

Physics

1. (b) $g_h = g \left(1 - \frac{2h}{R}\right)$

$$g_d = g \left(1 - \frac{d}{R}\right)$$

$$g_h = g_d$$

$$g \left(1 - \frac{2h}{R}\right) = g \left(1 - \frac{d}{R}\right)$$

$$d = 2R$$

$$= 2 \times 10 = 20 \text{ km}$$

2. (c) For a perfect rigid body elongation $\Delta l = 0$

$$y = \left(\frac{f}{A}\right) \frac{1}{\Delta l} \text{ becomes infinity}$$

3. (c) $\omega_1 = 0$

$$\omega_2 = 10 \text{ rad/sec}$$

$$t = 5 \text{ sec}$$

$$\theta = \left(\frac{\omega_1 + \omega_2}{2}\right) \times t$$

$$\theta = \frac{(0 + 10) \times 5}{2} = 25 \text{ rad}$$

4. (a)

$$V_b \cdot \rho_b = V_i \cdot \rho_i$$

$$\frac{V_i}{V_b} = \frac{\rho_b}{\rho_i} = \frac{0.917}{1} = 0.917$$

5. (c) From $E = A\sigma T^4$

$$E \propto A$$

Surface area is more for plate and less for sphere. Hence plate will cool the fastest and sphere the slowest

6. (a) In an adiabatic expansion as temperature decreases from ideal gas equation $PV = nRT$ the product of pressure and volume decreases

7. (c) $\frac{dW}{dQ} = 1 - \frac{1}{\gamma} = 1 - \frac{1}{(5/3)}$

$$= \frac{2}{5}$$

8. (c) $T = 2\pi \sqrt{\frac{m}{k_{\text{eff}}}}$

$$\frac{3}{2} = 2\pi \sqrt{\frac{12}{2k}}$$

$$\frac{9}{4} = 4\pi^2 \times \frac{12}{2k}$$

$$k \approx 105 \text{ N/m}$$

9. (d)

10. (c)

$$\phi_{ABCD} = \frac{\phi}{6} = \frac{q}{6\epsilon_0}$$

11. (a) $F = Eq$

$$= 20 \times 20 \times 10^{-6}$$

$$= 4 \times 10^{-4} \text{ V/m}$$

12. (c)

$$E = \frac{\lambda}{2\pi\epsilon_0 \cdot r}$$

$$= \frac{1}{4} \times \frac{10^{-2}}{10^{-2}} \times 18 \times 10^9 \times 5$$

$$= 2.25 \times 10^8 \text{ N/C}$$

$$13. (b) T = 2\pi \sqrt{\frac{I}{PE}}$$

$$14. (c) C_P - C_S = 6\mu\text{F}$$

$$2C - \frac{C}{2} = 6 \Rightarrow C = 4\mu\text{F}$$

$$15. (b) V_A = V_B > V_C$$

16. (a) Its radius increases

$$17. (c) \frac{1}{2}mv^2 = Vq$$

$$v = \sqrt{\frac{2Vq}{m}} = 2.1 \times 10^7$$

$$18. (a) R \propto \frac{1}{A}$$

Maximum when the battery is connected across $1 \text{ cm} \times \frac{1}{2} \text{ cm}$ faces

$$19. (c) V = E - ir = 10.4 \text{ V}$$

20. (a)

$$\frac{E_1}{E_2} = \frac{l_1}{l_2} \Rightarrow \frac{10}{E_2} = \frac{5}{2} \Rightarrow E_2 = 4 \text{ V}$$

21. (a) Red, Grey, Brown, Silver

22. (a) $R_{\text{eff}} = \frac{5}{6}r = \frac{5}{6}\Omega$

23. (d) $\text{Slope} = \frac{1}{R} = \frac{A}{\rho \times l}$

Less if the length of the wire is increased

24. (c) $B_{\text{net}} = B_1 + B_2 + B_3$

$$= \frac{3\mu_0 I}{8r} + \frac{\mu_0 I}{4\pi r} + 0$$

25. (a) $\frac{\mu_0 i \vec{dl} \times \vec{r}}{4\pi r^3}$

26. (c)

27. (c) $K.E = \frac{q^2 B^2 r}{2m}$ $K.E = \frac{q^2}{m}$

Minimum K.E is gained by deuteron

28. (b) $I \propto \frac{B}{T}$

$$\frac{I_2}{I_1} = \frac{B_2}{B_1} \times \frac{T_1}{T_2} \quad \frac{I_2}{8} = \frac{0.2}{0.6} \times \frac{4}{16} \quad I_2 = \frac{2}{3} \text{ Am}^{-1}$$

29. (d) $\frac{B}{M} = \frac{(\mu_0 IN/2r)}{N I \pi r^2}$

$$\frac{B}{M} = \alpha \frac{1}{r^3}$$

$$x = \frac{B}{M} (\text{let})$$

$$\frac{x_2}{x} = \left(\frac{r}{2r}\right)^3 = \frac{x}{8}$$

30. (d) Domains are all perfectly aligned

31. (b) $i = \frac{Blv}{R} = \frac{0.04 \times 2 \times 5}{3} = 133 \text{ mA}$

32. (b) $e = L \frac{di}{dt}$

$$= 0.2 \left(\frac{5-2}{0.5} \right)$$

$$= \frac{2}{5} \times 3 = 1.2 \text{ V}$$

33. (c)

$$V_0 = \sqrt{V_R^2 + (V_L - V_C)^2}$$

$$\begin{aligned}
 &= \sqrt{(60)^2 + (110 - 30)^2} \\
 &= 100 \\
 V_{\text{rms}} &= \frac{V_0}{\sqrt{2}} = \frac{100}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} \\
 &= 100 \left(\frac{\sqrt{2}}{2} \right) \\
 &= 100 \left(\frac{1.414}{2} \right) \\
 &= 70.7 \text{ V}
 \end{aligned}$$

34. (b)

$$\cos \phi = \frac{1}{\sqrt{3}}$$

$$\tan \phi = \frac{\sqrt{2}}{1}$$

$$\tan \phi = \frac{X_L}{R}$$

$$\begin{aligned}
 \sqrt{2} &= \frac{2}{R} \\
 R &= \frac{2}{\sqrt{2}} = \sqrt{2} \Omega
 \end{aligned}$$

35. (c)

$$\begin{aligned}
 V &= \frac{1}{2\pi\sqrt{LC}} \\
 &= \frac{1}{2\pi\sqrt{0.5 \times 10^{-3} \times 20 \times 10^{-6}}} \\
 &= 1592 \text{ Hz}
 \end{aligned}$$

36. (c) $I = \frac{E}{A}$

$$E = IA$$

$$P = \frac{2E}{C}$$

$$P = \frac{2IA}{C}$$

$$= \frac{2 \times 20 \times 25 \times 15}{3 \times 10^8}$$

$$= 5 \times 10^{-5} \text{ kg ms}^{-1}$$

37. (c) Moves away from the lens with a non-uniform acceleration.

