1. (a) Total Energy in SHM, $E = \frac{1}{2} m\omega^2 A^2 = 25$ J

at $\frac{A}{2}$, $U = PE = \frac{1}{2} m\omega^2 x^2$

$$U = \frac{1}{2} m\omega^2 \left(\frac{A}{2}\right)^2$$

$$k + U = E$$

$$k = \frac{1}{2} m\omega^2 A^2 \left(1 - \frac{1}{4}\right)$$

$$k = 25 \times \frac{3}{4} = 18.75 \text{ J}$$

2. (a) $V = IR = I\left(\frac{\rho l}{A}\right)$

$A \rightarrow 2A$

$I \rightarrow 2I$

$I = AneV_d$

$$V_d \propto \frac{I}{A}$$

3. (d) At highest point -

$$u\cos\theta = \frac{\sqrt{3}u}{2}$$

$$\theta = 30$$

$$T = \frac{2u\sin\theta}{g} = \frac{u}{g}$$

4. (a)

$\gamma = \frac{C_p}{C_v}$, Independent on T

5. (d) In semiconductors,

$T\uparrow, n_e$ in Conduction Band increases

$T\uparrow, R\downarrow$

6. (d) $f_c = \frac{1000\pi}{2\pi} = 500$ Hz

$$f_m = \frac{4\pi}{2\pi} = 2 \text{ Hz}$$

Upper side Band, USB $= f_c + f_m$

$USB = 502 HZ$

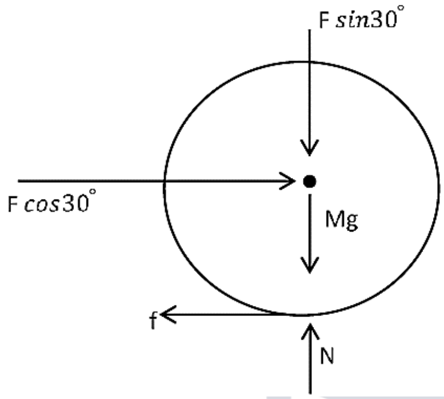
Lower side Band, $LSB = f_c - f_m$
$LSB = 498\ Hz$

7. **(d)** After A, $I = \frac{I_0}{2}$

After C, $I = \frac{I_0}{2}\cos^2 45^\circ = \frac{I_0}{4}$

After B, $I = \frac{I_0}{4}\cos^2 45^\circ = \frac{I_0}{8}$

8. **(d)** FBD of Sphere ⇒



$$N = Mg + F\sin 30^\circ$$
$$N = 700 + 200\sin 30^\circ$$
$$N = 800\ N$$

9. **(b)** $V_1 = V_2$

$$1000 \times \frac{4}{3}\pi r^3 = \frac{4}{3}\pi R^3$$

$R = 10r$

$E = U_1 - U_2$

$= 1000(T \times 4\pi r^2) - T \times 4\pi R^2$
$E = 4\pi T(1000 \times r^2 - 100r^2)$

$$E = 4 \times \frac{22}{7} \times 0.07 \times 900 \times 10^{-6}$$

$E = 7.92 \times 10^{-4}\ J$

10. **(c)** $W =$ Area of PV Graph

$$W = -\frac{1}{2} \times [50 + 10] \times 10^3 \times 150 \times 10^{-6}$$

$W = -4.5\ J$

$Q = \Delta U + W$

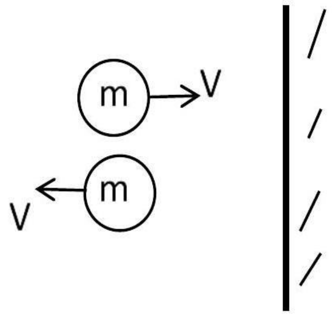
$0 = \Delta U - 4.5$

$= \Delta U = 4.5\ J$

11. (a)
12. (d) Rest mass of neutron is greater than proton.
13. (a) In Conductor, A portion of the Gravitational Potential Energy goes into generating eddy current.
14. (b) R, X_L, X_C have same unit i.e. ohm

$\frac{R}{\sqrt{x_L x_C}} \rightarrow \frac{ohm}{\sqrt{ohm}} \rightarrow$ Dimensionless

15. (d)



Change in momentum,

$|\overrightarrow{\Delta p}| = 2mV$

Average force,

$F_{avg} = N\frac{|\overrightarrow{\Delta p}|}{t}$

$F_{avg} = 100\left(\frac{2mV}{t}\right)$

$F_{avg} = \frac{200mV}{t}$

16. (c) $P = \frac{N}{t}\left(\frac{hc}{\lambda}\right)$

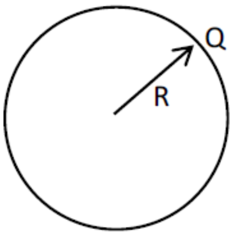
$15 \times 10^3 = 10^{16} \times \frac{6 \times 10^{-34} \times 3 \times 10^8}{\lambda}$

$\lambda = 1.2 \times 10^{-13}\ m$

$\lambda = 0.0012\ A^0$

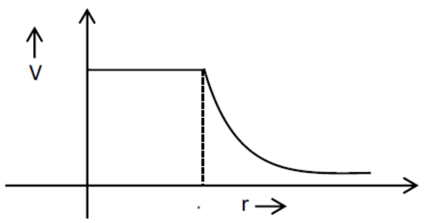
Corresponds to Gamma rays

17. (a)



$V_{in} = \frac{kQ}{R} \rightarrow$ Constant

$V_{out} = \frac{kQ}{r} \propto \frac{1}{r}$



18. **(b)** $W = MB(\cos\theta_1 - \cos\theta_2)$

$W = MB(\cos 0° - \cos 180°)$

$W = 2MB$

$W = 2 \times 5 \times 0.4$

$W = 4\ J$

19. **(4800)** Given

$$g\left(1 - \frac{d}{R}\right) = 4\frac{g}{\left(1 + \frac{h}{R}\right)^2}$$

$$1 - \frac{d}{R} = \frac{4}{(1+3)^2} = \frac{1}{4}$$

$$\frac{d}{R} = \frac{3}{4}$$

$$d = \frac{3R}{4} = \frac{3}{4} \times 6400$$

$d = 4800$ km

20. **(a)** Magnetic field in the Solenoid,

$B = \mu_0\mu_r nl$

Magnetic flux, $\phi = N(BA)$

$\phi = N(\mu_0\mu_r n/A)$

$$4\pi \times 10^{-6} = 400\left(4\pi \times 10^{-7}\mu_\nu \times \frac{400}{0.4} \times 0.4 \times 2 \times 10^{-4}\right)$$

$$\frac{1}{40} = \mu_r \times 8 \times 10^{-2}$$

$$\mu_r = \frac{100}{320} = \frac{5}{16}$$

21. (5) $V = \frac{c}{n}$

$n \rightarrow$ refractive index

$$n = \frac{c}{0.2c} = 5$$

$$n = \sqrt{\mu_r \varepsilon_r}$$

$$\varepsilon_r = n^2 = 25$$

$$\frac{\varepsilon_r}{n} = \frac{25}{5} = \frac{5}{1}$$

22. (10) On Rolling,

$$KE = \frac{1}{2}MV^2 + \frac{1}{2}I\omega^2$$

$$KE = \frac{1}{2}MV^2 + \frac{1}{2}\left(\frac{2}{5}MR^2\right)\left(\frac{V}{R}\right)^2$$

$$KE = \frac{7}{10}MV^2 = 7 \times 10^{-3}$$

$$V^2 = 10^{-2}$$

$$V = 10^{-1}\ m/s$$

$$V = 10\ cm/s$$

23. (7) Acceleration is constant,

$$v^2 = u^2 + 2as$$

$$v^2 = 2^2 + 2(b)(6)$$

$$v^2 = 28$$

$$\frac{1}{2}Mv^2 = \frac{1}{2} \times 500 \times 28$$

$$KE = 7\ kJ$$

24. (20) $T = 2\pi\sqrt{\frac{m}{k_{eq}}}$

$$T = 2\pi\sqrt{\frac{m}{2k}}$$

$$T = 2\pi\sqrt{\frac{0.49}{2 \times 2}}$$

$$T = 2\pi \times \frac{0.7}{2} = 0.7\pi$$

in 14πsec, $\frac{14\pi}{0.7\pi} = 20$

25. (242)

