Physics

1. (c) Given: Path length = 16 cm

$\therefore$ Amplitude $a = \frac{16}{2} = 8$ cm

Time period $T = 2\pi\sqrt{\frac{l}{g}}$

$= 2\pi\sqrt{\frac{1}{\pi^2}} = 2\pi \times \frac{1}{\pi} = 2s$

Maximum velocity $V_{max} = a\omega$

$= a \times \frac{2\pi}{T} = 8 \times \frac{2\pi}{2} = 8\pi cm/s$

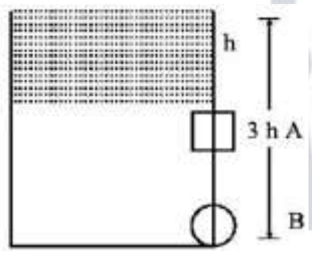
2. (c) Using equation of continuity

$A_1V_1 = A_2V_2$

$L^2\sqrt{2gh} = \pi r^2\sqrt{6gh}$

$L^4 gh = \pi^2 r^4 6gh$

$\therefore\ L = (\pi)^{\frac{1}{2}}(r)(3)^{\frac{1}{4}}$



3. (b) Voltage gain $= \frac{V_o}{V_i} = \frac{R_o \times I_c}{R_i \times I_B}$

$= \frac{2000 \times 1.5 \times 10^{-3}}{150 \times 20 \times 10^{-6}} = \frac{3}{3000 \times 10^{-6}} = 1000$

4. (b) Torque, $\tau = I\alpha$

$F \times R = \frac{MR^2}{2} \times \frac{\omega}{t}$

$\therefore$ Tangential force, $F = \frac{MR\omega}{2t}$

5. (b) $B_{centre} = \frac{\mu_0 \eta I}{R}$

Let at a distance x from the centre, magnetic

field becomes $\frac{B}{8}$

$\frac{B}{8} = \frac{\mu_0 \eta I R^2}{(R^2 + x^2)^{3/2}}$

$\Rightarrow \frac{\mu_0 \eta I}{8R} = \frac{\mu_0 \eta I R^2}{(R^2 + x^2)^{3/2}}$

or, $8R^3 = (R^2 + x^2)^{3/2}$

$\Rightarrow 2R = (R^2 + x^2)^{1/2}$

or $4R^2 = R^2 + x^2$ [from squarring both sides

of equation (i)]

$\Rightarrow 3R^2 = x^2$

$\therefore x = \sqrt{3R^2} = \sqrt{3}R$

6. (b) $I_{max} = (a_1 + a_2)^2$ and $I_{min} = (a_1 - a_2)^2$

$I_{max} + I_{min} = a_1^2 + a_2^2 + a_1^2 + a_2^2$

$= 2(a_1^2 + a_2^2) = 2(I_1 + I_2)$

7. (d) Alternating voltage, $e = e_0 \sin \omega t$

From question,

$e_0 = 200\sqrt{2}$ V, $\omega = 100$

$I_{rms} = \frac{v_{rms}}{X_c} = \frac{V_0 \omega C}{\sqrt{2}}$

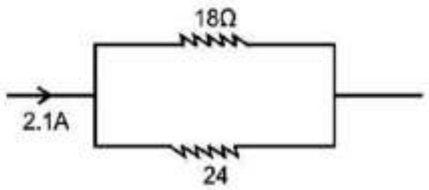
$= \frac{200\sqrt{2} \times 100 \times 10^{-6}}{\sqrt{2}}$

$= 2 \times 10^{-2} = 20$ mA

8. (c) $I = I_1 + I_2 = 2.1$ A

$18I_1 = 24I_2$

$\Rightarrow 3I_1 = 4I_2 = 4(2.1 - I_1)$



or, $7I_1 = 8.4$

$\therefore I_1 = \frac{8.4}{7} = 1.2$ A

9. (b)

10. (c) Tangential acceleration $= \alpha r$

Radial acceleration $= \frac{v^2}{r}$

$\therefore$ Ratio $= \frac{\alpha r}{v^2/r} = \frac{\alpha r^2}{v^2}$

11. (c) $d = \frac{\lambda}{2\mu \sin\alpha} = \frac{\lambda}{2N\cdot A}$

N.A. limit of resolution is decrease

12. (d) In amplitude modulation amplitude of the carrier wave changes according to information signal.

13. (b) Magnetisation of a paramagnetic $M_z = \frac{M_{ext}}{V}$

$M_z = \frac{CB}{T}$ ... (Paranmagnetic)

14. (d) $\frac{1}{\lambda} = R\left(\frac{1}{\eta_1} - \frac{1}{\eta_2}\right)$

$$\frac{1}{\lambda} = R\left[\frac{1}{1} - \frac{1}{16}\right] = \frac{15 - R}{16} \quad \therefore \lambda = \frac{16}{15R}$$

$P = \frac{h}{\lambda}$ or, $mv = \frac{h}{\lambda} \therefore v \frac{h}{m\lambda} = \frac{15hR}{m16}$

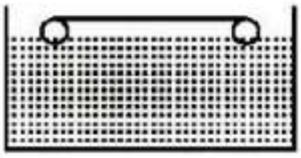
15. (a) Suspension tension, $T = \frac{f}{\ell}$

$mg = T.l$

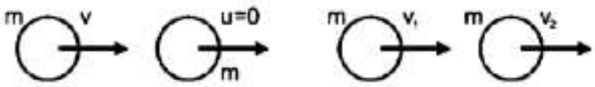
$\pi r^2 \rho g = Tl$

$$r^2 = \frac{T}{\pi\rho g}$$

$$r = \sqrt{\frac{T}{\pi\rho g}}$$



16. (c) Before collision After collision



$mV + 0 = mV_I + mV_2$

$V_1 + V_2 = V$

$$e = \frac{V_2 - V_1}{u_1 - u_2} e = \frac{V_2 - V_1}{V - 0}$$

Coefficient of restitution, e

$eV = V_2 - V_1$

$$eV + V = 2V_2 \Rightarrow V_2 = \frac{V(e+1)}{2}$$

$\therefore$ Ratio, $\frac{V_2}{V} = \frac{e+1}{2}$

17. (b) Distance travelled in one oscillation $= 4a$

Average velocity $= \frac{\text{total distance}}{\text{Time}}$

$$= \frac{4a}{T} = 4an\left[\therefore n = \frac{1}{T}\right]$$

18. (b) Given: $V_{in} = 220\text{ V}; V_{out} = 3.3 \times 10^3\text{ V}$

Power, $P = 4.4$ kW and no. of turns in secondary coil, $N_P = 600$

$$P = V_{in} \times I_{in} \Rightarrow I_{in} = \frac{4.4 \times 1000}{220}$$

$$= \frac{44 \times 10}{22} = 20\text{ A}; \frac{e_s}{e_p} = \frac{I_p}{I_s}$$

$$I_S = I_P = \frac{e_p}{e_S} = \frac{20 \times 220}{3.3 \times 1000} = \frac{44}{33} = \frac{4}{3}\text{ A}$$

19. (c) Using, $R = f\frac{\ell}{A}$

$$\frac{R_1}{R_2} = \frac{L_1}{L_2} \times \frac{A_2}{l_2} = \frac{L_1}{L_2} \times \frac{\frac{\pi d_2^2}{4}}{\frac{\pi d_1^2}{4}}$$

$\therefore$ Ratio $\frac{R_1}{R_2} = \frac{d_2^2}{d_1^2}$

20. (d)

$$C_P - C_V = R, \frac{C_P}{C_V} = \gamma \Rightarrow C_P = \gamma C_V$$

$$\gamma C_V - C_V = R \Longrightarrow C_V(r-1) = R$$

$$\therefore C_V = \frac{R}{(\gamma - 1)}$$

21. (b) From Jurin's law, $h \propto \frac{1}{r}$ or, $rh =$ constant

$r_1 h_1 = r_2 h_2 \qquad A_1 = \pi r_1^2$

$\frac{r_1}{r_2} = \frac{h_2}{h_1} \qquad A_2 = \pi r_2^2$

$3 = \frac{h_2}{h_1} = \frac{h_2}{h_1} \qquad \frac{\pi r_1^2}{9} = \pi r_2^2$

$h_2 = 3h_1 = 3h \qquad \frac{r_1^2}{r_2^2} = 9 \Rightarrow \frac{r_1}{r_2} = 3h$

22. (c) P-N junction diode with forward biosed mode.

