

PART-I PHYSICS

SECTION-A

- A small toy starts moving from the position of rest under a constant acceleration. If it travels a distance of 10 m in t s, the distance travelled by the toy in the next t s will be:
(a) 10 m (b) 20 m
(c) 30 m (d) 40 m
- At what temperature a gold ring of diameter 6.230 cm be heated so that it can be fitted on a wooden bangle of diameter 6.241 cm ? Both the diameters have been measured at room temperature (27°C). (Given: coefficient of linear thermal expansion of gold $\alpha_L = 1.4 \times 10^{-5} \text{ K}^{-1}$)
(a) 125.7°C (b) 91.7°C
(c) 425.7° (d) 152.7°C
- Two -point charges Q each are placed at a distance d apart. A third point charge q is placed at a distance x from mid-point on the perpendicular bisector. The value of x at which charge q will experience the maximum Coulomb's force is:
(a) $x = d$ (b) $x = \frac{d}{2}$
(c) $x = \frac{d}{\sqrt{2}}$ (d) $x = \frac{d}{2\sqrt{2}}$
- The speed of light in media 'A' and 'B' are $2.0 \times 10^{10} \text{ cm/s}$ and $1.5 \times 10^{10} \text{ cm/s}$ respectively. A ray of light enters from the medium B to A at an incident angle ' θ '. If the ray suffers total internal reflection, then
(a) $\theta = \sin^{-1}\left(\frac{3}{4}\right)$ (b) $\theta > \sin^{-1}\left(\frac{2}{3}\right)$
(c) $\theta < \sin^{-1}\left(\frac{3}{4}\right)$ (d) $\theta > \sin^{-1}\left(\frac{3}{4}\right)$
- In the following nuclear reaction,
 $D \xrightarrow{\alpha} D_1 \xrightarrow{\beta^-} D_2 \xrightarrow{\alpha} D_3 \xrightarrow{\gamma} D_4$
Mass number of D is 182 and atomic number is 74. Mass number and atomic number of D_4 respectively will be
(a) 174 and 71 (b) 174 and 69
(c) 172 and 69 (d) 172 and 71
- The electric field at the point associated with a light wave is given by
 $E = 200[\sin(6 \times 10^{15}t) + \sin(9 \times 10^{15}t)] \text{ Vm}^{-1}$
Given: $h = 4.14 \times 10^{-15} \text{ eVs}$
If this light falls on a metal surface having a work function of 2.50 eV, the maximum kinetic energy of the photoelectrons will be:
(a) 1.90 eV (b) 3.27 eV
(c) 3.60 eV (d) 3.42 eV
- A capacitor is discharging through a resistor R .
Consider in time t_1 , the energy stored in the capacitor reduces to half of its initial value and in time t_2 , the charge stored reduces to one eighth of its initial value. The ratio t_1/t_2 will be:
(a) $\frac{1}{2}$ (b) $\frac{1}{3}$
(c) $\frac{1}{4}$ (d) $\frac{1}{6}$
- Starting with the same initial conditions, an ideal gas expands from volume V_1 to V_2 in three different ways. The work done by the gas is W_1 if the process is purely isothermal. W_2 if the process is purely

adiabatic and W_3 if the process is purely isobaric. Then, choose the correct option

- (a) $W_1 < W_2 < W_3$ (b) $W_2 < W_3 < W_1$
(c) $W_3 < W_1 < W_2$ (d) $W_2 < W_1 < W_3$

9. Two long current carrying conductors are placed parallel to each other at a distance of 8 cm between them. The magnitude of magnetic field produced at mid-point between the two conductors due to current flowing in them is $300\mu\text{T}$. The equal current flowing in the two conductors is:

- (a) 30 A in the same direction.
(b) 30 A in the opposite direction.
(c) 60 A in the opposite direction.
(d) 300A in the opposite direction

10. The time period of a satellite revolving around earth in a given orbit is 7 hours. If the radius of orbit is increased to three times its previous value, then approximate new time period of the satellite will be:

- (a) 40 hours (b) 36 hours
(c) 30 hours (d) 25 hours

11. The TV transmission tower at a particular station has a height of 125 m. For doubling the coverage of its range, the height of the tower should be increased by:

- (a) 125 m (b) 250 m
(c) 375 (d) 500 m

12. The motion of a simple pendulum executing S.H.M. is represented by following equation.

$Y = A \sin(\pi t + \phi)$, where time is measured in second.