Current is in phase with EMF. Hence, Circuit is at Resonance.

$$I_{rms} = \frac{V_{rms}}{R} = \frac{220}{20}$$

$$I_{rms} = 11\ A$$

$$I_0 = \sqrt{2}I_{rms} = \sqrt{242}\ A$$

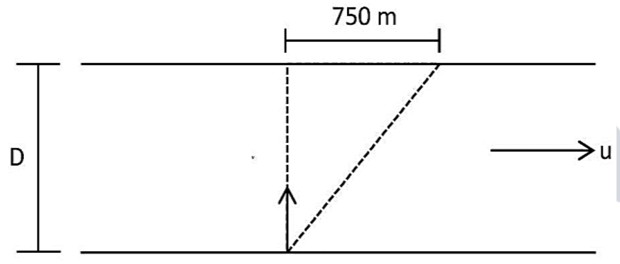
26. (3)

$$T = \frac{D}{V} = \frac{1}{4}hr$$

$$Drift = uT$$

$$\frac{750}{1000}kM = u \times \frac{1}{4}hr$$

$$u = 3\ km/hr$$



27. (27)

$$\frac{1}{\lambda_1} = R\left(\frac{1}{1^2} - \frac{1}{3^2}\right)$$

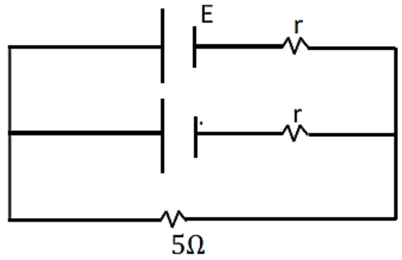
$$\lambda_1 = \frac{9}{8R}$$

$$\frac{1}{\lambda_2} = R\left(\frac{1}{1^2} - \frac{1}{2^2}\right)$$

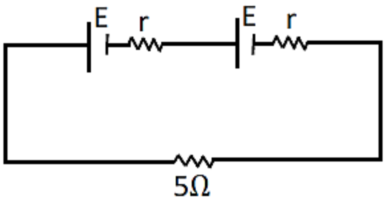
$$\lambda_2 = \frac{4}{3R}$$

$$\frac{\lambda_1}{\lambda_2} = \frac{27}{32}$$

28. (5)



(I)



(II)

$r_{eq} = \frac{r}{2}, r_{eq} = 2r$

$E_{eq} = \frac{r}{2}\left(\frac{E}{r} + \frac{E}{r}\right) = E, E_{eq} = 2E$

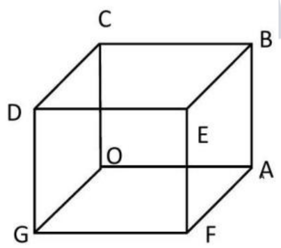
$I_1 = \frac{E}{5 + \frac{r}{2}}, I_2 = \frac{2E}{2r + 5}$

$I_1 = I_2$

$2r + 5 = 2\left(5 + \frac{r}{2}\right)$

$r = 5\Omega$

29. (640)



$\vec{E} \perp \vec{A}, \phi_{Top} = \phi_{Bottom} = \phi_{front} = \phi_{Back} = 0$

for OCDG, $x = 0, E = 0, \phi = 0$

for ABEF, $x = 0.2$ m

$E = 4000 \times (0.2)^2$

$E = 160$ V/m

$\phi = E(a^2) = 160 \text{ V/m} \times (0.2)^2 \text{ m}^2$

$\phi = 6.4$ V − m

$\phi = 640$ V − cm

30. (60) $Y = \frac{FL}{A\Delta L}$

$F = YA\left(\frac{\Delta L}{L}\right)$

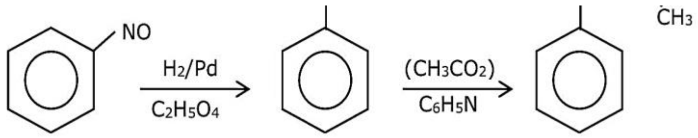
$F = \gamma A(\alpha\Delta T)$

$Mg = YA(\alpha\Delta T)$

$M \times 10 = 2 \times 10^{11} \times 3 \times 10^{-6} \times 2 \times 10^{-5} \times 50$

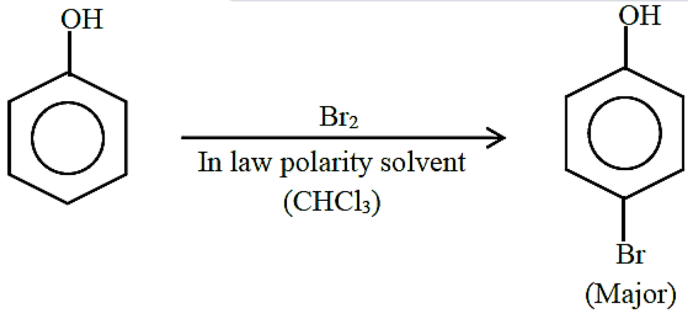
$M = 60$ kg

31. (a)
32. (a)



33. (c)

Phenol will give sooty flame while burning (aromatic compound)



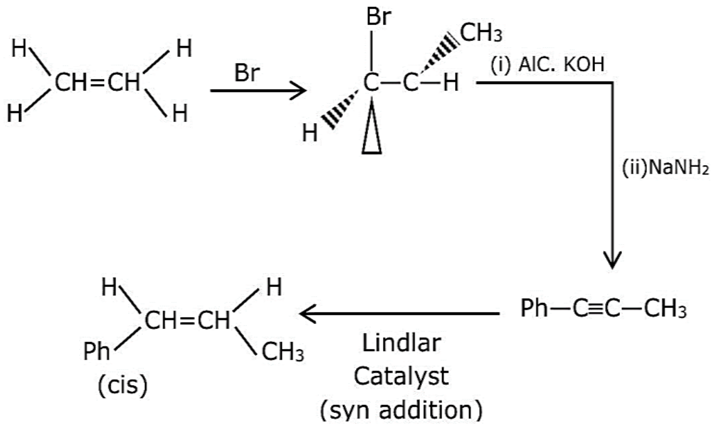
34. (d)

$$CuSO_4 + KI \longrightarrow \underset{\text{While ppt}}{Cu_2I_2} + \underset{\text{Violet}}{I_2} + K_2SO_4$$

