1. (b)

Emitter	Base	Collector
Moderate Size	Thin	Thick
Maximum Doping	Minimum Doping	Moderate Doping

2. (b)

$$E = \frac{hc}{\lambda} = \frac{1240}{350} = 3.54\text{eV}$$

If $E > \phi$, photo electrons will emit.

A will not emit and B will emit.

3. (a) At constant Pressure,

$$Q = nCpdT = 735\text{ J}$$

$$\Delta U = nCvdT = \frac{735}{(Cp/Cv)} = \frac{735}{8}$$

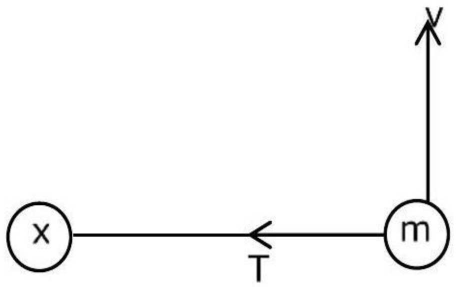
$$\Delta U = \frac{735}{(7/5)} = 525\text{ J}$$

4. (a)

$$L = mvr = [M^1\ L^2\ T^{-1}]$$
$$\tau = rF = [M^1\ L^2\ T^{-2}]$$
$$\text{Stress } = \frac{F}{A} = [M^1\ L^{-1}\ T^{-2}]$$

$$\text{Pressure Gradient} = \frac{dp}{dx} = [M^1\ L^{-2}\ T^{-2}]$$

5. (c)



$$T = \frac{mv^2}{\ell}$$

$400 = \frac{1 \times v^2}{1}$

$V = 20\ m/s$

6. (d) $d_{app} = \frac{d}{\mu} = \frac{h}{(5/3)}$

$\text{Shift} = h\frac{-3h}{5} = 30$

$h = 75\ cm$

7. (a)

$\alpha_v = \frac{NAB}{KR}\alpha\frac{N}{R}$

$\alpha_I = \frac{NAB}{K}\alpha N$

$N \uparrow, \alpha_I \uparrow, \frac{N}{R} \rightarrow \text{Constant}$

$\Delta\alpha_v = 0$

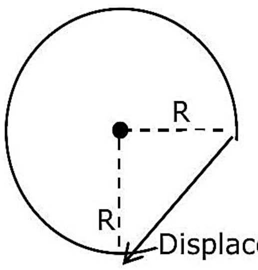
8. (b)

$w = \frac{2\pi}{T} = \frac{2\pi}{4} = \frac{\pi}{2}\text{rad/s}$

$\theta = wt$

$\theta = \frac{\pi}{2} \times 3$

$\theta = \frac{3\pi}{2}\text{rad}$



Displacement = $\sqrt{2}R = 10\sqrt{2}m$

9. (b)

$H = I^2Rt$

$\frac{H_1}{H_2} = \left(\frac{I_1}{I_2}\right)^2 = \left(\frac{4}{16}\right)^2$

$H_2 = 16H_1$

10. (c) Length Remains Same.

$\ell = N(2\pi r_1) = n(2\pi r_2)$

$$\frac{B_1}{B_2} = \frac{\left(N\frac{\mu_0 I}{2r_1}\right)}{\left(n\frac{\mu_0 I}{2r_2}\right)} = \frac{N}{n}\left(\frac{r_2}{r_1}\right) = \frac{N}{n}\left(\frac{N}{n}\right)$$

$$\frac{B_1}{B_2} = \left(\frac{N}{n}\right)^2$$

11. (a) $g_h = \frac{g}{\left(1+\frac{h}{R}\right)^2}$

$h = 9R$

$$g_h = \frac{g}{(1+9)^2} = \frac{g}{100}$$

$$w_h = \frac{mg}{100} = \frac{w}{100}$$

12. (c)

$$R\alpha\frac{n^2}{z}$$

$$\frac{R_1}{R_2} = \left(\frac{n_1}{n_2}\right)^2 = \left(\frac{2}{3}\right)^2$$

$$R_2 = \frac{9R}{4} = 2.25R$$

13. (b) For Adiabatic process,

$$P_1 V_1^\gamma = P_2 V_2^\gamma$$
$$\left(\frac{8}{27}\right)^\gamma = \frac{16}{81}$$
$$\left(\frac{2}{3}\right)^{3\gamma} = \left(\frac{2}{3}\right)^4$$
$$3\gamma = 4$$
$$\gamma = \frac{4}{3} = \frac{Cp}{Cv}$$

14. (d)

$$V = \sqrt{\frac{Y}{\rho}}$$

$$V = \sqrt{\frac{3.2 \times 10^{11}}{8 \times 10^3}} = \sqrt{0.4 \times 10^8}$$

$$V = \sqrt{40 \times 10^6}$$
$$V = 6.32 \times 10^3\ m/s$$

15. (d)

$v = u + at$

$$0 = 20 - \mu g(5)$$

$$\mu = \frac{2}{5} = 0.4$$

16. (a) Statement -1 is correct.

In Modulation Amplitude of carrier wave is increased.

17. (c) $\omega = 628 \mathrm{rad/s}$

$X_L = \omega L = 628 \times 5 \times 10^{-3}$

$X_L = 3.14\Omega$

18. (a) By Hooke's Law,

$$Y = \frac{FL}{A\Delta L}$$

$F, \Delta L \rightarrow$ Same

$$\frac{Y_1 A_1}{L_1} = \frac{Y_2 A_2}{L_2}$$

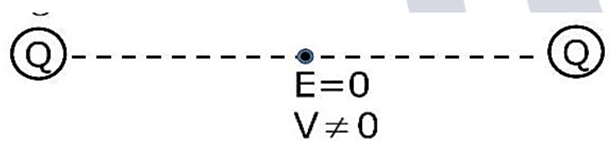
$$\frac{Y_1}{Y_2} = \frac{3 \times 10^{-5}}{2.5 \times 10^{-5}} \times \frac{5}{6} = \frac{1}{1}$$

19. (a)

20. (c) Electric field is a Vector Quantity.

Electric Potential is a Scalar Quantity.

Eg.



21. (25) $R = 80\Omega, X_L = 100\Omega, X_c = 40\Omega$

$$Z = \sqrt{R^2 + (x_L - X_C)^2}$$

$$Z = \sqrt{80^2 + 60^2} = 100\Omega$$

$$I_0 = \frac{V_0}{Z} = \frac{2500}{100} = 25 \text{ A}$$

22. (20)

In Range is same.

