

1. (a)

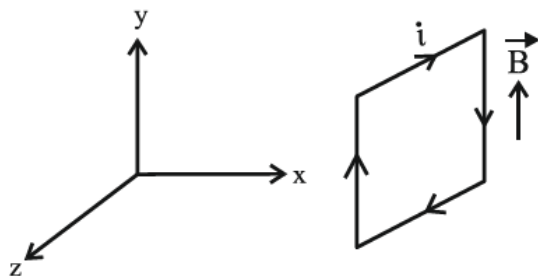
$$a = \frac{(m_1 - m_2)g}{(m_1 + m_2)} = \frac{g}{8}$$

$$8m_1 - 8m_2 = m_1 + m_2$$

$$7m_1 = 9m_2$$

$$\frac{m_1}{m_2} = \frac{9}{7}$$

2. (b)



$$M = i$$

$$= 5 \times (0.2) \times (0.1)(-\hat{i})$$

$$= 0.1(-\hat{i})$$

$$\vec{\tau} = \vec{M} \times \vec{B} = 0.1(-\hat{i}) \times (2 \times 10^{-3})(\hat{j})$$

$$= 2 \times 10^{-4}(-\hat{k}) \text{ N-m}$$

3. (c)

$$T = 2\pi \sqrt{\frac{\ell}{g}}$$

$$g = \frac{4\pi^2 \ell}{T^2}$$

$$\frac{\Delta g}{g} = \frac{\Delta \ell}{\ell} + \frac{2\Delta T}{T}$$

$$= \frac{0.2}{20} + 2\left(\frac{1}{40}\right)$$

$$= \frac{0.3}{20}$$

$$\text{Percentage change} = \frac{0.3}{20} \times 100 = 6\%$$

4. (b)

$$\text{In air } F = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r_2}$$

In medium

$$F' = \frac{1}{4\pi(K\epsilon_0)} \frac{q_1 q_2}{(r')^2} = \frac{25}{4\pi(5\epsilon_0)} \frac{q_1 q_2}{(r)^2} = 5 F$$

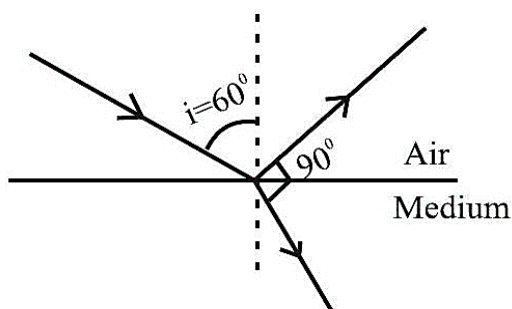
5. (d)

$$\langle P \rangle = IV \cos \phi$$

$$= \frac{20}{\sqrt{2}} \times \frac{10}{\sqrt{2}} \times \cos 60^\circ$$

$$= 50 \text{ W}$$

6. (a) By Brewster's law



At complete reflection refracted ray and reflected ray are perpendicular.

7. (a)

$$v = \sqrt{\frac{\gamma RT}{M}} = \sqrt{\frac{1.4 \times 8.3 \times 273}{32 \times 10^{-3}}}$$

$$= 314.8541 \approx 315 \text{ m/s}$$

8. (c)

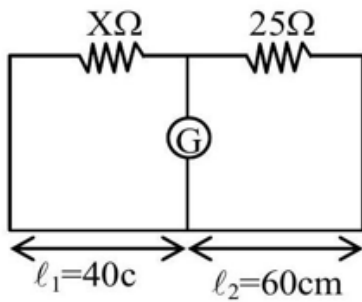
$$U = nC_V T$$

$$\Rightarrow U = n_1 C_{V_1} T + n_2 C_{V_2} T$$

$$\Rightarrow 8 \times \frac{3R}{2} \times T + 6 \times \frac{5R}{2} \times T$$

$$= 27RT$$

9. (d)



$$\frac{25}{r\ell_1} = \frac{X}{r\ell_2} \dots\dots\dots (i)$$

$$\frac{25}{2r\ell'_1} = \frac{X}{2r\ell'_2} \dots\dots\dots (ii)$$

From (i) and (ii)

$$\ell'_2 = \ell_2 = 40 \text{ cm}$$

10. (b)

$$\frac{1}{2} \epsilon_0 E^2 = \frac{B^2}{2\mu_0}$$

$$\because E = CB \text{ and } C = \frac{1}{\mu_0 \epsilon_0}$$

11. (c)

$$\text{Since } \frac{f}{2} < f_0$$

i.e. the incident frequency is less than threshold frequency. Hence there will be no emission of photoelectrons.

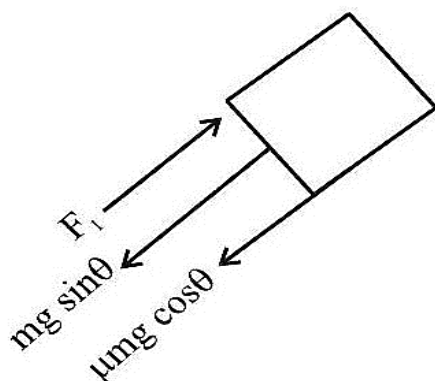
$$\Rightarrow \text{current} = 0$$

12. (d) ($5\sqrt{3}N$)

$$\text{Sol. } f_K = \mu mg \cos \theta$$

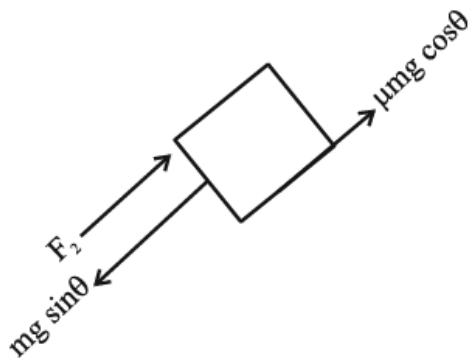
$$= 0.1 \times \frac{50 \times \sqrt{3}}{2}$$

$$= 2.5\sqrt{3} \text{ N}$$



$$F_1 = mg \sin \theta + f_k$$

$$= 25 + 2.5\sqrt{3}$$



$$F_2 = mg \sin \theta - f_k$$

$$= 25 - 2.5\sqrt{3}$$

$$\therefore F_1 - F_2 = 5\sqrt{3} \text{ N}$$

13. (c) $P = i^2 R$

$$P_{\text{int}} = I_{\text{int}}^2 R$$

$$P_{\text{final}} = (0.8 I_{\text{int}})^2 R$$

% change in power =

$$\frac{P_{\text{final}} - P_{\text{int}}}{P_{\text{int}}} \times 100 = (0.64 - 1) \times 100 = -36\%$$

14. (c)

The magnitude of resultant vector $R' = \sqrt{a^2 + b^2 + 2ab \cos \theta}$

Here $a = b = R$

$$\text{Then } R' = \sqrt{R^2 + R^2 + 2R^2 \cos \theta}$$

$$= R\sqrt{2\sqrt{1 + \cos \theta}}$$

$$= \sqrt{2}R \sqrt{2\cos^2 \frac{\theta}{2}}$$

$$= 2R\cos \frac{\theta}{2}$$

15. (a)

$$R_1 = \frac{R_2}{2}$$

$$R_0 (A_1)^{1/3} = \frac{R_0}{2} (A_2)^{1/3}$$

$$A_1 = \frac{1}{8} A_2$$

$$A_1 = \frac{192}{8} = 24$$

16. (a)

$$V_{\text{escape}} = \sqrt{\frac{2GM}{R}}$$

$$V_{\text{planet}} = \sqrt{\frac{2GM}{R}} = V$$

$$V_{\text{Moon}} = \sqrt{\frac{2GM \times 16}{144R}} = \frac{1}{3} \sqrt{\frac{2GM}{R}}$$

$$V_{\text{Moon}} = \frac{V_{\text{Planet}}}{3} = \frac{V}{3}$$

17. (a) Since density is negligible hence Buoyancy force will be negligible

At terminal velocity.

