

1. (a)

$$KE = \frac{f}{2} kT$$

Conceptual

2. (c)

$$Y = \overline{\overline{A} \cdot \overline{B}} = \overline{\overline{A}} + \overline{\overline{B}} = A + B$$

(De-Morgan's law)

3. (a)

$$t = \alpha x^2 + \beta x \text{ (differentiating wrt time)}$$

$$\frac{dt}{dx} = 2\alpha x + \beta$$

$$\frac{1}{v} = 2\alpha x + \beta$$

(differentiating wrt time)

$$-\frac{1}{v^2} \frac{dv}{dt} = 2\alpha \frac{dx}{dt}$$

$$\frac{dv}{dt} = -2\alpha v^3$$

4. (d)

$$\mu = \frac{\sin\left(\frac{A + \delta_m}{2}\right)}{\sin\frac{A}{2}}$$

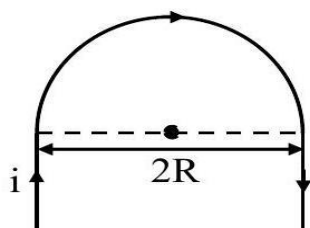
$$\frac{\cos\frac{A}{2}}{\sin\frac{A}{2}} = \frac{\sin\left(\frac{A + \delta_m}{2}\right)}{\sin\frac{A}{2}}$$

$$\sin\left(\frac{\pi}{2} - \frac{A}{2}\right) = \sin\left(\frac{A + \delta_m}{2}\right)$$

$$\frac{\pi}{2} - \frac{A}{2} = \frac{A}{2} + \frac{\delta_m}{2}$$

$$\delta_m = \pi - 2A$$

5. (d)



Note: Direction of magnetic field is in $+\hat{k}$

$$\vec{F} = i\vec{\ell} \times \vec{B}$$

$$\ell = 2R$$

$$\vec{F} = -2iR\hat{j}$$

6. (a)

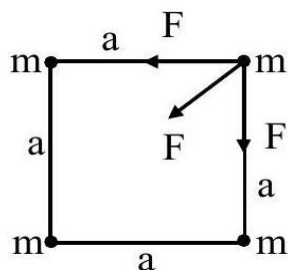
$$\frac{1}{\lambda} = \frac{13.6z^2}{hc} \left[\frac{1}{1^2} - \frac{1}{2^2} \right] \dots$$

$$\frac{1}{\lambda'} = \frac{13.6z^2}{hc} \left[\frac{1}{1^2} - \frac{1}{3^2} \right] \dots$$

On dividing (i) & (ii)

$$\lambda' = \frac{27}{32} \lambda$$

7. (b)



$$F_{\text{net}} = \sqrt{2} F + F'$$

$$F = \frac{Gm^2}{a^2} \text{ and } F' = \frac{Gm^2}{(\sqrt{2}a)^2}$$

$$F_{\text{net}} = \sqrt{2} \frac{Gm^2}{a^2} + \frac{Gm^2}{2a^2}$$

$$\left(\frac{2\sqrt{2} + 1}{32} \right) \frac{Gm^2}{L^2} = \frac{Gm^2}{a^2} \left(\frac{2\sqrt{2} + 1}{2} \right)$$

$$a = 4L$$

8. (d) $PV = nRT$

$$V = \left(\frac{nR}{P} \right) T$$

$$\text{Slope} = \frac{nR}{P}$$

$$\text{Slope} \propto \frac{1}{P}$$

$$(\text{Slope})_2 > (\text{Slope})_1$$

$$P_2 < P_1$$

9. (b)

$$R = \frac{\rho L}{\pi \frac{d^2}{4}}$$

$$\frac{\Delta R}{R} = \frac{\Delta L}{L} + \frac{2\Delta d}{d}$$

$$\frac{\Delta L}{L} = 0.1\% \text{ and } \frac{\Delta d}{d} = 0.1\%$$

$$\frac{\Delta R}{R} = 0.3\%$$

$$10. (a) U_E = \frac{1}{2} \epsilon_0 E^2$$

$$U_E = \frac{1}{2} \times 8.85 \times 10^{-12} \times (50)^2$$

$$= 1.106 \times 10^{-8} \text{ J/m}^3$$

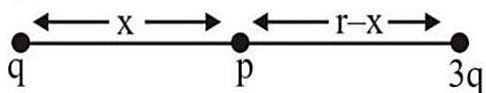
$$11. (a) F = ax^2 + bt^{1/2}$$

$$[a] = \frac{[F]}{[x^2]} = [M^1 L^{-1} T^{-2}]$$

$$[b] = \frac{[F]}{[t^{1/2}]} = [M^1 L^1 T^{-5/2}]$$

$$\left[\frac{b^2}{a} \right] = \frac{[M^2 L^2 T^{-5}]}{[M^1 L^{-1} T^{-2}]} = [M^1 L^3 T^{-3}]$$

12. (c)



$$(\vec{E}_{\text{net}})_P = 0$$

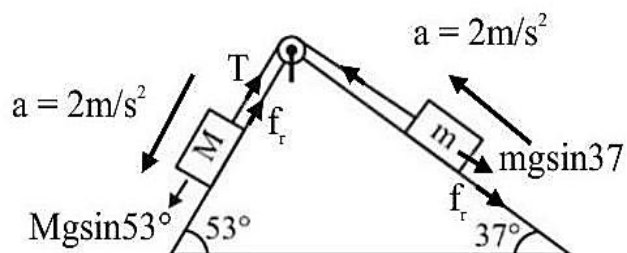
$$\frac{kq}{x^2} = \frac{k \cdot 3q}{(r-x)^2}$$

$$(r-x)^2 = 3x^2$$

$$r-x = \sqrt{3}x$$

$$x = \frac{r}{\sqrt{3} + 1}$$

13. (b)



For M block

$$10 g \sin 53^\circ - \mu(10 g) \cos 53^\circ - T = 10 \times 2$$

$$T = 80 - 15 - 20$$

$$T = 45 \text{ N}$$

For m block

$$T - m g \sin 37^\circ - \mu m g \cos 37^\circ = m \times 2$$

$$45 = 10 m$$

$$m = 4.5 \text{ kg}$$

14. (b)

$$\varepsilon = N \left(\frac{\Delta \phi}{t} \right)$$

$$\Delta \phi = (\Delta B) A$$

$$B_i = 5000 \text{ T},$$

$$B_f = 3000 \text{ T}$$

$$d = 0.02 \text{ m}$$

$$r = 0.01 \text{ m}$$

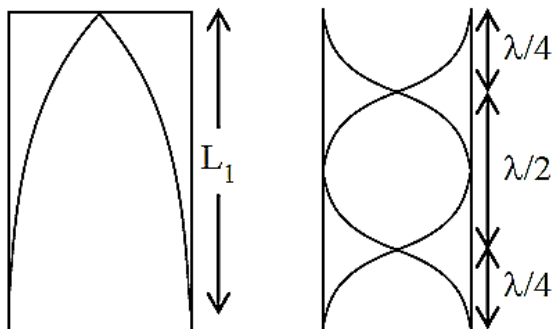
$$\Delta \phi = (\Delta B) A$$

$$= (2000) \pi (0.01)^2 = 0.2 \pi$$

$$\varepsilon = N \left(\frac{\Delta \phi}{t} \right) \Rightarrow 22 = N \left(\frac{0.2 \pi}{2} \right)$$