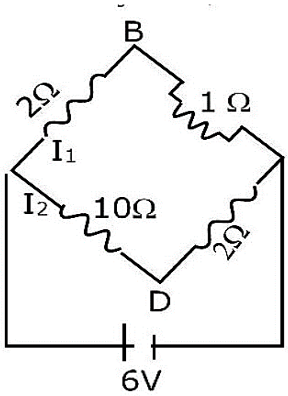
$\alpha + \beta = 90°$
$\alpha = 60°$
$\beta = 30°$

$$H_1 + H_2 = \frac{u_1^2 \sin^2 60°}{2g} + \frac{u_2^2 \sin^2 30°}{2g}$$

$$= \frac{u^2}{2g}\left(\frac{3}{4} + \frac{1}{4}\right) \quad [u_1 = u_2]$$

$$H_1 + H_2 = \frac{(40)^2}{20} = 80 \text{ m}$$

23. (1) At steady state, C → open Circuit

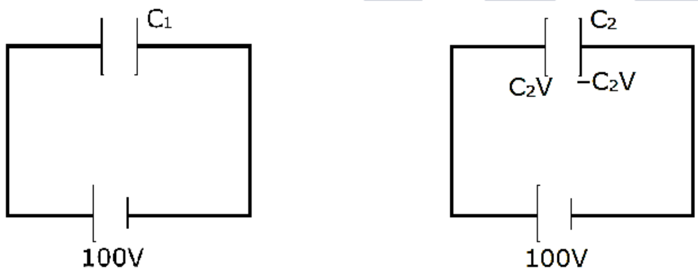


$$I_1 = \frac{6}{3} = 2 \text{ A}$$

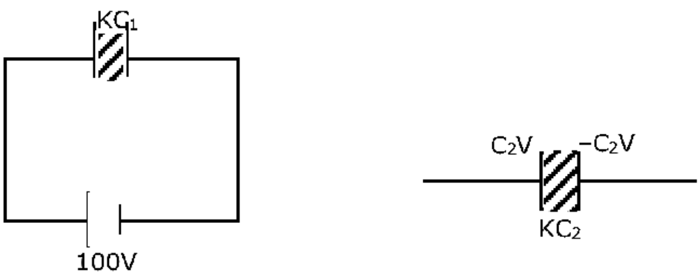
$$I_2 = \frac{6}{12} = \frac{1}{2} \text{ A}$$

$V_B + 2I_1 - 10I_2 = V_D$
$V_B - V_D = 5 - 4 = 1 \text{ V}$

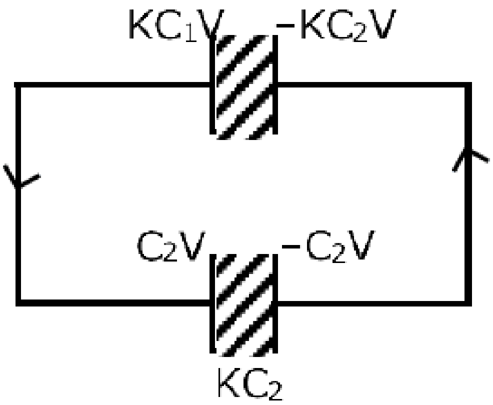
24. (55)



(1)



(2)



(3)

By charge conservation

$$Q_1 = Q_2$$
$$KC_1\,V + C_2\,V = (KC_1 + KC_2)V_{common}$$
$$V_{common} = \frac{(K+1)CV}{2KC} = \frac{K+1}{2\,K}\,V$$
$$V_{common} = \frac{11}{20} \times 100 = 55\text{ V}$$

25. (48)

$d = 0.35\text{ mm}, D = 7\text{ m}$

To Coincide, $n_1\left(\frac{\lambda_1 D}{d}\right) = n_2\left(\frac{\lambda_2 D}{d}\right)$

$$\frac{n_1}{n_2} = \frac{\lambda_2}{\lambda_1} = \frac{6}{8} = \frac{3}{4}$$

3^{rd} Maxima of λ_1 and 4^{th} Maxima of λ_2 will coincide.

$$Y = \frac{3\lambda_1 D}{d} = \frac{3 \times 800 \times 10^{-9} \times 7}{35 \times 10^{-5}}$$

$Y = 3 \times 160 \times 10^{-4}\text{ m}$

$Y = 48\text{ mm}$

26. (5)

$V = \sqrt{2\,g(20)}$

$eV = \sqrt{2gh}$

$$\frac{1}{e} = \sqrt{\frac{20}{h}}$$

$$h = 20e^2 = 20\left(\frac{1}{2}\right)^2$$

$$h = 5\ \mathrm{m}$$

27. (136)

$$BE = 13.6 \times \frac{z^2}{n^2}$$
$$BE = 13.6 \times \left(\frac{3}{3}\right)^2 = 13.6\mathrm{eV}$$
$$BE = 136 \times 10^{-1}\mathrm{eV}$$
$$x = 136$$

28. (300)

$$P_{used} = 0.7 \times 2000 = 1400\ \mathrm{W}$$
$$P = \frac{ms\Delta T}{t}$$
$$t = \frac{2 \times 4200 \times 50}{1400}$$
$$t = 300\mathrm{sec}$$

29. (5)

$$M_1 = M_2$$
$$S_1(\pi R_1^2 t_1) = S_2(\pi R_2^2 t_2)$$
$$\frac{R_1^2}{R_2^2} = \frac{5}{3} \times \frac{0.5}{1} = \frac{5}{6}$$
$$I = \frac{MR^2}{4}$$
$$\frac{I_1}{I_2} = \left(\frac{R_1}{R_2}\right)^2 = \frac{5}{6}$$

30. (20)

$$y_1 = 10\sin\left(\omega t + \frac{\pi}{3}\right)$$
$$y_2 = 10\left[\sin\omega t \times \frac{1}{2} + \frac{\sqrt{3}}{2}\cos\omega t\right]$$
$$y_2 = 10\sin\left(\omega t + \frac{\pi}{3}\right)$$

y_1 and y_2 are in same phase

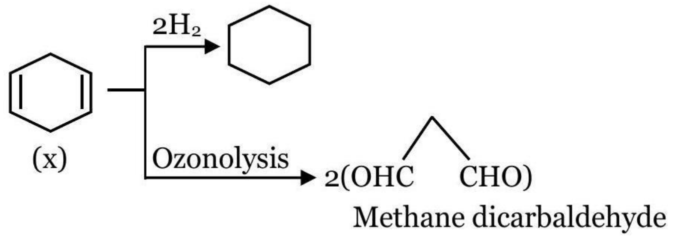
$$A_r = A_1 + A_2 = 20\ \mathrm{cm}$$

31. (a) Van Arkel method is used for refining of Ti, Zr, Hf, Bi, B

32. (b) d-block elements have more first I.E. than group 2 elements due to poor shielding of d-orbitals

33. (d)

34. (a)



35. (b)

(A) Value of i is different for both the solutions.

(B) True

(C) Osmosic takes place from hypotonic to hypertonic solution.

(D) $d = \frac{100}{\frac{1000}{4.09} \times \frac{32}{98}} \cong 1.26 gm/ml$

(E) Positively charged sol will be form.

36. (a) Due to back bonding Lewis acidic strength of Boron halides is $BI_3 > BBr_3 > BCl_3 > BF_3$

37. (c) $C_xH_y + \left(x + \frac{y}{4}\right) O_2 \rightarrow xCO_2 + \frac{y}{2} H_2O$

$x + \frac{y}{4} = 11 \; \frac{y}{2} = 4$

$x = 9 \; y = 8 (C_9H_8)$

38. (a) According to $(n + 1)$ rule Orbital has more value of $(n + l)$ has more energy. If value of some then orbital has more value of n has more energy

39. (d)

40. (d) Due to formation of Cu (II) met borate it gives blue colour

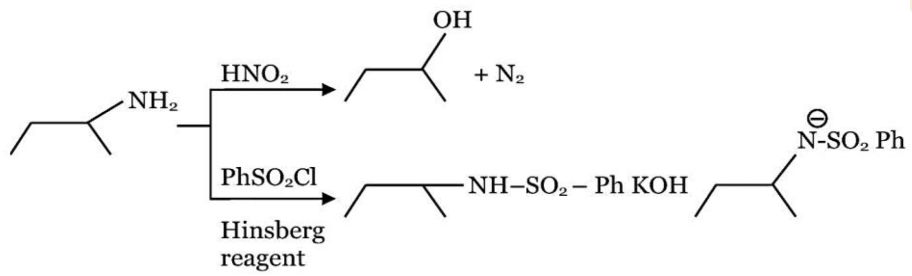
41. (a) Vernonal is a tranquilizer

Prontosil is a antibiotic drug

42. (a) Weak acid - weak base:-

Neither phenolphthalein nor methyl orange is suitable.

