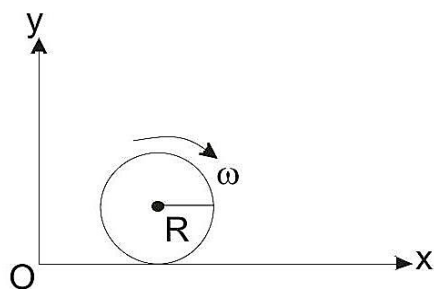


PART-I PHYSICS

SECTION-A

- Two balls A and B are placed at the top of 180 m tall tower. Ball A is released from the top at $t = 0$ s. Ball B is thrown vertically down with an initial velocity 'u' at $t = 2$ s. After a certain time, both balls meet 100 m above the ground. Find the value of 'u' in ms^{-1} . [use $g = 10 \text{ ms}^{-2}$]:
 (a) 10 (b) 15
 (c) 20 (d) 30
- A body of mass M at rest explodes into three pieces, in the ratio of masses 1: 1: 2. Two smaller pieces fly off perpendicular to each other with velocities of 30 ms^{-1} and 40 ms^{-1} respectively. The velocity of the third piece will be:
 (a) 15 ms^{-1} (b) 25 ms^{-1}
 (c) 35 ms^{-1} (d) 50 ms^{-1}
- The activity of a radioactive material is $2.56 \times 10^{-3} \text{ Ci}$. If the half life of the material is 5 days, after how many days the activity will become $2 \times 10^{-5} \text{ Ci}$?
 (a) 30 days (b) 35 days
 (c) 40 days (d) 25 days
- A spherical shell of 1 kg mass and radius R is rolling with angular speed ω on horizontal plane (as shown in figure). The magnitude of angular momentum of the shell about the origin O is $\frac{a}{3} R^2 \omega$. The value of a will be:



- (a) 2 (b) 3
 (c) 5 (d) 4

- A cylinder of fixed capacity of 44.8 litres contains helium gas at standard temperature and pressure. The amount of heat needed to raise the temperature of gas in the cylinder by 20.0°C will be:

(Given gas constant $R = 8.3 \text{ JK}^{-1} - \text{mol}^{-1}$)

- (a) 249 J (b) 415 J
 (c) 498 J (d) 830 J

- A wire of length L is hanging from a fixed support.

The length changes to L_1 and L_2 when masses 1 kg and 2 kg are suspended respectively from its free end. Then the value of L is equal to:

- (a) $\sqrt{L_1 L_2}$ (b) $\frac{L_1 + L_2}{2}$
 (c) $2 L_1 - L_2$ (d) $3 L_1 - 2 L_2$

- Given below are two statements: one is labelled as Assertion A and the other is labelled as Reason R.

Assertion A: The photoelectric effect does not take place, if the energy of the incident radiation is less than the work function of a metal.

Reason R: Kinetic energy of the photoelectrons is zero, if the energy of the incident radiation is equal to the work function of a metal.

In the light of the above statements, choose the most appropriate answer from the options given below.

- (a) Both A and R are correct and R is the correct explanation of A
- (b) Both A and R are correct but R is not the correct explanation of A
- (c) A is correct but R is not correct
- (d) A is not correct but R is correct

8. A particle of mass 500gm is moving in a straight line with velocity $v = bx^{5/2}$. The work done by the net force during its displacement from $x = 0$ to $x = 4$ m is: (Take $b = 0.25 \text{ m}^{-3/2} \text{ s}^{-1}$).

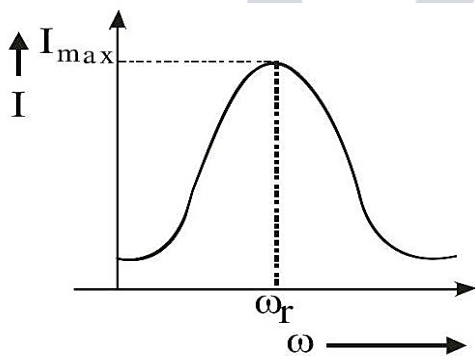
- (a) 2 J
- (b) 4 J
- (c) 8 J
- (d) 16 J

9. A charged particle moves along circular path in a uniform magnetic field in a cyclotron. The kinetic energy of the charged particle increases to 4 times its initial value. What will be the ratio of new radius to the original radius of circular path of the charged particle:

- (a) 1: 1
- (b) 1: 2
- (c) 2: 1
- (d) 1: 4

10. For a series LCR circuit, I vs ω curve is shown:

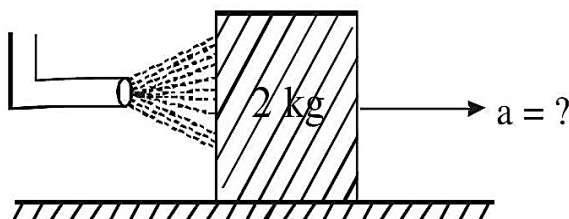
- (A) To the left of ω_r , the circuit is mainly capacitive.
- (B) To the left of ω_r , the circuit is mainly inductive.
- (C) At ω_r , impedance of the circuit is equal to the resistance of the circuit.
- (D) At ω_r , impedance of the circuit is 0.



