1. (d) Let they meet at time t.

$$t=\sqrt{\frac{2h}{g}}=\sqrt{\frac{2\times 80}{10}}$$

$= 4\text{sec}$

Time taken by ball B to meet A $= 2\text{sec}$

using $S=ut+\frac{1}{2}at^2$

$$-80=-u\times 2+\frac{1}{2}(-10)(2)^2$$

$u=30$

2. (b) Mass of pieces by $\frac{M}{4},\frac{M}{4},\frac{M}{2}$

conserving momentum

$\vec{P}_1+\vec{P}_2+\vec{P}_3=0$

$\overline{P}_3=-(\overline{P}_1+\overline{P}_2)$

As $\overline{P}_1 \& \overline{P}_2$ are perpendicular

so $P_3=\sqrt{P_1^2+P_2^2}$

$$P_3=(50)\frac{M}{4}$$

$$\& P_3=\frac{M}{2}v$$

so $v=25$

3. (b) $\frac{A}{A_0}=\frac{N}{N_0}$

$$\frac{2\times 10^{-5}}{2.56\times 10^{-3}}=\frac{N}{N_0}$$

$$\frac{N}{N_0}=\frac{1}{128}\Rightarrow N=\frac{N_0}{128}$$

After 7 half- life activity comes down to given value $T=7\times 5$

$= 35$ days

4. (c) $L_0=$ angular momentum of shell about O.

As shell is rolling

so $V_{cm}=\omega R$

$L_0 = mV_{cm}R + I\omega$

$= 1 \times \omega R \times R + \frac{2}{3}R^2\omega$

$= \frac{5}{3}R^2\omega$

so $a = 5$

5. **(c)** No of moles $= \frac{44.8}{22.4} = 2$

Gas is mono atomic so $C_v = \frac{3}{2}R$

$\Delta Q = nC_v\Delta T$

$= 2 \times \frac{3}{2}R(20)$

$= 60R$

$= 60 \times 8.3$

$= 498$ J

6. **(c)** By Hooke's Law

so $F \alpha \Delta L$

$\frac{F_1}{F_2} = \frac{\Delta L_1}{\Delta L_2}$

$\frac{10}{20} = \frac{(L_1 - L)}{(L_2 - L)}$

$L = 2\ L_1 - L_2$

7. **(b)** To free the electron from metal surface minimum energy required, is equal to the work function of that metal.

So, Assertion A, is correct.

$hv = w_0 + K.E_{max}$

if $hv = w_0$

$\Rightarrow K.E_{max} = 0$

Hence reason R, is correct, But R is not the correct explanation of A.

8. **(d)** By work energy theorem

work done by net force $= \Delta$ K.E.

$\Rightarrow w = \frac{1}{2}\ mv_f^2 - \frac{1}{2}\ mv_i^2$

$$w = \frac{1}{2} \times 0.5 \times (0.25)^2 \times (4)^5$$

$$w = 16\ J$$

9. **(c)** radius of paerticle in cyclotron

$$r = \frac{\sqrt{2mK.E}}{qB}$$

So ratio of new radius to original

$$\frac{r_n}{r_0} = \sqrt{\frac{(K.E.)_n}{(K.E)_0}} = \sqrt{4} \Rightarrow 2:1$$

10. **(c)** at $\omega_r, X_C = X_L$

$$\Rightarrow \frac{1}{\omega_r C} = \omega_r L$$

So if $\omega < \omega_r$ then x_C will increase and X_L will decrease.

Hence to left of ω_r circuit is capacitive

$$Z = \sqrt{R^2 + (X_C - X_L)^2}$$

at $\omega_r, Z = \sqrt{R^2 + 0^2} = R(c)$

11. **(c)**

$$F = \frac{dp}{dt} = v\frac{dm}{dt}$$

$$\Rightarrow Ma = 10 \times 1$$

$$\Rightarrow 2a = 10$$

$$a = 5\ m/sec^2$$

12. **(c)** By principle of homogenity

$$[P] = \left[\frac{a}{v^2}\right] \text{ and } [b] = [v]$$

$$\Rightarrow \left[\frac{a}{b}\right] = [PV](c)$$

13. **(c)** $(a^2 + b^2 + 2ab\cos\theta) = 4(a^2 + b^2 - 2ab\cos\theta)$

put $a = b$ we get

$$2a^2 + 2a^2\cos\theta = 8a^2 - 8a^2\cos\theta$$

$$\cos\theta = \frac{3}{5}$$

14. (a) $V_{escape} = \sqrt{\frac{2Gm}{R}} \Rightarrow \sqrt{\frac{2G\rho\times\frac{4}{3}\pi R^3}{R}}$

$V_{escape} \propto \sqrt{\rho R^2}$

$\therefore$ if ρ is 4 times and Radius is halved.

$\Rightarrow V_{esape}$ will remain same $\therefore$

15. (a) $B_H = B\cos\theta$

$\therefore B = \frac{B_H}{\cos\theta} = \frac{0.5G}{\cos 30^\circ} \Rightarrow \frac{G}{\sqrt{3}}$

16. (b) $V_P max = 4\ V_{wave}$

$\omega A = 4\left(\frac{\omega}{k}\right) \Rightarrow A = \frac{4\lambda}{2\pi}$

$\lambda = \frac{2\pi A}{4} \Rightarrow \frac{20\pi}{4} \Rightarrow 5\pi$

17. (a) $E \Rightarrow \frac{1}{2}(KC)v^2$

$\therefore$ % change

$$\Rightarrow \frac{\frac{1}{2}K_2CV^2 - \frac{1}{2}K_1CV^2}{\frac{1}{2}K_1CV^2} = \frac{K_2 - K_1}{K_1} \times 100$$

$$\Rightarrow \frac{15-10}{10} \times 100 = 50\%$$

18. (d) Distance travelled by particle before stopping

$$\frac{V^2}{2a} = S \Rightarrow \frac{v^2\ m}{2qE} \Rightarrow \frac{(200)^2 \times 100 \times 10^{-6}}{2 \times 40 \times 10^{-6} \times 10^5} = 0.5\ m$$

19. (d) Fringe width $\beta = \frac{D\lambda}{d}$

$\lambda_1 = 5000Å$

$$\beta_1 = \frac{D}{d}(5000 \times 10^{-10}) = 5 \times 10^{-4}\ m$$

$$\beta_2 = \frac{D}{(2\ d)}(6000 \times 10^{-10}) = x\ (\text{let})$$

Divide (II) & (I)

$$\frac{\beta_2}{\beta_1} = \frac{3000 \times 10^{-10}}{5000 \times 10^{-10}} = \frac{x}{5 \times 10^{-4}}$$

