1. **(c)** $F \propto I_1 I_2$

$$\frac{F_1}{F_{2I}} = \frac{1}{4}$$

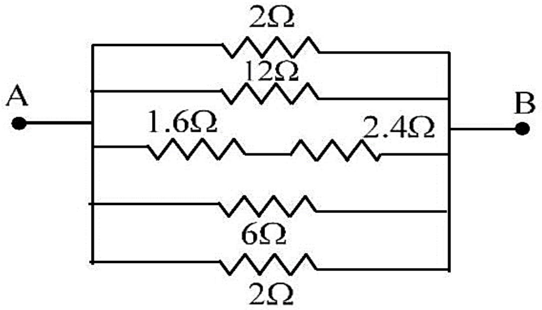
2. **(c)**

3. **(b)** $\eta = 1 - \frac{T_L}{T_H} = \frac{T_H - T_L}{T_H}$

Efficiently of Carnot's engine will be highest at $-273^{\circ} = 0\ K$

4. **(c)** Electric field inside material of conductor is zero

5. **(d)**



$$\frac{1}{R_{eq}} = \frac{1}{2} + \frac{1}{12} + \frac{1}{4} + \frac{1}{6} + \frac{1}{2}$$

$$= \frac{18}{12} = \frac{3}{2}$$

$$R_{eq} = \frac{2}{3}\Omega$$

6. **(d)** $\frac{2}{V_{av}} = \frac{1}{3} + \frac{1}{5} = \frac{8}{15}$

$$V_{av} = \frac{15}{4} = 3.75\ \text{kmhr}^{-1}$$

Ans. (d)

7. **(b)** $Z = \sqrt{R^2 + (X_L - X_e)^2}$

$$Z = \sqrt{(100)^2 + (200 - 100)^2}$$

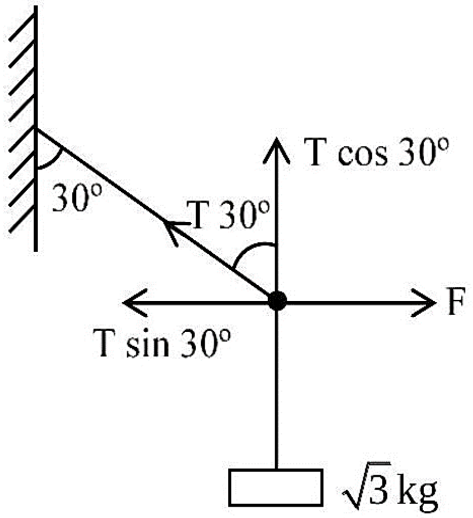
$$Z = 100\sqrt{2}\Omega$$

$$I_{rms} = \frac{V_{rms}}{Z} = \frac{200\sqrt{2}}{100\sqrt{2}} = 2\ A$$

8. **(d)** $I_{EF} = \frac{1}{2} \times \frac{5}{4\pi(5)^2}$

$$= \frac{1}{40\pi}\ w/m^2$$

9. **(a)**



$F = T\sin 30^{\circ}$

$\sqrt{3}\ g = T\cos 30^{\circ}$

$\tan 30^{\circ} = \frac{F}{\sqrt{3}} g$

$\frac{1}{\sqrt{3}} = \frac{F}{\sqrt{3}\ g}$

$F = 10\ N$

$T = \frac{F}{\sin 30^{\circ}} = 10 \times 2$

$T = 10 \times 2 = 20\ N$

10. (c)

11. (d) $KE = \frac{P^2}{2\ m}$

$P = \frac{h}{\lambda}$

$eV_1 = \frac{\left(\frac{h}{\lambda}\right)^2}{2\ m}$

$eV_2 = \frac{\left(\frac{h}{1.5\lambda}\right)^2}{2\ m}$

$\frac{V_1}{V_2} = (1.5)^2 = \frac{9}{4}$

12. (b) Average kinetic energy per molecule $= \frac{5}{2} KT$

Ratio $= \frac{1}{1}$

13. (c) $d\tan 60^\circ = 2\sqrt{3}$

$d = 2\ \text{cm}$

$$B = 3\left(\frac{\mu_0 I}{2\pi d}\right)\sin 60^\circ$$

$$B = \frac{3\times 2\times 10^{-7}\times 2}{2\times 10^{-2}}\times\frac{\sqrt{3}}{2}$$

$$B = 3\sqrt{3}\times 10^{-5}\ \text{T}$$

14. (d) Use work energy theorem

$\Delta KE = w_g$

$$\frac{1}{2}mv^2 - 0 = -[u_f - u_i]$$

$$\frac{1}{2}mv^2 = -\left[-\frac{GMm}{R} - \left(-\frac{GMm}{2R}\right)\right]$$

$$\frac{1}{2}mv^2 = \frac{GMm}{R} - \frac{GMm}{2R}$$

$$= \frac{GMm}{R}\left(\frac{2-1}{2}\right)$$

$$\frac{1}{2}mv^2 = \frac{GMm}{2R}$$

$$V = \sqrt{\frac{GM}{R}}$$

$$V = \sqrt{gR}(GM = gR^2)$$

15. (a) Torque $= N - m$

$$= kg\frac{m}{sec^2}\ m$$

$$= \frac{kgm}{sec^2}$$

Energy Density $= \frac{N-m}{m^3} = \frac{N}{m^2}$

$$= kg\frac{m}{sec^2}\times\frac{1}{m^2}$$

Pressure gradient $= \frac{\text{Pr essure}}{\text{length}} = \frac{F}{A-\text{length}}$

$$= kgm^{-2}sec^{-2}$$

Impulse $= \Delta P = kgm - s^{-1}$

16. (a) Nuclear density is independent of A

17. (d) $Y = \frac{F\ell}{A\Delta\ell}$

$$F = \frac{YA\Delta\ell}{\ell}$$

$$\left(\frac{A\Delta\ell}{\ell}\right)_1 = \left(\frac{A\Delta\ell}{\ell}\right)_2$$

$$\frac{\Delta\ell_2}{\Delta\ell_1} = \frac{A_1}{A_2} \times \frac{\ell_2}{\ell_1}$$