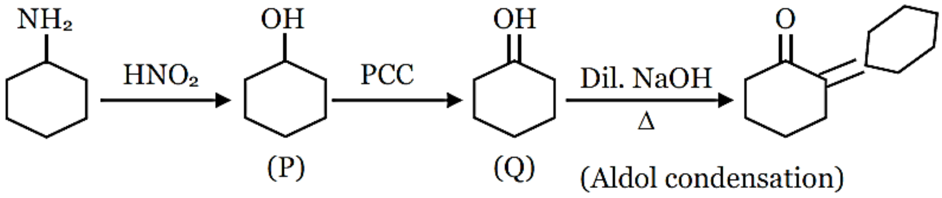
43. (d)



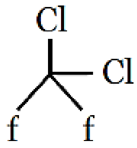
44. (a) Due to presence of CO_2 in air normal rain water is slightly acidic

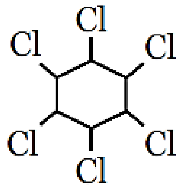
45. (a)

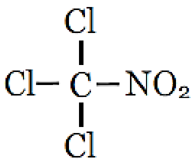
46. (b)

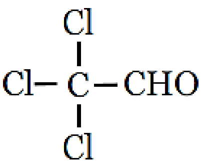


47. (b)

1\. Freon – 12 

2\. Gammaxene  or $C_6H_6Cl_6$

3\. Chloropicrin 

4\. Chloral 

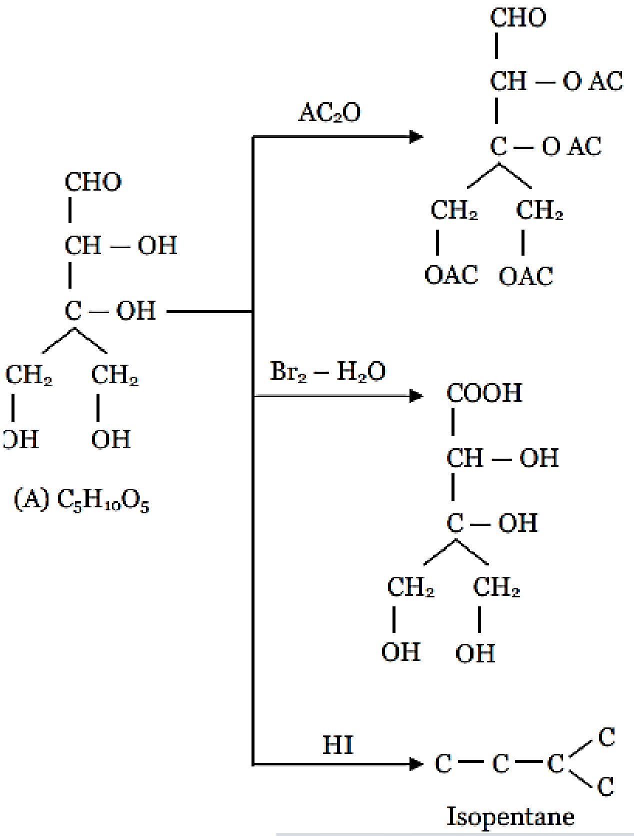
48. (b) Duma's method. The nitrogen containing organic compound, when heated with CuO in a atmosphere of CO_2, yields free N_2 in addition to CO_2 and H_2O.

$$C_xH_yN_z + \left(2x + \frac{y}{2}\right) CuO \rightarrow$$
$$xCOCO_2 + \frac{y}{2} H_2O + \frac{z}{2} N_2 + \left(2x + \frac{y}{2}\right) Cu$$

Traces of nitrogen oxides formed, if any, are reduced to nitrogen by passing the gaseous mixture over heated copper gauze.

49. (d)

50. (c)



51. (17)

$$t = \frac{1}{20}\ln 32$$

$$= \frac{2.303 \times 5 \times 0.3010}{20} = 17.33 \times 10^{-2}$$

$$\approx 17 \times 10^{-2}$$

52. (173)

53. (59)

$M^{2+} \rightarrow x\ M^{3+} \rightarrow (0.83 - x)$

$2x + 3(0.83 - x) = 2$

$x = 2.49 - 2 = 0.49$

$\%\text{ of } M^{2+} = \frac{0.49}{0.83} \times 100 = 59\%$

54. (25)

$$K = \frac{1}{5 \times 10^{-3}}$$

$$\Lambda_m = K \times \frac{1000}{M} = \frac{1}{5 \times 10^{-3}} \times \frac{1000}{0.8}$$

$$= \frac{1000}{40} \times 10^4 = 25 \times 10^4$$

55. (10) $1.434 \times 10^{-3} \mathrm{gm/L}$

$$= \frac{1.434 \times 10^{-3}}{107.9 + 35.5} \mathrm{M} = 10^{-5}\ \mathrm{m}$$

$\mathrm{Ksp} = \mathrm{S}^2 = 10^{-10} \Rightarrow -\log \mathrm{Ksp} = +10$

56. (480)

$$\begin{aligned}
&\mathrm{CFSE} = \left(-\frac{2}{5}x + \frac{3}{5}y\right)\Delta_0 \\
&-96 = \frac{-2}{5} \times 1 \times \Delta_0 \\
&\Delta_0 = 240\ \mathrm{kJ/mole} = \frac{240 \times 10^3}{\mathrm{NA/\ molecule}} \\
&\Delta_0 = \frac{hc}{\lambda abs} \\
&\frac{240 \times 10^3}{6 \times 10^{23}} = \frac{6.4 \times 10^{-34} \times 3 \times 10^8}{\lambda abs} \\
&\lambda ab = \frac{6.4 \times 3 \times 6 \times 10^{-3}}{240 \times 10^3}\ \mathrm{m} \\
&= 4.8 \times 10^{-7}\ \mathrm{m} \\
&= 4.8 \times 10^{-7} \times 10^9\ \mathrm{nm} \\
&= 480\ \mathrm{nm}
\end{aligned}$$

57. (2) K, Rb and Cs form stable super oxides but Cs has ionisation enthalpy less than 400 kJ.

58. (3)

59. (227)

$$\begin{aligned}
&2C + O_2 \rightarrow 2CO \\
&12\ g\ 48gm \\
&1\ \text{mole}\ 1.5\ \text{mole}
\end{aligned}$$

"C" is LR.