$x = 3 \times 10^{-4}$ m or 0.3 mm

20. (b) Frequency at 1000 nm $= \frac{3\times10}{1000\times10^{-9}} \Rightarrow 3\times10^{14}$ Hz available for channel band width

$$= \frac{2}{100}\times 3\times 10^{14} \Rightarrow 6\times10^{12}\ \text{Hz}$$

Bandwidth for 1 channel = 8000 Hz

∴ No. of channel

$$= \frac{6\times10^{12}}{8\times10^{3}} \Rightarrow \frac{600}{8}\times10^{7} = 75\times10^{7}$$

21. (15) $\frac{dQ}{dt} = i^2R = \frac{V^2}{R}$ (we know)

$\Rightarrow$ In 't' time, $\Delta Q = \left(\frac{V^2}{R}\right)t$

Given that, (for same source, v = same)

$$Q_0 = \frac{v^2}{R_1}\times 20 = \frac{V^2}{R_2}\times 60$$

$$\Rightarrow R_2 = 3R_1$$

If they are connected in parallel then Req $= \frac{R_2R_1}{R_1+R_2} = \frac{3R_1\cdot R_1}{3R_1+R_1} = \left(\frac{3R_1}{4}\right)$

To produce same heat, using equation

$$Q_0 = \frac{V^2}{R_1}\times 20 = \frac{v^2}{\left(\frac{3R_1}{4}\right)}\times t$$

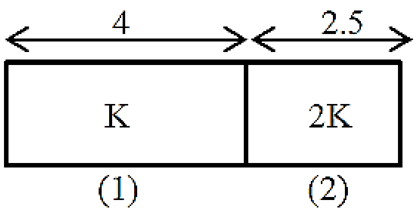
$$t = \frac{3\times 20}{4} = 15\ \text{min}$$

22. (43) $I = \left(\frac{1}{2}\varepsilon_0 E_0^2\right)C$

$$\Rightarrow E_0 \Rightarrow \sqrt{\frac{2I}{\varepsilon_0 C}} \Rightarrow \sqrt{\frac{2\times0.22}{8.85\times10^{-12}\times3\times10^{8}}} = 12.873$$

$$B \Rightarrow \frac{E_0}{C} \Rightarrow \frac{12.873}{3\times10^{8}} = 4.291\times10^{-8} = 43\times10^{-9}$$

23. (21)



$$\frac{\Delta Q}{\Delta t} = \left(\frac{1}{R}\right)\Delta T$$

R : Thermal resistivity

$$\therefore R_1 = \frac{L_1}{K_1 A} = \frac{L_1}{K(120)}$$

$L_1 = 4$ cm

$A = 120\ cm^2$

$$R_2 = \frac{2.5}{(2K)(120)}$$

Now, R_{eq} of this series combination

$R_{eq} = R_1 + R_2$

where $L_{eq} = 4 + 2.5 = 6.5$

$$\frac{L_{eq}}{K_{eq}(a)} = \frac{4}{K(120)} + \frac{\frac{5}{2}}{2K(120)}$$

$$\frac{6.5}{K_{eq}(120)} = \frac{4}{K(120)} + \frac{5}{4K(120)}$$

$$\frac{6.5}{K_{eq}} = \frac{21}{4K}$$

$$K_{eq} = \frac{26}{21}K = \left(1 + \frac{5}{21}\right)K$$

$\therefore a = 21$

24. (700) $A = 10$ cm

$\therefore$ Total Energy $= \frac{1}{2}KA^2$

By energy conservation we can final v at $x = 5$

$$\frac{1}{2}K(10)^2 = \frac{1}{2}K(5)^2 + \frac{1}{2}mv^2$$

$$V = \sqrt{\frac{75K}{m}}$$

Now, velocity is tripled through external mean so the amplitude of SHM will charge and so the total energy, (but potential) energy at this moment will remain same)

$$\therefore \frac{1}{2}K(5)^2 + \frac{1}{2}m\left(3\sqrt{\frac{75K}{m}}\right)^2 = \frac{1}{2}KA^2$$

$\Rightarrow 25K + 675K = KA^2$

$\therefore A = \sqrt{700}$

$\therefore x = 700$

25. (144) $1 = \rho\frac{\ell}{A}$

$$1 = \frac{\rho \times 31.4}{\frac{\pi(2.4)^2}{4}}$$

$$\frac{\pi(2.4)^2}{4} = \rho \times 314$$

$$\frac{2.4 \times 2.4}{4} = \rho \times 10$$

$$\frac{0.6 \times 2.4}{10} = \rho$$

$$\frac{1.44}{10} = \rho$$

$$0.144 = \rho$$

$$144 \times 10^{-3} = \rho$$

26. (102)

$$1 - \frac{Q_2}{Q_1} = 1 - \frac{T_2}{T_1}$$

$$\frac{Q_2}{Q_1} = \frac{T_2}{T_1}$$

$$\frac{225}{300} = \frac{T_2}{500}$$

$$\frac{500 \times 225}{300} = T_2$$

$$375 = T_2$$

$$102^\circ C = T_2$$

27. (3)

28. (2) $\Delta i_B = 100\mu A$

$$\beta = \frac{\Delta i_C}{\Delta i_B}$$

$$\Delta i_C = 10 \text{ mA}$$

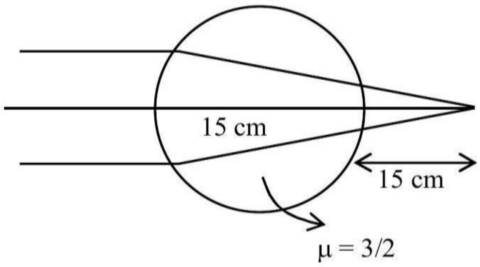
$$\text{power} = \beta^2 \times \frac{R_0}{R_{in}}$$

$$\text{Power} = \left(\frac{10}{0.1}\right)^2 \times \frac{2}{1}$$

Power = $100 \times 100 \times 2$

Gain = 2×10^4

29. (225)



$$\frac{\frac{3}{2}}{V} - \frac{1}{\infty} = \frac{\frac{3}{2} - 1}{15}$$

$$\frac{3}{2\,V} = \frac{1}{30}$$

$V = 45$ cm

$$\frac{1}{V} - \frac{\frac{3}{2}}{15} = \frac{1 - \frac{3}{2}}{\frac{2}{-15}}$$

$$\frac{1}{V} - \frac{1}{10} = \frac{1}{30}$$

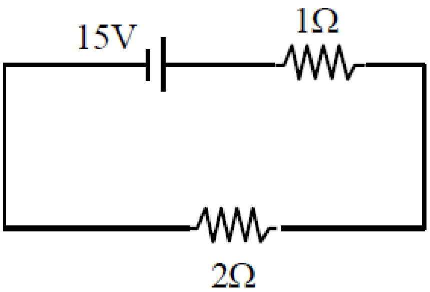
$$\frac{1}{V} = \frac{1}{10} + \frac{1}{30} = \frac{4}{30}$$