$$Mg = 6\pi\eta rv$$

$$V \propto \frac{1}{r} \text{ (as mass is constant)}$$

$$\text{Now, } \frac{v}{v'} = \frac{r'}{r}$$

$$r' = 2r$$

$$\text{So, } v' = \frac{v}{2}$$

18. (d)

$$\vec{F} = (6t\hat{i} + 6t^2\hat{j})\text{N}$$

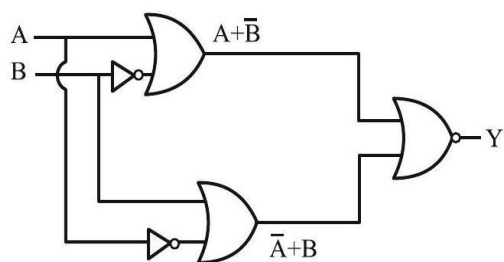
$$\vec{F} = ma = (6t\hat{i} + 6t^2\hat{j})$$

$$\vec{a} = \frac{\vec{F}}{m} = (3t\hat{i} + 3t^2\hat{j})$$

$$\vec{v} = \int_0^t \vec{a} dt = \frac{3t^2}{2}\hat{i} + t^3\hat{j}$$

$$P = \vec{F} \cdot \vec{v} = (9t^3 + 6t^5)\text{W}$$

19. (c)



If $A = 0; \bar{A} = 1$

$A = 1; \bar{A} = 0$

$B = 0; \bar{B} = 1$

$B = 1; \bar{B} = 0$

$$Y = \overline{(A + \bar{B}) + (\bar{A} + B)} = \overline{(1 + 1)} = 0$$

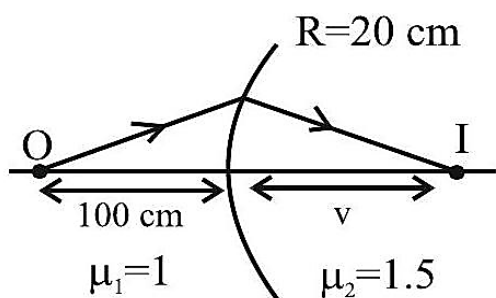
20. (b)

21. (3)

$$R_{eq} = \frac{4}{3} \Omega$$

$$\therefore P = \frac{V^2}{R_{eq}} = \frac{4}{4/3} = 3 \text{ W}$$

22. (200)



$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

$$\frac{1.5}{v} - \frac{1}{-100} = \frac{1.5 - 1}{20}$$

$$v = 100 \text{ cm}$$

Distance from object

$$= 100 + 100$$

$$= 200 \text{ cm}$$

23. (2)

$$\varepsilon = - \left(\frac{d\phi}{dt} \right) = 10t - 36$$

at $t = 2$, $\varepsilon = 16 \text{ V}$

$$i = \frac{\varepsilon}{R} = \frac{16}{8} = 2 \text{ A}$$

24. (12)

$$T = \left(\frac{2 m_1 m_2}{m_1 + m_2} \right) g = \frac{80}{3} \text{ N}$$

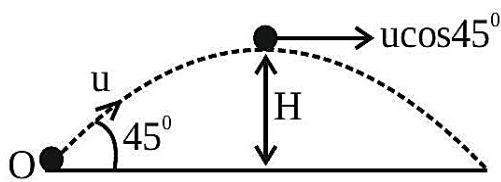
$$A = \pi r^2 = 16\pi \times 10^{-10} \text{ m}^2$$

$$\text{Strain} = \frac{\Delta \ell}{\ell} = \frac{F}{AY} = \frac{T}{AY}$$

$$= \frac{80/3}{16\pi \times 10^{-10} \times 2 \times 10^{11}} = \frac{1}{12\pi}$$

$$\alpha = 12$$

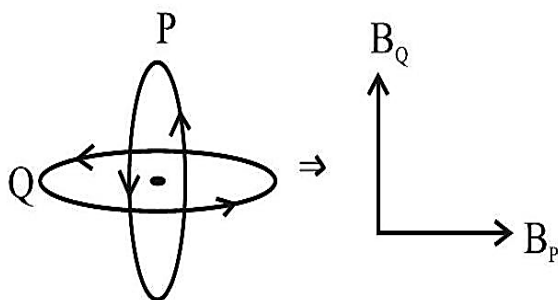
25. (8)



$$L = m u \cos \theta \frac{u^2 \sin^2 \theta}{2g}$$

$$= m u^3 \frac{1}{4\sqrt{2}g} \Rightarrow x = 8$$

26. (20)



$$B_P = \frac{\mu_0 N i_1}{2r} = \frac{\mu_0 \times 1 \times 100}{2\pi} = 2 \times 10^{-3} \text{ T}$$

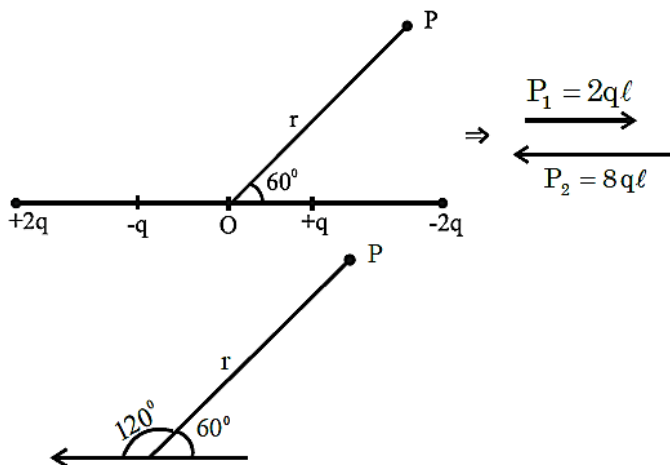
$$B_Q = \frac{\mu_0 N i_2}{2r} = \frac{\mu_0 \times 2 \times 100}{2\pi} = 4 \times 10^{-3} \text{ T}$$

$$B_{\text{net}} = \sqrt{B_P^2 + B_Q^2}$$

$$= \sqrt{20} \text{ mT}$$

$$x = 20$$

27. (27)



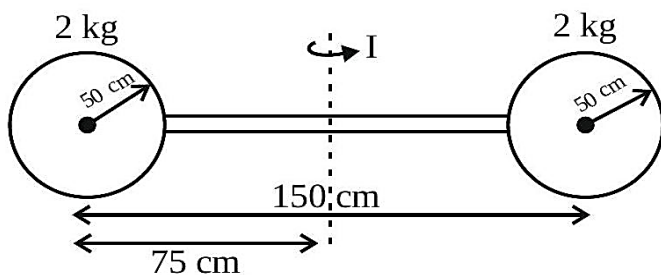
$$P_{\text{net}} = 6q\ell$$

$$V = \frac{K\vec{p} \cdot \vec{r}}{r^3} = \frac{9 \times 10^9 (6q\ell)}{r^2} \cos(120^\circ)$$

$$= -(27) \left(\frac{q\ell}{r^2} \right) \times 10^9 \text{ Nm}^2 \text{ C}^{-2}$$

$$\Rightarrow \alpha = 27$$

28. (53)



$$I = \left(\frac{2}{5} mR^2 + md^2 \right) \times 2$$

$$I = 2 \left(\frac{2}{5} \times 2 \times \left(\frac{1}{2} \right)^2 + 2 \times \left(\frac{3}{4} \right)^2 \right) = \frac{53}{20} \text{ kg} \cdot \text{m}^2$$

$$X = 53$$

29. (12)

$$k_{\text{eq}} = \frac{2k \cdot k}{3k} + k = \frac{5k}{3}$$

$$\text{Angular frequency of oscillation } (\omega) = \sqrt{\frac{k_{\text{eq}}}{m}}$$

$$(\omega) = \sqrt{\frac{5k}{3m}}$$

$$\text{Period of oscillation } (\tau) = \frac{2\pi}{\omega} = 2\pi \sqrt{\frac{3m}{5k}}$$

$$= \pi \sqrt{\frac{12m}{5k}}$$

30. (4)

