1. (b)

2. (b)

Torque $= F \times r_{\perp}$ Nm

Stress $= \dfrac{\text{Force}}{\text{Area}}$ N/m^2

Latent heat $= \dfrac{\text{Energy}}{\text{Mass}}$ JKg^{-1}

Power $= \dfrac{\text{Work}}{\text{Time}}$ Nms^{-1}

A-III, B-IV, C-II, D-I

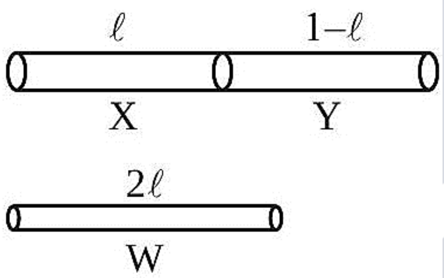
3. (c) Electric field between plates $E = \frac{q_1 - q_2}{2A\epsilon_0}$

$$V = Ed = \frac{q_1 - q_2}{2\,A\,\epsilon_0}\,d$$

$$V = \frac{q_1 - q_2}{2C}$$

4. (a)

5. (b)



$$\frac{R_X}{R_Y} = \frac{\ell_X}{\ell_Y}$$

When wire is stretched to double of its length, then resistance becomes 4 times

$$R_W = 4R_X = 2R_Y$$

$$\frac{R_X}{R_Y} = \frac{1}{2}$$

So. $\frac{\ell_X}{\ell_y} = \frac{1}{2}$

6. (a) Force of interaction $= I_1 \ell_1 B_{12}$

$$= \frac{\mu_0 I_1 I_2}{2\pi r}\ell_1$$

$$= \frac{4\pi \times 10^{-7} \times 6 \times 0.5}{2\pi \times 5 \times 10^{-2}}$$

$= 1.2 \times 10^{-5}$ towards X

7. (d) Time taken by ball to reach highest point $= \frac{u}{g}$

Frequency of throw $= \frac{g}{u} = n$

$\Rightarrow u = \frac{g}{n}$

$$H_{max} = \frac{u^2}{2g} = \frac{\left(\frac{g}{n}\right)^2}{2g}$$

$$\frac{g}{2n^2}$$

8. **(d)** Element X should be resistive with $R = 20\Omega$

Element Y should be inductive with $X_L = 20\Omega$

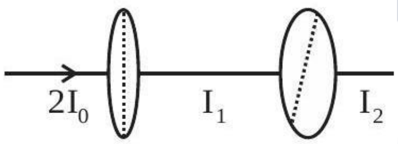
When X and Y are connector in series

$$Z = \sqrt{X_L^2 + R^2} = 20\sqrt{2}$$

$$I_0 = \frac{E_0}{Z} = \frac{100}{20\sqrt{2}} = \frac{5}{\sqrt{2}} \text{ A}$$

$$I_{rms} = \frac{I_0}{\sqrt{2}} = \frac{5}{2} \text{ A}$$

9. **(c)**



$$I_1 = \frac{1}{2}(2I_0) = I_0$$

$$I_2 = I_1 \cos^2 30^\circ$$

$$= I_0 \cdot \frac{3}{4} = \frac{3I_0}{4}$$

10. **(b)**

$$p = \sqrt{2mE} = \sqrt{2mqV}$$

$$\frac{p_\alpha}{p_p} = \sqrt{\frac{m_\alpha q_\alpha}{m_p q_p}} = \sqrt{\frac{4}{1} \times \frac{2}{1}}$$

$$= \frac{2\sqrt{2}}{1}$$

11. **(b)** $R \propto A^{1/3}$

$V = \frac{4}{3}\pi R^3 \propto A$

Mass $\propto$ A

So density is independent of A.

12. (a) $U_i = \frac{-GMm}{R}$

$U_f = -\frac{GMm}{4R}$

$\Delta U = U_f - U_i = \frac{3GMm}{4R}$

$= \frac{3}{4} mgR$

$= \frac{3}{4} \times 1 \times 10 \times 64 \times 10^5$

$= 48MJ$

13. (d)

For first $\frac{h}{2}$

$\frac{h}{2} = \frac{1}{2} g t_1^2$

For total height h

$h = \frac{1}{2} g(t_1 + t_2)^2$

$\frac{1}{\sqrt{2}} = \frac{t_1}{t_1 + t_2}$

$1 + \frac{t_2}{t_1} = \sqrt{2}$

$\frac{t_1}{t_2} = \frac{1}{\sqrt{2} - 1}$

$t_2 = (\sqrt{2} - 1)t_1$

14. (b) For equilibrium $m_2\, g = m_1\, g \sin\theta$

$\sin\theta = \frac{m_2}{m_1} = \frac{3}{5}$

$\cos\theta = \frac{4}{5}$

Normal force on $m_1 = 5\, g \cos\theta$

$= 5 \times 10 \times \frac{4}{5} = 40\text{ N}$

15. (c)

$P' = P + \frac{20}{100}P = 1.2P$

% change in KE $= \frac{K'-K}{K} \times 100$

$= \left(\dfrac{\frac{P'^2}{2\text{ m}} - \frac{P^2}{2\text{ m}}}{\frac{P^2}{2\text{ m}}}\right) \times 100$

$= [(1.2)^2 - 1] \times 100$

$= 44\%$

16. (c)

$\vec{\tau} = \vec{r} \times \vec{F} = \begin{vmatrix} i & j & k \\ 2 & 2 & 1 \\ 5 & 3 & -7 \end{vmatrix}$

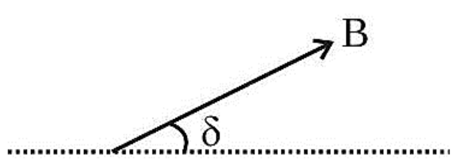
$= \text{i}(-14-3) - \text{j}(-14-5) + \hat{\text{k}}(6-10)$

$= -17\hat{\imath} + 19\hat{\jmath} - 4\hat{k}$

17. (b)

$$\begin{aligned} W_{DE} &= \frac{1}{2}(600+300)3\text{ J} \\ &= 1350\text{ J} \\ W_{EF} &= -300 \times 3 = -900\text{ J} \\ W_{DEF} &= 450\text{ J} \end{aligned}$$

18. (d)



$(\delta = 37^\circ)$

$B_V = B\sin\delta$

$6 \times 10^{-5} = B\frac{3}{5}$

$B = 10 \times 10^{-5}\text{ T}$

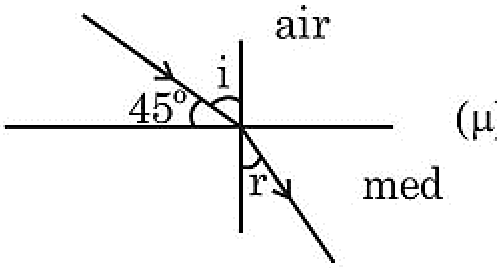
$= 10^{-4}\text{ T}$

19. (c)

$$V_{rms} = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3 \times 1.38 \times 10^{-23} \times 293}{5 \times 10^{-17}}}$$

