1. **(c)** Magnetic moment $m = IAN$

Magnetic moment of coil A $\rightarrow$

$$m_A = I_A \pi r_A^2 N_A$$
$$m_A = I_A \pi N_A (10)^2$$

Magnetic moment of coil B $\rightarrow$

$$m_B = I_B N_B \pi r_B^2$$
$$m_B = I_B N_B \pi (20)^2$$

Now $m_A = m_B$

$$I_A \cdot \pi N_A (100) = I_B N_B \pi 400$$
$$I_A N_A = 4 I_B N_B$$

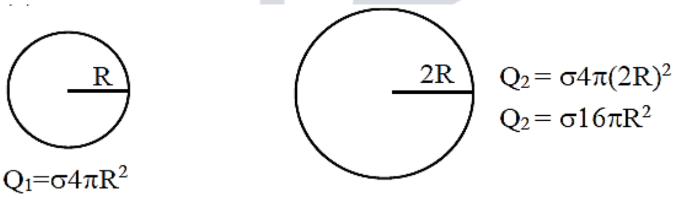
2. **(a)** Slope of curve P-t will represent the force so

$$F = \frac{dP}{dt} = \text{ slope}$$

Maximum slope $\rightarrow$ (c)

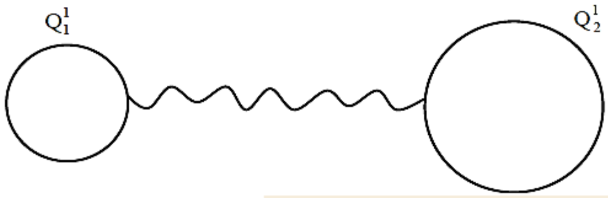
Minimum slope $\rightarrow$ (b)

3. **(c)**



$Q_1 = \sigma 4 \pi R^2$

Now



Charge will flow until voltage of both sphere become equal so

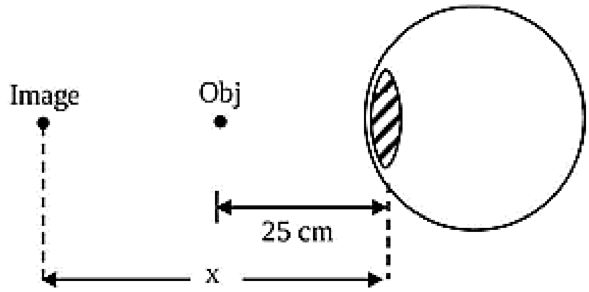
$$c = 4\pi\varepsilon_0 R$$
$$v_1{}' = v_2^1$$
$$\frac{Q_1}{c_1} = \frac{Q_2}{c_2} \Rightarrow \frac{Q_1{}'}{4\pi\varepsilon_0 R} = \frac{Q_2{}'}{4\pi\varepsilon_0 (2R)}$$
$$\Rightarrow 2Q_1{}' = Q_2{}' \ \ldots (1)$$
$$Q_1 + Q_2 = Q_1{}' + Q_2{}'$$
$$\sigma 20 \pi R^2 = Q_2{}' + \frac{Q_2{}'}{2} = \frac{3}{2} Q_2{}' \Rightarrow Q_2{}' = \frac{\sigma 40 \pi R^2}{3}$$
$$Q_2{}' = \frac{\sigma 40 \pi R^2}{3}$$

Now $\sigma' 4\pi(2R)^2 = \frac{\sigma 40\pi R^2}{3}$

$$\sigma' 16\pi R^2 = \frac{\sigma 40\pi R^2}{3}$$
$$\frac{\sigma'}{\sigma} = \frac{40}{3} \times \frac{1}{16} = \frac{5}{6}$$

4. **(d)** Power convex lens $= 2.0D$

Power of concave lens $= -1.0D$



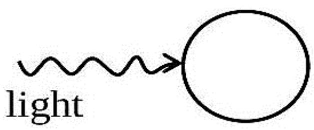
$x \rightarrow$ least distance of distinct vision
$f = \frac{1}{2} \times 100 = 50$ cm

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$
$$\frac{1}{50} = \frac{1}{(-x)} - \frac{1}{-25} \Rightarrow \frac{1}{50} - \frac{1}{25} = \frac{1}{(-x)}$$
$$\Rightarrow \frac{1-2}{50} = \frac{-1}{x}$$
$$\Rightarrow X = 50 \text{ cm}$$

5. **(b)** Power $= 20$mw

$t = 300$nsec

energy absorbed $= 300 \times 10^{-9} \times 20 \times 10^{-3} = 6 \times 10^{3} \times 10^{-12} = 6 \times 10^{-9}$ J



Pressure $= \frac{\text{Intensity}}{C} = \frac{\text{Power}}{\text{Area} \times C}$

Pressure $\times$ Area $= \frac{\text{Power}}{C}$

Force $= \frac{\text{Power}}{C} = \frac{20 \times 10^{-3}}{3 \times 10^{8}}$

$F = \frac{20}{3} \times 10^{-11}$ N

$F\Delta t = \Delta P$ (momentum)

$\frac{20}{3} \times 10^{-11} \times 300 \times 10^{-9} = P_f - P_i$

$20 \times 10^{-20} \times 100 = P_f$

$2 \times 10^{-17} = P_f$

6. (d)

(A) $x \propto t^2$

$\frac{dx}{dt} \propto 2t \Rightarrow V \propto t$ $A \to II$

(B) $x = x_0 e^{-\alpha t}$

$\frac{dx}{dt} = x_0 e^{-\alpha t}(-\alpha) = -\alpha(x_0 e^{-\alpha t})$

$V = -\alpha x_0 e^{-\alpha t}$ $B \to IV$

(C) $x \propto t \to V = \text{const}$

$x \propto -t \to V = -$ const $C \to III$

(D) $x \propto t \to V = \text{const}$ $D \to I$

7. (d) $PT^2 = \text{const.}$

$dV = V\gamma dT$

$\gamma = \frac{1}{V}\frac{dV}{dT}$

Using $PV = nRT$ and $PT^2 = \text{cont.}$

$\frac{nRT}{V} \cdot T^2 = \text{const}$

$V \propto T^3 \Rightarrow V = KT^3$

Now put in (a)

$\gamma = \frac{1}{KT^3} \times 3KT^2 = \frac{3}{T} \Rightarrow \gamma = \frac{3}{T}$

8. (a)

In isothermal process

$\Delta T = 0$

So $\Delta U = 0$

$\Delta Q = \omega + \Delta U$
$\Delta Q = \omega$

So heat will be used to do positive work

9. **(c)** $\Delta V = -\int_2^3 \vec{E} \cdot d\vec{r}$

$$V(c) - V(b) = -\int_2^3 \frac{-K}{r^2} \cdot dr$$

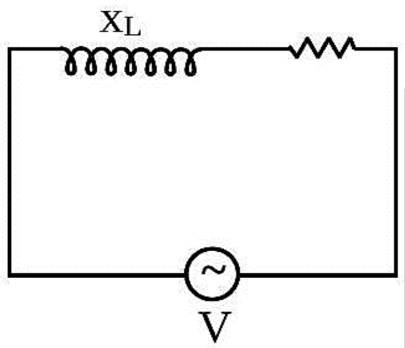
$$V(c) - 10 = -K\left(\frac{1}{r}\right)_2^3$$

$$V(c) - 10 = -6\left[\frac{1}{3} - \frac{1}{2}\right]$$

$$V - 10 = -6\left[\frac{2-3}{6}\right] = 1$$

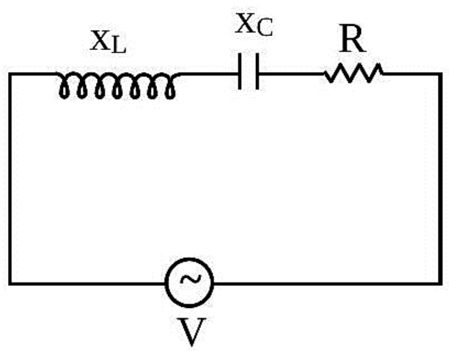
$$V = 11$$

10. **(c)**



Power factor $= \cos\phi = \frac{R}{Z}$

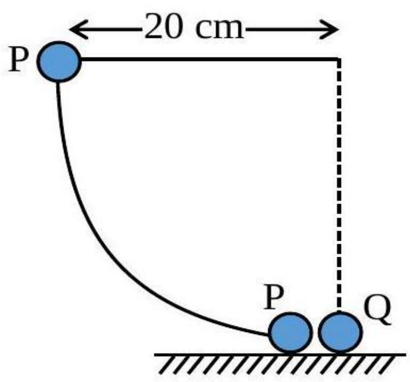
$$P_1 = \frac{R}{\sqrt{x_L^2 + R^2}} = \frac{R}{\sqrt{R^2 + R^2}} = \frac{R}{\sqrt{2}R} = \frac{1}{\sqrt{2}}$$



$$P_2 = \frac{R}{Z} = \frac{R}{\sqrt{(x_L - x_C)^2 + R^2}} = \frac{R}{R} = 1$$

$$\frac{P_1}{P_2} = \frac{1}{\sqrt{2}}$$

11. **(c)**

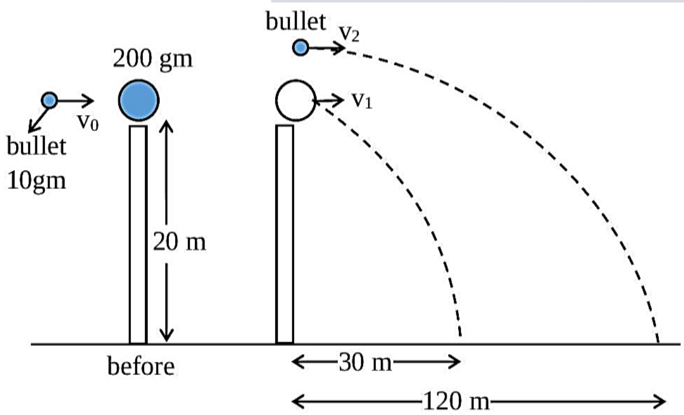


Energy conservation for ' P '

$$\mathrm{mgh} = \frac{1}{2}\mathrm{mV}^2$$
$$\mathrm{V} = \sqrt{2\mathrm{gh}}$$
$$\mathrm{V} = \sqrt{2 \times 10 \times 0.2}$$
$$\mathrm{V} = 2\ \mathrm{m/sec}$$

