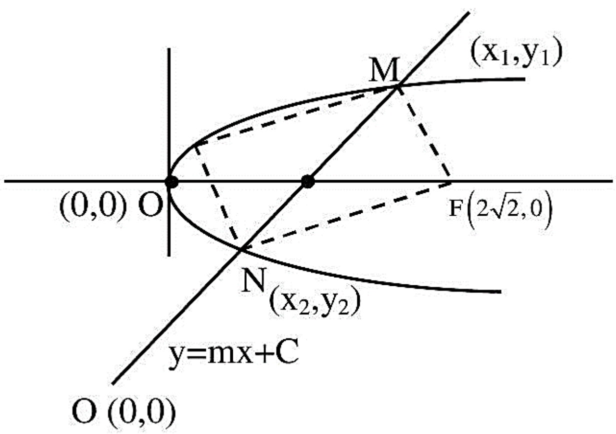
When $|x-1| = \sqrt{5-x^2}$

$$\Rightarrow (x-1)^2 = 5 - x^2$$
$$\Rightarrow x^2 - x - 2 = 0$$
$$\Rightarrow x = 2, -1$$

Required Area = Area of $\triangle$ ABC + Area of region BCD

$$= \frac{1}{2}\begin{vmatrix} 1 & 0 & 1 \\ 2 & 1 & 1 \\ -1 & 2 & 1 \end{vmatrix} + \frac{\pi}{4}(\sqrt{5})^2 - \frac{1}{2}(\sqrt{5})^2$$
$$= \frac{5\pi}{4} - \frac{1}{2}$$

77. (b)



$H: \frac{x^2}{4} - \frac{y^2}{4} = 1$

Focus (ae, 0)

$F(2\sqrt{2}, 0)$

Line L: $y = mx + c$ pass $(1,0)$

$o = m + C$

Line L is tangent to Hyperbola. $\frac{x^2}{4} - \frac{y^2}{4} = 1$

$C = \pm\sqrt{a^2\ m^2 - \ell^2}$

$C = \pm\sqrt{4m^2 - 4}$

From (1)

$-m = \pm\sqrt{4m^2 - 4}$

Squaring

$m^2 = 4\ m^2 - 4$

$4 = 3\ m^2$

$\frac{2}{\sqrt{3}} = m$ (as $m > 0$)

$C = -m$

$C = \frac{-2}{\sqrt{3}}\ y = \frac{2x}{\sqrt{3}} - \frac{2}{\sqrt{3}}$

$y^2 = 4x$

$\Rightarrow \left(\frac{2x - 2}{\sqrt{3}}\right)^2 = 4x$

$\Rightarrow x^2 + 1 - 2x = 3x$

$\Rightarrow x^2 - 5x + 1 = 0$

$y^2 = 4\left(\frac{\sqrt{3}y + 2}{2}\right)$

$y^2 = 2\sqrt{3}y + 4$

$\Rightarrow y^2 - 2\sqrt{3}y - 4 = 0$

Area

$\left|\frac{1}{2}\begin{vmatrix} 0 & x_1 & 2\sqrt{2} & x_2 & 0 \\ 0 & y_1 & 0 & y_2 & 0 \end{vmatrix}\right|$

$= \left|\frac{1}{2}\left[-2\sqrt{2}y_1 + 2\sqrt{2}y_2\right]\right|$

$= \sqrt{2}|y_2 - y_1| = \frac{(\sqrt{2})\sqrt{12+16}}{111}$

$= \sqrt{56}$

$= 2\sqrt{14}$

78. (b)

$f(x) = |x-1|\cos|x-2|\sin|x-1| + (x-3)\mid x^2 - 5x + 4\mid$

$= |x-1|\cos|x-2|\sin|x-1| + (x-3)|x-1||x-4|$

$= |x-1|[\cos|x-2|\sin|x-1| + (x-3)|x-4|]$

Non differentiable at $x = 1$ and $x = 4$.

79. (d) Total number of elements $= 2022$

$2022 = 2 \times 3 \times 337$

$\mathrm{HCF}(n, 2022) = 1$

is feasible when the value of ' n ' and 2022 has no common factor.