Choose the most appropriate answer from the options given below:

- (a) (A) and (D) only
- (b) (B) and (D) only
- (c) (A) and (C) only
- (d) (B) and (C) only

11. A block of metal weighing 2 kg is resting on a frictionless plane (as shown in figure). It is struck by a jet releasing water at a rate of 1 kg s^{-1} and at a speed of 10 ms^{-1} . Then, the initial acceleration of the block, in ms^{-2} , will be:



Plane

- (a) 3 (b) 6
(c) 5 (d) 4

12. In Vander Waals equation $\left[P + \frac{a}{V^2}\right][V - b] = RT$;

P is pressure, V is volume, R is universal gas constant and T is temperature. The ratio of constants $\frac{a}{b}$ is dimensionally equal to:

- (a) $\frac{P}{V}$ (b) $\frac{V}{P}$
(c) PV (d) PV^3

13. Two vectors $\vec{A}$ and $\vec{B}$ have equal magnitudes. If magnitude of $\vec{A} + \vec{B}$ is equal to two times the magnitude of $\vec{A} - \vec{B}$, then the angle between $\vec{A}$ and $\vec{B}$ will be :

- (a) $\sin^{-1}\left(\frac{3}{5}\right)$ (b) $\sin^{-1}\left(\frac{1}{3}\right)$
(c) $\cos^{-1}\left(\frac{3}{5}\right)$ (d) $\cos^{-1}\left(\frac{1}{3}\right)$

14. The escape velocity of a body on a planet 'A' is 12kms^{-1} . The escape velocity of the body on another planet 'B', whose density is four times and radius is half of the planet 'A', is:

- (a) 12kms^{-1} (b) 24kms^{-1}
(c) 36kms^{-1} (d) 6kms^{-1}

15. At a certain place the angle of dip is 30° and the horizontal component of earth's magnetic field is 0.5G. The earth's total magnetic field (in G), at that certain place, is:

- (a) $\frac{1}{\sqrt{3}}$ (b) $\frac{1}{2}$
(c) $\sqrt{3}$ (d) 1

16. A longitudinal wave is represented by $x = 10\sin 2\pi\left(nt - \frac{x}{\lambda}\right)$ cm. The maximum particle velocity will be four times the wave velocity if the determined value of wavelength is equal to:

- (a) 2π (b) 5π
(c) π (d) $\frac{5\pi}{2}$

17. A parallel plate capacitor filled with a medium of dielectric constant 10, is connected across a battery and is charged. The dielectric slab is replaced by another slab of dielectric constant 15. Then the energy of capacitor will:

- (a) increase by 50% (b) decrease by 15%
(c) increase by 25% (d) increase by 33%

18. A positive charge particle of 100mg is thrown in opposite direction to a uniform electric field of strength $1 \times 10^5 \text{NC}^{-1}$. If the charge on the particle is $40\mu\text{C}$ and the initial velocity is 200ms^{-1} , how much distance it will travel before coming to the rest momentarily:

- (a) 1 m (b) 5 m
(c) 10 m (d) 0.5 m

19. Using Young's double slit experiment, a monochromatic light of wavelength $5000\text{\AA}$ produces fringes of fringe width 0.5 mm. If another monochromatic light of wavelength $6000\text{\AA}$ is used and the separation between the slits is doubled, then the new fringe width will be:

- (a) 0.5 mm (b) 1.0 mm
(c) 0.6 mm (d) 0.3 mm

20. Only 2% of the optical source frequency is the available channel bandwidth for an optical communicating system operating at 1000 nm. If an audio signal requires a bandwidth of 8kHz, how many channels can be accommodated for transmission:

- (a) 375×10^7 (b) 75×10^7
(c) 375×10^8 (d) 75×10^9

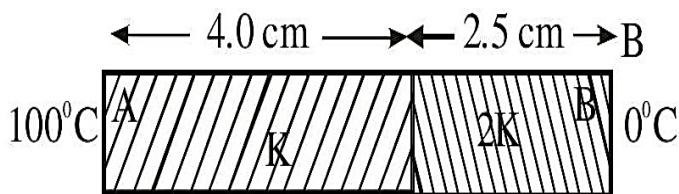
SECTION-B

21. Two coils require 20 minutes and 60 minutes respectively to produce same amount of heat energy when connected separately to the same source. If they are connected in parallel arrangement to the same source; the time required to produce same amount of heat by the combination of coils, will be min.