23. (a) Time period, $T = 1/V = \frac{2\pi m}{eB}$

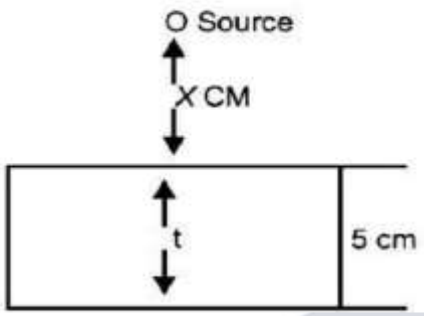
$$\Rightarrow B = \frac{2\pi m}{e} v$$

$$R = \frac{mv}{eB} = \frac{P}{eB} \left[\because F = \frac{mv^2}{R} = eB\right]$$

$$P = eBR = e \times \frac{2\pi m v^2}{2m} = 2\pi m v R$$

$$K \cdot E = \frac{P^2}{2m} = \frac{(2\pi m v R)^2}{2m} = 2\pi^2 m v^2 R^2$$

24.(b) According to question,



Let ' x ' be the distance of source from the surface of glass slab

time taken to travel light, from source to surface of the glass slab, (t_1) = time taken to travel in glass slab (t_2)

$t_1 = \frac{x}{3 \times 10^8}$ and, $t_2 = \frac{5}{\frac{3 \times 10^8}{1.6}}$

[$\because$ Velocity of light in glass medium, $= \frac{C}{M}$]

$$\therefore \frac{x}{3 \times 10^8} = \frac{5}{\frac{3 \times 10^8}{1.6}}$$

or, $x = 5 \times 1.6 = 8.0$ cm

25.(b) Fifth overtone, $2.4 = 6n$

$$A = 0.4m = \frac{\lambda}{2} \Rightarrow \lambda = 0.8$$

$\therefore$ Distance between successive node and antinode,

$$\frac{\lambda}{4} = \frac{0.8}{4} = 0.2 \text{ m}$$

26.(c) Here, $|A| = \sqrt{9 + 4 + 1} = \sqrt{14}$

$|B| = \sqrt{1 + 9 + 25} = \sqrt{35}$

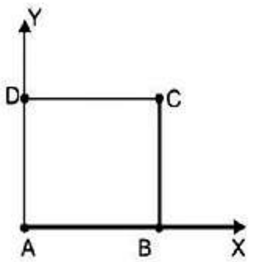
$| C = \sqrt{4 + 1 + 16} = \sqrt{21}$

Clearly, $\vec{B} = \vec{A} + \vec{C}$ and $B^2 = A^2 + C^2$

27.(a) Moment of inertia of the frame about X-axis.

$\frac{ml^2}{3}+\frac{ml^2}{3}+ml^2$

$I=\frac{5}{3}ml^2$



28.(d) Magnitude of unit vector = 1

$\sqrt{(0.8)^2+(b)^2+(0.4)^2}=1$

$\Rightarrow\sqrt{64+b^2+0.16}=1\Rightarrow\sqrt{0.80+b^2}=1$

$\Rightarrow 0.8+b^2=1\Rightarrow b^2=0.2\therefore b=\sqrt{0.2}$

29.(c) (Relation between magnetic permeability and susceptibility)

$B=(1+X)H$

$X=$ for paramagnetic positive and small

$X'=$ for diamagnetic negative and small

30. **(d)** Time period, $T=2\pi\sqrt{\frac{m}{k}}$

and, frequency, $n=\frac{1}{T}=\frac{1}{2\pi}\sqrt{\frac{k}{m}}25=\frac{1}{4\pi^2}\frac{k}{m}\Rightarrow k=100\pi^2m$

$kA=mg\Rightarrow A=\frac{mg}{k}$

$V_{max}=\omega A=\frac{2\pi}{T}A=2\pi nA$

$=\frac{2\pi5\times mg}{k}=\frac{10\pi\times m\times 10}{100\pi^2m}=\frac{1}{\pi}$

31. (b) Conservation of angular momentum,

$I_1\omega_1=I_2\omega_2$

or, $I\omega=2I\omega_1\Rightarrow\omega_1=\frac{\omega}{2}$

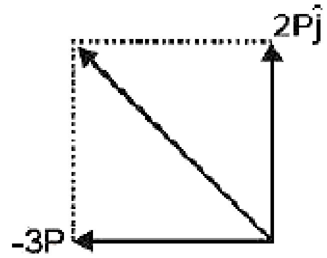
Original KE $=2\frac{1}{2}I\omega^2$

new wKE $=2\frac{1}{2}I_{\omega_1^2}=\frac{1}{2}2I\left(\frac{\omega}{2}\right)^2=\frac{I\omega^2}{4}$

Change in KE $=\frac{1}{2}I\omega^2-\frac{I\omega^2}{4}=\frac{I\omega^2}{4}$

32. (d) Magnitude of momentum of the 3rd part.

$\sqrt{9P^2 + 4P^2} = \sqrt{13}P$



33. (b) In the photocell, stopping potential directly proportional to frequency of incident radiation.

34. (b) Speed at top most point, $v = \sqrt{3rg}$

Centripetal acceleration, $a_c = \frac{v^2}{T} = \frac{3rg}{r} = 3\ g$

35. (a) Electric field intensity at a point outside a uniformly charged thin plane sheet $= \frac{\sigma}{2\varepsilon_0}$

So, it is independent of d

36. (a) Apparent frequency,

$n_a = n\left[\frac{v}{v - v_s}\right]$ i.e., frequency increase

As frequency increases, so wevelength decreases.

37. (d) According to question, the deflection in galvanometer falls to $\left(\frac{1}{4}\right)^{th}$ when it is shunted by 3Ω.

$\therefore I - \frac{I}{4} = \frac{3I}{4}$

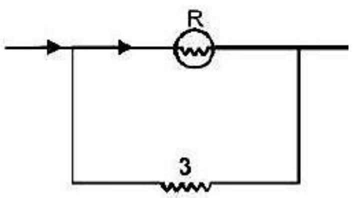
$\frac{R}{4} = \frac{3I}{4} \times 3$

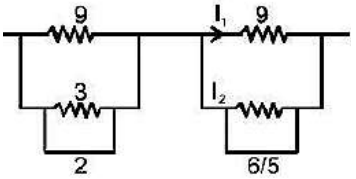
$\Rightarrow R = 9\Omega$

$I - I_1 = I_2$

$9I_1 = \frac{6}{5}I_2 \Rightarrow \frac{15}{2}I_1 = I_2^2$

$I = I_1 + \frac{15}{2}I_1 = \frac{17}{2}I_1$ or, $I_2 = \frac{2I}{17} = \frac{I}{8.5}$





38.(a) Let ' h ' be the height of the body above the surface of the earth.

$$\frac{GMm}{R}+\frac{1}{2}mu^2=0+-\frac{GMm}{R+h}$$

$$\frac{GM}{R+h}=\frac{Gm}{R}-\frac{u^2}{2}$$

$$\frac{GM}{(R+h)}=\frac{2Gm-Ru^2}{2R}$$

$$\frac{R+h}{GM}=\frac{2R}{2GM-Ru^2}$$

$$h=\frac{2GMR}{2GM-Ru^2}-R$$

$$=\frac{2GMR-2GMR+R^2u^2}{2GM-Ru^2}$$

$$=\frac{R^2u^2}{2GM-Ru^2}=\frac{Ru^2}{2gR-u^2}$$

39.(a) N_1 number of cap acitors each of capacity, C_1 charged to potential difference, 3 V are in series.