For a nucleus

Volume:

$$V = \frac{4}{3}\pi R^3$$

$$R = R_0(A)^{1/3}$$

$$V = \frac{4}{3}\pi R_0^3 A$$

$$\Rightarrow \frac{V_2}{V_1} = \frac{A_2}{A_1} = 4$$

31. (d)

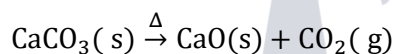
$[\text{Cr}(\text{H}_2\text{O})_6]^{3+}$ Contains Cr^{3+} : $[\text{Ar}]3d^3:t_{2g}^3e_g^0$

$[\text{Fe}(\text{H}_2\text{O})_6]^{3+}$ Contains Fe^{3+} : $[\text{Ar}]3d^5:t_{2g}^3e_g^2$

$[\text{Ni}(\text{H}_2\text{O})_6]^{2+}$ Contains Ni^{2+} : $[\text{Ar}]3d^8:t_{2g}^6e_g^2$

$[\text{V}(\text{H}_2\text{O})_6]^{3+}$ Contains V^{3+} : $[\text{Ar}]3d^2:t_{2g}^2e_g^0$

32. (a)



Let the weight of CaCO_3 be x gm

$$\therefore \text{weight of } \text{MgCO}_3 = (2.21 - x)\text{gm}$$

Moles of CaCO_3 decomposed = moles of CaO formed

$$\frac{x}{100} = \text{moles of } \text{CaO} \text{ formed}$$

$$\therefore \text{weight of } \text{CaO} \text{ formed} = \frac{x}{100} \times 56$$

Moles of MgCO_3 decomposed = moles of MgO formed

$$\frac{(2.21 - x)}{84} = \text{moles of } \text{MgO} \text{ formed}$$

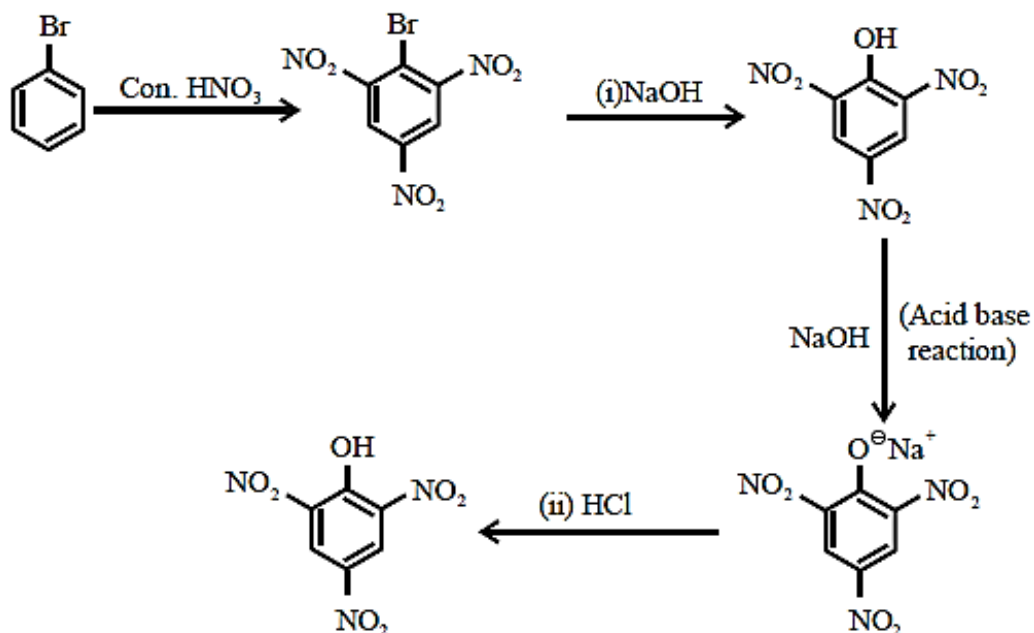
$$\therefore \text{weight of } \text{MgO} \text{ formed} = \frac{2.21 - x}{84} \times 40$$

$$\Rightarrow \frac{2.21 - x}{84} \times 40 + \frac{x}{100} \times 56 = 1.152$$

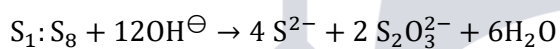
$\therefore x = 1.1886 \text{ g} = \text{weight of } \text{CaCO}_3$

& weight of $\text{MgCO}_3 = 1.0214 \text{ g}$

33. (a)

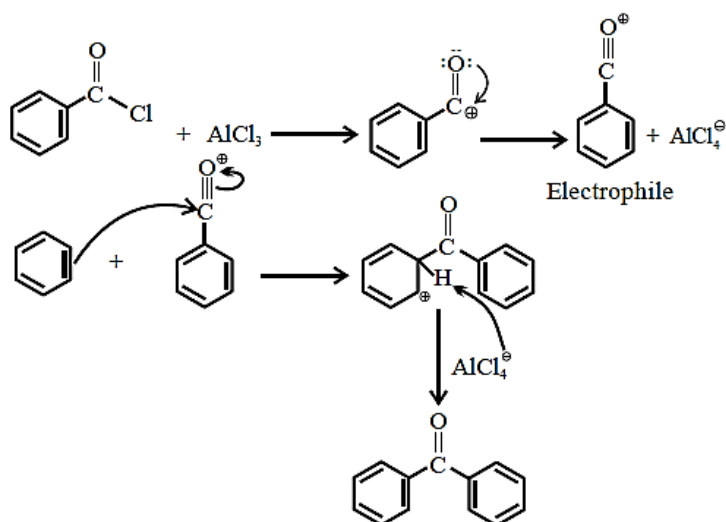


34. (a)

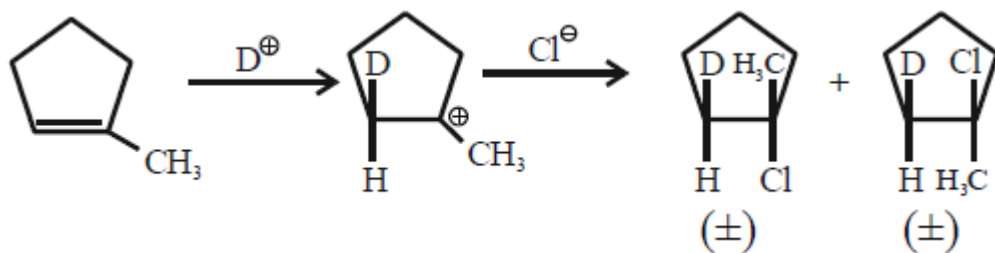


$\text{S}_2: \text{ClO}_4^-$ cannot undergo disproportionation reaction as chlorine is present in its highest oxidation state.

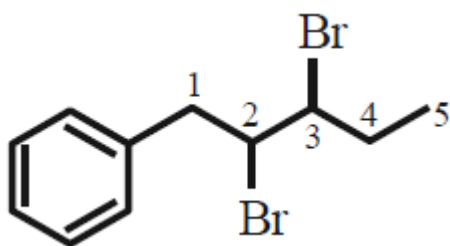
35. (d)



36. (c, d)



37. (c)