22. The intensity of the light from a bulb incident on a surface is 0.22 W/m^2 . The amplitude of the magnetic field in this light-wave is $\times 10^{-9} \text{ T}$

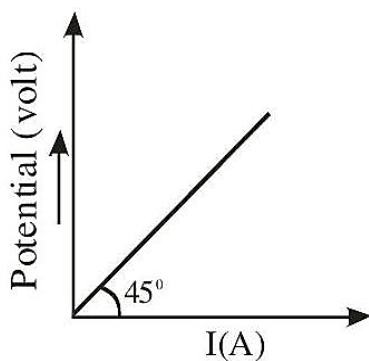
(Given: Permittivity of vacuum $\epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}$, speed of light in vacuum $c = 3 \times 10^8 \text{ ms}^{-1}$)

23. As per the given figure, two plates A and B of thermal conductivity K and 2 K are joined together to form a compound plate. The thickness of plates are 4.0 cm and 2.5 cm respectively and the area of cross-section is 120 cm^2 for each plate. The equivalent thermal conductivity of the compound plate is $(1 + \frac{5}{\alpha}) \text{ K}$, then the value of α will be



24. A body is performing simple harmonic with an amplitude of 10 cm. The velocity of the body was tripled by air Jet when it is at 5 cm from its mean position. The new amplitude of vibration is $\sqrt{x} \text{ cm}$. The value of x is

25. The variation of applied potential and current flowing through a given wire is shown in figure. The length of wire is 31.4 cm. The diameter of wire is measured as 2.4 cm. The resistivity of the given wire is measured as $x \times 10^{-3} \Omega \text{ cm}$. The value of x is [Take $\pi = 3.14$]



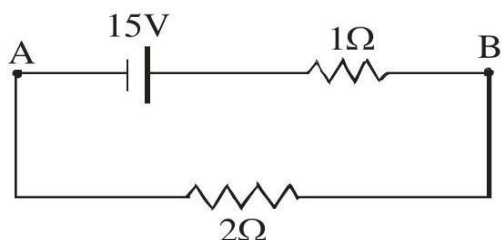
26. 300 cal. of heat is given to a heat engine and it rejects 225 cal. of heat. If source temperature is 227°C , then the temperature of sink will be $-^\circ \text{C}$.

27. $\sqrt{d_1}$ and $\sqrt{d_2}$ are the impact parameters corresponding to scattering angles 60° and 90° respectively, when an α particle is approaching a gold nucleus. For $d_1 = x d_2$, the value of x will be

28. A transistor is used in an amplifier circuit in common emitter mode. If the base current changes by $100 \mu \text{A}$, it brings a change of 10 mA in collector current. If the load resistance is $2 \text{ k}\Omega$ and input resistance is $1 \text{ k}\Omega$, the value of power gain is $x \times 10^4$. The value of x is

29. A parallel beam of light is allowed to fall on a transparent spherical globe of diameter 30 cm and refractive index 1.5. The distance from the centre of the globe at which the beam of light can converge is mm.

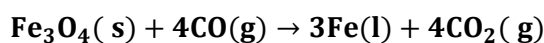
30. For the network shown below, the value $V_B - V_A$ is V.



PART-II CHEMISTRY

SECTION-A

31. Production of iron in blast furnace follows the following equation



when 4.640 kg of Fe_3O_4 and 2.520 kg of CO are allowed to react then the amount of iron (in g) produced is:

[Given: Molar Atomic mass (gmol^{-1}): Fe = 56 Molar Atomic mass (gmol^{-1}): O = 16

Molar Atomic mass (gmol^{-1}): C = 12

- (a) 1400 (b) 2200
(c) 3360 (d) 4200

32. Which of the following statements are correct?

- (A) The electronic configuration of Cr is $[\text{Ar}]3d^5 4s^1$.
(B) The magnetic quantum number may have a negative value.
(C) In the ground state of an atom, the orbitals are filled in order of their increasing energies.
(D) The total number of nodes are given by $n - 2$.