Now collision between P and Q is elastic and both have same mass then P will transfer all velocity to then Q. So velocity Q will be $2\ \mathrm{m/sec}$

12. (a)



Time to reach ground will be same for both

$$t = \sqrt{\frac{2\,h}{g}} = \sqrt{\frac{2 \times 20}{10}} = 2\sec$$

Range of bullet $= 120$

$120 = v_2(b) \Rightarrow v_2 = 60\ \mathrm{m/sec}$

Range of ball $= 30$

$30 = V_1(b) \Rightarrow v_1 = 15\ \mathrm{m/sec}$

Now apply momentum conservation

$P_i = P_f$

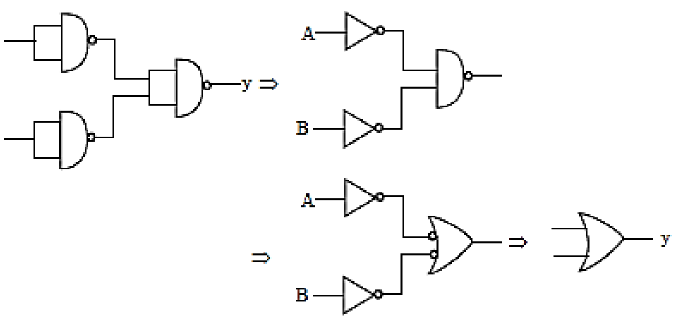
$P_{ball} + P_{bullet} = P_{ball} + P_{bullet}$

$0 + \left(\frac{10}{1000}\right) v_0 = \left(\frac{200}{1000}\right)(15) + \left(\frac{10}{1000} \times 60\right)$

$10 v_0 = 3000 + 600$

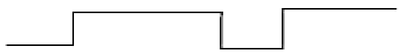
$v_0 = \frac{3600}{10} \Rightarrow v_0 = 360 \text{ m/sec}$

13. (c)



$y = A + B$

A	0	1	1	0	0	1	0
B	0	0	1	1	0	0	1
y	0	1	1	1	0	1	1



14. (c)

$Q = \alpha t - \beta t^2 + \gamma t^3$

$I = \frac{dQ}{dt} = \alpha - 2\beta t + 3\gamma t^2$

$\frac{dI}{dt} = 0 = 0 - 2\beta + 6\gamma t \Rightarrow t = \frac{2\beta}{6\gamma} = \frac{\beta}{2\gamma}$

$I_{min} = \alpha - 2\beta\left(\frac{\beta}{3\gamma}\right) + 3\gamma\left(\frac{\beta}{3\gamma}\right)^2$

$= \alpha - \frac{2\beta^2}{3\gamma} + \frac{\beta^2}{3\gamma}$

$I_{min} = \alpha - \frac{\beta^2}{3\gamma}$

15. (b)

$Y = 2\eta[1 + \sigma]$

and $Y = 3\,K[1 - 2\sigma]$

Now $2\eta(1 + \sigma) = 3\,K(1 - 2\sigma)$

$2\eta\sigma + 2\eta = 3\,K - 6\,K\sigma$

$(2n + 6\,K)\sigma = 3\,K - 2n$

$\sigma = \frac{3K - 2\eta}{2\eta + 6K}$

16. (d) We now

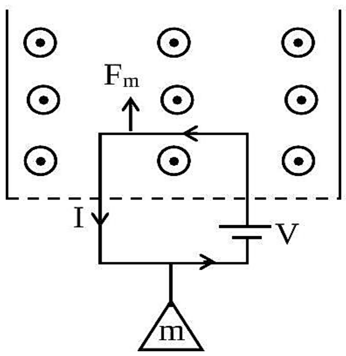
$V \propto \frac{z}{n}$

$$\frac{V_3}{V_7}=\frac{7}{3}$$

$$V_3=V_7\times\frac{7}{3}=3.6\times10^6\times\frac{7}{3}=1.2\times7\times10^6$$

$$V_3=8.4\times10^6\ \mathrm{m/s}$$

17. (c)



For balancing $\rightarrow l=10$ cm,

$B=10^3\mathrm{G}=0.1\ \mathrm{T}$,

$m=1\ \mathrm{g}$

$F_m=mg$

$I\ell B=mg$

$$\frac{V}{R}(0.1)(0.1)=\frac{1}{1000}\times10$$

$$\frac{V}{100R}=\frac{1}{100}$$

$$\frac{V}{10}=1\Rightarrow V=10\mathrm{Volt}$$

18. (b)

$$\vec{E}=\frac{A}{x^2}\hat{\imath}+\frac{B}{y^3}\hat{\jmath}$$

Unit of A $\rightarrow\frac{N}{C}\times m^2=Nm^2c^{-1}$

Unit of B $\rightarrow\frac{N}{C}\times m^3=Nm^3c^{-1}$

19. (a) $h=\frac{2\,T\cos\theta}{rgg}$

$$h\propto\frac{T}{\rho}$$

$$\frac{h_2}{h_1}=\frac{T_2}{T_1}\times\frac{\rho_1}{\rho_2}$$

$$\frac{h_2}{5\text{ cm}}=\frac{2\text{ T}}{\text{T}}\times\frac{\rho}{2\rho}=1$$

$h_2=5\text{ cm}=0.05\text{ m}$

20. (c) $V_{max}=V_m+V_c$

$120=V_c+V_m$

$V_{min}=V_c-V_m$

$80=V_c-V_m$

(a) + (b)

$200=2\,V_c\Rightarrow V_c=100$

$V_M=120-100=20\Rightarrow V_M=20$

$$\mu=\frac{V_m}{V_c}=\frac{20}{100}=0.2$$

Amplitude of side bond $=\frac{\mu A_c}{2}=0.2\times\frac{100}{2}=10\text{ V}$

21. (8) $x=A\sin(\omega t)$

Potential energy $U=\frac{1}{2}kx^2$

$$U=\frac{1}{2}\cdot K\cdot A^2\sin^2(\omega t)$$

$$\frac{dU}{dt}=\frac{KA^2}{2}\cdot 2\sin(\omega t)\cos(\omega t)\cdot\omega$$

Slope $=\frac{dU}{dt}=\frac{\omega KA^2}{2}\sin(2\omega t)$

→ Slope will be maximum for $\sin(2\omega t)$ will maximum

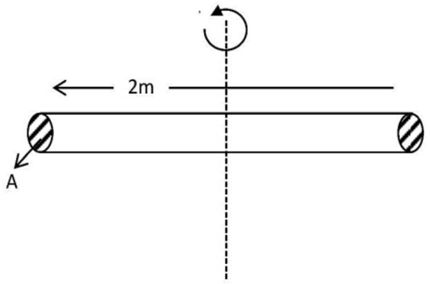
$$2\omega t=\frac{\pi}{2}$$

$$2\omega\cdot\frac{T}{\beta}=\frac{\pi}{2}$$

$$2\frac{2\pi}{T}\times\frac{T}{\beta}=\frac{\pi}{2}\Rightarrow\beta=8$$

Ans. = 8

22. (3)



density $= \mathrm{d}$

Area $= \mathrm{A}$

mass $\mathrm{m} = \mathrm{d}.\mathrm{A}\ell$

$\mathrm{m} = \mathrm{dA(b)} = 2\mathrm{Ad}$

K.E. $= \frac{1}{2} \mathrm{I}\, \omega^2$

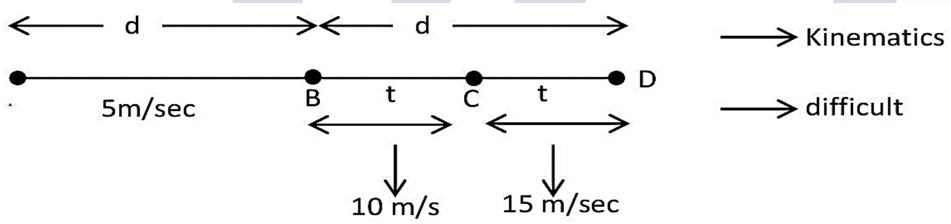
$$E = \frac{1}{2} \cdot \frac{m\ell^2}{12} \cdot \omega^2$$

$$E = \frac{1}{24} \cdot 2Ad \cdot (2)^2 \omega^2$$

$$\frac{24E}{8Ad} = \omega^2 \Rightarrow \omega = \sqrt{\frac{3E}{Ad}}$$

Ans. $\alpha = 3$

23. (50)