$$\frac{(\Delta\ell)_2}{0.2} = \frac{1}{2.4 \times 2.4} \times \frac{2}{1}$$

$$(\Delta\ell)_2 = 6.9 \times 10^{-2}\ \text{mm}$$

18. (c) $\delta_1 = \delta_2$ [For no deviation]

$$6(1.54 - 1) = A(1.72 - 1)$$

$$A = \frac{18}{4} = 4.5^\circ$$

19. (b) $20 \times 10^{-3} \times \frac{180}{60} \times 100 = 10\ \text{V}$

$$V = 0.6\ \text{ms}^{-1}$$

20. (a) $\omega = \sqrt{\frac{k}{m}}$

$$\frac{\omega_2}{\omega_1} = \sqrt{\frac{m_1}{m_2}} = \sqrt{\frac{1}{2}}$$

21. (54)

$$a = -\mu_k g = -3$$

$$v = u + at$$

$$v = 18 - 3 \times 2 = 12\ \text{ms}^{-1}$$

$$KE = \frac{1}{2}mvv^2 + \frac{1}{2}\left(\frac{mr}{2}\right)\left(\frac{v}{r}\right)^2$$

$$KE = \frac{3}{4}mv^2$$

$$KE = 3 \times 18 = 54\ \text{J}$$

22. (2) $\frac{2}{3} = \frac{\frac{x}{x+1}}{x}$

$$\frac{2}{3}=\frac{1}{x+1}$$

$$x=0.5=\frac{1}{2}$$

$$n=2$$

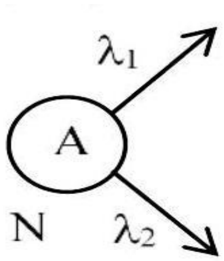
23. (125) $a=\omega^2 r$

$$a=\left(\frac{28\times 2\pi}{60}\right)^2\times 1.8$$

$$a=\frac{1936\times 1.8}{225}=\frac{1936}{125}\ ms^{-2}$$

$$x=125$$

24. (300)



$$\frac{dN}{dt}=-(\lambda_1+\lambda_2)N$$

$$\lambda_{eq}=\lambda_1+\lambda_2$$

$$\frac{1}{t_{1/2}}=\frac{1}{300}+\frac{1}{30}=\frac{11}{300}$$

$$t_{1/2}=\left(\frac{300}{11}\right)\sec$$

Ans: (300)

25. (313)

$$\frac{41^\circ-5^\circ}{95^\circ-5^\circ}=\frac{R-0}{100-0}$$

$$R=40^\circ C$$

$$R=313\ K$$

26. (1584)

$$E_{max}=NAB\omega$$

$$=100\times 14\times 10^{-2}\times 3\times\frac{360\times 2\pi}{60}$$

$= 1584\text{ V}$

Ans: (1584)

27. (4) $\frac{1}{2}\text{mvv}^2 = \text{pt}$

$$V = \sqrt{\frac{2pt}{m}}$$

$$\frac{dx}{dt} = \sqrt{\frac{2pt}{m}}$$

$$\int dx = \sqrt{\frac{2p}{m}} \int \sqrt{t}dt$$

$$x = \sqrt{\frac{2p}{m}} \left[t^{3/2}\right]_0^4$$

$$x = \frac{1}{3} \times 16\sqrt{p}$$

$\propto = 4$

28. (12) $\phi_{net} = -8 \times 3 + 2 \times 6$

$= -12$

$$\phi_{net} = \frac{q_{inside}}{\epsilon_0}$$

$\text{qinside} = -12\epsilon_0$

29. (3) $I = 4I_0 \cos^2 \frac{\phi}{2}$

$$\Delta\phi = \frac{2\pi}{\lambda} \times \Delta x$$

$$I_1 = 4I_0 \cos^2 \frac{\pi}{4} = 2I_0$$

$$I_2 = 4I_0 \cos^2 \frac{2\pi}{3} = I_0$$

$$\Rightarrow \frac{I_1 + I_2}{I_0} = 3$$

30. (88) $4v^2 = 50 - x^2$

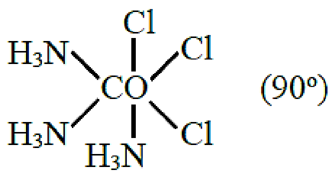
$$V = \frac{1}{2}\sqrt{50 - x^2}$$

$\omega = \frac{1}{2}$

$T = \frac{2\pi}{\omega} = 4\pi = \frac{88}{7}$

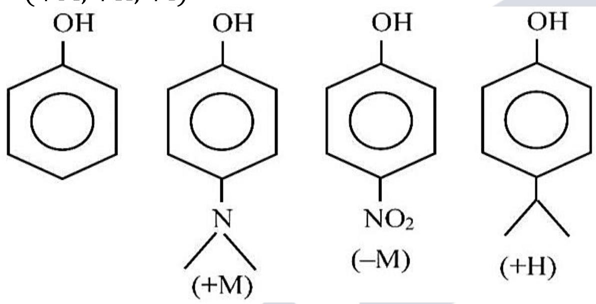
$x = (88)$

31. (a)



32. (c) Acidic strength $\propto (-M, -H, -I)$

$\propto \frac{1}{(+M, +H, +I)}$



$PKa \propto \frac{1}{\text{Acidic}}$ strength

A order of acidic strength: $c > a > d > b$

Order of PKa : $c < a < d < b$

33. (d) Mixture of CaF_2&Na_3AlF_6 decreasing the M.P. of Al_2O_3.