$A =$ Number which are divisible by 2 from $\{1,2,3 \ldots\ldots 2022\}$

$n(a) = 1011$

$B =$ Number which are divisible by 3 by 3

from $\{1,2,3 \ldots\ldots 2022\}$

$n(b) = 674$

$A \cap B =$ Number which are divisible by 6

from $\{1,2,3 \ldots\ldots\ldots 2022\}$

6,12,18 2022

$337 = n(A \cap B)$

$n(A \cup B) = n(a) + n(b) - n(A \cap B)$

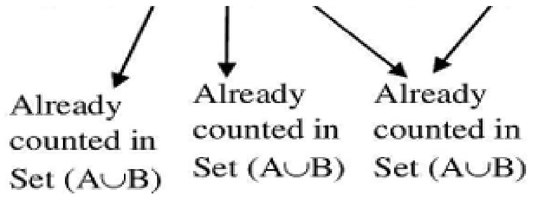
$= 1011 + 674 - 337$

$= 1348$

$C =$ Number which divisible by 337 from

$\{1 . 1022\}$

$C = \{337, 674, 1011, 1348, 1685, 20222\}$



Total elements which are divisible by 2 or 3 or 337 $= 1348 + 2 = 1350$

Favourable cases = Element which are neither divisible by 2,3 or 337

$= 2022 - 1350$

$= 672$

Required probability $= \frac{672}{2022} = \frac{112}{337}$

80. (d)
81. (16)

$7\cos^2\theta - 3\sin^2\theta - 2\cos^2 2\theta = 2$

$4\cos^2\theta + 3\cos 2\theta - 2\cos^2 2\theta = 2$

$2(1+\cos 2\theta) + 3\cos 2\theta - 2\cos^2 2\theta = 2$

$2\cos^2 2\theta - 5\cos 2\theta = 0$

$\cos 2\theta(2\cos 2\theta - 5) = 0$

$\cos 2\theta = 0$
$2\theta = (2n+1)\frac{\pi}{2}$

$\theta = (2n+1)\frac{\pi}{4}$

$S = \left\{\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}\right\}$

For all four values of θ

$x^2 - 2(\tan^2\theta + \cot^2\theta)x + 6\sin^2\theta = 0$

$\Rightarrow x^2 - 4x + 3 = 0$

Sum of roots of all four equations $= 4 \times 4 = 16$.

82. (4) $\sum x_1 = 15 \times 20 = 300$

$\frac{\sum x_1^2}{20} - (15)^2 = 9$

$\sum x_1^2 = 234 \times 20 = 4680$

$\frac{\sum (x_1 + \alpha)^2}{20} = 178 \Rightarrow \sum (x_1 + \alpha)^2 = 3560$

$\Rightarrow \sum x_1^2 + 2\alpha \sum x_1 + \sum \alpha^2 = 3560$

$4680 + 600\alpha + 20\alpha^2 = 3560$

$\Rightarrow \alpha^2 + 30\alpha + 56 = 0$

$\Rightarrow (\alpha + 28)(\alpha + 2) = 0$

$\alpha = -2, -28$

Square of maximum value of α is 4

83. (10)

$(\mathrm{a}, -4\mathrm{a}, -7) \perp$ to $(3, -1, 2\ \mathrm{b})$

$\mathrm{a} = 2\mathrm{b}$

$(\mathrm{a}, -4\mathrm{a}, -7) \perp$ to $(\mathrm{b}, \mathrm{a}, -2)$

$3\mathrm{a} + 4\mathrm{a} - 14\mathrm{b} = 0$

$\mathrm{ab} - 4\mathrm{a}^2 + 14 = 0$

From Equations (i) and (ii)

$2\mathrm{b}^2 - 16\mathrm{b}^2 + 14 = 0$

$\mathrm{b}^2 = 1$

$\mathrm{a}^2 = 4\ \mathrm{b}^2 = 4$

$$\frac{x+1}{5} = \frac{y-2}{3} = \frac{z}{1} = k$$

$\alpha = 5k - 1, \beta = 3k + 2, \gamma = k$

As (α, β, γ) satisfies $x - y + z = 0$

$5k - 1 - (3k + 2) + k = 0$

$k = 1$

$\therefore \alpha + \beta + \gamma = 9k + 1 = 10$

84. (16)