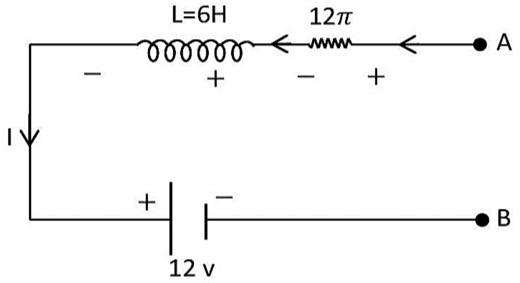
Avg. speed from B to D $\rightarrow V_{BD} = \frac{10+15}{2} = \frac{25}{2}$ m/sec

Now, $\frac{2}{V_{ay}} = \frac{1}{5} + \frac{2}{25}$

$$\frac{2}{V_{ag}} = \frac{7}{25} \Rightarrow V_{ag} = \frac{50}{7}$$

Ans. $x = 50$

24. (30)



$I = 2\text{ A}$

$\frac{dl}{dt} = -1\text{ A/sec}$

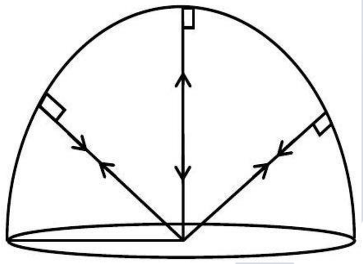
$V_A - IR - L\frac{dl}{dt} - 12 = V_B$

$V_A - 2(12) + 6(a) - 12 = V_B$

$V_A - V_B = 24 + 12 - 6 = 24 + 6 = 30$

Ans. 30

25. (4)



$\theta = 0^\circ$

Presses due reflecting surface $= \frac{2l}{C}$

Net force $= \frac{21}{C}$ Area

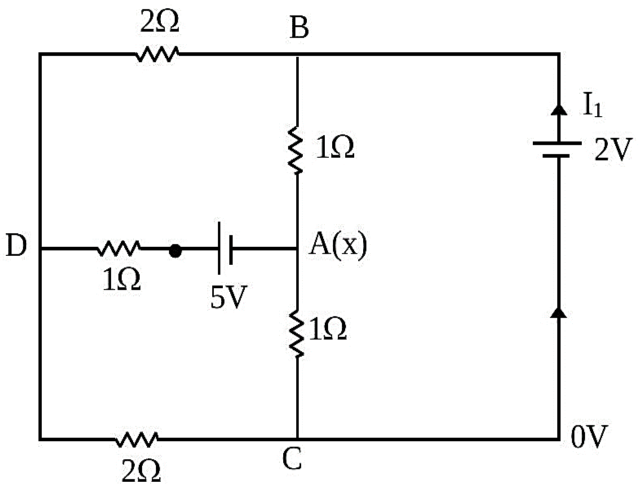
Now $I = \frac{\text{Power}}{\text{Area}} = \frac{\text{Power}}{4\pi r^2}$

From $F_{net} = \frac{2l}{C} \times$ Projected Area

$F_{net} = \frac{2}{C} \times \frac{\text{Power}}{4\pi r^2} \times \pi r^2$

$F_{net} = \frac{2 \times 24}{3 \times 10^8 \times 4} = 4 \times 10^{-8}$

26. (1.5)



Let at junction A → voltage = x

$V_A = x$

$V_D = y$

$V_C = 0$

$V_B = 2$

At junction 'A'

$$\frac{x-2}{1}+\frac{x-0}{1}+\frac{x+5-y}{1}=0$$

$3x - y + 3 = 0$

At junction ' D '

$$\frac{y-0}{2}+\frac{y-2}{2}+\frac{y-x-5}{1}=0$$

$4y - 2x = 12$

$2y - x = 6$

From (a) and (b)

$x = 0; y = 3$

So curent through 2 V cell is

$$I=\frac{3}{2}=1.5\text{ A}$$

27. (220) Pitch = 0.5 mm

$$\text{L.C.}=\frac{\text{pitch}}{\text{circular division}}=\frac{0.5\text{ mm}}{100}=0.005\text{ mm}$$

Zero error $= 6 \times$ L.C. $= 6 \times (0.005)$mm

Reading = main linear scale reading +n(L.C.) − zero error

$= 4(0.5\text{ mm}) + 46(0.005) - 6(0.005)$

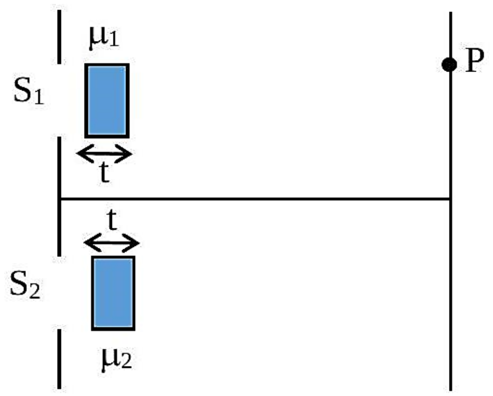
$= 2\text{ mm} + 40 \times 0.005\text{ mm}$

$= 2\text{ mm} + \frac{200}{1000}\text{ mm}$

$= 2.2\text{ mm}$

Rading $= 220 \times 10^{-2}$ mm

28. (10)



$\mu_1 = 1.51\ t = 0.1$ mm

$\mu_2 = 1.55\ \lambda = 4000\text{Å}$

Shifting central maxima
$\Delta x = [S_1P + (\mu_1 - 1)t] - [S_2P + (\mu_2 - 1)t]$

$0 = (S_1P - S_2P) + (\mu_1 - 1)t - (\mu_2 - 1)t$

$0 = \frac{yd}{D} + (\mu_1 - \mu_2)t$

$(\mu_2 - \mu_1)t = \frac{yd}{D}$

$(1.55 - 1.51)(0.1\text{ mm}) = y \times \frac{d}{D}$

$\frac{D}{d}(0.04 \times 0.1) \times 10^{-3} = y$

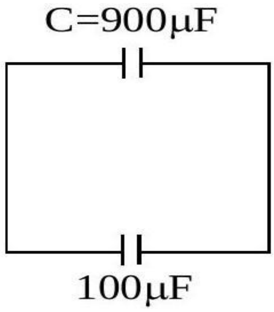
Now

Fringe width $\Rightarrow \beta = \frac{\lambda D}{d}$

No. of fringes shifted $= \frac{y}{\beta} = \frac{4\times10^{-6}}{4000\text{Å}} = 10$

Ans. 10

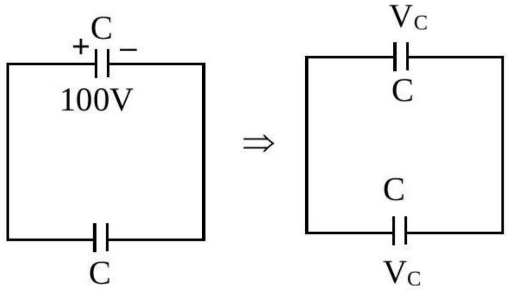
29. (225)



$Q = 900 \times 100 \mu C$

$Q = 9 \times 10^{-2} C$

Now



$$\Delta U = \frac{1}{2}\frac{C_1 \cdot C_2}{C_1 + C_2}(V_1 - V_2)^2$$
$$= \frac{1}{2} \times \frac{C \times C}{2C} \times (100 - 0)^2$$
$$= \frac{C}{4} \times 100 \times 100$$
$$= \frac{900}{4} \times 10^{-6} \times 10^4$$
$$= \frac{9}{4} = 2.25 \text{ J}$$
$$\Delta U = 225 \times 10^{-2} \text{ J}$$

30. (32)

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u} \quad dv = du = \frac{1 \text{ cm}}{20} = 0.05 \text{ cm (given)}$$
$$f^{-1} = v^{-1} + u^{-1}$$
$$(-1)f^{-2}df = (-1)v^{-2}dv - u^{-2}du$$
$$\frac{df}{f^2} = \frac{dv}{v^2} + \frac{du}{u^2}$$
$$\frac{1}{f} = \frac{1}{(-120)} + \frac{1}{-40}$$
$$\frac{1}{f} = \frac{1+3}{(-120)} = \frac{4}{-120} \Rightarrow f = -30 \text{ cm}$$

Put value of f, du, dv in (a)

$$\frac{df}{(30)^2} = \frac{0.05}{(120)^2} + \frac{0.05}{(40)^2}$$

$df = \frac{1}{32}$ cm so $K = 32$

31. (b) $8LiH + Al_2Cl_6 \rightarrow 2LiAlH_4 + 6LiCl$
32. (d) Ranitidine is an antacid it is an antihistamine and decrease the reaction of gastric juice in stomach
33. (c)
34. (c)

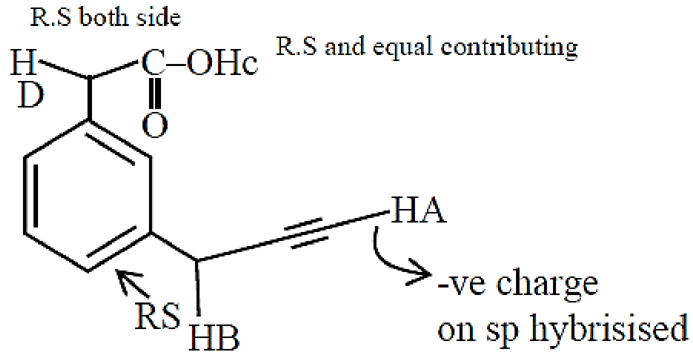
37 (K) s-block

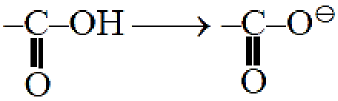
78 (pt) d-block

52 (Te) p-block

65(Tb) f-block

35. (d)





Equal contributing and resonance stablize So order $H_C > H_D > H_A > H_B$

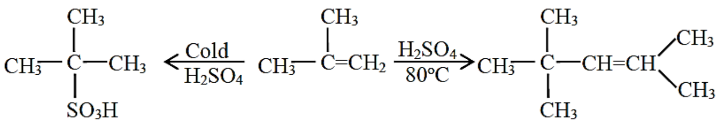
36. (d) fehling test gives positive result for aliphatic aldehyde While sodium nitroprasside gives blood red color with S and N.