≈ 15 mm/s

20. (c)



$i = 45°$

$D = i - r$

$15° = 45 - r \Rightarrow r = 30°$

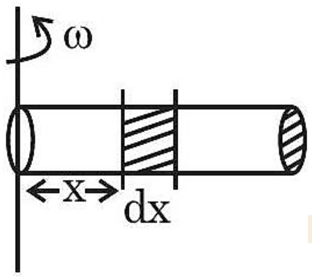
$n_1 \sin i = n_2 \sin r$

$1 \sin 45° = \mu \sin 30°$

$$\frac{1}{\sqrt{2}} = \mu \frac{1}{2}$$

$\mu = \sqrt{2} = 1.414$

21. (4)



$F = \int (dm)\omega^2 x$

$$= \int_0^L \left(\frac{m}{L} dx\right) \omega^2 x$$

$$= \frac{m}{L} \omega^2 \frac{L^2}{2}$$

$$= \frac{m\omega^2 L}{2}$$

$\omega = \sqrt{\frac{2}{mL}}\sqrt{F}$

$= \sqrt{\frac{2}{0.25 \times 0.5}}\sqrt{F}$

$= \sqrt{16}\sqrt{F}$

$= 4\sqrt{F}$

22. (84)

$P' = \mathbf{10}\%$ of 110 W

$= \frac{10}{100} \times 110$ W

$= 11$ W

$I_1 - I_2 = \frac{P'}{4\pi r_1^2} - \frac{P'}{4\pi r_2^2}$

$= \frac{11}{4\pi}\left[\frac{1}{1} - \frac{1}{25}\right]$

$= \frac{11}{4\pi} \times \frac{24}{25}$

$= \frac{264}{\pi} \times 10^{-2} = 84 \times 10^{-2}\ \mathrm{W/m^2}$

23. (50)

$T = \frac{mv^2}{\ell} = \frac{10 \times v^2}{0.5} = 20v^2$

$T_{max} =$ Breaking stress $\times$ Area

$= 5 \times 10^8 \times 10^{-4} = 5 \times 10^4$

$20\ V^2 = 5 \times 10^4$

$V = \sqrt{\frac{1}{4}10^4} = 50$ m/s

24. (25)

$F_V + F_B = mg$($v =$ constant)

$F_V = mg - F_B$

$= \rho_B Vg - \rho_L Vg$

$= (\rho_B - \rho_L)Vg$

$= (8-1.3)\times 10^{+3}\times \frac{0.3\times 10^{-3}}{8\times 10^{3}}\times 10$

$= \frac{6.7\times 0.3}{8}\times 10^{-2}$ ($g = 10$)

$= \frac{67\times 3}{8}\times 10^{-4} = 25.125\times 10^{-4}$

25.125

25. **(2)** Frequencies present in output of square law device $2f_c$, $f_c + f_m$, f_c, $f_c - f_m$, $2f_m$, f_m After passing through band bass filte.

$f_c + f_m, f_c, f_c - f_m$

Band width $= 2f_m$

$= \frac{2\omega_m}{2\pi} = \frac{6.28\times 10^6}{3.14}$

$= 2\text{MHz}$

26. **(15)**

$V_w = \sqrt{\frac{T}{\mu}}$

$60 = \sqrt{\frac{T}{10\times 10^{-3}}\times 0.5}$

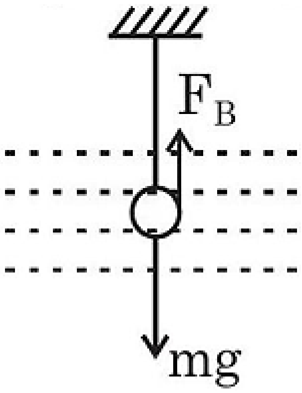
$T = \frac{(60)^2\times 10^{-2}}{0.5} = 72\ \text{N}$

$\Delta\ell = \frac{F\ell}{AY} = \frac{72\times 0.5}{2\times 10^{-6}\times 1.2\times 10^{11}}$

$= \frac{72\times 5}{24}\times 10^{-5} = 15\times 10^{-5}$

Ans. 15

27. **(5)** $mg' = mg - F_B$



$g' = \frac{mg - F_B}{m}$

$= \frac{\rho_B Vg - \rho_w Vg}{\rho_B V}$

$= \left(\frac{\rho_B - \rho_w}{\rho_B}\right) g \; T = 2\pi\sqrt{\frac{\ell}{g}}$

$= \frac{5-1}{5} \times g$

$= \frac{4}{5}\, g$

$\frac{T'}{T} = \sqrt{\frac{g}{g'}} = \sqrt{\frac{g}{\frac{4}{5}g}} = \sqrt{\frac{5}{4}}$

$T' = T\sqrt{\frac{5}{4}} = \frac{10}{2}\sqrt{5}$

$T' = 5\sqrt{5}$

28. (480)

$\varepsilon - IR - V_z = 0$

$20 - IR - 6 = 0$

$IR = 12$

$25 \times 10^{-3} R = 12$

$R = \frac{12}{25 \times 10^{-3}} = 480\Omega$

29. (9)

$N = N_0 e^{-\lambda t}$

$$\frac{N_B}{N_A}=\frac{e^{-\lambda_2 t}}{e^{-\lambda_1 t}}=e^{-\lambda_2 t}\cdot e^{\lambda_1 t}$$

$e^1=e^{(\lambda_1-\lambda_2)t}$

$(\lambda_1-\lambda_2)t=1$

$$t=\frac{1}{\lambda_1-\lambda_2}=\frac{1}{25\lambda-16\lambda}=\frac{1}{9\lambda}$$

30. (10) Energy stored in capacitor

$$=\frac{1}{2}CV^2=\frac{1}{2}500\times10^{-6}\times10^4$$
$$=\frac{5}{2}\ J$$

Current will be maximum when whole energy of capacitor becomes energy of inductor.

$$\frac{1}{2}LI^2=\frac{5}{2}$$

$$I=\sqrt{\frac{5}{L}}=\sqrt{\frac{5}{50\times10^{-3}}}=10\ A.$$

31. (c)

32. (b) Energy order of subshell decided by $(n+\lambda)$ rule.