Moles of CO formed = 1

Volume of CO $= 1 \times 22.7$

$= 227 \times 10^{-1}$ L

60. (5) $XeF_2, I_3{}^-, C_3O_2, CO_2, BeCl_2$

61. (c)

62. (c)

$$S = \left\{(a,b) \mid a,b \in R - \{0\}, 2 + \frac{a}{b} > 0\right\} \&$$
$$T = \{(a,b) \mid a,b \in R, a^2 - b^2 \in Z\}$$

For S, $2 + \frac{a}{b} > 0 \Rightarrow \frac{a}{b} > -2$

Let $(-1,2) \in S \left(\because -\frac{1}{2} > -2\right)$

$\&(2,-1) \notin S \left(\because \frac{2}{-1}\right.$ not greater than -2)

So, S is not symmetric

For T,

If $(a, b) \in T \Rightarrow a^2 - b^2 \in Z$

$\Rightarrow -(a^2 - b^2) \in Z$

$\Rightarrow b^2 - a^2 \in Z$

$\Rightarrow (b, a) \in T$

So, T is symmetric

63. (c)

$$\alpha > 0$$
$$I = \int_0^\alpha \frac{x}{\sqrt{x+2} - \sqrt{x}} dx = \frac{16 + 2\sqrt{2}}{15}$$
$$I = \int_0^\alpha \frac{x(\sqrt{x+\alpha} + \sqrt{x})}{\alpha} dx$$
$$= \frac{1}{\alpha}\left[\int_0^\alpha x\sqrt{x+\alpha}\sqrt{x+\alpha}dx + \int_0^\alpha x^{3/2}dx\right]$$

$$I_1 = \int_0^{\alpha} (x + \alpha - \alpha)\sqrt{x + \alpha}dx$$

$$= \int_0^{\alpha} (x + \alpha)^{3/2} - \alpha \int_0^{\alpha} (x + \alpha)^{1/2} dx$$

$$= \frac{2}{5}\left[(x + \alpha)^{5/2}\right]^{\alpha} - \frac{\alpha(b)}{3}\left[(x + \alpha)^{3/2}\right]_0^{\alpha}$$

$$= \frac{2}{5}\left[(2\alpha)^{5/2} - \alpha^{5/2}\right] - \frac{2\alpha}{3}\left[(2\alpha)^{3/2} - \alpha^{3/2}\right]$$

$$= \frac{2}{5}(2\alpha)^{5/2} - \frac{2}{5}\alpha^{5/2} - \frac{(2\alpha)^{5/2}}{3} + \frac{2\alpha^{5/2}}{3}$$

$$= (2\alpha)^{5/2}\left[\frac{2}{5} - \frac{1}{3}\right] + 2\alpha^{5/2}\left[\frac{1}{3} - \frac{1}{5}\right]$$

$$= (2\alpha)^{5/2}\left[\frac{1}{15}\right] + 2\alpha^{5/2}\left[\frac{2}{15}\right]$$

$$= \frac{(2\alpha)^{5/2}}{15} + \frac{4\alpha^{5/2}}{15}$$

$$= \frac{4\alpha^{5/2}}{15}[\sqrt{2} + 1]$$

$$I_2 = \int_0^{\alpha} x^{3/2}dx = \frac{2}{5}\left[x^{5/2}\right]_0^{\alpha} = \frac{2}{5}\alpha^{5/2}$$

$$I = \frac{1}{\alpha}(I_1 + I_2)$$

$$I = \frac{1}{\alpha}\left[\frac{4\alpha^{5/2}(\sqrt{2} + 1)}{15} + \frac{2}{5}\alpha^{5/2}\right]$$

$$= \frac{2\alpha^{5/2}}{15\alpha}[2(\sqrt{2} + 1) + 3]$$

$$= \frac{2}{15}\alpha^{3/2}[2\sqrt{2} + 5]$$

$$\frac{16 + 20\sqrt{2}}{15} = \frac{2}{15}\alpha^{3/2}[2\sqrt{2} + 5]$$

$$\alpha^{3/2} = 2\sqrt{2}$$

$$\alpha^3 = 8$$

$$\alpha = 2$$

64. (c)

$$Z = \frac{i - 1}{\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}}$$

$$i - 1 = \sqrt{2}\left(\frac{-1}{\sqrt{2}} + \frac{i}{\sqrt{2}}\right) = \sqrt{2} \cdot e^{i \cdot \frac{3\pi}{4}}$$

$$z = \frac{\sqrt{2} \cdot e^{i \cdot \frac{3\pi}{4}}}{e^{i \cdot \frac{3\pi}{4}}}$$

$$= \sqrt{2} \cdot e^{i\left(\frac{3\pi}{4} - \frac{\pi}{3}\right)}$$

$$= \sqrt{2}e^{\frac{5\pi}{12}i}$$

$$= \sqrt{2}\left(\cos\left(\frac{5\pi}{12}\right) + i\sin\left(\frac{5\pi}{12}\right)\right)$$

65. (b)

$$(3y^2 - 5x^2)ydx + 2x(x^2 - y^2)dy = 0$$
$$2x(x^2 - y^2)dy = (5x^2 - 3y^2)ydx$$
$$\Rightarrow \frac{dy}{dx} = \frac{(5x^2 - 3y^2)y}{2x(x^2 - y^2)}$$

Let $y = tx$

$$\frac{dy}{dx} = t + x\frac{dt}{dx}$$
$$t + x\frac{dt}{dx} = \frac{(5 - 3t^2)t}{2(1 - t^2)}$$
$$x\frac{dt}{dx} = t\left[\frac{5 - 3t^2 - 2 + 2t^2}{2(1 - t^2)}\right]$$

$$= \frac{t}{2}\left[\frac{3 - t^2}{1 - t^2}\right]$$
$$\frac{1}{3}\int \frac{3(1 - t^2)dt}{3t - t^3} = \int \frac{dx}{2x}$$
$$\frac{1}{3}\ln|3t - t^3| = \frac{1}{2}\ln|x| + c$$
$$2\ln|3t - t^3| = 3\ln|x| + 6c$$
$$(3t - t^3)^2 = x^3\lambda$$
$$\left(\frac{3y}{x} - \frac{y^3}{x^3}\right)^2 = x^3\lambda$$
$$(3yx^2 - y^3)^2 = x^9\lambda$$
$$x = 1 \Rightarrow y = 1$$
$$(3 - 1)^2 = 1 \times \lambda$$
$$\lambda = 4$$
$$(3yx^2 - y^3)^2 = 4x^9$$

Let $x = 2$

$$(3y(b) \times 4 - (y(2)^3)^2) = 4(2)^9$$

Taking square root both the sides,

$$|(y(b))^3 - 12y(b)| = 2(2)^{9/2}$$
$$= 2(2)^4\sqrt{2} = 32\sqrt{2}$$

66. (b)

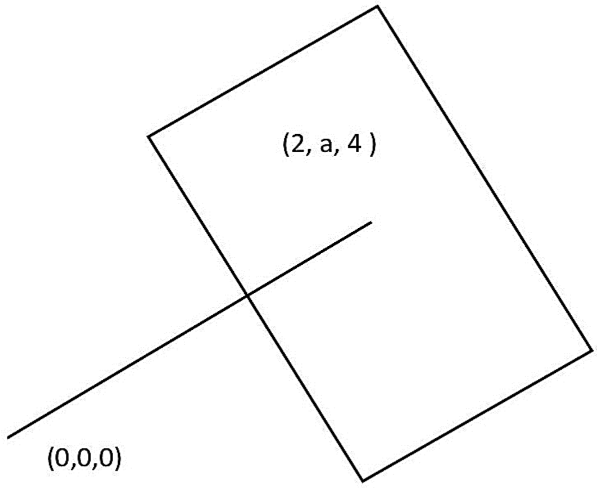
$$\lim_{x\to\infty} \frac{(\sqrt{3x+1} + \sqrt{3x-1})^6 + (\sqrt{3x+1} - \sqrt{3x-1})^6}{\left(x + \sqrt{x^2-1}\right)^6 + \left(x - \sqrt{x^2-1}\right)^6}$$

Taking height power common

$$\lim_{x\to\infty}\frac{x^6\left[\left(\sqrt{3+\frac{1}{x}}+\sqrt{3-\frac{1}{x}}\right)^6+\left(\sqrt{3+\frac{1}{x}}-\sqrt{3-\frac{1}{x}}\right)^6\right]}{x^6\left[\left(1+\sqrt{1-\frac{1}{x^2}}\right)^6+\left(1-\sqrt{1-\frac{1}{x^2}}\right)^6\right]}$$

$$=\frac{(\sqrt{3}+\sqrt{3})^6+(\sqrt{3}-\sqrt{3})^6}{(1+1)^6+(1-1)^6}$$

$$=\frac{(2\sqrt{3})^6}{2^6}=(\sqrt{3})^6=27$$



67. (d)