In electrolytic definning, pure metal is always deposited at the cathode

34. (b) A. $CHCl_3 + C_6H_5NH_2 \rightarrow$ Distillation (III)

B. $C_6H_{14} + C_5H_{12} \rightarrow$ fractional distillation (IV)

C. $C_6H_5NH_2 \rightarrow H_2O \rightarrow$ Steam distillation (I)

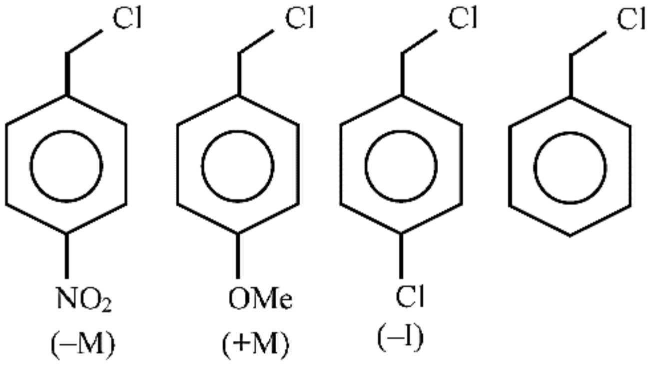
D. Organic compound in $H_2O \rightarrow$ Differential extraction (II)

35. (b)

$$\underset{0.01\,mol}{\left[Co(NH_3)_5SO_4\right]}Br + \underset{excess}{AgNO_3} \rightarrow \underset{0.01\,mol}{AgBr}\downarrow$$

$$\underset{0.02\,mol}{\left[Co(NH_3)_5Br\right]}SO_4 + \underset{excess}{BaCl_2} \rightarrow \underset{0.01\,mol}{BaSO_4}\downarrow$$

36. (b)



B > D > C > A

37. (d)

38. (d) Due to strong hydrogen bonding present in boric acid, boric acid present in solid form.

39. (c)

40. (b)

Nessler's reagent

$K_2HgI_4 + KOH$

41. (b)

42. (b) Max e^-that can be accommodated in shell $= 2n^2$

$(n = 4)$

$2(4)^2 = 32$

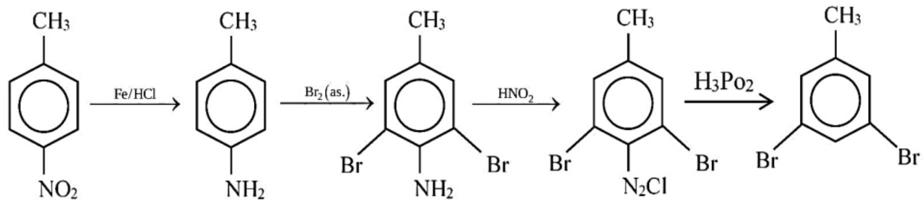
43. (d)

At node $\psi_{2\,s} = 0$

$$2 - \frac{r_0}{a_0} = 0$$

$r_0 = 2a_0$

44. (c)



45. (a)

Complex	Hyloridisation
(A) $Ni(CO)_4$	sp^3
(B) $[Cu(NH_3)_4]^{+2}$	dsp^2
(C) $[Fe(NH_3)_6]^{+2}$	d^2sp^3
(D) $[Fe(H_2O)_6]^{+2}$	$sp^3\ d^2$

46. (c)

47. (b) Due to smaller size, Be^{+2}. will show more polarising power, hence, Be will have maximum covalent character & most soluble in organic solvent.

48. (b)

$$2KMnO_4 + 10I^- + 16H^+ \rightarrow 2Mn^{+2} + 8H_2O + 5I_2$$

neutral/faintly alkaline sol ^{n}.

$$2MnO_4^- + I^- + H_2O \rightarrow 2MnO_2 + 2OH^- + IO_3^-$$

49. (c) $H_2O > H_2S > H_2Se > H_2Te$

50. (d) Clean water have BOD value less than 5ppm while highly polluted water have. BOD value of 17ppm or more.

51. (3)

52. (150)

$$q = 0$$

$$\Delta U = W = nCv\Delta T$$

$$= 1 \times 20 \times [T_2 - 300] = -3000$$

$$= T_2 - 300 = -150$$

$$= T_2 = 150 \text{ K}$$

53. (6)

54. (243)

$$\Delta T_f = i \cdot kf \cdot m$$

$$m = \frac{38}{98} \times \frac{1000}{62}$$

$$\Delta T_f = 2.67 \times 1.8 \times \frac{38}{98} \times \frac{1000}{62}$$

$$\Delta T_f = 30.05$$

$$\text{F.P.} = 273 - 30 = 243 \text{ K}$$

55. (150)

$$\text{Molarity} = \frac{\text{VolumeStrength}}{11.35}$$

$$\text{Strength (g/ lit)} = \text{Molarity} \times \text{mol. Wt}$$

$$= \frac{50}{11.35} \times 34 == 150\text{gm/lit}$$

56. (275)

$$x + y^{+2} \rightarrow y + x^{+2}$$

$$E^0 \text{ Cell} = E^0 \text{ Cathode} - E^o \text{ Anode}$$

E° Cell $= 0.36 - (-2.36) = 2.72$ V

$$E_{Cell} = 2.72 - \frac{0.06}{2}\log\frac{x^{+2}}{y^{+2}}$$

$$E_{Cell} = 2.72 - \frac{0.06}{2}\log\frac{0.001}{0.01}$$

$= 2.72 + 0.03 = 2.75$ V

$= 275 \times 10^{-2}$ V

57. (1350)

$$K = \frac{2.303}{540}\log\frac{100}{40}$$

$$K = \frac{2.303}{540} \times 0.4$$

$$t_{90} = \frac{2.303 \times 540}{2.303 \times 0.4}\log\frac{100}{10}$$

$t_{90} = 1350$

58. (3)

The yield of SO_3 at equilibrium will be due to:

B. Increasing pressure

C. Adding more SO_2

D. Adding more O_2

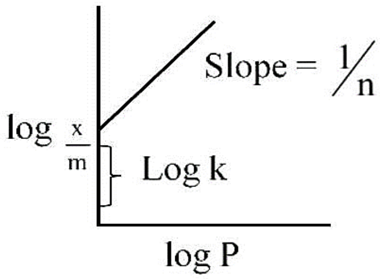
59. (4)

$$d = \frac{z \times M}{N_0 \times a^3}$$

$$4 = \frac{z \times 72}{6 \times 10^{23} \times 125 \times 10^{-24}}$$

$Z = 4.166 \cong 4$

60. (16)



$$\text{Slope } = \tan 45^\circ = 1$$
$$\log K = 0.6020 = \log 4$$
$$K = 4$$
$$\frac{x}{m} = KP^{1/n}$$
$$\frac{x}{m} = 4(0.4)1 = 16 \times 10^{-1}$$

61. (b)