$$N = 70$$

15. (d)



$$\frac{\lambda}{4} = L_1 \quad 2\left(\frac{\lambda}{2}\right) = \lambda$$

$$v = f\lambda \quad f_2 = \frac{2v}{2L_2}$$

$$v = f_1(4L_1) \quad f_2 = \frac{v}{L_2}$$

$$f_1 = \frac{v}{4L_1}$$

$$f_1 = f_2$$

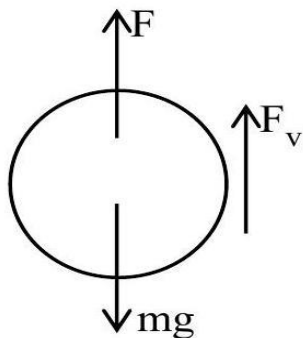
$$\frac{v}{4L_1} = \frac{v}{L_2}$$

$$\Rightarrow L_2 = 4L_1$$

$$60 = 4 \times L_1$$

$$L_1 = 15 \text{ cm}$$

16. (b)



$$mg - F_B - F_v = ma$$

$$\left(\rho \frac{4}{3}\pi r^3\right)g - \left(\rho_L \frac{4}{3}\pi r^3\right)g - 6\pi\eta r v = m \frac{dv}{dt}$$

$$\text{Let } \frac{4}{3}\pi R^3 g(\rho - \rho_L) = K_1 \text{ and } \frac{6\pi\eta r}{m} = K_2$$

$$\frac{dv}{dt} = K_1 - K_2 v$$

$$\int_0^v \frac{dv}{K_1 - K_2 v} = \int_0^t dt$$

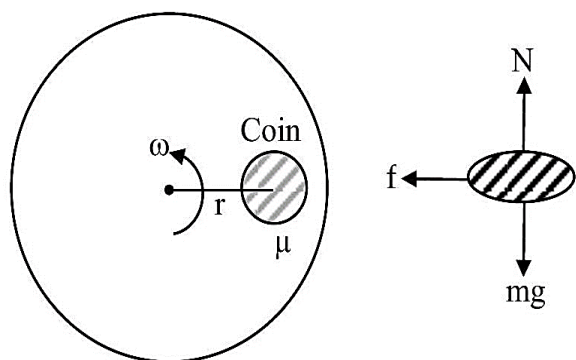
$$-\frac{1}{K_2} \ln[K_1 - K_2 v]_0^v = t$$

$$\ln\left(\frac{K_1 - K_2 V}{K_1}\right) = -K_2 t$$

$$K_1 - K_2 v = K_1 e^{-K_2 t}$$

$$v = \frac{K_1}{K_2} [1 - e^{-K_2 t}]$$

17. (c)



$$N = mg$$

$$f = m\omega^2 r$$

$$f = \mu N$$

$$\mu mg = mr\omega^2$$

$$\omega = \sqrt{\frac{\mu g}{r}}$$

18. (b) Series

$$R_{eq} = R_1 + R_2$$

$$2R(1 + \alpha_{eq}\Delta\theta) = R(1 + \alpha_1\Delta\theta) + R(1 + \alpha_2\Delta\theta)$$

$$2R(1 + \alpha_{eq}\Delta\theta) = 2R + (\alpha_1 + \alpha_2)R\Delta\theta$$

$$\alpha_{eq} = \frac{\alpha_1 + \alpha_2}{2}$$

Parallel:

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$\frac{1}{\frac{R}{2}(1 + \alpha_{eq}\Delta\theta)} = \frac{1}{R(1 + \alpha_1\Delta\theta)} + \frac{1}{R(1 + \alpha_2\Delta\theta)}$$

$$\frac{2}{1 + \alpha_{eq}\Delta\theta} = \frac{2}{1 + \alpha_1\Delta\theta} + \frac{2}{1 + \alpha_2\Delta\theta}$$

$$\frac{2}{1 + \alpha_{eq} \Delta\theta} = \frac{1 + \alpha_2 \Delta\theta + 1 + \alpha_1 \Delta\theta}{(1 + \alpha_1 \Delta\theta)(1 + \alpha_2 \Delta\theta)}$$

$$2[(1 + \alpha_1 \Delta\theta)(1 + \alpha_2 \Delta\theta)]$$

$$= [2 + (\alpha_1 + \alpha_2) \Delta\theta][1 + \alpha_{eq} \Delta\theta]$$

$$2[1 + \alpha_1 \Delta\theta + \alpha_2 \Delta\theta + \alpha_1 \alpha_2 \Delta\theta]$$

=

$$2 + 2\alpha_{eq} \Delta\theta + (\alpha_1 + \alpha_2) \Delta\theta + \alpha_{eq} (\alpha_1 + \alpha_2) \Delta\theta^2$$

Neglecting small terms

$$2 + 2(\alpha_1 + \alpha_2) \Delta\theta = 2 + 2\alpha_{eq} \Delta\theta + (\alpha_1 + \alpha_2) \Delta\theta$$

$$(\alpha_1 + \alpha_2) \Delta\theta = 2\alpha_{eq} \Delta\theta$$

$$\alpha_{eq} = \frac{\alpha_1 + \alpha_2}{2}$$

19. (b)

$$|\vec{p}_1| = |\vec{p}_2|$$

$$KE = \frac{p^2}{2M}; p \text{ same}$$

$$KE \propto \frac{1}{m}$$

$$\frac{KE_1}{KE_2} = \frac{p^2/2M_1}{p^2/2M_2} = \frac{M_2}{M_1}$$

20. (c) $E = \phi + K_{\max}$

$$\phi = \frac{hc}{\lambda_0}$$

$$K_{\max} = eV_0$$

$$8e = \frac{hc}{\lambda} - \frac{hc}{\lambda_0}$$

$$2e = \frac{hc}{3\lambda} - \frac{hc}{\lambda_0}$$

on solving (i) & (ii)

$$\lambda_0 = 9\lambda$$

21. (5)

$$\vec{F} = q(\vec{v} \times \vec{B})$$

$$5e\hat{k} = e(3\hat{i} + 5\hat{j}) \times (B_0\hat{i} + 2B_0\hat{j})$$

$$5e\hat{k} = e(6B_0\hat{k} - 5B_0\hat{k})$$

$$\Rightarrow B_0 = 5 \text{ T}$$

22. (2)