$V = 7.5$

$V = 22.5$

$v = 225$ mm

30. (10)



$$i = \frac{15}{3} = 5 \text{ A}$$

$15 - 5(1) = 10$ Volt

31. (c)

Moles of $Fe_3O_4 = \frac{4.640\times10^3}{232} = 20$

Moles of $CO = \frac{2.52\times10^3}{28} = 90$

So limiting Reagent $= Fe_3O_4$

So moles of Fe formed $= 60$

Weight of Fe $= 60 \times 56 = 3360$gms

32. (d)

(A) $Cr = [Ar]3\ d^5 4\ s^1$

(B) $m = -\ell$ to $+\ell$

(C) According to Aufbau principle, orbitals are filled in order of their increasing energies.

(D) Total nodes $= n - 1$

33. (c) $LiCl > NaCl > KCl > CsCl$ (Covalent character)
34. (d) In deionized water no common ion effect will take place so maximum solubility
35. (d) As equal & similar charge particle will repel each other, hence will never precipitate.
36. (a) $_{78}Pt = [Xe]4f^{14}5\ d^9\ 6\ s^1$ (Exceptional electronic configuration)
37. (d) For ZnS, KCN is used as depressant.

For Gold and silver ⇒ leaching [Cyanide process]

38. (c) In option (a) and (c) reducing action of hydrogen peroxide is shown.

In option (a) it is in acidic medium, in option (b) it is in basic medium.

or

For reducing ability H_2O_2 changes to O_2, i.e. oxidize, so in option 'A' & ' C ' O_2 is formed but ' A ' is in acidic medium so option - C correct.

39. (a)

Metal	Li	Na	K	Rb	Cs
Colour	Crimson red	Yellow	Violet	Red Violet	Blue
λ/nm	670.8	589.2	766.5	780.0	455.5

40. (a)

Caesium is used in devising photoelectric cells.

Boron fibres are used in making bullet-proof vest.

Silicones being surrounded by non-polar alkyl groups are water repelling in nature.

Gallium is less toxic and has a very high boiling point, so it is used in high temperature thermometers.

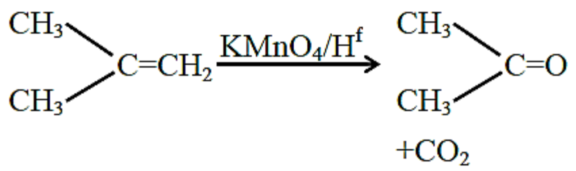
41. (b)

$P_4 + 3NaOH + 3H_2O \rightarrow PH_3 + 3NaH_2PO_2$

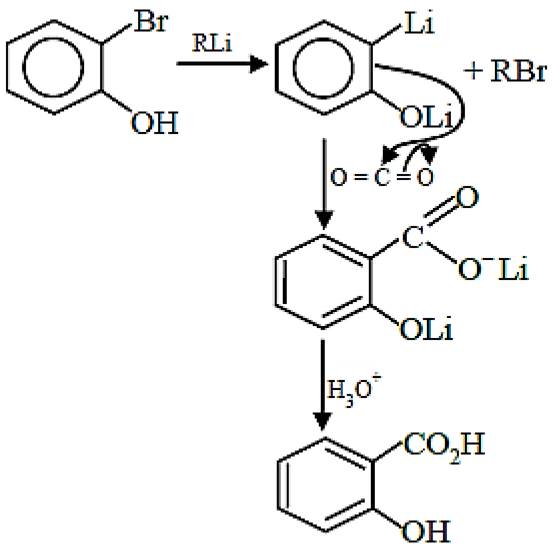
oxoacid $= H_3PO_2$ (hypo phosphorus acid) or (phosphinic acid)

42. (a) $CaCO_3 + H_2SO_4 \rightarrow CaSO_4 + H_2O + CO_2$

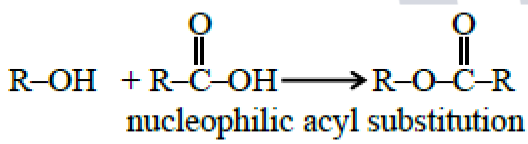
43. (d)



44. (b)

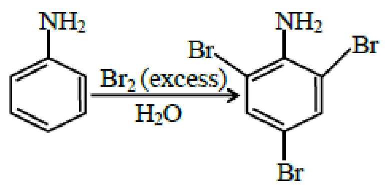


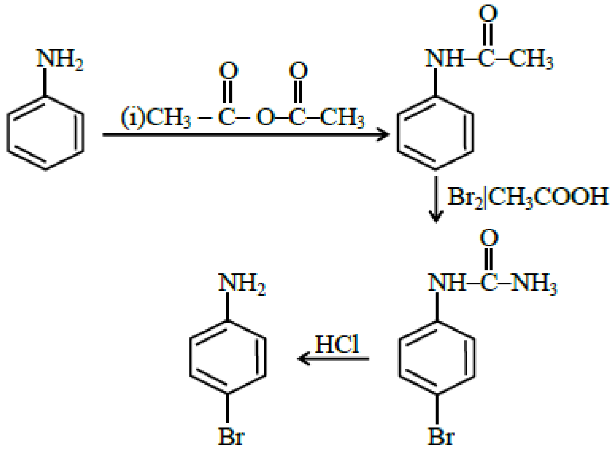
45. (a)



electron with drawing group on carboxylic acid will increase the rate of esterification

46. (c)





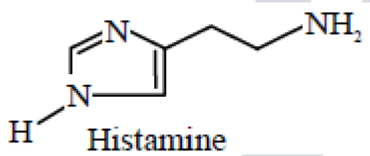
47. (c) Buna - N is synthetic rubber which can be stretched and retains its original status on releasing the force.

48. (b)

DNA contains $\Rightarrow \beta - D - 2 -$ deoxyribose

RNA contains $\Rightarrow \beta - D$ - ribose

49. (c)



Histamine is nitrogenous compound it does not contain sulpher.

50. (c) Phenol are weakly acidic. Phenol is more acidic than alcohol & H_2O statement (I) is correct. (II) is incorrect.