SoNa + N + C + S → NaSCN (Sodium thiocyanate)

$SCN^- + Fe^{3+} \rightarrow [Fe(SCN)]^{2+}$ Ferric thiocyanate (Blood red color)

Confims presence of N and S

37. (b)



38. (a)

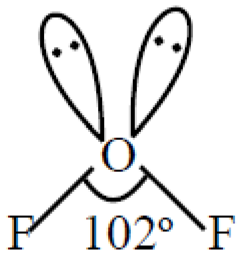
$$Na_2SO_3 + HCl \rightarrow NaCl + H_2O + SO_2 \uparrow$$
$$K_2Cr_2O_7 + H_2SO_4 + SO_2 \rightarrow {}_2\; K_2SO_4 + \underset{\text{green}}{Cr_2(SO_4)_3} + H_2O$$

39. (d)

$Zn^{+2}, CO^{+2}, Ni^{+2}$, IVth group

Fe^{+3} = IIIrd group

40. (c) OF_2

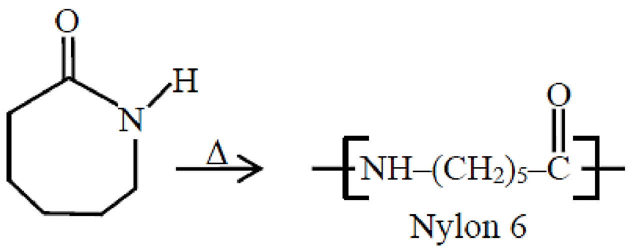


2 l.pe$^-$ in 'O'

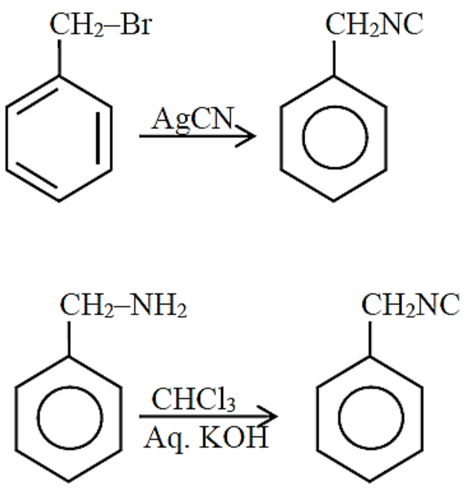
bond angle

102° bent/V shape

41. (b)



42. (d)



43. (c)

$$NO_2 \xrightarrow{h\upsilon} \underset{(A)}{NO} + \underset{(B)}{O}$$

$$O \xrightarrow{O_2} O_3$$

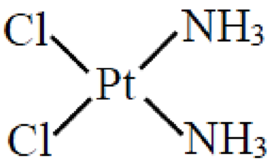
44. (b) Seliwanoff's test - Test to differentiate for ketose and aldose.

In this keto hexose are more rapidly dehydrated to form 5-hydroxy methyl furfural when heated in acidic medium which on condensation with resorcinol, as result brown red colored complex is formed.

45. (d)

Molecule	l.pe of C.M. $^{-}$
IF_7	o
ICl_4^-	2
XeF_6	1
XeF_2	3

46. (a) Cis plating



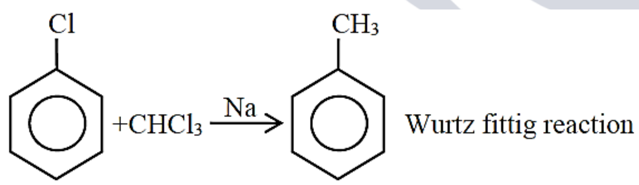
is used as Anticancer agent

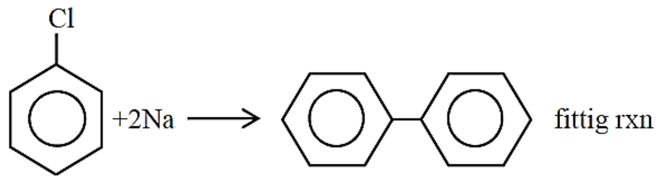
47. (b) $BeSO_4$&$MgSO_4$ are soluble in water
$CaSO_4$ is partially soluble
$SrSO_4$&$BaSO_4$ is insoluble

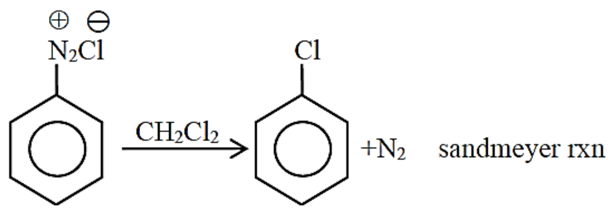
48. (c) order of ligand strength

$S^{2-} < C_2O_4^{2-} < NH_3 < en < CO$

49. (a)

(A) 

(B) 

(C) 

(D) $C_2H_5Cl + NaI \rightarrow C_2H_5I + NaCl$ Finkelstein rxn

50. (a) The copper ore contains iron, it is mixed with silica before heating in reverberatory furnace, feO of slags off as $FeSiO_3$

$FeO + SiO_2 \rightarrow FeSiO_3$

51. (186)

$$[H^+]_{mix} = \frac{(600 \times 0.01) + (400 \times 0.01 \times 2)}{1000}$$

$$= \frac{6+8}{1000} = 14 \times 10^{-3}$$

$pH = -\log(14 \times 10^{-3})$

$= 3 - \log 2 - \log 7$

$= 3 - 0.30 - 0.84$

$pH = 1.86$

52. (789)

$E_{photon} = 6.626 \times 10^{-34 \times 2 \times 10^{12} \times 6.023 \times 10^{23}}$

$= 79.81 \times 10$

$= 798.1 \approx 798$

53. (10)

Cell reaction:-

$H_2 + 2Fe^{3+} \rightarrow 2H^+ + 2Fe^{2+}$

$$E_{cell} = 0.771 - \frac{2.303RT}{2\ F} \log \frac{[Fe^{2+}]^2[H^+]^2}{[Fe^{3+}]^2}$$

$0.712 = 0.771 - 0.03\log(x)^2$

$$\frac{0.059}{2}\log(x)^2 = 0.059$$

$\log x = 1$

$$x = \frac{[Fe^{2+}]}{[Fe^{3+}]} = 10$$

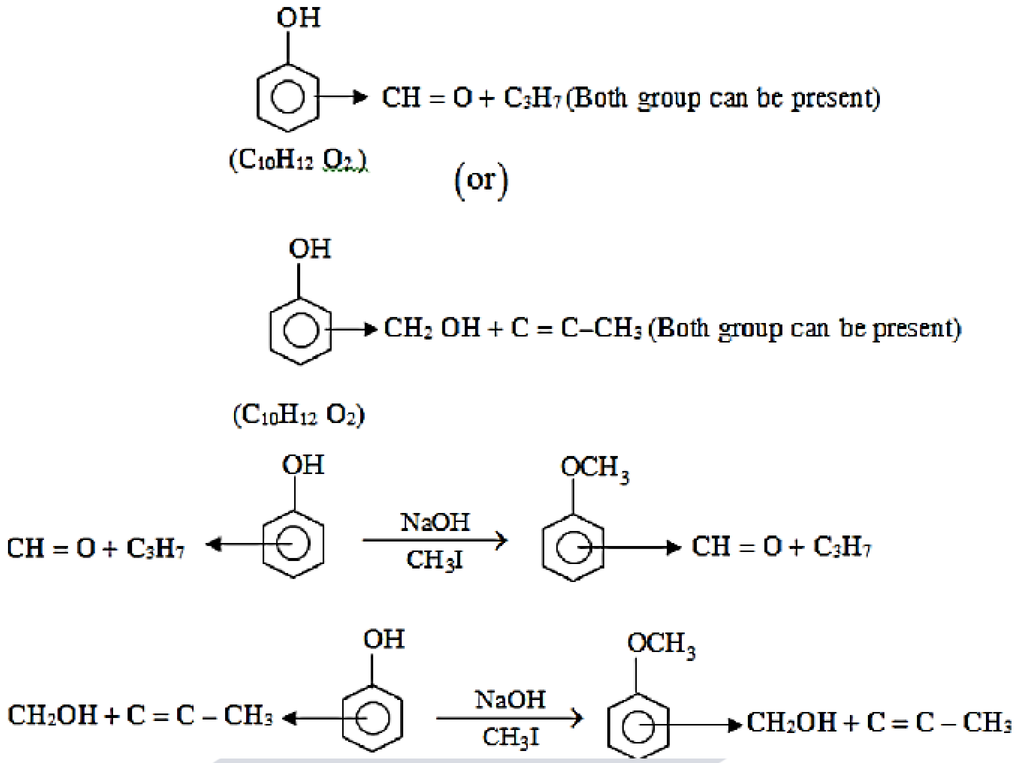
54. (3) $4H^+ + MnO_4^- + 3e^- \rightarrow MnO_2 + 2H_2O$
55. (1362)