$\vec{n}=(2,a,4)$

Plane is

$2x+ay+4z=4+a^2+16$

$=20+a^2$

$A\left(\frac{20+a^2}{2},0,0\right)$

$B\left(0,\frac{20+a^2}{a},0\right)$

$C\left(0,0,\frac{20+a^2}{4}\right)$

$\frac{1}{6}\times\frac{(20+a^2)^3}{8a}=144=2^4\times3^2$

$(20+a^2)^3=2^8 3^3 a$

$20+a^2=(4a)^{\frac{1}{3}}(12)$

$a=2$ satisfies above equation

So, $2x+2y+4z=24$

$X+Y+2z=12$

(A) (0,6,3)

(B) (0,4,4)

(C) (2,2,4)

(D) (3,0,4)

68. (c) $x^2 - 6x + 10 = (x-3)^2 + 1 \geq 1$

So, $x = 3$ is the only element in the Domain

So, $\alpha x^2 + \beta x + \sin^{-1}(x^2 - 6x + 10) + \cos^{-1}(x^2 - 6x + 10) = 0$

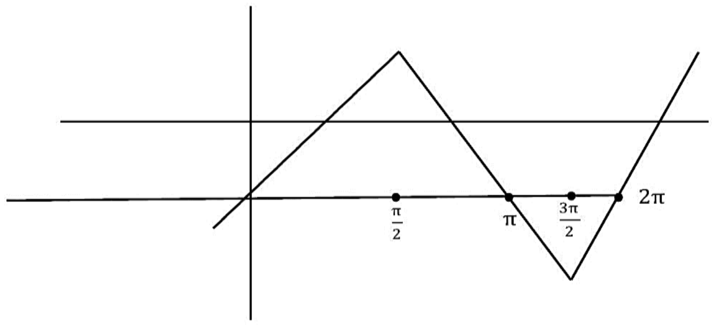
$9\alpha + 3\beta + \frac{\pi}{2} = 0$

$\mathrm{Sin}^{-1}(\sin\theta) - \cos^{-1}(\sin\theta) > 0$

$\sin^{-1}(\sin\theta) - \left(\frac{\pi}{2} - \sin^{-1}(\sin\theta)\right) > 0$

$2\sin^{-1}(\sin\theta) > \frac{\pi}{2}$

$\sin^{-1}(\sin\theta) > \frac{\pi}{4}$



So, $\theta \in \left(\frac{\pi}{4}, \frac{3\pi}{4}\right)$

$\alpha - \beta = \frac{3\pi}{4} - \frac{\pi}{4} = \frac{\pi}{2}$

& $9\alpha + 3\beta = \frac{-\pi}{2}$

$3\alpha + \beta = \frac{-\pi}{6}$

Adding (a) & (b)

$4\alpha = \frac{\pi}{2} - \frac{\pi}{6} = \frac{2\pi}{6}$

$\alpha = \frac{\pi}{12}$

69. (d)

$m_A = 40\ S \cdot d_A = \alpha > 0\ n_A = 100$
$m_B = 55\ S \cdot d_B = 30 - \alpha n_B = n$
$m_{AVB} = 50\ \ \text{Variance AVB} = 350 n_{AVB} = 100 + n$
$A = \{x_1, \ldots \ldots, x_{100}\} B = \{y_1 \ldots, y_n\}$
$\sum x_i = 4000$
$\sum y_i = 55n$
$\sum (x_i + y_i) = 50(100 + n)$
$4000 + 55n = 5000 + 50n$

Using formula of standard deviation

$5n = 1000n = 200$

$\alpha^2 = \frac{\sum x_i{}^2}{100} - (40)^2\ (30 - \alpha)^2 \frac{\sum y_i^2}{200} - (55)^2$

$\sum x_i{}^2 = 100(1000 + \alpha^2)$

$\sum y_i^2 = 200((55)^2 + (30 - \alpha)^2)$

$350 = \frac{\sum\left(x_i^2 + y_i^2\right)}{300} - (50)^2$

$\sum x_i{}^2 + \sum y_i^2 = ((50)^2 + 350)300$

$160000 + 100\alpha^2 + 200(55)^2 + 200(30 - \alpha)^2$

$(50)^2 300 + 350 \times 300$

$1600 + \alpha^2 + 6050 + 2(30 - \alpha)^2 = 7500 + 1050$

$\alpha^2 + 1800 - 120\alpha + 2\alpha^2 - 900 = 0$

$3\alpha^2 - 120\alpha + 900 = 0$

$\alpha^2 - 40\alpha + 300 = 0$

$(\alpha - 10)(\alpha - 30) = 0$

$\alpha = 10$ or $\alpha = 30$

if $\alpha = 10 \text{Var}\, A = 100 \& \text{Var} B = 400$

$\text{Var}_A + \text{V}_{\text{arB}} = 500$

70. (d)

$f(x) = |x^2 - x + 1| + [x^2 - x + 1]$

$x \in [-1,2]$ Here $x^2 - x + 1 > 0, \forall x \in R$

Minimum value of $x^2 - x + 1$ occurs at $a = \frac{1}{2} \in [-1,2]$

So, $\operatorname{Minf}(x)=f\left(\frac{1}{2}\right)$

$=\frac{3}{4}+\left[\frac{3}{4}\right]=\frac{3}{4}$

71. (b)

$F_1 F_2=2ae=(1+\sqrt{2})-(1-\sqrt{2})=2\sqrt{2}$

$ae=\sqrt{2}$

$e=\sqrt{2}$

$\Rightarrow a=1 \Rightarrow b=1 (\because e=\sqrt{2})$

L.L.R. $=\frac{2\,b^2}{a}=\frac{2(1)^2}{1}=2$

72. (d) $a_7=3 a_1 a_4$ minimum

$a+6d=3$

$a(a+3\,d) \rightarrow$ minimum

$S_n=0 \Rightarrow \frac{n}{2}[na_1+(n-1)d]=0$

$2a_1+(n-1)d=0$

Let $a(a+3d)$ is minimum

$f(d)=(3-6\,d)(3-6\,d+3\,d)$

$f(d)=(3-6d)(3-3d)$

$=18d^2-27d+9$ is minimum at $d=\frac{27}{2\times18}=\frac{9\times3}{2\times9\times2}=\frac{3}{4}$

So, $d=\frac{3}{4}$

$a_1+6d=3$

$a_1=3-6\left(\frac{3}{4}\right)=3-\frac{9}{2}=-\frac{3}{2}$

Putting $a_1=\frac{-3}{2}$ & $d=\frac{3}{4}$ in (a)

$2\left(\frac{-3}{2}\right)+(n-1)\left(\frac{3}{4}\right)=0$

$\frac{3}{4}(n-1)=3$

$n-1=4$

$n=5$

$ni-4a_{n(n+2)}$

$n=5$ so $n!=5!=120$

$\&a_{5(7)}=a_{35}=\frac{-3}{2}+(34)\left(\frac{3}{4}\right)$

$=\frac{-3}{2}+\frac{51}{2}$

$=\frac{48}{2}=24$

$5!-4(24)=24$

73. (d)