$\cos\alpha = \cos 60$ $\quad$ $\cos\beta = \cos 45$

$\ell = \frac{1}{2}$ $\quad$ $m = \frac{1}{\sqrt{2}}$

$1^2 + m^2 + n^2 = 1$ $\quad$ $n^2 = 1 - \frac{3}{4} = \frac{1}{4}$

$\Rightarrow \frac{1}{4} + \frac{1}{2} + n^2 = 1$ $\quad$ $n = \frac{1}{2}$

Direction of $\vec{v}$ is $\frac{1}{2}\hat{\mathbf{i}} + \frac{1}{\sqrt{2}}\hat{\mathbf{j}} + \frac{1}{2}\hat{k}$

Equation of plane through $(\sqrt{2}, -1, 1)$ & Normla to $\vec{v}$ is $\frac{1}{2}(x - \sqrt{2}) + \frac{1}{\sqrt{2}}(y + 1) + \frac{1}{2}(z - 1) = 0$

It passes through (a, b, c)

$$(a - \sqrt{2}) + \sqrt{2}(b + 1) + (c - 1) = 0$$
$$\Rightarrow a + \sqrt{2}b + c = \sqrt{2} - \sqrt{2} + 1$$
$$\Rightarrow a + \sqrt{2}b + c = 1$$

62. (b) If $\log_a b, \log_c a, \log_b c \to$ G.P.

$(\log_c a)^2 = \log_a b \times \log_b c$

$(\log_c a)^2 = \log_a c$

$\Rightarrow (\log_c a)^2 = \frac{1}{\log_c a}$

$\Rightarrow (\log_c a)^3 = 1$

$\Rightarrow \log_c a = 1$

$a = c$

$$\text{If } a^3 b^3 c^3 \to \text{ A.P}$$
$$2b^3 = a^3 + c^3$$
$$\text{If } a = c$$
$$\Rightarrow a = b = c$$

For AP

$$A = \frac{a + 4a + a}{3} \quad D = \frac{a - 8a + a}{10}$$

$$A = 2a$$

$$S_{20} = \frac{20}{2}\left[2 \times 2a + (20-1)\left(\frac{-3a}{5}\right)\right]$$

$$= 10\left[4a - \frac{57a}{5}\right]$$

$$= 10\left[-\frac{37a}{5}\right] = -444$$

$$\Rightarrow a = \frac{444 \times 5}{37 \times 10}$$

$$\Rightarrow a = 6$$

$$abc = 6 \times 6 \times 6 = 216$$

63. (c) $a_1 = 1, a_2, a_3, \ldots . a_n$ be consecative natural numbers.

$$\tan^{-1}\left(\frac{1}{1 + a_1 a_2}\right) + \tan^{-1}\left(\frac{1}{1 + a_2 a_3}\right) + \cdots .. + \tan^{-1}\left(\frac{1}{1 + a_{2021} a_{2022}}\right)$$

$$\Rightarrow T_K = \tan^{-1}\left(\frac{1}{1 + K(K+1)}\right)$$

$$= \tan^{-1}\left(\frac{K + 1 - K}{1 + K(K+1)}\right)$$

$$= \tan^{-1}(K+1) - \tan^{-1} K$$

$$T_1 = \tan^{-1} 2 - \tan^{-1} 1$$

$$T_2 = \tan^{-1} 3 - \tan^{-1} 2$$

$$T_3 = \tan^{-1} 4 - \tan^{-1} 3$$

$$T_{2021} = \tan^{-1}(2022) - \tan^{-1}(2021)$$

On adding

$$\Sigma T_n = \tan^{-1}(2022) - \tan^{-1}(a)$$

$$\sum_{n=1}^{2021} T_n = \tan^{-1}(2022) - \frac{\pi}{4}$$

64. (c)

$((\vec{a}+\vec{b})\times(\vec{a}\times\vec{b}))\times(\vec{a}-\vec{b}) -= 8\hat{i}-40\hat{j}-24\hat{k}$

$\Rightarrow (\vec{a}\times(\vec{a}\times\vec{b})+\vec{b}\times(\vec{a}\times\vec{b}))\times(\vec{a}-\vec{b})$

$\Rightarrow ((\vec{a}\cdot\vec{b})\vec{a}-(\vec{a}\cdot\vec{a})\vec{b}+(\vec{b}\cdot\vec{b})\vec{a}-(\vec{b}\cdot\vec{a})\vec{b})\times(\vec{a}-\vec{b})$

$\Rightarrow 0-(\vec{a}\cdot\vec{b})(\vec{a}\times\vec{b})-a^2(\vec{b}\times\vec{a})+0-b^2(\vec{a}\times\vec{b})-(\vec{a}\cdot\vec{b})\vec{b}\times\vec{a} = 8\hat{i}-40\hat{j}-24\hat{k}$

$\Rightarrow (a^2-b^2)(\vec{a}\times\vec{b}) = 8\hat{i}-40\hat{j}-24\hat{k}$

$\left((\lambda^2+4+9)-(1+\lambda^2+4)\right)(\vec{a}\times\vec{b})$

$8(\vec{a}\times\vec{b}) = 8(\hat{i}-5\hat{j}-3\hat{k})$

$\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ \lambda & 2 & -3\\ 1 & -\lambda & 2\end{vmatrix} = \hat{i}-5\hat{j}-3\hat{k}$

$\hat{i}(4-3\lambda)-\hat{j}(2\lambda+3)+\hat{k}(-\lambda^2-2) = \hat{i}-5\hat{j}-3\hat{k}$

$\Rightarrow\ 4-3\lambda = 1\ 2\lambda+3 = 5\ -\lambda^2-2 = -3$

$3\lambda = 3\ \lambda^2 = 1$

$\lambda = 1\ \lambda = 1$

$|\lambda(\vec{a}+\vec{b})\times(\vec{a}-\vec{b})| = |(\vec{a}+\vec{b})\times(\vec{a}-\vec{b})|^2$

$\Rightarrow\ |-\vec{a}\times\vec{b}+\vec{b}\times\vec{a}|^2 = |2(\vec{a}\times\vec{b})|^2 = 4(1+25+9) = 140$

65. (a) $x^2-px+\frac{5}{4}p = 0$

Roots are rational

$D =$ A perfect square

$p^2-4(a)\dfrac{5}{4}p$

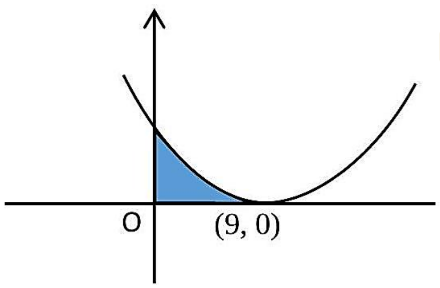
$p^2-5p =$ A perfect square

for $p = 0, p = 5, p = 9$ the D is a perfect square

$\therefore$ maximum integral of p is 9 .