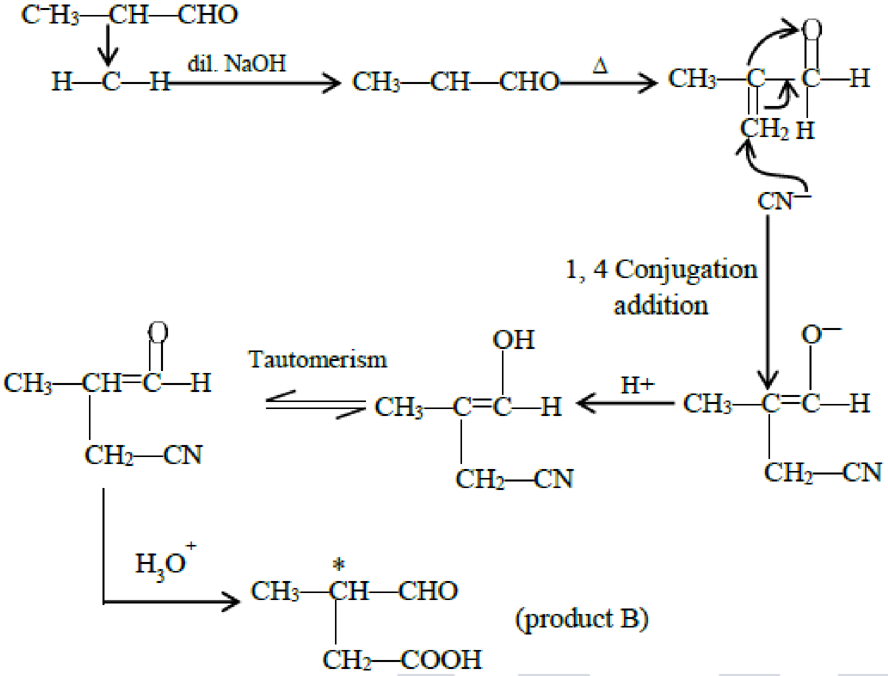
$$I_2 \xrightarrow[\text{Hypo sol}^n.]{Na_2S_2O_3} Na_2S_4O_6 + NaI$$

' M ' Electrolysis & liquation is method of purification where as hydraulic washing, leading, froth flotation are method of can conbration.

35. (b)



36. (b)



Carboxylic acid will give CO_2 gas with $NaHCO_3$ solutions

37. (3) Methods involved in concentration of one are

(i) Hydraulic Washing

(ii) Froth Flotation

(iii) Magnetic Separation

(iv) Leaching

38. (d) From protein, only ∝-Amino acid is possible

39. (c)

$Na = 4f^4 5\,d^0 6\,s^2$

$Na^{+2} = 4f^4 5d^0 6s^0$

40. (d)

XeF_4	Sq. planar
SF_4	see saw
NH_4^+	Tetrahedral
BrF_3	Bent ' T′ shaped.

41. (a)

$$ClO + NO_2 \longrightarrow \underset{(x)}{ClO.NO_2} \underset{(y)}{\xrightarrow{H_2O}} \underset{(z)}{HOCl} + \underset{3}{HNO_3}$$

42. (c)
43. (a)

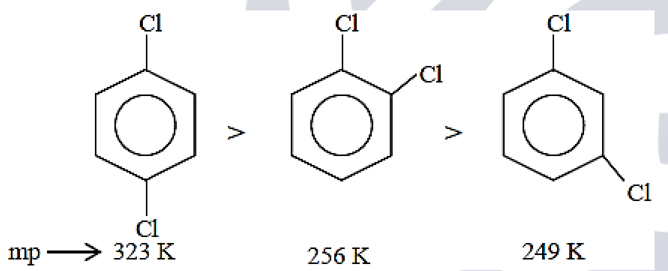
$$CoCl_2 + H_2O \longrightarrow \underset{\text{Pink}}{[Co(H_2O)_6]^{2+}} \xrightarrow{\text{conc. HCl}} \underset{\text{Blue Tetrahderal}}{[CoCl_4]^{2-}}$$

44. (a)

$$2NaO\overset{+1}{Cl} + H_2O_2 \longrightarrow 2Na\overset{-1}{Cl} + H_2O + O_2$$

H_2O_2 acts as reducing agent.

45. (a) Non polar end will be towards non polar solvent
46. (b)



47. (d) Leaser is charge on canter atom more will be the basicity.
48. (c) Alitame has 2000 has times more sweetner as compare to cane sugar.
49. (d) Brine soln gives H_2/OH^-at cathode &Cl_2 at anode.
50. (a)

$$\lambda_H = \lambda_{He^+}$$

$$R_H \times (1)^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) = R_H \times (2)^2\left(\frac{1}{(2)^2} - \frac{1}{(4)^2}\right)$$

$$\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) = \left(\frac{4}{4}\right) - \left(\frac{4}{16}\right)$$

$$\frac{1}{n_1^2} - \frac{1}{n_2^2} = \frac{1}{1} - \frac{1}{4}$$

$n_1 = 1 : n_2 = 2$ for H-atom

51. (4) Hypophosphoric acid is $H_4P_2O_6$ oxidation state of P is +4.

52. (610)

$$
\begin{tikzcd}
\frac{1}{2} Cl_2 \arrow[r] \arrow[d, "\frac{1}{2}\ BE"'] & Cl^- \text{ aq.} \\
Cl_{(g)} \arrow[r, "\Delta H_{eg.}\ .\ \text{Of } Cl_{(g)}"'] & Cl_{(g)} \arrow[u, "\Delta H_{Hyd.}\ \text{Of } Cl_{(g)}"']
\end{tikzcd}
$$

$\Delta H_{eg.}$. Of $Cl_{(g)}$

$$\Delta H^\circ_\gamma = \frac{1}{2} \times BE + \Delta H_{eg} + \Delta H_{Hyd}$$
$$= \frac{1}{2} \times 240 + (-350) + (-380)$$
$$\Rightarrow 120 - 350 - 380$$
$$\Rightarrow -610$$

53. (6)

$\Delta G^\circ = -RT\ell nK$

$-nFE^\circ_{\text{cell}} = -RT \times 2.303(\log_{10} K)$

Net reaction $\rightarrow Pd^{2+}(aq.) + 4Cl^-(aq.) \rightleftharpoons PdCl_4^{2-}(aq.)$

$E_{cell} = E_{cathod} - E^\circ$ anode

$E_{cell} = 0.83 - 0.65$

From equation (1)

Also $n = 2$

$\log K = 6$

54. (44)

44gm of CO_2 contains 12 g carbon.

0.792gm of CO_2 contains $\frac{0.792 \times 12}{44}$ g of carbon

$$\% \text{ of carbon } = \frac{0.216}{0.492} \times 100$$
$$= 43.9\% = 44\%$$

55. (4)

$Zn + 2HCl \longrightarrow ZnCl_2 + H_2$

No. of moles of Zn $= \frac{11.5}{65.3} =$ No. of moles of H_2

No. of H_2 liberated $= 0.176 \times 22.7$Lt.

$= 3.99$ L $= 4$Lt.