Without dielectric

$$Q = \frac{A\epsilon_0}{d}V$$

with dielectric

$$Q = \frac{A\epsilon_0 V}{d - t + \frac{t}{K}}$$

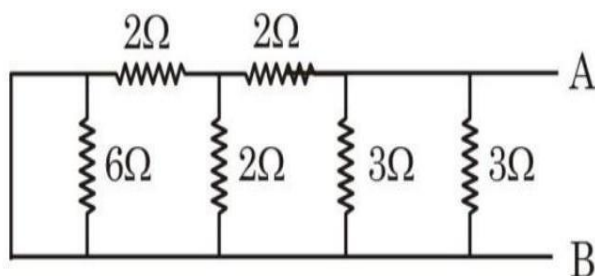
given

$$\frac{A\epsilon_0 V}{d - t + \frac{t}{K}} = (1.25) \frac{A\epsilon_0 V}{d}$$

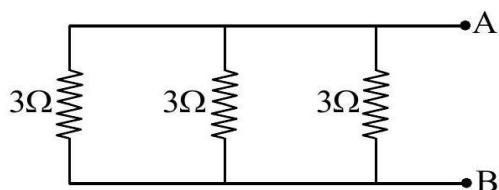
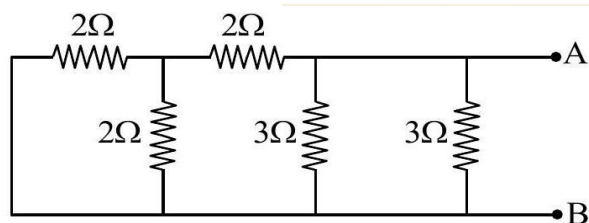
$$\Rightarrow 1.25 \left(3 + \frac{2}{K} \right) = 5$$

$$\Rightarrow K = 2$$

23. (1)



6Ω is short circuit



$$R_{eq} = 3 \times \frac{1}{3} = 1\Omega$$

24. (6) Using work energy theorem

$$W = \Delta KE = 0 - \left(\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 \right)$$

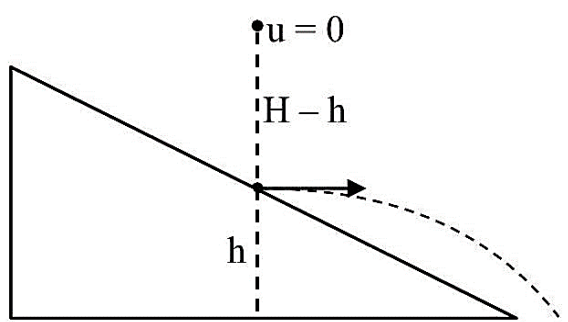
$$W = 0 - \frac{1}{2}mv^2 \left(1 + \frac{K^2}{R^2} \right)$$

$$= -\frac{1}{2} \times 50 \times 0.4^2 \left(1 + \frac{1}{2} \right) = -6 \text{ J}$$

Absolute work = +6 J

$$W = -6 \text{ J} \mid |W| = 6 \text{ J}$$

25. (2)



Total time of flight = T

$$T = \sqrt{\frac{2h}{g}} + \sqrt{\frac{2(H-h)}{g}}$$

$$\text{For max. time} = \frac{dT}{dh} = 0$$

$$\sqrt{\frac{2}{g}} \left(\frac{-1}{2\sqrt{H-h}} + \frac{1}{2\sqrt{h}} \right) = 0$$

$$\sqrt{H-h} = \sqrt{h}$$

$$h = \frac{H}{2} \Rightarrow \frac{H}{h} = 2$$

26. (13)

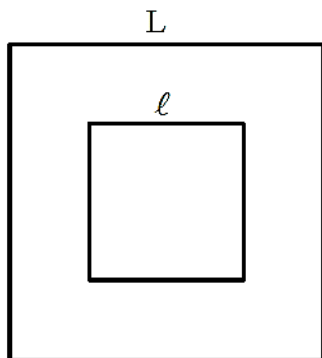
$$\text{For incoherent wave } I_1 = I_A + I_B \Rightarrow I_1 = I_0 + 9I_0 \mid I_1 = 10I_0$$

$$\text{For coherent wave } I_2 = I_A + I_B + 2\sqrt{I_A I_B} \cos 60^\circ$$

$$I_2 = I_0 + 9I_0 + 2\sqrt{9I_0^2} \cdot \frac{1}{2} = 13I_0$$

$$\frac{I_1}{I_2} = \frac{10}{13}$$

27. (128)



Flux linkage for inner loop.

$$\phi = B_{\text{center}} \cdot \ell^2$$

$$= 4 \times \frac{\mu_0 i}{4\pi \frac{L}{2}} (\sin 45 + \sin 45) \ell^2$$

$$\phi = 2\sqrt{2} \frac{\mu_0 i}{\pi L} \ell^2$$

$$M = \frac{\phi}{i} = \frac{2\sqrt{2} \mu_0 \ell^2}{\pi L} = 2\sqrt{2} \frac{\mu_0}{\pi}$$

$$= 2\sqrt{2} \frac{4\pi}{\pi} \times 10^{-7}$$

$$= 8\sqrt{2} \times 10^{-7} \text{ H}$$

$$= \sqrt{128} \times 10^{-7} \text{ H}$$

$$x = 128$$

28. (18)

$$\beta = \frac{-\Delta P}{\frac{\Delta V}{V}}$$

$$\Delta P = -\beta \frac{\Delta V}{V}$$

$$\rho gh = -\beta \frac{\Delta V}{V}$$

$$10^3 \times 10 \times h = -9 \times 10^8 \times \left(-\frac{0.02}{100} \right)$$

$$\Rightarrow h = 18 \text{ m}$$

29. (7)

$$V = \omega \sqrt{A^2 - x^2}$$

$$\text{at } x = \frac{2A}{3}$$

$$v = \omega \sqrt{A^2 - \left(\frac{2A}{3}\right)^2} = \frac{\sqrt{5} A \omega}{3}$$

New amplitude = A'

$$v' = 3v = \sqrt{5} A \omega = \omega \sqrt{(A')^2 - \left(\frac{2A}{3}\right)^2}$$

$$A' = \frac{7A}{3}$$

30. (1)

$$\begin{aligned} E &= \Delta mc^2 \\ &= 0.4 \times 10^{-3} \times (3 \times 10^8)^2 \\ &= 3600 \times 10^7 \text{ kWs} \\ &= \frac{3600 \times 10^7}{3600} \text{ kWh} = 1 \times 10^7 \text{ kWh} \end{aligned}$$

31. (d)

Statement I and II are False

Noble gases have low boiling points

Noble gases are held together by weak dispersion forces.

32. (a)

$$K_c = \frac{\text{Products ion conc.}}{\text{Reactants ion conc.}}$$

$$K_c = \frac{[\text{FeSCN}^{2+}]}{[\text{Fe}^{3+}][\text{SCN}^-]}$$

33. (d) $(\text{CH}_3)_2\text{CO} + \text{CS}_2$ Exhibits positive deviations from Raoult's Law

34. (b) Lead sulphate-white

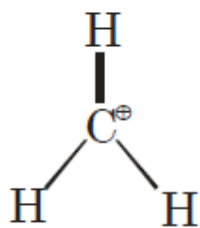
Ammonium sulphide-soluble

Lead iodide-Bright yellow

Ammonium arsinomolybdate-yellow

35. (a) Mn, Ni and Cd metals used in battery industries.