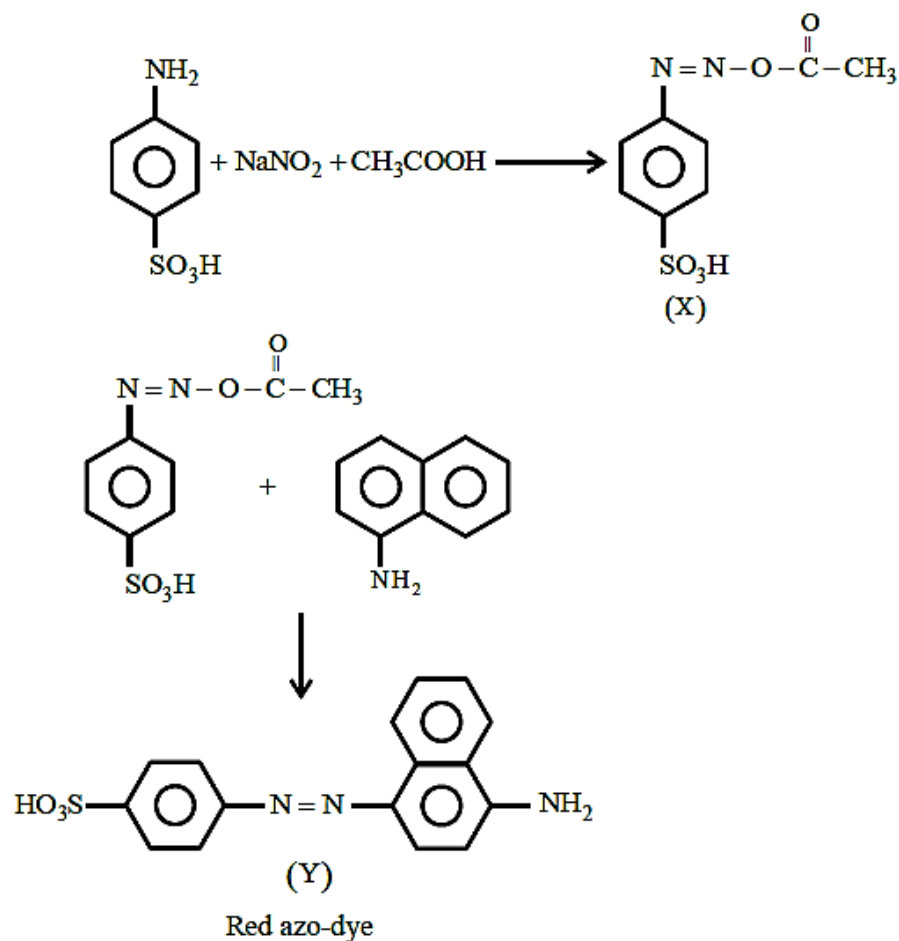
2, 3-dibromo -1-phenylpentane

38. (d)

 $[\text{Ni}(\text{CO})_4] \rightarrow$ diamagnetic, sp^3 hybridisation, number of unpaired electrons = 0

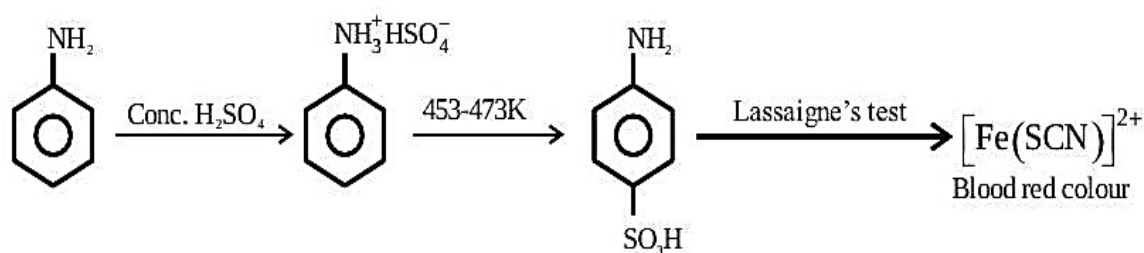
 $[\text{NiCl}_4]^{2-}, \rightarrow$ paramagnetic, sp^3 hybridisation, number of unpaired electrons = 2

39. (d)



This is known as Griess-Ilosvay test.

40. (d)



41. (b)

$$A_{(g)} \rightleftharpoons B_{(g)} + \frac{C}{2} (g)$$

$$t = t_{eq} (1 - \alpha) \propto \frac{\alpha}{2}$$

$$P_B = \frac{\alpha}{(1 + \frac{\alpha}{2})} \cdot P, P_A = \frac{(1 - \alpha)}{(1 + \frac{\alpha}{2})} \cdot P, P_C = \frac{\frac{\alpha}{2}}{(1 + \frac{\alpha}{2})} \cdot P$$

$$K_P = \frac{P_B \cdot P_C^{\frac{1}{2}}}{P_A}$$

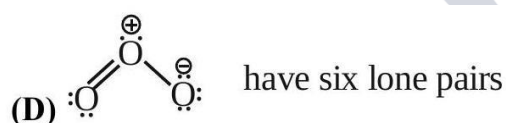
$$= \frac{(\alpha)^{\frac{3}{2}} (P)^{\frac{1}{2}}}{(1 - \alpha)(2 + \alpha)^{\frac{1}{2}}}$$

42. (d)

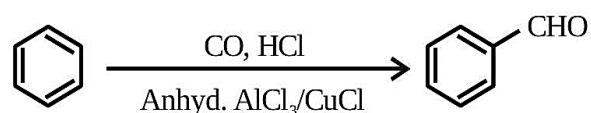
(A) All group 16 elements form oxides of the EO_2 and EO_3 type where $E = S, Se, Te$ or Po .

(B) SO_2 is reducing while TeO_2 is an oxidising agent.

(C) The reducing property increases from H_2S to H_2Te down the group.



43. (c)



Gatterman-Koch reaction

44. (b) $AgCl < CoCl_2 < BaCl_2 < KCl$ (ionic character)

Reason: Ag^+ has pseudo inert gas configuration.

45. (d) Steam distillation technique is applied to separate substances which are steam volatile and are immiscible with water.

46. (d) In trivalent state most of the compounds being covalent are hydrolysed in water. Trichlorides on hydrolysis in water form tetrahedral $[M(OH)_4]^-$ species, the hybridisation state of element M is sp^3 . In case of aluminium, acidified aqueous solution forms octahedral $[Al(H_2O)_6]^{3+}$ ion.

47. (b)

$$_{19}K_1 s^2, 2 s^2, 2p^6, 3 s^2, 3p^6, 4 s^1$$

Outermost orbital of potassium is 4 s orbital $n = 4, l = 0, m_l = 0, s = \pm \frac{1}{2}$.

48. (d)

(A) Mn_2O_7 is green oil at room temperature.

(B) V_2O_4 dissolve in acids to give VO^{2+} salts.

(C) CrO is basic oxide

(D) V_2O_5 is amphoteric it reacts with acid as well as base.

49. (d)

CH_3 shows +M and +I

$-Cl$ shows +M and $-I$ but inductive effect dominates.

$-NO_2$ shows $-M$ and $-I$.

Electrophilic substitution $\propto \frac{1}{-M \text{ and } -I}$

$\propto +M \text{ and } +I$

Hence, order is $B > A > C > D$.

50. (b) In general, along the period from left to right, size decreases and metallic character decrease.

In general, down the group, size increases and metallic character increases.

$B' < A'$ (size) $C' > A'$ (size)

$D' < C'$ (size) $D' > B'$ (size)

$B' < A'$ (metallic character)

$D' < C'$ (metallic character)

$B'^+ < A'^+$ (size)

$D'^+ < C'^+$ (size)

$\therefore$ C statement is incorrect.

51. (0)

$$\therefore \mu = 1.2D = q \times d$$

$$\Rightarrow 1.2 \times 10^{-10} \text{ esu} \text{Å} = q \times 1 \text{Å}$$

$$\therefore q = 1.2 \times 10^{-10} \text{ esu}$$

52. (399)

$$r = k[A]$$

So, order of reaction = 1

$$t_{1/2} = 120 \text{ min}$$

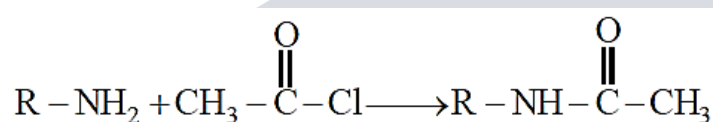
For 90% completion of reaction

$$\Rightarrow k = \frac{2.303}{t} \log \left(\frac{a}{a-x} \right)$$

$$\Rightarrow \frac{0.693}{t_{1/2}} = \frac{2.303}{t} \log \frac{100}{10}$$

$$\therefore t = 399 \text{ min.}$$

53. (2)

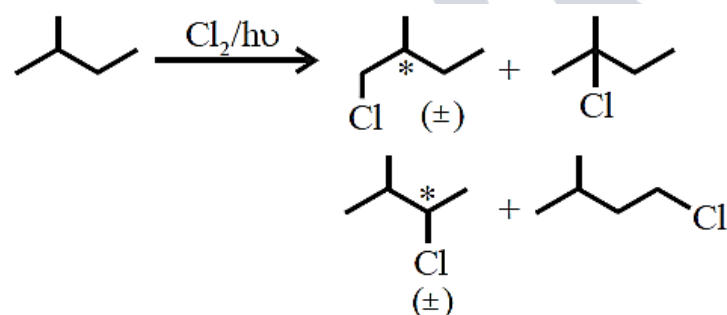


Gain in molecular weight after acylation with one $-\text{NH}_2$ group is 42.