56. (2520)

$$\text{In}\left(\frac{K_2}{K_1}\right)=\frac{Ea}{R}\left[\frac{1}{T_1}-\frac{1}{T_2}\right]$$
$$\text{Log}\left(\frac{0.05}{0.03}\right)=\frac{Ea}{2.3\times 8.3}\left[\frac{1}{200}-\frac{1}{300}\right]$$
$$[0.70-0.48]=\frac{Ea}{2.3\times 8.3}\left[\frac{300-200}{300\times 200}\right]$$
$$0.22=\frac{Ea}{2.3\times 8.3}\left[\frac{1}{600}\right]$$
$$Ea=0.22\times 2.3\times 8.3\times 600$$
$$=2519.88\ \text{J}$$
$$\approx 2520$$

57. (1)

$$K_C=1\times 10^{13}$$
$$SO_2(g)+\frac{1}{2}O_2 \rightleftharpoons SO_3(g)$$
$$\Delta n=\frac{-1}{2} \qquad K_P=2\times 10^{12}$$
$$KP_P=K(RT)\Delta^{ng} \qquad P=1\ \text{atm}$$
$$2\times 10^{12}=KC_C(0.082\times 300)^{-1/2} \qquad T=27^\circ C$$
$$K_C=1\times 10^{13}$$

58. (555)

Number of moles of gas X $=\frac{0.6}{20}=0.03$

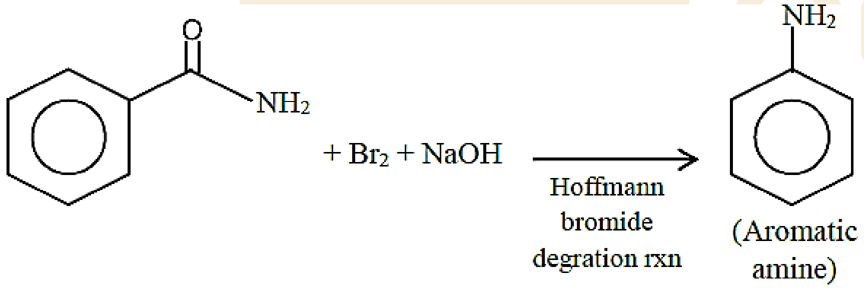
Number of moles of gas Y $=\frac{0.45}{45}=0.01$

Total number of moles $=0.03+0.01=0.04$ mole

Partial pressure of gas X $=$ Mole fraction $\times$ Total pressure

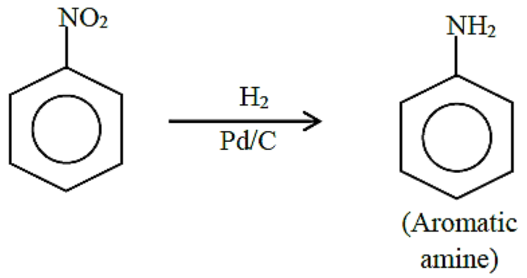
$$=\frac{0.03}{0.04}\times 740=555$$

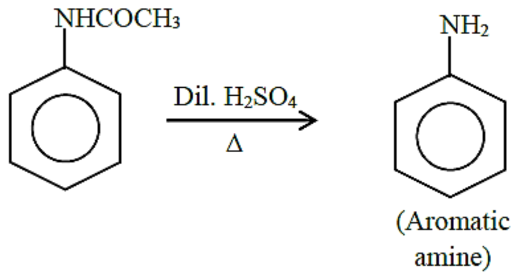
59. (3)



(a)

(b) (2) In Gabriel phthalimide synthesis chloro-benzene is poor substrok for S_{N_2}, Hence reaction will not observed.

(c) 

(d) 

60. (62250)

$$\pi = CRT$$

$$\frac{400\ \text{Pa}}{10^5} = \frac{\frac{2.5\ \text{g}}{M_\circ}}{250/1000} \times 0.083 \frac{\text{L}-\text{bar}}{\text{Kmol}} \times 300\ \text{K}$$

$$M_\circ = 62250$$

61. (c) Normal to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ at point $(a\cos\theta, b\sin\theta)$ is $\frac{ax}{\cos\theta} - \frac{by}{\sin\theta} = a^2 - b^2$

Its distance from origin is

$$d = \frac{|a^2 - b^2|}{\sqrt{a^2\sec^2\theta + b^2\text{cosec}^2\theta}}$$

$$d = \frac{|a^2 - b^2|}{\sqrt{a^2 + b^2 + 2ab + (a\tan\theta - b\cot\theta)^2}}$$

$$d \frac{|(a-b)(a+b)|}{\sqrt{a^2 + b^2 + 2ab + (a\tan\theta - b\tan\theta)^2}}$$

$$d_{max} = \frac{|(a-b)(a+b)|}{a+b} = |a - b|$$

$\because d_{max} = 1$

$|2 - b| = 1$

$2 - b = 1 [\because b < 2]$

$b = 1$

Eccentricity $= \sqrt{1 - \frac{b^2}{a^2}} = \sqrt{1 - \frac{1}{4}} = \frac{\sqrt{3}}{2}$

$e = \frac{\sqrt{3}}{2}$

62. (17)

$f(x) + \int_3^x \frac{f(t)}{t} dt = \sqrt{x+1}, x \geq 3$

Differentiate both side w.r.t. x

$f^1(x) + \frac{f(x)}{x} = \frac{1}{2\sqrt{x+1}}$

Above eqn. is linear differential equation

$\text{I.f.} = e^{\int \frac{1}{x} dx} = e^{\ln x} = x$

Solution is

$f(x) \cdot x = \int \frac{x}{2\sqrt{x+1}} dx + C$

$f(x) \cdot x = \frac{1}{2} \int \left(\frac{x+1}{\sqrt{x+1}} - \frac{1}{\sqrt{x+1}} \right) dx + C$

$f(x) \cdot x = \frac{1}{2} \int \left(\sqrt{x+1} - \frac{1}{\sqrt{x+1}} \right) dx + C$

$f(x) \cdot x = \frac{1}{2} \left[\frac{2}{3} (x+1)^{3/2} - 2\sqrt{x+1} \right] + C$

$\because f(c) = 2$

than

$2.3 = \frac{1}{2} \left[\frac{2}{3} \times 8 - 2 \times 2 \right] + C$

$6 = \frac{1}{2} \left[\frac{16}{3} - 4 \right] + C$

$6 = \frac{2}{3} + C$

$C = \frac{16}{3}$

$f(x) \cdot x = \frac{1}{2} \left[\frac{2}{3} (x+1)^{3/2} - 2\sqrt{x+1} \right] + \frac{16}{3}$

Put $x = 8$

$f(8) \cdot 8 = \frac{1}{2} \left[\frac{2}{3} \times 27 - 2 \times 3 \right] + \frac{16}{3}$