Choose the most appropriate answer from the options given below:

- (a) (A), (C) and (D) only (b) (A) and (B) only
(c) (A) and (C) only (d) (A), (B) and (C) only

33. Arrange the following in the decreasing order of their covalent character:

- (A) LiCl
(B) NaCl
(C) KCl
(D) CsCl

Question: Choose the most appropriate answer from the options given below:

- (a) (A) > (C) > (B) > (D)

(b) $(B) > (A) > (C) > (D)$

(c) $(A) > (B) > (C) > (D)$

(d) $(A) > (B) > (D) > (C)$

34. The solubility of AgCl will be maximum in which of the following?

(a) 0.01M KCl

(b) 0.01M HCl

(c) 0.01M AgNO_3

(d) Deionised water

35. Which of the following is a correct statement?

(a) Brownian motion destabilises sols.

(b) Any amount of dispersed phase can be added to emulsion without destabilising it.

(c) Mixing two oppositely charged sols in equal amount neutralises charges and stabilises colloids.

(d) Presence of equal and similar charges on colloidal particles provides stability to the colloidal solution.

36. The electronic configuration of Pt (atomic number 78) is:

(a) $[\text{Xe}]4f^{14}5d^96s^1$

(b) $[\text{Kr}]4f^{14}5d^{10}$

(c) $[\text{Xe}]4f^{14}5d^{10}$

(d) $[\text{Xe}]4f^{14}5d^86s^2$

37. In isolation of which one of the following metals from their ores, the use of cyanide salt is not commonly involved?

(a) Zinc

(b) Gold

(c) Silver

(d) Copper

38. Which one of the following reactions indicates the reducing ability of hydrogen peroxide in basic medium?

(a) $\text{HOCl} + \text{H}_2\text{O}_2 \rightarrow \text{H}_3\text{O}^+ + \text{Cl}^- + \text{O}_2$

(b) $\text{PbS} + 4\text{H}_2\text{O}_2 \rightarrow \text{PbSO}_4 + 4\text{H}_2\text{O}$

(c) $2\text{MnO}_4^- + 3\text{H}_2\text{O}_2 \rightarrow 2\text{MnO}_2 + 3\text{O}_2 + 2\text{H}_2\text{O} + 2\text{OH}^-$

(d) $\text{Mn}^{2+} + \text{H}_2\text{O}_2 \rightarrow \text{Mn}^{4+} + 2\text{OH}^-$

39. Match the List-I with List- II.

List-I (Metal)	List-II (Emitted light wavelength (nm))
(A) Li	(I) 670.8
(B) Na	(II) 589.2
(C) Rb	(III) 780.0
(D) Cs	(IV) 455.5

Choose the most appropriate answer from the options given below:

- (a) (A)-(I), (B)-(II), (C)-(III), (D)-(IV)
 (b) (A)-(III), (B)-(II), (C)-(I), (D)-(IV)
 (c) (A)-(III), (B)-(I), (C)-(II), (D)-(IV)
 (d) (A)-(IV), (B)-(II), (C)-(I), (D)-(III)

40. Match the List-I with List- II.

List-I (Metal)	List-II Application
(A) Cs	(I) High temperature thermometer
(B) Ga	(II) Water repellent sprays
(C) B	(III) Photoelectric cells
(D) Si	(IV) Bullet proof vest

Choose the most appropriate answer from the option given below:

- (a) (A)-(III), (B)-(I), (C)-(IV), (D)-(II)
 (b) (A)-(IV), (B)-(III), (C)-(II), (D)-(I)

(c) (A)-(II), (B)-(III), (C)-(IV), (D)-(I)

(d) (A)-(I), (B)-(IV), (C)-(II), (D)-(III)

41. The oxoacid of phosphorus that is easily obtained from a reaction of alkali and white phosphorus and has two P – H bonds, is:

- (a) Phosphonic acid (b) Phosphinic acid
(c) Pyro phosphorus acid (d) Hypo phosphoric acid

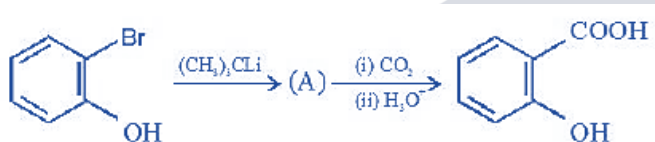
42. The acid that is believed to be mainly responsible for the damage of Taj Mahal is

- (a) Sulfuric acid (b) Hydrofluoric acid
(c) Phosphoric acid (d) Hydrochloric acid

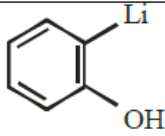
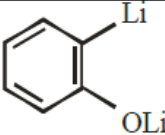
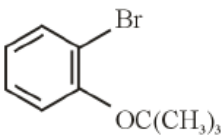
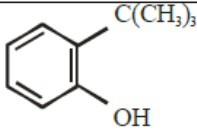
43. Two isomers 'A' and 'B' with molecular formula C_4H_8 give different products on oxidation with $KMnO_4$ in acidic medium. Isomer 'A' on reaction with $KMnO_4/H^+$ results in effervescence of a gas and gives ketone. The compound 'A' is

- (a) But-1-ene (b) cis-But-2-ene
(c) trans-But-2ene (d) 2-methyl propene

44.