The length of pendulum is:

- (a) 97.23 cm (b) 25.3 cm
(c) 99.4 cm (d) 406.1 cm

13. A vessel contains 16 g of hydrogen and 128 g of oxygen at standard temperature and pressure. The volume of the vessel in cm^3 is:

- (a) 72×10^5 (b) 32×10^5
(c) 27×10^4 (d) 54×10^4

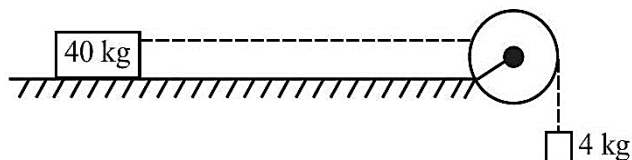
14. Given below are two statements:

Statement I: The electric force changes the speed of the charged particle and hence changes its kinetic energy: whereas the magnetic force does not change the kinetic energy of the charged particle.

Statement II: The electric force accelerates the positively charged particle perpendicular to the direction of electric field. The magnetic force accelerates the moving charged particle along the direction of magnetic field. In the light of the above statements, choose the most appropriate answer from the options given below:

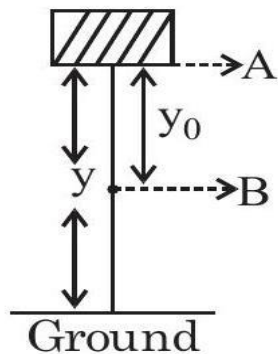
- (a) Both Statement I and Statement II are correct.
(b) Both Statement I and Statement II are incorrect.
(c) Statement I is correct but Statement II is incorrect.
(d) Statement I is incorrect but Statement II is correct.

15. A block of mass 40 kg slides over a surface, when a mass of 4 kg is suspended through an inextensible massless string passing over frictionless pulley as shown below. The coefficient of kinetic friction between the surface and block is 0.02 . The acceleration of block is. (Given $g = 10 \text{ ms}^{-2}$.)



- (a) 1 ms^{-2} (b) $1/5 \text{ ms}^{-2}$
(c) $4/5 \text{ ms}^{-2}$ (d) $8/11 \text{ ms}^{-2}$

16. In the given figure, the block of mass m is dropped from the point 'A'. The expression for kinetic energy of block when it reaches point 'B' is:



- (a) $\frac{1}{2} mgy_0^2$ (b) $\frac{1}{2} mgy^2$
(c) $mg(y - y_0)$ (d) mgy_0

17. A block of mass M placed inside a box descends vertically with acceleration ' a '. The block exerts a force equal to one-fourth of its weight on the floor of the box. The value of ' a ' will be:

- (a) $\frac{g}{4}$ (b) $\frac{g}{2}$
(c) $\frac{3g}{4}$ (d) g

18. If the electric potential at any point (x, y, z) m in space is given by $V = 3x^2$ volt. The electric field at the point $(1, 0, 3)$ m will be:

- (a) $3V\text{m}^{-1}$, directed along positive x-axis.
(b) $3V\text{m}^{-1}$, directed along negative x-axis.
(c) $6V\text{m}^{-1}$, directed along positive x-axis.
(d) $6V\text{m}^{-1}$, directed along negative x-axis.

19. The combination of two identical cells, whether connected in series or parallel combination provides the same current through an external resistance of 2Ω . The value of internal resistance of each cell is:

- (a) 2Ω (b) 4Ω
(c) 6Ω (d) 8Ω

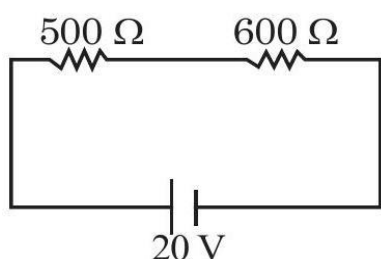
20. A person can throw a ball upto a maximum range of 100 m. How high above the ground he can throw the same ball?

- (a) 25 m (b) 50 m
(c) 100 m (d) 200 m

SECTION-B

21. The vernier constant of Vernier callipers is 0.1 mm and it has zero error of $(-0.05)\text{cm}$. While measuring diameter of a sphere, the main scale reading is 1.7 cm and coinciding vernier division is 5. The corrected diameter will be $\times 10^{-2}$ cm

22. A small spherical ball of radius 0.1 mm and density 10^4 kg m^{-3} falls freely under gravity through a distance h before entering a tank of water. If after entering the water the velocity of ball does not change and it continues to fall with same constant velocity inside water, then the value of h will be m.
(Given $g = 10 \text{ ms}^{-2}$, viscosity of water = $1.0 \times 10^{-5} \text{ N - sm}^{-2}$).
23. In an experiment to determine the velocity of sound in air at room temperature using a resonance is observed when the air column has a length of 20.0 cm for a tuning fork of frequency 400 Hz is used. The velocity of the sound at room temperature is 336 ms^{-1} . The third resonance is observed when the air column has a length of cm.
24. Two resistors are connected in series across a battery as shown in figure. If a voltmeter of resistance 2000Ω is used to measure the potential difference across 500Ω resistor, the reading of the voltmeter will be V.