38. (d) $n = \frac{\sin\left(\frac{A+d_m}{2}\right)}{\sin\frac{A}{2}}$

$$\cot \frac{A}{2} = \frac{\sin \left(\frac{A + d_m}{2} \right)}{\sin \frac{A}{2}}$$

$$\frac{\cos \frac{A}{2}}{\sin \frac{A}{2}} = \frac{\sin \left(\frac{A + d_m}{2} \right)}{\sin \frac{A}{2}}$$

$$\sin \left(90 - \frac{A}{2} \right) = \sin \left(\frac{A + d_m}{2} \right)$$

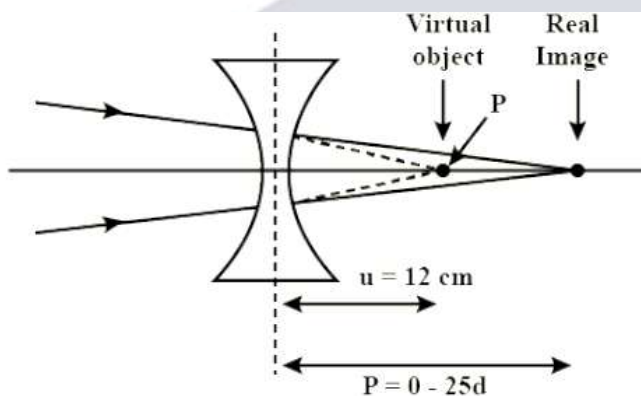
$$90 - \frac{A}{2} = \frac{A + d_m}{2}$$

$$180 - A - A = d_m$$

$$180 - 2A = d_m$$

39. (d) By using lens formula

$$\frac{11}{-16v} - \frac{1}{(+12)} \Rightarrow \frac{1}{v} = \frac{1}{2} - \frac{1}{16} = \frac{4-3}{16} \Rightarrow v = 48 \text{ cm}$$



40. (c) $I = \frac{I_0}{2} (\cos^2 \theta)^2$

41. (a) $\theta = \frac{d}{D}$

42. (d) $\text{Shift} = (M - 1)t \frac{D}{d} = 0.25 \text{ mm}$

43. (b) $E = 13.6 \text{ V } \lambda = \frac{12.27}{\sqrt{13.6}} = \frac{12.27}{3.68} = 3.33 \text{ \AA}$

44. (a) Stopping potential same

So, frequencies same ($r_1 = r_2$)

Currents are different

So, intensity are different

$$I_1 \neq I_2$$

45. (c) $\tan^3$

46. (b) $mvr = \frac{nh}{2\pi}$

$$n = 3$$

47. (c) A' will be maximum

B' will be Minimum

48. (b)

49. (c) A neutron in the nucleus decays emitting an electron

50. (c) Fraction of remaining element

$$\left(1 - \frac{N}{N_0}\right) \times 100 = \left(\frac{1}{2}\right)^{t/T} \times 100 = 0.25$$

The fraction that will decay in 30 years is 0.75

51. (d) The potential difference across the capacitor is peak voltage.

$$V_{\max} = V_{\text{rms}} \times \sqrt{2} = 220\sqrt{2} \text{ V}$$

52. (b) $P = 0, Q = 1$

53. (b) A vacancy created when an electron leaves a covalent bond.

$$54. (b) B = \frac{\mu_0 i}{2\pi r}$$

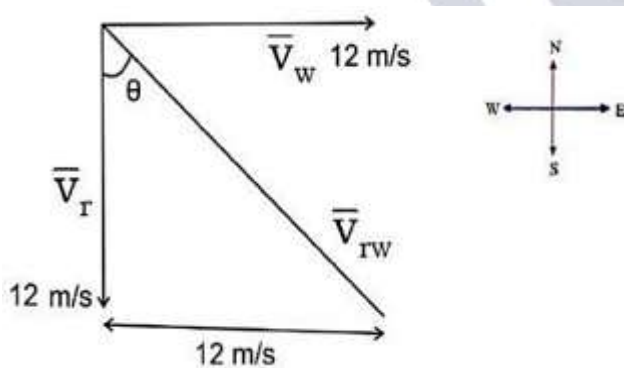
$$B \propto \frac{1}{r}$$

$$55. (d) d = \frac{m}{v} = \frac{m}{\pi r^2 l}$$

$$\frac{\Delta d}{d} \times 100\% = \frac{\Delta m}{m} \times 100\% + 2 \frac{\Delta r}{r} \times 100\% + \frac{\Delta l}{l} \times 100\% = 4$$

$$56. (c) t = \frac{t_1 t_2}{t_1 + t_2} = 12$$

57. (b)



$$\tan \theta = \frac{|V_r|}{|V_w|} = \frac{12}{12} = 1$$

$$\theta = 45^\circ \text{ towards east}$$

58. (a) The net force on the particle (directed towards the centre) is tension (T) in the string

59. (b)

$$P = \frac{1}{2} \frac{mv^2}{t} = \frac{1}{2} \frac{m \times (at)^2}{t}$$

pat

$$60. (a) I_z = I_x + I_y = 0.5 \text{Kgm}^2$$

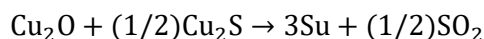
$$I = mK^2$$

$$K = 0.5 \text{ m} = 50 \text{ cm}$$

Chemistry

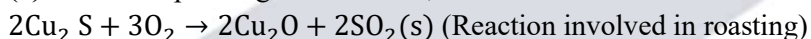
61. (d) The formula of copper pyrites is CuFeS_2 . In auto reduction, the sulphide ores of the metals which are less electropositive like Hg, Pb and Cu are heated in the air so as to convert a part of it to its oxides. These oxides then react with the remaining sulphide ore in the absence of air, to give the metal and sulphur dioxide.

The corresponding reaction is:



62. (b)

63. (a) The corresponding reactions are;



So here X is SO_2 (sulphur dioxide)

And Y is SO_2Cl_2 (Sulfuryl chloride)

64. (a)

65. (a) The hydrides of group 15, 16 below the 3rd period, follows Drago's rule. The rule states that due to a large energy difference between the atomic orbitals, these compounds do not exhibit hybridization. Thus, PH_3 will not exhibit hybridization and here the bond formation takes place due to the overlap of pure p-orbitals and s-orbitals. PH_3 has a lone pair on the central P atom, which is absent in PH_4 . Thus in PH_3 , there will be bond pair - lone pair repulsion and this is the reason why the bond angle in PH_3 is less than that of PH_4^+ .