$q = 9$

$\{(x,y); 0\le y\le (x-9)^2, 0\le x\le 9\}$



Area $= \int_0^9 (x-9)^2dx$

$\Rightarrow \left.\dfrac{(x-9)^3}{3}\right]_0^3$

$\Rightarrow 0-\dfrac{(0-9)^3}{3}$

$\Rightarrow \frac{9\times 9\times 9}{3}$

$= 243$

66. (d)

LHL

$\lim_{\delta\to 0} g(h(-\delta))\ \delta > 0$
$\lim_{\delta\to 0} g(-2+1)$

$\Rightarrow g(-1) = 1$

RHL

$\lim_{\delta\to 0} g(h(\delta))$
$\lim_{\delta\to 0} g(2\times 0-1)$
$\lim_{\delta\to 0} g(-1)$
$\lim_{x\to 1} g(h(x-1) = 1$

67. (a) Let $a_1 = n\ a_2 = n+1\ a_3 = n+2$

$$\bar{x} = \frac{n+(n+1)+(n+2)+\cdots..n+99}{100}$$

$$= \frac{100n + \frac{100\times 99}{2}}{100} = n + \frac{99}{2}$$

Mean deviation about the mean

$$\frac{1}{100}\sum|x_i - \bar{x}|$$

$$\Rightarrow \frac{1}{100}\left(\frac{99}{2}+\frac{97}{2}+\frac{95}{2}+\cdots....+\frac{97}{2}+\frac{99}{2}\right)$$

$$\Rightarrow \frac{2}{100}\left(\frac{99}{2}+\frac{97}{2}+\frac{95}{2}+\cdots.....50\text{ terms }\right)$$

$$\Rightarrow \frac{2}{100}\times\frac{1}{2}\times(50)^2 = \frac{50\times 50}{100} = 25$$

It is 25 irrespective of the value of

$\therefore n \in N$

$\Rightarrow S = N$

68. (c) $\begin{vmatrix} 1 & -1 & 1 \\ 2 & 2 & \alpha \\ 3 & -1 & 4 \end{vmatrix} = 0$

$1(8+\alpha)+1(8-3\alpha)+1(-2-6) = 0$

$\Rightarrow 8 + \alpha + 8 - 3\alpha - 8 = 0$

$-2\alpha = -8$

$\alpha = 4$

$D_1 = 0$

$$\Rightarrow \begin{vmatrix} 5 & -1 & 1 \\ 8 & 2 & 4 \\ \beta & -1 & 4 \end{vmatrix} = 0$$

$5(8+4) + 1(32 - 4\beta) + 1(-8 - 2\beta) = 0$

$60 + 32 - 4\beta - 8 - 2\beta = 0$

$\Rightarrow -6\beta = -84$

$\beta = 14$

Equation having roots as $\alpha \& \beta$

$x^2 - 18x + 56 = 0$

69. (c)

$$\begin{aligned} &\vec{b} \cdot \vec{c} = (2\vec{a} \times \vec{b}) \cdot \vec{b} - 3\vec{b} \cdot \vec{b} \\ &= 0 - 3\,b^2 \\ &= -3 \times 16 = -48 \\ &\vec{b} \cdot \vec{c} = -48 \end{aligned}$$

70. (d)

$$\begin{aligned} &f'(x) = x^2 + 2\,b + ax = 0 \\ &g'(x) = x^2 + a + 2bx = 0 \\ &x = 1 \text{ is the common root} \\ &1 + 2\,b + a = 0 \\ &2\,b + a + 1 + 6 = 6 \\ &2\,b + a + 7 = 6 \end{aligned}$$

71. (b)

$$\begin{aligned} &(P^T)^T = aP^T(a-1)I \\ &P = a(aP + (a-1)I) + (a-1)I \\ &= a^2P + (a^2 - a)I + (a-1)I \\ &= a^2P + (a^2 - a + a - 1)I \\ P &= a^2P + (a^2 - 1)I\ P = (1 - a^2) = (a^2 - 1)I \\ |\ \mathrm{Adj}P\ | &= |P|^{3-1} = (-1)^2 \\ &= 1 \end{aligned}$$

72. (b)

73. (d)

$$\lim_{x\to\infty}\frac{3}{n}\left\{\underset{(2+4/n)^2}{\underset{\uparrow}{4}}\quad(2+1/n)^2+\left(2+\frac{2}{n}\right)^2+\cdots.+\left(3-\frac{1}{n}\right)^2\right\}$$

$$\Rightarrow \sum_{r=0}^{n-1}\frac{3}{n}\left(2+\frac{r}{n}\right)^2$$

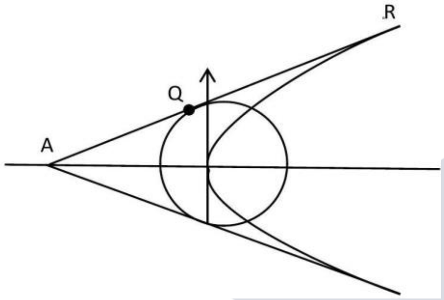
$$\Rightarrow \int_0^1 3(2+x)^2dx$$

$$\Rightarrow [(2+x)^3]_0^1$$

$$\Rightarrow (2+1)^3-(2+0)^3$$

$$\Rightarrow 27-8=19$$

74. (b)