25. A potential barrier of 0.4 V exists across a p-n junction. An electron enters the junction from the n-side with a speed of $6.0 \times 10^5 \text{ ms}^{-1}$. The speed with which electron enters the p side will be $\frac{x}{3} \times 10^5 \text{ ms}^{-1}$ the value of x is (Given mass of electron = $9 \times 10^{-31} \text{ kg}$, charge on electron = $1.6 \times 10^{-19} \text{ C}$.)
26. The displacement current of $4.425\mu\text{A}$ is developed in the space between the plates of parallel plate capacitor when voltage is changing at a rate of 10^6 Vs^{-1} . The area of each plate of the capacitor is 40 cm^2 . The distance between each plate of the capacitor is $x \times 10^{-3} \text{ m}$. The value of x is, (Permittivity of free space, $\epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}$)
27. The moment of inertia of a uniform thin rod about a perpendicular axis passing through one end is I_1 . The same rod is bent into a ring and its moment of inertia about a diameter is I_2 . If $\frac{I_1}{I_2}$ is $\frac{x\pi^2}{3}$, then the value of x will be
28. The half-life of a radioactive substance is 5 years. After x years a given sample of the radioactive substance gets reduced to 6.25% of its initial value of x is
29. In a double slit experiment with monochromatic light, fringes are obtained on a screen placed at some distance from the plane of slits. If the screen is moved by $5 \times 10^{-2} \text{ m}$ towards the slits, the change in fringe width is $3 \times 10^{-3} \text{ cm}$. If the distance between the slits is 1 mm, then the wavelength of the light will be nm.
30. An inductor of 0.5mH, a capacitor of $200\mu\text{F}$ and a resistor of 2Ω are connected in series with a 220 V ac source. If the current is in phase with the emf, the frequency of ac source will be $_ \times 10^2 \text{ Hz}$.

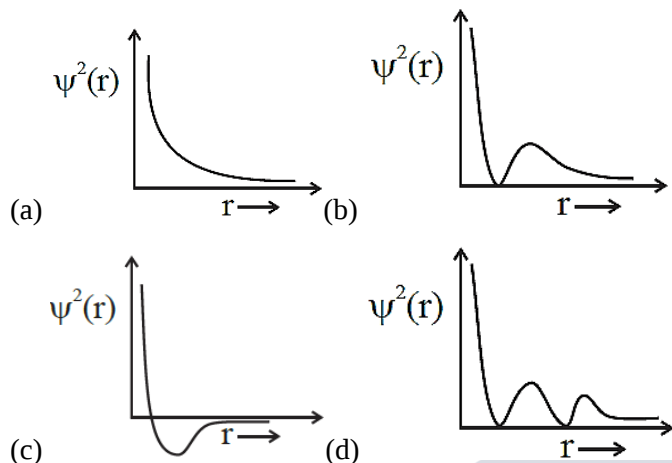
PART-II CHEMISTRY

SECTION-A

31. Using the rules for significant figures, the correct answer for the expression $\frac{0.02858 \times 0.112}{0.5702}$ will be:

- (a) 0.005613 (b) 0.00561
(c) 0.0056 (d) 0.006

32. Which of the following is the correct plot for the probability density $\psi^2(r)$ as a function of distance 'r' of the electron from the nucleus for 2s orbital?



33. Consider the species CH_4 , NH_4^+ and BH_4^- . Choose the correct option with respect to the these species:

- (a) They are isoelectronic and only two have tetrahedral structures
(b) They are isoelectronic and all have tetrahedral structures
(c) Only two are isoelectronic and all have tetrahedral structures
(d) Only two are isoelectronic and only two have tetrahedral structures

34. 4.0 moles of argon and 5.0 moles of PCl_5 are introduced into an evacuated flask of 100 litre capacity at 610 K. The system is allowed to equilibrate. At equilibrium, the total pressure of mixture was found to be 6.0 atm. The K_p for the reaction is [Given: $R = 0.082 \text{ L atm K mol}^{-1}$]

- (a) 2.25 (b) 6.24
(c) 12.13 (d) 15.24

35. A 42.12% (w/v) solution of NaCl causes precipitation of a certain sol in 10 hours. The coagulating value of NaCl for the sol is

[Given: Molar mass: $\text{Na} = 23.0 \text{ g mol}^{-1}$, $\text{Cl} = 35.5 \text{ g mol}^{-1}$]

- (a) 36 mmol L^{-1} (b) 36 mol L^{-1}
(c) 1440 mol L^{-1} (d) 1440 mmol L^{-1}

36. Given below are two statements. One is labelled as Assertion A and the other is labelled as Reason R.

Assertion A: The first ionization enthalpy for oxygen is lower than that of nitrogen.