66. (b) In XeF_6 there are 6 bond pairs and one lone pair. Thus the shape of XeF_6 is distorted octahedral.

In XeOF_4 , Xe is sp^3d hybridized and the shape is square pyramidal.

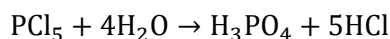
In XeO_3 , Xe is sp^3 hybridized and the shape is trigonal pyramidal.

In XeF_4 , Xe is sp^3d^2 hybridized. It has 4 bond pairs and two lone pairs. Thus, the shape is square planar.

67. (a) (a) Phosphorous pentachloride (PCl_5) has a trigonal bipyramidal structure. Here, the five bonds are not of equal lengths as the axial bonds are slightly longer than the equatorial bonds.

(b) PCl_5 solid exists as $[\text{PCl}_4]^+$ and $[\text{PCl}_6]^-$ and they have tetrahedral and octahedral structures respectively.

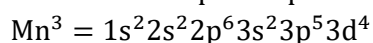
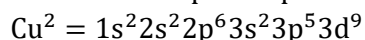
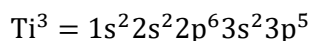
(c) On hydrolysis, PCl_5 gives H_3PO_4 which is tribasic.



In H_3PO_4 , P is in its +5-oxidation state. So, it is in its highest oxidation state and hence is not a good reducing agent.

68. (c) The ions which contain unpaired electrons exhibit paramagnetic behaviour.

The electronic configuration of the ions are given below:



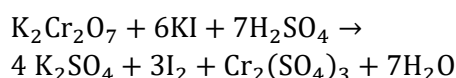
Thus, all of them have unpaired electrons. Hence, they will show paramagnetic behaviour.

In option (B) V^3 does not possess any unpaired electrons.

In option (C) Zn^2 does not possess any unpaired electrons.

Similarly, in option (D) Zn^2 does not possess any unpaired electrons.

69. (a) The balanced redox reaction will be:



3 moles of I_2 requires 1 mole of $\text{K}_2\text{Cr}_2\text{O}_7$

Hence, 6 moles of I_2 will require 2 moles of $\text{K}_2\text{Cr}_2\text{O}_7$

70. (a) CuCl_2 is more stable than Cu_2Cl_2 . The size of Cu^{2+} is smaller than Cu^+ . The hydration enthalpy depends on the size of the cation. Smaller the size of the cation more will be the hydration enthalpy. Hence, more will be stability.

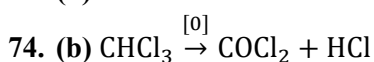
71. (d) $\text{C}_2\text{O}_4^{2-}$ is the oxalate ligand. It binds through two oxygen atoms and thus, is a bidentate ligand.

Hence the co-ordination number of Fe in $[\text{Fe}(\text{C}_2\text{O}_4)_3]^{3-}$ is 6.

SCN is the thiocyanate ligand. It is a monodentate ligand. Hence the co-ordination number of Co in the complex ion $[\text{Co}(\text{SCN})_4]^{2-}$ is 4

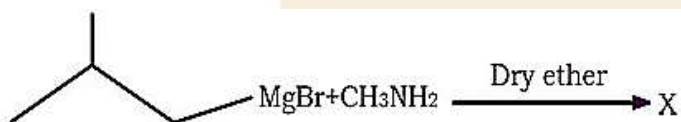
72. (d)

73. (d)



75. (a) Rate of SN^1 reaction is directly proportional to stability of carbocation or Reactivity of SN^1 reaction is influenced by stability of carbocation.

76. (d)



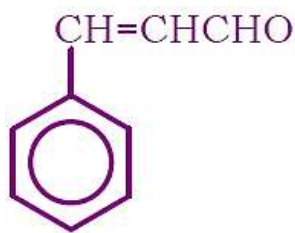
x = isobutane and it has two isomers.

77. (d) Primary alkyl halides/benzyl halides reacts with alkoxide/phenoxide through SN^2 mechanism gives ethers. Vinyl and aryl halides least reactive towards SN^1

78. (c)

79. (d)

80. (c)



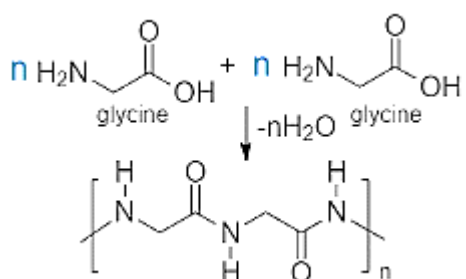
81. (c)
 82. (c) Aldehydes and ketones containing alpha hydrogens will undergo aldol condensation
 83. (b)
 84. (b)
 85. (b)
 86. (c)
 87. (b)
 88. (b) The monomer of polythene is ethane (C_2H_4)

The monomer of polystyrene is styrene ($C_6H_5CH=CH_2$)

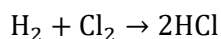
The monomer of neoprene is chloroprene ($CH_2 = C(Cl) - CH = CH_2$)

The monomer of Terylene is 1,4 -benzene dicarboxylic acid and 1,2 -ethanediol. Thus, it possesses intermolecular hydrogen bonding and has the strongest intermolecular forces among the given options.

89. (b) Glycine can undergo condensation polymerisation. Given below is an example:



90. (a)
 91. (b) Antagonists impede the action of drugs by occupying the enzyme sites themselves and hence is the odd one out.
 92. (b) The corresponding balanced reaction is:



Again, $0.4gH_2 = 0.4/2 \text{ mol} = 0.2 \text{ mol}$

$7.1gCl_2 = 7.1/71 \text{ mol} = 0.1 \text{ mol}$

So, Cl_2 is the limiting reagent here.

Hence 0.2 moles of HCl will be formed.

Again, we need to find out the volume of HCl formed in the STP condition.

1 mol of gas in STP occupies 22.7L

Thus, 0.2 mol of HCl will occupy = 4.54 L

93. (d) Photoelectric effect is the emission of electrons from a certain metal surface when it is irradiated with photons or light. The minimum frequency required to eject an electron from a metal surface is called threshold frequency. (ν_0)

The equation for the photoelectric effect is:

$$h\nu = h\nu_0 + KE, \text{ where } \nu = \text{frequency of the incident radiation}$$

Here $h\nu_0$ is the work function. The number of electrons ejected does not depend on work function.

