Physics

1. (a)

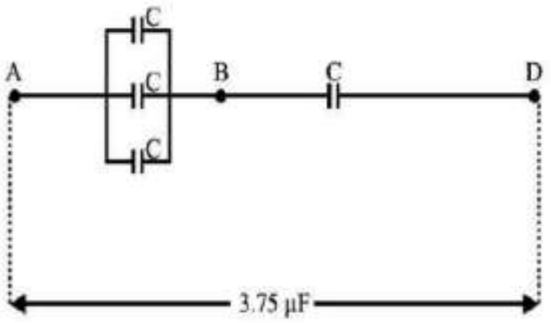
$$\text{Using } v = n\lambda, \frac{1}{\lambda} = \frac{n}{v}$$
$$\Rightarrow \frac{1}{\lambda} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$
$$\Rightarrow v = Rc\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$
$$\therefore v_2 = Rc\left(\frac{1}{2^2} - \frac{1}{3^2}\right) = Rc\left(\frac{1}{4} - \frac{1}{9}\right) .. \quad (i)$$

$$v_1 = Rc\left(\frac{1}{2^2}\right) = \frac{Rc}{4}$$
$$v_3 = Rc\left(\frac{1}{3^2}\right) = \frac{Rc}{9}$$
$$\Rightarrow v_1 - v_3 = Rc\left(\frac{1}{4} - \frac{1}{9}\right)$$

From eqs. (i) and (ii),

$v_1 - v_3 = v_2 \Rightarrow v_1 - v_2 = v_3$

2. (a) Net capacitance between A and B $C' = C + C + C = 3C$



Net capacitance between A and D

$C\ eq = \frac{3C \times C}{3C + C} = \frac{3C}{4}$

$$\therefore C = \frac{4C_{eq}}{3} = \frac{4 \times 3.75}{3} = 5.00\mu F$$

3. (c) $S = \frac{G}{8}$ (Given) .... (1)

And, $S = \frac{G}{n-1}$ ……. (ii)

From eqs. (i) and (ii),

$$\frac{G}{8} = \frac{G}{n-1} \Rightarrow 8 = n - 1$$

$\therefore n = 8 + 1 = 9$

Since, the range of galvanometer is increased by 9 times, therefore its sensitivity reduces to $\frac{S}{9}$

4. (b) From question, $1_X = 40$ cm, $I_R = 60$ cm. using principle of metre-bridge,

$\frac{X}{R} = \frac{I_X}{I_R} = \frac{40}{60} = \frac{2}{3}$........ (i)

When 30Ω resistance is connected in series with the smaller of the two resistances

$$\frac{X+30}{R} = \frac{60}{40} = \frac{3}{2}$$

$\Rightarrow R = \frac{2(X+30)}{3}$......... (ii)

From eqs. (i) and (ii),

$$\frac{X}{2\left(\frac{X+30}{3}\right)} = \frac{2}{3} \Rightarrow \frac{3X}{2(X+30)} = \frac{2}{3}$$

or, $9X = 4X + 120 \Rightarrow 5X = 120$

$\therefore X = 24\Omega$

5. (b) Given $d = 0.2 \times 10^{-3}$ m, $D = 2$ m and $\lambda = 5 \times 10^{-7}$ m

From $B = \frac{\lambda D}{d} = \frac{5\times10^{-7}\times2}{0.2\times10^{-3}} = \frac{5\times10^{-7}}{10^{-4}} \therefore B = 510^{-3}$ m

Distance between 1st minima on either side

$= 5 \times 10^{-3} + 5 \times 10^{-3} = 10 \times 10^{-3} = 10^{-2}$ m

6. (d) Given, $R = 18\Omega, Z = 33\Omega, V_{rms} = 220$ V

$\lambda = 5 \times 10^{-7}$ m

Power consumed in an AC circuit

$$P = e_{III}, i_{IIIs} \cdot \cos\phi = e_{IUI} \cdot \frac{e_{rms}}{Z} \cdot \frac{R}{Z}$$

$$\left[\because \cos\phi = \frac{R}{Z}\right]$$

$$= \frac{220 \times 220 \times 18}{33 \times 33} = 20 \times 20 \times 2 = 800 \text{ W}$$

7. (a) Effective capacity of the series combination of capacitors

$\frac{1}{C_1} = \frac{1}{C} + \frac{1}{C} = \frac{2}{5}$ or, $C_1 = \frac{C}{2}$

Effective capacity of the series combination of capacitors with dielectric material

$$\frac{1}{C_1} = \frac{1}{C} + \frac{1}{KC}; \frac{1}{C_2} = \frac{1}{C}\left[1 + \frac{1}{K}\right]$$

or, $C_2 = \frac{C}{\left(1+\frac{1}{K}\right)} = \frac{CK}{(K+1)}$

$\therefore$ Change in effective capacitance

$\Delta C = C_2 - C_1$

$= \frac{CK}{(K+1)} - \frac{C}{2} = C\left[\frac{K}{K+1} - \frac{1}{2}\right]$

$= C\left[\frac{2K-K-1}{2(K+1)}\right] = \frac{C}{2}\left[\frac{K-1}{K+1}\right]$

8. (b) Polarising angle, $\tan\theta = \mu$

Also, $M = \frac{c}{v}$

or, $\cot\theta = \frac{v}{c} \therefore \theta = \cot^{-1}\frac{v}{c}$

9. (b) Resultant intensity for two coherent sources,

$I_R = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$

For two identical light waves, $I_1 = I_2 = I$

$\therefore I_R = 4I\cos^2\frac{\phi}{2}$ or, $1_R \propto \cos^2\frac{\phi}{2}$

10. (a) As we know, $\beta_{dc} = \frac{\alpha_{dc}}{1-\alpha_{dc}}$

$\therefore (1-\alpha_{dc}) = \frac{\alpha_{dc}}{\beta_{dc}}$ (i)

Also, $\frac{\beta_{dc}-\alpha_{dc}}{\alpha_{dc}\cdot\beta_{dc}} = \frac{\beta_{dc}\left(1-\frac{\alpha_{dc}}{\beta_{dc}}\right)}{\alpha_{dc}\beta_{dc}}$

From equation (i)

$\frac{1-\frac{\alpha_{dc}}{\beta_{dc}}}{\alpha_{dc}} = \frac{1-(1-\alpha_{dc})}{\alpha_{dc}}$

$= \frac{1-1+\alpha_{dc}}{\alpha_{dc}} = 1$

11. (b) According to question,

$N_0 = 10{,}000$ disintegeration /min

$N_t = 2500$ disintegeration /min

$t = 4$ min

From the radioactive decay law,

$\frac{Nt}{N_0} = e^{-\lambda t}$

or, $\frac{2500}{10000} = e^{-\lambda\times 4}$

$\Rightarrow \frac{1}{4}e^{-4\lambda} \Rightarrow e^{4\lambda} = 4$

$\Rightarrow 4\lambda = \log_e 4 \Rightarrow 4\lambda = \log_e 2^2$

$\Rightarrow 4\lambda = 2\log_e 2$

$\therefore \lambda = 0.5\log_e 2$

12. (c) $\because$ Number of waves in glass slab = number of waves in water column

$\therefore \mu_g \cdot h_g = \mu_w \cdot h_w$

h_g = thickness of slab and h_w = height of water column.

or, $\mu_w = \frac{\mu_g \cdot h_g}{hw} = \frac{1.5 \times 6}{7} = 1.286$

13. (c) Energy difference between two states

$\Delta E = E_2 - E_1 = \frac{-13.6}{2^2} - \left(\frac{-13.6}{1^2}\right)$

$\Delta = \frac{13.6}{1^2} - \frac{13.6}{2^2}\,\Delta = 13.6\left[\frac{4-1}{4}\right] = 13.6 \times \frac{3}{4}$

$\therefore \Delta E = 10.2\text{eV}$

Since, the energy is radiaed in form of photons,

$\therefore$ Energy of photons $= h\nu = 10.2\text{eV}$

From Einstein's photoelectric equation,

$h\nu = \phi_0 + eV_s$

$10.2\text{eV} = 4.2\text{eV} + eV_s$

$\Rightarrow 6\text{eV} = eV_s$

$\therefore V_s = 6\text{ V}$

14. (d) Magnetic moment $(M_0) = \frac{e}{2me} \times L$

Where, L = orbital angular momentum.

And $L = \frac{nh}{2\pi}$ .... (i)

$\Rightarrow L \propto n$ (ii)

n = principal quantum number

h = planck's constant.

$\therefore M_0 \propto n.$

15. (a) Photodiode is a reversed biased $p - n$ junction.

16. (c) According to question, $I = 2\text{ kg m}^2$

$\omega_0 = 60 \text{rad/s}, \omega = 0$

$t = 5 \text{ min} = 5 \times 60 = 300 \text{ s}$

using, $\omega = \omega_0 + \alpha t \Rightarrow \alpha = \frac{\omega - \omega_0}{t}$

$= \frac{0-60}{300} = \frac{-60}{300} = \frac{-1}{5} \text{rad/s}^2$

For $t = 2$ min

$\omega = \omega_0 + \alpha t$

$= 60 - \frac{1}{5} \times 120 = 60 - 24 \Rightarrow \omega = 36 \text{rad/s}$

Angular momentum,

$L = 1\omega = 2 \times 36 = 72 \text{ kg m}^2/\text{s}$

17. (d) Standard equation of wave motion,

$Y = A\sin\left[2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right) + \frac{\pi}{4}\right]$

When compare the given equation with standard equation we get,

Amplitude, $A = 3$ m

Wavelength, $\lambda = 10$ m

18. (c) Rate of energy radiation.