Reason R: The four electrons in 2p orbitals of oxygen experience more electron-electron repulsion.

In the light of the above statements, choose the correct answer from the options given below.

- (a) Both A and R are correct and R is the correct explanation of A.
(b) Both A and R are correct but R is NOT the correct explanation of A.
(c) A is correct but R is not correct.

(d) A is not correct but R is correct

37. Match List I with List II.

List I Ore	List II Composition
A. Siderite	I. FeCO_3
B. Malachite	II. $\text{CuCO}_3 \cdot \text{Cu(OH)}_2$
C. Sphalerite	III. ZnS
D. Calamine	IV. ZnCO_3

Choose the correct answer from the options given below:

- (a) A-I, B-II, C-III, D-IV (b) A-III, B-IV, C-II, D-I
(c) A-IV, B-III, C-I, D-II (d) A-I, B-II, C-IV, D-III

38. Given below are two statements.

Statement I: In $\text{CuSO}_4 \cdot 5\text{H}_2\text{O}$, Cu – O bonds are present.

Statement II: In $\text{CuSO}_4 \cdot 5\text{H}_2\text{O}$, ligands coordinating with Cu(II) ion are O- and S-based ligands.

In the light of the above statements, choose the correct answer from the options given below

- (a) Both Statement I and Statement II are correct
(b) Both Statement I and Statement II are incorrect
(c) Statement I is correct but Statement II is incorrect
(d) Statement I is incorrect but Statement II is correct

39. Amongst baking soda, caustic soda and washing soda carbonate anion is present in:

- (a) washing soda only.
(b) washing soda and caustic soda only.
(c) washing soda and baking soda only.
(d) baking soda, caustic soda and washing soda.

40. Number of lone pair (s) of electrons on central atom and the shape of BrF_3 molecule respectively, are:

- (a) 0, triangular planar. (b) 1, pyramidal.
(c) 2, bent T-shape. (d) 1, bent T-shape

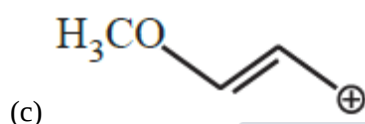
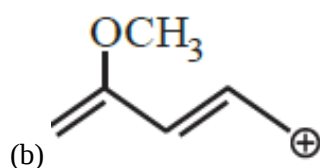
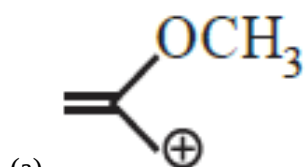
41. Aqueous solution of which of the following boron compounds will be strongly basic in nature?

- (a) NaBH_4 (b) LiBH_4
(c) B_2H_6 (d) $\text{Na}_2\text{B}_4\text{O}_7$

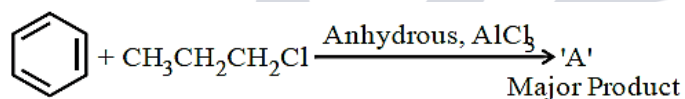
42. Sulphur dioxide is one of the components of polluted air. SO_2 is also a major contributor to acid rain. The correct and complete reaction to represent acid rain caused by SO_2 is:

- (a) $2\text{SO}_2 + \text{O}_2 \rightarrow 2\text{SO}_3$
 (b) $\text{SO}_2 + \text{O}_3 \rightarrow \text{SO}_3 + \text{O}_2$
 (c) $\text{SO}_2 + \text{H}_2\text{O}_2 \rightarrow \text{H}_2\text{SO}_4$
 (d) $2\text{SO}_2 + \text{O}_2 + 2\text{H}_2\text{O} \rightarrow 2\text{H}_2\text{SO}_4$

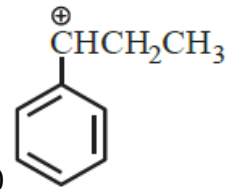
43. Which of the following carbocations is most stable?