$2\alpha+9\beta+8\gamma=0$

$10\alpha+3\beta+4\gamma=0$

$8\alpha+8\beta+8\gamma=0$

$\alpha+\beta+\gamma=0$

$\gamma=-\alpha-\beta$

$2\alpha+9\beta-8\alpha-8\beta=0$

$\beta=6\alpha$

$\gamma=-\alpha-6\alpha=-7\alpha$

$(\alpha, 6\alpha, -7\alpha)$ Satisfies the above system of equation

$2\alpha+4(6\alpha)+3(-7\alpha)=5$

$5\alpha=5$

$\alpha=1$

$\beta=6$

$\gamma=-7$

$6\alpha+9\beta+7\gamma=6+54-49=11$

74. (c)

$\vec{r}\times\vec{b}=\vec{c}\times\vec{b}$
$\vec{r}\times\vec{b}-\vec{c}\times\vec{b}=\vec{0}$
$(\vec{r}-\vec{c})\times\vec{b}=\vec{0}$
$\vec{r}-\vec{c}=\lambda\vec{b}$
$\vec{r}=\lambda\vec{b}+\vec{c}=(\lambda+5)\hat{\imath}-(\lambda+3)\hat{\jmath}+(2\lambda+3)\hat{k}$
$\vec{r}\cdot\vec{a}=0$
$1(\lambda+5)-2(\lambda+3)+3(2\lambda+3)=0$
$5\lambda+8=0\Rightarrow\lambda=\frac{-8}{5}$
$\vec{r}=\frac{17}{5}\hat{\imath}-\frac{7}{5}\hat{\jmath}+\frac{1}{5}\hat{k}$
$25|\vec{r}|^2=17^2+7^2+1^2=339$

75. (d)

$y-\text{intercept}=\frac{-12}{\alpha_1}=1$

$\alpha_1=-12\ \&\ \vec{n}=(8,\alpha_1,\alpha_2)$

$\vec{\ell}=(2,3,5)$

$\vec{n}\cdot\vec{\ell}=0$ ($\because$ plane P& line L are parallel)

$16+3\alpha_1+5\alpha_2=0$

$16-36+5\alpha_2=0$

$5\alpha_2=20$

$\alpha_2=4$
$8x-12y+4z+12=0$

$\Rightarrow 2x-3y+z+3=0$

$(-2,3,-4)$ is a point on the line L distance bet ^{n} the point $(-2,3,-4)$ and the plane P is :-

$d=\frac{|-4-9-4+3|}{\sqrt{2^2+3^2+1^2}}$

$=\frac{14}{\sqrt{14}}=\sqrt{14}$

76. (a) Let $A(4,1,-3)\ \&\ B(2,4,3)$

$\vec{n}=\overrightarrow{AB}=(-2,3,6)$

Plane P is :

$-2(x-1)+3(y+1)+6(z+5)=0$

$-2x+2+3y+3+6z+30=0$

$2x-3y-6z=35$

Distance of P from the point $(3,-2,2)$ is

$$=\frac{|6+6-12-35|}{\sqrt{2^2+3^2+6^2}}$$

$$=\frac{35}{7}=5$$

77. (d)

$((p\wedge q)\Rightarrow(r\vee q))\wedge((p\wedge r)\Rightarrow q)$
$(p\wedge q)\Rightarrow(r\vee q)$
$\sim(p\wedge q)\vee(r\vee q)$
$\sim p\vee\sim q\vee r\vee q$
$\&(p\wedge r)\Rightarrow q$
$\sim(p\wedge r)\vee q$
$\sim p\vee\sim r\vee q$
$(\sim p\vee\sim q\vee r\vee q)\wedge(\sim p\vee\sim r\vee q)$
$\equiv\sim p\vee\sim r\vee q$
If $r=p$
$\sim p\vee\sim p\vee q\rightarrow$ Not tautology
If $r=\sim p$

$\sim p\vee p\vee q\rightarrow$ tautology
If $r=q$
$\sim p\vee\sim q\vee q\rightarrow$ tautology
If $r=\sim q$
$\sim p\vee q\vee q\rightarrow$ Not tautology

78. (d)

$$x^2+y^2-\left(\frac{1+a}{2}\right)x-\left(\frac{1-a}{2}\right)y=0$$

$$x\left(x-\left(\frac{1+a}{2}\right)\right)+y\left(y-\left(\frac{1-a}{2}\right)\right)=0$$

$$y-y_1=m(x-x_1)\ (x_1,y_1)=\left(\frac{1+a}{2},\frac{1-a}{2}\right)$$

$\&x+y=0$

$-x-y_1=mx-mx_1$

$mx_1-y_1=(1+m)x$

$$x=\frac{mx_1-y_1}{1+m}\ \&\ y=\frac{y_1-mx_1}{1+m}$$

$$\frac{\frac{y_1}{2}-y}{\frac{x_1}{2}-x}=-\frac{1}{m}$$

$$\frac{\frac{y_1}{2}-\left(\frac{y_1-mx_1}{1+m}\right)}{\frac{x_1}{2}-\left(\frac{mx_1-y_1}{1+m}\right)}=-\frac{1}{m}$$

$$=\frac{(1+m)y_1-2y_1+2mx_1}{(1+m)x_1-2mx_1+2y_1}=-\frac{1}{m}$$

$$\frac{m(y_1+2x_1)-y_1}{-mx_1+x_1+2y_1}=-\frac{1}{m}$$

$$m^2(y_1+2x_1)-my_1=mx_1-x_1-2y_1$$

$$m^2(y_1+2x_1)-(y_1+x_2)m+x_1+2y_1=0$$

$$D>0$$

$$(y_1+x_1)^2-4(y_1+2x_1)(x_2+2y_1)>0$$

$$x_1=\frac{1+a}{2}, y_1=\frac{1-a}{2}$$

$$x_1+y_1=1$$

$$y_1+2x_1=\frac{1-a}{2}+1+a=\frac{3}{2}-\frac{a}{2}=\frac{3-a}{2}$$

$$x_1+2y_1=\frac{1+a}{2}+1-a=\frac{3}{2}+\frac{a}{2}=\frac{3+a}{2}$$

$$1-4\left(\frac{3-a}{2}\right)\left(\frac{3+a}{2}\right)>0$$

$$1-(9-a^2)>0$$

$$a^2-8>0$$

$$a^2>8 \rightarrow (8,\infty)$$

79. (a)

$$\sqrt{x}\phi(x)=\int_{\frac{\pi}{4}}^{x}\left(4\sqrt{2}\sin t-3\phi'(t)\right)dt$$

Differentiating w.r.t. x,

$$\frac{1}{2\sqrt{x}}\phi(x)+\sqrt{x}\phi'(x)=4\sqrt{2}\sin x-3\phi'(x)$$

$$(\sqrt{x}+3)\phi'(x)+\frac{1}{2\sqrt{x}}\phi(x)=4\sqrt{2}\sin x$$

$$\phi'(x)+\frac{1}{2\sqrt{x}(\sqrt{x}+3)}\phi(x)=\frac{4\sqrt{2}\sin x}{\sqrt{x}+3}$$

Put $x=\left(\frac{\pi}{4}\right)$

$$\phi'\left(\frac{\pi}{4}\right)+0=\frac{4\sqrt{2}\times\frac{1}{\sqrt{2}}}{\sqrt{\frac{\pi}{4}}+3}\left(\because \phi\left(\frac{\pi}{4}\right)=0\right)$$

$$\phi'\left(\frac{\pi}{4}\right)=\frac{4\times 2}{\sqrt{\pi}+6}=\frac{8}{\sqrt{\pi}+6}$$