51. (152)

Assuming ideal behaviour $P = \frac{dRT}{M}$

$P = \frac{100}{760}$ atm, $T = 257 + 273 = 530$ K

$d = 0.46 gm/L$

So $M = \frac{0.46 \times 0.082 \times 530}{100} \times 760 = 151.93 \approx 152$

52. (117) Given data is for 1 moles and asked for 5 moles so value is $23.4 \times 5 = 117$ kJ

53. (5)

$M = d \times V = 1.02 \times 1.2 = 1.224 gm$

Moles of acetic acid $= 0.0204$ moles in 2 L

So molality $= 0.0102$ mol/kg

Now $\Delta T_f = i \times K_f \times M$

$i = 1 + \alpha$ for acetic acid

$0.0198 = (1 + \alpha) \times 1.85 \times 0.0102$

$\alpha = 0.04928$

$\cong 5\%$

54. (127) At anode

$2H_2O \rightarrow O_2(g) + 4H^+ + 4e^-$

At cathode

$2H^+ + 2e^- \rightarrow H_2(g)$

Now number of gm eq. $= \frac{i \times t}{96500}$

$$= \frac{0.1 \times 2 \times 60 \times 60}{96500}$$

$= 0.00746$

$$V_{O_2} = \frac{0.00746}{4} \times 22.7 = 0.0423$$

$$V_{H_2} = \frac{0.00746}{2} \times 22.7 = 0.0846$$

$V_{Total} \approx 127$ml or cc

55. (1)

$$\ln\left(\frac{K_2}{K_1}\right) = \frac{E_a}{R}\left(\frac{1}{T_1} - \frac{1}{T_2}\right)$$

$$\ln\left(\frac{K_2}{K_1}\right) = \frac{532611}{8.3} \times \left(\frac{10}{310 \times 300}\right)$$

where K_2 is at 310 K& K_1 is at 300 K

$$\ell n\left(\frac{K_2}{K_1}\right) = 6.9$$

$= 3 \times \ln 10$

$$\ln \frac{K_2}{K_1} = \ln 10^3$$

$K_2 = K_1 \times 10^3$

$K_1 = K_2 \times 10^3$

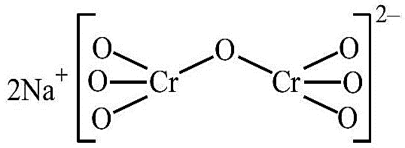
Final JEE-Main Exam June, 2022/29-06-2022/Morning Session

So K = 1

56. (6)

$4FeCr_2O_4 + 8Na_2CO_3 + 7O_2 \rightarrow 8Na_2CrO_4 + 2Fe_2O_3 + 8CO_2$

$2Na_2CrO_4 + 2H^+ \rightarrow \underbrace{Na_2Cr_2O_7}_{B} + 2Na^+ + H_2O$



57. (0) $3MnO_4^{2-} + 4H^+ \longrightarrow 2\overset{+7}{Mn}OO_4^- + \overset{+4}{Mn}O_2 + 2H_2O$

$\overset{+7}{Mn}$ = no. of unpaired electrons is ' 0 '

$\mu = 0$ B.M.

58. (64) Meq of H_2SO_4 used by $NH_3 = 12.5 \times 1 \times 2 = 25$ % of N in the compound $= \frac{25\times10^{-3}\times14\times100}{0.55} = 63.6$

or

Meq. of H_2SO_4 = Meq. of NH_3
$12.5 \times 1 \times 2 = 25$ meq. of NH_3

$= 25$ millimoles of NH_3

So Millimoles of ' N ' = 25

Moles of ' N ' $= 25 \times 10^{-3}$

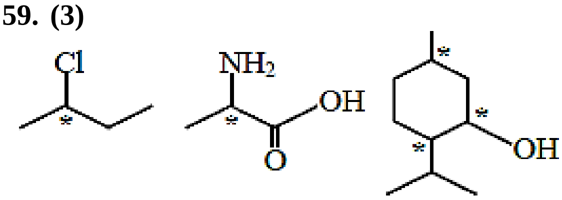
wt. of N $= 14 \times 25 \times 10^{-3}$

$$\% \text{ N} = \frac{14 \times 25 \times 10^{-3}}{0.55} \times 100$$

$= 63.66$

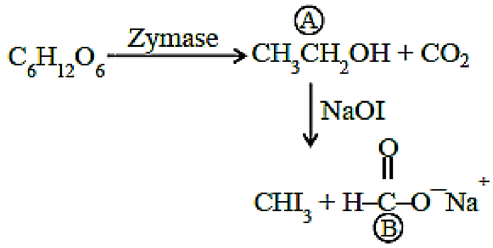
$\approx 64\%$

59. (3)



Number of compounds containing asymmetric carbons are three.

60. (1)



no. of carbon atoms present in B is 1

61. (c) Let matrix A is singular then $|A| = 0$

Number of singular matrix = All entries are same + only two prime number are used in matrix

$= 10 + 10 \times 9 \times 2$

$= 190$

Required probability $= \frac{190}{10^4} = \frac{19}{10^3}$

62. (a)

$$y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$\Rightarrow x\frac{dv}{dx} = \sqrt{v^2 + 16}$$

$$\Rightarrow \int \frac{dv}{\sqrt{v^2 + 16}} = \int \frac{dx}{x}$$

$$\Rightarrow \ln\left|v + \sqrt{v^2 + 16}\right| = \ln x + \ln C$$

$$\Rightarrow y + \sqrt{y^2 + 16x^2} = Cx^2$$

As $y(1) = 3 \Rightarrow C = 8$

$\Rightarrow y(2) = 15$

63. (c)

$$\frac{a-2}{3} = \frac{b-4}{-1} = \frac{c-7}{4} = \frac{-2(6-4+28-2)}{3^2+1^2+4^2}$$

$$\Rightarrow a = \frac{-84}{13} + 2, b = \frac{28}{13} + 4, C = \frac{-112}{13} + 7$$

$\Rightarrow 2a + b + 2c = -6$

64. (c)

$$\cdot\begin{cases} f(x) = x^3 - 3x, x \le -1 \\ 2, -1 < x < 2 \\ x^2 + 2x - 6, 2 < x < 3 \\ 9, 3 \le x < 4 \\ 10, 4 \le x < 5 \\ 11, x = 5 \\ 2x + 1, x > 5 \end{cases}$$

Clearly f(x) is not differentiable at

$x = 2,3,4,5 \Rightarrow m = 4$

$I = \int_{-2}^{-1} (x^3 - 3x)dx + \int_{-1}^{2} 2 \cdot dx = \frac{27}{4}$

65. (a)