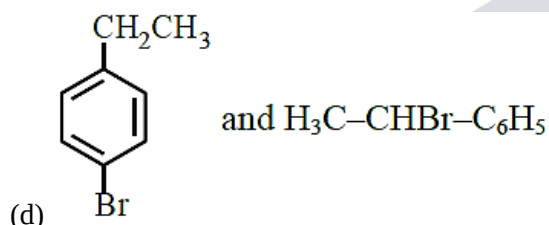
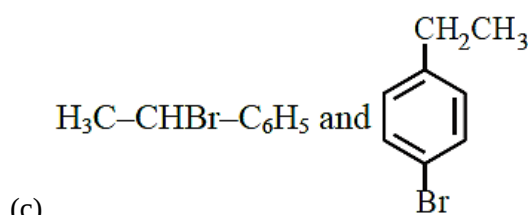
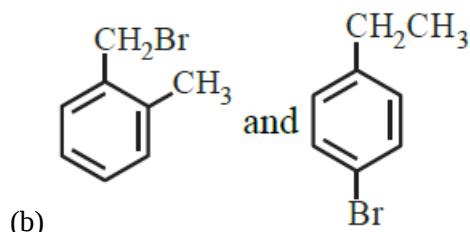
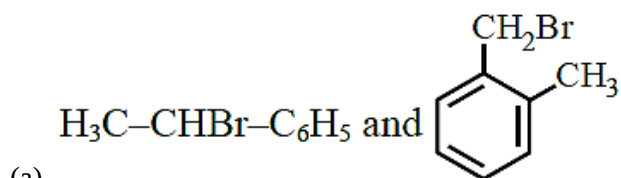
44.



The stable carbocation formed in the above reaction is :

- (a) $\text{CH}_3\text{CH}_2\text{CH}_2^+$
 (b) CH_3CH_2^+
 (c) $\text{CH}_3 - \text{CH}^+ - \text{CH}_3$
 (d) 

45. Two isomers (A) and (B) with Molar mass 184 g/mol and elemental composition C, 52.2%; H, 4.9% and Br 42.9% gave benzoic acid and p bromobenzoic acid, respectively on oxidation with KMnO_4 . Isomer 'A' is optically active and gives a pale -yellow precipitate when warmed with alcoholic AgNO_3 . Isomer 'A' and 'B' are, respectively:



46. In Friedel-Crafts alkylation of aniline, one gets:

- (a) alkylated product with ortho and para substitution.
- (b) secondary amine after acidic treatment.
- (c) an amide product.
- (d) positively charged nitrogen at benzene ring.

47. Given below are two statements: one is labelled as Assertion A and the other is labelled as Reason R.

Assertion A: Dacron is an example of polyester polymer.

Reason R: Dacron is made up of ethylene glycol and terephthalic acid monomers.

In the light of the above statements, choose the most appropriate answer from the options given below.

- (a) Both A and B are correct and R is the correct explanation of A.
- (b) Both A and B are correct but R is NOT the correct explanation of A.
- (c) A is correct but R is not correct.
- (d) A is not correct but R is correct.

48. The structure of protein that is unaffected by heating is:

- (a) secondary structure
- (b) tertiary structure
- (c) primary structure
- (d) quaternary structure

49. The mixture of chloroxylenol and terpineol is an example of:

- (a) antiseptic (b) pesticide
(c) disinfectant (d) narcotic analgesic

50. A white precipitate was formed when BaCl_2 was added to water extract of an inorganic salt. Further, a gas 'X' with characteristic odour was released when the formed white precipitate was dissolved in dilute HCl. The anion present in the inorganic salt is:

- (a) I^- (b) SO_3^{2-}
(c) S^{2-} (d) NO_2^-

SECTION-B

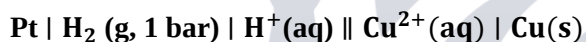
51. A box contains 0.90 g of liquid water in equilibrium with water vapour at 27°C . The equilibrium vapour pressure of water at 27°C is 32.0 Torr. When the volume of the box is increased, some of the liquid water evaporates to maintain the equilibrium pressure. If all the liquid water evaporates, then the volume of the box must be litre. [nearest integer] (Given: $R = 0.082 \text{ L atm K}^{-1} \text{ mol}^{-1}$) (Ignore the volume of the liquid water and assume water vapours behave as an ideal gas.)