Mole of dissolved $CO_2 = 0.2 \times 300 = 60mmol$

$V_{CO_2} = 60 \times 10^{-3} \times 22.7$

$= 1362ml$

56. (4)



57. (623)

For Ist order: -

$$t = \frac{1}{2.011 \times 1^{-3}} \times 2.303 \times \log\frac{7}{2}$$

$$= \frac{2.303 \times (0.845 - 0.301)}{2.011 \times 10^{-3}}$$

$$= 622.9 \approx 623$$

58. (100)

$$\Delta T_b = 373.52 - 373 = 0.52$$

$$\Delta T_b = iK_b m \; i = 1$$

$$0.52 = 0.52 \times \frac{2/x}{20} \times 1000$$

$$x = 100\text{gm/mol}$$

59. (0)

$$\Delta U = 0$$

process is Isothermal

60. (148.322)

Molar mass $= 12 + 2 + 71$

$= 85$

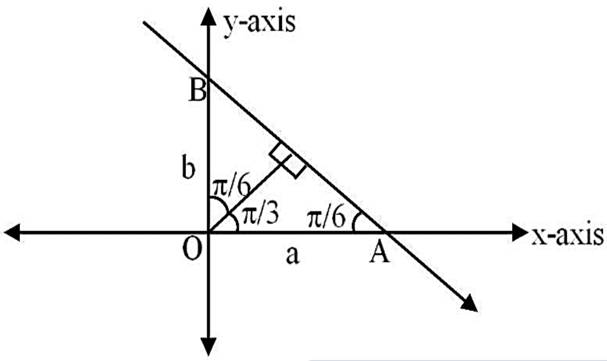
mmoles of DCM $= 671.141 \times 2.6 \times 10^{-3}$

mass of solution $= 1.49 \times 671.141$

PPM $= \frac{671.141 \times 2.6 \times 10^{-3} \times 85 \times 10^{-3}}{1.49 \times 671.141} \times 10^6$

148.322

61. (a)



In $\triangle$ AOB

$\tan\frac{\pi}{6} = \frac{OB}{OA} = \frac{b}{a}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{b}{a}$

$\Rightarrow a = \sqrt{3}\, b$

$\because$ area of triangle $\triangle$ OAB $= \frac{1}{2} \times ab = \frac{98}{3} \times \sqrt{3}$

$\Rightarrow \frac{\sqrt{3}\, b^2}{2} = \frac{98}{\sqrt{3}}$

$\Rightarrow b^2 = \frac{98}{3} \times 2$

$\Rightarrow b = \sqrt{\frac{196}{3}}$

$a = \sqrt{196}$

$a^2 - b^2 = 196 - \frac{196}{3} = \frac{588 - 196}{3}$

$\Rightarrow a^2 - b^2 = \frac{392}{3}$

62. (d) $R = \{(a, b), (b, c)\}$

For symmetric relation (b, a), (c, b) must be added in R

For transitive relation (a, c), (a, a), (b, b), (c, c), (c, a) must be added in R

So, minimum number of element = 7

63. (d) Unbiased die. Marked with $-2,-1,0,1,2,3$

Product of outcomes is positive if

All time get positive number, 3 time positive and 2 time negative, 1 time positive and 4 time negative.

P (Product of the outcomes is positive) = $\underbrace{{}^5C_5\left(\frac{3}{6}\right)^5}_{\text{All positive}}+\underbrace{{}^5C_3\left(\frac{3}{6}\right)^3\left(\frac{2}{6}\right)^2}_{\text{3positive, ,negative}}+\underbrace{{}^5C_1\left(\frac{3}{6}\right)\left(\frac{2}{6}\right)^4}_{\text{1positive, 4 negative}}$

$$=\frac{3^5}{6^5}+\frac{10\times3^3\times2^2}{6^5}+\frac{5\times3\times2^4}{6^5}$$

$$=\frac{1563}{6^5}=\frac{521}{2592}$$

64. (d)

$\vec{a}=\alpha\vec{b}-\hat{n},\vec{b}\cdot\vec{c}=12$

$\vec{c}\times(\vec{a}\times\vec{b})=(\vec{c}\cdot\vec{b})\vec{a}-(\vec{c}\cdot\vec{a})\vec{b}$

$\vec{c}\times(\vec{a}\times\vec{b})=12\vec{a}-(\vec{c}\cdot\vec{a})\vec{b}$

$\because\vec{a}=\alpha\vec{b}-n$

$\vec{c}.\vec{a}=\alpha\vec{c}.\vec{b}-\vec{c}.n$

$\overline{c.\vec{a}}=12\alpha$

Equation (b) put in equation (a)

$\vec{c}\times(\vec{a}\times\vec{b})=12\vec{a}-12\alpha\vec{b}$

$|\vec{c}\times(\vec{a}\times\vec{b})|=12|\vec{a}-\alpha\vec{b}|$ $[\because\vec{a}-\alpha\vec{b}=-n$ then $|\vec{a}-\alpha\vec{b}|=1]$

$\Rightarrow|\vec{c}\times(\vec{a}\times\vec{b})|=12$

65. (c)

66. (b) Equation of normal of the parabola $x=4y^2$

At a point $P\left(\frac{t^2}{16},\frac{2t}{16}\right)$ is

$y+tx=\frac{2t}{16}+\frac{1}{16}t^3$

$\because$ Normal pass through $Q(0,33)$ then

$$33=\frac{t}{8}+\frac{t^3}{16}$$
$$\Rightarrow t^3+2t-528=0$$
$$\Rightarrow (t-8)(t^2+8+166)=0$$
$$\Rightarrow t=8$$

Point P is $(4,1)$

Given parabola is $y^2=4(x+y)$

$y^2-4y=4x$

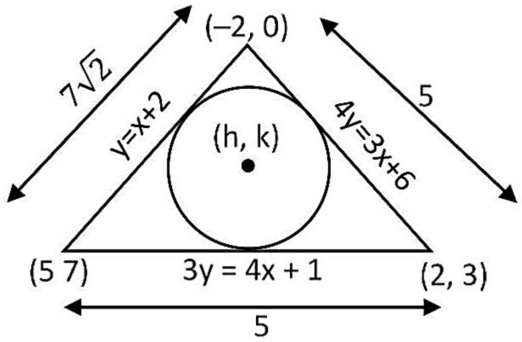
$(y-2)^2=4(x+1)$

directrix is $x+1=-1$

$x=-2$

Distance of $P(4,1)$ from the directrix $x=-2$ is 6.

67. (d)



In centre of triangle is (h,k)

$$=\left(\frac{5(-2)+2\times 7\sqrt{2}+5\times 5}{5+5+7\sqrt{2}},\frac{3\times(7\sqrt{2})+0\times 5+7\times 5}{5+5+7\sqrt{2}}\right)$$
$$=\left(\frac{14\sqrt{2}+15}{10+7\sqrt{2}},\frac{21\sqrt{2}+35}{10+7\sqrt{2}}\right)$$

So, $h+k=\frac{14\sqrt{2}+15}{10+7\sqrt{2}}+\frac{21\sqrt{2}+35}{10+7\sqrt{2}}$

$$h+k=\frac{35\sqrt{2}+50}{7\sqrt{2}+10}=\frac{5(7\sqrt{2}+10)}{7\sqrt{2}+10}=5$$

$\Rightarrow h+k=5$

68. (a) Given curve is $y=54x^5-135x^4-70x^3+180x^2+210x$

$$\frac{dy}{dx}=270x^4-540x^3-210x^2+360x+210$$

$\because$ Normal is parallel to $x+90y+2=0$

Then tangent is $\perp^r$ to $x+90y+2=0$

Then $(270x^4 - 540x^3 - 210x^2 + 360x + 210)\left(\frac{-1}{90}\right) = -1$

$270x^4 - 540x^3 - 210x^2 + 360x + 120 = 0$

$\Rightarrow 9x^4 - 18x^3 - 7x^2 + 12x + 4 = 0$

$\Rightarrow (x-1)(x-2)(3x+1)(3x+2) = 0$

$\Rightarrow x = 1, 2, -\frac{1}{3}, -\frac{2}{3}$

Number of points are 4

69. (c) given that $a_n = \frac{-2}{4n^2 - 16n + 15}$

$a_1 + a_2 + a_3 + \cdots . a_{25} = \sum_{n=1}^{25} \frac{-2}{(2n-3)(2n-5)}$

$= \sum_{n=1}^{25} \frac{(2n-5)-(2n-3)}{(2n-3)(2n-5)}$

$= \sum_{n=1}^{25} \left(\frac{1}{2n-3} - \frac{1}{(2n-5)}\right)$

$= \frac{1}{-1} - \frac{1}{-3}$

$+ \frac{1}{1} - \frac{1}{-1}$

$+ \frac{1}{3} - \frac{1}{1}$

$\vdots \quad \vdots$

$\frac{1}{47} - \frac{1}{45}$

$= \frac{1}{47} + \frac{1}{3}$

$= \frac{3+47}{141} = \frac{50}{141}$

70. (d)

$$\tan 15^\circ + \frac{1}{\tan 75^\circ} + \frac{1}{\tan 105^\circ} + \tan 195^\circ = 2a$$
$$\Rightarrow \tan 15^\circ + \frac{1}{\cot 15^\circ} - \frac{1}{\cot 15^\circ} + \tan 15^\circ = 2a$$
$$\Rightarrow \tan 15^\circ + \tan 15^\circ - \tan 15^\circ + \tan 15^\circ = 2a$$
$$\Rightarrow 2\tan 15^\circ = 2a$$
$$\Rightarrow a = \tan 15^\circ$$
$$a + \frac{1}{a} = \tan 15^\circ + \frac{1}{\tan 15^\circ}$$
$$= \tan 15^\circ + \cot 15^\circ$$
$$= 2 - \sqrt{3} + 2 + \sqrt{3}$$
$$\Rightarrow a + \frac{1}{a} = 4$$