$A\Rightarrow 3\ d\Rightarrow n+1=5$

$B\Rightarrow 4p\Rightarrow n+\lambda=5$

$C\Rightarrow 4\ d\Rightarrow n+\ell\Rightarrow 6$

$D\Rightarrow 3\ s\Rightarrow (n+\ell)=4$

$D<A<B<C$

33. (d)

$C(S)+O_2(g)\rightarrow CO_2(g)+400\ kJ$

1 g mole

$C(s)+\frac{1}{2}O_2(g)\rightarrow CO(g)+100\ kJ$

0.6×1000

$=600gm$

$600\times\frac{60}{100}$ (Pure Carbon)

$=360gm=\frac{360}{12}=30$ mole (Pure Carbon)

Carbon converted into $CO_2 = \left(30 - 30 \times \frac{60}{100}\right)$

$= 12$ mole

and carbon converted in $CO = 30 \times \frac{60}{100} = 18$ mole

Energy generated during II equation

$= 18 \times 100$

$= 1800$ kJ

Energy generated during Ist reaction.

$= 12 \times 400$

$= 4800$

Total $= 1800 + 4800 = 6600$ kJ

34. (b) $HCl + H_2SO_4$

$$[H^+] = \frac{(0.01 \times 200) + (0.01 \times 2 \times 400)}{600}$$

$$= \frac{2+8}{600} = \frac{10}{600} = \frac{1}{60}$$

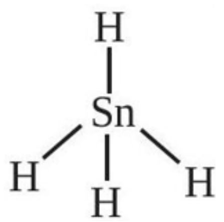
$$pH = -\log\left[\frac{1}{60}\right]$$

$= 1.78$

35. (a) More the critical temp. of gas greater is the ease of liquefaction hence greater is the adsorption.

36. (c) Liquation process is used for metal having low melting point such as tin in which they are heated and brought to molten state and made to flow down the slope while impurities with higher melting point left on the top.

37. (c) SnH_4 is non planar molecular hydride



Tetrahedral shape, sp^3 hybridisation

38. (b) Gypsum $(CaSO_4 \cdot 2H_2O)$ is used to enhance setting time in portland cement.

39. (b)

40. (b)

41. (a) There is unsymmetric filling of e_g subset of Cu^{+2} ion, while there is symmetrical distribution in t_{2g} set, if the complex has same ligand there will be equal repulsion which leads to symmetrical bond length along t_{2g}, but due to uneven filling of electron in e_g subset, either octahedral will be elongated or compressed.

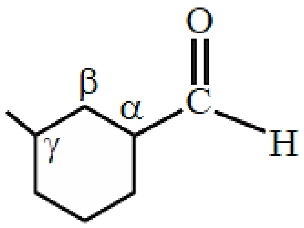
42. (c)

$N_2(g) + O_2(g) \rightarrow 2NO(g)$

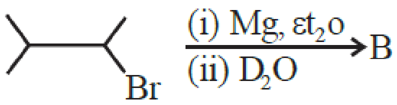
$2NO(g) + O_2(g) \rightarrow 2NO_2(g)$

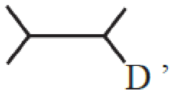
NO_2 damage plant leaves

43. (a)



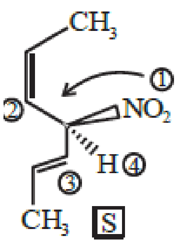
44. (c)



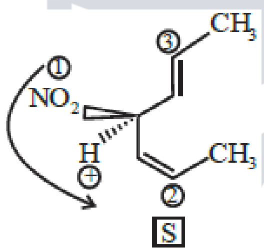


chiral

45. (c)



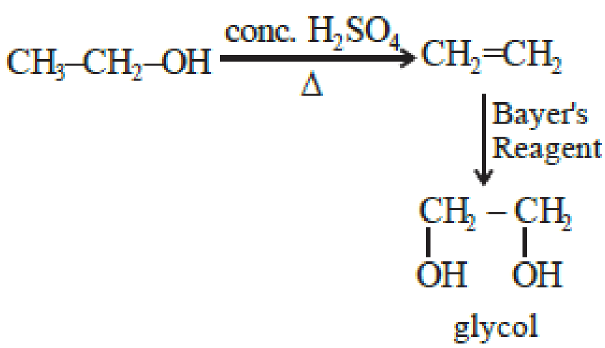
(A)



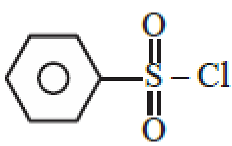
(B)

Having same configuration.

46. (c)



47. (a)



B.S.C (Benzene sulphonyl chloride) is known's Hinsberg Reagent

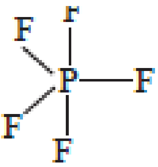
48. (d) Rayon is semisynthetic polymer.
49. (d) Amylose is water soluble.
50. (c) Phenolphthalein is weak acid give colour in basic medium.
51. (80)

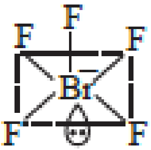
$O_2 + Ne$

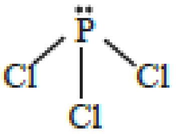
Xgm 200gm

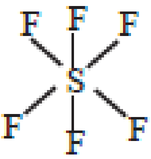
$$P_{total} = \mathbf{25}\ \text{bar} ; P_{Ne} = \mathbf{20}$$
$$P_{O_2} + P_{Ne} = 25$$
$$P_{O_2} = 25 - 20 = 5\text{bar}$$
$$5 = \frac{\frac{x}{32}}{\frac{x}{32} + \frac{200}{20}} \times 25$$
$$\frac{1}{5} = \frac{\frac{x}{32}}{\frac{x}{32} + 10}$$
$$\frac{1}{5} = \frac{x \times 32}{32(x + 320)}$$
$$5x = x + 320$$
$$4x = 320$$
$$x = \frac{320}{4} = 80\text{gm}$$

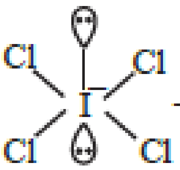
52. (4)

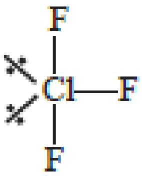
 $\rightarrow sp^3d$ Hybridisation

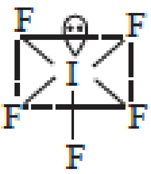
 $\rightarrow sp^3d^2$ Hybridisation

 $\rightarrow sp^3$ Hybridisation

 $\rightarrow sp^3d^2$ Hybridisation

 $\rightarrow sp^3d^2$ Hybridisation

 $\rightarrow sp^3d$ Hybridisation

 $\rightarrow sp^3d^2$ Hybridisation

53. (80) Mass of $C_2H_5OH = 62.5 \times 0.8 = 50$ g

$\Delta T_f = K_f \times m$

$$0.9 = 2 \times \frac{1.8 \times 1000}{M_w \times 50}$$

$$M_w = \frac{2 \times 1.8 \times 1000}{0.9 \times 50} = 80$$

54. (34)