52. 2.2 g of nitrous oxide (N_2O) gas is cooled at a constant pressure of 1 atm from 310 K to 270 K causing the compression of the gas from 217.1 mL to 167.75 mL. The change in internal energy of the process, ΔU is $-x \text{ J}$. The value of 'x' is [nearest integer]

(Given: atomic mass of N = 14 g mol^{-1} and of O = 16 g mol^{-1} . Molar heat capacity of N_2O is $100 \text{ J K}^{-1} \text{ mol}^{-1}$)

53. Elevation in boiling point for 1.5 molal solution of glucose in water is 4 K. The depression in freezing point for 4.5 molal solution of glucose in water is 4 K. The ratio of molal elevation constant to molal depression constant (K_b/K_f) is

54. The cell potential for the given cell at 298 K



is 0.31 V. The pH of the acidic solution is found to be 3, whereas the concentration of Cu^{2+} is 10^{-x} M . The value of x is

(Given: $E_{\text{Cu}^{2+}/\text{Cu}}^\ominus = 0.34 \text{ V}$ and $\frac{2.303RT}{F} = 0.06 \text{ V}$)

55. The equation

$$k = (6.5 \times 10^{12} \text{ s}^{-1}) e^{-26000 \text{ K}/T}$$

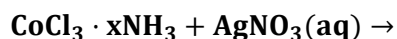
is followed for the decomposition of compound A.

The activation energy for the reaction is kJ mol^{-1} . [nearest integer]

(Given: $R = 8.314 \text{ J K}^{-1} \text{ mol}^{-1}$)

56. Spin only magnetic moment of $[\text{MnBr}_6]^{4-}$ is B.M. (round off to the closest integer)

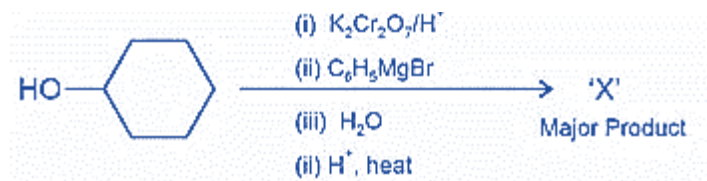
57. For the reaction given below:



If two equivalents of AgCl precipitate out, then the value of x will be

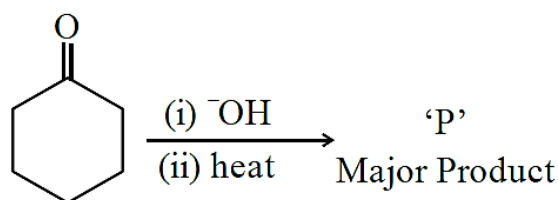
58. The number of chiral alcohol(s) with molecular formula $C_4H_{10}O$ is

59. In the given reaction



the number of sp^2 hybridised carbon (s) in compound 'X' is

60. In the given reaction,



The number of π electrons present in the product 'P' is

PART-III MATHEMATICS

SECTION-A

61. Let α be a root of the equation $1 + x^2 + x^4 = 0$. Then the value of $\alpha^{1011} + \alpha^{2022} - \alpha^{3033}$ is equal to:

- (a) 1 (b) α
(c) $1 + \alpha$ (d) $1 + 2\alpha$

62. Let $\arg(z)$ represent the principal argument of the complex number z . The, $|z| = 3$ and $\arg(z - 1) - \arg(z + 1) = \frac{\pi}{4}$ intersect:

- (a) Exactly at one point
(b) Exactly at two points
(c) Nowhere
(d) At infinitely many points.

63. Let $A = \begin{pmatrix} 2 & -1 \\ 0 & 2 \end{pmatrix}$. If $B = I - {}^5C_1(\text{adj } A) + {}^5C_2(\text{adj } A)^2 - \dots - {}^5C_5(\text{adj } A)^5$, then the sum of all elements of the matrix B is:

- (a) -5 (b) -6
(c) -7 (d) -8

64. The sum of the infinite series $1 + \frac{5}{6} + \frac{12}{6^2} + \frac{22}{6^3} + \frac{35}{6^4} + \frac{51}{6^5} + \frac{70}{6^6} + \dots$ is equal to:

- (a) $\frac{425}{216}$ (b) $\frac{429}{216}$
(c) $\frac{288}{125}$ (d) $\frac{280}{125}$

65. The value of $\lim_{x \rightarrow 1} \frac{(x^2-1)\sin^2(\pi x)}{x^4-2x^3+2x-1}$ is equal to:

- (a) $\frac{\pi^2}{6}$ (b) $\frac{\pi^2}{3}$
(c) $\frac{\pi^2}{2}$ (d) π^2

66. Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be a function defined by $f(x) = (x-3)^{n_1}(x-5)^{n_2}$, $n_1, n_2 \in \mathbb{N}$. The, which of the following is NOT true?

- (a) For $n_1 = 3, n_2 = 4$, there exists $\alpha \in (3,5)$ where f attains local maxima.
(b) For $n_1 = 4, n_2 = 3$, there exists $\alpha \in (3,5)$ where f attains local minima.
(c) For $n_1 = 3, n_2 = 5$, there exists $\alpha \in (3,5)$ where f attains local maxima.
(d) For $n_1 = 4, n_2 = 6$, there exists $\alpha \in (3,5)$ where f attains local maxima.