$\frac{Q}{t} = \sigma A T^4$ i.e., power, $P = \sigma A T^4$

$\therefore A \propto \frac{1}{T^4}$

If body radiates same power, then

$\frac{A_2}{A_1} = \frac{T_1^4}{T_2^4} \Rightarrow \frac{4\pi r_2^2}{4\pi r_1^2} = \frac{T_1^4}{T_2^4}$

$\therefore \frac{r_2}{r_1} = \left(\frac{T_1}{T_2}\right)^2$

19. (c) For open pipe first overtone, $v_1 = \frac{v}{L}$

For closed pipe first overtone, $v_1 = \frac{3v}{4L}$

$\therefore v_1 - v_1 = \frac{V}{L} - \frac{3V}{4L} = 3$

or, $\frac{V}{4L} = 3 \therefore \frac{V}{L} = 12$

When length of open pipe is made $\frac{L}{3}$, Fundamental frequency $v = \frac{V}{2\left(\frac{L}{3}\right)} = \frac{3V}{2L}$

When length of closed pipe is made 3 times,

Fundamental frequency $v' = \frac{V}{4(3L)} = \frac{V}{12L}$

Beats produced $= v - v'$

$$= \frac{3V}{2L} - \frac{V}{12L} = \frac{17}{12}, \frac{V}{L} \left[\because \frac{V}{L} = 12\right]$$

$$= \frac{17}{12} \times 12 = 17$$

20. **(b)** Maximum tension in the rope $= m(g + a)$

Stress in the rope, $T = \frac{m(g+a)}{\pi r^2}$

$\therefore T = \frac{m(g+a)}{\pi r^2} = \frac{m(g+a)}{\pi\left(\frac{d}{2}\right)^2}$ or, $T = \frac{4m(g+a)}{\pi d^2} \Rightarrow d^2 = \frac{4m(g+a)}{\pi T}$

$$\therefore d = \left[\frac{4m(g + a)}{\pi T}\right]^{1/2}$$

21. (b) Given, mass, $m = 2$ kg, $v = 6$ m/s and force constant $K = 36$ N/m Kinetic energy of rolling solid sphere

$$= \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

$$= \frac{1}{2}mv^2 + \frac{1}{2} \times \frac{2}{5}mr^2\omega^2$$

$$= \frac{1}{2}mv^2 + \frac{1}{5}mv^2 = \frac{7}{10}mv^2$$

The potential energy of the spring on maximum compression $x = \frac{1}{2}kx^2$

$$\therefore \frac{1}{2}kx^2 = \frac{7}{10}mv^2$$

$$\Rightarrow x^2 = \frac{14}{10}\frac{mv^2}{k} = \frac{14}{10} \times \frac{2 \times (6)^2}{36} = 2.8$$

or, $x = \sqrt{2.8}$ m

22.(d) According to question,

$\omega_0 = 0, \omega = 24$rad/s and $t = 8$ s

using $\omega = \omega_0 + \alpha t$

$$\alpha = \frac{\omega - \omega_0}{t} = \frac{24}{8} = 3\text{rad/s}^2$$

Substituting the given values, we get Now using,

$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2 = 0 + \frac{1}{2} \times 3 \times (8)^2$$

$$= \frac{3 \times 64}{2} = 96\text{rad}$$

23.(a) Fundamental frequency of the wire

$$f = \frac{1}{2L_1}\sqrt{\frac{T}{m}} = \frac{1}{2L_1}\sqrt{\frac{T}{\pi r_1^2 \rho}} = \frac{1}{2L_1 r_1}\sqrt{\frac{T}{\pi\rho}}$$

First overtone of the first wire,

$$f_1 = 2f = \frac{2}{2L_1 r_1}\sqrt{\frac{T}{\pi\rho}} = \frac{1}{L_1 r_1}\sqrt{\frac{T}{\pi\rho}} \ldots\ldots \text{(i)}$$

Second overtone of the second wire

$$f_2 = \frac{3}{2L_2 r_2}\sqrt{\frac{T}{\pi\rho}} \ldots\ldots \text{(ii)}$$

$\because f_1 = f_2$

$$\therefore \frac{1}{L_1 r_1}\sqrt{\frac{T}{\pi\rho}} = \frac{3}{2L_2 r_2}\sqrt{\frac{T}{\pi\rho}}$$

$\because f_1 = f_2$

$\therefore 3L_1 r_1 = 2L_2 r_2$

$$\Rightarrow \frac{L_1}{L_2} = \frac{2}{3} \cdot \frac{r_2}{r_1} = \frac{2}{3} \cdot \frac{r_2}{2r_2} = \frac{1}{3} [\because r_1 = 2r_2]$$

24.(a) According to question,

force $F = 150$ dyne 105×10^{-5} N and

Surface tension $T = 7 \times 10^{-2}$ N/m

$\because$ Circumference of the capillary $\times$ surface tension = upward force

$\therefore 2\pi r T = F$

or, $2\pi r = \frac{F}{T} = \frac{105 \times 10^{-5}}{7 \times 10^{-2}} = 15 \times 10^{-3}$ m

$= 1.5 \times 10^{-2} = 1.5$ cm

25.(c) For rigid diatomic molecule, $\frac{C_p}{C_v} = \frac{7}{5}$

$\therefore C_v = \frac{5}{7} C_p \quad \ldots.$ (i)

Also, $C_p - C_v = R$

or, $C_p - \frac{5}{7} C_p = R \Rightarrow \frac{2}{7} C_p = R$

$n = \frac{2}{7} = 0.2857$

26.(a) Ideal gas equation, $pV = nRT$

$pV = \frac{m'}{M}RT$ here, m' is the mass of the gas and M molecular weight $p = \frac{m'}{V}\frac{RT}{M}$

$\therefore p = \frac{\rho RT}{M}$

$\therefore \rho = \frac{m'}{V}$ density of the gas

$\rho = \frac{pM}{RT} = \frac{pM}{NkT}$, N is Avogadro, number $\rho = \frac{pm}{KT}$, where $m = \frac{M}{N}$ mass of each molecule.

27.(d) Let radius of big drop $= R$

and of small drop $= r$

Volume of big drop $= n($ Volume of small drop $)$

$\frac{4}{3}\pi R^3 = n \cdot \frac{4}{3}\pi r^3$

$R^3 = nr^3 \Rightarrow R = n^{1/3}.r$

Surface energy of n drops,

$E_2 = n \times 4\pi r^2 \times T$

Surface energy of big drop,

$E_1 = 4\pi R^2 T$

$\therefore \frac{E_2}{E_1} = \frac{nr^2}{R^2} = \frac{nr^2}{(n^{1/3} \cdot r)^2}$

$= \frac{nr^2}{n^{2/3} \cdot r^2} = n^{1/3}[\because R = n^{1/3} \cdot r]$

or, ratio of energy, $E_2 : E_1 = \sqrt[3]{n} : 1$

28.(a) Binding energy on the surface of the earth

$E_1 = \frac{GMm}{R}$ …… (i)

Binding energy of revolving satellite at a height h from the earth surface,

$E_2 = \frac{GMm}{2(R+h)}$ ….. (ii)

From eqs. (i) and (ii),

$\frac{E_1}{E_2} = \frac{2(R+h)}{R}$

29.(d) Given, particle velocity, $v = \pi m/s$ and time period $T = 16$ s

Displacement of the particle, x = Asin ωt Velocity of the particle,

$v = \frac{dx}{dt} - A\omega\cos\omega t$ ..... (i)

Angular velocity,

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{16} = \frac{\pi}{8} = \text{rad/s}$$

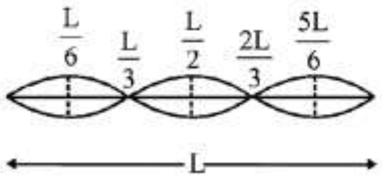
Now from eq. (i),

$$\pi = A \times \frac{\pi}{8} \times \cos\frac{\pi}{8} \times 2$$

$$1 = \frac{A}{8}\cos\frac{\pi}{4} = \frac{A}{8} \cdot \frac{1}{\sqrt{2}}$$

$$\therefore A = 8\sqrt{2}\ \text{m}$$

30. **(d)** The figure represents string vibrating in second overtone between two bridges



Clearly, amplitude of vibration is maximum at

$\frac{L}{6}, \frac{L}{2}, \frac{5L}{6}$

31. **(c)** Acceleration due to gravity varies with depth d as,

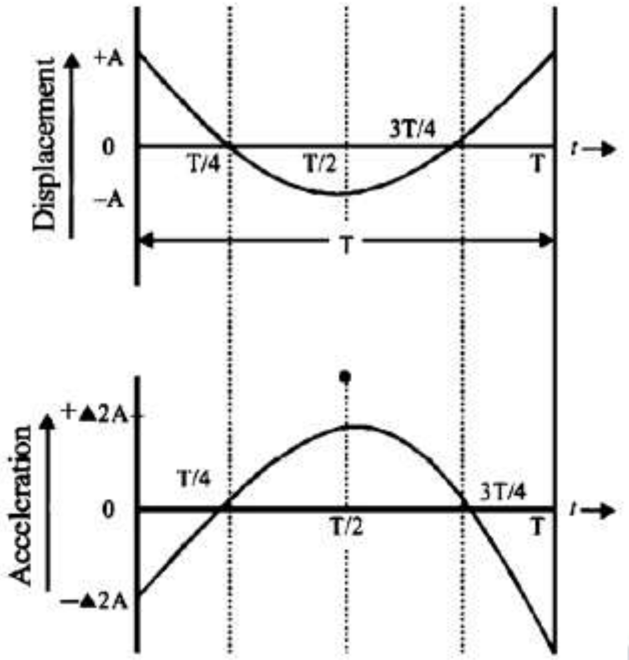
$$g' = g\left(1 - \frac{d}{R}\right)$$

And according to question, $g' = \frac{g}{n}$

$$g' = \frac{g}{n} = g\left(1 - \frac{d}{R}\right) \Rightarrow \frac{1}{n} = 1 - \frac{d}{R}$$

or, $\frac{d}{R} = 1 - \frac{1}{n} = \frac{n-1}{n} \therefore d = R\left(\frac{n-1}{n}\right)$

32. **(d)** The relation of displacement and acceleration with time in SHM are shown below,



Clearly, the phase difference between displacement and acceleration is π.

33.(d) Fundamental frequency, for a closed pipe

$$v_1 = \frac{V}{4L} = 100 \text{ Hz}$$

Fundamental frequency, for an open pipe

$$v_1 = \frac{V}{2L} = 200 \text{ Hz}$$

In a pipe open at both the ends, all multiples of the fundamental frequency are produced.

$\therefore 1 \times 200, 2 \times 200, 3 \times 200 \ldots$

i.e., $200, 400, 600 \ldots$

34.(a) The total mechanical energy remains conserved, kinetic energy changes into potential energy and vice-versa. At the highest point potential energy is maximum and at the lowest point its velocity and hence kinetic energy is maximum.

35.(a) Potential energy of a simple pendulum

$$= \frac{1}{2} M\omega^2 A^2 = \frac{1}{2} M \cdot \frac{g}{L} \cdot A^2 \left(\therefore \omega = \sqrt{\frac{g}{L}}\right)$$

36.(b) According to the Einstein's photoelectric equation,

$KE_{max} = hv_0 - \phi_0$

Initially, $hv = 0.4 + \phi_0$ (i)

and when the frequency of incident radiation is increased by 30% then

$1.3hv = 0.9 + \phi_0$ (ii)

From eqs. (i) and (ii)

$0.3\phi_0 = 0.9 - 1.3(0.4)$

$\therefore \phi_0 = \frac{0.38}{0.3} = 1.267eV$

37. (d) In LC parallel resonant circuit, at resonating frequency, current is minimum graph (4) correctly depicts.