At anode

$Cu \rightarrow Cu^{2+} + 2e^-$

At cathode

$2Ag^+ + 2e^- \rightarrow 2Ag$

Cell reaction $\rightarrow Cu + 2Ag^+ \rightarrow Cu^{2+} + 2Ag$

$$E_{cell} = E^0_{cell} - \frac{0.06}{2}\log\frac{[Cu^{2+}]}{[Ag^+]^2}$$

$$0.43 = E^0_{cell} - \frac{0.06}{2}\log\frac{(0.001)}{(0.01)^2}$$

$$E^0_{cell} = 0.46$$

$$E^0_{cell} = E^0_{Ag^+/Ag} - E^0_{Cu^{2+}/Cu}$$

$$0.46 = 0.80 - E^0_{Cu^{2+}/Cu}$$

$$E^0_{Cu^{2+}/Cu} = 0.34\text{volt}^2$$

$$E^0_{Cu^{2+}/Cu} = 34 \times 10^{-2}$$

55. (1)

$$t = \frac{1}{\lambda}\ln\left(\frac{a}{a-x}\right)$$

$$100 = \frac{30}{\ln 2}\ln\left(\frac{1}{w}\right)$$

$$\frac{1}{w} = 10$$

$W = 0.1 \times \mu g$

Ans. $1 \times 10^{-1}\mu g$

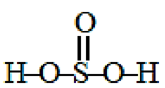
56. (9)

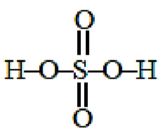
Coordination no. = 6

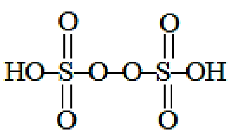
Oxidation state = 3

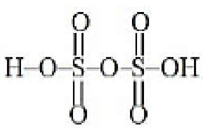
$6 + 3 = 9$

57. (1)


(H_2SO_3)


(H_2SO_4)


$(H_2S_2O_8)$


$(H_2S_2O_7)$

58. (3)

$R(OH)_x \rightarrow H_2$

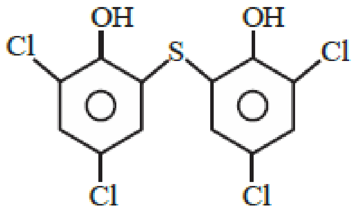
PoAC on H –

$$x\left(\frac{1.84\times10^{-3}}{92}\right)=\frac{1.344}{22.4}\times2$$

$$x=\frac{1.344\times2\times92\times1000}{1.84\times22400}=6$$

$$x=6$$

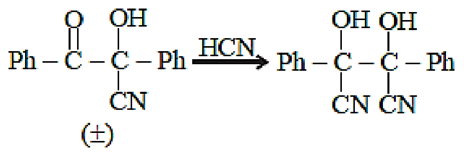
59. (4) Bithinol



Mathpix 2D

Chlorine atoms = 4

60. (3)



3 stereoisomers

61. (d) $|z-1/z|=2$

$\left|\,|z|-\frac{1}{|z|}\right|\leq\left|z-\frac{1}{z}\right|\leq|z|+\frac{1}{|z|}$ Let $|z|=r$

$\left|r-\frac{1}{r}\right|\leq2\leq r+\frac{1}{r}$

$\left|r-\frac{1}{r}\right|\leq2\,\&\,r+\frac{1}{r}\geq2$ always true

$r-\frac{1}{r}\geq-2\,\&\,r-\frac{1}{r}\leq2$

$r^2-1\leq2r$

$r^2-2r\leq1$

$(r-1)^2\leq2$

$r-1\leq\sqrt{2}$

$\therefore|z|_{max}=1+\sqrt{2}$

62. (c)

63. (c) $x+y+z=6$

$2x+5y+\alpha z=\beta$

$x+2y+3z=14$

$\mathrm{x}+\mathrm{y}=6-\mathrm{z}$

$x+2y=14-3z$

On solving

$\mathrm{x}=\mathrm{z}-2 \Rightarrow \mathrm{y}=8-2\mathrm{z}$ in (2)

$2(\mathrm{z}-2)+5(8-2\mathrm{z})+\alpha \mathrm{z}=\beta$

$(\alpha-8)\mathrm{z}=\beta-36$ For having infinite solutions

$\alpha-8=0\ \&\beta-36=0$

$\alpha=8, \beta=36$

$(\alpha+\beta=44)$

64. (d) $\mathrm{f(x)}=\begin{cases}\frac{\ln(1+5x)-\ln(1+\alpha x)}{x} & ; x \neq 0 \\ 10 & ; x=0\end{cases}$

$$\lim_{x\to 0}\frac{\ln(1+5x)-\ln(1+\alpha x)}{x}=10$$

Using expension

$$\lim_{x\to 0}\frac{(5x+\cdots..)-(\alpha x+\cdots..)}{x}=10$$

$5-\alpha=10 \Rightarrow \alpha=-5$

65. (a)

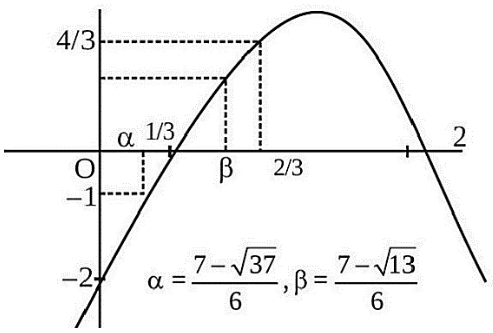
$$\mathrm{I}=\int_0^1[2x-|3x^2-3x-2x+2|+1]dx$$

$$\mathrm{I}=\int_0^1[2x-|(3x-2)(x-1)|]dx+\int_0^1 1dx$$

$$\mathrm{I}=\int_0^{2/3}\left[\left(2x-(3x^2-5x+2)\right)\right]dx+\int_{2/3}^1\left(2x+(3x^2-5x+2)\right)dx+1$$

$$\mathrm{I}=\int_0^{2/3}[-3x^2+7x-2]dx+\int_{2/3}^1(3x^2-3x+2)dx+1$$

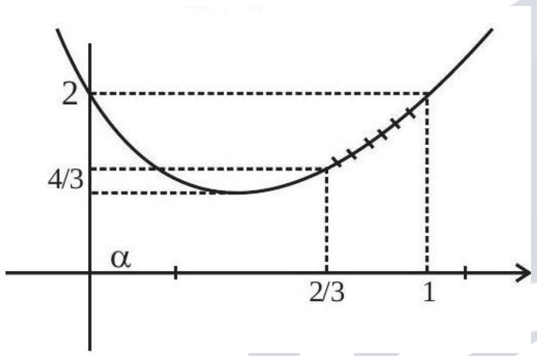
$y=-3x^2+7x-2$



$\int_0^{\alpha}(-2)dx+\int_{\alpha}^{1/3}(-1)dx+\int_{1/3}^{\beta}0dx+\int_{\beta}^{2/3}1\cdot dx$

$=-2\alpha-\left(\frac{1}{3}-\alpha\right)+\frac{2}{3}-\beta=-\alpha-\beta+\frac{1}{3}$