$f(8) \cdot 8 = \frac{1}{2} [12] + \frac{16}{3}$

$f(8)\cdot 8 = 6 + \frac{16}{3} = \frac{34}{3}$

$12f(8) = 17$

63. (c) $C_1 : |z| = 4$ then $z\bar{z} = 16$

$z + \frac{1}{z} = z + \frac{\bar{z}}{16}$

$= x + iy + \frac{x - iy}{16}$

$z + \frac{1}{z} = \frac{17x}{16} + i\frac{15y}{16}$

Let $X = \frac{17x}{16}$, $Y = \frac{15}{16}y$

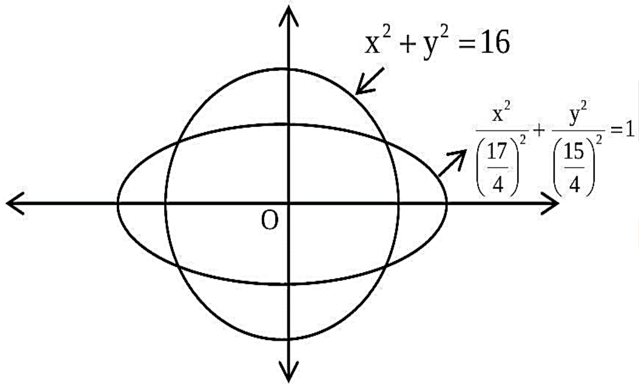
$\frac{X}{\left(\frac{17}{16}\right)} = x, \frac{Y}{\left(\frac{15}{16}\right)} = y$

$\because x^2 + y^2 = 16$

$\frac{X^2}{\left(\frac{17}{16}\right)^2} + \frac{Y^2}{\left(\frac{15}{16}\right)^2} = 16$

$\Rightarrow C_2 : \frac{x^2}{\left(\frac{17}{4}\right)^2} + \frac{y^2}{\left(\frac{15}{4}\right)^2} = 1$

(Ellipse)



Curve C_1 and C_2 intersect at 4 point.

64. (c)

65. (b) Total length of wire $= 20$ m

area of square $(A_1) = \left(\frac{\ell_1}{4}\right)^2$

area of circle $(A_2) = \pi\left(\frac{\ell_2}{2\pi}\right)^2$

Let $S = 2A_1 + 3A_2$

$$S = \frac{\ell_1^2}{8} + \frac{3\ell_2^2}{4\pi}$$

$\because\ \ell_1 + \ell_2 = 20$ then

$$1 + \frac{d\ell_2}{d\ell_1} = 0$$

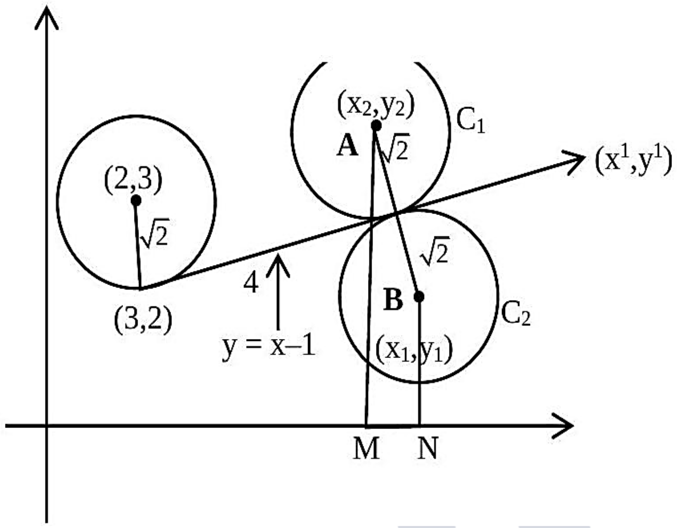
$$\frac{d\ell_2}{d\ell_1} = -1$$

$$\frac{ds}{d\ell_1} = \frac{\ell_1}{4} + \frac{6\ell_2}{4\pi} \cdot \frac{d\ell_2}{d\ell_1} = 0$$

$$= \frac{\ell_1}{4} = \frac{6\ell_2}{4\pi}$$

$$= \frac{\pi\ell_1}{\ell_2} = \frac{6}{1}$$

66. (a)



(x', y') point lies on line $y = x - 1$ have distance 4 unit from (3,2).

$$x' = \frac{4}{\sqrt{2}} + 3 = 2\sqrt{2} + 3$$

$$y' = \frac{4}{\sqrt{2}} + 2 = 2\sqrt{2} + 2$$

Slope of line AB is -1 .

i.e. $= \tan\theta = -1$ then $\sin\theta = \frac{1}{\sqrt{2}}, \cos\theta = -\frac{1}{\sqrt{2}}$

for point A and B

$$x = \pm\sqrt{2}\left(\frac{-1}{\sqrt{2}}\right) + (2\sqrt{2} + 3)$$

$$y = \pm\sqrt{2}\left(\frac{1}{\sqrt{2}}\right) + (2\sqrt{2} + 2)$$

for point A we take +ve sign

$(x_2, y_2) = (2\sqrt{2} + 2,\ 2\sqrt{2} + 3)$

for point B we take -ve sign

$(x_1, y_1) = (2\sqrt{2} + 4, 2\sqrt{2} + 1)$

$MN = |x_2 - x_1| = 2$

$AM + BN = 2\sqrt{2} + 3 + 2\sqrt{2} + 1 = 4 + 4\sqrt{2}$

area of trapezium $= \frac{1}{2} \times 2 \times (4 + 4\sqrt{2})$

$= 4(1 + \sqrt{2})$

67. (b) Probability $= \frac{{}^3C_2 + {}^6C_2}{{}^2C_2 + {}^3C_2 + {}^4C_2 + {}^5C_2 + {}^6C_2}$

$$= \frac{10 + 15}{1 + 3 + 6 + 10 + 15}$$
$$= \frac{5}{7}$$

68. (a) equation of parabola which have focus $\left(-\frac{1}{2}, 0\right)$ and directrix $y = -\frac{1}{2}$ is

$$\left(x + \frac{1}{2}\right)^2 = \left(y + \frac{1}{4}\right)$$
$$y = f(x) = (x^2 + x)$$

$\because S = \left\{x \in R: \tan^{-1}\left((\sqrt{f(x)}) + \sin^{-1}(\sqrt{f(x) + 1}) = \frac{\pi}{2}\right)\right\}$

$\tan^{-1}(\sqrt{f(x)}) + \sin^{-1}(\sqrt{f(x) + 1}) = \frac{\pi}{2}$

$f(x) \geq 0 \& \sqrt{f(x) + 1}$ can not greater then 1 , so f(x) must be 0

i.e. $f(x) = 0$

$\Rightarrow x^2 + x = 0$

$x(x + 1) = 0$

$x = 0, x = -1$

S contain 2 element.

69. (b)

$|\vec{a}+\vec{b}+\vec{c}|=|\vec{a}+\vec{b}-\vec{c}|, \vec{b}\cdot\vec{c}=0$

$|\vec{a}+\vec{b}+\vec{c}|^2=|\vec{a}+\vec{b}-\vec{c}|^2$

$|\vec{a}|^2+|\vec{b}|^2+|\vec{c}|^2+2\vec{a}\cdot\vec{b}+2\vec{b}\cdot\vec{c}+2\vec{a}\cdot\vec{c}$

$=|\vec{a}|^2+|\vec{b}|^2+|\vec{c}|^2+2\vec{a}\cdot\vec{b}-2\vec{b}\cdot\vec{c}-2\vec{a}\cdot\vec{c}$

$2\vec{a}\cdot\vec{b}+2\vec{b}\cdot\vec{c}+2\vec{a}\cdot\vec{c}=2\vec{a}\cdot\vec{b}-2\vec{b}\cdot\vec{c}-2\vec{a}\cdot\vec{c}$

$\vec{a}\cdot\vec{b}+\vec{a}\cdot\vec{c}=\vec{a}\cdot\vec{b}-\vec{a}\cdot\vec{c}$