$y^2=16x$

circle

Tangent $x^2+y^2=8$

$y=mx+\frac{4}{m}$ Tagent

$y=mx\pm 2\sqrt{2}\sqrt{1+m^2}$

$\frac{4}{m}=\pm 2\sqrt{2}\sqrt{1+m^2}$

$\frac{16}{m^2}=8+8m^2$

$8m^4+8m^2=16$

$m^4+m^2=2$

$m^2=1,-2$

Let $m=1$

$\underline{m}>1$

$\therefore y=x+4$

Point of tangecy at parabola

$Q\left(\frac{4}{m^2}, \frac{8}{m}\right)$

$Q(4,8)$

Point of tangency at circle

eq^h at tangent at $R(x_1, y_1)$

is $T = 0$

$xx_1 + yy_1 = 8$

Comparison with

$x - y + 4 = 0$

$\frac{x_1}{1} = \frac{y_1}{-1} = -\frac{8}{4}$

$x_1 = -2\ y_1 = 2$

$R(-2,2)$

Now $QR^2 = \sqrt{(4+2)^2 + (8-2)^2}$

$QR^2 = 72$

75. (b) $_{Eq^n}$ **of plane**

$$\begin{vmatrix} x-2 & y-k & 3+1 \\ 1-2 & 1-k & 2+1 \\ -1-2 & k-k & 0+1 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} x-2 & y-k & 3+1 \\ -1 & 1-k & 3 \\ -3 & 0 & 1 \end{vmatrix} = 0$$

$$(x-2)(1-k-0) - (y-k)(-1+9) + (3+1)(0+3-3k) = 0$$

$$\Rightarrow (1-k)x - 8y + (3-3k)z - 2 + 2k + 8k + 3 - 3k = 0$$

$$(1-K)x - 8y + (3-3k)z + 7K + 1 = 0$$

Plane is parallel to the line L :

$$\therefore (1-k)1 - 8.1 + (3-3k)(-1) = 0$$

$$\Rightarrow 1 - k - 8 - 3 + 3k = 0$$

$$2k = 10$$

$$k = 5$$

$$\frac{k^2+1}{(k-1)(k-2)}$$

$$= \frac{25+1}{(5-1)(5-2)} = \frac{26}{4 \times 3} = \frac{13}{6}$$

76. (d)

$3-x \geq 0 \quad 2+x \geq 0$
$x \leq 3 \quad x \geq -2$
$x \in [-2,3]$

Now, $f(-2)=\sqrt{3+2}=\sqrt{5}$

$$f(c)=\sqrt{2+3}=\sqrt{5}$$
$$f(x)=\sqrt{3-x}+\sqrt{2+x}$$
$$f'(x)=\frac{1}{2\sqrt{3-x}}x-1+\frac{1}{2\sqrt{2+x}}=0$$
$$\Rightarrow \frac{1}{\sqrt{3-x}}=\frac{1}{\sqrt{2+x}}$$
$$\Rightarrow 3-x=2+x$$
$$\Rightarrow 2x=1$$
$$x=1/2$$

$$f\left(\frac{1}{2}\right)=\sqrt{3-1/2}+\sqrt{2+\frac{1}{2}}$$
$$=\sqrt{\frac{5}{2}}+\sqrt{\frac{5}{2}} \Rightarrow 2\sqrt{\frac{5}{2}} \Rightarrow \sqrt{10}$$

Range $=[\sqrt{5},\sqrt{10}]$

77. (b)

$y=Vx$

$$\frac{dy}{dx}=V+\frac{xdv}{dx}$$

$$v+\frac{xdv}{dx}=-\frac{x^2+3v^2x^2}{3x^2+v^2x^2}$$

$$x\frac{dv}{dx}=-\frac{1+3v^2}{3+v^2}-v$$

$$x\frac{dv}{dx}=-\frac{1+3v^2+3v+v^3}{3+v^2}$$

$$\int \frac{3+v^2}{1+3v^2+3v+v^3}dv=-\int \frac{dx}{x}$$

$$\Rightarrow \int \frac{3+v^2}{(1+v)^3} dv = -\ln x + C$$

Let $v+1=t$

$$\int \frac{3+(t-1)^2}{t^3} dt = -\ln x + c$$

$$\Rightarrow \int \frac{t^2-2t+4}{t^3} dt$$

$$\Rightarrow \int \left(\frac{1}{t}-\frac{2}{t^2}+\frac{4}{t^3}\right) dt = -\ln x + c$$

$$\Rightarrow \ln t + \frac{2}{t} - \frac{4}{2t^2} = -\ln x + C$$

$$\Rightarrow \ln\left(\frac{y}{x}+1\right)^{-1} \frac{2}{\frac{y}{x}+1} - \frac{2(y/x+1)^2}{x} = -\ln x + C$$

$$\Rightarrow \ln\left(\frac{y+x}{x}\right) + \frac{2x}{y+x} - \frac{2x^2}{(x+y)^2} = -\ln x + c$$

$$\Rightarrow \ln|x+y| + \frac{2x}{(x+y)^2}(x+y-x) = C$$

$$\Rightarrow \ln|x+y| + \frac{2xy}{(x+y)^2} = C$$

78. **(b)** at $y=1$, Both curve intersect

$$\Rightarrow \left.\begin{array}{l} ax^2+2bx+c=0 \\ dx^2+2ex+f=0 \end{array}\right\} \text{Common Root}$$

Given a, b, c are in G.P

$\Rightarrow D = 4b^2 - 4ac = 0$ for the first equation

$\Rightarrow$ Both the Root are equal

$\because$ sum of the roots $= -2\frac{b}{a}$

$$\alpha+\alpha = -2\frac{b}{a}$$

$$\alpha = -\frac{b}{a}$$

It satisfies the second equation also

$$d\left(-\frac{b}{a}\right)^2+2e\left(-\frac{b}{a}\right)+f=0$$
$$d\left(\frac{b^2}{a^2}\right)-\frac{2eb}{a}+f=0$$
$$d\left(\frac{ac}{a^2}\right)-2e\frac{b}{a}+f=0$$
$$\frac{d}{a}-\frac{2eb}{ac}+\frac{f}{c}=0$$
$$\frac{d}{a}-\frac{2eb}{b^2}+\frac{f}{c}=0$$
$$\Rightarrow 2\frac{e}{b}=\frac{d}{a}+\frac{f}{c}\Rightarrow \frac{d}{a},\frac{e}{b},\frac{f}{c}\text{ arein AP}$$

79. (a)

$$P\rightarrow(\sim Q\wedge R)$$
$$\sim PV(\sim Q\wedge R)$$
$$\Rightarrow((\sim P)V(\sim Q))\wedge((\sim P)vR)$$