$y=3x^2-3x+2$



When $x\in\left(\frac{2}{3},1\right)$

$3x^2-3x+2\in\left(\frac{4}{3},2\right)$

$[3x^2-3x+2]=1$

$\therefore\int_{2/3}^{1}[3x^2-3x+2]dx=1\left(1-\frac{2}{3}\right)=\frac{1}{3}$

Hence $I=\left(\frac{1}{3}-(\alpha+\beta)\right)+\left(\frac{1}{3}\right)+1$

$=\frac{5}{3}-\left(\frac{7-\sqrt{37}}{6}+\frac{7-\sqrt{13}}{6}\right)$

$=\frac{-2}{3}+\frac{\sqrt{37}+\sqrt{13}}{6}$

$=\frac{\sqrt{37}+\sqrt{13}-4}{6}$

66. (b) $a_0=0, a_1=0$

$a_{n+2}=3a_{n+1}-2a_{n+1}: n\geq 0$

$a_{n+2} - a_{n+1} = 2(a_{n+1} - a_n) + 1$

$n = 0\ a_2 - a_1 = 2(a_1 - a_0) + 1$

$n = 1\ a_3 - a_2 = 2(a_2 - a_1) + 1$

$n = 2\ a_4 - a_3 = 2(a_3 - a_2) + 1$

$n = n\ a_{n+2} - a_{n+1} = 2(a_{n+1} - a_n) + 1$

$(a_{n+2} - a_1) - 2(a_{n+1} - a_0) - (n+1) = 0$

$a_{n+2} = 2a_{n+1} + (n+1)$

$n \to n - 2$

$a_n - 2a_{n-1} = n - 1$

Now $a_{25}a_{23} - 2a_{25}a_{22} - 2a_{23}a_{24} + 4a_{22}a_{24}$

$= (a_{25} - 2a_{24})(a_{23} - 2a_{22}) = (24)(22) = 528$

67. (b) $\sum_{x=1}^{20}(r^2+1)r\,!$

$$\sum_{x=1}^{20}((r+1)^2 - 2r)r!$$
$$\sum_{x=1}^{20}((r+1)(r+1)! - r.r!) - \sum_{r=1}^{20} r \cdot r!$$
$$\sum_{x=1}^{20}((r+1)(r+1)! - r.r!) - \sum_{r=1}^{20}((r+1)! - r!)$$
$$= (21 \cdot 21 - 1) - (|\,21 - 1)$$
$$= 20 \cdot 21! = 22! - 2.21!$$

68. (a)

$I(x) = \int \sec^2 x \cdot \sin^{-2022} xdx - 2022\int \sin^{-2022} xdx$ II $I = \tan x \cdot (\sin x)^{-2022} + \int (2022)\tan x \cdot (\sin x)^{-2023}\cos xdx$

$-2022\int (\sin x)^{-2022}dx$

$I(x) = (\tan x)(\sin x)^{-2022} + C$

At $X = \pi/4, 2^{1011} = \left(\frac{1}{\sqrt{2}}\right)^{-2022} + C \therefore C = 0$

Hence $I(x) = \frac{\tan x}{(\sin x)^{2022}}$

$$I(\pi/6) = \frac{1}{\sqrt{3}\left(\frac{1}{2}\right)^{2022}} = \frac{2^{2022}}{\sqrt{3}}$$

$$I(\pi/3) = \frac{\sqrt{3}}{\left(\frac{\sqrt{3}}{2}\right)^{2022}} = \frac{2^{2022}}{(\sqrt{3})^{2021}} = \frac{1}{3^{1010}} I\left(\frac{\pi}{6}\right)$$

$$3^{1010} I(\pi/3) = I(\pi/6)$$

69. (a) $\frac{dy}{dx} = \frac{x+y-2}{x-y} = \frac{(x-1)+(y-1)}{(x-1)-(y-1)}$

$$x - 1 = X, y - 1 = Y$$

$$\frac{dY}{dX} = \frac{X+Y}{X-Y}$$

$$Y = VX \frac{dY}{dX} = V + X\frac{dV}{dX}$$

$$V + X\frac{dV}{dX} = \frac{1+V}{1-V} \; X\frac{dV}{dX} = \frac{V^2+1}{1-V}$$

$$\int \frac{1-V}{1+V^2} dV = \int \frac{dX}{X}$$

$$\int \frac{dV}{1+V^2} - \frac{1}{2}\int \frac{2VdV}{1+V^2} = \int \frac{dX}{X}$$

$$\tan^{-1} V - \frac{1}{2}\ln(1+V^2) = \ln X + c$$

$$\tan^{-1}\left(\frac{Y}{X}\right) - \frac{1}{2}\ln\left(1+\frac{Y^2}{X^2}\right) = \ln(X) + c$$

$$\tan^{-1}\left(\frac{y-1}{x-1}\right) - \frac{1}{2}\ln\left(1+\frac{(y-1)^2}{(x-1)^2}\right) = \ln(x-1) + c$$

Passes through (2,1)

$$0 - \frac{1}{2}\ln 1 = \ln 1 + c \therefore c = 0$$

Passes through $(k+1, 2)$

$$\therefore \tan^{-1}\left(\frac{1}{k}\right) - \frac{1}{2}\ln\left(1+\frac{1}{k^2}\right) = \ln k$$

$$2\tan^{-1}\left(\frac{1}{k}\right) = \ln\left(\frac{1+k^2}{k^2}\right) + 2\ln k$$

$$2\tan^{-1}\left(\frac{1}{k}\right) = \ln(1+k^2)$$

70. (b) $\frac{dy}{dx} + \left(\frac{2x^2+11x+13}{x^3+6x^2+11x+6}\right)y = \frac{x+3}{x+1}$

$\int p(x)dx$ I.F. $= e^{\int p(x)dx}$

$$\int p(x)dx = \int \frac{(2x^2+11x+13)dx}{(x+1)(x+2)(x+3)}$$

Using partial fraction

$$\frac{2x^2+11x+13}{(x+1)(x+2)(x+3)} = \frac{A}{x+1} + \frac{B}{x+2} + \frac{C}{x+3}$$

$$A = \frac{4}{2} = 2$$

$$B = 1$$

$$C = -1$$

$$\because \int p(x)dx = A\ln(x+1) + B\ln(x+2) + c\ln(x+3)$$

$$= \ln\left(\frac{(x+1)^2(x+2)}{x+3}\right)$$

$$\text{I.F.} = e^{\int p(x)dx} = \frac{(x+1)^2(x+2)}{(x+3)}$$

Solution $y(IF) = \int Q.(IF)dx$

$$y\left(\frac{(x+1)^2(x+2)}{x+3}\right) = \int \left(\frac{x+3}{x+1}\right)\frac{(x+1)/(x+2)}{(x+3)}dx$$

$$y\left(\frac{(x+1)^2(x+2)}{x+3}\right) = \frac{x^3}{3} + \frac{3x^2}{2} + 2x + c$$