$\vec{a}\cdot\vec{c}=0$ (B is incorrect)

$|\vec{a}+\lambda\vec{c}|^2\geq|\vec{a}|^2$

$|\vec{a}|^2+\lambda^2|\vec{c}|^2+2\lambda\vec{a}\cdot\vec{c}\geq|\vec{a}|^2$

$=\lambda^2c^2\geq 0$

True $\forall\lambda\in R$ (A is correct)

70. (d) $\int_{\frac{\pi}{3}}^{\frac{\pi}{2}}\frac{(2+3\sin x)}{\sin x(1+\cos x)}dx$

$$=\int_{\frac{\pi}{3}}^{\frac{\pi}{2}}\frac{2}{\sin x+\sin x\cos x}dx+\int_{\frac{\pi}{3}}^{\frac{\pi}{2}}\frac{3}{1+\cos x}dx$$

$$=I_1+I_2$$

$$I_1=\int_{\frac{\pi}{3}}^{\frac{\pi}{2}}\frac{2dx}{\sin x(1+\cos x)}=2\int_{\frac{\pi}{3}}^{\frac{\pi}{2}}\frac{\left(1+\tan^2\frac{x}{2}\right)dx}{2\tan\frac{x}{2}\times\left(1+\frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}\right)}$$

$$=2\int_{\frac{\pi}{3}}^{\frac{\pi}{2}}\frac{\left(1+\tan^2\frac{x}{2}\right)\left(1+\tan^2\frac{x}{2}\right)dx}{2\tan\frac{x}{2}\times 2}$$

$$=2\int_{\frac{\pi}{3}}^{\frac{\pi}{2}}\frac{\sec^2\frac{x}{2}\left(1+\tan^2\frac{x}{2}\right)}{4\tan\frac{x}{2}}dx$$

Let, $\tan\frac{x}{2}=t$ then $\sec^2\frac{x}{2}\times\frac{1}{2}dx=dt$

$$=2\int_{\frac{1}{\sqrt{3}}}^{1}\frac{1+t^2}{2t}dt$$

$$=\left[\ell nt+\frac{t^2}{2}\right]_{\frac{1}{\sqrt{3}}}^{1}$$

$$=\left[\frac{1}{2}-\ln\frac{1}{\sqrt{3}}-\frac{1}{6}\right]$$

$$I_1=\left[\ln\sqrt{3}+\frac{1}{3}\right]$$

$$I_2=3\int_{\frac{\pi}{3}}^{\frac{\pi}{2}}\frac{dx}{1+\cos x}=3\int_{\frac{\pi}{3}}^{\frac{\pi}{2}}\frac{1-\cos x}{\sin^2 x}dx$$

$$I_2 = 3[\operatorname{cosec} x - \cot x]_{\frac{\pi}{3}}^{\frac{\pi}{2}} = 3 - \sqrt{3}$$

$$I_1 + I_2 = \ln\sqrt{3} + \frac{1}{3} + 3 - \sqrt{3}$$

$$= \frac{10}{3} + \ln\sqrt{3} - \sqrt{3}$$

71. (d) Let $\vec{b}_1 =< -2,0,1 > \vec{a}_1 = (5, \lambda, -\lambda)$

$\vec{b}_2 =< 1,1,-1 > \vec{a}_2 = (-1,1,4)$

Normal vector of both line is $\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -2 & 0 & 1 \\ 1 & 1 & -1 \end{vmatrix}$

$\hat{i}(-1) - \hat{j}(a) + \hat{k}(-2)$

$\vec{b}_1 \times \vec{b}_2 =< -1,-1,-2 >$

$\vec{a}_1 - \vec{a}_2 =< 6, \lambda - 1, -\lambda - 4 >$

Shortest distance $d = \left|\frac{(\vec{a}_2 - \vec{a}_1) \times (\vec{b}_1 \times \vec{b}_2)}{|\vec{b}_1 \times \vec{b}_2|}\right|$

$$2\sqrt{6} = \left|\frac{< 6, \lambda - 1, -\lambda - 4 > \times < -1,-1,-2 >}{\sqrt{(1)^2 + (1)^2 + (2)^2}}\right|$$

$$12 = |-6 - \lambda + 1 + 2\lambda + 8|$$

$$|\lambda + 3| = 12$$

$$\lambda = 9, -15$$

$$\lambda = 9 (\because \lambda \geq 0)$$

$\because (\alpha, \beta, \gamma)$ lies on line L then

$$\frac{\alpha - 5}{-2} = \frac{\beta - 9}{0} = \frac{\gamma + 9}{1} = K$$

$\alpha = 5 - 2K, \beta = 9K, \gamma = -9 + K$

$\alpha + 2\gamma, = 5 - 2K - 18 + 2K = -13 \neq 24$

Therefore $\alpha + 2\gamma = 24$ is not possible.

72. (c)

$x + y + z = 6$

$\alpha x + \beta y + 7z = 3$

$x + 2y + 3z = 14$

equation (3) - equation (1)

$y + 2z = 8$

$y = 8 - 2z$

From (a) $x = -2 + z$

Value of x and y put in equation (2)

$\alpha(-2+z) + \beta(8-2z) + 7z = 3$

$-2\alpha + \alpha z + 8\beta - 2\beta z + 7z = 3$

$(\alpha - 2\beta + 7)z = 2\alpha - 8\beta + 3$

if $\alpha - 2\beta + 7 \neq 0$ then system has unique solution

if $(\alpha - 2\beta + 7 = 0)$ and $2\alpha - 8\beta + 3 \neq 0$ then system has no solution

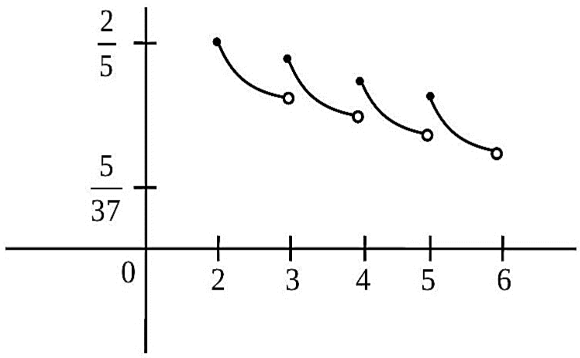
if $(\alpha - 2\beta + 7 = 0)$ and $2\alpha - 8\beta + 3 = 0$ then system has infinite solution

73. (c)

$$f(x) = \frac{[x]}{1+x^2},\ x = \in [2,6]$$

$$f(x) = \begin{cases} \dfrac{2}{1+x^2} & x \in [2,3) \\ \dfrac{3}{1+x^2} & x \in [3,4) \\ \dfrac{4}{1+x^2} & x \in [4,5) \\ \dfrac{5}{1+x^2} & x \in [5,6) \end{cases}$$

$\because$ f(x) is $\downarrow$ in $x \in [2,6)$



range is $\left(\frac{5}{37}, \frac{2}{5}\right]$

74. (b) $(a, b)\ R\ (c, d) \Leftrightarrow ad(b-c) = bc(a-d)$

For reflexive

(a, b) R (a, b)

$\Rightarrow ab(b-a) \neq ba(a-b)$

R is not reflexive

For symmetric:

$(a,b)R(c,d) \Rightarrow ad(b-c) = bc(a-d)$

then we check

$(c, d)\ R(a,b) \Rightarrow cb(d-a) = ad(c-b)$

$\Rightarrow cb(a-d) = ad(b-c)$

R is symmetric:

For transitive:

$\because$ $(2,3)R(3,2)$ and $(3,2)R(5,30)$

But $(2,3)$ is not related to $(5,30)$

R is not transitive.