80. (d)

$$y=\frac{x^2+2x+1}{x^2-8x+12}$$
$$x^2y-8xy+12y=x^2+2x+1$$
$$x^2(y-1)-(8y+2)x+12y-1=0$$
$$D\geq 0$$
$$(8y+2)^2-4(y-1)(12y-1)\geq 0$$
$$4(4y+1)^2-4(y-1)(12y-1)\geq 0$$
$$16y^2+8y+1-(12y^2-13y+1)\geq 0$$
$$4y^2+21y\geq 0$$
$$4y\left[y+\frac{21}{4}\right]\geq 0$$

$$\left(-\infty,\frac{-21}{4}\right]\cup[0,\infty)$$

81. (204)

82. (84) $|A|=2$

$$\left|\mathrm{Adj}\left(2\mathrm{Adj}(2\,A^{-1})\right)\right|=2^{84}$$

$$|2\mathrm{Adj}(2\,A^{-1})|^{n-1}=2^{84}$$

$$(2^n|\mathrm{Adj}(2\,A^{-1})|)^{n-1}=2^{84}$$

$$(2^n|2^{n-1}\mathrm{Adj}(A^{-1})|)^{n-1}=2^{84}$$

$$(2^n\times(2^{n-1})^n|\mathrm{Adj}(A^{-1})|)^{n-1}=2^{84}$$

$$\left(2^n\times 2^{n(n-1)}\times|A^{-1}|\right)^{n-1}=2^{84}$$

$$\left(2^n\times 2^{n(n-1)}\times\left(\frac{1}{2}\right)^{n-1}\right)^{n-1}=2^{84}$$

$$\left(2^{n+n^2-n-n+1}\right)^{n-1}=2^{84}$$

$$\left(2^{n^2-n+1}\right)^{n-1}=2^{84}$$

$$(n^2-n+1)(n+1)=84$$

83. (98)

$$\left(\frac{x^{5/2}}{2}-\frac{4}{x^{\ell}}\right)^9$$

$$T_{r+1}={}^9C_r\left(\frac{x^{5/2}}{2}\right)^{9-r}\left(\frac{-4}{x^{\ell}}\right)^r$$

$= {}^9C_r\left(\frac{1}{2}\right)^{9-r}(-4)^r x^{\frac{45-5r}{2}-\ell r}$

For constant term

$$= {}^9C_r\left(\frac{1}{2}\right)^{9-r}(-4)^r = -84$$
$$= {}^9C_r\left(\frac{1}{2}\right)^{9-r}(-1)^r 2^{2r} = -84$$
$$= {}^9C_r 2^{r-9}2^{2r}(-1)^r = -84$$
$$= {}^9C_r 2^{3r-9}(-1)^r = -84$$
$$\Rightarrow r = 3$$
$$\frac{45-5r}{2} - \ell r = 0$$
$$\frac{45-15}{2} - 3\ell = 0$$
$$15 = 3\ell$$
$$\ell = 5$$

For coefficient of x^{-15} is

$$\frac{45-5r}{2} - 5r = -15$$

$$45 - 5r - 10r = -30$$

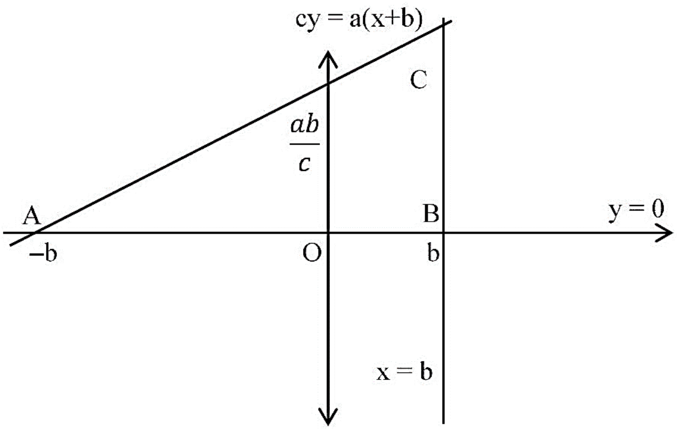
$$75 = 15r$$

$$r = 5$$

For coefficient of x^{-15} is ${}^9C_5\left(\frac{1}{2}\right)^4(-4)^5$

$$\frac{9\times8\times7\times6}{4\times3\times2\times1}\times\frac{1}{2^4}\times 2^{10}\times(-1)$$
$$= 9\times2\times7\times2^6\times(-1)$$
$$= 2^7(-63) = 2^\alpha\beta$$
$$\alpha = 7, \beta = -63$$
$$|\alpha\ell - \beta| = |7(5) + 63| = |35 + 63|$$
$$|\alpha\ell - \beta| = 98.$$

84. (146) tangent at $P(b, c) my^2 = 2ax$ is



$$\text{area} = \left|\frac{1}{2} \times 2\,b \times \frac{2ba}{c}\right| = 16$$

$$\frac{2\,b^2 a}{C} = 16$$

$$\frac{b^2 a}{C} = 8$$

$\because$ P(b, c) lies on $y^2 = 2ax$

$C^2 = 2ab$

$$\Rightarrow \frac{b^4 a^2}{c^2} = 64$$

$$\Rightarrow \frac{b^4 a^2}{2ab} = 64$$

$$\Rightarrow b^3 a = 128$$

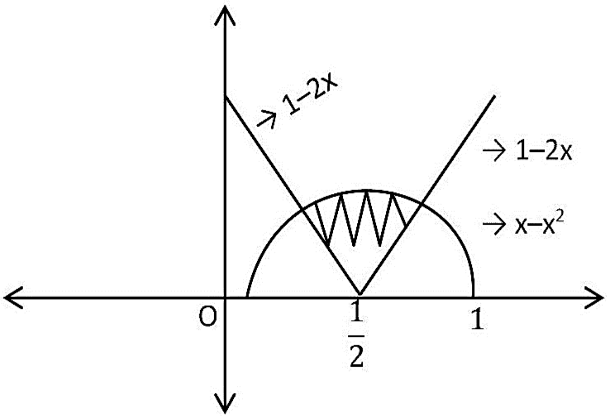
$$\Rightarrow a = \frac{128}{b^3}$$

a can be 128,16,2 then

$S = \{2,16,128\}$

$\sum_{a \in s} a = 146$

85. (125) $|2x - 1| \le y \le |x^2 - x|, 0 \le x \le 1$



$x - x^2 = 1 - 2x$

$x^2 - 3x + 1 = 0$

$x = \frac{3 \pm \sqrt{9-4}}{2} = \frac{3 \pm \sqrt{5}}{2}$

$x = \frac{3-\sqrt{5}}{2}$ as $0 < x < \frac{1}{2}$

$\text{Area} = 2\int_{\frac{3-\sqrt{5}}{2}}^{1/2} [(x - x^2) - (1 - 2x)]dx$

$2\int_{\frac{3-\sqrt{5}}{2}}^{1/2} [3x - x^2 - 1]dx$

$= 2\left[\frac{3x^2}{2} - \frac{x^3}{3} - x\right]_{\frac{3-\sqrt{5}}{2}}^{1/2}$

For $x = \frac{3-\sqrt{5}}{2}$,

$x^2 = 3x - 1$

$x^3 = 3x^2 - x$

$= 3(3x - 1) - x$

$= 8x - 3$

$\frac{3}{2}x^2 - \frac{1}{3}x^3 - x = \frac{3}{2}(3x - 1) - \frac{1}{3}(8x - 3) - x$