In the given conversion the compound A is:

(A)	
(B)	
(C)	
(D)	

45. Given below are two statements:

Statement I: The esterification of carboxylic acid with an alcohol is a nucleophilic acyl substitution.

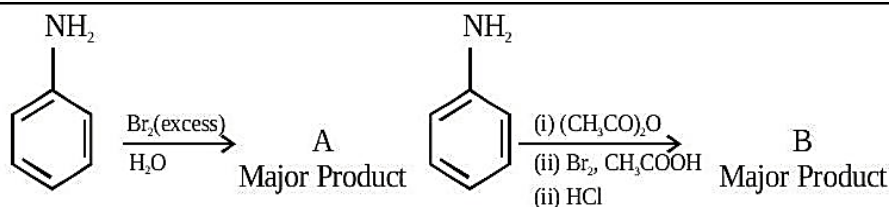
Statement II: Electron withdrawing groups in the carboxylic acid will increase the rate of esterification reaction.

Choose the most appropriate option:

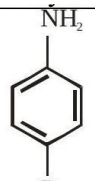
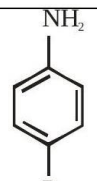
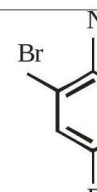
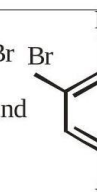
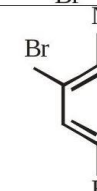
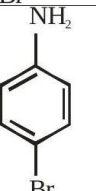
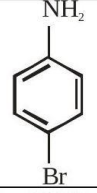
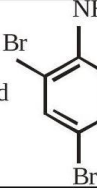
- (a) Both Statement I and Statement II are correct.

- (b) Both Statement I and Statement II are incorrect.
- (c) Statement I is correct but Statement II is incorrect.
- (d) Statement I is incorrect but Statement II is correct.

46.



Consider the above reaction, the product A and product B respectively are

(A)	 and 
(B)	 and 
(C)	 and 
(D)	 and 

47. The polymer, which can be stretched and retains its original status on releasing the force is

- (a) Bakelite (b) Nylon 6,6
(c) Buna-N (d) Terylene

48. Sugar moiety in DNA and RNA molecules respectively are

- (a) β -D-2-deoxyribose, β -D-deoxyribose
(b) β -D-2-deoxyribose, β -D-ribose
(c) β -D-ribose, β -D-2-deoxyribose
(d) β -D-deoxyribose, β -D-2-deoxyribose

49. Which of the following compound does not contain sulphur atom?

- (a) Cimetidine (b) Ranitidine
(c) Histamine (d) Saccharin

50. Given below are two statements.

Statement I: Phenols are weakly acidic.

Statement II: Therefore, they are freely soluble in NaOH solution and are weaker acids than alcohols and water.

Choose the most appropriate option:

- (a) Both Statement I and Statement II are correct.
- (b) Both Statement I and Statement II are incorrect.
- (c) Statement I is correct but Statement II is incorrect.
- (d) Statement I is incorrect but Statement II is correct

SECTION-B

51. Geraniol, a volatile organic compound, is a component of rose oil. The density of the vapour is 0.46 g L^{-1} at 257°C and 100 mmHg. The molar mass of geraniol is (Nearest Integer)

[Given $R = 0.082 \text{ L atm K}^{-1} \text{ mol}^{-1}$]

52. 17.0 g of NH_3 completely vapourises at -33.42°C and 1 bar pressure and the enthalpy change in the process is 23.4 kJ mol^{-1} . The enthalpy change for the vapourisation of 85 g of NH_3 under the same conditions is kJ.