Total increase in molecular weight = 84

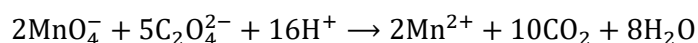
$$\therefore \text{Number of amino group in } x = \frac{84}{42} = 2$$

54. (6)



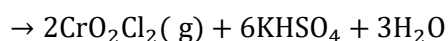
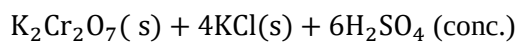
Number of isomeric products = 6

55. (8)



$\therefore$ Number of moles of H^+ ions required by 1 mole of MnO_4^- to oxidise oxalate ion to CO_2 is 8

56. (6)



This reaction is called chromyl chloride test.

Here oxidation state of Cr is +6.

57. (11)

Specific gravity (density) = 1.54 g/cc.

Volume = 1 L = 1000ml

Mass of solution = 1.54×1000

= 1540 g

% purity of H_2SO_4 is 70%

So weight of $\text{H}_3\text{PO}_4 = 0.7 \times 1540 = 1078 \text{ g}$

Mole of $\text{H}_3\text{PO}_4 = \frac{1078}{98} = 11$

Molarity = $\frac{11}{1 \text{ L}} = 11$

58. (4)

Conductivity (S m^{-1})

$\left. \begin{array}{l} 2.1 \times 10^3 \\ 1.2 \times 10 \\ 3.91 \\ 1 \times 10^3 \end{array} \right\}$ conductors at 298.15 K

1×10^{-16} Insulator at 298.15 K

$\left. \begin{array}{l} 1.5 \times 10^{-2} \\ 1 \times 10^{-7} \end{array} \right\}$ Semiconductor at 298.15 K

Therefore number of conductors is 4.

59. (5) Vitamins A, D, E, K and B_{12} are stored in liver and adipose tissue.

60. (28721)

It is isothermal reversible expansion, so work done negative

$$W = -2.303nRT \log\left(\frac{V_2}{V_1}\right)$$

$$= -2.303 \times 5 \times 8.314 \times 300 \log\left(\frac{100}{10}\right)$$

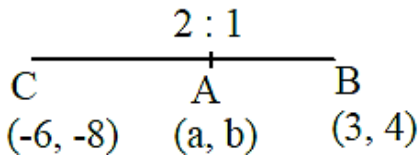
$$= -28720.713 \text{ J}$$

$$\equiv -28721$$

61. (d) After giving 2 apples to each child 15 apples left now 15 apples can be distributed in $^{15+3-1}C_2 = {}^{17}C_2$ ways

$$= \frac{17 \times 16}{2} = 136$$

62. (c) A(a, b), B(3, 4), C(-6, -8)



$$\Rightarrow a = 0, b = 0 \Rightarrow P(3, 5)$$

Distance from P measured along $x - 2y - 1 = 0$

$$\Rightarrow x = 3 + r \cos \theta, y = 5 + r \sin \theta$$

$$\text{Where } \tan \theta = \frac{1}{2}$$

$$r(2 \cos \theta + 3 \sin \theta) = -17$$

$$\Rightarrow r = \left| \frac{-17\sqrt{5}}{7} \right| = \frac{17\sqrt{5}}{7}$$

63. (b)

$$z_1 + z_2 = 5$$

$$z_1^3 + z_2^3 = 20 + 15i$$

$$z_1^3 + z_2^3 = (z_1 + z_2)^3 - 3z_1z_2(z_1 + z_2)$$

$$z_1^3 + z_2^3 = 125 - 3z_1z_2(5)$$

$$\Rightarrow 20 + 15i = 125 - 15z_1z_2$$

$$\Rightarrow 3z_1z_2 = 25 - 4 - 3i$$

$$\Rightarrow 3z_1z_2 = 21 - 3i$$

$$\Rightarrow z_1 \cdot z_2 = 7 - i$$

$$\Rightarrow (z_1 + z_2)^2 = 25$$

$$\Rightarrow z_1^2 + z_2^2 = 25 - 2(7 - i)$$

$$\Rightarrow 11 + 2i$$

$$(z_1^2 + z_2^2)^2 = 121 - 4 + 44i$$

$$\Rightarrow z_1^4 + z_2^4 + 2(7 - i)^2 = 117 + 44i$$

$$\Rightarrow z_1^4 + z_2^4 = 117 + 44i - 2(49 - 1 - 14i)$$

$$\Rightarrow |z_1^4 + z_2^4| = 75$$

64. (b)

$$\therefore (y - 2) = m(x - 8)$$

$$\Rightarrow \text{x-intercept}$$

$$\Rightarrow \left(\frac{-2}{m} + 8 \right)$$

$$\Rightarrow \text{y-intercept}$$

$$\Rightarrow (-8m + 2)$$

$$\Rightarrow OA + OB = \frac{-2}{m} + 8 - 8m + 2$$

$$f'(m) = \frac{2}{m^2} - 8 = 0$$

$$\Rightarrow m^2 = \frac{1}{4}$$

$$\Rightarrow m = \frac{-1}{2}$$

$$\Rightarrow f\left(\frac{-1}{2}\right) = 18$$

$$\Rightarrow \text{Minimum} = 18$$

65. (c)

$$f(x) = \int_{-x}^x (|t| - t^2)e^{-t^2} dt$$

$$\Rightarrow f'(x) = 2 \cdot (|x| - x^2)e^{-x^2}$$

$$g(x) = \int_0^{x^2} t^{\frac{1}{2}} e^{-t} dt$$

$$g'(x) = xe^{-x^2} (2x) - 0$$

$$f'(x) + g'(x) = 2xe^{-x^2} - 2x^2e^{-x^2} + 2x^2e^{-x^2}$$

Integrating both sides w.r.t.x

$$f(x) + g(x) = \int_0^\alpha 2xe^{-x^2} dx$$

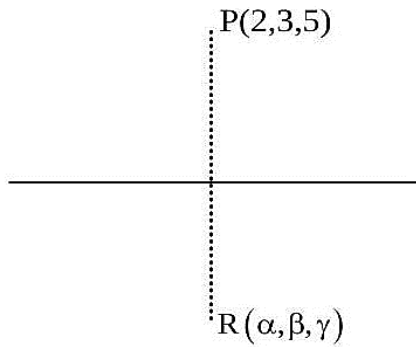
$$x^2 = t$$

$$\Rightarrow \int_0^{\sqrt{\alpha}} e^{-t} dt = [-e^{-t}]_0^{\sqrt{\alpha}}$$

$$= -e^{(\log_e(9))^{-1}+1}$$

$$\Rightarrow 9(f(x) + g(x)) = \left(1 - \frac{1}{9}\right) 9 = 8$$

66. (b)