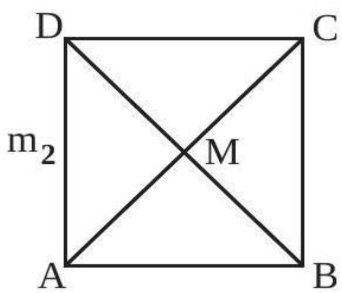
Passes through $(0,1) C = \frac{2}{3}$

Now put $x = 1$

$$\Rightarrow y(1) = \frac{3}{2}$$

71. (b) $m_1 m_2 = -1$

$$a^2 + 11a + 3\left(m_1^2 + \frac{1}{m_1^2}\right) = 220$$



Eq. of AC

$$AC = (\cos\alpha - \sin\alpha) + (\sin\alpha + \cos\alpha)y = 10$$

$BD = (\sin\alpha - \cos\alpha)x + (\sin\alpha - \cos\alpha)y = 0$

$(10(\cos\alpha - \sin\alpha), 10(\sin\alpha - \cos\alpha))$

Slope of AC $= \left(\frac{\sin\alpha - \cos\alpha}{\sin\alpha + \cos\alpha}\right) = \tan\theta = M$

Eq. of line making an angle π_4 with AC

$$m_1,\ m_2 = \frac{m \pm \tan\frac{\pi}{4}}{1 \pm m\tan\frac{\pi}{4}}$$

$= \frac{m+1}{1-m}$ or $\frac{m-1}{1+m}$

$$\frac{\frac{\sin\alpha - \cos\alpha}{\sin\alpha + \cos\alpha} + 1}{1 - \left(\frac{\sin\alpha - \cos\alpha}{\sin\alpha + \cos\alpha}\right)}, \frac{\frac{\sin\alpha - \cos\alpha}{\sin\alpha + \cos\alpha} - 1}{1 + \frac{\sin\alpha - \cos\alpha}{\sin\alpha + \cos\alpha}}$$

$m_1,\ m_2 = \tan\alpha, \cot\alpha$

mid point of AC&BD

$= M(5(\cos\alpha - \sin\alpha), 5(\cos\alpha + \sin\alpha))$

B $(10(\cos\alpha - \sin\alpha), 10(\cos\alpha + \sin\alpha))$

$a = AB = \sqrt{2}BM = \sqrt{2}(5\sqrt{2}) = 10$

$a = 10$

$$\because a^2 + 11a + 3\left(m_1^2 + \frac{1}{m_1 2}\right) = 220$$

$100 + 110 + 3(\tan^2\alpha + \cot^2\alpha) = 220$

Hence $\tan^2\alpha = 3, \tan^2\alpha = \frac{1}{3} \Rightarrow \alpha = \frac{\pi}{3}$ or $\frac{\pi}{6}$

Now $72(\sin^4\alpha + \cos^4\alpha) + a^2 - 3a + 13$

$$= 72\left(\frac{9}{16} + \frac{1}{16}\right) + 100 - 30 + 13$$

$$= 72\left(\frac{5}{8}\right) + 83 = 45 + 83 = 128$$

72. (a) $2\cos\left(\frac{x^2+x}{6}\right) = 4^x + 4^{-x}$

L.H.S $\leq 2.$ & R.H.S. ≥ 2

Hence L.H.S $= 2$& R.H.S $= 2$

$$2\cos\left(\frac{x^2 + x}{6}\right) = 2\ 4^x + 4^{-x} = 2$$

Check $x = 0$ Possible hence only one solution

73. (b) $A(\alpha, -2)$: $B(\alpha, 6)$: $C\left(\frac{\alpha}{4}, -2\right)$

since AC is perpendicular to AB.

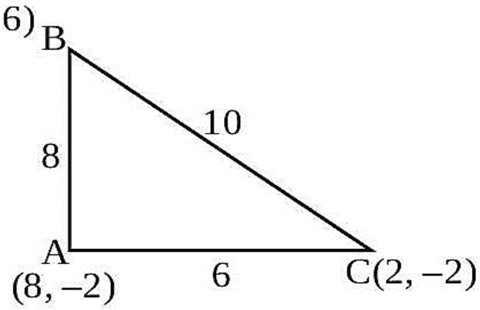
So, $\triangle$ ABC is right angled at A.

Circumcentre = mid point of BC. $= \left(\frac{5\alpha}{8}, 2\right)$

$$\therefore \frac{5\alpha}{8} = 5 \& \frac{\alpha}{4} = 2$$

$\alpha = 8$

(8,6)



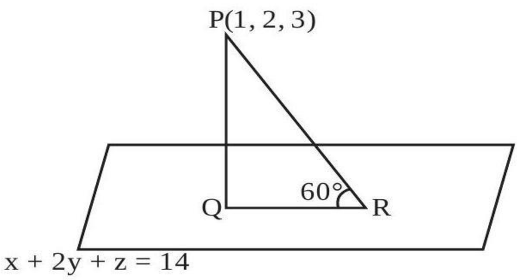
Area $= \frac{1}{2}(6)(8) = 24$

Perimeter $= 24$

Circumradius $= 5$

Inradius $= \frac{\Delta}{s} = \frac{24}{12} = 2$

74. (b)



Length of perpendicular

$$PQ = \left|\frac{1 + 4 + 3 - 14}{\sqrt{6}}\right| = \sqrt{6}$$

$QR = (PQ)\cot 60^\circ = \sqrt{2}$

$\therefore$ Area of $\triangle$ PQR $= \frac{1}{2}(PQ)(QR) = \sqrt{3}$

75. (d) $A(2,3,9); B(5,2,1); C(1,\lambda,8); D(\lambda,2,3)$

$$[\overrightarrow{AB} \quad \overrightarrow{AC} \quad \overrightarrow{AD}] = 0$$

$$\begin{vmatrix} 3 & -1 & -8 \\ -1 & \lambda-3 & -1 \\ \lambda-2 & -1 & -6 \end{vmatrix} = 0$$

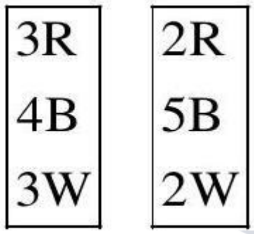
$$\Rightarrow [-6(\lambda-3)-1] - 8(1-(\lambda-3)(\lambda-2)) + (6+(\lambda -2) = 0$$

$$3(-6\lambda+17) - 8(-\lambda^2+5\lambda-5) + (\lambda+4) = 8$$

$$8\lambda^2 - 57\lambda + 95 = 0$$

$$\lambda_1\lambda_2 = \frac{95}{8}$$

76. (b)