38. (a) Energy of photon

$E_p = \frac{hc}{\lambda_p} \Rightarrow \lambda_p = \frac{hc}{E_p}$

$E_e = mc^2 = pc \Rightarrow p = \frac{E_p}{c} [\therefore mc = p]$

$\therefore \lambda_e = \frac{h}{p} = \frac{hc}{E_e}$

$\therefore E_p = E_e$ (Given)

$\therefore \lambda_p \propto \lambda_e.$

39. (b) Note: We have considered that the battery is kept disconnected from the capacitor. When dielectric is introduced inside the parallel plate capacitor, then potential difference.

$V = \frac{V_0}{K}$ Also, energy decreases, i.e., $U = \frac{U_0}{K}$

40. (b) According to question,

$\rho = 40 \times 10^{-8} \Omega m$

$A = 8 \times 10^{-6}\ m^2; I = 0.2A$

Resistance, $R = \frac{\rho I}{A}$

$\Rightarrow \frac{R}{I} = \frac{\rho}{A} = \frac{40 \times 10^{-8}}{8 \times 10^{-6}} = 5 \times 10^{-2}$

$\therefore$ potential gradient of the were

$\frac{V}{I} = \frac{IR}{I} = 0.2 \times 5 \times 10^{-2} = 10^{-2}\ V/m$

41. (c) According to question,

$\omega_0 = \omega, \omega = \frac{\omega}{4}; \theta = 2\pi n$

using, $\omega^2 = \omega_0^2 - 2\alpha\theta$

Putting the given values

$\left(\frac{\omega}{4}\right)^2 = \omega^2 - 2\alpha n(2\pi)$

$2\alpha n(2\pi) = \omega^2 - \frac{\omega^2}{16} \Rightarrow 2\pi n = \frac{15}{16}\left(\frac{\omega^2}{2\alpha}\right)$

When the fan is switched off,

$\omega = 0, \omega_0 = \omega, \theta = 2\pi n'$

$\Rightarrow 0 = \omega^2 - 2\alpha n'(2\pi)$

$\therefore 2\pi n' = \frac{\omega^2}{2\alpha}$ or, $n' = \frac{16}{15}n$

42.(d) From conservation of angular momentum, as net torque on the system is zero

$$I_1\omega_1 = (I_1 + I_2)\omega_2$$
$$\Rightarrow \frac{\omega_2}{\omega_1} = \frac{I_1}{I_1 + I_2}$$

Energy lost $\Delta E = E_1 - E_2$

$$= \frac{1}{2}I_1\omega_1^2 - \frac{1}{2}(I_1 + I_2)\omega_2^2$$

$$= \frac{1}{2}\omega_1^2\left[I_1 - (I_1 + L_2)\frac{\omega_2^2}{\omega_1^2}\right]$$

$$= \frac{1}{2}\omega_1^2\left[I_1 - (I_1 + I_2)\frac{I_1^2}{(I_1 + I_2)^2}\right]$$

$$\left[\because \frac{\omega_2}{\omega_1} = \frac{I_1}{I_1 + I_2}\right]$$

$$= \frac{1}{2}\omega_1^2\left[\frac{I_1^2 + I_1I_2 - I_1^2}{I_1 + I_2}\right]$$

or, $\Delta E = \frac{1}{2}\left[\frac{I_1I_2}{I_1+I_2}\right]\omega_1^2$

43.(a) Let the distance be x when velocity is u and acceleration α.
And the distance y when velocity is v and acceleration β.

If ω is the angular frequency, then

$\alpha = \omega^2 x$ and $\beta = \omega^2 y$

$\therefore \alpha + \beta = \omega^2(x + y)$ ..... (i)

Also, $u^2 = \omega^2A^2 - \omega^2x^2$

and $v^2 = \omega^2A^2 - \omega^2y^2$

$\Rightarrow v^2 - u^2 = \omega^2(x^2 - y^2)$

$v^2 - u^2 = \omega^2(x - y)(x + y)$ ..... (ii)

From eqs. (i) and (ii),

$v^2 - u^2 = (x - y)(\alpha + \beta)$

$\therefore x - y = \frac{v^2 - u^2}{\alpha+\beta}$ or $y - x = \frac{u^2 - v^2}{\alpha+\beta}$

44.(a) From Doppler's effect, when the observer is moving towards the source and source is stationary, then the apparent frequency

$$n' = n\left(\frac{v + v_0}{v}\right)$$

When the observer is moving away from the source and the source is stationary, then the apparent frequency.

$$n'' = -n\left(\frac{v - v_0}{v}\right)$$

From eqs. (i) and (ii),

$$n' - n'' = \frac{n}{v}(v + v_0 - v + v_0) = \frac{2nv_0}{v}$$

45.(b) Let L_0 be the original length of the wire, after heating temperature T length becomes ' L '. α be the coefficient of thermal expension. Increase in the length of the rod

$$\Delta L = \alpha L_0 T$$

$$L = L_0[1 + \alpha T]$$

Also, Young's modulus,

$$Y = \frac{FL_0}{A\Delta L}$$

Using equation (i) and substituting in equation (ii), we get

$$Y = \frac{F}{A} \cdot \frac{L_0(1 + \alpha T)}{\Delta L}$$

or, $\Delta L = \frac{FL_0(1+\alpha T)}{A \cdot Y}$

From eqs. (i) and (ii),

$$\frac{FL_0(1 + \alpha t)}{AY} = \alpha L_0 T$$

$$\therefore F = \frac{YA\alpha T}{(1 + \alpha T)}$$

46.(c) Induced emf in coil P

$$|e_p| = M \cdot \frac{dI_Q}{dt}$$

Putting $e_p = 15mV = 15 \times 10^{-2}$ V

and $\frac{dI_Q}{dt} = 10$ A/s

$$15 \times 10^{-3} = M \times 10$$

or, $M = 15 \times 10^{-4}$H

Magnetic flux linked with coil Q

$\phi_Q = MI_p = 15 \times 10^{-4} \times 1.8$

$[\because I_p = 1.8\ A]$

$= 27.0 \times 10^{-4}$

$= 2.7 \times 10^{-3} = 2.7mWb$

47.(c) In YDSE, position of a minima

$$y = \frac{(2n-1)\lambda D}{d}$$

Here, $y = \frac{d}{2}, n = 2$

Substituting $y = \frac{d}{2}$ and $n = 2$ in eq. (i) $\frac{d}{2} = \frac{D}{d}\left(\frac{2\times2-1}{2}\right)\lambda \Rightarrow \frac{d}{2} = \frac{D}{d}\frac{3}{2}\lambda$

$$\therefore \lambda = \frac{d^2}{3D}$$

48. **(c)** The process of superimposing the low frequency signal on a high frequency wave is called modulation.

49.(d) From question, $L = 3\ cm = 3 \times 10^{-2}\ m$ $A = 2\ cm^2 = 2 \times 10^4\ m^2$

$M = 3Am^2$

Intensity of magnetisation, $Im = \frac{M}{L\times A}$

$$= \frac{3\ A - m^2}{3 \times 2 \times 10^{-6}\ m^3}$$

$$= \frac{1}{2} \times 10^6\ A/m = 5 \times 10^5\ A/m$$

50.(c) Magnetic induction inside the solenoid

$B = \frac{\mu_0 NI}{L}$ (i)

Magnetic flux, $\phi = BA$

or, $\phi = \frac{\mu_0 NI.A}{L}$

Magnetic moment $= NIA = \frac{\phi L}{\mu_0}$

From question,

$L = 60$ cm and $\phi = 1.57 \times 10^{-6}$ Wb (given)

$$M = \frac{1.57 \times 10^{-6} \times 0.6}{4 \times 3.14 \times 10^{-7}} = 0.75\ A$$

$[\because \mu_0 = 4\pi \times 10^{-7}]$

Chemistry

51. (c) Work done in a chemical reaction,

$W = -\Delta n_e RT$

Where, Δn_g = number of moles of gaseous products - number of moles of gaseous reactants

Reaction involved in combustion of ethane is,

$$C_2H_6(g) + \frac{7}{2}O_2(g) \longrightarrow 2CO_2(g) + 3H_2O(l)$$

$\therefore\ \Delta n_g = 2 - 4.5 = -2.5$

$W = (2.5\text{ mol})(8.314\text{JK}^{-1}\text{ mol}^{-1}) \times 300\text{ K}$

$= 6235.5\text{ J} = 6.2355\text{KJ}$

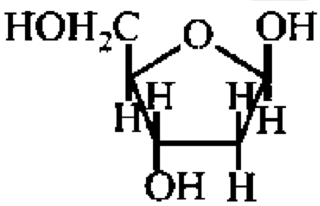
1 mole of C_2H_6 = 30 g of C_2H_6

Work done during combustion of 30 g of C_2H_6 = 6.2355 kJ

$\therefore$ Work done during combustion of 90 g of

$$C_2H_6 = \frac{6.2355 \times 90}{30} = 18.7065\text{ kJ}$$
$$= 18.71\text{ kJ}$$

52. (c) The sugar molecule present in DNA is 2'-deoxyribose.



β-D-deoxyribose used in DNA
or
D-2-deoxyribose

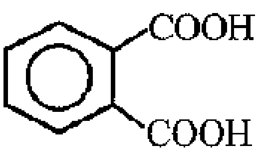
53. (a) Molality

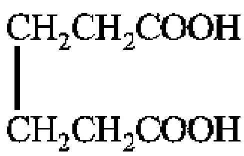
$$(m) = \frac{\text{Number of moles of solute}}{\text{Mass of solvent (in kg)}}$$

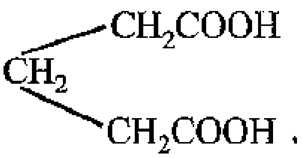
$$\Rightarrow \frac{\frac{15.2}{60}}{0.150} = \frac{0.2533}{0.15}$$

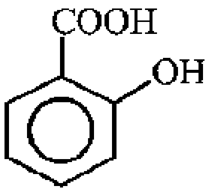
$\Rightarrow 1.689\text{ mol kg}^{-1}$

54. (d)


Phthalic acid


Adipic acid


Glutaric acid

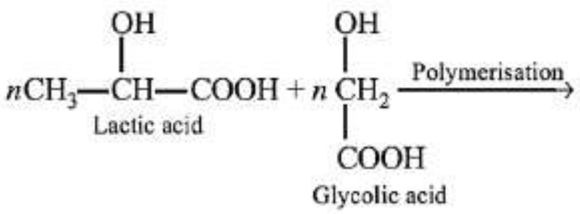

Salicylic acid

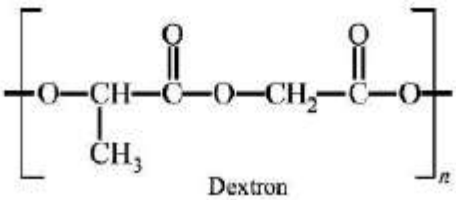
∴ Salicylic acid contains both -OH and -COOH groups

55. (b)

56. (a) Due to the smaller size of lithium, it has the highest Hydration Enthalpy which compensate its I.E. and shows negative E° value. Therefore, it acts as powerful reducing agent and hence weakest oxidising agent.