$$\therefore C_{eq}=\frac{C_1}{N_1};V=3V$$

$$E=\frac{1}{2}CV^2=\frac{1}{2}\frac{C_1}{N_1}9V^2$$

N_2 number of capacitors each of capacity, C_2 charged to potential difference, V are in parallel,

$$\therefore C_{eq=}N_2C_2;V=V$$

$$E=\frac{1}{2}CV^2=\frac{1}{2}C_2N_2V^2$$

From question, total energy stored in both the combination is same.

$$\therefore \frac{9}{2}\frac{C_1}{N_1}V^2=\frac{C_2N_2V^2}{2}\Rightarrow C_1=C_2\frac{N_2N_1}{9}$$

40. **(b)** Heat energy incident on the surface, $Q_i=1000\ J/m$

Coefficient of absorption, $a=0.8$ coefficient of reflection, $r=0.1$ Heat energy transmitted in 5 minutes, $Q_t=?$

$$\because 1=r+a+t\Rightarrow t=1-0.1-0.8=0.1$$

$Q_t=Q_i\times t\times T(t=$ cofficient of transmitance $)$

$$Q_t=0.1\times 1000\times 5=500\ J$$

41. (c) $Y=\frac{Fl}{A\Delta l},\Delta l=\frac{Fl}{YA}$

$$m=\rho V=\rho\times A\times l$$

$A \propto m\frac{\Delta lP}{\Delta lQ} = \frac{A_2}{A_1} = \frac{m_2}{m_1}$

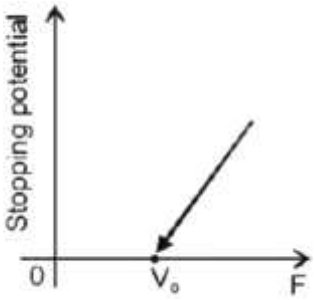
42.(b) Percentage error in the measurement of x

$\frac{\Delta x}{x} = 2\frac{\Delta a}{a} + 2\frac{\Delta b}{b} + \frac{\Delta c}{c}$

$= 2 \times 2 + 2 \times 3 + 4$

$= 4 + 6 + 4 = 14\%$

43.(a) The variation of stopping potential corresponding to the frequency of incident radiation (F) as shown below.



44.(b) In compound microscope, the focal length and aperture of the objective used is respectively large and small.

45.(b) de-Broglie wavelength

$\lambda = \frac{h}{p}\frac{h}{\sqrt{2mK \cdot E}}$

or, $\lambda^2 = \frac{h^2}{2m(K \cdot E)}$

$\therefore$ K. E. $= \frac{h^2}{2m\lambda^2}$

46.(b) Frequency of waves produced in string when tension in string T,

$n = \frac{1}{2L}\sqrt{\frac{T}{m}}$

When tension in string is doubled i.e., 2T.

$n' = \frac{1}{2L}\sqrt{\frac{2T}{m}} = \sqrt{2}\frac{1}{2L}\sqrt{\frac{T}{m}}$

Clearly, $n' = \sqrt{2}n$

47.(b) $\because Y = \frac{FL}{Al} \quad \therefore l\frac{FL}{AY}$

Work done / increase in energy

$= \frac{YAL^2}{2L} = \frac{F^2L}{2AY}$

48. **(a)** According to question,

$$\frac{B_1}{B_2}=\frac{25}{2}\Rightarrow\frac{\frac{\mu_0}{4\pi}\frac{Md_1}{(d_1^2-l^2)^2}}{\frac{\mu_0}{4\pi}\frac{Md_2}{(d_2^2-l^2)}}=\frac{25}{2}$$

$$\frac{d_1}{d_2}\times\frac{(d_2^2-l^2)^2}{(d_1^2-l^2)^2}=\frac{25}{2}$$

$$d_1=10\text{ cm}, d_2=20\text{ cm}$$

$$\frac{10}{20}\times\left(\frac{20^2-l^2}{10^2-l^2}\right)=\frac{25}{2}$$

$$400-l^2=5(100-l^2)$$

$$4l^2=100\Rightarrow t^2=25\text{ or, } l=5\text{ cm}$$

49.(d) According to question, speed of satellite in its orbit.

$v_c=\frac{1}{4}v_e.$

(v_e = escape speed)

$$\sqrt{\frac{GM}{R+h}}=\frac{1}{4}\sqrt{\frac{2GM}{(R)}};\sqrt{\frac{GM}{(R+h)}}=\frac{1}{16}\times\frac{2Gm}{(R)}$$

$$R+h=8(R)$$

$$R+h=8R$$

$$7R=h$$

50.(c) In case of closed organ pipe, fundamental frequency

$n\frac{V}{4L}=\frac{332}{4\times83\times10^{-2}}$ or, $n=100$ Hz

Hence frequency of overtones $n_1:n_2:n_3:$

$-----=1:3:3$ i.e., 100,300,500,700 and 900 Hz

So, number of possible natural oscillations of air column whose frequencies lie below 1000 are 5.

Chemistry

51. (b)

52.(a) $\Delta U=q+W$

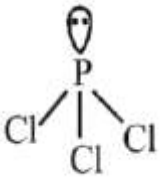
According to first law of thermodynamics

$$=q+(-P_{ex}\cdot\Delta V)(\because W=-P_{ex}\cdot\Delta V)$$

$$\Delta U=q_p-P_{ex}\cdot\Delta V$$

53. (b) Zinc is used for coating iron surface. Because zinc get oxidized first when comes in contact with moisture and hence iron surface is protected from corosion.

54. (b) PCl_3 - has 3 sigma bond and 1 tone pair. $3 + 1 = 4$



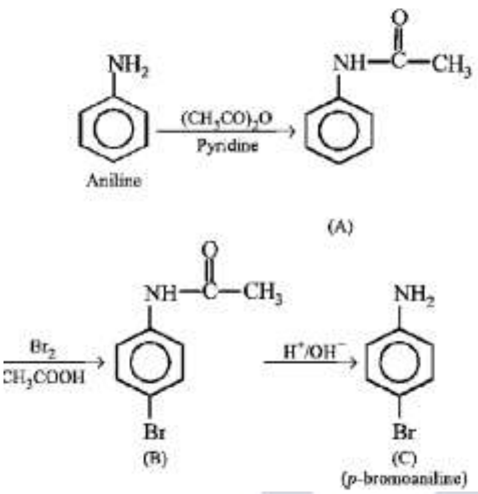
Hence, hybridization = sp^3

55. (b) Zn forms amphoteric oxide ZnO

$ZnO + 2HCl \longrightarrow 2nCl_2 + H_2O$ (acid)

$ZnO + 2NaOH \longrightarrow Na_2ZnO_2 + H_2O$ (Base)

56. (c)



57. (c) $-CH_3$ is electron donating group which shows + I effect.

58. (a) Polonium crystallizes as simple cube. Iron crystallizes as BCC unit cell. Copper and gold crystallize as FCC unit cell.

59. (c) $\underset{\text{(phosphonic acid)}}{4H_3PO_3} \longrightarrow 3H_3PO_4 + PH_3$

$H_3PO_3 \longrightarrow H_3PO_4$ (oxidation)

$H_3PO_3 \longrightarrow PH_3$ (reduction)

60. (b) In $[AuCl_4]^{1-}$ Let the oxidation number of

$Au = x$

$x + 4(-1) = -1$

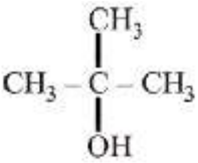
$x - 4 = -1$

$x = -1 + 4$

$x = +3$

61. (a)

62. (d)



3° alcohol reacts with lucas reagent (HCl + anhydrous $ZnCl_2$) immediately & gives two separate layers.