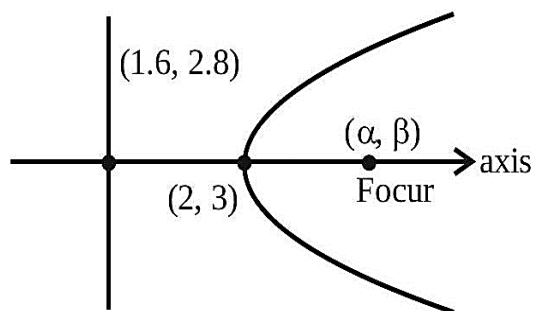
$$\therefore \overrightarrow{PR} \perp (2,3,4)$$

$$\therefore \overrightarrow{PR} \cdot (2,3,4) = 0$$

$$(\alpha - 2, \beta - 3, \gamma - 5) \cdot (2,3,4) = 0$$

$$\Rightarrow 2\alpha + 3\beta + 4\gamma = 4 + 9 + 20 = 33$$

67. (d)



$$\text{Slope of axis} = \frac{1}{2}$$

$$y - 3 = \frac{1}{2}(x - 2)$$

$$\Rightarrow 2y - 6 = x - 2$$

$$\Rightarrow 2y - x - 4 = 0$$

$$2x + y - 6 = 0$$

$$4x + 2y - 12 = 0$$

$$\alpha + 1.6 = 4 \Rightarrow \alpha = 2.4$$

$$\beta + 2.8 = 6 \Rightarrow \beta = 3.2$$

Ellipse passes through (2.4, 3.2)

$$\Rightarrow \frac{\left(\frac{24}{10}\right)^2}{a^2} + \frac{\left(\frac{32}{10}\right)^2}{b^2} = 1$$

$$\text{Also } 1 - \frac{b^2}{a^2} = \frac{1}{2} = \frac{b^2}{a^2} = \frac{1}{2} \Rightarrow a^2 = 2b^2$$

$$\text{Put in (a)} \Rightarrow b^2 = \frac{328}{25}$$

$$\Rightarrow \left(\frac{2b^2}{a} \right)^2 = \frac{4b^2}{a^2} \times b^2 = 4 \times \frac{1}{2} \times \frac{328}{25} = \frac{656}{25}$$

68. (c)

$$\frac{dT}{dt} = -k(T - 80)$$

$$\int_{160}^T \frac{dT}{(T - 80)} = \int_0^t -K dt$$

$$[\ln |T - 80|]_{160}^T = -kt$$

$$\ln |T - 80| - \ln 80 = -kt$$

$$\ln \left| \frac{T - 80}{80} \right| = -kt$$

$$T = 80 + 80e^{-kt}$$

$$120 = 80 + 80e^{-k \cdot 15}$$

$$\frac{40}{80} = e^{-k \cdot 15} = \frac{1}{2}$$

$$\therefore T(45) = 80 + 80e^{-k \cdot 45}$$

$$= 80 + 80(e^{-k \cdot 15})^3$$

$$= 80 + 80 \times \frac{1}{8}$$

$$= 90$$

69. (d)

$1 + d, 1 + 7d, 1 + 43d$ are in GP

$$(1 + 7d)^2 = (1 + d)(1 + 43d)$$

$$1 + 49d^2 + 14d = 1 + 44d + 43d^2$$

$$6d^2 - 30d = 0$$

$$d = 5$$

$$S_{20} = \frac{20}{2} [2 \times 1 + (20 - 1) \times 5]$$

$$= 10[2 + 95]$$

$$= 970$$

70. (b)

$$-f: \mathbb{R} \rightarrow (0, \infty)$$

$$\lim_{x \rightarrow \infty} \frac{f(7x)}{f(x)} = 1$$

$\therefore f$ is increasing

$$\therefore f(x) < f(5x) < f(7x)$$

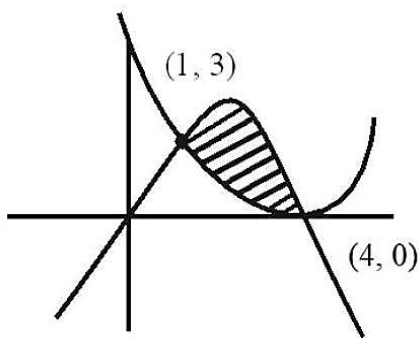
$$\therefore \frac{f(x)}{f(x)} < \frac{f(5x)}{f(x)} < \frac{f(7x)}{f(x)}$$

$$1 < \lim_{x \rightarrow \infty} \frac{f(5x)}{f(x)} < 1$$

$$\therefore \left[\frac{f(5x)}{f(x)} - 1 \right]$$

$$\Rightarrow 1 - 1 = 0$$

71. (c)



$$\text{Area} = \left| \int_1^4 \left[(4x - x^2) - \frac{(x-4)^2}{3} \right] dx \right|$$

$$\text{Area} = \left| \frac{4x^2}{2} - \frac{x^3}{3} - \frac{(x-4)^3}{9} \right|_1^4$$

$$= \left| \left(\frac{64}{2} - \frac{64}{3} - \frac{4}{2} + \frac{1}{3} - \frac{27}{9} \right) \right|$$

$$\Rightarrow (27 - 21) = 6$$

72. (c)

a, b, 68, 44, 48, 60

$$\text{Mean} = 55 \quad a > b$$

$$\text{Variance} = 194 \quad a + 3b$$

$$\frac{a + b + 68 + 44 + 48 + 60}{6} = 55$$

$$\Rightarrow 220 + a + b = 330$$

$$\therefore a + b = 110 \dots (1)$$

Also,

$$\sum \frac{(x_i - \bar{x})^2}{n} = 194$$

$$\Rightarrow (a - 55)^2 + (b - 55)^2 + (68 - 55)^2 + (44 - 55)^2$$

$$+ (48 - 55)^2 + (60 - 55)^2 = 194 \times 6$$

$$\Rightarrow (a - 55)^2 + (b - 55)^2 + 169 + 121 + 49 + 25 = 1164$$

$$\Rightarrow (a - 55)^2 + (b - 55)^2 = 1164 - 364 = 800$$

$$a^2 + 3025 - 110a + b^2 + 3025 - 110b = 800$$

$$\Rightarrow a^2 + b^2 = 800 - 6050 + 12100$$

$$a^2 + b^2 = 6850 \dots (2)$$

Solve (1) & (2);

$$a = 75, b = 35$$

$$\therefore a + 3b = 75 + 3(35) = 75 + 105 = 180$$

73. (a)

$$f(x) = e^{x^3 - 3x + 1}$$

$$f'(x) = e^{x^3 - 3x + 1} \cdot (3x^2 - 3)$$

$$= e^{x^3 - 3x + 1} \cdot 3(x - 1)(x + 1)$$

$$\text{For } f'(x) \geq 0$$

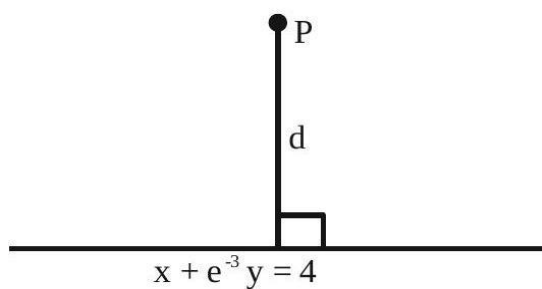
$\therefore f(x)$ is increasing function

$$\therefore a = e^{-\infty} = 0 = f(-\infty)$$

$$b = e^{-1+3+1} = e^3 = f(-1)$$

$$P(2b + 4, a + 2)$$

$$\therefore P(2e^3 + 4, 2)$$



$$d = \frac{(2e^3 + 4) + 2e^{-3} - 4}{\sqrt{1 + e^{-6}}} = 2\sqrt{1 + e^6}$$

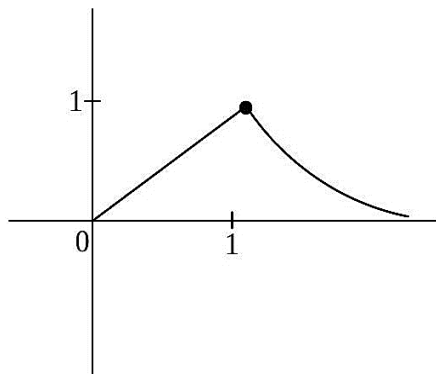
74. (c)

$$f: (0, \infty) \rightarrow \mathbb{R}$$

$$f(x) = e^{-|\log_e x|}$$

$$f(x) = \frac{1}{e^{|\ln x|}} = \begin{cases} \frac{1}{e^{-\ln x}}; 0 < x < 1 \\ \frac{1}{e^{\ln x}}; x \geq 1 \end{cases}$$

$$\begin{cases} \frac{1}{x} = x; 0 < x < 1 \\ \frac{1}{x}, x \geq 1 \end{cases}$$



$m = 0$ (No point at which function is not continuous)

$n = 1$ (Not differentiable)

$$\therefore m + n = 1$$

75. (d)

Take $e^{\sin x} = t (t > 0)$

$$\Rightarrow t - \frac{2}{t} = 2$$

$$\Rightarrow \frac{t^2 - 2}{t} = 2$$

$$\Rightarrow t^2 - 2t - 2 = 0$$

$$\Rightarrow t^2 - 2t + 1 = 3$$

$$\Rightarrow (t - 1)^2 = 3$$

$$\Rightarrow t = 1 \pm \sqrt{3}$$

$$\Rightarrow t = 1 \pm 1.73$$

$$\Rightarrow t = 2.73 \text{ or } -0.73 \text{ (rejected as } t > 0 \text{)}$$

$$\Rightarrow e^{\sin x} = 2.73$$

$$\Rightarrow \log_e e^{\sin x} = \log_e 2.73$$

$$\Rightarrow \sin x = \log_e 2.73 > 1$$

So no solution.