The number of electrons ejected increases with the increase in the intensity of incident light, an increase in the intensity of incident light means that the number of photons incident per unit surface area of the metal increases. (Provided the incident photons has its frequency more than the threshold frequency)

The energy of the ejected electrons increases with an increase in the frequency of incident light.

94. (d) The last element of *p*-Block in 6th period is $Rn - 86$. It's outer electronic configuration is $4f^{14}5d^{10}6s^26p^6$. It is a Noble Gas. Thus,

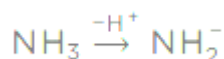
95. (d) The correct option is CNH_2^-

Conjugate base:

Species formed by the loss of a proton from an acid is called the conjugate base of that acid.

[Conjugate base has one proton less than the acid]

When H^+ is removed from NH_3 we get NH_2^- .



So, NH_2^- is the conjugate base of NH_3 acid.

96. (c) Since temperature and pressure are constant, we can assume it to be in STP condition.

Now, let the total volume be 100 L

$$\text{Moles of He present} = \frac{25}{22.4} = 1.12 \text{ mol}$$

$$\text{Mass of He present} = 1.12 \times 4 = 4.48 \text{ g}$$

$$\text{Moles of } CH_4 \text{ present} = \frac{75}{22.4} = 3.35 \text{ mol}$$

$$\text{Mass of } CH_4 \text{ present} = 3.35 \times 16 = 53.60 \text{ g}$$

$$\text{Hence, the mass percentage of } CH_4 = \frac{53.60}{58.08} \times 100 = 92.28\%$$

97. (c) In NO_2 , the steric number of N is two (two sigma bonds). Thus, it is sp hybridized. Hence, the percentage s-character is $= 1/2 \times 100 = 50\%$.

In O_3 , the steric number of N is three (three sigma bonds). Thus, it is sp^2 hybridized. Hence the percentage s-character is $= 1/3 \times 100 = 33.33\%$.

In NH_4 , the steric number of N is four (four sigma bonds). Thus, it is sp^3 hybridized.

Hence, the percentage s-character is $= 1/4 \times 100 = 25\%$

98. (d)

99. (a) In thermodynamics, we are concerned about entropy change due to the transfer of heat.

Which we can write in the form, $\Delta S = q/T$, here q is constant.

So, $\Delta S \propto 1/T$

We are given, $T_1 > T_2$, so the actual relation between the entropies will be $\Delta S \propto 1/T$

100. (b)

101. (b)

102. (d) Potassium generally forms three oxides: K_2O , K_2O_2 and KO_2 . K_2O_3 is not formed in normal conditions and hence, is the answer.

103. (a) Amphoteric metals can react with both acids and bases.

104. (a)

105. (b)

106. (c)

107. (a)

$M_{O.96}^\circ$

No. of M^{+2} ions = x

No. of M^{+3} ions = $0.96 - x$

Total positive charges = Total negative charge (in magnitude)

$$x(2) + (0.96 - x)(3) = 1(2)$$

$$2x + 2.88 - 3x = 2$$

$$-x = 2 - 2.88$$

$$\therefore x = 0.88$$

$$\begin{aligned} \text{No. of } M^{+3} \text{ ions} &= 0.96 - 0.88 \\ &= 0.08 \end{aligned}$$

$$\text{Percentage of } M^{+3} = \frac{0.08}{0.96} \times 100$$

$$= 8.33\%$$

108. (b)

For FCC

$$\text{Atomic radius } (r) = \frac{\sqrt{2}a}{4}$$

$$\sqrt{2} \times 10^{-10} = \frac{\sqrt{2}a}{4}$$

$$a = \frac{4 \times \sqrt{2} \times 10^{-10}}{\sqrt{2}}$$

$$a = 4 \times 10^{-10} \text{ m}$$

Volume of unit cell = a^3

$$= (4 \times 10^{-10})^3$$

$$= 64 \times 10^{-30}$$

$$= 6.4 \times 10^{-29} \text{ m}^3$$

109. (d) An intrinsic semiconductor is a semiconductor where we do not use any dopant, i.e. it is an undoped semiconductor. Silicon has 4 valence electrons whereas gallium has only 3 valence electrons. Thus, the dopant has less number of valence electrons in this case. Hence, silicon doped with gallium forms a p-type semiconductor.
110. (b) The value of the constant A in the Debye - Huckel - Onsager equation, $\lambda_m = \lambda_m^0 - A\sqrt{C}$ depends on temperature, charges of the ions, the dielectric constant of the solvent and also the viscosity of the solvent. Here, since we are not given the solvent, we will assume that the solvent is the same for each case. Here, the deciding factor is the charge on the ions. Again, in NH_4Cl and NaBr , the charges on the ions are the same. Hence the pair of electrolytes that will possess the same value for the constant (A) will be NH_4Cl , NaBr .
111. (a) When solute particle concentration is same then they are isotonic
112. (d)

$$i = 1 + \alpha \left(\frac{1}{n} - 1 \right)$$

$$i = 1 + 0.8 \left(\frac{1}{2} - 1 \right)$$

$$i = 1 - 0.4 = 0.6$$

$$\Delta T_b = k_b \times \frac{W}{m} \times \frac{100}{W(\text{gm})} \times i$$

$$0.3 = 0.52 \times \frac{2.5}{m} \times \frac{1000}{100} \times 0.6$$

$$\text{Molar mass of } x(m) = \frac{0.52 \times 2.5 \times 10 \times 0.6}{0.3}$$

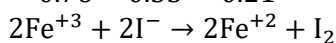
$$= 26$$

113. (a) $E_{\text{Fe}^{+3}/\text{Fe}^{+2}}^0 = +0.76$ (cathode)

$$E_{\text{I}_2/\text{I}^-}^0 = +0.55$$
 (Anode)

$$E_{\text{cell}}^0 = E_{\text{C}}^0 - E_{\text{A}}^0$$

$$= 0.76 - 0.55 = 0.21$$



$$E_{\text{Cell}}^0 = \frac{0.059}{n} \log k_c$$

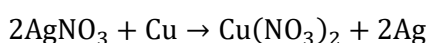
$$0.21 = \frac{0.059}{2} \log k_c$$

$$\log k_c = 7$$

$$k_c = 10^7$$

114. (d)

115. (b) Platinum and gold lie at the bottom of the reactivity series. Thus it does not react with HNO_3 and HCl . Again, Zn is more reactive than copper. Thus, copper will be displaced by Zn, if ZnSO_4 is placed in a copper vessel. But Ag is less reactive than copper. Thus, copper will displace silver. The corresponding reaction is: $\square$