80. (b) Let $x=I_1+f_1$

$$(8\sqrt{3}+13)^{13}=I_1+f_1$$
$$(8\sqrt{3}-13)^{13}=f_1{}'(\text{ let })$$

On Subtaction

$$(8\sqrt{3}+13)^{13}-(8\sqrt{b}-13)^{13}=I+f_1-f_1{}'$$

$$2\left[{}^{13}C_1(8\sqrt{3})^{12}\right]13+{}^{13}C_3(8\sqrt{3})^{10}13^3$$

$$+---0]=1+0$$

$\because$ I = Even Number

$[x]$ = Even

similarly,

Let $y=I_2+f_2$

$$(7\sqrt{2}+9)^9=I_2+f_2$$

$$(7\sqrt{2}-9)^9=f_2'$$

On Subtaction

$$(7\sqrt{2}+9)^9-(7\sqrt{2}-9)^9=I_2+f_2-f_2^1$$

$$2\left[{}^9C_1(7\sqrt{2})^8\cdot 9+{}^9C_3(7\sqrt{2})^7 9^2---\right]=I_2+0$$

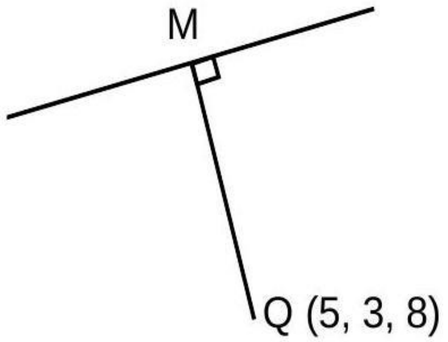
I_2 = Even

$\because [x]+[y]=$ Even + Even
$\quad\quad = $ Even

81. (158) The Direction ratio of line

$$\begin{vmatrix} i & j & k \\ 1 & 3 & -2 \\ 1 & -1 & 2 \end{vmatrix} = i(6-2) - j(2+2) + k(-1-3)$$

Equation of line L



$$\frac{x-2}{1} = \frac{y-3}{-1} = \frac{z-1}{-1} = \lambda(\alpha d)$$

Let $M(\lambda+2, -\lambda+3, -\lambda+1)$

DR's of MQ is $< \lambda+2-5, -\lambda+3-3, -\lambda+1-8 >$
$\because L \perp MQ$

$$\Rightarrow (\lambda-3)(a) + (-\lambda)(-1) + (-\lambda-7)(-1) = 0$$

$$\Rightarrow \lambda - 3 + \lambda + \lambda + 7 = 0$$

$$\Rightarrow 3\lambda = -4 \Rightarrow \lambda = -\frac{4}{3}$$

$$\therefore M\left(-\frac{4}{3}+2, \frac{+4}{3}+3, \frac{4}{3}+1\right) = \left(\frac{2}{3}, \frac{13}{3}, \frac{7}{3}\right)$$

$MQ = \alpha$

$$\therefore 3\alpha^2 = 3 \times \left(\left(5-\frac{2}{3}\right)^2 + \left(3-\frac{13}{3}\right)^2 + \left(8-\frac{7}{5}\right)^2\right)$$

$$= 3\left(\frac{169}{9} + \frac{16}{9} + \frac{289}{9}\right) \Rightarrow \frac{474}{9} = 158$$

82. (14)

$$p = 1 \cdot \frac{1}{6}$$

$$q = \left({}^6C_1 \cdot \frac{1}{6} \cdot \frac{1}{6} \cdot \frac{1}{6} \cdot \frac{5}{6}\right)\frac{4!}{3!} = \frac{5}{216} \times 4 = \frac{5}{54}$$

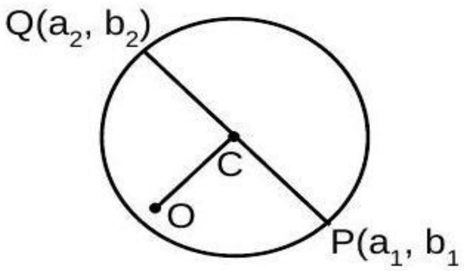
$$\frac{p}{q} = \frac{1/6}{5/54} = \frac{9}{5}$$

$m = 9$

$n = 5$

$m + n = 9 + 5 = 14$

83. (24)



OC is $\perp^r$ to both CP & CQ
$\Rightarrow$ PQ is a Diameter

Area of $\triangle$ OCP $= \frac{\sqrt{35}}{2}$

$$\frac{1}{2} \times CP \times OC = \frac{\sqrt{35}}{2}$$

$$CP \times \sqrt{2+3} = \sqrt{35}$$

$$CP = \sqrt{7} \Rightarrow \text{radius} = \sqrt{7}$$

$$\text{Now } OP^2 = OC^2 + PC^2$$

$$a_1^2 + b_1^2 = 2 + 3 + 7 = 12$$

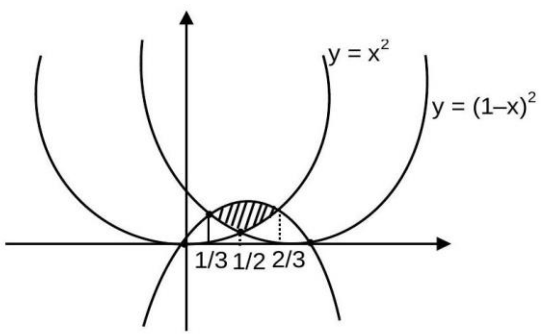
Similarly

$$OQ^2 = OC^2 + CQ^2$$

$$a_1^2 + b_1^2 = 2 + 3 + 7 = 12$$

$$\therefore a_1^2 + a_2^2 + b_1^2 + b_2^2 = 24$$

84. (25)