57. (a) Monomers used in preparation of dextron are lactic acid and glycolic acid.





58. (a) All the hydrides, except water (H_2O) of group 16 elements acts as a reducing -agents.

59. (d) Processes such as irradiation, addition of salts and heat, antioxidants, emulsifiers are used to preserve the food.

60. (d) Electron releasing groups increase electron density at N -atom hence, such substituents increase basic nature of aromatic amines. Option (a), (b), (c) are electron releasing group whereas (d): C_6H_5 is EWG, thus decreases the basic strength.

61. (d) (+) 2-methyl butane-1-ol and (-) 2-methyl butan $-1-$ ol are enantiomer. They are nonsuperimposable mirror images of each other. Hence, they are optically active. The optical activity of a compound can be confirmed by the value of specific rotation.

62. (b) Haematite - Fe_2O_3

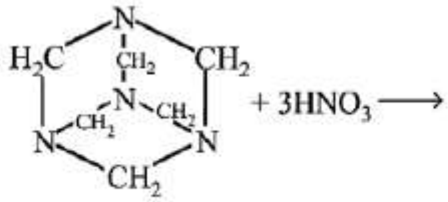
Magnesite - $MgCO_3$

Magnetite - Fe_3O_4

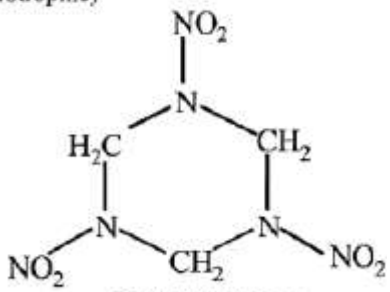
Siderite $-FeCO_3$

∴ Magnesite is the mineral of magnesium (Mg).

63. (c) Urotropine gives highly explosive cyclonite on nitration.



Hexamethylene tetramine (urotropine)



Cyclotrimethylene trinitramine (cyclonite)

$+ 3CH_2O + NH_3$

64.(b) Work done during compression,

$W = p_{ext}\,\Delta V$

Given, $p_{ext} = 100 KPa, T = 300\ K$

$\Delta V = V_2 - V_1$

$= (10 - 1) dm^3 = 9 dm^3$

$= 0.99\ m^3$

$\therefore W = 100 KPa (0.99) m^3$

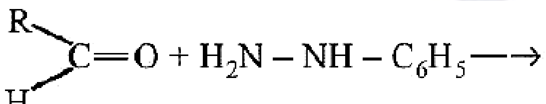
$= 99\ kJ.$

65.(b) Nitrogen because of its small size, high electronegativity, high ionisation energy, absence of vacant d-orbitals has tendency to form $p\pi - p\pi$ multiple bonds $N = N$

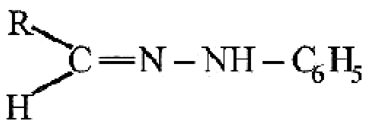
66.(b) Statement (b) is incorrect. Hoffmann bromamide degradation is used to synthesise primary amine.

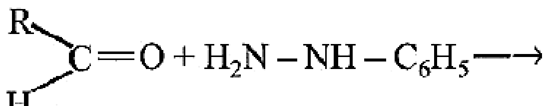
67.(d) Zirconium (Zr) with atomic number 40 and Hafnium (Hf) with atomic number 72 belongs to period 5 th and 6 th respectively.

68. **(b)** Aldehydes or ketones on treatment $C_6H_5 - NH - NH_2$ (phenylhydrazine) gives phenylhydrazone. This is a nucleophilic addition reaction.

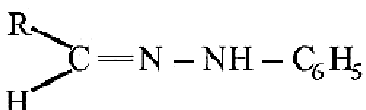


Aldehyde





Ketone



Phenyl hydrazone

69. **(d)** The solubility of NaBr changes slightly with temperature.

70. (b) The reaction of ethanol with water:

$$2C_2H_5OH(l) + 2Na(s) \longrightarrow$$
$$2C_2H_5\bar{O}Na^+ + H_2(g) \uparrow$$

2 mole of Na(46 g) = 1 mole of H_2

$= 2\ g = 2 \times 10^{-3}$ kg

71. (d)

72. (c) Aluminum is refined by Hoope's process.

73. (a) Oxidation state of Mn in MnO_4^{2-} is +6.

$Mn(+6) = 1s^2 2s^2 2p^6, 3s^2 3p^6 3d^1, 4s^0$

Hence, manganate ion MnO_4^{2-} is paramagnetic due to presence of unpaired electron. Also, MnO_4^{2-} is green in colour.

74. (a) Osmotic pressure, $\pi = CRT$ or $\pi = \frac{w}{MV}RT$ where, C = concentration of solution. Given, w = 34.2 g, V = 1 L, M = 342 g mol^{-1} T = 20°C, C = 20 + 273 = 293°K

$$\therefore \pi = \frac{w}{MV}RT$$

$$= \frac{342.2\ g}{342\ g\ mol^{-1}}$$

$$\times \frac{0.082\ L\ atm\ K^{-1}\ mol^{-1} \times 293\ K}{1\ L}$$

= 2.40 atm.

75. (a) In R-S configuration, priority sequence is decided by the atoms directly attached to the chiral carbon are arranged in decreasing atomic number. From the given groups, Sulphur (S) has the highest atomic number i.e., 16 therefore it has highest priority.

76. (b) Bithional is an antiseptic, which is mixed to medication soaps to impart antiseptic properties.

Whereas chloramphenicol is antibiotic, cimetidine is antacid and chlordiazepoxide is transquilizer.

77. (c) In the preparation of sulphuric acid (H_2SO_4) by lead chamber process, mixture containing SO_2 air and NO is treated with steam (H_2O). In this reaction, NO acts as a catalyst.

$$2SO_2 + \underset{\text{(from air)}}{O_2} + 2H_2O + \underset{\text{Catalyst}}{[NO]} \longrightarrow$$
$$2H_2SO_4 + \underset{\text{Catalyst}}{[NO]}$$

78. (b) Coordination number of Fe in $K_3[Fe(CN)_6]$ is 6 as it is bonded with six CN ligands.

Let x be the oxidation state of Cr

$\therefore x + 6(-1) = -3$

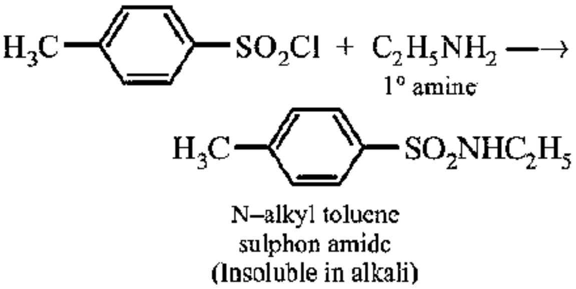
$\therefore x = +3$

79. (a) Pseudo first order reaction e.g., inversion of cane sugar is an example of

$$\underset{\substack{\text{Sucrose}\\ \text{(Cane sugar)}}}{C_{12}H_{22}O_{11}} + H_2O \xrightarrow{H^+} \underset{\text{Glucose}}{C_6H_{12}O_6} + \underset{\text{Fructose}}{C_6H_{12}O_6}$$

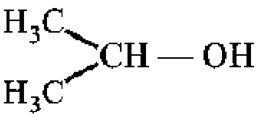
The concentration of water remains constant

80. **(a)** 1°, 2° and 3° amines can be distinguished from each other by p-toluenesulphonyl chloride (Hinsberg reagent). Among which 1° amine on sulphonation gives a product, which is insoluble in acid.



81. **(c)** Variation of rate constant k with temperature T(K) is given by Arrhenius equation.

$k = Ae^{-E_a/RT} = \frac{A}{e^{Ea/RT}}$ ...... (i)

82.(a) Isopropyl alcohol 

will give positive iodoform test.

83.(a) According to first law of thermodynamics, $\Delta U = q + W$

where, ΔU = Internal energy

q = Heat ; w = Work done

For isothermal process, $\Delta T = 0, \Delta U = 0$

$\therefore q = -W$

84. **(b)** Alkyl chlorides or bromides reacts with NaI in dry acetone to give alkyl iodide. This reaction is known as Finkelstein reaction. $R - X + NaI \longrightarrow RI + NaX[X = Cl, Br]$ e.g., $C_2H_5Br + NaI \longrightarrow C_2H_5I + NaBr$

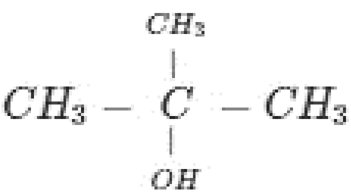
85.(c) Fullerenes are the allotropes of carbon structure having cage like with general formula, C_{2n} (e.g., C_{60}, C_{70} etc). C_{60} or the bucky ball consists of 60 C -atoms in which each C-atom in C_{60} is sp^2-hybridised.

86. **(b)** S.I Unit of conductivity (κ) is Sm^{-1}

87.(a) Alk. $KMnO_4$ is called Baeyer's reagent.

88. **(b)** The chief consitituent of Pyrex glass is SiO_2.

89. **(c)** For isomeric alcohols, the boiling point decreases with increase in branching of carbon chain. Therefore, tert butyl alcohol has lowest boiling point.



Tert-butyl alcohol

90. **(c)** Relation between heat of reaction ($\Delta Hr°$) and bond enthalpies of reactants and products is

$\Delta H_r = \Sigma BE_{reactants} - \Sigma BE_{products}$

$\therefore \Delta H^\circ_{reaction} = \Sigma H^\circ_{product\ bonds} - \Sigma H^\circ$ reactant bonds

91. **(a)** Given, $k = 7 \times 10^{-4}\ s^{-1}, [A]_0 = 0.08M$

$t_{1/2}=\frac{0.693}{k}=\frac{0.0693}{7\times10^{-4}\ s}=990\ s$

92.(a) Bakelite is highly cross -linked polymer. Which is used in making handles of cookers and frying pans, electrical goods, etc.