75. (b)

76. (a) Four term of G.P. $\frac{a}{r^3}, \frac{a}{r}, ar, ar^3$

$$\frac{a}{r^3} + \frac{a}{r} + ar + ar^3 = 126$$

$$\frac{a}{r^3} \cdot \frac{a}{r} \cdot ar \cdot ar^3 = 1296$$

$$a^4 = 1296$$

$$a = 6$$

$$\frac{6}{r^3} + \frac{6}{r} + 6r + 6r^3 = 126$$

$$\left(r + \frac{1}{r}\right) + r^3 + \frac{1}{r^3} = 21$$

$$\left(r + \frac{1}{r}\right) + \left(r + \frac{1}{r}\right)^3 - 3\left(r + \frac{1}{r}\right) = 21$$

Let $r + \frac{1}{r} = t$

$$t^3 - 2t = 21$$

$$\Rightarrow t = 3$$

$$r + \frac{1}{r} = 3$$

$$r^2 - 3r + 1 = 0$$

$$r = \frac{3 \pm \sqrt{9-4}}{2}$$

$$r = \frac{3 \pm \sqrt{5}}{2}$$

Sum of common ratio $= \frac{9}{4} + \frac{5}{4} + \frac{3\sqrt{5}}{2} + \frac{9}{4} + \frac{5}{4} - \frac{3\sqrt{5}}{2}$

$= 7$

77. (c)

$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & -1 \\ 0 & 12 & -3 \end{bmatrix}$

$A^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & -1 \\ 0 & 12 & -3 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & -1 \\ 0 & 12 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & -1 \\ 0 & 12 & -3 \end{bmatrix}$

$A^3 = A^4 = A^5 \ldots = A$

$(A + I)^{11} = {}^{11}C_0\, A^{11} + {}^{11}C_1\, A^{10} + {}^{11}C_2\, A^9 + \cdots {}^{11}C_{11} I$

$== ({}^{11}C_0 + {}^{11}C_1 + {}^{11}C_2 + \cdots {}^{11}C_{10})A + I$

$= (2^{11} - 1)A + I$

$= 2047\, A + I$

Sum of diagonal element $= 2047(1 + 4 - 3) + 3$

$= 4097$

78. (b)

$$\begin{aligned}
&\sqrt{x^2 - 4x + 3} + \sqrt{x^2 - 9} = \sqrt{4x^2 - 14x + 6} \\
&\sqrt{(x-3)(x-1)} + \sqrt{(x-3)(x+3)} = \sqrt{4x^2 - 12x - 2x + 6} \\
&\Rightarrow \sqrt{x-3}(\sqrt{x-1} + \sqrt{x+3}) = \sqrt{4x(x-3) - 2(x-3)} \\
&\Rightarrow \sqrt{x-3}(\sqrt{x-1} + \sqrt{x+3}) = \sqrt{(4x-2)(x-3)} \\
&\Rightarrow \sqrt{x-3}(\sqrt{x-1} + \sqrt{x+3} - \sqrt{2(2x-1)}) = 0 \\
&\sqrt{x-3} = 0 \text{ or } \sqrt{x-1} + \sqrt{x+3} - \sqrt{2(2x-1)} = 0 \\
&x = 3 \text{ or } \sqrt{x-1} + \sqrt{x+3} = \sqrt{2(2x-1)} \\
&x - 1 + x + 3 + 2\sqrt{(x-1)(x+3)} = 4x - 2 \\
&\Rightarrow 2\sqrt{(x-1)(x+3)} = 2x - 4 \\
&\Rightarrow (x-1)(x+3) = (x-2)^2 \\
&\Rightarrow x^2 + 2x - 3 = x^2 + 4 - 4x \\
&\Rightarrow 6x = 7 \\
&x = \frac{7}{6} (\text{ not possible })
\end{aligned}$$

Number of real root $= 1$

79. (c)

$\sin^{-1}\frac{\alpha}{17} + \cos^{-1}\frac{4}{5} - \tan^{-1}\frac{77}{36} = 0, 0 < \alpha < 13$

$$\sin^{-1}\frac{\alpha}{17}=\tan^{-1}\frac{77}{36}-\tan^{-1}\frac{3}{4}$$

$$=\tan^{-1}\left(\frac{\frac{77}{36}-\frac{3}{4}}{1+\frac{77}{36}\cdot\frac{3}{4}}\right)$$

$$\sin^{-1}\frac{\alpha}{17}=\tan^{-1}\left(\frac{8}{15}\right)=\sin^{-1}\left(\frac{8}{17}\right)$$

$$\frac{\alpha}{17}=\frac{8}{17}$$

$\alpha=8$

$\sin^{-1}(\sin 8)+\cos^{-1}(\cos 8)$

$=3\pi-8+8-2\pi$

$=\pi$

80. (d)

$\alpha\in(0,1),\beta=\log_e(1-\alpha)$

$$P_n(x)=x+\frac{x^2}{2}+\frac{x^3}{3}+\cdots..+\frac{x^n}{n},x\in(0,1)$$

$$\int_0^\alpha\frac{t^{50}-1+1}{1-t}dt$$

$$-\int_0^\alpha\frac{1-t^{50}}{1-t}dt+\int_0^\alpha\frac{1}{1-t}dt$$

$$-\int_0^\alpha(1+t+t^2+\cdots..t^{49})dt-[\ln(1-t)]_0^\alpha$$

$$-\left[t+\frac{t^2}{2}+\frac{t^3}{3}+\cdots..\frac{t^{50}}{50}\right]_0^\alpha-\ln(1-\alpha)$$

$$-\left[\alpha+\frac{\alpha^2}{2}+\frac{\alpha^3}{3}+\cdots..+\frac{\alpha^{50}}{50}\right]-\ln(1-\alpha)$$

$-P_{50}(\alpha)-\ln(1-\alpha)$

$-(\beta+P_{50}(\alpha))$

81. (2)

$$T_{r+1}={}^{30}C_r\left(x^{\frac{2}{3}}\right)^{30-r}\left(\frac{2}{x^3}\right)^4$$

$$={}^{30}C_r2^rx^{\frac{60-11r}{3}}$$

$$\frac{60-11r}{3}<0$$

$11r>60$

$$r>\frac{60}{11}$$

$r=6$

$T_7={}^{30}C_6 2^6x^{-2}$ then

$\beta={}^{30}C_6\times 2^6\in N$

$\alpha=2$

82. **(72)**

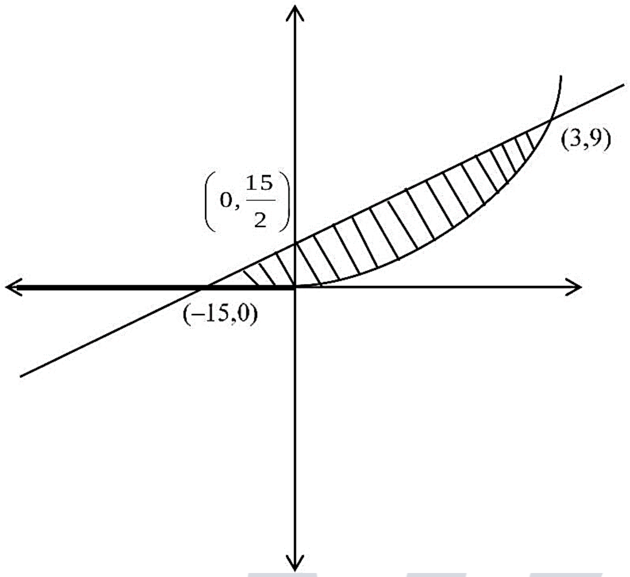
$$f(x)=\frac{x+|x|}{2}=\begin{cases}x & x\ge 0\\ 0 & x<0\end{cases}$$