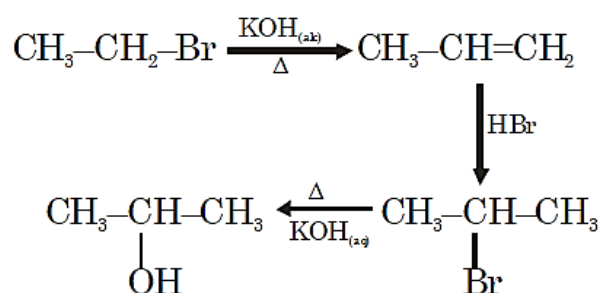
36. (c)



Six electron species

37. (b) Conductivity of electrolytic cell is affected by concentration of electrolyte, nature of electrolyte and nature of solvent.

38. (d)



39. (d) As per NCERT, Assertion (A) and Reason (R) is correct but Reason (R) is not the correct explanation.

40. (b)

Element	$\Delta_{\text{eg}} H(\text{kJ/mol})$
F	-333
S	-200
Br	-325
Ar	+96

41. (d)

A. CrO_4^{2-} is tetrahedral

B. $2\text{Na}_2\text{CrO}_4 + 2\text{H}^+ \rightarrow \text{Na}_2\text{Cr}_2\text{O}_7 + 2\text{Na}^+ + \text{H}_2\text{O}$

C. As per NCERT, green manganate is paramagnetic with 1 unpaired electron.

D. Statement is correct

E. Statement is correct

42. (b) Principle used in chromatography is adsorption.

43. (a)

$$\begin{aligned}
 A &\rightarrow B + C \\
 P_i &0 \quad 0 \\
 P_i - x &x \quad x \\
 P_t &= P_i + x \\
 P_i - x &= P_i - P_t + P_i \\
 &= 2P_i - P_t \\
 K &= \frac{2.303}{t} \log \frac{P_i}{2P_i - P_t}
 \end{aligned}$$

44. (a) Phenol is more acidic than ethanol because conjugate base of phenoxide is more stable than ethoxide.

45. (a) 7-Hydroxyheptan-2-one is correct IUPAC name

46. (d)

B. VBT does not explain stability of complex

C. Hybridisation of $[\text{Ni}(\text{CN})_4]^{-2}$ is dsp^2 .

47. (b) Molecular orbital should have maximum overlap

Symmetry about the molecular axis should be similar

48. (b)

Glucose $\xrightarrow[\Delta]{\text{NaHCO}_3}$ no reaction

Glucose $\xrightarrow[\Delta]{\text{HNO}_3}$ saccharic acid

Glucose $\xrightarrow[\Delta]{\text{HI}}$ n-hexane

Glucose $\xrightarrow[\Delta]{\text{Br}_2}$ Gluconic acid

49. (c) SnO_2 and PbO_2 are amphoteric

50. (b)

51. (78)

CaF_2 does not evolve any gas with concentrated H_2SO_4 .

$\text{NaBr} \rightarrow$ evolve Br_2

$\text{NaNO}_3 \rightarrow$ evolve NO_2

$\text{KI} \rightarrow$ evolve I_2

52. (28) NH_3 act as WFL with Ni^{2+}

$\text{Ni}^{2+} = 3 \text{ d}^8$

↑↓	↑↓	↑↓	↑	↑
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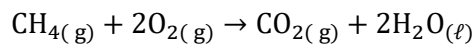
No. of unpaired electron = 2

$$\mu = \sqrt{n(n+2)} = \sqrt{8} = 2.82 \text{ BM}$$

$$= 28.2 \times 10^{-1} \text{ BM}$$

$$x = 28$$

53. (50)



$$n_{\text{CO}_2} = \frac{22}{44} = 0.5 \text{ moles}$$

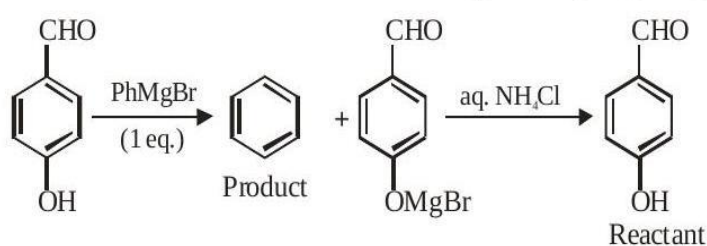
So moles of CH_4 required = 0.5 moles

i.e. 50×10^{-2} mole

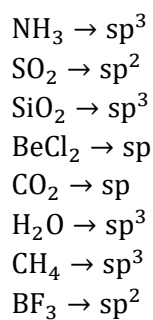
$$x = 50$$

54. (0)

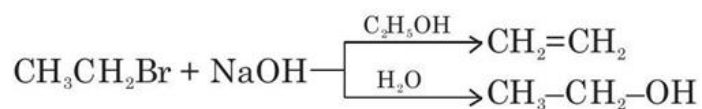
Product benzene has zero hydroxyl group



55. (4)

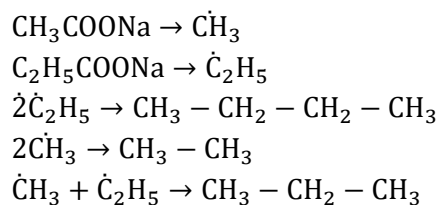


56. (10)

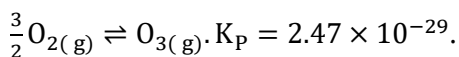


Total number of hydrogen atom in A and B is 10

57. (3)



58. (163)



$$\Delta_r G^\ominus = -RT \ln K_p$$

$$= -8.314 \times 10^{-3} \times 298 \times \ln(2.47 \times 10^{-29})$$

$$= -8.314 \times 10^{-3} \times 298 \times (-65.87)$$

$$= 163.19 \text{ kJ}$$

59. (494)

$$E = \frac{1240}{\lambda(\text{nm})} \text{ eV}$$

$$= \frac{1240}{242} \text{ eV}$$

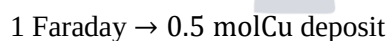
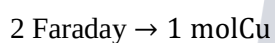
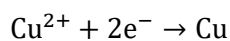
$$= 5.12 \text{ eV}$$

$$= 5.12 \times 1.6 \times 10^{-19}$$

$$= 8.198 \times 10^{-19} \text{ J/atom}$$

$$= 494 \text{ kJ/mol}$$

60. (5)



$$0.5 \text{ mol} = 0.5 \text{ g atom} = 5 \times 10^{-1}$$

$$x = 5$$

61. (a)

$$f(x) = (a + b - 2c)x^2 + (b + c - 2a)x + (c + a - 2b)$$

$$f(x) = a + b - 2c + b + c - 2a + c + a - 2b = 0$$

$$f(a) = 0$$

$$\therefore \alpha \cdot 1 = \frac{c + a - 2b}{a + b - 2c}$$

$$\alpha = \frac{c + a - 2b}{a + b - 2c}$$

$$\text{If, } -1 < \alpha < 0$$

$$-1 < \frac{c+a-2b}{a+b-2c} < 0$$

$$b+c < 2a \text{ and } b > \frac{a+c}{2}$$

therefore, b cannot be G.M. between a and c.

$$\text{If, } 0 < \alpha < 1$$

$$0 < \frac{c+a-2b}{a+b-2c} < 1$$

$$b > c \text{ and } b < \frac{a+c}{2}$$

Therefore, b may be the G.M. between a and c.