$\frac{a \cdot c}{|\vec{c}|} = \frac{10}{3}$

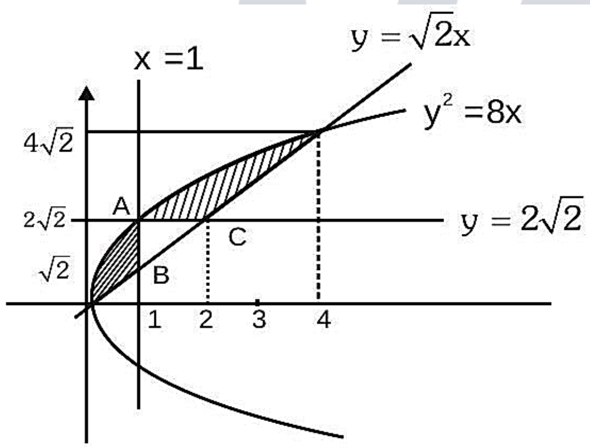
$\Rightarrow \frac{\alpha + 6 + 2}{\sqrt{1+4+4}} = \frac{10}{3} \Rightarrow \alpha = 2$

and $\begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 3 & -\beta & 4 \\ 1 & 2 & -2 \end{vmatrix} = -6\hat{\imath} + \hat{\jmath} + \hat{k}$

$\Rightarrow 2\beta - 8 = -6 \Rightarrow \beta = 1$

$\Rightarrow \alpha + \beta = 3$

66. (c)



Area of $\triangle$ ABC $= \frac{1}{2}(\sqrt{2}) \cdot 1 = \frac{\sqrt{2}}{2}$

So required Area $= \int_0^4 (\sqrt{8x} - \sqrt{2}x)dx - \frac{\sqrt{2}}{2}$

$= \frac{32\sqrt{2}}{3} - 8\sqrt{2} - \frac{\sqrt{2}}{2} = \frac{13\sqrt{2}}{6}$

67. (b)

$$\begin{vmatrix} 2 & 1 & -1 \\ 1 & -3 & 2 \\ 1 & 4 & \delta \end{vmatrix} = 0$$

$\Rightarrow \delta = -3$

And $\begin{vmatrix} 7 & 1 & -1 \\ 1 & -3 & 2 \\ K & 4 & -3 \end{vmatrix} = 0 \Rightarrow K = 6$

$\Rightarrow \delta + K = 3$

Alternate

$2x + y - z = 7$

$x - 3y + 2z = 1$

$x + 4y + \delta z = k$

Equation (2) +(3)

We get $2x + y + (2 + \delta)z = 1 + K$

For infinitely solution

Form equation (1) and (4)

$2 + \delta = -1 \Rightarrow \delta = -3$

$1 + k = 7 \Rightarrow k = 6$

$\delta + k = 3$

68. (a) $X^2 = 1 - 2i \Rightarrow \alpha^2 = 1 - 2i,\ \beta^2 = 1 - 2i$

Hence $\alpha^8 = \beta^8$

$|\alpha^8 + \beta^8| = |2\alpha^8| = 2|\alpha^2|^4$

$= 2\sqrt{5}^4 = 50$

69. (c)

$p \vee q \Rightarrow q$

$\Rightarrow \sim (p \vee q) \vee q$

$\Rightarrow (\sim p \wedge \sim q) \vee q$

$\Rightarrow (\sim p \vee q) \wedge (\sim q \vee q)$

$\Rightarrow (\sim p \vee q) \wedge t = \sim p \vee q$

Now by taking option C

$(p \wedge q) \Rightarrow \sim p \vee q$

$\Rightarrow \sim p \vee \sim q \vee \sim p \vee q$

$\Rightarrow t$

70. (a) $A = \begin{pmatrix} 1 & 2 & 2^2 \\ 1/2 & 1 & 2 \\ 1/2^2 & 1/2 & 1 \end{pmatrix}$

$A^2 = 3A$

$A^3 = 3^2 A$

$A^2 + A^3 + \cdots . A^{10}$

$= 3A + 3^2 A + \cdots + 3^9 A = \frac{3(3^9 - 1)}{3 - 1} A$

$= \frac{3^{10} - 3}{2} A$

71. (d) $A = \{1,2,3\}$

$R = \{(1,1), (1,2), (1,3)(2,1), (2,2), (2,3)(3,1),$

$(3,2)(3,3)\}$

72. (b)

$a_2 = 1, a_3 = 3\overline{a_4 = 6}$

$a_n = \frac{n(n-1)}{2}$

$S = \sum_{n=2}^{\infty} \frac{n(n-1)}{2(7^n)}$

$S = \frac{1}{7^2} + \frac{3}{7^3} + \frac{6}{7^4} + \frac{10}{7^5} + \frac{15}{7^5} + \cdots \ldots$

$\frac{S}{7} = \frac{1}{7^3} + \frac{3}{7^4} + \frac{6}{7^5} + \frac{10}{7^6} + \cdots$

$6\frac{S}{7} = \frac{1}{7^2} + \frac{2}{7^3} + \frac{3}{7^4} + \frac{4}{7^5} + \cdots$

$6\frac{S}{7^2} = \frac{1}{7^3} + \frac{2}{7^4} + \frac{3}{7^5} + \cdots$

$6\frac{S}{7} \cdot \frac{6}{7} = \frac{1}{7^2} + \frac{1}{7^3} + \cdots = \frac{1/7^2}{1 - 1/7}$

$6 \times 6\frac{S}{7^2} =\cdot \frac{1}{7 \times 6}$

$S = \frac{7}{6^3} = \frac{7}{216}$

Alternate

$a_{n+2} = 2a_{n+1} - a_n + 1$

$\Rightarrow \frac{a_{n+2}}{7^{n+2}} = \frac{2}{7}\frac{a_{n+1}}{7^{n+1}} - \frac{1}{49}\frac{a_n}{7^n} + \frac{1}{7^{n+2}}$

$\Rightarrow \sum_{n=2}^{\infty} \frac{a_{n+2}}{7^{n+2}} = \frac{2}{7}\sum_{n=2}^{\infty} \frac{a_{n+1}}{7^{n+1}} - \frac{1}{49}\sum_{n=2}^{\infty} \frac{a_n}{7^n} + \sum_{n=2}^{\infty} \frac{1}{7^{n+2}}$

Let $\sum_{n=2}^{\infty} \frac{a_n}{7^n} = p$

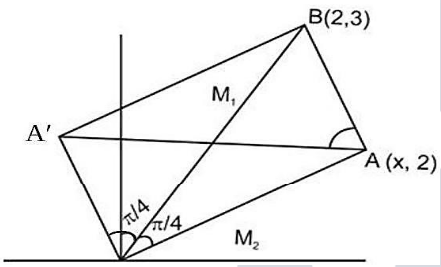
$\Rightarrow \left(p - \frac{a_2}{7^2} - \frac{a_3}{7^3}\right) = \frac{2}{7}\left(p - \frac{a_2}{7^2}\right) - \frac{1}{49}p + \frac{1/7^4}{1 - \frac{1}{7}}$