116. (d) 60% completion

$$K = \frac{2.303}{t} \log \frac{[R_0]}{[R]}$$

$$K = \frac{2.303}{50} \log \frac{100}{40}$$

$$K = \frac{2.303}{50} \times 0.397$$

93.6% completion

$$K = \frac{2.303}{t} \log \frac{[R_0]}{[R]}$$

$$\frac{2.303}{50} \times 0.397 = \frac{2.303}{t} \log \frac{100}{6.4}$$

t = 150 min

117. (b)

$$-\frac{1}{3} \frac{d((B))}{dt} = +\frac{1}{4} \frac{d((C))}{dt}$$

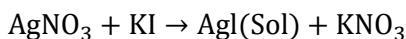
$$\frac{-d((B))}{dt} = +\frac{3}{4} \frac{d((C))}{dt}$$

$$= \frac{+3}{4} (2.8 \times 10^{-3}) \text{ molL}^{-1} \text{ S}^{-1}$$

118. (c) The expression of rate constant given in the question ($k = PZe^{(-E_a/RT)}$) is according to Arrhenius theory.

To speed up the reaction, we will have to decrease the value of E_a i.e. activation energy and will have to increase the value of temperature (T) and the number of collisions (Z).

119. (d) Given reaction is,



Here, the amount of AgNO_3 present is 0.1 M.

Amount of KI present is 0.2 M.

Since KI is excess here, thus, sol obtained is a negative sol with I adsorbed on AgI.

120. (a)

Mathematics

121. (b)

122. (b)

123. (b)

124. (b)

$$y = (2x)^{(n+1)} + (3x)^{(-n)}$$

$$\Rightarrow dy/dx = 2(n+1)x^n - (3nx)^{(-n-1)}$$

$$\Rightarrow (d^2y)/(dx^2) = 2n(n+1)x^{(n-1)} + 3n(n+1)x^{(-n-2)}$$

$$\Rightarrow x^2(d^2y)/(dx^2) = n(n+1)[(2x)^{(n+1)} + 3/x^n]$$

$$\Rightarrow x^2(d^2y)/(dx^2) = n(n+1)y$$

125. (d)

126. (a)

127. (a) Let one side of the cube be x and surface area be A

$$\text{So, } dx = 5\% = 5x/100$$

$$\text{Then, } A = (6x)^2$$

$$\Rightarrow dA/dx = 12x$$

$$\Rightarrow dA = (12x)dx$$

$$\Rightarrow dA = (12x)(5x/100)$$

$$\Rightarrow dA = 10A/100$$

$$\Rightarrow dA = 10\%$$

128. (b)

129. (c)

130. (a)

131. (b)

132. (d)

133. (d)

134. (b) To find the area of the region bounded by the curve $y^2 = 8x$ and the line $y = 2x$, we start by determining the points of intersection.

First, equate $y^2 = 8x$ and $y = 2x$:

$$(2x)^2 = 8x$$

$$4x^2 = 8x$$

$$4x^2 - 8x = 0$$

$$4x(x - 2) = 0$$

$$x = 0 \text{ or } x = 2$$

When $x = 0$, $y = 2(0) = 0$. So one point of intersection is $(0,0)$.

When $x = 2$, $y = 2(2) = 4$. So the other point of intersection is $(2,4)$.

Now, we need to find the area between these two curves from $x = 0$ to $x = 2$.

The area A is given by:

$$A = \int_0^2 (y_{\text{upper}} - y_{\text{lower}}) dx$$

For $y^2 = 8x$:

$$y = \sqrt{8x}$$

For $y = 2x$:

$$\sqrt{8x} = 2x$$

$$8x = 4x^2$$

$$4x^2 - 8x = 0$$

$$4x(x - 2) = 0$$

$$x = 0 \text{ or } x = 2$$

Thus, the upper curve $y = \sqrt{8x}$ and the lower curve $y = 2x$.

The area A is:

$$A = \int_0^2 (\sqrt{8x} - 2x) dx$$

Compute the integral:

$$A = \left[\frac{2\sqrt{8x}}{3} - x^2 \right]_0^2$$

$$A = \left(\frac{2\sqrt{16}}{3} - 4 \right) - \left(\frac{2\sqrt{0}}{3} - 0 \right)$$

$$A = \left(\frac{2 \cdot 4}{3} - 4 \right) - 0$$

$$A = \left(\frac{8}{3} - \frac{12}{3} \right)$$

$$A = \frac{-4}{3}$$

However, area cannot be negative, so we take the absolute value:

$$A = \frac{4}{3}$$

Therefore, the area of the region bounded by the curve $y^2 = 8x$ and the line $y = 2x$ is $\frac{4}{3}$ square units.

135. (c) Let $I = \int_{-\pi/2}^{\pi/2} \frac{\cos x}{1+e^x} dx$

Then, $I = \int_0^{\pi/2} \left(\frac{\cos x}{1+e^x} + \frac{\cos(-x)}{1+e^{-x}} \right) dx$

(As $\int_{-a}^a f(x) dx = \int_0^a (f(x) + f(-x)) dx$)

$$\Rightarrow I = \int_0^{\pi/2} \left(\frac{\cos x}{1+e^x} + \frac{\cos x}{1+e^{-x}} \right) dx$$

$$\Rightarrow I = \int_0^{\pi/2} \left(\frac{1}{1+e^x} + \frac{e^x}{1+e^x} \right) \cos x dx$$

$$\Rightarrow I = \int_0^{\pi/2} \left(\frac{1+e^x}{1+e^x} \right) \cos x dx$$

$$\Rightarrow I = \int_0^{\pi/2} \cos x dx$$

$$\Rightarrow I = (\sin x)_0^{\pi/2} = \sin \frac{\pi}{2} - \sin 0 = 1$$

136. (a) Solution and Explanation

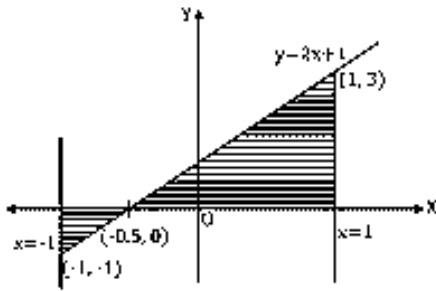
$$\left(y = \left[\left(\frac{C_2 + C_3}{C_1} \right) e^{C_4} \right] e^x = Ae^x, \right)$$

$$\text{where } A = \left(\left(\frac{C_2 + C_3}{C_1} \right) e^{C_4} \right)$$

Order = Number of independent arbitrary constants = 1

137. (a)