63.(a) Nitric oxide

64.(c) Moles of electron

$$= \frac{\text{Charge}}{\text{F}} = \frac{\text{Current} \times \text{Time}}{96500}$$

$$\frac{2 \times 20 \times 60}{96500} = 0.02487$$

or 2.487×10^{-2} mole^{-}

65.(b) Molarity $= \frac{\text{No. of moles of solute}}{\text{Volume of solution in (L)}}$

Urea (molar mass) $= 60$ g/mol

molarity $= \frac{15 \times 1000}{60 \times 500} = \frac{15}{6 \times 5} = \frac{1}{2}$

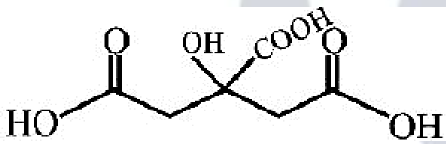
$= 0.5$ mol dm^{-3}

66. (c)

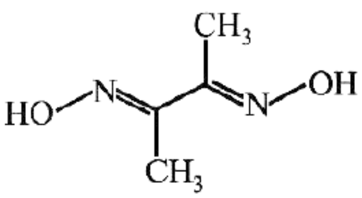
67.(c) CH_3CH_2COOH is most reactive towards esterification as bulkier group near the site of reaction, slows down esterification.

68. (d) Molarity $= \frac{\text{No. of moles of solute}}{\text{Vol. of Solution in dm}^3}$

69.(a) Citric acid is a tricarboxylic acid.



70.(b) No. of donor atom $= 2$

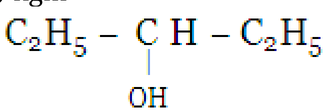


71. (c) Nitrous oxide (N_2O) has the lowest oxidation state (zero) as it is a neutral molecule.

72.(b) Formaldehyde is most reactive towards addition reaction.

73.(d) $La(OH)_3 (Z = 57)$ is the most basic hydroxide due to lanthanide contration.

74.(c) kgm^{-3}



75. (c)

Because haloform is given by compound containing

$CH_3-\overset{O}{\overset{\|}{C}}-$ group or $R-\underset{OH}{\underset{|}{CH}}-CH_3$ only

76. (a) Work done is as: -

$W = -PdV$

$W = -P(V_2 - V_1)$

$V_1 = 10 dm^3 = 10^{-2}\ m^3$

$V_2 = 2\ m^3$

$P = 101.325 \times 10^3 pa$

$W = -101.325 \times 10^3 (1.99)$

$= -201.6\ kJ$

77. (a) ZnS because it shows Frenkel defect.

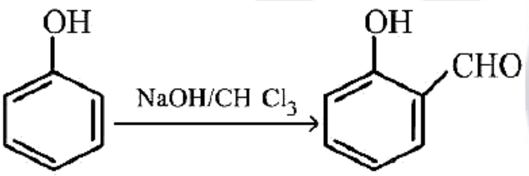
78. (c) The self- ionization of water molecules takes place to produce hydronium ion and hydroxide ion is known as auto-pyrolysis of water.

$H_2O(l) + H_2O(l) \rightleftharpoons H_3O^+(aq) + OH^-(aq)$.

In this reaction, one of the water molecules acts as an acid and the other acts as a base.

79. (b) Hall's process is used to purify the ore of aluminium.

80. (b)



This reaction is known as Reimer-Tiemann reaction

81. (a) Be-belongs to second period which exhibits anomalous properties.

82.(b)

$$\underset{\text{(Excess)}}{NH_3} + NaOCl \longrightarrow \underset{\text{(Hydrazine)}}{NH_2-NH_2} + NaCl + H_2O$$

83.(b) $1.2\ g\ mL^{-1}$

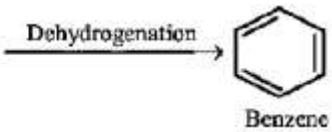
84. (c) Nomex is used to manufacture clothes for firefighters.

85.(b) Titanium is obtained in pure form by Van Arkel method.

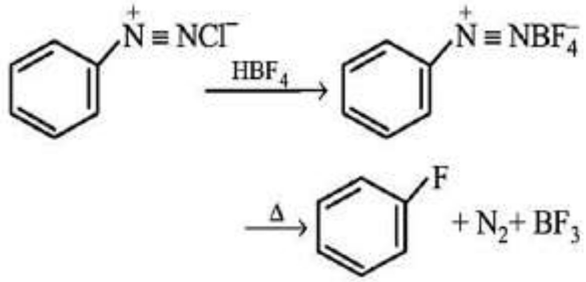
86. (d) Bromo pheniramine is an Antihistamine.

87.(d)

$CH_3-CH_2-CH_2-CH_2-CH_2-CH_3$
Hexane



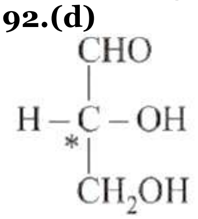
88. **(d)** Bismuth does not exhibit allotrophy.
89. **(b)** In Balz- Schiemann reaction aryl fluoride is prepared from diazonium salts and fluoroboric acid.



90. **(a)** $\Delta T_b = \frac{K_b \times W_2 \times 1000}{W_1 \times M_2}$

$M_2 = \frac{K_b \times W_2}{\Delta T_b \times W_1}$

91. (a) Nitrogen exists as N_2 (diatomic molecule), not as tetra atomic molecule.
92.(d)



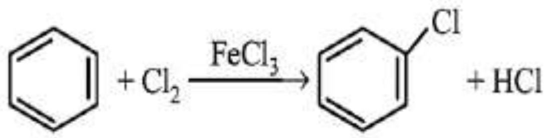
glyceraldehyde

asymmetric carbon atom

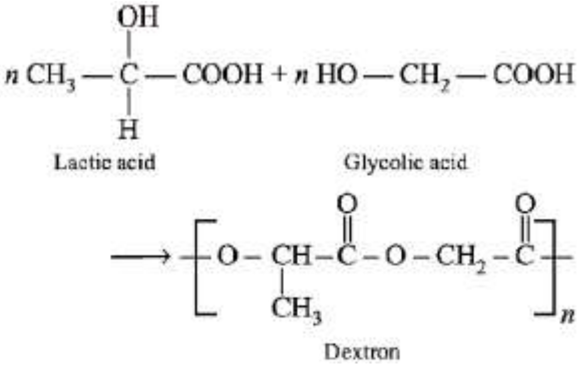
93.(c) Ti^{4+}, Cu^{1+} are colourless compound

Ti: $[Ar]4s^2 3d^2$ Cu: $[Ar]4s^1 3d^{10}$
$Ti^{4\ 4}$: $[Ar]4s^0 3d^0$ Cu^{11}: $[Ar]4s^0 3d^{10}$

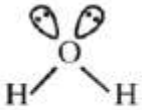
94.(c) It is an Electrophilic addition reaction.