$\because a_2 = 1, a_3 = 3$

$\Rightarrow p - \frac{1}{49} - \frac{3}{343} = \frac{2}{7}p - \frac{2}{7^3} - \frac{p}{49} + \frac{1}{6.7^3}$

$\Rightarrow p = \frac{7}{216}$

73. (c)



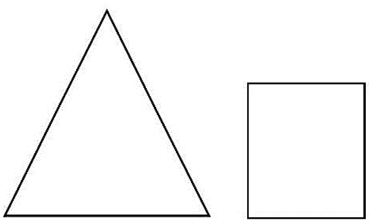
$M_1 = 3/2 \ M_2 = 2/x$

$\tan \pi/4 = \left|\frac{3/2 - 2/x}{1 + 6/2x}\right| = 1$

$\Rightarrow x_1 = 10, \ x_2 = -2/5$

$\Rightarrow AA^1 = 52/5$

74. (b)



$3a = x \ 4b = 22 - x$

$a = 2/13$

$$A_T=\frac{\sqrt{3}}{4}a^2+b^2$$

$$=\frac{\sqrt{3}}{4}x^2/9+\frac{(22-x)^2}{16}$$

$$\frac{\mathrm{dA}}{\mathrm{dx}}=0\Rightarrow \mathrm{x}\left(\frac{\sqrt{3}}{2\times 9}+\frac{1}{8}\right)-\frac{22}{8}=0$$

$$\Rightarrow x\left(\frac{4\sqrt{3}+9}{36}\right)=\frac{11}{2}$$

$$\mathrm{a}=\mathrm{x}/3$$

$$a=\left(\frac{11/2}{\frac{4\sqrt{3}+9}{36}}\right)\left(\frac{1}{3}\right)=\frac{66}{4\sqrt{3}+9}$$

75. (d)

$$-1\le\frac{2\sin^{-1}\left(\frac{1}{4x^2-1}\right)}{\pi}\le 1$$

$$-\pi/2\le\sin^{-1}\frac{1}{4x^2-1}\le\pi/2$$

Always $-1\le\frac{1}{4x^2-1}\le 1$

$$x\in\left(\infty,\frac{1}{\sqrt{2}}\right)\cup\left[\frac{1}{\sqrt{2}},\infty\right)$$

76. (d) General term

$$\mathrm{T}_{\mathrm{r}+1}=\frac{\underline{10}}{|\mathrm{r}_1|\mathrm{r}_2 \mid \mathrm{r}_3}(3)^{\mathrm{r}_1}(-2)^{\mathrm{r}_2}(5)^{\mathrm{r}_3}(\mathrm{x})^{3\mathrm{r}_1+2\mathrm{r}_2-5\mathrm{r}_3}$$

$$3\mathrm{r}_1+2\mathrm{r}_2-5\mathrm{r}_3=0$$

$$\mathrm{r}_1+\mathrm{r}_2+\mathrm{r}_3=10$$

from equation (1) and (2)

$$\mathrm{r}_1+2(10-\mathrm{r}_3)-5\mathrm{r}_3=0$$

$$\mathrm{r}_1+20=7\mathrm{r}_3$$

$$(\mathrm{r}_1,\mathrm{r}_2,\mathrm{r}_3)=(1,6,3)$$

constant term $=\frac{\underline{10}}{1163}(3)^1(-2)^6(5)^3$

$$=2^9\cdot 3^2\cdot 5^4\cdot 7^1$$

$$l=9$$

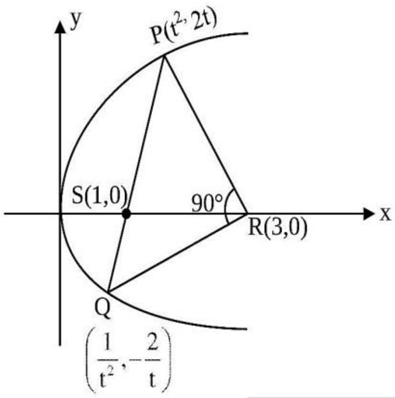
77. (d) $I=\int_0^5 \cos\left(\pi x-\pi\left[\frac{x}{2}\right]\right) dx$

$\Rightarrow \mathrm{I}=\int_0^2 \cos(\pi \mathrm{x}) \mathrm{dx}+\int_2^4 \cos(\pi \mathrm{x}-\pi) \mathrm{dx}+\int_4^5 \cos(\pi \mathrm{x}-2 \pi) \mathrm{dx}$

$\Rightarrow \mathrm{I}=\left[\frac{\sin \pi \mathrm{x}}{\pi}\right]_0^2+\left[\frac{\sin(\pi \mathrm{x}-\pi)}{\pi}\right]_2^4+\left[\frac{\sin(\pi \mathrm{x}-2 \pi)}{\pi}\right]_4^5$

$\Rightarrow \mathrm{I}=0$

78. (b) PQ is focal chord



$\mathrm{m}_{\mathrm{PR}} \cdot \mathrm{m}_{\mathrm{PQ}}=-1$

$\frac{2 \mathrm{t}}{\mathrm{t}^2-3} \times \frac{-2 / \mathrm{t}}{\frac{1}{\mathrm{t}^2}-3}=-1$

$\left(\mathrm{t}^2-1\right)^2=0$

$\Rightarrow \mathrm{t}=1$

$\Rightarrow$ P&Q must be end point of latus rectum:

$\mathrm{P}(1,2) \& \mathrm{Q}(1,-2)$

$\therefore \frac{2 \mathrm{~b}^2}{\mathrm{a}}=4 \& \mathrm{ae}=1$

$\because$ We know that $\mathrm{b}^2=\mathrm{a}^2\left(1-\mathrm{e}^2\right)$

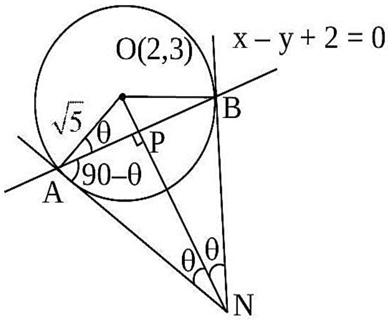
$\therefore \mathrm{a}=1+\sqrt{2}$

$\because \mathrm{e}^2=1-\frac{\mathrm{b}^2}{\mathrm{a}^2}$

$\therefore \mathrm{e}^2=3-2 \sqrt{2}$

$\frac{1}{\mathrm{e}^2}=3+2 \sqrt{2}$

79. (c) $OP=\left|\frac{2-3+2}{\sqrt{2}}\right|$



$$\mathrm{OP} = \frac{3}{\sqrt{2}}$$

$$\mathrm{AP} = \sqrt{\mathrm{OA}^2 - \mathrm{OP}^2}$$

$$= \frac{1}{\sqrt{2}}$$

$$\tan\theta = 3$$

$$\therefore \sin\theta = \frac{3}{\sqrt{10}} = \frac{\mathrm{AP}}{\mathrm{AN}}$$