$= \frac{9x - 3}{2} - \frac{(8x - 3)}{3} - x$

$= \frac{27x - 9 - (16x - 6)}{6} - x$

$= \frac{11x - 3}{6} - x$

$= \frac{5x - 3}{6}$

For $x = \frac{1}{2}$,

$$\frac{3}{2}x^2 - \frac{1}{3}x^3 - x = \frac{3}{2}\left(\frac{1}{4}\right) - \frac{1}{3}\left(\frac{1}{8}\right) - \frac{1}{2}$$
$$= \frac{9 - 1 - 12}{24}$$
$$= \frac{-4}{24} = -\frac{1}{6}$$

$$\text{Area } = 2\left[-\frac{1}{6}-\left(\frac{\frac{5(3-\sqrt{5})}{2}-3}{6}\right)\right]$$

$$= 2\left[-\frac{1}{6}-\left(\frac{15-5\sqrt{5}-6}{12}\right)\right]$$

$$= 2\left[-\frac{1}{6}-\left(\frac{9-5\sqrt{5}}{12}\right)\right]$$

$$= 2\left[\frac{-2-9+5\sqrt{5}}{12}\right]$$

$$= \frac{5\sqrt{5}-11}{6}$$

$$(6\,A+11)^2 = (5\sqrt{5})^2 = 125$$

86. (5040)

$$\left(\frac{4x}{5}\times\frac{5}{2x^2}\right)^9$$

General term is

$$T_{r+1} = {}^9C_r\left(\frac{4n}{r}\right)^{9-r}\left(\frac{5}{2n^2}\right)^r$$

$$= {}^9C_r\left(\frac{4}{5}\right)^{9-r}\left(\frac{5}{2}\right)^r x^{9-r-2r}$$

For coefficient of term x^{-6} $9-r-2r=-6$

$15 = 3r$

Coefficient of term $^{-6}\ {}^9C_5\left(\frac{4}{5}\right)^4\left(\frac{5}{2}\right)^5$

$$= \frac{9\times 8\times 7\times 6}{4\times 3\times 2\times 1}\times 5\times 2^3$$

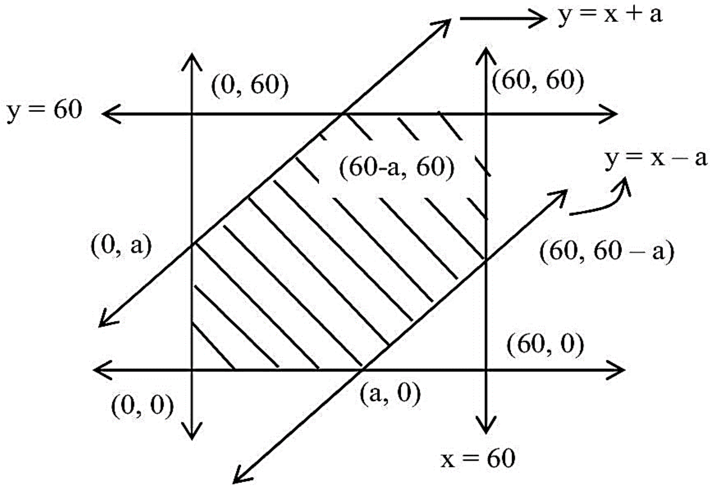
$$= 9\times 2\times 7\times 5\times 8$$

$$= 5040$$

87. (10)

$|x-y| < a \to -a < x-y \;\&\; x-y < a$

$x, y \in [0,60]$



$$P(A)=\frac{\text{Shaded area}}{\text{Total area}}=\frac{(60)^2-\left[\frac{1}{2}(60-a)^2+\frac{1}{2}\times(60-a)^2\right]}{(60)^2}$$

$$P(A)=\frac{(60)^2-(60-a)^2}{(60)^2}$$

$$\frac{11}{36}=\frac{120a-a^2}{3600}$$

$$1100=120a-a^2$$

$$a^2-120a+1100=0$$

$$a^2-110a-10a+1100=0$$

$$a(a-110)-10(a-110)=0=$$

$$(a-10)(a-110)=0$$

Ans. $a=10$

$(\because \text{ for } a=110, P(A)=1)$

88. (45)

$$^{2n+1}P_{n-1}:{}^{2n-1}P_n=11:21$$
$$\frac{(2n+1)!}{(n+2)!}\times\frac{(n-1)!}{(2n-1)!}=\frac{11}{21}$$
$$\Rightarrow\frac{(2n+1)(2n)}{(n+2)(n+1)n}=\frac{11}{21}$$
$$\Rightarrow\frac{2(2n+1)}{(n+2)(n+1)}=\frac{11}{21}$$
$$\Rightarrow 42(2n+1)=11(n^2+3n+2)$$
$$\Rightarrow 84n+42=11n^2+33n+22$$
$$\Rightarrow 11n^2-51n-20=0$$
$$\Rightarrow n=5$$
$$n^2+n+15=25+5+15=45$$

89. (3)

$|\vec{a}| = \sqrt{31}\; 4|\vec{b}| = |\vec{c}| = 2$

$2(\vec{a} \times \vec{b}) = 3(\vec{c} \times \vec{a})$

$\vec{b} \wedge \vec{c} = \frac{2\pi}{3}$

$\vec{a} \times 2\vec{b} = 3\vec{c} \times \vec{a} = -\vec{a} \times 3\vec{c}$

$\vec{a} \times (2\vec{b} + 3\vec{c}) = \vec{o}$

$\vec{a} = \lambda(2\vec{b} + 3\vec{c})$

$|\vec{a}|^2 = \lambda^2|2\vec{b} + 3\vec{c}|^2$

$|\vec{a}|^2 = \lambda^2\left(4|\vec{b}|^2 + 9|\vec{c}|^2 + 12|\vec{b}||\vec{c}|\cos\theta\right)$

$31 = \lambda^2\left(1 + 9(2)^2 + 12|\vec{b} \parallel \vec{c}|\cos\frac{2\pi}{3}\right)$

$31 = \lambda^2\left(1 + 36 - 6 \times \frac{1}{2} \times 2\right)$

$31 = \lambda^2(31)$

$\lambda^2 = 1$

$\Rightarrow \lambda = \pm 1$

$\vec{a} = \pm(2\vec{b} + 3\vec{c})$

$\vec{a} \times \vec{c} = \pm(2\vec{b} + 3\vec{c}) \times \vec{c}$

$= \pm 2(\vec{b} \times \vec{c})$

$|\vec{a} \times \vec{c}|^2 = 4|\vec{b} \times \vec{c}|^2 = 3$

$\vec{a} \cdot \vec{b} = \mp 1$

$\left(\frac{|\vec{a} \times \vec{c}|}{\vec{a} \cdot \vec{b}}\right)^2 = 3$

90. (6952)

$1^2 - 2.3^2 + 3.5^2 - 4.7^2 + 5.9^2 - \cdots + 15.29^2$

$S = \underbrace{15.29^2 - 14.27^2} + \cdots + \underbrace{3.5^2 - 2.3^2} + 1^2$

$(n+1)(2n+1)^2 - n(2n-1)^2$

$n(4n^2 + 4n + 1) + 4n^2 + 4n + 1 - n(4n^2 - 4n + 1)$

$= 12n^2 + 4n + 1$

$S = [\Sigma 12n^2 + 4n + 1 \text{ for } n = 2,4,6,8,10,12,14] + 1$

$$S_1 = \sum_{k=1}^{7} 12(2k)^2 + 4(2k) + 1$$

$$= \sum_{k=1}^{7} [48k^2 + 8k + 1]$$

$$= 48\sum_{k=1}^{7} k^2 + 8\sum_{k=1}^{7} k + \sum_{k=1}^{7} 1$$

$$= \frac{48(7)(8)(15)}{6} + \frac{8(7)(8)}{2} + 7 = 6951$$

$S = 6952$