53. 1.2 mL of acetic acid is dissolved in water to make 2.0 L of solution. The depression in freezing point observed for this strength of acid is 0.0198°C . The percentage of dissociation of the acid is

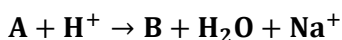
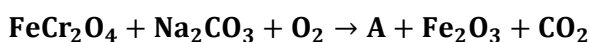
(Nearest integer) [Given: Density of acetic acid is 1.02 g mL^{-1} Molar mass of acetic acid is 60 g mol^{-1} $K_f(\text{H}_2\text{O}) = 1.85 \text{ K kg mol}^{-1}$]

54. A dilute solution of sulphuric acid is electrolysed using a current of 0.10 A for 2 hours to produce hydrogen and oxygen gas. The total volume of gases produced at STP is cm^3 . (Nearest integer) [Given: Faraday constant $F = 96500 \text{ C mol}^{-1}$ at STP, molar volume of an ideal gas is 22.7 L mol^{-1}]

55. The activation energy of one of the reactions in a biochemical process is $532611 \text{ J mol}^{-1}$. When the temperature falls from 310 K to 300 K, the change in rate constant observed is $k_{300} = x \times 10^{-3} k_{310}$. The value of x is

[Given: $\ln 10 = 2.3$ $R = 8.3 \text{ J K}^{-1} \text{ mol}^{-1}$]

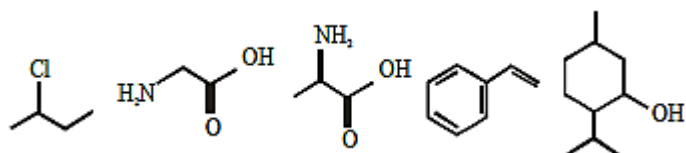
56. The number of terminal oxygen atoms present in the product B obtained from the following reaction is



57. An acidified manganate solution undergoes disproportionation reaction. The spin-only magnetic moment value of the product having manganese in higher oxidation state is B.M. (Nearest integer)

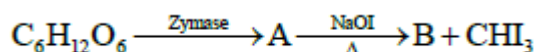
58. Kjeldahl's method was used for the estimation of nitrogen in an organic compound. The ammonia evolved from 0.55 g of the compound neutralised 12.5 mL of $1 \text{ M H}_2\text{SO}_4$ solution. The percentage of nitrogen in the compound is (Nearest integer)

59. Observe structures of the following compounds



The total number of structures/compounds which possess asymmetric carbon atoms is

60.



The number of carbon atoms present in the product B is

PART-III MATHEMATICS

SECTION-A

61. The probability that a randomly chosen 2×2 matrix with all the entries from the set of first 10 primes, is singular, is equal to:

- (a) $\frac{133}{10^4}$ (b) $\frac{18}{10^3}$
(c) $\frac{19}{10^3}$ (d) $\frac{271}{10^4}$

62. Let the solution curve of the differential equation $x \frac{dy}{dx} - y = \sqrt{y^2 + 16x^2}$, $y(1) = 3$ be $y = y(x)$. Then $y(2)$ is equal to:

- (a) 15 (b) 11
(c) 13 (d) 17

63. If the mirror image of the point $(2, 4, 7)$ in the plane $3x - y + 4z = 2$ is (a, b, c) , the $2a + b + 2c$ is equal to :

- (a) 54 (b) 50
(c) -6 (d) -42

64. Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be a function defined by:

$$f(x) = \begin{cases} \max\{t^3 - 3t\}; x \leq 2 \\ t \leq x \\ x^2 + 2x - 6; 2 < x < 3 \\ [x - 3] + 9; 3 \leq x \leq 5 \\ 2x + 1; x > 5 \end{cases}$$

Where $[t]$ is the greatest integer less than or equal to t . Let m be the number of points where f is not differentiable and $I = \int_{-2}^2 f(x) dx$. Then the ordered pair (m, I) is equal to:

- (a) $(3, \frac{27}{4})$ (b) $(3, \frac{23}{4})$
(c) $(4, \frac{27}{4})$ (d) $(4, \frac{23}{4})$

65. Let $\vec{a} = \alpha \hat{i} + 3\hat{j} - \hat{k}$, $\vec{b} = 3\hat{i} - \beta \hat{j} + 4\hat{k}$ and $\vec{c} = \hat{i} + 2\hat{j} - 2\hat{k}$ where $\alpha, \beta \in \mathbb{R}$, be three vectors. If the projection of $\vec{a}$ on $\vec{c}$ is $\frac{10}{3}$ and $\vec{b} \times \vec{c} = -6\hat{i} + 10\hat{j} + 7\hat{k}$, then the value of $\alpha + \beta$ equal to:

- (a) 3 (b) 4
(c) 5 (d) 6

66. The area enclosed by $y^2 = 8x$ and $y = \sqrt{2}x$ that lies outside the triangle formed by $y = \sqrt{2}x$, $x = 1$, $y = 2\sqrt{2}$, is equal to:

- (a) $\frac{16\sqrt{2}}{6}$ (b) $\frac{11\sqrt{2}}{6}$
(c) $\frac{13\sqrt{2}}{6}$ (d) $\frac{5\sqrt{2}}{6}$