93.(b) Electron gain enthalpy becomes less negative on moving from chlorine to iodine. However, negative electron gain enthalpy of fluorine is less than that of chlorine due to small sized of fluorine atom. It has very high inter electronic repulsion in the relatively small 2p orbitals. Hence, incoming electron experience less attraction from the nucleus.

∴ Chlorine has the highest value of electron gain enthalpy.

94.(b) Half of the volume occupied in water is empty or unoccupied. Therefore, 10 cm^3 of the actual volume is occupied by water molecules present in 20 cm^3 of water.

95.(d) EAN = Z (atomic number of the metal) - number of electrons lost in the ion formation + number of electrons gained from the donor atoms of the ligands.

Complex	Oxidation state of metal ion	Atomic no.	Coordination no.	EAN
$[Pt(NH_3)_6]^{4+}$	+4	76	6	$(78-4)+(6\times 2) = 86(Rn)$
$[Fe(CN)_6]^{4-}$	+2	26	6	$(26-2)+(6\times2) = 36(Kr)$
$[Zn(NH_3)_4]^{2+}$	+2	30	4	$(30-2)+(4\times2) = 36(Kr)$
$[Cu(NH_3)_4]^{2+}$	+2	29	4	$(29-2)+(4\times\ \ 8) = 35(Br)$

$[Cu(NH_3)_4]^{2+}$ is an exception to EAN rule.

96.(d) The reaction involved for lead accumulator during discharging i.e., when cell is in the use are

At anode: $Pb(s)+$

$HSO_4^-(aq)\longrightarrow PbSO_4(s)+H^++2e^-$

At cathode: $PbO_2(s)+3H^+(aq)+$

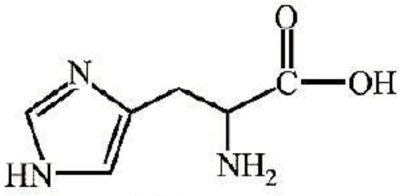
$HSO_4^-(aq)+2e^-\longrightarrow PbSO_4(s)+2H_2O(l)$

Overall reaction: $Pb(s)+PbO_2(s)+2H^+$

$+2HSO_4^-(aq)\longrightarrow 2PbSO_4(s)+2H_2O(l)$

97. (d) For the aqueous solution of urea, van't Hoff factor has the lowest value.

98. (a)



Histidine

It is basic in nature as it contains more number of $-NH_2$ groups than -COOH groups.

99.(a) Ar (Noble gas) does not form diatomic molecules. Due to presence of completely filled valence shell, these gases are highly stable.

100. **(d)** Stachyose contains 24 carbon atoms in its structure.

Mathematics

101. **(b)** We have, $\tan 2\theta = 1$.

$\Rightarrow \tan 2\theta = \tan\frac{\pi}{4}. \Rightarrow 2\theta = n\pi + \frac{\pi}{4}.$

$\Rightarrow \theta = \frac{n}{2}\pi + \frac{\pi}{8}.$

Also, the value of $\tan 2\theta$ is positive. So, θ lies in 1 st and 3 rd quadrants.

$\therefore \theta = \frac{\pi}{8} \& \frac{9\pi}{8}$, are the two principal solutions.

102. **(a)** The objective function is given as, minimize, $z = 4x_1 + 5x_2$

Subject to constraints, $2x_1 + x_2 \geq 7$,

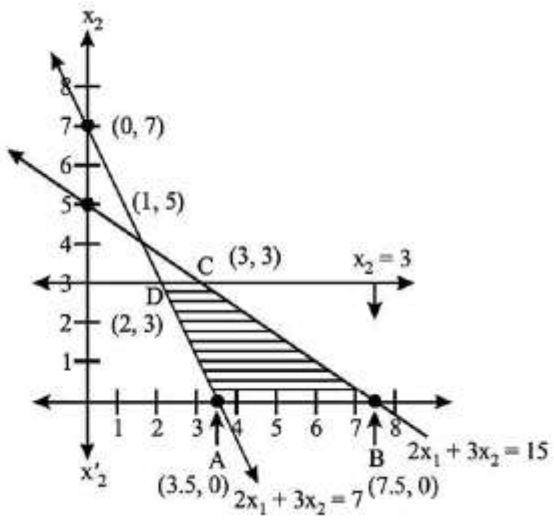
$2x_1 + 3x_2 \leq 15, x_2 \leq 3$ and $x_1, x_2 \geq 0$

For line $2x_1 + x_2 = 7$

x_1	0	1	2	3
x_2	7	5	3	1

For line $2x_1 + 3x_2 = 15$

x_1	0	3	6
x_2	5	3	1

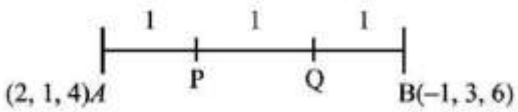


Now, the value of z at corner points are calculated as:

Corner points	$z = \mathbf{4x_1 + 5x_2}$
A(3.5,0)	$z = 4 \times 3.5 + 5 \times 0 = 14$ (minimum)
B(7.5,0)	$z = 4 \times 7.5 + 5 \times 0 = 30$
C(3,3)	$z = 4 \times 3 + 5 \times 3 = 27$
D(2,3)	$z = 4 \times 2 + 5 \times 3 = 23$

Hence, the minimum value of z is 14 at point (3.5,0) which lies on X-axis.

103. **(d)** Let P&Q are the points with z-coordinates as z_1 and z_2 respectively which trisect the line segment AB.



Then, coordinates of P

$$= \left(\frac{1 \times (-1) + 2 \times 2}{1+2}, \frac{1 \times 3 + 2 \times 1}{1+2}, \frac{1 \times 6 + 2 \times 4}{1+2}\right)$$

Now, z-coordinate of P $= \frac{1 \times 6 + 2 \times 4}{1+2}$

i.e. $z_1 = \frac{6+8}{3} = \frac{14}{3}$

And coordinates of Q

$$= \left(\frac{2 \times (-1) + 1 \times 2}{2+1}, \frac{2 \times 3 + 1 \times 1}{2+1}, \frac{2 \times 6 + 1 \times 4}{2+1}\right)$$

So, z -coordinate of Q $= \frac{2 \times 6 + 1 \times 4}{2+1}$

i.e. $z_2 = \frac{12+4}{3} = \frac{16}{3}$

Hence, $z_1 + z_2 = \frac{14}{3} + \frac{16}{3} = \frac{30}{3} = 10$

104. **(b)** Since, $f(x) = \frac{\log x}{x}$

After differentiating on both sides w.r.t.x, we get

$$f'(x) = \frac{x \cdot \frac{1}{x} - \log x \cdot 1}{x^2} = \frac{1 - \log x}{x^2}$$

For maximum or minimum value of f(x), put $f'(x) = 0$

$$\Rightarrow \frac{1 - \log x}{x^2} = 0 \Rightarrow \log x = 1 \Rightarrow x = e$$

Now, $f''(x) = \frac{3+2\log x}{x^3}$

$$\therefore\ f''(e) = -\frac{1}{e^3} < 0$$

After substituting $x = e$ in eq. (i), we get

$$f(e) = \frac{\log e}{e} = \frac{1}{e}$$

Hence, maximum value of f(x) is $\frac{1}{e}$ at $x = e$.

105. **(b)** $\int_0^1 x\tan^{-1} x dx$

$$= \left[\tan^{-1} x \int \ xdx\right]_0^1 - \int_0^1 \left(\frac{d}{dx}(\tan^{-1} x) \int \ xdx\right) dx$$

$$= \left[\tan^{-1} x \frac{x^2}{2}\right]_0^1 - \int_0^1 \left(\frac{1}{1+x^2} \cdot \frac{x^2}{2}\right) dx$$

$$= \left(\frac{1}{2}\tan^{-1} 1 - 0\right) - \frac{1}{2}\int_0^1 \frac{1+x^2-1}{1+x^2} dx$$

$$= \frac{1}{2}\left(\frac{\pi}{4}\right) - \frac{1}{2}\int_0^1 \left(1 - \frac{1}{1+x^2}\right) dx$$

$$= \frac{\pi}{8} - \frac{1}{2}[x - \tan^{-1} x]_0^1$$

$$= \frac{\pi}{8} - \frac{1}{2}[1 - \tan^{-1} 1 - 0 + 0]$$

$$= \frac{\pi}{8} - \frac{1}{2}\left[1 - \frac{\pi}{4}\right] = \frac{\pi}{4} - \frac{1}{2}$$

106. **(b)** The truth table is given below

p	**q**	**$\sim$ p**	**p $\vee$ q**	**$\sim$ p $\wedge$ q**	**(p $\vee$ q) $\wedge\sim$ q**
T	T	F	T	F	F
T	F	F	T	F	F

F	T	T	T	T	T
F	F	T	F	F	F

$\therefore\ (\sim p \wedge q) \equiv (p \vee q) \wedge \sim p$

107. **(a)** Given, $g(x) = f^{-1}(x)$

$f(g(x)) = x$

On differentiating both sides w.r.t. ' x ', we get $f'(g(x)) \cdot g'(x) = 1$

$$\therefore\ \frac{1}{1+(g(x))^4} g'(x) = 1 \left[\because f'(x) = \frac{1}{1+x^4}\right]$$

$\Rightarrow g'(x) = 1 + [g(x)]^4$

108. **(b)** Consider $A = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 3 & 0 \\ 5 & 2 & -1 \end{bmatrix}$

So, $|A| = \begin{vmatrix} 1 & 0 & 0 \\ 3 & 3 & 0 \\ 5 & 2 & -1 \end{vmatrix}$

$= 1(3 \times (-1) - 0) - 0(3 \times (-1) - 0 - 0) + 0(3 \times 2 - 5 \times 3)$

$= 1 \times (-3) - 0 - 0 = -3$

Now, adj A

$$= \begin{bmatrix} (3 \times (-1) - 0) & -3 \times (-1) - 0) & (3 \times 2 - 5 \times 3) \\ -(0-0) & (1 \times (-1) - 0) & -(2 \times 1 - 5 \times 0) \\ (3 \times 0 - 0) & -(1 \times 0 - 0) & (3 \times 1 - 0) \end{bmatrix}^T$$

$$= \begin{bmatrix} -3 & 3 & -9 \\ 0 & -1 & -2 \\ 0 & 0 & 3 \end{bmatrix}$$

$$\text{adj } A = \begin{bmatrix} -3 & 0 & 0 \\ 3 & -1 & 0 \\ -9 & -2 & 3 \end{bmatrix}$$