$$\Rightarrow \mathrm{AN} = \frac{\sqrt{5}}{3} = \mathrm{BN}$$

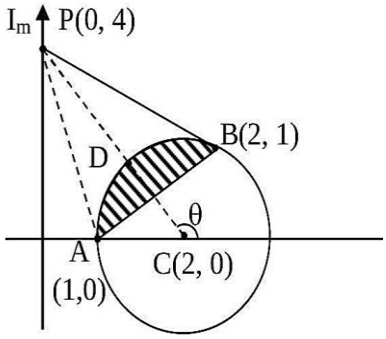
Area of $\triangle$ ANB $= \frac{1}{2}\cdot(\mathrm{AN}^2)\sin 2\theta = \frac{1}{6}$

80. (b)

$$\bar{x} = \frac{\sum x_i}{5} = \frac{24}{5} \Rightarrow \sum x_i = 24$$
$$\sigma^2 = \frac{\sum x_i^2}{5} - \left(\frac{24}{5}\right)^2 = \frac{194}{25}$$
$$\Rightarrow \sum x_i^2 = 154$$
$$x_1 + x_2 + x_3 + x_4 = 14$$
$$\Rightarrow x_5 = 10$$
$$\sigma^2 = \frac{x_1^2 + x_2^2 + x_3^2 + x_4^2}{4} - \frac{49}{4} = a$$
$$x_1^2 + x_2^2 + x_3^2 + x_4^2 = 4a + 49$$
$$x_5^2 = 154 - 4a - 49$$
$$\Rightarrow 100 = 105 - 4a \Rightarrow 4a = 5$$
$$4a + x_5 = 15$$

81. (26)

$$|z - 2| \leq 1$$



$(x-2)^2 + y^2 \le 1$

&

$z(1+i) + \bar{z}(1-i) \le 2$

Put $z = x + iy$

$\therefore x - y \le 1$

$PA = \sqrt{17}, PB = \sqrt{13}$

Maximum is PA& Minimum is PD
Let $D(2+\cos\theta, 0+\sin\theta)$

$\therefore m_{cp} = \tan\theta = -2$

$\cos\theta = -\frac{1}{\sqrt{5}}, \sin\theta = \frac{2}{\sqrt{5}}$

$\therefore D\left(2 - \frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}}\right)$

$\Rightarrow z_1 = \left(2 - \frac{1}{\sqrt{5}}\right) + \frac{2i}{\sqrt{5}}$

$|z_1| = \frac{25 - 4\sqrt{5}}{5}$ &$z_2 = 1$

$\therefore |z_2|^2 = 1$

$\therefore 5(|z_1|^2 + |z_2|^2) = 30 - 4\sqrt{5}$

$\therefore \alpha = 30$

$\beta = -4$

$\therefore \alpha + \beta = 26$

82. (2) $\frac{dy}{dx} + \frac{\sqrt{2}}{2\cos^4 x - \cos 2x} y = x e^{\tan^{-1}(\sqrt{2}\cot 2x)}$

$\int \frac{dx}{2\cos^4 x - \cos 2x}$

$$= \int \frac{dx}{\cos^4 x + \sin^4 x} = \int \frac{\operatorname{cosec}^4 x dx}{1 + \cot^4 x}$$

$$= -\int \frac{t^2 + 1}{t^4 + 1} dt = -\int \frac{\left(1 + \frac{1}{t^2}\right)}{\left(t - \frac{1}{t}\right)^2 + 2} dt = \frac{-1}{\sqrt{2}} \tan^{-1}\left(\frac{t - \frac{1}{t}}{\sqrt{2}}\right)$$

$\operatorname{Cot} x = t$

$$= -\frac{1}{\sqrt{2}} \tan^{-1}(\sqrt{2} \cot 2x)$$

$\therefore \text{IF} = e^{-\tan^{-1}(\sqrt{2} \cot 2x)}$

$$y e^{-\tan^{-1}(\sqrt{2} \cot 2x)} = \int x dx$$

$$y e^{-\tan^{-1}(\sqrt{2} \cot 2x)} = \frac{x^2}{2} + c$$

$$y\left(\frac{\pi}{4}\right) = \frac{\pi^2}{32} + c \Rightarrow c = 0$$

$$y = \frac{x^2}{2} e^{\tan^{-1}(\sqrt{2} \cot 2x)}$$

$$y\left(\frac{\pi}{3}\right) = \frac{\pi^2}{18} e^{\tan^{-1}\left(\sqrt{2} \cot \frac{2\pi}{3}\right)}$$

$$= \frac{\pi^2}{18} e^{-\tan^{-1}\left(\sqrt{\frac{2}{3}}\right)}$$

$$\alpha = \sqrt{\frac{2}{3}} \Rightarrow 3\alpha^2 = 2$$

83. (26) Points $P(1,2,-1)$ and $Q(2,-1,3)$ lie on same side of the plane.

Perpendicular distance of point P from plane is

$$\left|\frac{-1 + 2 - 1 - 1}{\sqrt{1^2 + 1^2 + 1^2}}\right| = \frac{1}{\sqrt{3}}$$

Perpendicular distance of point Q from plane is

$$= \left|\frac{-2 - 1 + 3 - 1}{\sqrt{1^2 + 1^2 + 1^2}}\right| = \frac{1}{\sqrt{3}}$$

$\Rightarrow \overrightarrow{PQ}$ is parallel to given plane. So, distance between P and Q = distance between their foot of perpendiculars.