A: Drown ball from boy II is black

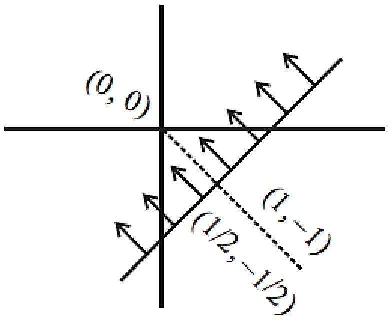
B: Red ball transferred

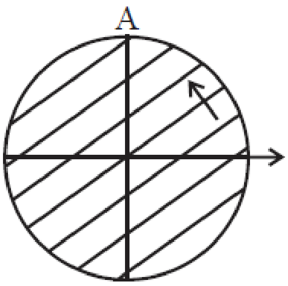
$$P\left(\frac{B}{A}\right) = \frac{P(A\cap B)}{P(a)}$$

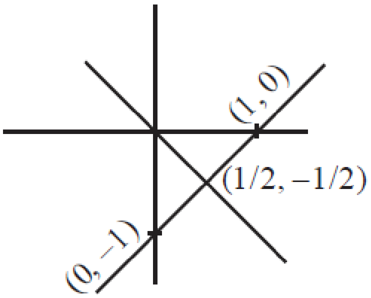
$$= \frac{\frac{3}{9}\times\frac{5}{10}}{\frac{3}{9}\times\frac{5}{10}+\frac{4}{9}\times\frac{6}{10}+\frac{3}{9}\times\frac{5}{10}}$$

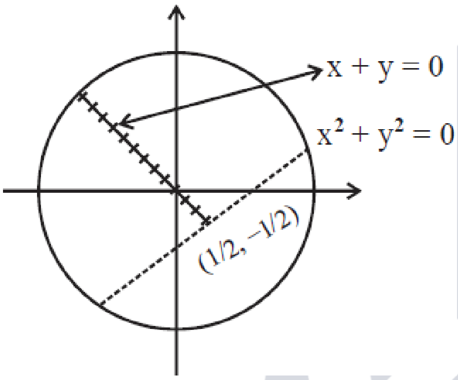
$$= \frac{15}{15+24+15} = \frac{15}{54} = \frac{5}{18}$$

77. (b) $|z-1+i| \geq |z|; |z| < 2; |z+i| = |z-1|$









Hence

$$w = 2x + iy \in S$$
$$2x \le \frac{1}{2} \therefore x \le \frac{1}{4}$$

Now

$$(2x)^2 + (2x)^2 < 4$$
$$x^2 < \frac{1}{2} \Rightarrow x \in \left(\frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$$
$$\therefore x \in \left(\frac{-1}{\sqrt{2}}, \frac{1}{4}\right]$$

78. (c) $|\vec{a}\,||\,\vec{b}\,||\,\vec{c}| = 14$

$$\vec{a}\wedge\vec{b} = \vec{b}\wedge\vec{c} = \vec{c}\wedge\vec{a} = \theta = \frac{2\pi}{3}$$

So, $\vec{a}\cdot\vec{b} = -\frac{1}{2}ab, \vec{b}\cdot\vec{c} = -\frac{1}{2}bc, \vec{a}\cdot\vec{c}. = -\frac{1}{2}ac$

(let)

$$(\vec{a}\times\vec{b})\cdot(\vec{b}\times\vec{c}) = (\vec{a}\cdot\vec{b})(\vec{b}\cdot\vec{c}) - (\vec{a}\cdot\vec{c})(\vec{b}\cdot\vec{b})$$

$= \frac{1}{4}ab^2c + \frac{1}{2}ab^2c = \frac{3}{4}ab^2c$

Similarly

$(\vec{b} \times \vec{c}) \cdot (\vec{c} \times \vec{a}) = \frac{3}{4}abc^2$

$(\vec{c} \times \vec{a}) \cdot (\vec{a} \times \vec{b}) = \frac{3}{4}a^2bc$

$168 = \frac{3}{4}abc(a + b + c)$

So, $(a + b + c) = 16$

79. (c) $f(x) = \sin^{-1}\left(\frac{x^2-3x+2}{x^2+2x+7}\right)$ Domain

$\frac{x^2-3x+2}{x^2+2x+7} \geq -1$ and $\frac{x^2-3x+2}{x^2+2x+7} \leq 1$

$2x^2 - x + 9 \geq 0$ and $5x \geq -5 \Rightarrow x \geq -1$

$x \in R$

Hence Domain $x \in [-1, \infty)$

80. (b)

$(p \to q) \vee (p \to r)$

$(\sim p \vee q) \vee (\sim p \vee r)$

$= \sim p \vee (q \vee r)$

$= p \to (q \vee r) \equiv (3)$ is true.

Now (1) $(p \wedge \sim r) \to q$

$\sim (p \wedge \sim r) \vee q = (\sim p \vee r) \vee q$

$= \sim p \vee (r \vee q) = p \to (q \vee r)$

(4) $(p \wedge \sim q) \to r = p \to (q \vee r)$

81. (96) Let, mean $= m = np$

& variance $= v = npq, p + q = 1$

Sum $= m + v = \frac{165}{2}$

Product $= mv = 1350$

On solving,

$m = np = 60 \& v = npq = \frac{45}{2} \therefore q = \frac{3}{8} \therefore P = \frac{5}{8}$

Hence n = 96

82. (16) $Pn = \alpha^n - \beta^n\ x^2 - x - 4 = 0$

$$\frac{P_{15}P_{16} - P_{14}P_{16} - P_{15}^2 + P_{14}P_{15}}{P_{13}P_{14}}$$

As $P_n - P_{n-1} = (\alpha^n - \beta^n) - (\alpha^{n-1} - \beta^{n-1})$

$= \alpha^{n-2}(\alpha^2 - \alpha) - \beta^{n-2}(\beta^2 - \beta)$

$= 4(\alpha^{n-2} - \beta^{n-2})$

$P_n - P_{n-1} = 4P_{n-2}$

Hence Expression (1)

$$\frac{P_{16}(P_{15} - P_{14}) - P_{15}(P_{15} - P_{14})}{P_{13}P_{14}}$$

$$= \frac{(P_{15} - P_{14})(P_{16} - P_{15})}{P_{13}P_{14}} = \frac{(4P_{13})(4P_{14})}{P_{13}P_{14}} = 16$$

83. (10) $X = \begin{bmatrix}1\\1\\1\end{bmatrix}; A = \begin{bmatrix}-1 & 2 & 3\\0 & 1 & 6\\0 & 0 & -1\end{bmatrix}$