138. (c)

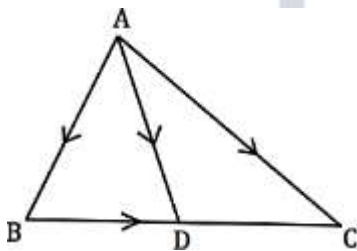


Area bounded by $y = 2x + 1$ with x-axis

$$= (1/2)(1/2)(1) + (1/2)(3/2)(3)$$

$$= 5/2 \text{ sq. units.}$$

139. (d)



$$\vec{AB} = \hat{i} + \hat{j} + \hat{k}$$

$$\vec{AC} = \hat{i} + 3\hat{j} + 5\hat{k}$$

$$\Rightarrow \vec{BC} = \vec{AC} - \vec{AB} = 2\hat{j} + 4\hat{k}$$

Since D is the mid-point of $\vec{BC}$,

$$\vec{BD} = \frac{\vec{BC}}{2} = \hat{j} + 2\hat{k}$$

Now, in $\triangle ABD$,

$$\vec{AD} = \vec{AB} + \vec{BD} = \hat{i} + 2\hat{j} + 3\hat{k}$$

$$\therefore |\vec{AD}| = \sqrt{1^2 + 2^2 + 3^2} = \sqrt{14}$$

140. (c)

141. (d)

Given, $\left(\frac{dy}{dx} = \frac{2x}{y}\right)$

$$\Rightarrow ydy = 2xdx$$

$$\Rightarrow \int ydy = \int 2xdx$$

$$\Rightarrow y^2/2 = x^2 + A, \text{ where } A \text{ is a constant.}$$

The above equation represents a hyperbola.

142. (b)

$$|\vec{a} \times \vec{b}|^2 + |\vec{a} \cdot \vec{b}|^2 = 144$$

$$|\vec{a}|^2 |\vec{b}|^2 \sin^2 \theta + |\vec{a}|^2 |\vec{b}|^2 \cos^2 \theta = 144$$

$$|\vec{a}|^2 |\vec{b}|^2 = 144$$

$$|\vec{a}| |\vec{b}| = 12$$

$$4 |\vec{b}| = 12$$

$$|\vec{b}| = 3$$

143. (c) Signs of x-coordinate, y-coordinate and z-coordinate are, +, -, + respectively.

$\therefore (1, -3, 4)$ lies in the fourth octant.

144. (a)

145. (b)

$$\text{Let } \left(\frac{x-3}{2} = \frac{y-3}{3} = \frac{z+5}{6} = t\right)$$

$$\Rightarrow (x, y, z) = (2t+3, 3t+3, 6t-5)$$

$\therefore$ d.r.'s of the line perpendicular to $\left(\frac{x-3}{2} = \frac{y-3}{3} = \frac{z+5}{6}\right)$ and

joining $(2t+3, 3t+3, 6t-5)$

and $(1, 2, -4)$ is $(2t+2, 3t+1, 6t-1)$

$$\therefore 2(2t+2) + 3(3t+1) + 6(6t-1) = 0$$

$$\Rightarrow t = -1/49$$

$$\therefore \text{Distance} = \sqrt{(2t+2)^2 + (3t+1)^2 + (6t-1)^2}$$

$$= \sqrt{49t^2 + 2t + 6}$$

$$= \sqrt{\frac{1}{49} - \frac{2}{49} + 6} = \frac{\sqrt{293}}{7}$$

146. (d) We have, the equation of line as

$$\frac{x-2}{3} = \frac{y-3}{4} = \frac{z-4}{5}$$

This line is parallel to the vector $\vec{b} = 3\hat{i} + 4\hat{j} + 5\hat{k}$

Equation of plane is $2x - 2y + z = 5$

Normal to the plane is $\vec{n} = 2\hat{i} - 2\hat{j} + \hat{k}$

Its angle between line and plane is ' θ '.

$$\sin \theta = \frac{|\vec{b} \cdot \vec{n}|}{|\vec{b}||\vec{n}|} = \frac{|(3\hat{i} + 4\hat{j} + 5\hat{k}) \cdot (2\hat{i} - 2\hat{j} + \hat{k})|}{\sqrt{3^2 + 4^2 + 5^2}\sqrt{4 + 4 + 1}}$$

$$\text{Then, } = \frac{|6 - 8 + 5|}{\sqrt{50}\sqrt{9}} = \frac{3}{15\sqrt{2}} = \frac{1}{5\sqrt{2}}$$

$$\sin \theta = \frac{\sqrt{2}}{10}$$

147. (a) Given $\alpha = \beta = \pi/3$

Let acute angle made by z-axis be γ

Then, $\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$

$$\Rightarrow \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2 + \cos^2 \gamma = 1$$

$$\Rightarrow \frac{1}{4} + \frac{1}{4} + \cos^2 \gamma = 1$$

$$\Rightarrow \cos^2 \gamma = \frac{1}{2}$$

$$\Rightarrow \cos \gamma = \pm \frac{1}{\sqrt{2}}$$

$$\Rightarrow \gamma = \frac{\pi}{4}$$

[$\therefore \gamma$ is acute]

148. (b)

Corner Points	Corresponding value of $Xp + q; p, q > 0$
(0,3)	3q
(1,1)	p + q
(3,0)	3p

So, condition of p and q, so that the minimum of Z occurs at (3,0) and (1,1) is

$$p + q = 3p \Rightarrow 2p = q$$

$$\therefore p = \frac{q}{2}$$

So, condition of p and q, so that the minimum of Z occurs at (3,0) and (1,1) is

$$p + q = 3p \Rightarrow 2p = q$$

$$\therefore p = \frac{q}{2}$$

149. (d) y Intercept of $x + y = 5$ is (0,5)

y-intercept of $x + 3y = 9$ is (0,3)

The intersection point of $x + y = 5$ and $x + 3y = 9$ is (3,2)

Therefore, the corner points are (0,5), (0,3), (3,2)

At (0,5), $Z = 35$

At (0,3), $Z = 21$

At (3,2), $Z = 47$

So, $Z_{\max} = 47$ at (3,2).