71. (c)

$$\log_{\cos x} \cot x + 4\log_{\sin x} \tan x = 1, x \in \left(0, \frac{\pi}{2}\right)$$
$$\Rightarrow \log_{\cos x} \frac{\cos x}{\sin x} + 4\log_{\sin x} \frac{\sin x}{\cos x} = 1$$
$$\Rightarrow 1 - \log_{\cos x} \sin x + 4 - 4\log_{\sin x} \cos x = 1$$
$$\Rightarrow 4 = \log_{\cos x} \sin x + 4\log_{\sin x} \cos x$$

Let $\log_{\cos x} \sin x = t$

$$\Rightarrow 4 = t + \frac{4}{t}$$
$$\Rightarrow t^2 - 4t + 4 = 0$$
$$\Rightarrow (t-2)^2 = 0$$
$$\Rightarrow t = 2$$
$$\Rightarrow \log_{\cos x} \sin x = 2$$
$$\Rightarrow \sin x = \cos^2 x$$
$$\Rightarrow \sin x = 1 - \sin^2 x$$
$$\Rightarrow \sin^2 x + \sin x - 1 = 0$$

$$\Rightarrow \sin x = \frac{-1 \pm \sqrt{1+4}}{2}$$

$\Rightarrow \sin x = \frac{-1+\sqrt{5}}{2}, \frac{-1-\sqrt{5}}{2}$ $\because$ $x \in \left(0, \frac{\pi}{2}\right)$ then $\frac{-1-\sqrt{5}}{2}$ not possible

$$\Rightarrow x = \sin^{-1}\left(\frac{-1+\sqrt{5}}{2}\right)$$

$\because \alpha = -1, \beta = 5$ then

$\alpha + \beta = 4$

72. (c)

$x + y + kz = 2$

$2x + 3y - z = 1$

$3x + 4y + 2z = k$

Have Infinitely many solution then

$$\begin{vmatrix} 1 & 1 & k \\ 2 & 3 & -1 \\ 3 & 4 & 2 \end{vmatrix} = 0$$

$1(10) - 1(7) + k(8 - 9) = 0$

$\Rightarrow 10 - 7 - k = 0$

$\Rightarrow k = 3$

For $k = 3$

$4x + 5y = 7$

$7x + 8y = 10$

has unique solution and solution is $(-2,3)$.

Hence solution is unique and satisfying $x + y = 1$

73. (d) equation of l_1 is $\frac{x-2}{2} = \frac{y-6}{1} = \frac{z-2}{-2}$

Let l_2 is $\frac{x+1}{2} = \frac{y+4}{-3} = \frac{z}{2}$

Point on l_1 is $a = (2,6,2)$, direction $\vec{p} =< 2,1,-2 >$

Point on l_2 is $b = (-1,-4,0)$ direction $\vec{q} =< 2,-3,2 >$

Shortest distance between l_1 and l_2 $= \left|\frac{\vec{a}-\vec{b})\cdot(\vec{p}\times\vec{q})}{|\vec{p}\times\vec{q}|}\right|$

$$\because \vec{p} \times \vec{q} = \begin{vmatrix} \hat{i} & \hat{j} & k \\ 2 & 1 & -2 \\ 2 & -3 & 2 \end{vmatrix} = \hat{i}(-4) - \hat{j}(8) + k(-8)$$

$$= \left|\frac{\langle 3,10,2\rangle \cdot \langle -4,-8,-8\rangle}{\sqrt{16 + 64 + 64}}\right|$$

$$= \left|\frac{-12 - 80 - 16}{\sqrt{144}}\right|$$

$$= \frac{108}{12}$$

$$= 9$$

Shortest distance between the lines is 9

74. (d)

$A = \begin{bmatrix} m & n \\ p & q \end{bmatrix}, d = |A| = mq - np$

$A - d(\text{Adj}.A) = \begin{bmatrix} m & n \\ p & q \end{bmatrix} - d\begin{bmatrix} q & -n \\ -p & m \end{bmatrix}$

$= \begin{bmatrix} m - dq & n + dn \\ p + pd & q - dm \end{bmatrix}$

$|A - d(\text{Adj } A)| = (m - dq)(q - dm) - (n + dn)(p + pd) = 0$

$\Rightarrow mq - m^2 d - dq^2 + d^2 qm = np(1 + d)^2$

$\Rightarrow (mq - m^2 d - dq^2 + d^2 qm) = (mq - d)(1 + d)^2$

$\Rightarrow mq - m^2 d - dq^2 + d^2 qm = mq + mqd^2 + 2mqd - d(1 + d)^2$
$\Rightarrow d(1 + d)^2 = m^2 d + dq^2 + 2mqd$
$\Rightarrow (1 + d)^2 = (m + q)^2$

75. (c) $\frac{3(e-1)^2}{e}\int_1^2 x^2 e^{[x]+[x^3]}dx$

Let $I = \int_1^2 x^2 e^{[x]+[x^3]}dx$

$I = \int_1^2 x^2 e^{1+[x^3]}dxdx$

$\Rightarrow I = e\int_1^2 x^2 e^{[x^3]}dx$

Let $x^3 = t$

$3x^2 dx = dt$

$I = \frac{e}{3}\int_1^8 e^{[t]d}$

$\Rightarrow I = \frac{e}{3}\left[\int_1^2 edt + \int_2^3 e^2 dt + \int_3^4 e^3 dt + \cdots .. + \int_7^8 e^7 dt\right]$

$\Rightarrow I = \frac{e}{3}[e + e^2 + e^3 + \cdots .. + e^7]$

$\Rightarrow I = \frac{e}{3}\left[\frac{e(e^7 - 1)}{e - 1}\right]$

Therefore $\frac{3(e-1)}{e}\int_1^2 x^2 e^{[x]+[x^3]}dx = \frac{3(e-1)}{e} \times \frac{e^2}{3}\frac{(e^7-1)}{e-1}$

$\Rightarrow \frac{3(e - 1)}{e}\int_1^2 x^2 e^{[x]+[x^3]}dx = e^8 - e$

76. (c)$\because \overrightarrow{OP}$ makes angle α, β, γ with positive directions of the co-ordinate axes then $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$.

Point on planes are $a(1,2,3), b(2,3,4)$ and $c(1,5,7)$.

$\because \overrightarrow{ab} =< 1,1,1 >$

$\overrightarrow{ac} = \langle 0,3,4 >$

normal vector of plane $= \begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 0 & 3 & 4 \end{bmatrix}$

$= \hat{i}(a) - \hat{j}(d) + \hat{k}(c)$

$=< 1, -4,3 >$

direction cosine of normal is $= \left\langle \pm\frac{1}{\sqrt{26}}, \pm\frac{4}{\sqrt{26}}, \pm\frac{3}{\sqrt{26}} \right\rangle$

then direction cosine of $\overrightarrow{op}$ is $\left\langle -\frac{1}{\sqrt{26}}, \frac{4}{\sqrt{26}}, -\frac{3}{\sqrt{26}} \right\rangle$

$\left(\because \beta \in \left(0, \frac{\pi}{2}\right)\right)$

Hence $\alpha \in \left(\frac{\pi}{2}, \pi\right)$ and $\gamma \in \left(\frac{\pi}{2}, \pi\right)$

77. (b)

$$x^0(1+x)^{500} + x(1+x)^{499} + x^2(1+x)^{498} + \cdots + x^{500}$$
$$= (1+x)^{500} \frac{\left[\left(\frac{x}{1+x}\right)^{501} - 1\right]}{\frac{x}{1+x} - 1}$$
$$= \frac{(1+x)^{500}(x^{501} - (1+x)^{501})}{(1+x)^{501}\left(\frac{-1}{x+x}\right)}$$
$$= (1+x)^{501} - x^{501}$$

Coefficient of x^{301} in above expression is ${}^{501}C_{301}$ or ${}^{501}C_{200}$.