62. (d)

$$\text{Put } x = 1$$

$$\therefore a = 1$$

$$b = \lim_{x \rightarrow 0} \frac{\int_0^x \frac{\ln(1+t)}{1+t^{2024}} dt}{x^2}$$

Using L' HOPITAL Rule

$$b = \lim_{x \rightarrow 0} \frac{\ln(1+x)}{(1+x^{2024})} \times \frac{1}{2x} = \frac{1}{2}$$

$$\text{Now, } cx^2 + dx + e = 0, x^2 + x + 4 = 0$$

$$(D < 0)$$

$$\therefore \frac{c}{1} = \frac{d}{1} = \frac{e}{4}$$

63. (a)

$$\frac{x^2}{9} + \frac{y^2}{25} = 1$$

$$a = 3, b = 5$$

$$e = \sqrt{1 - \frac{9}{25}} = \frac{4}{5} \therefore \text{foci} = (0, \pm be) = (0, \pm 4)$$

$$\therefore e_H = \frac{4}{5} \times \frac{15}{8} = \frac{3}{2}$$

Let equation hyperbola

$$\frac{x^2}{A^2} - \frac{y^2}{B^2} = -1$$

$$\therefore B \cdot e_H = 4 \therefore B = \frac{8}{3}$$

$$\therefore A^2 = B^2(e_H^2 - 1) = \frac{64}{9} \left(\frac{9}{4} - 1 \right) \therefore A^2 = \frac{80}{9}$$

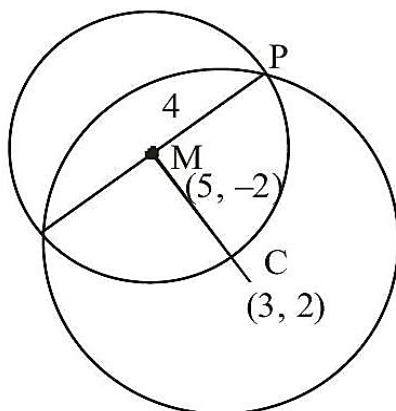
$$\therefore \frac{x^2}{\frac{80}{9}} - \frac{y^2}{\frac{64}{9}} = -1$$

$$\text{Directrix: } y = \pm \frac{B}{e_H} = \pm \frac{16}{9}$$

$$PS = e \cdot PM = \frac{3}{2} \left| \frac{14}{3} \cdot \sqrt{\frac{2}{5} - \frac{16}{9}} \right|$$

$$= 7 \sqrt{\frac{2}{5} - \frac{8}{3}}$$

64. (c)



$$2x + 3y = 12$$

$$3x - 2y = 5$$

$$13x = 39$$

$$x = 3, y = 2$$

Center of given circle is (5, -2)

$$\text{Radius } \sqrt{25 + 4 - 13} = 4$$

$$\therefore CM = \sqrt{4 + 16} = 5\sqrt{2}$$

$$\therefore CP = \sqrt{16 + 20} = 6$$

65. (d)

$$y^2 \leq 4x, x < 4$$

$$\frac{xy(x-1)(x-2)}{(x-3)(x-4)} > 0$$

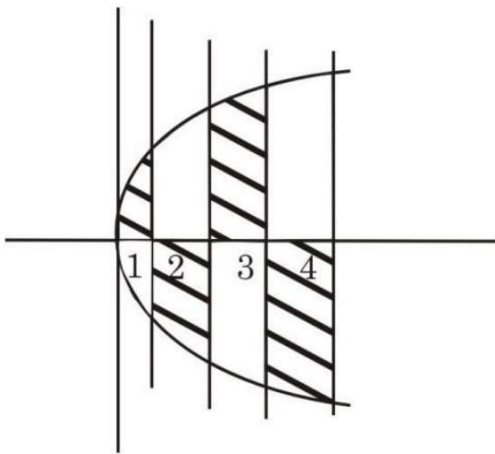
Case - I: $y > 0$

$$\frac{x(x-1)(x-2)}{(x-3)(x-4)} > 0$$

$$x \in (0,1) \cup (2,3)$$

Case - II : $y < 0$

$$\frac{x(x-1)(x-2)}{(x-3)(x-4)} < 0, x \in (1,2) \cup (3,4)$$



$$\text{Area} = 2 \int_0^4 \sqrt{x} dx$$

$$= 2 \cdot \frac{2}{3} [x^{3/2}]_0^4 = \frac{32}{3}$$

66. (d)

$$f(x) = \frac{4x+3}{6x-4}$$

$$g(x) = \frac{4\left(\frac{4x+3}{6x-4}\right) + 3}{6\left(\frac{4x+3}{6x-4}\right) - 4} = \frac{34x}{34} = x$$

$$g(x) = x \therefore g(g(g(d))) = 4$$

67. (d)

$$\lim_{x \rightarrow 0} \frac{e^{2|\sin x|} - 2|\sin x| - 1}{x^2}$$

$$\lim_{x \rightarrow 0} \frac{e^{2|\sin x|} - 2|\sin x| - 1}{|\sin x|^2} \times \frac{\sin^2 x}{x^2}$$

$$\text{Let } |\sin x| = t$$

$$\lim_{t \rightarrow 0} \frac{e^{2t} - 2t - 1}{t^2} \times \lim_{x \rightarrow 0} \frac{\sin^2 x}{x^2}$$

$$= \lim_{t \rightarrow 0} \frac{2e^{2t} - 2}{2t} \times 1 = 2 \times 1 = 2$$

68. (d)

$$D = \begin{vmatrix} 1 & -2 & 1 \\ 2 & \alpha & 3 \\ 3 & -1 & \beta \end{vmatrix}$$

$$= 1(\alpha\beta + 3) + 2(2\beta - 9) + 1(-2 - 3\alpha)$$

$$= \alpha\beta + 3 + 4\beta - 18 - 2 - 3\alpha$$

For infinite solutions $D = 0, D_1 = 0, D_2 = 0$ and

$$D_3 = 0$$

$$D = 0$$

$$\alpha\beta - 3\alpha + 4\beta = 17$$

$$D_1 = \begin{vmatrix} -4 & -2 & 1 \\ 5 & \alpha & 3 \\ 3 & -1 & \beta \end{vmatrix} = 0$$

$$D_2 = \begin{vmatrix} 1 & -4 & 1 \\ 2 & 5 & 3 \\ 3 & 3 & \beta \end{vmatrix} = 0$$

$$\Rightarrow 1(5\beta - 9) + 4(2\beta - 9) + 1(6 - 15) = 0$$

$$13\beta - 9 - 36 - 9 = 0$$

$$13\beta = 54, \beta = \frac{54}{13} \text{ put in (a)}$$

$$\frac{54}{13}\alpha - 3\alpha + 4\left(\frac{54}{13}\right) = 17$$

$$54\alpha - 39\alpha + 216 = 221$$

$$15\alpha = 5 \Rightarrow \alpha = \frac{1}{3}$$

$$\text{Now, } 12\alpha + 13\beta = 12 \cdot \frac{1}{3} + 13 \cdot \frac{54}{13}$$