Hence, $A^{-1} = \frac{1}{|A|} \text{adj } A = \frac{1}{-3} \begin{vmatrix} -3 & 0 & 0 \\ 0 & -1 & 0 \\ -9 & -2 & 3 \end{vmatrix}$

109. **(a)** $\int \frac{1}{\sqrt{9-16x^2}} dx$

$$= \int \frac{1}{\sqrt{3^2 - (4x)^2}} dx = \frac{1}{4} \int \frac{1}{\sqrt{\left(\frac{3}{4}\right)^2 - (x)^2}} dx$$

$$= \frac{1}{4} \sin^{-1} \frac{4x}{3} + c$$

$$\left[\because \int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1} \frac{x}{a} + c\right]$$

As, $\int \frac{1}{\sqrt{9-16x^2}} dx = \alpha \sin^{-1(\beta x)} + c$

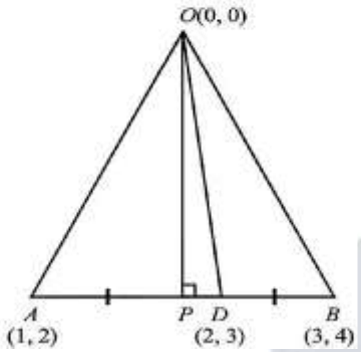
$\therefore\ \alpha \sin^{-1}(\beta x) + c = \frac{1}{4} \sin^{-1}\left(\frac{4}{3}x\right) + c$

After comparing on both sides, we get

$\alpha = \frac{1}{4}$ and $\beta = \frac{4}{3}$

Hence, $\alpha + \frac{1}{\beta} = \frac{1}{4} + \frac{1}{\frac{4}{3}} = \frac{1}{4} + \frac{3}{4} = 1$

110. (d)



Since, $O(0,0), A(1,2)$ and $B(3,4)$ are the vertices of $\triangle$ OAB

Consider that OP and OD are altitude and median of $\triangle$ OAB, respectively.

Then, Coordinates of D

$= \left(\frac{1+3}{2}, \frac{2+4}{2}\right) = (2,3)$

So, equation of OD is $(y - 0) = \left(\frac{3-0}{2-0}\right)(x - 0)$

Hence, $y = \frac{3}{2}x \Rightarrow 3x - 2y = 0$

Now, slope of OP $= \frac{-1}{\text{Slope of AB}}$

$\frac{-1}{\left(\frac{3-1}{4-2}\right)} = -1[\text{As}, OP \perp AB]$

$= \frac{1}{\left(\frac{3-1}{4-2}\right)} = -1$

$\Rightarrow 3x^2 + xy - 2y^2 = 0$

$= e^{\lim_{x\to 0}} \left(\frac{2\tan x}{1 - \tan x}\right)\frac{1}{x}\left[\because \lim_{x\to 0} \frac{\tan x}{x} = 1\right]$

$\therefore$ Equation of OP is $(y - 0) = -1(x - 0)$

$\therefore\ y = -x \Rightarrow x + y = 0$

Hence, joint equation of OP and OD is:

$(x+y)(3x-2y)=0$

111. **(c)**

112. **(c)** We have,

$A(\operatorname{adj} A)=\begin{bmatrix}10 & 0\\ 0 & 10\end{bmatrix}=10\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}=10I$

As, $A(\operatorname{adj} A)=|A|I$

After comparing on both sides, we get

$|A|=10$

113. **(b)** We have, $\frac{dy}{dx}=\tan\left(\frac{y}{x}\right)+\left(\frac{y}{x}\right)$ .... (i)

Since, it is homogeneous differential equation After putting, $y=V_x$, we get

$$\frac{dy}{dx}=V+x\frac{dV}{dx}$$

$$V+x\frac{dV}{dx}=\tan V+V \quad \text{........ form (i)}$$

$$\Rightarrow \quad x\frac{dV}{dx}=\tan V$$

$$\Rightarrow \quad \frac{1}{\tan V}dV=\frac{1}{x}dx$$

After integrating on both sides, we get

$$\int \frac{1}{\tan V}dV=\int \frac{1}{x}dx$$

$$\Rightarrow \quad \int \cot V dV=\log x+\log c$$

$$\Rightarrow \quad \log\sin V=\log(xc)$$

$$\Rightarrow \quad \sin v=xc$$

Hence, $\sin\left(\frac{y}{x}\right)=x$

114. **(c)** We have, $\sin^2 A+\sin^2 B=\sin^2 C$

$\Rightarrow a^2+b^2=c^2$ (From Sine rule)

$\therefore$ $\triangle$ ABC is right angled triangle and $\angle ACB=90^\circ$

So, area of $(\triangle ACB)=\frac{1}{2}ab$

By Sine rule's we get

$$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$$

$$\Rightarrow \frac{a}{\sin A}=\frac{b}{\sin B}=\frac{10}{1}$$

Hence, $a=10\sin A$

and $b=10\sin B$

$$\text{Area of } \triangle ACB \quad =\frac{1}{2}(10\sin A)(10\sin B)[\text{ From (i)}]$$

$$=50\sin A\sin B$$

As, maximum value of $\sin A \sin B = \frac{1}{2}$

Hence, maximum value of area of $\triangle ACB$

$$= 50 \times \frac{1}{2} = 25$$

115. **(a)** We have $x = f(t)$ and $y = g(t)$

After differentiating on both sides w.r.t ' t ', we get

$$\frac{dx}{dt} = f'(t) \text{ and } \frac{dy}{dt} = g'(t)$$

As, $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$

$$\Rightarrow \frac{dy}{dx} = \frac{g'(t)}{f'(t)}$$

Now, differentiating again both sides w.r.t. ' x ', we get

$$\frac{d^2y}{dx^2} = \frac{f'(t) \cdot g''(t) - g'(t) \cdot f''(t)}{(f'(t))^2} \cdot \frac{dt}{dx}$$
$$= \frac{f'(t) \cdot g''(t) - g'(t) \cdot f''(t)}{(f'(t))^2} \cdot \frac{1}{f'(t)}$$
$$= \frac{f'(t) \cdot g''(t) - g'(t) \cdot f''(t)}{(f'(t))^3}$$

116. **(b)** Consider $(x_1, y_1, z_1) \equiv (-3, 2, -5)$

As, the line is equally inclined to coordinate axes.

$l = -1, m = 1$ and $n = -1$

Since, the equation of line passing through (x_1, y_1, z_1) and direction cosines l, m, n is

$$\frac{x - x_1}{l} = \frac{y - y_1}{m} = \frac{z - z_1}{n}$$

Hence, $\frac{x+3}{-1} = \frac{y-2}{1} = \frac{z+5}{-1}$

117. **(d)** We have, $\int_0^{\frac{\pi}{2}} \log \cos x dx = \frac{\pi}{2} \log \frac{1}{2}$ .... (i)

$$= \int_0^{\frac{\pi}{2}} \log \sec x dx = \int_0^{\frac{\pi}{2}} \log \left(\frac{1}{\cos x}\right) dx$$

$$= -\int_0^{\frac{\pi}{2}} \log(\cos x) dx = -\int_0^{\frac{\pi}{2}} \log(\cos x) dx$$
$$= -\frac{\pi}{2} \log\left(\frac{1}{2}\right) \quad \text{From (i)]}$$
$$= \frac{\pi}{2} \log 2$$

118. **(c)** Suppose X be a random variable which denotes the no. of heads in tossing a coin three times. X can take value 0,1,2,3.

As, y = ₹2x. So, Y can take the values ₹0, ₹ 2, ₹ 4 and ₹6

$$\therefore \ P(y=0)=P(0 \text{ head })=\frac{1}{8}$$

$$P(y=2)=P(1 \text{ head })=\frac{3}{8}$$

$$P(y=4)=P(2 \text{ heads })=\frac{3}{8}$$

$$P(y=6)=P(3 \text{ heads })=\frac{1}{8}$$

Hence, expected gain

$$=0\left(\frac{1}{8}\right)+2\left(\frac{3}{8}\right)+4\left(\frac{1}{8}\right)+6\left(\frac{1}{8}\right)$$

$$=\frac{6+12+6}{8}=3$$

119. **(c)**

120. **(c)** Let $\mathbf{n_1}$ and $\mathbf{n_2}$ are normals to the planes r. $(m\hat{\mathbf{i}}-\hat{\mathbf{j}}+2\hat{\mathbf{k}})+3=0$ and r. $(2\hat{\mathbf{i}}-m\hat{\mathbf{j}}-\hat{\mathbf{k}})-5=0$, respectively.

Now, $\theta=\frac{\pi}{3}$ is angle between the planes.

As, $\cos\theta=\left|\frac{\mathbf{n_1}\cdot\mathbf{n_2}}{|\mathbf{n_1}||\mathbf{n_2}|}\right|$

So, $\cos\frac{\pi}{3}=\frac{(m\hat{i}-\hat{j}+2\hat{k})\cdot(2\hat{i}-m\hat{j}-\hat{k})}{\left(\sqrt{m^2+(-1)^2+2^2}\right)\left(\sqrt{2^2+(-m)^2+(-1)^2}\right)}$

$$\Rightarrow \frac{1}{2}=\left|\frac{2\,m+m-2}{\sqrt{m^2+1+4}\sqrt{4+m^2+1}}\right|=\left|\frac{3\,m-2}{\sqrt{(m^2+5)^2}}\right|$$

$$\Rightarrow \ \pm\frac{1}{2}=\frac{3\,m-2}{m^2+5}$$

$$\Rightarrow m^2+5=6\,m-4 \text{ or } -m^2-5=6\,m-4$$

$$\Rightarrow m^2-6\,m-9=0 \text{ or } m^2+6\,m+1=0$$

$$\Rightarrow (m-3)^2=0 \ \text{ or } \ m^2+6m+1=0$$

As $m^2+6\,m+1=0$ does not give any real values.

Hence, $(m-3)^2=0 \Rightarrow m=3$

121. **(c)** We have, $O(0,0,0), P(2,3,4), Q(1,2,3)$ and $R(x,y,z)$ are coplanar.