$$g(x)=\begin{cases}x^2 & x\ge 0\\ x & x<0\end{cases}$$

$$Fog(x)=f\{g(x)\}=\begin{cases}g(x) & g(x)\ge 0\\ 0 & g(x)<0\end{cases}$$

$$fog(x)=\begin{cases}x^2 & x\ge 0\\ 0 & x<0\end{cases}$$

given lines are $2y-x=15$ and $y=0$



$$\text{Area}=\int_0^3\left(\frac{x+15}{2}-x^2\right)dx+\frac{1}{2}\times\frac{15}{2}\times 15$$

$$=\left.\frac{x^2}{4}+\frac{15x}{2}-\frac{x^3}{3}\right]_0^3+\frac{225}{4}$$

$$=\frac{9}{4}+\frac{45}{2}-9+\frac{225}{4}$$

Area $=72$

83. **(710)** 4-digit number which are less then 2800 are 1000 - 2799

Number which are divisible by 3

$$2799=1002+(n-1)3$$
$$n=600$$

Number which are divisible by 11 in 1000 - 2799

$=$ (Number which are divisible by 11 in $1-2799$)

(Number which are divisible by 11 in 1 - 999)

$= \left[\frac{2799}{11}\right] - \left[\frac{999}{11}\right]$

$= 254 - 90$

$= 164$

Number which are divisible by 33 in 1000 - 2799

= (Number which are divisible by 33 in $1 - 2799$) $-$ (Number which are divisible by 33 in 1 - 999)

$= \left[\frac{2799}{33}\right] - \left[\frac{999}{33}\right]$

$= 84 - 30 = 54$

total number $= n(c) + n(11) - n(33)$

$= 600 + 164 - 54 = 710$

84. (5)

x_i	f_i	$d_i = x_i - 5$	$(f_i d_i)^2$	$f_i d_i$
2	3	-3	27	-9
3	6	-2	24	-12
4	16	-1	16	-16
5	α	0	0	0
6	9	1	9	9
7	5	2	20	10
8	6	3	54	18

$$\sigma^2 = \frac{\sum f_i d_i^2}{\sum f_i} - \left(\frac{\sum f_i d_i}{\sum f_i}\right)^2$$

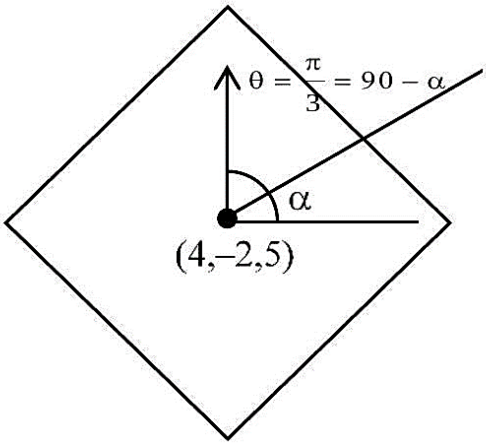
$$= \frac{150}{45 + \alpha} - 0 = 3$$

$\Rightarrow 150 = 135 + 3\alpha$

$\Rightarrow 3\alpha = 15$

$\Rightarrow \alpha = 5$

85. (9)



$\cos\theta = \frac{\langle 1,1,2\rangle \cdot \langle 2,-1,1\rangle}{6}$

$= \frac{2-1+2}{6} = \frac{1}{2}$

$\theta = \frac{\pi}{3}$

then $\frac{\pi}{2} - \alpha = \frac{\pi}{3}$

$\alpha = \frac{\pi}{6}$

$(\tan^2\theta)(\cot^2\alpha) = (\sqrt{3})^2 \times (\sqrt{3})^2 = 9$

86. (2997)

87. (36)

$|\vec{a} \times \vec{b}|^2 = |\vec{a}|^2|\vec{b}|^2 - (\vec{a}\cdot\vec{b})^2$

$48 = 14 \times 6 - (\vec{a}\cdot\vec{b})^2$

$(\vec{a}\cdot\vec{b})^2 = 84 - 48$

$(\vec{a}\cdot\vec{b})^2 = 36$

88. (180)

Point on line L is $(2\lambda+1, -\lambda-1, \lambda+3)$

If above point is intersection point of line L and plane then

$2(2\lambda+1) + (-\lambda-1) + 3(\lambda+3) = 16$

$\lambda = 1$

Point $P = (3, -2, 4)$

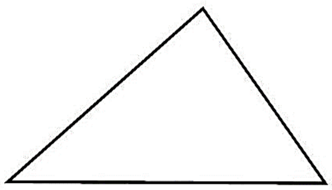
Dr of QR $= < 2\lambda, -\lambda, \lambda+6 >$

Dr of L $= < 2, -1, 1 >$

$4\lambda + \lambda + \lambda + 6 = 0$

$\lambda = -1$

$Q = (-1, 0, 2)$



$\overrightarrow{QR} = 2\hat{i} - \hat{j} - 5\hat{k}$

$\overrightarrow{QP} = 4\hat{i} - 2\hat{j} + 2\hat{k}$

$$\overrightarrow{QR} \times \overrightarrow{QP} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & -5 \\ 4 & -2 & 2 \end{vmatrix} = -12\hat{i} - 24\hat{j}$$

$$\alpha = \frac{1}{2} \times \sqrt{144 + 576}$$

$$\alpha^2 = \frac{720}{4} = 180$$

$\alpha^2 = 180$

89. (8)

Given that

$a_5 = 2a_7$

$a_1 + 4\,d = 2(a_1 + 6\,d)$

$a_1 + 8\,d = 0$

$a_1 + 10d = 18$

$a_1 = -72, d = 9$

$a_{18} = a_1 + 17d = -72 + 153 = 81$

$a_{10} = a_1 + 9d = 9$

$$12\left(\frac{\sqrt{a_{11}} - \sqrt{a_{10}}}{d} + \frac{\sqrt{a_{12}} - \sqrt{a_{11}}}{d} + \cdots \ldots . \frac{\sqrt{a_{18}} - \sqrt{a_{17}}}{d}\right)$$

$$= 12\left(\frac{\sqrt{a_{18}} - \sqrt{a_{10}}}{d}\right)$$

$$= \frac{12 \times (9 - 3)}{9} = 8$$

90. (9)

$5^{99} = 5^4 5^{95}$
$= 625(5^5)^{19}$
$= 625(3125)^{19}$
$= 625(3124 + 1)^{19}$
$= 625(11\lambda + 1)$
$= 11\lambda \times 625 + 625$
$= 11\lambda \times 625 + 616 + 9$
$= 11 \times k + 9$
Remainder $= 9$