$S = \frac{a_1}{2} + \frac{a_2}{2^2} + \frac{a_3}{2^3} + \cdots.$

$$\frac{S}{2} = \frac{a_1}{2^2} + \frac{a_2}{2^3} + \cdots$$

$$\frac{S}{2} = \frac{a_1}{2} + d\left(\frac{1}{2^2} + \frac{1}{2^3} + \cdots\right)$$

$$\frac{S}{2}=\frac{a_1}{2}+d\left(\frac{\frac{1}{4}}{1-\frac{1}{2}}\right)$$

$\therefore S=a_1+d=a_2=4$

Or $4a_2=16$

85. (84)

$$\frac{T_5}{T_{n-3}}=\frac{{}^{n}C_4\left(2^{1/4}\right)^{n-4}\left(3^{-1/4}\right)^4}{{}_{n-4}\left(2^{1/4}\right)^4\left(3^{-1/4}\right)^{n-4}}=\frac{\sqrt[4]{6}}{1}$$

$\Rightarrow 2^{\frac{n-8}{4}}3^{\frac{n-8}{4}}=6^{1/4}$

$\Rightarrow 6^{n-8}=6$

$\Rightarrow n-8=1\Rightarrow n=9$

$$T_6={}^{9}C_5\left(2^{1/4}\right)^4\left(3^{-1/4}\right)^5=\frac{84}{\sqrt[4]{3}}$$

$\therefore \alpha=84$

86. (282)

$$A=\begin{bmatrix}a_{11} & a_{12} & a_{13}\\ a_{21} & a_{22} & a_{23}\\ a_{31} & a_{32} & a_{33}\end{bmatrix} a_{ij}\in\{0,1\}$$

$\sum a_{ij}=2,3,5,7$

Total matrix $={}^{9}C_2+{}^{9}C_3+{}^{9}C_5+{}^{9}C_7$

$=282$

87. (4)

$$\Delta=\begin{vmatrix}P! & (P+1)! & (P+2)!\\ (P+1)! & (P+2)! & (P+3)!\\ (P+2)! & (P+3)! & (P+4)!\end{vmatrix}$$

$$\Delta=P!\,(P+1)!\,(P+2)!\begin{vmatrix}1 & 1 & 1\\ P+1 & P+2 & P+3\\ (P+2)(P+1) & (P+3)(P+2) & (P+4)(P+3)\end{vmatrix}$$

$\Delta=2P!\,(P+1)!\,(P+2)!$

Which is divisible by $P^{\alpha}\&(P+2)^{\beta}$

$\therefore \alpha=3,\beta=1$

Ans. 4

88. (286)

$$\frac{1}{2.3.4}+\frac{1}{3.4.5}+\cdots\ldots+\frac{1}{100.101.102}=\frac{k}{101}$$

$$\frac{4-2}{2.3\cdot 4}+\frac{5-3}{3\cdot 4\cdot 5}+\cdots\ldots+\frac{102-100}{100.101.102}=\frac{2k}{101}$$

$$\frac{1}{2.3}-\frac{1}{3.4}+\frac{1}{3.4}-\frac{1}{4.5}+\cdots\ldots+\frac{1}{100.101}-\frac{1}{101.102}=\frac{2k}{101}$$

$$\frac{1}{2.3}-\frac{1}{101.102}=\frac{2k}{101}$$

$$\therefore 2k=\frac{101}{6}-\frac{1}{102}$$

$\therefore 34k=286$

89. (11)

$S=\{4,6,9\}\ T=\{9,10,11\ldots.1000\}$

$A\{a_1+a_2+\cdots\ldots+a_k: K\in N\}\&a_i\in S$

Here by the definition of set ' A '

$A=\{a: a=4x+6y+9z\}$

Except the element 11, every element of set T is of of the form $4x+6y+9z$ for some $x, y, z\in W$

$\therefore T-A=\{11\}$

Ans. 11

90. (12) Image of centre $c_1\equiv(1,3)$ in $x-y+1=0$ is given by

$$\frac{x_1-1}{1}=\frac{y_1-3}{-1}=\frac{-2(1-3+1)}{1^2+1^2}$$

$\Rightarrow x_1=2, y_1=2$

$\therefore$ Centre of circle $c_2\equiv(2,2)$

$\therefore$ Equation of c_2 be $x^2+y^2-4x-4y+\frac{38}{5}=0$

Now radius of c_2 is $\sqrt{4+4-\frac{38}{5}}=\sqrt{\frac{2}{5}}=r$

$(\text{ radius of } c_1)^2=(\text{ radius of } c_2)^2$

$$\Rightarrow 10-\alpha=\frac{2}{5}\Rightarrow\alpha=\frac{48}{5}$$

$$\therefore\alpha+6r^2=\frac{48}{5}+\frac{12}{5}=12$$