So, $\begin{vmatrix} x-0 & y-0 & z-0 \\ 2-0 & 3-0 & 4-0 \\ 1-0 & 2-0 & 3-0 \end{vmatrix}=0$

$$\Rightarrow \begin{vmatrix} x & y & z \\ 2 & 3 & 4 \\ 1 & 2 & 3 \end{vmatrix}=0$$

$\Rightarrow x(9-8)-y(6-4)+z(4-3)=0$

$\Rightarrow\ x-2y+z=0$

122. **(a)** We have, pair of line is $px^2-qy^2=0$ ... (i)

After comparing eq. (i) with $ax^2+2hxy+by^2=0$, we get $a=p, b=-q, h=0$

As slopes of pair of lines represented by

$ax^2+2hxy+by^2=0$

are real and distinct iff $h^2-ab>0$

So, $0+pq>0$

Hence, $pq>0$

123. **(d)** Consider p, q, r, s, m and n are the position vectors of P, Q, R, S, M and N be respectively.

As, M and N are mid-points of PQ and RS respectively

So, $m=\frac{p+q}{2}$ and $n=\frac{r+s}{2}$ .... (i)

Now, PS + QR

$= s-p+r-q=(r+s)-(p+q)$

$= 2n-2m=2\text{ MN}$ [From eq. (i)]

124. **(c)** We have, pair of lines

$kx^2+5xy+y^2=0$ ..... (i)

After comparing eq. (i) with

$ax^2+2hx+by^2=0$, we get
$a=k, b=1$ and $2h=5$

Suppose m_1 and m_2 are two slopes of pair of lines.

Then $m_1+m_2=\frac{-2h}{b}=-5$ and $m_1m_2=\frac{a}{b}=k$

As, $(m_1-m_2)^2=(m_1+m_2)^2-4m_1m_2$

$\Rightarrow\quad (1)^2=(-5)^2-4k$
$\Rightarrow\quad 4k=24\Rightarrow k=6$

125. **(c)** We have, vector r with dc 's l, m, n is equally inclined to the coordinate axes.

So, $l = m = n$ ...... (i)
As, $l^2 + m^2 + n^2 = 1$
$\therefore l^2 + l^2 + l^2 = 1$ [from eq. (i)]
$\Rightarrow 3l^2 = 1$
$\Rightarrow l = \pm\frac{1}{\sqrt{3}}$
Therefore, $l = m = n = \pm\frac{1}{\sqrt{3}}$
$\therefore$ vector $\mathbf{r} = |\mathbf{r}|\left(\pm\frac{1}{\sqrt{3}}\hat{\mathbf{i}} \pm \frac{1}{\sqrt{3}}\hat{\mathbf{j}} \pm \frac{\mathbf{1}}{\sqrt{3}}\hat{\mathbf{k}}\right)$

Hence, total number of required vectors $= 2^3 = 8$

126. **(a)** Consider $I = \int \frac{1}{(x^2+4)(x^2+9)}dx$

$$= \int \frac{1}{5}\left(\frac{1}{x^2+4} - \frac{1}{x^2+9}\right)dx$$
$$= \frac{1}{5}\left[\int \frac{1}{x^2+2^2}dx - \int \frac{1}{x^2+3^2}dx\right]$$
$$= \frac{1}{5}\left[\frac{1}{2}\tan^{-1}\frac{x}{2} - \frac{1}{3}\tan^{-1}\frac{x}{3}\right] + C$$
$$= \frac{1}{10}\tan^{-1}\frac{x}{2} - \frac{1}{15}\tan^{-1}\frac{x}{3} + C$$

We have, $I = A\tan^{-1}\frac{x}{2} + B\tan^{-1}\frac{x}{3} + C$
So, $A\tan^{-1}\frac{x}{2} + B\tan^{-1}\frac{x}{3} + C$

$$= \frac{1}{10}\tan^{-1}\frac{x}{2} - \frac{1}{15}\tan^{-1}\frac{x}{3} + C$$

After comparing on both sides, we get

$A = \frac{1}{10}$ and $B = \frac{-1}{15}$

Hence, $A - B = \frac{1}{10} + \frac{1}{15} = \frac{15+10}{150} = \frac{1}{6}$

127. **(a)** It is given that, α and β are the roots of the equation
$x^2 + 5|x| - 6 = 0$

Here, $|x|^2 + 6|x| - |x| - 6 = 0$

$\Rightarrow |x|(|x| + 6) - 1(|x| + 6) = 0$

$\Rightarrow (|x| + 6)(|x| - 1) = 0$

$|x| = -6, 1$

As, modulus is always positive.

Therefore, $|x| = 1 \Rightarrow x = \pm 1$

Consider, $\alpha = 1$ and $\beta = -1$

Hence,

$$|\tan^{-1}\alpha - \tan^{-1}\beta| = |\tan^{-1}1 - \tan^{-1}(-1)|$$
$$= \left|\frac{\pi}{4} - \left(-\frac{\pi}{4}\right)\right| = \left|\frac{\pi}{2}\right|$$

128. **(c)** We have, $x = a\left(t - \frac{1}{t}\right)$ and $y = a\left(t + \frac{1}{t}\right)$

Then, $y^2 - x^2 = \left[a^2\left(t + \frac{1}{t}\right)^2 - a^2\left(t - \frac{1}{t}\right)^2\right]$

$\Rightarrow y^2 - x^2 = 4a^2$

After differentiating on both sides w.r.t. ' x ', we get

$$2y\frac{dy}{dx} - 2x = 0 \Rightarrow 2\left(y\frac{dy}{dx} - x\right) = 0$$

Hence, $\frac{dy}{dx} = \frac{x}{y}$

129. **(c)** Suppose that slope of the curvey $= \sqrt{x-1}$ is m_1

$$\therefore m_1 = \frac{dy}{dx} = \frac{d}{dx}\sqrt{x-1}\frac{1}{2\sqrt{x-1}}$$

slope of the line $2x + y - 5 = 0$ is m_2

$$\therefore m_2 = \frac{dy}{dx} = \frac{d}{dx}(5 - 2x) = -2$$

As, lines are perpendicular if $m_1 m_2 = -1$

So, $\frac{1}{2\sqrt{x-1}} \cdot (-2) = -1 \Rightarrow \sqrt{x-1} = 1$

$\Rightarrow x - 1 = 1 \Rightarrow x = 2$

After substituting $x = 2$ in $y = \sqrt{x-1}$, we get $y = 1$ Hence, required point is $(2,1)$.

130. **(d)** Consider, $I = \int \sqrt{\frac{x-5}{x-7}}\,dx$

$$= \int \sqrt{\frac{(x-5)(x-5)}{(x-7)(x-5)}}dx$$

$$= \int \frac{x-5}{\sqrt{x^2-12x+35}}dx$$

$$= \frac{1}{2}\int \frac{2x-12+2}{\sqrt{x^2-12x+35}}$$

$$= \frac{1}{2}\int \frac{2x-12}{\sqrt{x^2-12x+35}}dx + \int \frac{1}{\sqrt{x^2-12x+35}}dx$$

$$= \sqrt{x^2-12x+35} + \int \frac{1}{\sqrt{(x^2-12x+36-1)}}dx + C$$

$$= \sqrt{x^2-12x+35} + \int \frac{1}{\sqrt{(x-6)^2-1^2}}dx + C$$

$$I = \sqrt{x^2-12x+35} + \log\left|x-6+\sqrt{x^2-12x+35}\right| + C$$

As,

$$I = A\sqrt{x^2-12x+35} + \log\left|x-6+\sqrt{x^2-12x+35}+C\right|$$

Hence, $A = 1$

131. **(b)** We have, mean $= 18$ and variance $= 12$

So, $np = 18$ and $npq = 12$

$$\therefore \frac{npq}{np} = \frac{12}{18} \Rightarrow q = \frac{2}{3}$$

Therefore, $p = 1 - q = 1 - \frac{2}{3} = \frac{1}{3}$

After putting, $p = \frac{1}{3}$ in $np = 8$, we get

$$n\left(\frac{1}{3}\right) = 18 \Rightarrow n = 54$$

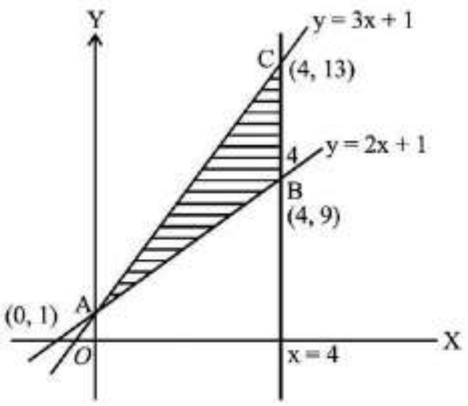
Hence, total number of possible value of $X = n + 1 = 54 + 1 = 55$

132. **(d)** For line $y = 2x + 1$,

two points are$(0,1)$ and $(4,9)$

For line $y = 3x + 1$,

two points are $(0,1)$ and $(4,13)$



Hence, area of shaded region

$$= \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \frac{1}{2}\begin{vmatrix} 0 & 1 & 1 \\ 4 & 9 & 1 \\ 4 & 13 & 1 \end{vmatrix}$$

$$= \frac{1}{2}[0 - 0 + 4(13 - 9)] = 8 \text{ sq units}$$

133. **(d)** Given, X is the number of defective pens obtained. Two pens are defective. So, X have possible values 0,1,2

$$\text{Now, } P(X = 0) = \frac{{}^4C_2}{{}^6C_2} = \frac{4 \times 3}{6 \times 5} = \frac{6}{15}$$

$$P(X = 1) = \frac{{}^2C_1 \times {}^4C_1}{{}^6C_2} = \frac{8}{15}$$

$$P(X = 2) = \frac{{}^2C_2}{{}^6C_2} = \frac{1 \times 2}{6 \times 5} = \frac{1}{15}$$

$$E(X^2) = \frac{8}{15} + \frac{2^2}{15} = \frac{12}{15} = \frac{4}{5}$$

$$\text{Standard deviation } = \sqrt{E(X^2) - [E(X)]^2}$$

$$= \sqrt{\left(\frac{4}{5}\right) - \left(\frac{2}{3}\right)^2}$$

$$= \sqrt{\frac{16}{45}} = \frac{4}{3\sqrt{5}}$$

134. **(a)** Let r be the radius of spherical ball

$\because$ Volume of spherical ball $V = \frac{4}{3}\pi r^3$ …. (i)

Now, $288\pi = \frac{4}{3}\pi r^3$

$\Rightarrow r^3 = 72 \times 3 = 8 \times 27$

$\Rightarrow r = 6$

After differentiating eq. (i) w. r. t. 't', we get

$$\frac{dV}{dt}=4\pi r^2\frac{dr}{dt}$$
$$\Rightarrow 4\pi=4\pi r^2\frac{dr}{dt}\left[\because \frac{dV}{dt}=4\pi cm^3/s\right]$$
$$\Rightarrow 1=(6)^2\frac{dr}{dt}$$
$$\Rightarrow \frac{dr}{dt}=\frac{1}{36}$$