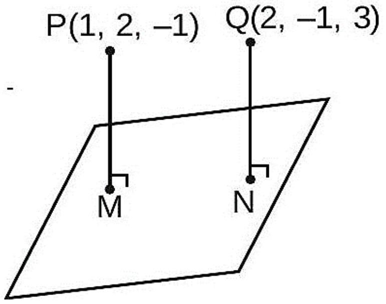
$$\Rightarrow |\overrightarrow{PQ}| = \sqrt{(1-2)^2 + (2+1)^2 + (-1-3)^2}$$

$$= \sqrt{26}$$

$$|\overrightarrow{PQ}|^2 = 26 = d^2$$

Alternate

$$-x+y+z-1=0$$



$M(x_1, y_1, z_1)$

$$\frac{x_1-1}{-1}=\frac{y_1-2}{1}=\frac{z_1+1}{1}=\frac{1}{3}$$

$$x_1=\frac{2}{3}, y_1=\frac{7}{3}, z_1=\frac{-2}{3}$$

$$M\left(\frac{2}{3}, \frac{7}{3}, \frac{-2}{3}\right)$$

$N(x_2, y_2, z_2)$

$$\frac{x_2-2}{-1}=\frac{y_2+1}{1}=\frac{z_2-3}{1}=\frac{1}{3}$$

$$x_2=\frac{5}{3}, y_2=\frac{-2}{3}, z_2=\frac{10}{3}$$

$$N=\left(\frac{5}{3}, \frac{-2}{3}, \frac{10}{3}\right)$$

$$d^2=1^2+3^2+4^2=26$$

84. (32)

$$3\cos^2 2\theta+6\cos 2\theta-10\cos^2\theta+5=0$$

$$3\cos^2 2\theta+6\cos 2\theta-5(1+\cos 2\theta)+5=0$$

$$3\cos^2 2\theta+\cos 2\theta=0$$

$\text{Cos}\,2\theta=0$ OR $\cos 2\theta=-1/3$

$$\theta\in[-4\pi, 4\pi]$$

$$2\theta=(2n+1)\cdot\frac{\pi}{2}$$

$\therefore\ \theta=\pm\pi/4.\pm3\pi/4\ \dots\dots..\pm15\pi/4$

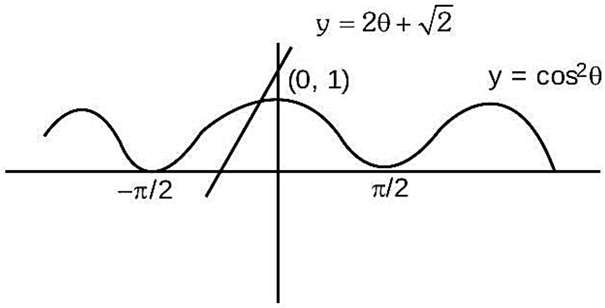
Similarly, $\cos 2\theta=-1/3$ gives 16 solution

85. (1)

$2\theta - \cos^2\theta + \sqrt{2} = 0$

$\Rightarrow \cos^2\theta = 2\theta + \sqrt{2}$

$y = 2\theta + \sqrt{2}$



Both graphs intersect at one point.

86. (29)

$50\tan\left(3\tan^{-1}\frac{1}{2} + 2\cos^{-1}\frac{1}{\sqrt{5}}\right)$

$+4\sqrt{2}\tan\left(\frac{1}{2}\tan^{-1}2\sqrt{2}\right)$

$= 50\tan\left(\tan^{-1}\frac{1}{2} + 2\left(\tan^{-1}\frac{1}{2} + \tan^{-1}2\right)\right)$

$+4\sqrt{2}\tan\left(\frac{1}{2}\tan^{-1}2\sqrt{2}\right)$

$= 50\tan\left(\tan^{-1}\frac{1}{2} + 2\cdot\frac{\pi}{2}\right) + 4\sqrt{2}\times\frac{1}{\sqrt{2}}$

$= 50\left(\tan\tan^{-1}\frac{1}{2}\right) + 4$

$= 25 + 4 = 29$

87. (3395)

$f(x) = (c+1)x^2 + (1-c^2)x + 2k$

$\&f(x+y) = f(x) + f(y) - xy\ \forall xy \in R$

$\lim_{y\to 0}\frac{f(x+y) - f(x)}{y} = \lim_{y\to 0}\frac{f(y) - xy}{y} \Rightarrow f'(x) = f'(0) - x$

$f(x) = -\frac{1}{2}x^2 + f'(0)\cdot x + \lambda$ but $f(0) = 0 \Rightarrow \lambda = 0$

$f(x) = -\frac{1}{2}x^2 + (1-c^2)\cdot x$

$\therefore$ as $f'(0) = 1 - c^2$

Comparing equation (1) and (2)

We obtain, $c = -\frac{3}{2}$

$\therefore\ f(x) = -\frac{1}{2}x^2 - \frac{5}{4}x$

Now $\left|2\sum_{x=1}^{20} f(x)\right| = \sum_{x=1}^{20} x^2 + \frac{5}{2} \cdot \sum_{x=1}^{20} x$

$= 2870 + 525$

$= 3395$

88. (88)

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

Given $e^2 = 1 + \frac{b^2}{a^2} \Rightarrow \frac{11}{4} = 1 + \frac{b^2}{a^2} \Rightarrow b^2 = \frac{7}{4}a^2$

$\therefore \frac{x^2}{(a)^2} - \frac{y^2}{\left(\frac{\sqrt{7}}{2}a\right)^2} = 1$ Now given

$2a + 2 \cdot \frac{\sqrt{7}a}{2} = 4(2\sqrt{2} + \sqrt{14})$

$a(2 + \sqrt{7}) = 4\sqrt{2}(2 + \sqrt{7})$

$a = 4\sqrt{2} \Rightarrow a^2 = 32$

$b^2 = \frac{7}{4} \times 16 \times 2 = 56$

89. (28)

$P_1 : \vec{r} \cdot (2\hat{\imath} + \hat{\jmath} - 3\hat{k}) = 4$

$P_1 : 2x + y - 3z = 4$

$P_2 \begin{vmatrix} x-2 & y+3 & z-2 \\ 0 & 1 & -5 \\ -1 & -1 & 0 \end{vmatrix} = 0$

$\Rightarrow -5x + 5y + z + 23 = 0$

Let a, b, c be the d'rs of line of intersection

Then $a = \frac{16\lambda}{15}; b = \frac{13\lambda}{15}; c = \frac{15\lambda}{15}$

$\therefore \alpha = 13 : \beta = 15$

90. (18915)

$b_i \in \{1,2,3 \ldots \ldots \ldots 100\}$

Let $A =$ set when $b_1 b_2 b_3$ are consecutive

$$n(A) = \frac{97 + 97 + \cdots \ldots + 97}{98 \text{ times}} = 97 \times 98$$

Similarly when $\mathrm{b}_2\ \mathrm{b}_3\ \mathrm{b}_4$ are consecutive

$\mathrm{N(a)} = 97 \times 98$

$$\mathrm{n(A \cap B)} = \frac{97 + 97 + - - - - - - - 97}{98 \text{ times}} = 97 \times 98$$

Similarly, when $\mathrm{b}_2\ \mathrm{b}_3\ \mathrm{b}_4$ are consecutive

$\mathrm{n(b)} = 97 \times 98$

$\mathrm{n(A \cap B)} = 97$

$\mathrm{n(AUB)} = \mathrm{n(a)} + \mathrm{n(b)} - \mathrm{n(A \cap B)}$

Number of permutation $= 18915$