78. (d) $\left(\frac{dy}{dx}\right) - \frac{3x^5\tan^{-1}(x^3)}{(1+x^6)^{3/2}} y = 2x\exp\left(\frac{x^3 - \tan^{-1} x^3}{\sqrt{(1+x^6)}}\right)$

above equation is linear differential equation.

$$\text{I.F.} = e^{\int \frac{-3x^5\tan^{-1}(x^3)}{(1+x^6)^{3/2}} dx}$$

$$= e^{-\int \frac{3x^2 \cdot x^3 \tan^{-1}(x^3)}{(1+x^6)^{3/2}} dx}$$

Let $\tan^{-1}(x^3) = t$ then

$$\frac{3x^2 \cdot dx}{1+x^6} = dt$$

$$= e^{-\int \frac{t\tan t}{\sqrt{1+\tan^2 t}} dt}$$

$$= e^{-\int \frac{t\tan t}{\sec t} dt}$$

$$= e^{-\int t\sin t\,dt}$$

$$= e^{-[-t\cos t + \sin t]}$$

$$= e^{t\cos t - \sin t}$$

$$\text{I.F.} = e^{\frac{\tan^{-1} x^3}{\sqrt{1+x^6}} - \frac{x^3}{\sqrt{1+x^6}}}$$

Solution is

$$y\left(e^{\frac{\tan^{-1}x^3}{\sqrt{1+x^6}}-\frac{x^3}{\sqrt{1+x^6}}}\right)=\int 2xe^{\left(\frac{x^3-\tan^{-1}x^3}{\sqrt{1+x^6}}\right)}\cdot e^{\frac{\tan^{-1}x^3-x^3}{\sqrt{1+x^6}}}dx$$

$$y\left(e^{\frac{\tan^{-1}x^3-x^3}{\sqrt{1+x^6}}}\right)=\int 2xdx=x^2+c$$

above eq. is passing through $(0,0)$ then $c=0$

$$y=x^2e^{\frac{x^3-\tan^{-1}x^3}{\sqrt{1+x^6}}}$$

Put $x=1$ then

$$y(a)=e^{\frac{1-\frac{\pi}{4}}{\sqrt{2}}}=e^{\frac{4-\pi}{4\sqrt{2}}}$$

$$\Rightarrow y(a)=\exp\left(\frac{4-\pi}{4\sqrt{2}}\right)$$

79. (b) $\left(ax^3+\frac{1}{bx^{1/3}}\right)^{15}$

general term is $T_{r+1}={}^{15}C_r(ax^3)^{15-r}\left(\frac{1}{bx^{1/3}}\right)^r$

$$\Rightarrow T_{r+1}={}^{15}C_r\frac{a^{15-r}}{b^r}x^{45-3r}-\frac{r}{3}$$

For coefficient of $x^{15}\Rightarrow 45-3r-\frac{r}{3}=15$

$$30=\frac{10r}{3}$$

$r=9$

Coefficient of x^{15} is $={}^{15}C_9a^6\,b^{-9}$

$\because$ general term of $\left(ax^{1/3}-\frac{1}{bx^3}\right)^{15}$ is

$$T_{r+1}={}^{15}C_r\left(ax^{1/3}\right)^{15-r}\left(\frac{-1}{bx^3}\right)^r$$

For coefficient of $x^{-15}\Rightarrow\frac{15-r}{3}-3r=-15$

$\Rightarrow 15-r-9r=-45$

$\Rightarrow 60=10r$

$r=6$

Coefficient of x^{-15} is $={}^{15}C_6a^9\,b^{-b}$

$\because$ both coefficient are equal then

$${}^{15}C_9a^6\,b^{-9}={}^{15}C_6a^9\,b^{-6}$$

$\Rightarrow a^6 b^{-9} = a^9 b^{-6}$

$\Rightarrow a^3 b^3 = 1$

$\Rightarrow ab = 1$

80. (c)

$f: R \to (0, \infty)$
$5f(x+y) = f(x) \cdot f(y)$
Put $x = 3, y = 0$ then
$5f(c) = f(c)f(0)$
$\Rightarrow f(0) = 5$

Put $x = 1, y = 1$ then $5f(b) = f^2(a)$

Put $x = 1, y = 2$ then $5f(c) = f(a)f(b)$

$$5 \times 320 = \frac{f^3(a)}{5} = f(a) = 20$$
$\Rightarrow f(b) = 80$

Put $x = 2, y = 2$ then $5f(d) = f(b)f(b)$

$$f(d) = \frac{80 \times 80}{5} = 1280$$

Put $x = 2, y = 3$ then $5f(5) = f(b) \cdot f(b)$

$$f(5) = \frac{80 \times 320}{5} = 5120$$

$$\sum_{n=0}^{5} F(n) = f(0) + f(a) + \cdots . + f(5)$$
$$= 5 + 20 + 80 + 320 + 1280 + 5120$$
$$= 5(1 + 2^2 + 2^4 + 2^6 + 2^8 + 2^{10})$$
$$= 6825$$

81. (9)

$z = 1 + i,\ \bar{z} = 1 - i,\ i\bar{z} = 1 + i$

$$z_1 = \frac{1 + \bar{z}}{\bar{z}(1 - z) + \frac{1}{z}}$$

$$z_1 = \frac{i + z}{\bar{z} - z\bar{z}) + \frac{1}{z}}$$

$$z_1 = \frac{i + 2}{1 - i - 2 + \frac{1 - i}{2}}$$

$$z_1 = \frac{i + 2}{-\frac{1}{2} - \frac{3i}{2}}$$

$$Z_1 = \frac{-2(i + 2)}{(1 + 3i)} \times \frac{(1 - 3i)}{(1 - 3i)}$$

$z_1 = \frac{-2(5-5i)}{10}$

$z_1 = -1 + i$

$\arg.(z_1) = \pi - \tan^{-1}\left(\frac{1}{1}\right) = \pi - \frac{\pi}{4} = \frac{3\pi}{4}$

$\therefore \arg(z_1) = \frac{12}{\pi} \times \frac{3\pi}{4} = 9$

82. (315)

Plane $P_1 : \vec{r} \cdot (3\hat{i} - 5\hat{j} + \hat{k}) = 7$

$P_2 : \vec{r} \cdot (\lambda\hat{i} + \hat{j} - 3k) = 9$

angle between plane is same as angle between their normal.

angle between normal θ then

$\operatorname{Cos}\theta = \frac{\langle 3,-5,1\rangle \cdot \langle \lambda, 1, -3\rangle}{\sqrt{9+25+1}\sqrt{\lambda^2+1+9}}$

$\operatorname{Cos}\theta = \frac{3\lambda - 5 - 3}{\sqrt{35}\sqrt{\lambda^2+10}}$

$\because\ \theta = \sin^{-1}\left(\frac{2\sqrt{6}}{5}\right)$ then

$\sin\theta = \frac{2\sqrt{6}}{5}$

$\cos\theta = \frac{1}{5}$

from equation (a)

$\frac{3\lambda - 8}{\sqrt{35}\sqrt{\lambda^2+10}} = \frac{1}{5}$

$\Rightarrow \frac{(3\lambda-8)^2}{35(\lambda^2+10)} = \frac{1}{25}$

$\Rightarrow 5(3\lambda-8)^2 = 7(\lambda^2+10)$

$\Rightarrow 5(9\lambda^2 - 48\lambda + 64) = 7\lambda^2 + 70$

$\Rightarrow 38\lambda^2 - 240\lambda + 250 = 0$

$\Rightarrow 19\lambda^2 - 120\lambda + 125 = 0$

$\Rightarrow \lambda = 5, \frac{25}{19}$

$\lambda_1 = \frac{25}{19}, \lambda_2 = 5$

Point $(38\lambda_1, 10\lambda_2, 2) \equiv (50,50,2)$

distance of $(50,50,2)$ from plane P_1 is

$$d = \left|\frac{3 \times 50 - 5 \times 50 + 2 - 7}{\sqrt{9 + 25 + 1}}\right|$$

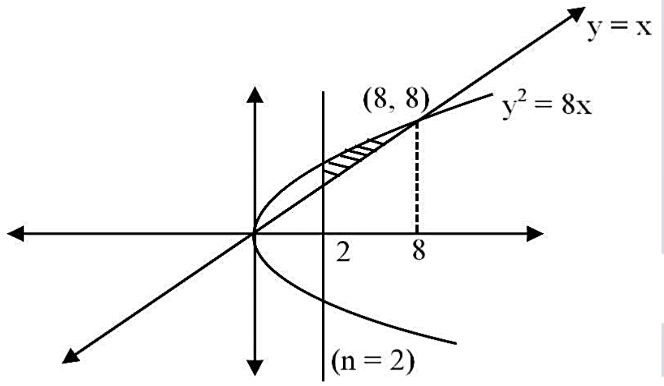
$$d = \left|\frac{150 - 250 + 2 - 7}{\sqrt{35}}\right|$$

$$d = \left|\frac{105}{\sqrt{35}}\right|$$

$$d = 3\sqrt{35}$$

$$d^2 = 315$$

83. (22) area $(\alpha) = \int_2^8 (2\sqrt{2}\sqrt{x} - x)dx$



$$= \left[2\sqrt{2} \cdot \frac{2}{3}x^{3/2} - \frac{x^2}{2}\right]_2^8$$

$$= \frac{4\sqrt{2}}{3}[8 \times 2\sqrt{2} - 2\sqrt{2}] - 30$$

$$= \frac{28 \times 4}{3} - 30$$

$$= \frac{112}{3} - 30$$

$$\alpha = \frac{22}{3}$$

$$3\alpha = 22$$

84. $(\mathbf{26})$

Let $\sum_{n=0}^{\infty} \frac{n^3((2n)!)+(2n-1)n!}{(n!)((2n)!)}$

$$= \sum_{n=0}^{\infty} \frac{n^3(2n)!}{n!\,(2n)!} + \frac{(2n-1)n!}{n!\,(2n)!}$$

$= S_1 + S_2$

Let $S_1 = \sum_{n=0}^{\infty} \frac{n^3(2n)!}{n!(2n)!} = \sum_{n=0}^{\infty} \frac{n^3}{n!} = \sum_{n=1}^{\infty} \frac{n^2}{(n-1)!}$