$\because$ Surface area of spherical ball, $s=4\pi r^2$ After differentiating on both sides, w.r.t. 't', we get

$$\frac{ds}{dt}=4\times 2\pi r\frac{dr}{dt}$$
$$\Rightarrow \frac{ds}{dt}=8\times\pi\times 6\times\frac{1}{36}$$

Hence, $\frac{ds}{dt}=\frac{4\pi}{3}\ cm^2/s$

135. **(b)** We have, $f(x)=\begin{cases}\log(\sec^2 x)^{\cot^2 x}, & \text{for } x\neq 0\\ k, & \text{for } x=0\end{cases}$

As, f(x) is continuous at $x=0$

So,

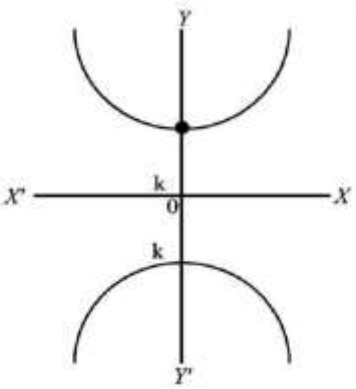
$$f(0)=\lim_{x\to 0}\left[\log(\sec^2 x)^{\cot^2 x}\right]$$
$$=\lim_{x\to 0}\left[\cot^2 x\log(\sec^2 x)\right]$$
$$=\lim_{x\to 0}\frac{\log(1+\tan^2 x)}{\tan^2 x}=\lim_{x\to 0}\frac{1}{1+\tan^2 x}=1$$

136. **(a)** After replacing ' $\vee$' by ' $\wedge$ ' and ' $\wedge$ ' by ' $\vee$ ', we get dual of the statement $\sim p\wedge(q\vee c)$ is $\sim p\vee(q\wedge t)$

137. **(a)** Let vertex of parabola be $(0,k)$ as axis of parabola is Y -axis

So, equation of parabola is

$$(x-0)^2=4a(y-k)$$
$$\Rightarrow x^2=4ay-4ak$$



After, differentiating both sides w. r. t., ' x ', we get

$2x = 4a\frac{dy}{dx}$

Therefore, $\frac{1}{2a} = \frac{1}{x}\frac{dy}{dx}$

Again, differentiating on both sides w. r. t. ' x ', we get

$\frac{d}{dx}\left(\frac{1}{x}, \frac{dy}{dx}\right) = \frac{d}{dx}\left(\frac{1}{2a}\right)$

$\Rightarrow \frac{1}{x} \cdot \frac{d^2y}{dx^2} + \frac{dy}{dx}\left(-\frac{1}{x^2}\right) = 0$

Hence, $x\frac{d^2y}{dx^2} - \frac{dy}{dx} = 0$

138. **(a)** $\int_0^3 [x]dx = \int_0^1 0dx + \int_1^2 1dx + \int_2^3 2dx$

$= [x]_1^2 + 2[x]_2^3$
$= (2-1) + 2(3-2) = 3$

139. **(c)** Objective function of a LPP defined over convex set attains its optimum value at at least one of the corner points.

140. **(d)** Consider, $A = \begin{bmatrix} \alpha & 14 & -1 \\ 2 & 3 & 1 \\ 6 & 2 & 3 \end{bmatrix}$

So,

$|A| = \begin{bmatrix} \alpha & 14 & -1 \\ 2 & 3 & 1 \\ 6 & 2 & 3 \end{bmatrix} = \alpha(9-2) - 14(6-6) - 1(4-18)$

$= 7\alpha + 14$

As, inverse of matrix A does not exists.

Therefore, $|A| = 0$

$\Rightarrow 7\alpha + 14 = 0$

Hence, $\alpha = -2$

141. **(a)** We have, $f(x) = \begin{cases} x, & \text{for } x \le 0 \\ 0, & \text{for } x > 0 \end{cases}$

LHL at $x = 0 = \lim_{x\to 0^-} f(x) = \lim_{x\to 0} x$

and RHL at $x = 0 = \lim_{x\to 0^+} f(x) = \lim_{x\to 0} 0 = 0$
Now, $f(0) = 0$

So, LHL = RHL = f(0)

Hence, $f(x)$ is continuous at $x = 0$

Here, $f'(x) = 1$ for $x \le 0$,

0 for $x > 0$

Thus, f(x) is not differentiable at $x = 0$

142. **(a)** Equation of plane which passes through

$\vec{a} = -\hat{i} + \hat{j} + 2\hat{k}$ as it is perpendicular to

$\hat{n} = \hat{i} + \hat{j} + \hat{k}$ will be $\vec{r} \cdot \hat{n} = \vec{a} \cdot \hat{n}$

$\vec{r} \cdot (\hat{i} + \hat{j} + \hat{k}) = (-\hat{i} + \hat{j} + 2\hat{k}) \cdot (\hat{i} + \hat{j} + \hat{k})$

$= -1 + 1 + 2 = 2$

Hence, $\vec{r} \cdot (\hat{i} + \hat{j} + \hat{k}) = 2$

143. **(b)** We have, probability that a person will develop immunity after vaccination is 0.8 Hence, probability that all 8 persons develop immunity $= (0.8)^8$

144. **(a)** The coordinates of given point is $(2,3,\lambda)$. So, equation of the plane is

$$\begin{aligned}
&\vec{r} \cdot (3\hat{i} + 2\hat{j} + 6\hat{k}) = 13 \\
&\Rightarrow (x\hat{i} + y\hat{j} + z\hat{k}) \cdot (3\hat{i} + 2\hat{j} + 6\hat{k}) = 13 \\
&\Rightarrow 3x + 2y + 6z - 13 = 0
\end{aligned}$$

Therefore, distance of the plane from the given point $(2,3,\lambda)$ will be

$\left|\frac{3\times2+2\times3+6\times\lambda-13}{\sqrt{3^2+2^2+6^2}}\right| = 5[\text{ Given }]$

$$\Rightarrow \pm5 = \frac{6\lambda - 1}{\sqrt{49}}$$

$\Rightarrow \pm35 = 6\lambda - 1$

$\Rightarrow 35 = 6\lambda - 1$ or $-35 = 6\lambda - 1$

Hence, $\lambda = 6, -\frac{17}{3}$

145. **(a)**

$$\begin{aligned}
&\cos^{-1}\left(\cot\frac{\pi}{2}\right) + \cos^{-1}\left(\sin\frac{2\pi}{3}\right) \\
&= \cos^{-1}(0) + \cos^{-1}\left(\frac{\sqrt{3}}{2}\right) \\
&= \cos^{-1}\left(\cos\frac{\pi}{2}\right) + \cos^{-1}\left(\cos\frac{\pi}{6}\right) \\
&= \frac{\pi}{2} + \frac{\pi}{6} = \frac{2\pi}{3}
\end{aligned}$$

146. **(b)** Given, $xdy + 2ydx = 0$

$$\therefore \frac{dy}{dy} + \frac{2dx}{x} = 0$$

After integrating on both sides, we get

$\int \frac{1}{y} dy + 2\int \frac{1}{x} dx = \log C$
$\Rightarrow \log y + 2\log x = \log C$
$\Rightarrow yx^2 = C$
If $x = 2$ then $y = 1$
So, $C = 1 \times 2^2 = 4$

Hence, particular solution will be $x^2y = 4$.

147. **(d)** It is given that, $A(2,3,5), B(-1,3,2)$ and $C(\lambda, 5, \mu)$ are three vertices of ΔABC

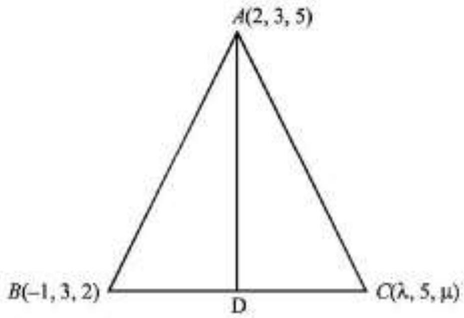
Let D be the mid-point of BC

So, coordinates of D $= \left(\frac{\lambda-1}{2}, \frac{5+3}{2}, \frac{\mu+2}{2}\right)$

$= \left(\frac{\lambda - 1}{2}, 4, \frac{\mu + 2}{2}\right)$

So, direction ratios of

$AD = \left(\frac{\lambda - 1}{2} - 2, 4 - 3, \frac{\mu + 2}{2} - 5\right)$
$= \left(\frac{\lambda - 5}{2}, 1, \frac{\mu - 8}{2}\right)$



As, AD is equally inclined to both the coordinates axes.

Therefore, $\frac{\lambda-5}{2} = 1 = \frac{\mu-8}{2}$

$\Rightarrow \frac{\lambda - 5}{2} = 1 \Rightarrow \lambda = 7$

And, $1 = \frac{\mu-8}{2} \Rightarrow \mu = 10$

148. **(b)** $p(3 < x \le 5) = p(x = 4) + p(x = 5) = (0.62 - 0.48) + (0.85 - 0.62) = 0.14 + 0.23 = 0.37$

149. **(b)** Since, $\frac{x-1}{2} = \frac{y+1}{2} = \frac{z-1}{4} = \lambda$

and $\frac{x-3}{1} = \frac{y-6}{2} = \frac{z}{1}$

Now, any point on the line (i) is $P(2\lambda + 1,$

$2\lambda - 1, 4\lambda + 1)$

$\therefore \frac{2\lambda+1-3}{1} = \frac{2\lambda-1-6}{2} = \frac{4\lambda+1}{1}$ [from (ii)]

So, $4\lambda - 4 = 2\lambda - 7 \Rightarrow 2\lambda = -3$

Hence, point of intersection P is

$= \left(2 \times \left(-\frac{3}{2}\right) + 1, 2 \times \left(-\frac{3}{2}\right) - 1, 4 \times \left(-\frac{3}{2}\right) + 1\right)$

$\equiv (-2, -4, -5)$

150. **(b)** Consider, $I = \int \frac{\sec^8 x}{\operatorname{cosec} x} dx = \int \frac{\sin x}{\cos^8 x} dx = \int \tan x \cdot \sec^7 x dx$

$= \int \sec^6 x \cdot \sec x \tan x dx$

Let $\sec x = t \Rightarrow \sec x \cdot \tan x dx = dt$,

So, $I = \int t^6 dt = \frac{t^7}{7} + c = \frac{\sec^7 x}{7} + c$