$x^2 = (1-x)^2$ $\quad$ $x^2 = 2x - 2x^2$ $\quad$ $(1-x)^2 + 2x - 2x^2$

$x^3 = 1 + x^2 - 2x$ $\quad$ $3x^1 = 2x$ $\quad$ $1 + x^2 - 2x = 2x$

$x = \frac{1}{2}$ $\quad$ $x(3x-2) = 0$ $\quad$ $\Rightarrow 3x^2 - 4x + 1 = 0$

$x = 0, \frac{2}{3}$ $\quad$ $\Rightarrow 3x^2 - 3x - x + 1 = 0$

$\Rightarrow 3x(x-1) - 1(x-1) = 0$

$x = 1, \frac{1}{3}$

Required Area

$$A = \int_{\frac{1}{3}}^{\frac{1}{2}} \{(2x - 2x^2) - (1-x)^2\}dx + \int_{\frac{1}{2}}^{\frac{2}{3}} \left((2x - 2x^2) - x^2\right)dx$$

$$\Rightarrow \left[x^2 - \frac{2x^3}{3} + \frac{(1-x)^3}{3}\right]_{\frac{1}{3}}^{\frac{1}{2}} + (x^2 - x^3)_{1/2}^{2/3}$$

$$\Rightarrow \left(\frac{1}{4} - \frac{2}{3} \cdot \frac{1}{8} + \frac{1}{8.3}\right) - \left(\frac{1}{9} - \frac{2}{3} \cdot \frac{1}{27} + \frac{8}{27.3}\right) + \left(\frac{4}{9} - \frac{8}{27}\right) - \left(\frac{1}{4} - \frac{1}{8}\right)$$

$$\Rightarrow -\frac{1}{24} - \frac{1}{9} - \frac{6}{3 \times 27} + \frac{4}{9} - \frac{8}{27} + \frac{1}{8}$$

$$\Rightarrow -\frac{1}{24} + \frac{3}{9} - \frac{10}{27} + \frac{3}{24} = \frac{-27 + 216 - 240 + 81}{24 \times 27} = \frac{297 - 267}{24 \times 27} = A$$

$$540\ A = 540 \times \frac{30}{24 \times 27} = 25$$

85. (151)

8^{th} common term of the series

$S_1 = 3 + 7 + 11 + 15 + 19 +$

$S_2 = 1 + 6 + 11 + 16 + 21 +$

First common term $= 11$

common diff of the AP of common terms

$=$ L.C.M of $\{4,5\}$

$= 20$

$\therefore$ AP

11,31,51,

$T_8 = 11 + (8-1)20$

$= 11 + 140$

$T_8 = 151$

86. (1) $f(n)$ for every $m, n \in A$ with $m \cdot n \in A$ is equal to

LHL

RHL

$\lim_{h\to 0} g(H(1-h-1))$

$\lim_{h\to 0} g(H(1+h-1))$

$\lim_{h\to 0} g(2(-1)-f(-h))$

$\lim_{h\to 0} g(H(h))$

$\lim_{h\to 0} g\left(-2-\frac{1-h}{|-h|}\right)$

$\Rightarrow \lim_{h\to 0}\ g(2(0)+(h))$

$\Rightarrow g\left(-2-\frac{-1}{1}\right)$

$g\ (0-1)$

$\Rightarrow 2(-1)=+1$

$\Rightarrow 1$

$\therefore \lim_{h\to 0}\ g(H(x-1)=1$

87. (1)

$$I=\int \sqrt{\sec 2x-1}dx$$

$$\Rightarrow \int \sqrt{\frac{1-\cos 2x}{\cos 2x}}dx$$

$$\Rightarrow \int \frac{\sqrt{2}\sin x}{\sqrt{2\cos^2 x-1}}dx$$

Let $\sqrt{2}\cos x = t$

$-\sqrt{\alpha}\sin x dx = dt$

$$I=\int \frac{-dt}{\sqrt{t^2-1}}=\ln\left|t+\sqrt{t'-1}\right|+c$$

$$\Rightarrow -\ln\left|\sqrt{2}\cos x+\sqrt{2\cos^2 x-1}\right|+c$$

$$\Rightarrow \frac{-1}{2}\ln\left|(\sqrt{2}\cos x+\sqrt{\cos 2x})^2\right|+c$$

$$\Rightarrow \frac{-1}{2}\ln\left|2\cos^2 x+\cos 2x+2\sqrt{2}\cos x\sqrt{\cos 2x}\right|+c$$

$$\Rightarrow \frac{-1}{2}\ln\left|1+\cos 2x+\cos 2x+2\sqrt{2}\sqrt{\cos 2x}\times\sqrt{\frac{1+\cos x}{2}}\right|+c$$

$$\Rightarrow \frac{-1}{2}\ln\left|2\cos 2x+1+2\sqrt{\cos 2x(1+\cos 2x)}\right|+c$$

$$\Rightarrow \frac{-1}{2}\ln\left|\cos 2x+\frac{1}{2}\sqrt{\cos 2x(1+\cos 2x)}\right|+c$$

$$\alpha=\frac{-1}{2}\ \beta=\frac{1}{2}$$

$$\therefore \beta-\alpha=\frac{1}{2}-\left(\frac{-1}{2}\right)=1$$

88. (13)

$$x^2-5ax+1=0$$
$$x^2-ax-5=0$$
$$-\ \ +\ \ +$$
$$-4ax+6=0$$

$x=\frac{6}{4a}=\frac{3}{2a}$ (common root)

$$\therefore \left(\frac{3}{2a}\right)^2-5a\left(\frac{3}{2a}\right)+1=0$$

$$\Rightarrow 9-30a^2+4a^2=0$$

$$\Rightarrow 26a^2=9$$

$$a^2=\frac{9}{26}\Rightarrow a=\frac{3}{\sqrt{26}}=\frac{3}{\sqrt{2\beta}}$$

$$\beta=13$$

89. (23)

$x^{\frac{1}{50}}=12\ y^{\frac{1}{50}}=18$

Remainder when $x+y$ is division by 25.

$x=12^{50}\ y=18^{50}$

$x+y=12^{50}+18^{50}$

$=6^{50}(2^{50}+3^{50})$

$=(5+1)^{50}((2^2)^{25}+(3^2)^{25})$

$= (25\lambda_1 + 1)((5-1)^{25} + (10-1)^{25})$

$= (25\lambda_1 + 1)(25(\lambda_2 + \lambda_3) - 2)$

$= (25\lambda_1 + 1)(25\text{ K} - 2)$

$\Rightarrow 25\lambda_1 \cdot 25\text{ K} - 50\lambda_1 + 25\text{ K} - 2$

$\Rightarrow 25n_1 - 2$

$\Rightarrow 25n_2 + 23$

Remainder $= 23$

90. (240)