76. (b)

$$a = \sin^{-1}(\sin 5) = 5 - 2\pi$$

$$\text{and } b = \cos^{-1}(\cos 5) = 2\pi - 5$$

$$\therefore a^2 + b^2 = (5 - 2\pi)^2 + (2\pi - 5)^2$$

$$= 8\pi^2 - 40\pi + 50$$

77. (d)

$$\therefore {}^6C_m + 2({}^6C_{m+1}) + {}^6C_{m+2} > {}^8C_3$$

$${}^7C_{m+1} + {}^7C_{m+2} > {}^8C_3$$

$${}^8C_{m+2} > {}^8C_3$$

$$\therefore m = 2$$

$$\text{And } {}^{n-1}P_3: {}^nP_4 = 1:8$$

$$\frac{(n-1)(n-2)(n-3)}{n(n-1)(n-2)(n-3)} = \frac{1}{8}$$

$$\therefore n = 8$$

$$\therefore {}^nP_{m+1} + {}^{n+1}C_m = {}^8P_3 + {}^9C_2$$

$$= 8 \times 7 \times 6 + \frac{9 \times 8}{2}$$

$$= 372$$

78. (a)

$$\text{Let probability of tail is } \frac{1}{3}$$

$$\Rightarrow \text{Probability of getting head} = \frac{2}{3}$$

$$\therefore \text{Probability of getting 2 tails and 1 head}$$

$$= \left(\frac{1}{3} \times \frac{2}{3} \times \frac{1}{3}\right) \times 3$$

$$= \frac{2}{27} \times 3$$

$$= \frac{2}{9}$$

79. (a)

$$\text{Let } A = \begin{bmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{bmatrix}$$

$$\text{Given } A \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 2 \end{bmatrix}$$

$$\therefore \begin{bmatrix} x_1 + z_1 \\ x_2 + z_2 \\ x_3 + z_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 2 \end{bmatrix}$$

$$\therefore x_1 + z_1 = 2$$

$$x_2 + z_2 = 0$$

$$x_3 + z_3 = 0$$

$$\text{Given } A \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -4 \\ 0 \\ 4 \end{bmatrix}$$

$$\therefore \begin{bmatrix} -x_1 + z_1 \\ -x_2 + z_2 \\ -x_3 + z_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \\ 4 \end{bmatrix}$$

$$\Rightarrow -x_1 + z_1 = -4$$

$$-x_2 + z_2 = 0$$

$$-x_3 + z_3 = 4$$

$$\text{Given } A \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix}$$

$$\therefore \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix}$$

$$\therefore y_1 = 0, y_2 = 2, y_3 = 0$$

$$\therefore \text{from (2), (3), (4), (5), (6) and (7)}$$

$$x_1 = 3x, x_2 = 0, x_3 = -1$$

$$y_1 = 0, y_2 = 2, y_3 = 0$$

$$z_1 = -1, z_2 = 0, z_3 = 3$$

$$\therefore A = \begin{bmatrix} 3 & 0 & -1 \\ 0 & 2 & 0 \\ -1 & 0 & 3 \end{bmatrix}$$

$$\therefore \text{Now } (A - 3I) \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}$$

$$\therefore \begin{bmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}$$

$$\begin{bmatrix} -z \\ -y \\ -x \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$$[z = -1], [y = -2], [x = -3]$$

$$80. (c) L_2 = \frac{x+4}{3} = \frac{y-4}{2} = \frac{z-3}{0}$$

$$\therefore S.D = \frac{\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ 2 & -3 & 2 \\ 3 & 2 & 0 \end{vmatrix}}{|\vec{n}_1 \times \vec{n}_2|}$$

$$= \frac{\begin{vmatrix} 5 & -5 & -7 \\ 2 & -3 & 2 \\ 3 & 2 & 0 \end{vmatrix}}{|\vec{n}_1 \times \vec{n}_2|}$$

$$= \frac{141}{|-4\hat{i} + 6\hat{j} + 13\hat{k}|}$$

$$= \frac{141}{\sqrt{16 + 36 + 169}}$$

$$= \frac{141}{\sqrt{221}}$$

81. (15)

$$\begin{aligned} & \int_0^{\pi} \frac{x^2 \sin x \cdot \cos x}{\sin^4 x + \cos^4 x} dx \\ &= \int_0^{\frac{\pi}{2}} \frac{\sin x \cdot \cos x}{\sin^4 x + \cos^4 x} (x^2 - (\pi - x)^2) dx \\ &= \int_0^{\frac{\pi}{2}} \frac{\sin x \cdot \cos x (2\pi x - \pi^2)}{\sin^4 x + \cos^4 x} dx \\ &= 2\pi \int_0^{\frac{\pi}{2}} \frac{x \sin x \cos x}{\sin^4 x + \cos^4 x} dx - \pi^2 \int_0^{\frac{\pi}{2}} \frac{\sin x \cos x}{\sin^4 x + \cos^4 x} dx \\ &= 2\pi \cdot \frac{\pi}{4} \int_0^{\frac{\pi}{2}} \frac{\sin x \cos x}{\sin^4 x + \cos^4 x} dx - \pi^2 \int_0^{\frac{\pi}{2}} \frac{\sin x \cos x}{\sin^4 x + \cos^4 x} dx \\ &= -\frac{\pi^2}{2} \int_0^{\frac{\pi}{2}} \frac{\sin x \cos x}{\sin^4 x + \cos^4 x} dx \\ &= -\frac{\pi^2}{2} \int_0^{\frac{\pi}{2}} \frac{\sin x \cos x dx}{1 - 2\sin^2 x \times \cos^2 x} \\ &= -\frac{\pi^2}{2} \int_0^{\frac{\pi}{2}} \frac{\sin 2x}{2 - \sin^2 2x} dx \\ &= -\frac{\pi^2}{2} \int_0^{\frac{\pi}{2}} \frac{\sin 2x}{1 + \cos^2 2x} dx \end{aligned}$$