95. **(d)** -OH, Atomic mass of oxygen is more than that of C& N
96.(b) Dextron



97.(a)



Distorted tetrahedral

98. **(a)** Fluorine

$R-H+F_2 \xrightarrow{\text{dark}} R-F+HF$

(alkane)

(highly exothermic)

99.(b) Combustion of ethanol is as: -

$$\underset{1}{C_2H_5OH} + \underset{3}{3O_2} \longrightarrow \underset{2}{2CO_2} + \underset{3}{3H_2O}$$

$0.138 \text{ kg} = 138/46 = 3 \text{ mole}$

138 g

$3C_2H_5OH(l) + 9O_2(g) \rightarrow 6CO_2(g) + 9H_2O(l)$

$\Delta n = 6-9 = -3$

$\text{Work} = -\Delta nRT$

$= -(-3) \times 8.314 \times 300 = 7482 \text{ J}$

100. **(c)** Acc to Arrhenius equation.

$$K = Ae^{-E_a/RT}$$

$$\ln k = \ln A - \frac{E_a}{RT}$$

$$\log k = \log A - \frac{E_a}{2.303R} \times \frac{1}{T}$$

$y = mx + C$

$y = \log k; x = \frac{1}{T};\ \text{Slope } = \frac{-E_a}{2.303R}$

$m = \frac{-E_a}{2.303R}$

Mathematics

101. **(c)** Here, $\int_0^k \frac{dx}{2+18x^2} = \frac{\pi}{24}$

$$\Rightarrow \frac{\pi}{24} = \frac{1}{18}\int_0^k \frac{dx}{\left(\frac{1}{9}\right)+x^2} = \frac{1}{18}\int_0^k \frac{dx}{\left(\frac{1}{3}\right)^2+x^2}$$

$$\Rightarrow \frac{\pi}{24} = \frac{1}{18}\times\frac{1}{\left(\frac{1}{3}\right)}\tan^{-1}\left[\frac{x}{\left(\frac{1}{3}\right)}\right]_0^k$$

$$\Rightarrow \frac{\pi}{24} = \frac{3}{18}\tan^{-1}[3x]_0^k = \frac{1}{6}[\tan^{-1}3k - \tan^{-1}0]$$

$$\Rightarrow \frac{\pi}{24} = \frac{1}{6}[\tan^{-1}3k - 0]$$

$$\therefore \frac{6\pi}{24} = \tan^{-1}3k \Rightarrow \tan\frac{\pi}{4} = 3k$$

$$\Rightarrow 1 = 3\,K \Rightarrow K = \frac{1}{3}$$

102. **(d)** $\because y^2 = -16x$

Comparing it with $y^2 = -4ax$

$\therefore\ a = 4$

Parametric equations are

$x = -at^2, y = 2at$

or $x = -4\left(\frac{1}{2}\right)^2, y = 2(4)\left(\frac{1}{2}\right)$

or $x = -1, y = 4$

103. **(d)** $\int \frac{dx}{\sin x\cos^2 x} = \int \frac{\sin^2 x+\cos^2 x}{\sin x\cos^2 x}dx = \int \frac{\sin x}{\cos^2 x}dx + \int \frac{dx}{\sin x} = \int \tan x\sec x + \int \operatorname{cosec} x dx$

$= \sec x + \log|\operatorname{cosec} x - \cot x| + c$

104. **(d)** Since, $\log_{10}\left(\frac{x^3-y^3}{x^3+y^3}\right) = 2$

$\therefore\ \log(x^3-y^3) - \log(x^3+y^3) = 2$

$\Rightarrow\ \log(x^3-y^3) = 2 + \log(x^3+y^3)$

Differentiating b/s w.r.t. x

$$\Rightarrow \frac{1}{x^3-y^3}\left[3x^2 - 3y^2\frac{dy}{dx}\right] = \frac{1}{x^3+y^3}\left[3x^2 + 3y^2\frac{dy}{dx}\right]$$

$$\Rightarrow \frac{3x^2}{x^3-y^3} - \frac{3y^2}{x^3-y^3}\frac{dy}{dx} = \frac{3x^2}{x^3+y^3} + \frac{3y^2}{x^3+y^3}\frac{dy}{dx}$$

$$\Rightarrow \frac{3x^2}{x^3-y^3}-\frac{3x^2}{x^3+y^3}=\left[\frac{3y^2}{x^3+y^3}+\frac{3y^2}{x^3-y^3}\right]\frac{dy}{dx}$$

$$\Rightarrow 3x^2\left[\frac{1}{x^3-y^3}-\frac{1}{x^3+y^3}\right]=3y^2\left[\frac{1}{x^3+y^3}+\frac{1}{x^3-y^3}\right]\frac{dy}{dx}$$

$$\Rightarrow 3x^2\left[\frac{2y^3}{(x^3-y^3)(x^3+y^3)}\right]=3y^2\Big)\left[\frac{2x^3}{(x^3+y^3x^3-y)(}\right]\frac{dy}{dx}$$

$$\Rightarrow \frac{y}{x}=\frac{dy}{dx}$$

105. **(a)** Here $f(x)=\frac{x^2-4}{x-2}=\frac{(x-2(x+2)}{(x-2)}=x+2$

$\therefore$ Range is R

106. **(c)** $f(x)=\begin{cases} x^2+\alpha & \text{if } x\geq 0 \\ 2\sqrt{x^2+1}+\beta & \text{if } x<0 \end{cases}$

is continuous at $x=0$

$$\therefore f(0)=\lim_{x\to 0^-} f(x)$$

$$\Rightarrow\ 0+\alpha=\lim_{x\to 0^-} 2\sqrt{x^2+1}+\beta$$

$$\Rightarrow \alpha=2+\beta\Rightarrow \alpha-\beta=2$$

$$f\left(\frac{1}{2}\right)=\left(\frac{1}{2}\right)^2+\alpha=2\Rightarrow \alpha=2-\frac{1}{4}=\frac{7}{4}$$

$$\therefore \beta=\frac{7}{4}-2=\frac{-1}{4}$$

$$\therefore \alpha^2+\beta^2=\left(\frac{7}{4}\right)^2+\left(\frac{-1}{4}\right)^2=\frac{49+1}{16}=\frac{50}{16}=\frac{25}{8}$$

107. **(b)** Since, $y=(\tan^{-1}x)^2$

$$\Rightarrow \frac{dy}{dx}=\frac{2\tan^{-1}x}{1+x^2}$$

$$\Rightarrow (1+x^2)\frac{dy}{dx}=2\tan^{-1}x=2\sqrt{y}$$

$$\Rightarrow (1+x^2)^2\left(\frac{dy}{dx}\right)^2=4y$$

Again differentiating b/s with respect to x, we get:

$$2(1+x^2)(2x)\left(\frac{dy}{dx}\right)^2+2\left(\frac{dy}{dx}\right)\frac{d^2y}{dx^2}(1+x^2)^2=4\frac{dy}{dx}$$

$$\Rightarrow 4x(1+x^2)\left(\frac{dy}{dx}\right)^2+2(1+x^2)^2\left(\frac{dy}{dx}\right)\frac{d^2y}{dx^2}=4\frac{dy}{dx}$$

$$\Rightarrow 4x(1+x^2)\frac{dy}{dx}+2(1+x^2)^2\frac{d^2y}{dx^2}=4$$

$$\Rightarrow (x^2+1)^2\frac{d^2y}{dx^2}+2x(1+x^2)\frac{dy}{dx}=2$$

108. **(c)** $5x+y-1=0$ coincides

$5x^2+xy-kx-2y+2=0$

$\therefore a=5, b=0, h=\frac{1}{2}, g=-\frac{k}{2}, f=-1, c=2$

If the above equation represents a pair of straight lines, then

$$\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix}=0 \Rightarrow \begin{vmatrix} 5 & \frac{1}{2} & -\frac{k}{2} \\ \frac{1}{2} & 0 & -1 \\ -\frac{k}{2} & -1 & 2 \end{vmatrix}=0$$

$$\therefore 5(0-1)-\frac{1}{2}\left(1-\frac{k}{2}\right)-\frac{k}{2}\left(\frac{-1}{2}-0\right)=0$$

$$\therefore -5-\frac{1}{2}+\frac{k}{4}+\frac{k}{4}=0 \Rightarrow \frac{k}{2}=\frac{11}{2} \Rightarrow k=11$$

109. **(b)** $(A^2-5A)A^{-1}=A^2A^{-1}-5AA^{-1}=A\cdot AA^{-1}-5I=A-5I$

$$=\begin{bmatrix} 1 & 2 & 3 \\ -1 & 1 & 2 \\ 1 & 2 & 4 \end{bmatrix}-\begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}=\begin{bmatrix} -4 & 2 & 3 \\ -1 & -4 & 2 \\ 1 & 2 & -1 \end{bmatrix}$$

110. **(c)** Let a, b, c be d. rs of desired line which is also perpendicular to the given lines.