$$= 4 + 54 = 58$$

69. (c)

$$\frac{dx}{dy} = \frac{x}{y} \left(\ln \left(\frac{x}{y} \right) + 1 \right)$$

$$\text{Let } \frac{x}{y} = t \Rightarrow x = ty$$

$$\frac{dx}{dy} = t + y \frac{dt}{dy}$$

$$t + y \frac{dt}{dy} = t(\ln(t) + 1)$$

$$y \frac{dt}{dy} = t \ln(t) \Rightarrow \frac{dt}{t \ln(t)} = \frac{dy}{y}$$

$$\Rightarrow \int \frac{dt}{t \cdot \ln(t)} = \int \frac{dy}{y}$$

$$\Rightarrow \int \frac{dp}{p} = \int \frac{dy}{y} \text{ let } \ln t = p$$

$$\frac{1}{t} dt = dp$$

$$\Rightarrow \ln p = \ln y + c$$

$$\ln(\ln t) = \ln y + c$$

$$\ln\left(\ln\left(\frac{x}{y}\right)\right) = \ln y + c$$

$$\text{at } x = e, y = 1$$

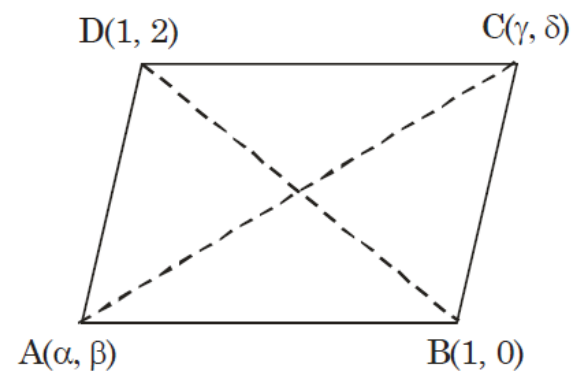
$$\ln\left(\ln\left(\frac{e}{1}\right)\right) = \ln(a) + c \Rightarrow c = 0$$

$$\ln\left|\ln\left(\frac{x}{y}\right)\right| = \ln y$$

$$\left|\ln\left(\frac{x}{y}\right)\right| = e^{\ln y}$$

$$\left|\ln\left(\frac{x}{y}\right)\right| = y$$

70. (d)



Let E is mid point of diagonals

$$\frac{\alpha + \gamma}{2} = \frac{1 + 1}{2} \quad \& \quad \frac{\beta + \delta}{2} = \frac{2 + 0}{2}$$

$$\alpha + \gamma = 2 \quad \beta + \delta = 2$$

$$2(\alpha + \beta + \gamma + \delta) = 2(2 + 2) = 8$$

71. (a)

$$\frac{dy}{dx} = \frac{\sin x + y \cos x}{\sin x \cdot \cos x \left(\frac{1}{\cos x} - \sin x \cdot \frac{\sin x}{\cos x} \right)}$$

$$= \frac{\sin x + y \cos x}{\sin x (1 - \sin^2 x)}$$

$$\frac{dy}{dx} = \sec^2 x + y \cdot 2(\operatorname{cosec} 2x)$$

$$\frac{dy}{dx} - 2\operatorname{cosec}(2x) \cdot y = \sec^2 x$$

$$\frac{dy}{dx} + p \cdot y = Q$$

$$\text{I.F.} = e^{\int p dx} = e^{\int -2\operatorname{cosec}(2x) dx}$$

$$\text{Let } 2x = t$$

$$2 \frac{dx}{dt} = 1$$

$$dx = \frac{dt}{2}$$

$$= e^{-\int \operatorname{cosec}(t) dt}$$

$$= e^{-\ln \left| \tan \frac{t}{2} \right|}$$

$$= e^{-\ln |\tan x|} = \frac{1}{|\tan x|}$$

$$y(\text{IF}) = \int Q(\text{IF}) dx + c$$

$$\Rightarrow y \frac{1}{|\tan x|} = \int \sec^2 x \cdot \frac{1}{|\tan x|} + c$$

$$y \cdot \frac{1}{|\tan x|} = \int \frac{dt}{|t|} + c \text{ for } \tan x = t$$

$$y \cdot \frac{1}{|\tan x|} = \ln |t| + c$$

$$y = |\tan x| (\ln |\tan x| + c)$$

$$\text{Put } x = \frac{\pi}{4}, y = 2$$

$$2 = \ln 1 + c \Rightarrow c = 2$$

$$y = |\tan x|(\ln |\tan x| + 2)$$

$$y\left(\frac{\pi}{3}\right) = \sqrt{3}(\ln \sqrt{3} + 2)$$

72. (d)

$$\vec{p} \times \vec{b} - \vec{c} \times \vec{b} = \vec{0}$$

$$(\vec{p} - \vec{c}) \times \vec{b} = \vec{0}$$

$$\vec{p} - \vec{c} = \lambda \vec{b} \Rightarrow \vec{p} = \vec{c} + \lambda \vec{b}$$

$$\text{Now, } \vec{p} \cdot \vec{a} = 0 \text{ (given)}$$

$$\text{So, } \vec{c} \cdot \vec{a} + \lambda \vec{a} \cdot \vec{b} = 0$$

$$(3 - 3 - 8) + \lambda(12 + 1 - 14) = 0$$

$$\lambda = -8$$

$$\vec{p} = \vec{c} - 8\vec{b}$$

$$\vec{p} = -31\hat{i} - 11\hat{j} - 52\hat{k}$$

$$\text{So, } \vec{p} \cdot (\hat{i} - \hat{j} - \hat{k})$$

$$= -31 + 11 + 52$$

$$= 32$$

73. (d) General term of the sequence,

$$T_r = \frac{r}{1 - 3r^2 + r^4}$$

$$T_r = \frac{r}{r^4 - 2r^2 + 1 - r^2}$$

$$T_r = \frac{r}{(r^2 - 1)^2 - r^2}$$

$$T_r = \frac{r}{(r^2 - r - 1)(r^2 + r - 1)}$$

$$T_r = \frac{\frac{1}{2}[(r^2 + r - 1) - (r^2 - r - 1)]}{(r^2 - r - 1)(r^2 + r - 1)}$$

$$= \frac{1}{2} \left[\frac{1}{r^2 - r - 1} - \frac{1}{r^2 + r - 1} \right]$$

Sum of 10 terms,

$$\sum_{r=1}^{10} T_r = \frac{1}{2} \left[\frac{1}{-1} - \frac{1}{109} \right] = \frac{-55}{109}$$

74. (d) A vector in the direction of the required line can be obtained by cross product of

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 5 \\ -1 & 3 & 2 \end{vmatrix}$$