Let $\cos 2x = t$

82. (36)

$$(a^2 + b^2)x^2 - 2b(a + c)x + b^2 + c^2 = 0$$

$$\Rightarrow a^2x^2 - 2abx + b^2 + b^2x^2 - 2bcx + c^2 = 0$$

$$\Rightarrow (ax - b)^2 + (bx - c)^2 = 0$$

$$\Rightarrow ax - b = 0, bx - c = 0$$

$$\Rightarrow a + b > c \quad b + c > a \quad c + a > b$$

$$a + ax > bx \mid ax + bx > a \quad ax^2 + a > ax$$

$$a + ax > ax^2 \quad ax + ax^2 > a \quad x^2 - x + 1 > 0$$

$$x^2 - x - 1 < 0 \mid x^2 + x - 1 > 0 \text{ always true}$$

$$\frac{1 - \sqrt{5}}{2} < x < \frac{1 + \sqrt{5}}{2}$$

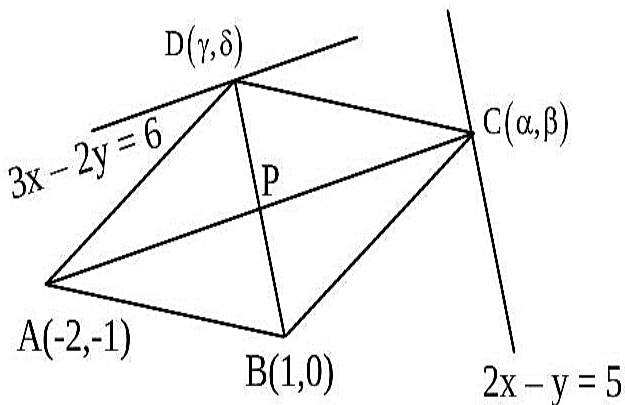
$$x < \frac{-1 - \sqrt{5}}{2}, \text{ or } x > \frac{-1 + \sqrt{5}}{2}$$

$$\Rightarrow \frac{\sqrt{5} - 1}{2} < x < \frac{\sqrt{5} + 1}{2}$$

$$\Rightarrow \alpha = \frac{\sqrt{5} - 1}{2}, \beta = \frac{\sqrt{5} + 1}{2}$$

$$12(\alpha^2 + \beta^2) = 12 \left(\frac{(\sqrt{5} - 1)^2 + (\sqrt{5} + 1)^2}{4} \right) = 36$$

83. (32)



$$P \equiv \left(\frac{\alpha - 2}{2}, \frac{\beta - 1}{2} \right) \equiv \left(\frac{\gamma + 1}{2}, \frac{\delta}{2} \right)$$

$$\frac{\alpha - 2}{2} = \frac{\gamma + 1}{2} \text{ and } \frac{\beta - 1}{2} = \frac{\delta}{2}$$

$$\Rightarrow \alpha - \gamma = 3 \dots (1), \beta - \delta = 1 \dots (2)$$

Also, (γ, δ) lies on $3x - 2y = 6$

$$3\gamma - 2\delta = 6 \dots (6)$$

and (α, β) lies on $2x - y = 5$

$$\Rightarrow 2\alpha - \beta = 5.$$

Solving (a), (b), (c), (d)

$$\alpha = -3, \beta = -11, \gamma = -6, \delta = -12$$

$$|\alpha + \beta + \gamma + \delta| = 32$$

84. (25)

$$(x+3)^{n-1} + (x+3)^{n-2}(x+2) + (x+3)^{n-3}$$

$$(x+2)^2 + \dots + (x+2)^{n-1}$$

$$\sum \alpha_r = 4^{n-1} + 4^{n-2} \times 3 + 4^{n-3} \times 3^2 + \dots + 3^{n-1}$$

$$= 4^{n-1} \left[1 + \frac{3}{4} + \left(\frac{3}{4}\right)^2 + \dots + \left(\frac{3}{4}\right)^{n-1} \right]$$

$$= 4^{n-1} \times \frac{1 - \left(\frac{3}{4}\right)^n}{1 - \frac{3}{4}}$$

$$= 4^n - 3^n = \beta^n - \gamma^n$$

$$\beta = 4, \gamma = 3$$

$$\beta^2 + \gamma^2 = 16 + 9 = 25$$

85. (7)

$$|A| = 2$$

$$\begin{aligned} \underbrace{\text{adj}(\text{adj}(\text{adj} \dots (a)))}_{2024 \text{ times}} &= |A|^{(n-1)2024} \\ &= |A|^{2^{2024}} \\ &= 2^{2^{2024}} \end{aligned}$$

$$2^{2024} = (2^2)2^{2022} = 4(8)^{674} = 4(9-1)^{674}$$

$$\Rightarrow 2^{2024} \equiv 4 \pmod{9}$$

$$\Rightarrow 2^{2024} \equiv 9m + 4, m \leftarrow \text{even}$$

$$2^{9m+4} \equiv 16 \cdot (2^3)^{3m} \equiv 16 \pmod{9}$$

$$\equiv 7$$

86. (38)

$$(\vec{a} + \vec{b}) \times \vec{c} = 2(\vec{a} \times \vec{b}) + 24\hat{j} - 6\hat{k}$$

$$(5\hat{i} + \hat{j} + 4\hat{k}) \times \vec{c} = 2(7\hat{i} - 7\hat{j} - 7\hat{k}) + 24\hat{j} - 6\hat{k}$$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 5 & 1 & 4 \\ x & y & z \end{vmatrix} = 14\hat{i} + 10\hat{j} - 20\hat{k}$$

$$\Rightarrow \hat{i}(z - 4y) - \hat{j}(5z - 4x) + \hat{k}(5y - x) = 14\hat{i} + 10\hat{j} - 20\hat{k}$$

$$z - 4y = 14, 4x - 5z = 10, 5y - x = -20$$

$$(a - b + i) \cdot \vec{C} = -3$$

$$(2\hat{i} + 3\hat{j} - 2\hat{k}) \cdot \vec{c} = -3$$

$$2x + 3y - 2z = -3$$

$$\therefore x = 5, y = -3, z = 2$$

$$|\vec{C}|^2 = 25 + 9 + 4 = 38$$

87. (81)

$$ax^2 \left(1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots \right) - b \left(x - \frac{x^2}{2} + \frac{x^3}{3} - \dots \right)$$

$$\therefore \lim_{x \rightarrow 0} \frac{+cx \left(1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \dots \right)}{x^3 \cdot \frac{\sin x}{x}}$$

$$= \lim_{x \rightarrow 0} \frac{(c - b)x + \left(\frac{b}{2} - c + a \right)x^2 + \left(a - \frac{b}{3} + \frac{c}{2} \right)x^3 + \dots}{x^3} = 1$$

$$c - b = 0, \frac{b}{2} - c + a = 0$$

$$a - \frac{b}{3} + \frac{c}{2} = 1 \quad a = \frac{3}{4} \quad b = c = \frac{3}{2}$$

$$a^2 + b^2 + c^2 = \frac{9}{16} + \frac{9}{4} + \frac{9}{4}$$

$$16(a^2 + b^2 + c^2) = 81$$

88. (22)

$$\frac{x - 4}{12} = \frac{x + 6}{4} = \frac{z + 2}{6}$$

$$\frac{x - 4}{6} = \frac{y + 6}{2} = \frac{z + 2}{3} = 21$$

$$\left(21 \times \frac{6}{7} + 4, \frac{2}{7} \times 21 - 6, \frac{3}{7} \times 21 - 2 \right)$$

$$= (22, 0, 7) = (a, b, c)$$

$$\therefore \sqrt{324 + 144 + 16} = 22$$

89. (9)

$$\sec^2 x \frac{dx}{dy} + e^{2y} \tan^2 x + \tan x = 0$$

$$\left(\text{Put } \tan x = t \Rightarrow \sec^2 x \frac{dx}{dy} = \frac{dt}{dy} \right)$$

$$\frac{dt}{dy} + e^{2y} \times t^2 + t = 0$$

$$\frac{dt}{dy} + t = -t^2 \cdot e^{2y}$$

$$\frac{1}{t^2} \frac{dt}{dy} + \frac{1}{t} = -e^{2y}$$

$$\left(\text{Put } \frac{1}{t} = u \quad \frac{-1}{t^2} \frac{dt}{dy} = \frac{du}{dy} \right)$$

$$\frac{-du}{dy} + u = -e^{2y}$$

$$\frac{du}{dy} - u = e^{2y}$$

$$\text{I.F.} = e^{-\int dy} = e^{-y}$$

$$ue^{-y} = \int e^{-y} \times e^{2y} dy$$

$$\frac{1}{\tan x} \times e^{-y} = e^y + c$$

$$x = \frac{\pi}{4}, y = 0, c = 0$$

$$x = \frac{\pi}{6}, y = \alpha$$

$$\sqrt{3}e^{-\alpha} = e^{\alpha} + 0$$

$$e^{2\alpha} = \sqrt{3}$$

$$e^{8\alpha} = 9$$

90. (66)

$$R = \{(3,2), (6,4), (9,6), (12,8), \dots \dots (99,66)\}$$

$$n(R) = 33$$

$$\therefore 66$$