67. Let f be a real valued continuous function on $[0, 1]$ and $f(x) = x + \int_0^1 (x-t)f(t)dt$. Then which of the following points (x, y) lies on the curve $y = f(x)$?

- (a) (2,4) (b) (1,2)
(c) (4,17) (d) (6,8)

68. If $\int_0^2 (\sqrt{2x} - \sqrt{2x-x^2})dx =$

$$\int_0^1 \left(1 - \sqrt{1-y^2} - \frac{y^2}{2}\right) dy + \int_1^2 \left(2 - \frac{y^2}{2}\right) dy + I$$

- (a) $\int_0^1 (1 + \sqrt{1-y^2})dy$
(b) $\int_0^1 \left(\frac{y^2}{2} - \sqrt{1-y^2} + 1\right) dy$
(c) $\int_0^1 (1 - \sqrt{1-y^2})dy$
(d) $\int_0^1 \left(\frac{y^2}{2} + \sqrt{1-y^2} + 1\right) dy$

69. If $y = y(x)$ is the solution of the differential equation $(1 + e^{2x}) \frac{dy}{dx} + 2(1 + y^2)e^x = 0$ and $y(0) = 0$, then

$6 \left(y'(0) + (y(\log_e \sqrt{3}))^2 \right)$ is equal to:

- (a) 2 (b) -2
(c) -4 (d) -1

70. Let $P: y^2 = 4ax$, $a > 0$ be a parabola with focus S . Let the tangents to the parabola P make an angle of $\frac{\pi}{4}$ with the line $y = 3x + 5$ touch the parabola P at A and B . Then the value of a for which A, B and S are collinear is:

- (a) 8 only (b) 2 only
(c) $\frac{1}{4}$ only (d) any $a > 0$

71. Let a triangle ABC be inscribed in the circle $x^2 - \sqrt{2}(x+y) + y^2 = 0$ such that $\angle BAC = \frac{\pi}{2}$. If the length of side AB is $\sqrt{2}$, then the area of the $\triangle ABC$ is equal to:

- (a) $(\sqrt{2} + \sqrt{6})/3$ (b) $(\sqrt{6} + \sqrt{3})/2$
(c) $(3 + \sqrt{3})/4$ (d) 1

72. Let $\frac{x-2}{3} = \frac{y+1}{-2} = \frac{z+3}{-1}$ lie on the plane $px - qy + z = 5$, for some $p, q \in \mathbb{R}$. The shortest distance of the plane from the origin is:

- (a) $\sqrt{\frac{3}{109}}$ (b) $\sqrt{\frac{5}{142}}$
(c) $\sqrt{\frac{5}{71}}$ (d) $\sqrt{\frac{1}{142}}$

73. The distance of the origin from the centroid of the triangle whose two sides have the equations $x - 2y + 1 = 0$ and $2x - y - 1 = 0$ and whose orthocenter is $(\frac{7}{3}, \frac{7}{3})$ is:

- (a) $\sqrt{2}$ (b) 2
(c) $2\sqrt{2}$ (d) 4

74. Let Q be the mirror image of the point P(1, 2, 1) with respect to the plane $x + 2y + 2z = 16$. Let T be a plane passing through the point Q and contains the line $\vec{r} = -\hat{k} + \lambda(\hat{i} + \hat{j} + 2\hat{k}), \lambda \in \mathbb{R}$. Then, which of the following points lies on T?

- (a) (2, 1, 0) (b) (1, 2, 1)
(c) (1, 2, 2) (d) (1, 3, 2)

75. Let A, B, C be three points whose position vectors respectively are:

$$\vec{a} = \hat{i} + 4\hat{j} + 3\hat{k}$$

$$\vec{b} = 2\hat{i} + \alpha\hat{j} + 4\hat{k}, \alpha \in \mathbb{R}$$

$$\vec{c} = 3\hat{i} - 2\hat{j} + 5\hat{k}$$

If α is the smallest positive integer for which $\vec{a}, \vec{b}, \vec{c}$ are non-collinear, then the length of the median, in $\triangle ABC$, through A is:

- (a) $\frac{\sqrt{82}}{2}$ (b) $\frac{\sqrt{62}}{2}$
(c) $\frac{\sqrt{69}}{2}$ (d) $\frac{\sqrt{66}}{2}$