$\therefore 2a-2b+c=0$
$a-2b+2c=0$

$$\Rightarrow \frac{a}{\begin{vmatrix} -2 & 1 \\ -2 & 2 \end{vmatrix}}=\frac{-b}{\begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix}}=\frac{c}{\begin{vmatrix} 2 & -2 \\ 1 & -2 \end{vmatrix}}$$

$$\Rightarrow \frac{a}{-4+2}=\frac{-b}{4-1}=\frac{c}{-4+2}$$

$$\Rightarrow \frac{a}{-2}=\frac{b}{-3}=\frac{c}{-2}$$

$\therefore$ direction ratios are $<-2,-3,-2>$
Hence equation of desired line is

$$\frac{x-3}{-2}=\frac{y+1}{-3}=\frac{z-2}{-2} \text{ or } \frac{x-3}{2}=\frac{y+1}{3}=\frac{z-2}{2}$$

111. **(c)** The word HULULULU contains 4U, 3 L & 1 H. Consider 3L together i.e. we have to arrange 6 units which contains 4 U.

Hence number of possible arrangements

$$=\frac{6!}{4!}=6\times 5=30$$

Number of ways of arranging all letters of given

word $=\frac{8!}{3!4!}=\frac{8\times7\times6\times5}{3\times2}=8\times7\times5$

Hence required probability $=\frac{30}{8\times7\times5}=\frac{6}{8\times7}=\frac{3}{28}$

112. **(b)** $9+99+999+\cdots\ldots\ldots$ upto 10 terms $=(10-1)+(100-1)+(1000-1)$ $+\cdots\ldots\ldots$ upto 10 terms $=(10+100+1000+\cdots\ldots\ldots\ldots$ upto 10 terms $)-(1+1+\cdots$ upto 10 times $)$

$$=\frac{10[(10)^{10}-1]}{10-1}-10$$

$$=\frac{10(10^{10}-1)}{9}-10=\frac{10^{11}-10-90}{9}$$

$$=\frac{10^{11}-100}{9}=\frac{100(10^{9}-1)}{9}$$

113. **(b)** We know that if $A+B+C=\pi$, then $\tan A+\tan B+\tan C=\tan A\tan B\tan C$

$$\Rightarrow \frac{1}{\tan B\tan C}+\frac{1}{\tan A\tan C}+\frac{1}{\tan A\tan B}=1 \Rightarrow \cot B\cot C+\cot A\cot C+\cot A\cot B=1$$

114. **(d)** $\int \frac{dx}{\sqrt{16-9x^2}}=\frac{1}{3}\int \frac{dx}{\sqrt{\left(\frac{16}{9}\right)-x^2}}=\frac{1}{3}\int \frac{dx}{\sqrt{\left(\frac{4}{3}\right)^2-x^2}}$

$$=\frac{1}{3}\sin^{-1}\left[\frac{x}{\left(\frac{4}{3}\right)}\right]+C \therefore A=\frac{1}{3} \text{ and } B=\frac{3}{4}$$

$$\therefore A+B=\frac{1}{3}+\frac{3}{4}=\frac{13}{12}$$

115. **(a)** $\int e^x\left[\frac{2+\sin 2x}{1+\cos 2x}\right]dx=\int e^x\left[\frac{2(1+\sin x\cos x)}{2\cos^2 x}\right]=\int e^x[\sec^2 x+\tan x]dx=e^x\tan x+c$

116. **(d)** A coin is tossed 3 times. $\therefore$ possibilities are

$HHH \rightarrow X=3-0=3$

$TTT \rightarrow X=3-0=3$

$HHT \rightarrow X=2-1=1$

$HTH \rightarrow X=2-1=1$

$THH \rightarrow X=2-1=1$

$HTT \rightarrow X=2-1=1$

$TTH \rightarrow X=2-1=1$

$THT \rightarrow X=2-1=1$

$$\therefore P(X=1)=\frac{6}{8}=\frac{3}{4}$$

117. **(d)** $2\sin\left(\theta+\frac{\pi}{3}\right)=\cos\left(\theta-\frac{\pi}{6}\right)$

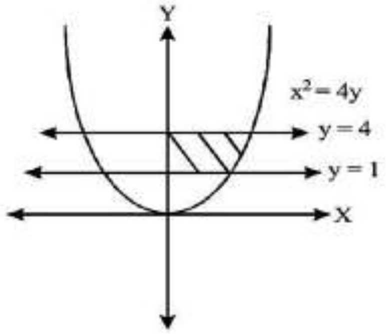
$$\Rightarrow 2\left[\sin\theta\cos\frac{\pi}{3}+\cos\theta\sin\frac{\pi}{3}\right]=\cos\theta\cos\frac{\pi}{6}+\sin\theta\sin\frac{\pi}{6}$$

$$\Rightarrow 2\left[\frac{\sin\theta}{2}+\cos\theta\left(\frac{\sqrt{3}}{2}\right)\right]=\cos\theta\left(\frac{\sqrt{3}}{2}\right)+\sin\theta\left(\frac{1}{2}\right)$$

$\Rightarrow \sin\theta + \sqrt{3}\cos\theta = \frac{\sqrt{3}}{2}\cos\theta + \frac{1}{2}\sin\theta$

$\Rightarrow \frac{1}{2}\sin\theta = \frac{-\sqrt{3}}{2}\cos\theta \Rightarrow \tan\theta = -\sqrt{3}$

118. (b)



We have $x^2 = 4y \Rightarrow x = 2\sqrt{y}$

$\therefore$ area between $x = 2\sqrt{y}, y = 1 \& y = 4$

$$= \int_1^4 2\sqrt{y}dy = 2\left[\frac{y^{3/2}}{3/2}\right]_1^4$$

$$= 2\left(\frac{2}{3}\right)[y\sqrt{y}]_1^4 = \frac{4}{3}(8-1) = \frac{28}{3}$$

119. (d) $f(x) = \frac{e^{x^2} - \cos x}{x^2}$

Since $f(x)$ is continuous at $x = 0$

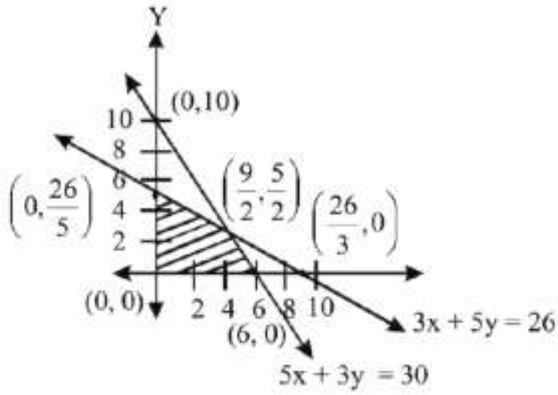
$$\Rightarrow f(0) = \lim_{x\to 0} f(x)$$

$$\Rightarrow f(0) = \lim_{x\to 0} \frac{(e^{x^2} - 1) - (\cos x - 1)}{x^2}$$

$$\Rightarrow f(0) = \lim_{x\to 0} \frac{e^{x^2} - 1}{x^2} - \lim_{x\to 0} \frac{-2\sin^2\frac{x}{2}}{x^2}$$

$$\Rightarrow f(0) = 1 + 2\lim_{x\to 0}\left[\frac{\sin\frac{x}{2}}{\frac{x}{2}}\right]^2 \times \frac{1}{4} = 1 + \frac{2}{4} = \frac{3}{2}$$