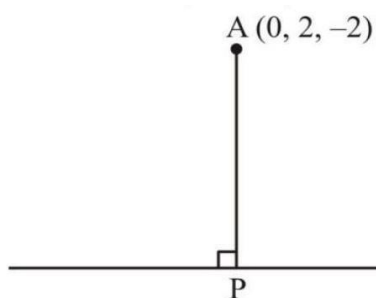
$$= -9\hat{i} - 9\hat{j} + 9\hat{k}$$

Required line,

$$\vec{r} = (5\hat{i} - 4\hat{j} + 3\hat{k}) + \lambda'(-9\hat{i} - 9\hat{j} + 9\hat{k})$$

$$\vec{r} = (5\hat{i} - 4\hat{j} + 3\hat{k}) + \lambda(\hat{i} + \hat{j} - \hat{k})$$

Now distance of $(0, 2, -2)$



$$\text{P.V. of P} \equiv (5 + \lambda)\hat{i} + (\lambda - 4)\hat{j} + (3 - \lambda)\hat{k}$$

$$\overrightarrow{AP} = (5 + \lambda)\hat{i} + (\lambda - 6)\hat{j} + (5 - \lambda)\hat{k}$$

$$\overrightarrow{AP} \cdot (\hat{i} + \hat{j} - \hat{k}) = 0$$

$$5 + \lambda + \lambda - 6 - 5 + \lambda = 0$$

$$\lambda = 2$$

$$|\overrightarrow{AP}| = \sqrt{49 + 16 + 9}$$

$$|\overrightarrow{AP}| = \sqrt{74}$$

75. (a) Let $\sin^{-1} \alpha = A, \sin^{-1} \beta = B, \sin^{-1} \gamma = C$

$$A + B + C = \pi$$

$$(\alpha + \beta)^2 - \gamma^2 = 3\alpha\beta$$

$$\alpha^2 + \beta^2 - \gamma^2 = \alpha\beta$$

$$\frac{\alpha^2 + \beta^2 - \gamma^2}{2\alpha\beta} = \frac{1}{2}$$

$$\Rightarrow \cos C = \frac{1}{2}$$

$$\sin C = \gamma$$

$$\cos C = \sqrt{1 - \gamma^2} = \frac{1}{2}$$

$$\gamma = \frac{\sqrt{3}}{2}$$

76. (d) Probability of drawing first red and then white = $\frac{10}{75} \times \frac{30}{75} = \frac{4}{75}$

77. (d) Let $g(x) = ax + b$

Now function $f(x)$ is continuous at $x = 0$

$$\therefore \lim_{x \rightarrow 0^+} f(x) = f(0)$$

$$\Rightarrow \lim_{x \rightarrow 0} \left(\frac{1+x}{2+x} \right)^{\frac{1}{x}} = b$$

$$\Rightarrow 0 = b$$

$$\therefore g(x) = ax$$

Now, for $x > 0$

$$f'(x) = \frac{1}{x} \cdot \left(\frac{1+x}{2+x} \right)^{\frac{1}{x}-1} \cdot \frac{1}{(2+x)^2}$$

$$+ \left(\frac{1+x}{2+x} \right)^{\frac{1}{x}} \cdot \ln \left(\frac{1+x}{2+x} \right) \cdot \left(-\frac{1}{x^2} \right)$$

$$\therefore f'(a) = \frac{1}{9} - \frac{2}{3} \cdot \ln \left(\frac{2}{3} \right)$$

$$\text{And } f(-1) = g(-1) = -a$$

$$\therefore a = \frac{2}{3} \ln \left(\frac{2}{3} \right) - \frac{1}{9}$$

$$\therefore g(c) = 2 \ln \left(\frac{2}{3} \right) - \frac{1}{3}$$

$$= \ln \left(\frac{4}{9 \cdot e^{1/3}} \right)$$

78. (c)

$$f(0) = \begin{vmatrix} 0 & 1 & 1 \\ 2 & 0 & 6 \\ 0 & 4 & -2 \end{vmatrix} = 12$$

$$f'(x) = \begin{vmatrix} 3x^2 & 4x & 3 \\ 3x^2 + 2 & 2x & x^3 + 6 \\ x^3 - x & 4 & x^2 - 2 \end{vmatrix} +$$

$$\begin{vmatrix} x^3 & 2x^2 + 1 & 1 + 3x \\ 6x & 2 & 3x^2 \\ x^3 - x & 4 & x^2 - 2 \end{vmatrix} +$$

$$\begin{vmatrix} x^3 & 2x^2 + 1 & 1 + 3x \\ 3x^2 + 2 & 2x & x^3 + 6 \\ 3x^2 - 1 & 0 & 2x \end{vmatrix}$$

$$\therefore f'(0) = \begin{vmatrix} 0 & 0 & 3 \\ 2 & 0 & 6 \\ 0 & 4 & -2 \end{vmatrix} + \begin{vmatrix} 0 & 1 & 1 \\ 0 & 2 & 0 \\ 0 & 4 & -2 \end{vmatrix} + \begin{vmatrix} 0 & 1 & 1 \\ 2 & 0 & 6 \\ -1 & 0 & 0 \end{vmatrix}$$

$$= 24 - 6 = 18$$

$$\therefore 2f(0) + f'(0) = 42$$

79. (d) 3 bad apples, 15 good apples.

Let X be no of bad apples

$$\text{Then } P(X = 0) = \frac{{}^{15}C_2}{{}^{18}C_2} = \frac{105}{153}$$

$$P(X = 1) = \frac{{}^3C_1 \times {}^{15}C_1}{{}^{18}C_2} = \frac{45}{153}$$

$$P(X = 2) = \frac{{}^3C_2}{{}^{18}C_2} = \frac{3}{153}$$

$$E(X) = 0 \times \frac{105}{153} + 1 \times \frac{45}{153} + 2 \times \frac{3}{153} = \frac{51}{153}$$

$$= \frac{1}{3}$$

$$\text{Var}(X) = E(X^2) - (E(X))^2$$

$$= 0 \times \frac{105}{153} + 1 \times \frac{45}{153} + 4 \times \frac{3}{153} - \left(\frac{1}{3}\right)^2$$

$$= \frac{57}{153} - \frac{1}{9} = \frac{40}{153}$$

80. (b) $ax^2 + 2(a+1)x + 9a + 4 < 0 \forall x \in R \therefore a < 0$

81. (176)

$$I = \int_0^{\frac{\pi}{2}} \sin 2x \cdot (\cos x)^{\frac{11}{2}} \left(1 + (\cos x)^2\right)^{\frac{5}{2}} dx$$

Put $\cos x = t^2 \Rightarrow \sin x dx = -2t dt$

$$\therefore I = 4 \int_0^1 t^2 \cdot t^{11} \sqrt{1+t^5} (t) dt$$

$$I = 4 \int_0^1 t^{14} \sqrt{1+t^5} dt$$

Put $1+t^5 = k^2$

$$\Rightarrow 5t^4 dt = 2k dk$$

$$\therefore I = 4 \cdot \int_1^{\sqrt{2}} (k^2 - 1)^2 \cdot k \frac{2k}{5} dk$$

$$I = \frac{8}{5} \int_1^{\sqrt{2}} k^6 - 2k^4 + k^2 dk$$

$$I = \frac{8}{5} \left[\frac{k^7}{7} - \frac{2k^5}{5} + \frac{k^3}{3} \right]_1^{\sqrt{2}}$$