76. The probability that a relation R from $\{x, y\}$ to $\{x, y\}$ is both symmetric and transitive, is equal to:

- (a) $\frac{5}{16}$ (b) $\frac{9}{16}$
(c) $\frac{11}{16}$ (d) $\frac{13}{16}$

77. The number of values of $a \in \mathbb{N}$ such that the variance of 3, 7, 12a, 43 - a is a natural number is:

- (a) 0 (b) 2
(c) 5 (d) infinite

78. From the base of a pole of height 20 meter, the angle of elevation of the top of a tower is 60° . The pole subtends an angle 30° at the top of the tower. Then the height of the tower is:

- (a) $15\sqrt{3}$ (b) $20\sqrt{3}$
(c) $20 + 10\sqrt{3}$ (d) 30

79. Negation of the Boolean statement $(p \vee q) \Rightarrow ((\sim r) \vee p)$ is equivalent to:

- (a) $p \wedge (\sim q) \wedge r$ (b) $(\sim p) \wedge (\sim q) \wedge r$
(c) $(\sim p) \wedge q \wedge r$ (d) $p \wedge q \wedge (\sim r)$

80. Let $n \geq 5$ be an integer. If $9^n - 8n - 1 = 64\alpha$ and $6^n - 5n - 1 = 25\beta$, then $\alpha - \beta$ is equal to:

- (a) $1 + {}^nC_2(8 - 5) + {}^nC_3(8^2 - 5^2) + \dots + {}^nC_n(8^{n-1} - 5^{n-1})$
(b) $1 + {}^nC_3(8 - 5) + {}^nC_4(8^2 - 5^2) + \dots + {}^nC_n(8^{n-2} - 5^{n-2})$
(c) ${}^nC_3(8 - 5) + {}^nC_4(8^2 - 5^2) + \dots + {}^nC_n(8^{n-2} - 5^{n-2})$
(d) ${}^nC_4(8 - 5) + {}^nC_5(8^2 - 5^2) + \dots + {}^nC_n(8^{n-3} - 5^{n-3})$

SECTION-B

81. Let $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}, \vec{b} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{c}$ be a vector such that $\vec{a} + (\vec{b} \times \vec{c}) = \vec{0}$ and $\vec{b} \cdot \vec{c} = 5$. Then, the value of $3(\vec{c} \cdot \vec{a})$ is equal to

82. Let $y = y(x)$, $x > 1$, be the solution of the differential equation $(x - 1) \frac{dy}{dx} + 2xy = \frac{1}{x-1}$, with $y(2) = \frac{1+e^4}{2e^4}$. If $y(3) = \frac{e^{\alpha+1}}{\beta e^{\alpha}}$, then the value of $\alpha + \beta$ is equal to
83. Let 3, 6, 9, 12, ... upto 78 terms and 5, 9, 13, 17, ... upto 59 terms be two series. Then, the sum of the terms common to both the series is equal to
84. The number of solutions of the equation $\sin x = \cos^2 x$ in the interval $(0, 10)$ is_.
85. For real numbers $a, b (a > b > 0)$, let
 $\text{Area}\{(x, y): x^2 + y^2 \leq a^2 \text{ and } \frac{x^2}{a^2} + \frac{y^2}{b^2} \geq 1\} = 30\pi$
 and
 $\text{Area}\{(x, y): x^2 + y^2 \geq b^2 \text{ and } \frac{x^2}{a^2} + \frac{y^2}{b^2} \leq 1\} = 18\pi$
 Then the value of $(a - b)^2$ is equal to
86. Let f and g be twice differentiable even functions on $(-2, 2)$ such that $f\left(\frac{1}{4}\right) = 0, f\left(\frac{1}{2}\right) = 0, f(1) = 1$ and $g\left(\frac{3}{4}\right) = 0, g(1) = 2$. Then, the minimum number of solutions of $f(x)g''(x) + f'(x)g'(x) = 0$ in $(-2, 2)$ is equal to_.
87. Let the coefficients of x^{-1} and x^{-3} in the expansion of $\left(2x^{\frac{1}{5}} - \frac{1}{x^{\frac{1}{5}}}\right)^{15}$, $x > 0$, be m and n respectively. If r is a positive integer such $mn^2 = {}^{15}C_r \cdot 2^r$, then the value of r is equal to
88. The total number of four -digit numbers such that each of the first three digits is divisible by the last digit, is equal to
89. Let $M = \begin{bmatrix} 0 & -\alpha \\ \alpha & 0 \end{bmatrix}$, where α is a non-zero real number and $N = \sum_{k=1}^{49} M^{2k}$. If $(I - M^2)N = -2I$, then the positive integral value of α is
90. Let $f(x)$ and $g(x)$ be two real polynomials of degree 2 and 1 respectively. If $f(g(x)) = 8x^2 - 2x$, and $g(f(x)) = 4x^2 + 6x + 1$, then the value of $f(2) + g(2)$ is