120. (a)



Corner Points	Value of $z = \mathbf{2x+y}$
(0,0)	$z = 0$
(6,0)	$z = 2(6) + 0 = 12$
$\left(\frac{9}{2},\frac{5}{2}\right)$	$z = 2\left(\frac{9}{2}\right) + \frac{5}{2} = 11.5$
$\left(0,\frac{26}{5}\right)$	$z = 2(0) + \frac{26}{5} = 5.2$

121. **(c)** $|\vec{a}| = 1, |\vec{b}| = 2, |\vec{c}| = 3$

$\vec{a}\cdot\vec{b} = \vec{b}\cdot\vec{c} = \vec{a}\cdot\vec{c} = 0$

Now, $[\vec{a}+\vec{b}+\vec{c}\vec{b}-\vec{a}\vec{c}]$

$= (\vec{a}+\vec{b}+\vec{c})\cdot[(\vec{b}-\vec{a})\times\vec{c}] = (\vec{a}+\vec{b}+\vec{c})\cdot(\vec{b}\times\vec{c}-\vec{a}\times\vec{c})$

$= [\vec{a}\vec{b}\vec{c}] - [\vec{b}\vec{a}\vec{c}] = 2[\vec{a}\vec{b}\vec{c}] = 2\vec{a}\cdot(\vec{b}\times\vec{c})$

$= 2|\vec{a}|\cdot|\vec{b}\times\vec{c}|\cos 0^\circ$ ($\because \vec{a}$ & $(\vec{b}\times\vec{c})$ are parallel)

$= 2|\vec{a}|\cdot|\vec{b}\times\vec{c}| = 2|\vec{a}\,\|\,\vec{b}\,\|\,\vec{c}|\sin 90^\circ$

$= 2(1)(2)(3) = 12$

122. **(d)** $\overrightarrow{PQ} = (-1, y-5, 4-x)$

$\overrightarrow{QR} = (2, 8-y, -4)$

$\because$ P, Q, R are collinear

$\therefore \frac{-1}{2} = \frac{y-5}{8-y} = \frac{4-x}{-4}$

$\Rightarrow -8+y = 2y-10$ & $4 = 8-2x$

$\Rightarrow y = 2$ & $x = 2$

$\therefore x+y = 4$

123. **(a)** $\because ax^2 + 2hxy + 2by^2 = 0$

Let the slope of one line is m.

In slope of other line = 2 m.

We know that

$m + 2m = \frac{-2h}{b}$ & $m \times 2m = \frac{a}{b}$.

$\Rightarrow 3m = \frac{-2h}{b}$ & $2m^2 = \frac{a}{b}$.

$\Rightarrow m = \frac{-2h}{b} \Rightarrow m^2 = \frac{4h^2}{9b^2}$.

$\therefore 2\left(\frac{4h^2}{9b^2}\right) = \frac{a}{b}. \Rightarrow 8h^2 = 9ab$

124. **(a)** $y - 1 = (-1)(x + 3) \Rightarrow x + y + z = 0$

125. **(b)** p: Hema gets the admission in good college

q: Hema gets 95% marks

$\therefore$ given statement can be written as:

$p \rightarrow q$

$\therefore$ its negation is $p \wedge \sim q$

126. **(a)** $\cos 1^\circ \cos 2^\circ \cos 3^\circ \ldots \ldots \cos 179^\circ = 0$

Since $\cos 90^\circ = 0$

$\therefore$ required product $= 0$

127. **(c)** Planes $x - cy - bz = 0$

$cx - y + az = 0$

$bx + ay - z = 0$

Since the given planes pass through a straight line.

$\therefore$ planes are concurrent

$$\begin{vmatrix} 1 & -c & -b \\ c & -1 & a \\ b & a & -1 \end{vmatrix} = 0$$

$1(1 - a^2) + c(-c - ab) - b(ac + b) = 0$

$1 - a^2 - c^2 - abc - abc - b^2 = 0$

$a^2 + b^2 + c^2 + 2abc = 1$

$a^2 + b^2 + c^2 = 1 - 2abc$

128. **(c)** $x^2 - y^2 + x + 3y - 2 = 0$

Comparing the above equation with

$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$

we get

$a = 1, h = 0, b = -1, g = \frac{1}{2}, f = \frac{3}{2}, c = -2$

$\therefore$ req. point of intersection is:

$$\left(\frac{hf - bg}{ab - h^2}, \frac{gh - af}{ab - h^2}\right)$$

$$\equiv \left(\frac{0 + \frac{1}{2}}{-1}, \frac{0 - \frac{3}{2}}{-1}\right) \equiv \left(-\frac{1}{2}, \frac{3}{2}\right)$$

129. **(b)** When we get 1, number of positive divisors are 1

When we get 2, number of positive divisors are 2

When we get 3, number of positive divisors are 2

When we get 4, number of positive divisors are 3

When we get 5, number of positive divisors are 2

When we get 6, number of positive divisors are 4

Hence range of random variable X is $\{1,2,3,4\}$

130. **(b)** P (getting perfect square in atleast one throw) $= 1 -$ P (not getting perfect square in any throw)

$$= 1 - \left(\frac{4}{6} \times \frac{4}{6} \times \frac{4}{6} \times \frac{4}{6}\right)$$

$$= 1 - \left(\frac{2}{3}\right)^4 = 1 - \frac{16}{81} = \frac{65}{81}$$

131. **(b)** $\int_0^{\pi/4} x\sec^2 x dx$

$$= \left[x\int \sec^2 x dx\right]_0^{\pi/4} - \int_0^{\pi/4} \left[\frac{d}{dx} x \int \sec^2 x dx\right] dx$$

$$= [x \cdot \tan x]_0^{\pi/4} - \int_0^{\pi/4} [\tan x] dx$$

$$= [x \cdot \tan x]_0^{\pi/4} - [\log|\sec x|]_0^{\pi/4}$$

$$= \left[\frac{\pi}{4} - 0\right] - \left[\log\left|\sec\frac{\pi}{4}\right| - \log|\sec 0|\right]$$

$$= \frac{\pi}{4} - [\log\sqrt{2} - \log 1]$$

$$= \frac{\pi}{4} - \log\sqrt{2}$$

132. **(c)** $\because$ a, b, c are in A.P.

$\therefore 2b = a + c$ ……. (i)

Now, $\cos^2\left(\frac{C}{2}\right) + \cos^2\left(\frac{A}{2}\right)$

$= a\frac{[1+\cos C]}{2} + c\frac{[1+\cos A]}{2}$

$= \frac{a + c + a\cos C + C\cos A}{2}$

$= \frac{a+c+b}{2}$ $[\because b = a\cos C + c\cos A]$

$= \frac{2b+b}{2} = \frac{3b}{2}$ [Using equation (i)]

133. **(a)** $x = e^{\theta}(\sin\theta - \cos\theta) \& y = e^{\theta}(\sin\theta + \cos\theta)$

$\therefore \frac{dx}{d\theta} = e^{\theta}(\cos\theta + \sin\theta) + (\sin\theta - \cos\theta)e^{\theta}$

$= e^{\theta}[2\sin\theta]$

$\& \frac{dy}{d\theta} = e^{\theta}(\cos\theta - \sin\theta) + (\sin\theta + \cos\theta)e^{\theta} = e^{\theta}[2\cos\theta]$

$\therefore \frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{e^{\theta}[2\cos\theta]}{e^{\theta}[2\sin\theta]}$

$\Rightarrow \frac{dy}{dx} = \cot\theta$

$\Rightarrow \left.\frac{dy}{dx}\right|_{\theta=\frac{\pi}{4}} = \cot\frac{\pi}{4} = 1$