$= \sum_{n=1}^{\infty} \frac{n^2 - 1 + 1}{(n-1)!}$

$= \sum_{n=2}^{\infty} \frac{(n+1)}{(n-2)!} + \sum_{n=1}^{\infty} \frac{1}{(n-1)!}$

$= \sum_{n=2}^{\infty} \frac{(n-2)+3}{(n-2)!} + \sum_{n=1}^{\infty} \frac{1}{(n-1)!}$

$= \sum_{n=3}^{\infty} \frac{1}{(n-3)!} + 3\sum_{n=2}^{\infty} \frac{1}{(n-2)!} + \sum_{n=1}^{\infty} \frac{1}{(n-1)!}$

$S_1 = e + 3e + e = 5e$

$\because S_2 = \sum_{n=0}^{\infty} \frac{(2n-1)n!}{n!\,(2n)!}$

$= \sum_{n=0}^{\infty} \frac{2n-1}{(2n)!}$

$= \sum_{n=1}^{\infty} \frac{1}{(2n-1)!} - \sum_{n=0}^{\infty} \frac{1}{(2n)!}$

$= \left(\frac{1}{1!} + \frac{1}{3!} + \frac{1}{5!} + \cdots.\right) - \left(1 + \frac{1}{2!} + \frac{1}{4!} + \cdots..\right)$

$= -1 + \frac{1}{1!} - \frac{1}{2!} + \frac{1}{3!} - \frac{1}{4!} + \frac{1}{5!} - \cdots.$

$= -\left(1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} + \cdots.\right)$

$= -e^{-1}$

$S_1 + S_2 = 5e - \frac{1}{e} = ae + \frac{b}{e} + c$

Compare both side

$a = 5, b = -1, c = 0$

$a^2 - b + c = 25 + 1 + 0 = 26$

85. (15)

give line is $x - 3y + 2z - 1 = 0 = 4x - y + z$

Direction of line $\vec{a} = \begin{vmatrix} \hat{i} & \hat{j} & k \\ 1 & -3 & 2 \\ 4 & -1 & 1 \end{vmatrix} = \hat{i}(-1) - \hat{j}(-7) + k(11)$

$\Rightarrow \vec{a} =< -1,7,11 >$

. Line is $\perp^r$ to the plane then direction of line is parallel to normal of plane.

$\vec{n} = < -1, 7, 11 >$

Equation of plane is

$-1(x-1) + 7(y-1) + 11(z-2) = 0$

$-x + 7y + 11z + 1 - 7 - 22 = 0$

$\Rightarrow \ -x + 7y + 11z = 28$

$\Rightarrow \ -\frac{1}{28}x + \frac{7}{28}y + \frac{11}{28}z = 1$

$A = -\frac{1}{28}, B = \frac{7}{28}, C = \frac{11}{28}$

$140(C - B + A) = 140\left(\frac{11}{28} - \frac{7}{28} - \frac{1}{28}\right)$

$= \frac{140 \times 3}{28} = 15$

86. (21)
87. (3125)

$f^1(x) = \frac{3x+2}{2x+3}, x \in R - \left\{-\frac{3}{2}\right\}$
$f^2(x) = f^1 0 f^1(x) = f^1\left(\frac{3x+2}{2x+3}\right)$

$= \frac{3\left(\frac{3x+2)}{2x+3)}\right) + 2}{2\left(\frac{3x+2}{2x+3}\right) + 3}$

$= \frac{9x + 6 + 4x + 6}{6x + 4 + 6x + 9}$

$f^2(x) = \frac{13x + 12}{12x + 13}$

$f^3(x) = f^1 o f f^2(x)$

$= f^1\left(\frac{13x+12}{12x+13}\right)$

$= \frac{3\left(\frac{13x+12}{12x+13}\right) + 2}{2\left(\frac{13x+12}{12x+13}\right) + 3}$

$= \frac{39x + 36 + 24x + 26}{26x + 24 + 36x + 39}$

$f^3(x) = \frac{63x + 62}{62x + 63}$

$$f^4(x) = f^1\left(\frac{63x + 62}{62x + 63}\right)$$

$$= \frac{3\left(\frac{63x + 62}{62x + 63}\right) + 2}{2\left(\frac{63x}{62x} + \frac{62}{63}\right) + 2}$$

$$f^4(x) = \frac{313x + 312}{312x + 313}$$

$$f^5(x) = f^1\left(\frac{313x + 312}{312x + 313}\right)$$
$$= \frac{3\left(\frac{313x + 312}{312x + 313}\right) + 2}{2\left(\frac{313x + 312}{312x + 313}\right) + 3}$$

$$f^5(x) = \frac{1563x + 1562}{1562x + 1563}$$

$\because$ $a = 1563, b = 1562$

$a + b = 3125$

88. (37) Mean of 7 observations $= \frac{\sum_{i=1}^{7} x_i}{7}$

$\Rightarrow \sum_{i=1}^{7} x_i = 7 \times 8 = 56$

Variance $= \frac{\Sigma x_i^2}{n} - (\overline{x})^2$

$\Sigma x_i^2 = 7(16 + 64) = 560$

If 14 is removed then

Mean $= a = \frac{\sum_{i=1}^{7} x_i - 14}{6} \Rightarrow 6a = 56 - 14$

$\Rightarrow a = 7$

Variance $= b = \frac{\sum_{i=1}^{7} x_i^2 - (14)^2}{6} - 49$

$\Rightarrow 6b = 560 - 196 - 294$

$\Rightarrow 6b = 70$

$\Rightarrow 3\,b = 35$

$\therefore a + 3b - 5 = 7 + 35 - 5 = 37$

89. (3240)

Case - I

$f(6) = S$ i.e. 1 option

$f(5) =$ any 5 element subset A of S i.e. ${}^{6}C_5 = 6$ options

$f(d) =$ any 4 element subset B of A i.e. ${}^{5}C_4 = 5$ options

$f(c) =$ any 3 element subset C of B i.e. ${}^{4}C_3 = 4$ options

$f(b) =$ any 2 element subset D of C i.e. ${}^{3}C_2 = 3$ options

$f(a) =$ any 1 element subset E of D or empty subset i.e. 3 options

Total function $= 6 \times 5 \times 4 \times 3 \times 2 \times 3 = 1080$

Case - II

$f(6) = S$

$f(5) =$ any 4 element subset A of S i.e. ${}^{6}C_4 = 15$ options

$f(d) =$ any 3 element subset B of A i.e. ${}^{4}C_3 = 4$ options

$f(c) =$ any 2 element subset C of B i.e. ${}^{3}C_2 = 2$ options

$f(b) =$ any 1 element subset D of C i.e. ${}^{2}C_1 = 2$ options

$f(a) =$ empty subset i.e. 1 options

Total function $= 15 \times 4 \times 3 \times 2 \times 1 = 360$

Case - III

$f(6) = S$

$f(5) =$ any 5 element subset A of S i.e. ${}^{6}C_5 = 6$ options

$f(d) =$ any 3 element subset B of A i.e. ${}^{5}C_3 = 10$ options

$f(c) =$ any 2 element subset C of B i.e. ${}^{3}C_2 = 3$ options

$f(b) =$ any 1 element subset D of C i.e. ${}^{2}C_1 = 2$ options

$f(a) =$ empty subset i.e. 1 options

Total function $= 6 \times 10 \times 3 \times 2 \times 1 = 360$

Case - IV

$f(6) = S$

$f(5) =$ any 5 element subset A of S i.e. ${}^{6}C_5 = 6$ options

$f(d) =$ any 4 element subset B of A i.e. ${}^{5}C_4 = 5$ options

$f(c) =$ any 2 element subset C of B i.e. ${}^{4}C_2 = 6$ options

$f(b) =$ any 1 element subset D of C i.e. ${}^{2}C_1 = 2$ options

$f(a) = $ empty subset i.e. 1 options

Total function $= 6 \times 5 \times 6 \times 2 \times 1 = 360$

Case - V

$f(6) = S$

$f(5) = $ any 5 element subset A of S i.e. $^6C_5 = 6$ options

$f(d) = $ any 4 element subset B of A i.e. $^5C_4 = 5$ options

$f(c) = $ any 3 element subset C of B i.e. $^4C_3 = 4$ options

$f(b) = $ any 1 element subset D of C i.e. $^3C_1 = 3$ options

$f(a) = $ empty subset i.e. 1 options

Total function $= 6 \times 5 \times 4 \times 3 \times 1 = 360$

Case - VI

$f(6) = $ any 5 element subset A of S i.e. $^6C_5 = 6$ options

$f(5) = $ any 4 element subset B of A i.e. $^5C_4 = 5$ options

$f(d) = $ any 3 element subset C of B i.e. $^4C_3 = 4$ options

$f(c) = $ any 2 element subset D of C i.e. $^3C_2 = 3$ options

$f(b) = $ any 1 element subset E of D i.e. $^2C_1 = 2$ options

$f(a) = $ empty subset i.e. 1 options

Total function $= 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$

Total number of such functions $= 1080 + (4 \times 360) + 720 = 3240$

90. (12) $\lim_{x\to 0} \frac{48}{x^4}\int_0^x \frac{t^3}{t^6+1} = \left(\frac{0}{0}\right)$ form

Using L Hopital Rule

$$= \lim_{x\to 0} \frac{48 \times \frac{x^3}{x^6+1}}{4x^3}$$

$$= \lim_{x\to 0} \frac{48}{4(x^6+1)}$$

$$= \frac{48}{4}$$

$$= 12$$