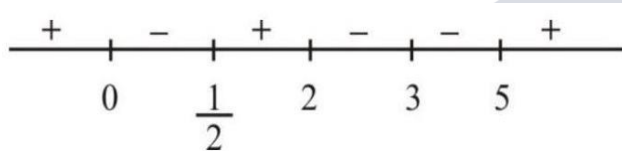
$$I = \frac{8}{5} \left[\frac{8\sqrt{2}}{7} - \frac{8\sqrt{2}}{5} + \frac{2\sqrt{2}}{3} - \frac{1}{7} + \frac{2}{5} - \frac{1}{3} \right]$$

$$I = \frac{8}{5} \left[\frac{22\sqrt{2}}{105} - \frac{8}{105} \right]$$

$$\therefore 525 \cdot I = 176\sqrt{2} - 64$$

82. (27)

$$f'(x) = (e^x - 1)^{11} (2x - 1)^5 (x - 2)^7 (x - 3)^{12} (2x - 10)^{61}$$



Local minima at $x = \frac{1}{2}, x = 5$

Local maxima at $x = 0, x = 2$

$$\therefore p = 0 + 4 = 4, q = \frac{1}{2} + 5 = \frac{11}{2}$$

$$\text{Then } p^2 + 2q = 16 + 11 = 27$$

83. (3734)

We have III, TT, D, S, R, B, U, O, N

Number of words with selection (a, a, a, b)

$$= {}^8C_1 \times \frac{4!}{3!} = 32$$

Number of words with selection (a, a, b, b)

$$= \frac{4!}{2!2!} = 6$$

Number of words with selection (a, a, b, c)

$$= {}^2C_1 \times {}^8C_2 \times \frac{4!}{2!} = 672$$

Number of words with selection (a, b, c, d)

$$= {}^9C_4 \times 4! = 3024$$

$$\therefore \text{total} = 3024 + 672 + 6 + 32$$

$$= 3734$$

84. (12)

$$\frac{x}{1} = \frac{y}{1} = \frac{z-1}{0} = r \rightarrow Q(r, r, 1)$$

$$\frac{x}{1} = \frac{y}{-1} = \frac{z+1}{0} = k \rightarrow R(k, -k, -1)$$

$$\overrightarrow{PQ} = (a-r)\hat{i} + (a-r)\hat{j} + (a-1)\hat{k}$$

$$a = r + a - r = 0.$$

$$2a = 2r \rightarrow a = r$$

$$\overrightarrow{PR} = (a-k)\hat{i} + (a+k)\hat{j} + (a+1)\hat{k}$$

$$a - k - a - k = 0 \Rightarrow k = 0$$

As, $PQ \perp PR$

$$(a-r)(a-k) + (a-r)(a+k) + (a-1)(a+1) = 0$$

$$a = 1 \text{ or } -1$$

$$12a^2 = 12$$

85. (118)

$$(1+x)(1-x^2) \left(1 + \frac{3}{x} + \frac{3}{x^2} + \frac{1}{x^3} \right)^5$$

$$= (1+x)(1-x^2) \left(\left(1 + \frac{1}{x} \right)^3 \right)^5$$

$$= \frac{(1+x)^2(1-x)(1+x)^{15}}{x^{15}}$$

$$= \frac{(1+x)^{17} - x(1+x)^{17}}{x^{15}}$$

$$= \text{coeff}(x^3) \text{ in the expansion} \approx \text{coeff}(x^{18}) \text{ in}$$

$$(1+x)^{17} - x(1+x)^{17}$$

$$= 0 - 1$$

$$= -1$$

$$\text{coeff}(x^{-13}) \text{ in the expansion} \approx \text{coeff}(x^2) \text{ in}$$

$$(1+x)^{17} - x(1+x)^{17}$$

$$= \binom{17}{2} - \binom{17}{1}$$

$$= 17 \times 8 - 17$$

$$= 17 \times 7$$

$$= 119$$

$$\text{Hence Answer} = 119 - 1 = 118$$

86. (3)

$$(\sqrt{2})^x = 2^x \Rightarrow x = 0 \Rightarrow \alpha = 1$$

$$z = \frac{\pi}{4} (1 + i)^4 \left[\frac{\sqrt{\pi} - \pi i - i - \sqrt{\pi}}{\pi + 1} + \frac{\sqrt{\pi} - i - \pi i - \sqrt{\pi}}{1 + \pi} \right]$$

$$= -\frac{\pi i}{2} (1 + 4i + 6i^2 + 4i^3 + 1)$$

$$= 2\pi i$$

$$\beta = \frac{2\pi}{\frac{\pi}{2}} = 4$$

$$\text{Distance from } (1, 4) \text{ to } 4x - 3y = 7$$

$$\text{Will be } \frac{15}{5} = 3$$

87. (51)

$$\text{foci} \equiv (\pm 5, 0); \frac{2b^2}{a} = \sqrt{50}$$

$$ae = 5 \quad b^2 = \frac{5\sqrt{2}a}{2}$$

$$b^2 = a^2(1 - e^2) = \frac{5\sqrt{2}a}{2}$$

$$\Rightarrow a(1 - e^2) = \frac{5\sqrt{2}}{2}$$

$$\Rightarrow \frac{5}{e}(1 - e^2) = \frac{5\sqrt{2}}{2}$$

$$\Rightarrow \sqrt{2} - \sqrt{2}e^2 = e$$

$$\Rightarrow \sqrt{2}e^2 + e - \sqrt{2} = 0$$

$$\Rightarrow \sqrt{2}e^2 + 2e - e - \sqrt{2} = 0$$

$$\Rightarrow \sqrt{2}e(e + \sqrt{2}) - 1(1 + \sqrt{2}) = 0$$

$$\Rightarrow (e + \sqrt{2})(\sqrt{2}e - 1) = 0$$

$$\therefore e \neq -\sqrt{2}; e = \frac{1}{\sqrt{2}}$$

$$\frac{x^2}{b^2} - \frac{y^2}{a^2 b^2} = 1 \quad a = 5\sqrt{2}$$

$$a^2 b^2 = b^2 (e_1^2 - 1) \Rightarrow e_1^2 = 51$$

88. (48)

$$\vec{b} \cdot \vec{c} = (2\vec{a} \times \vec{b}) \cdot \vec{b} - 3|b|^2$$

$$|b||c|\cos\alpha = -3|b|^2$$

$$|c|\cos\alpha = -12, \text{ as } |b| = 4$$

$$\vec{a} \cdot \vec{b} = 2$$

$$\cos\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}$$

$$|c|^2 = |(2\vec{a} \times \vec{b}) - 3\vec{b}|^2$$

$$= 64 \times \frac{3}{4} + 144 = 192$$

$$|c|^2 \cos^2\alpha = 144$$

$$192 \cos^2\alpha = 144$$

$$192 \sin^2\alpha = 48$$

89. (16)

All elements are included

90. (5)

$$f(a) + f(1-a) = 1.$$

$$M = \int_{f(a)}^{f(1-a)} (1-x) \cdot \sin^4 x (1-x) dx$$

$$M = N - M \quad 2M = N$$

$$\alpha = 2; \quad \beta = 1$$