$X^TA^KX = 33$

$$[1 \quad 1 \quad 1]\begin{bmatrix}-1 & 2 & 3\\0 & 1 & 6\\0 & 0 & -1\end{bmatrix}^k\begin{bmatrix}1\\1\\1\end{bmatrix} = 33$$

$$[1 \quad 1 \quad 1]\begin{bmatrix}-1 & 2 & 3\\0 & 1 & 6\\0 & 0 & -1\end{bmatrix}\begin{bmatrix}1\\1\\1\end{bmatrix} = 33$$

$$\text{As } A^2 = \begin{bmatrix}-1 & 2 & 3\\0 & 1 & 6\\0 & 0 & -1\end{bmatrix}\begin{bmatrix}-1 & 2 & 3\\0 & 1 & 6\\0 & 0 & -1\end{bmatrix} = \begin{bmatrix}1 & 0 & 6\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$$

$$A^4 = \begin{bmatrix}1 & 0 & 6\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}\begin{bmatrix}1 & 0 & 6\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix} = \begin{bmatrix}1 & 0 & 12\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$$

$$A^8 = \begin{bmatrix}1 & 0 & 24\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$$

$$A^{10} = \begin{bmatrix}1 & 0 & 6\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}\begin{bmatrix}1 & 0 & 24\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix} = \begin{bmatrix}1 & 0 & 30\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$$

$$\text{for K} \rightarrow \text{Even } A^K = \begin{bmatrix}1 & 0 & 3K\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$$

$X^TA^KX = 33$ (This is not correct)

$[1 \quad 1 \quad 1]\begin{bmatrix}1 & 0 & 3\,K\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}\begin{bmatrix}1\\1\\1\end{bmatrix}$

$= [1 \quad 1 \quad 3\,K+1]\begin{bmatrix}1\\1\\1\end{bmatrix} = [3\,K+3]$

$\therefore 3\,K+3 = 33 \therefore K = 10$

But it should be dropped as 33 is not matrix

If K is odd

$X^T A^K X = 33$

$X^T A A^{K-1} X = 33$

$[1 \quad 1 \quad 1]\begin{bmatrix}-1 & 2 & 3\\0 & 1 & 6\\0 & 0 & -1\end{bmatrix}\begin{bmatrix}1 & 0 & 3k-3\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}\begin{bmatrix}1\\1\\1\end{bmatrix} = 33$

$[-1 \quad 3 \quad 8]\begin{bmatrix}3k-2\\1\\1\end{bmatrix} = [33]$

$[-3k+13] = [33]$

$k = 20/3$ (not possible)

84. (6)
85. (221)

$$\sum_{K=1}^{10} K\left({}^{10}C_K\right)^2$$

$$\sum_{K=1}^{10} \left(K \cdot {}^{10}C_K\right)^2 = \sum_{K=1}^{10} \left(10 \cdot {}^{9}C_{K-1}\right)^2$$

$$= 100 \sum_{K=1}^{10} {}^{9}C_{K-1} \cdot {}^{9}C_{10-K}$$

$$= 100\left({}^{18}C_9\right) = 100\left(\frac{18!}{9!\,9!}\right)$$

$$\Rightarrow 4862000 = 22000\,L$$

Hence $L = 221$

86. (79) $f(x) = 4|2x+3| + 9\left[x+\frac{1}{2}\right] - 12[x+20]$

$x \in (-20,20)$

$f(x)$ is not Diff. at $x = I \in \{-19,-18,\ldots.0,\ldots 19\} = 39$

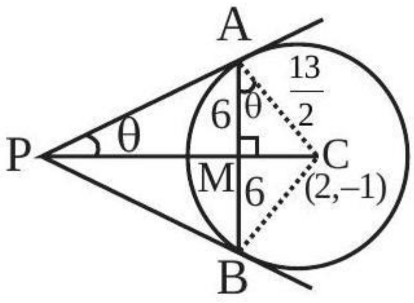
at $x = I + \frac{1}{2}$, $f(x)$ Non diff. at 39 points

Check at $x = \frac{-3}{2}$ Discount at $x = \frac{-3}{2}$ $\therefore$ N.R(1)

No. of point of non-differentiabilty

$= 39 + 39 + 1 = 79$

87. (195)
88. (72)



$$\cos\theta = \frac{6}{\frac{13}{2}} = \frac{12}{13}$$

$$\sin\theta = \frac{5}{13}$$

$\mathrm{PM} = \mathrm{AM}\cot\theta$

$\mathrm{PM} = 6\left(\frac{12}{5}\right) \therefore 5(\mathrm{PM}) = 72$

89. (14)

$|\vec{a}+\vec{b}|^2 = |\vec{a}|^2 + 2|\vec{b}|^2; \vec{a}\cdot\vec{b} = 3$

As $|\vec{a}|^2 + |\vec{b}|^2 + 2\vec{a}\cdot\vec{b} = |\vec{a}|^2 + 2|\vec{b}|^2$

$|\vec{b}|^2 = 2\vec{a}\cdot\vec{b} = 6$

$|\vec{a}\times\vec{b}|^2 = 75$

$|\vec{a}|^2|\vec{b}|^2 - (\vec{a}\cdot\vec{b})^2 = 75$

$6|\vec{a}|^2 - 9 = 75 \Rightarrow |\vec{a}|^2 = 14$

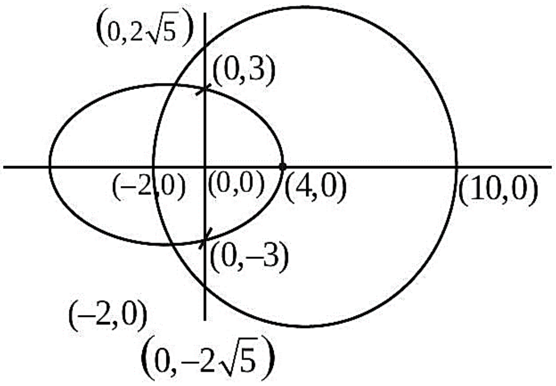
90. (27)

$S: \frac{(x-3)^2}{16} + \frac{(y-4)^2}{9} \leq 1; x, y \in \{1,2,3, \ldots \ldots$

$T: (x-7)^2 + (y-4)^2 \leq 36x, y \in R$

Let $x - 3 = x: y - 4 = y$

$S: \frac{x^2}{16} + \frac{y^2}{9} \leq 1; x \in \{-2, -1, 0, 1, \ldots \ldots$ $T: (x-4)^2 + y^2 \leq 36; y \in \{-3, -2, -1, 0, \ldots \ldots$



$S \cap T = (-2,0), (-1,0), \ldots\ldots (4,0) \rightarrow (7)$

$(-1,1), (0,1), \ldots\ldots\ldots (3,1) \rightarrow (5)$

$(-1,-1), (0,-1), \ldots\ldots\ldots (3,-1) \rightarrow (5)$

$(-1,2), (0,2), (1,2), (2,2) \rightarrow (4)$

$(-1,-2), (0,-2), (1,-2), (2,-2) \rightarrow (4)$

$(0,3)(0,-3) \rightarrow (2)$