150. (b) Given $n = 10$

Probability of odd number, $p = 1/2$

$\therefore q = 1/2$

Required probability = $P(X \geq 1)$

$= 1 - P(X = 0)$

$= 1 - {}^{10}C_0(1/2)^{10-0}(1/2)^0$

$= 1 - 1/2^{10}$

$= 1 - 1/1024$

$= 1023/1024$

151. (a)

152. (a) Let $P(A | E_1) = x$

By Bayes' theorem,

$$P(E_2 | A) = \frac{P(E_2)P(A | E_2)}{P(E_1)P(A | E_1) + P(E_2)P(A | E_2)}$$

$$\Rightarrow \frac{1}{2} = \frac{\left(\frac{1}{2}\right)\left(\frac{2}{3}\right)}{\left(\frac{1}{2}\right)x + \left(\frac{1}{2}\right)\left(\frac{2}{3}\right)}$$

$$\Rightarrow x = \frac{2}{3}$$

$$\therefore P(E_1 | A) = \frac{P(E_1)P(A | E_1)}{P(E_1)P(A | E_1) + P(E_2)P(A | E_2)}$$

$$= \frac{\left(\frac{1}{2}\right)\left(\frac{2}{3}\right)}{\left(\frac{1}{2}\right)\left(\frac{2}{3}\right) + \left(\frac{1}{2}\right)\left(\frac{2}{3}\right)} = \frac{1}{2}$$

153. (b)

Required probability = $P(A'BC) + P(AB'C) + P(ABC')$

$= 1/2 \times 1/4 \times 1/3 + 1/2 \times 3/4 \times 1/3 + 1/2 \times 1/4 \times 2/3$

$= 1/24 + 1/8 + 1/12$

$= (1 + 3 + 2)/24$

$= 1/4$

154. (d) $n(A) = 2$

Given, $2^{(n(A) \cdot n(B))} = 1024$

$$\Rightarrow (2)^{(2 \cdot n(B))} = (2)^{10}$$

$$\Rightarrow 2 \cdot n(B) = 10$$

$$\Rightarrow n(B) = 5$$

155. (a)

156. (a)

$$\begin{aligned}\tan A + \cot A &= 2 \\ \Rightarrow (\tan A + \cot A)^2 &= 4 \\ \Rightarrow \tan^2 A + \cot^2 A + 2 \tan A \cot A &= 4 \\ \Rightarrow \tan^2 A + \cot^2 A &= 2 \\ \Rightarrow (\tan^2 A + \cot^2 A)^2 &= 4 \\ \Rightarrow \tan^4 A + \cot^4 A + 2 \tan^2 A \cot^2 A &= 4 \\ \Rightarrow \tan^4 A + \cot^4 A &= 2\end{aligned}$$

157. (c) Total number of subsets of A is $2^{n(A)} = 2^6 = 64$

Number of subsets of A which contain at least two elements is

$$\begin{aligned}64 - ({}^6C_0 + {}^6C_1) \\ = 64 - (1 + 6) \\ = 57\end{aligned}$$

158. (d)

$$|z + 1| = |z - 1|$$

Let $z = x + iy$

$$\text{Then, } |x + iy + 1| = |x + iy - 1|$$

$$\begin{aligned}\Rightarrow \sqrt{(x+1)^2 + y^2} &= \sqrt{(x-1)^2 + y^2} \\ \Rightarrow (x+1)^2 + y^2 &= (x-1)^2 + y^2 \\ \Rightarrow x^2 + 2x + 1 + y^2 &= x^2 + 1 - 2x + y^2 \\ \Rightarrow 4x &= 0\end{aligned}$$

$\Rightarrow x = 0$ which represents y - axis.

159. (a)

$$\begin{aligned}{}^{16}C_9 + {}^{16}C_{10} - {}^{16}C_6 - {}^{16}C_7 \\ = {}^{16}C_9 + {}^{16}C_{10} - {}^{16}C_{10} - {}^{16}C_9 = 0 \\ (\because {}^nC_r = {}^nC_{n-r})\end{aligned}$$

160. (a) Number of terms in the expansion of $(x_1 + x_2 + \dots + x_r)^n$ is ${}^{n+r-1}C_n$. $\Rightarrow$ Number of terms in $(x + y + z)^{10} = {}^{12}C_{10} = 66$.

161. (c) To find the smallest positive integer for which $P(n): 2^n < n!$ is true, put $n = 2, 3, 4, 5$ in $P(n)$. $n! = n \times (n-1) \times (n-2) \dots \times 3 \times 2 \times 1$

Calculations:

Given, $P(n): 2^n < n!$

Find the smallest positive integer for which $P(n): 2^n < n!$ is true, put $n = 2, 3, 4, 5$ in $P(n)$. $n! = n \times (n-1) \times (n-2) \dots \times 3 \times 2 \times 1$

Put $n = 2$

$$\text{L.H.S} = 2^n = 2^2 = 4$$

$$\text{R.H.S} = n! = 2! = 2$$

Here, $4 < 2$, which is not possible

Hence, $n = 2$ is not the smallest positive integer for which $P(n): 2^n < n!$ is true.

Put $n = 3$

$$\text{L.H.S} = 2^n = 2^3 = 8$$

$$R.H.S = n! = 3! = 6$$

Here, $8 < 6$, which is not possible

Hence, $n = 3$ is not the smallest positive integer for which $P(n): 2^n < n!$ is true.

Put $n = 4$

$$L.H.S = 2^n = 2^4 = 16$$

$$R.H.S = n! = 4! = 24$$

Here, $16 < 24$, which is possible

Hence, $n = 4$ is positive integer for which $P(n): 2^n < n!$ is true.

Put $n = 5$

$$L.H.S = 2^n = 2^5 = 32$$

$$R.H.S = n! = 5! = 120$$

Here, $32 < 120$, which is possible

Hence, $n = 5$ is positive integer for which $P(n): 2^n < n!$ is true.

But $4 < 5$

Hence, $n = 4$ is the smallest positive integer for which $P(n)$ is true.

162. (a) Product of slopes = -1

$$\Rightarrow ll'' + mm' = 0$$

163. (b)

$$x^2 = 4ay$$

Given parabola passes through $(2,1)$.

$$\Rightarrow 2^2 = 4a$$

$$\Rightarrow a = 1$$

Length of latus rectum = $4a = 4$.

164. (c)

$$S_n = n^2 + n$$

$$S_1 = 1 + 1 = 2 = T_1$$

$$S_2 = 2^2 + 2 = 6 = T_1 + T_2$$

$$\therefore T_2 = S_2 - S_1 = 4$$

Common difference, $d = T_2 - T_1$

$$= 4 - 2$$

$$= 2$$

165. (c) Negation: For some real numbers x and y , $x + y \neq y + x$.