67. If the system of linear equations

$$2x + y - z = 7$$

$$x - 3y + 2z = 1$$

$$x + 4y + \delta z = k, \text{ where } \delta, k \in \mathbb{R}$$

has infinitely many solutions, then $\delta + k$ is equal to:

- (a) -3 (b) 3
(c) 6 (d) 9

68. Let α and β be the roots of the equation $x^2 + (2i - 1)x = 0$. Then, the value of $|\alpha^8 + \beta^8|$ is equal to:

- (a) 50 (b) 250
(c) 1250 (d) 1500

69. Let $\Delta \in \{\wedge, \vee, \Rightarrow, \Leftrightarrow\}$ be such that $(p \wedge q) \Delta ((p \vee q) \Rightarrow q)$ is a tautology. Then Δ is equal to :

- (a) $\wedge$ (b) $\vee$
(c) $\Rightarrow$ (d) $\Leftrightarrow$

70. Let $A = [a_{ij}]$ be a square matrix of order 3 such that $a_{ij} = 2^{j-i}$, for all $i, j = 1, 2, 3$. Then, the matrix $A^2 + A^3 + \dots + A^{10}$ is equal to:

- (a) $\left(\frac{3^{10}-3}{2}\right)A$ (b) $\left(\frac{3^{10}-1}{2}\right)A$
(c) $\left(\frac{3^{10}+1}{2}\right)A$ (d) $\left(\frac{3^{10}+3}{2}\right)A$

71. Let a set $A = A_1 \cup A_2 \cup \dots \cup A_k$, where $A_i \cap A_j = \phi$ for $i \neq j$, $1 \leq i, j \leq k$. Define the relation R from A to A by $R = \{(x, y) : y \in A_i \text{ if and only if } x \in A_i, 1 \leq i \leq k\}$. Then, R is:

- (a) reflexive, symmetric but not transitive
(b) reflexive, transitive but not symmetric
(c) reflexive but not symmetric and transitive
(d) an equivalence relation

72. Let $\{a_n\}_{n=0}^\infty$ be a sequence such that $a_0 = a_1 = 0$ and $a_{n+2} = 2a_{n+1} - a_n + 1$ for all $n \geq 0$. Then, $\sum_{n=2}^\infty \frac{a_n}{7^n}$ is equal to

- (a) $\frac{6}{343}$ (b) $\frac{7}{216}$
(c) $\frac{8}{343}$ (d) $\frac{49}{216}$

73. The distance between the two points A and A' which lie on $y = 2$ such that both the line segments AB and $A'B$ (where B is the point $(2, 3)$) subtend angle $\frac{\pi}{4}$ at the origin, is equal to:

- (a) 10 (b) $\frac{48}{5}$
(c) $\frac{52}{5}$ (d) 3

74. A wire of length 22 m is to be cut into two pieces.

One of the pieces is to be made into a square and the other into an equilateral triangle. Then, the length of the side of the equilateral triangle, so that the combined area of the square and the equilateral triangle is minimum, is :

- (a) $\frac{22}{9+4\sqrt{3}}$ (b) $\frac{66}{9+4\sqrt{3}}$
(c) $\frac{22}{4+9\sqrt{3}}$ (d) $\frac{66}{4+9\sqrt{3}}$

75. The domain of the function $\cos^{-1}\left(\frac{2\sin^{-1}\left(\frac{1}{4x^2-1}\right)}{\pi}\right)$ is:

- (a) $\mathbb{R} - \left\{-\frac{1}{2}, \frac{1}{2}\right\}$
(b) $(-\infty, -1] \cup [1, \infty) \cup \{0\}$
(c) $(-\infty, \frac{-1}{2}) \cup (\frac{1}{2}, \infty) \cup \{0\}$
(d) $(-\infty, \frac{-1}{\sqrt{2}}] \cup [\frac{1}{\sqrt{2}}, \infty) \cup \{0\}$

76. If the constant term in the expansion of $\left(3x^3 - 2x^2 + \frac{5}{x^5}\right)^{10}$ is $2^k \cdot l$, where l is an odd integer, then the value of k is equal to:

- (a) 6 (b) 7
(c) 8 (d) 9

77. $\int_0^5 \cos\left(\pi\left(x - \left[\frac{x}{2}\right]\right)\right) dx$,

Where $[t]$ denotes greatest integer less than or equal to t , is equal to:

- (a) -3 (b) -2
(c) 2 (d) 0

78. Let PQ be a focal chord of the parabola $y^2 = 4x$ such that it subtends an angle of $\frac{\pi}{2}$ at the point $(3, 0)$. Let the line segment PQ be also a focal chord of the ellipse $E: \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, a^2 > b^2$. If e is the eccentricity of the ellipse E , then the value of $\frac{1}{e^2}$ is equal to:

- (a) $1 + \sqrt{2}$ (b) $3 + 2\sqrt{2}$
(c) $1 + 2\sqrt{3}$ (d) $4 + 5\sqrt{3}$

79. Let the tangent to the circle $C_1: x^2 + y^2 = 2$ at the point $M(-1, 1)$ intersect the circle $C_2: (x - 3)^2 + (y - 2)^2 = 5$, at two distinct points A and B. If the tangents to C_2 at the points A and B intersect at N, then the area of the triangle ANB is equal to:

- (a) $\frac{1}{2}$ (b) $\frac{2}{3}$
(c) $\frac{1}{6}$ (d) $\frac{5}{3}$

80. Let the mean and the variance of 5 observations x_1, x_2, x_3, x_4, x_5 be $\frac{24}{5}$ and $\frac{194}{25}$ respectively. If the mean and variance of the first 4 observation are $\frac{7}{2}$ and a respectively, then $(4a + x_5)$ is equal to:

- (a) 13 (b) 15
(c) 17 (d) 18

SECTION-B

81. Let $S = \{z \in \mathbb{C} : |z - 2| \leq 1, z(1 + i) + \bar{z}(1 - i) \leq 2\}$. Let $|z - 4i|$ attains minimum and maximum values, respectively, at $z_1 \in S$ and $z_2 \in S$. If $5(|z_1|^2 + |z_2|^2) = \alpha + \beta\sqrt{5}$, where α and β are integers, then the value of $\alpha + \beta$ is equal to

82. Let $y = y(x)$ be the solution of the differential equation

$$\frac{dy}{dx} + \frac{\sqrt{2}y}{2\cos^4 x - \cos 2x} = xe^{\tan^{-1}(\sqrt{2}\cot 2x)}, 0 < x <$$

$$\pi/2 \text{ with } y\left(\frac{\pi}{4}\right) = \frac{\pi^2}{32}.$$

If $y\left(\frac{\pi}{3}\right) = \frac{\pi^2}{18}e^{-\tan^{-1}(\alpha)}$, then the value of $3\alpha^2$ is equal to

83. Let d be the distance between the foot of perpendiculars of the points $P(1, 2 - 1)$ and $Q(2, -1, 3)$ on the plane $-x + y + z = 1$. Then d^2 is equal to

84. The number of elements in the set $S = \{\theta \in [-4\pi, 4\pi] : 3\cos^2 2\theta + 6\cos 2\theta - 10\cos^2 \theta + 5 = 0\}$ is

85. The number of solutions of the equation $2\theta - \cos^2 \theta + \sqrt{2} = 0$ is R is equal to

86. $50\tan\left(3\tan^{-1}\left(\frac{1}{2}\right) + 2\cos^{-1}\left(\frac{1}{\sqrt{5}}\right)\right) + 4\sqrt{2}\tan\left(\frac{1}{2}\tan^{-1}(2\sqrt{2})\right)$ is equal to

87. Let $c, k \in \mathbb{R}$. If $f(x) = (c + 1)x^2 + (1 - c^2)x + 2k$ and $f(x + y) = f(x) + f(y) - xy$, for all $x, y \in \mathbb{R}$, then the value of $12(f(1) + f(2) + f(3) + \dots + f(20))$ is equal to

88. Let $H: \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1, a > 0, b > 0$, be a hyperbola such that the sum of lengths of the transverse and the conjugate axes is $4(2\sqrt{2} + \sqrt{14})$. If the eccentricity H is $\frac{\sqrt{11}}{2}$, then value of $a^2 + b^2$ is equal to

89. Let $P_1: \vec{r} \cdot (2\hat{i} + \hat{j} - 3\hat{k}) = 4$ be a plane. Let P_2 be another plane which passes through the points $(2, 3, 2), (2, -2, -3)$ and $(1, -4, 2)$. If the direction ratios of the line of intersection of P_1 and P_2 be $16, \alpha, \beta$, then the value of $\alpha + \beta$ is equal to

90. Let $b_1 b_2 b_3 b_4$ be a 4-element permutation with $b_i \in \{1, 2, 3, 100\}$ for $1 \leq i \leq 4$ and $b_i \neq b_j$ for $i \neq j$, such that either b_1, b_2, b_3 are consecutive integers or b_2, b_3, b_4 are consecutive integers. Then the number of such permutations $b_1 b_2 b_3 b_4$ is equal to