166. (a) Mean, $\bar{x} = \frac{6 \cdot 8 \cdot 10}{5} = \frac{40}{5} = 8$

Standard deviation, $\sigma = \sqrt{\frac{1}{5}(4 + 1 + 0 + 1 + 4)}$

Because, $\sigma = \sqrt{\frac{1}{n} \sum (x_i - \bar{x})^2}$

$$\Rightarrow \sigma = \sqrt{\frac{10}{5}} = \sqrt{2}$$

167. (a)

168. (c) $R = \{(1,1)\}$ on set $\{1,2,3\}$

Clearly, R is symmetric and transitive.

169. (b) Given that, $f(x) = x^2 - 4x + 5$

Let $y = x^2 - 4x + 5$

$$\Rightarrow y = x^2 - 4x + 4 + 1 = (x - 2)^2 + 1$$

$$\Rightarrow (x - 2)^2 = y - 1 \Rightarrow x - 2 = \sqrt{y - 1}$$

$$\Rightarrow x = 2 + \sqrt{y - 1}$$

$$\therefore y - 1 \geq 0, y \geq 1$$

Range = $[1, \infty]$

170. (c)

171. (a) For f to be defined,

$$x - 1 \geq 0 \text{ and } -1 \leq \sqrt{(x - 1)} \leq 1$$

$$\Rightarrow x \geq 1 \text{ and } 0 \leq x - 1 \leq 1$$

$$\Rightarrow x \geq 1 \text{ and } 1 \leq x \leq 2$$

$$\Rightarrow x \in [1, 2]$$

Hence, domain of f is $[1, 2]$.

172. (d) $\pi/3 \notin [-1, 1]$ which is the domain of $\sin^{-1} x, \cos^{-1} x$ So, $\cos(\sin^{-1} \pi/3 + \cos^{-1} \pi/3)$ does not exist.

173. (a)

174. (d)

$$A = \{a, b, c\}$$

$$n(A) = 3$$

Number of binary operations is

$$n(A)^{(n(A))^2} = 3^{(3^2)} = 3^9$$

175. (b)

176. (c)

177. (d) Transpose of a Matrix:

The new matrix obtained by interchanging the rows and columns of the original matrix is called as the transpose of the matrix. For example: $A = \begin{bmatrix} a & b \\ x & y \end{bmatrix} \Rightarrow A' = \begin{bmatrix} a & x \\ b & y \end{bmatrix}$. It is denoted by A' or A^T .

Properties of Transpose of a Matrix:

The transpose of the product of two matrices is equivalent to the product of their transposes in reversed order:

$$(AB)' = B'A'$$

$$(ABC)' = C'B'A'$$

$$(A')' = A$$

Calculation:

It is given that B is a skew-symmetric matrix.

$$\therefore B' = -B$$

Now, consider the transpose of the product matrix $A'BA$.

$$\begin{aligned}(A'BA)' &= A'B'(A')' \\ &= A'(-B)A [\because B' = -B] \\ &= -(A'BA)\end{aligned}$$

Since the transpose is equal to its negative, $A'BA$ is a Skew-Symmetric matrix.

178. (b)

$$\begin{aligned}|\text{adj } A| &= |A|^{3-1} = 25 \\ \therefore |A \cdot \text{adj. } A| &= 5 \times 25 = 125.\end{aligned}$$

179. (d)

180. (c)

181. (b)

182. (c)

183. (c)

184. (a)

185. (d)

186. (b)

187. (d)

188. (b)

189. (d)

190. (c)

191. (b)

192. (d)

193. (b)

194. (c)

195. (c)

196. (b)

197. (d)

198. (d)

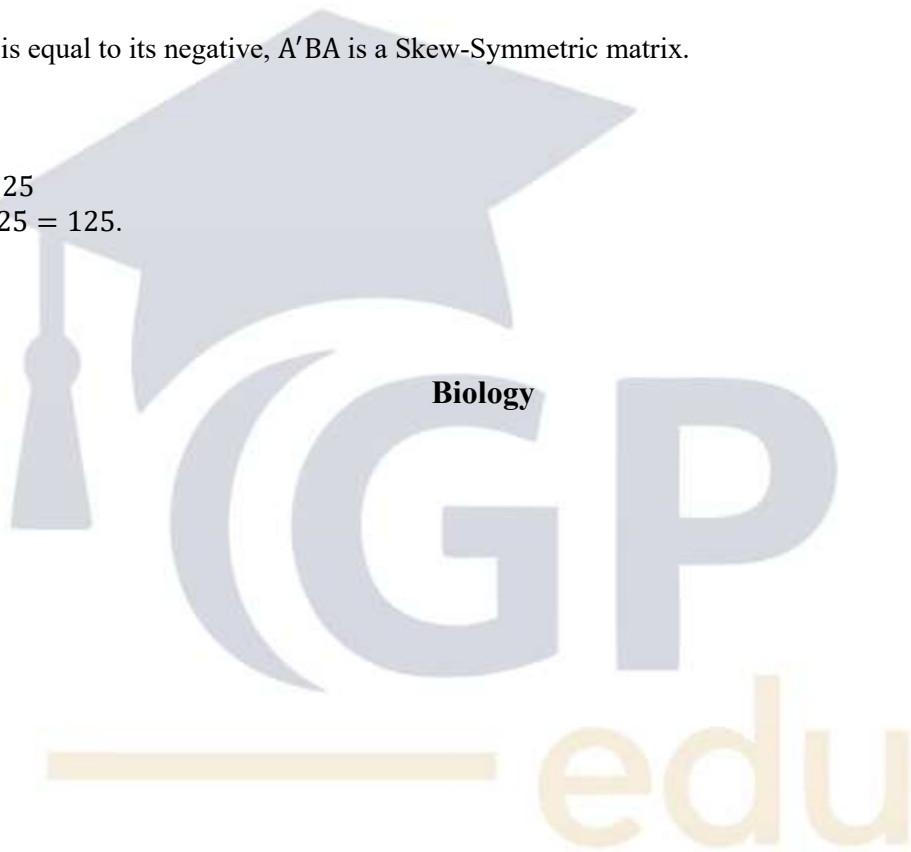
199. (d)

200. (d)

201. (a)

202. (d)

Biology



203. (a)
204. (d)
205. (d)
206. (a)
207. (c)
208. (c)
209. (c)
210. (c)
211. (a)
212. (b)
213. (d)
214. (c)
215. (b)
216. (d)
217. (c)
218. (b)
219. (c)
220. (b)
221. (b)
222. (b)
223. (d)
224. (d)
225. (d)
226. (c)
227. (c)
228. (c)
229. (d)
230. (b)
231. (d)
232. (c)
233. (d)
234. (c)
235. (d)
236. (a)
237. (d)
238. (a)
239. (c)
240. (c)