134. **(b)** $\sin x + \sin 3x + \sin 5x = 0$

$\sin 5x + \sin x + \sin 3x = 0$

$2\sin 3x \cdot \cos 2x + \sin 3x = 0$

$\therefore \sin 3x[2\cos 2x + 1] = 0$

$\therefore \sin 3x = 0$ or $2\cos 2x + 1 = 0$

$\Rightarrow \sin 3x = \sin n\pi$

$\Rightarrow 3x = n\pi$

$\Rightarrow x = \frac{n\pi}{3}$

also $2\cos 2x = -1$

$\Rightarrow \cos 2x = -1/2$

$\Rightarrow \cos 2x = -\cos\pi/3$

$\cos 2x = \cos(\pi - \pi/3)$

$\Rightarrow \cos 2x = \cos\frac{2\pi}{3}$

$\Rightarrow 2x = 2n\pi \pm \frac{2\pi}{3}$

$x = \frac{n\pi}{3}, x = n\pi \pm \frac{\pi}{3}$

$x \in \left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$ gives

$x = \pi, \frac{2\pi}{3} \& \frac{4\pi}{3}$

135. **(c)** $\tan^{-1} 2x + \tan^{-1} 3x = \frac{\pi}{4}$

$\Rightarrow \tan^{-1}\left(\frac{2x + 3x}{1 - 6x^2}\right) = \frac{\pi}{4}$

$\Rightarrow \frac{5x}{1 - 6x^2} = \tan\frac{\pi}{4}$

$\Rightarrow \frac{5x}{1 - 6x^2} = 1$

$\Rightarrow 5x = 1 - 6x^2$

$\Rightarrow 6x^2 + 5x - 1 = 0 \Rightarrow 6x^2 + 6x - x - 1 = 0$

$\Rightarrow 6x(x + 1) - 1(x + 1) = 0$

$\Rightarrow (x + 1)(6x - 1) = 0$

$\therefore x = -1, x = \frac{1}{6}$

When $x = \frac{1}{6}$, given equation is satisfied.

When $x = -1$, we get sum of two negative angles, hence discarded.

$\therefore x = \frac{1}{6}$

136. **(c)** Here $A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 5 \\ 2 & 4 & 7 \end{bmatrix}$

we know that,

$a_{31}A_{31} + a_{32}A_{32} + a_{33}A_{33} = |A|$

$= +1(7 - 20) - 2(7 - 10) + 3(4 - 2)$

$= -13 + 6 + 6 = -1$

137. **(b)** p = The weather is fine

q = My friends will come and we go for a picnic.

$\therefore$ given statement can be written as : $p \rightarrow q$

$\therefore$ its contrapositive is: $\sim q \rightarrow \sim p$

i.e. If my friends do not come or we do not go for picnic, then weather will not be fine.

138. **(d)** $f(x) = \frac{x}{x^2+1}$

$$\Rightarrow f'(x) = \frac{(x^2+1)(1)-(x)(2x)}{(x^2+1)^2}$$

$$\Rightarrow f'(x) = \frac{1-x^2}{(x^2+1)^2}$$

$\because$ f(x) is increasing function.

$\therefore f'(x) > 0$

$$\Rightarrow \frac{1-x^2}{(x^2+1)^2} > 0$$

Here $x^2 + 1 \neq 0$, $\quad x^2 \neq -1$

$1 - x^2 > 0$, $\quad x^2 < 1$

$x \in (-1,1)$

139. **(a)** $X = 4^n - 3n - 1$ $n \in N$

$\&Y = 9(n-1)$ $n \in N$

$\Rightarrow X = \{0,9,54,243, \dots \dots\}$

$\&Y = \{0,9,18,27,36,45,54, \dots\dots\}$

$\therefore X \cap Y = X$

140. **(b)**

$$\begin{aligned} & p \wedge (\sim p \wedge q) \\ & = (p \wedge \sim p) \wedge (p \wedge q) \\ & = F \wedge (p \wedge q) \\ & = F \end{aligned}$$

A contradiction.

141. **(a)** Line $y = 4x - 5 \rightarrow$ slope of line $m = 4$ .... (i)

curve $y^2 = ax^3 + b$

$\therefore$ differentiating w.r.t. ' x '

$$2y\frac{dy}{dx} = 3ax^2$$

$\frac{dy}{dx} = \frac{3ax^2}{2y}$ = slope of tangent

$$\therefore \left.\frac{dy}{dx}\right|_{(2,3)} = \frac{3a \times 4}{2 \times 3} = 2a$$

$\therefore$ from (i) and (ii), we get

$4 = 2a \Rightarrow a = 2$

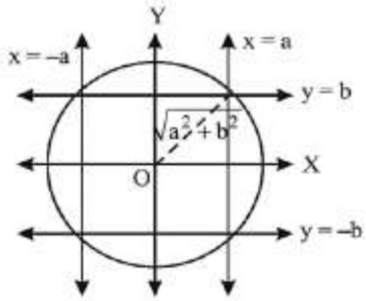
Since, (2,3) is a point on the curve: $y^2 = ax^3 + b$.

$\therefore (3)^2 = 2(2)^3 + b$

$\Rightarrow b = -7$

$\therefore 7a + 2b = 7 \times 2 + 2(-7) = 0$

142. (b)



Centre $= (0,0)$& radius $= r = \sqrt{a^2 + b^2}$

$\therefore$ equation of circle

$x^2 + y^2 = a^2 + b^2$

143. (a) $f(x) = x\log x$

$\therefore f'(x) = 1 + \log x$

For minimum value $f'(x) = 0 \Rightarrow 1 + \log x = 0$

$\Rightarrow \log x = -1 \Rightarrow x = \frac{1}{e}$

min value $= f\left(\frac{1}{e}\right) = \frac{1}{e} \cdot \log\left(\frac{1}{e}\right)$

$= \frac{1}{e}\Big(\log 1 - \log e \neq \frac{1}{e}(0 - 1) = -\frac{1}{e}$

144. (d) $n = 10, p = 0.4, q = 0.6$

$\because E(x) = np = 4$

$\& V(x) = npq = 10(0.4)(0.6) = 2.4$

Now, $V(x) = E(x^2) - [E(x)]^2$

$\Rightarrow 2.4 = E(x^2) - (4)^2$

$\Rightarrow E(x^2) = 18.4$

145. (a) $\frac{dx}{dy} = \cos(x + y)$

$\Rightarrow \frac{dy}{dx} = \frac{1}{\cos(x + y)}$

Put $x + y = V$

Differentiating w. r. t. ' x '

$$1+\frac{dy}{dx}=\frac{dV}{dx}$$

$$\Rightarrow \frac{dy}{dx}=\frac{dV}{dx}-1$$

$$\Rightarrow \frac{dV}{dx}-1=\frac{1}{\cos V}$$

$$\Rightarrow \frac{dV}{dx}=\frac{1}{\cos V}+1$$

$$\Rightarrow \frac{dV}{dx}=\frac{1+\cos V}{\cos V}$$

$$\Rightarrow \frac{\cos V}{(1+\cos V)}dV=dx$$

Integrate both sides, we get :

$$\int \frac{(1+\cos V)-1}{1+\cos V}dV=\int dx \Rightarrow \int\left[1-\frac{1}{2\cos^2\frac{V}{2}}\right]dV=\int dx$$

$$\Rightarrow V-\frac{1}{2}\frac{\tan\frac{V}{2}}{\frac{1}{2}}=x+C_1$$

$$\Rightarrow x+y-\tan\left(\frac{x+y}{2}\right)=x+C_1$$

$$\Rightarrow \tan\left(\frac{x+y}{2}\right)=y+C\ [\because C=-C_1]$$

146. **(d)** We have $\vec{r}\cdot(p\hat{i}-\hat{j}+2\hat{k})+3=0$ and

$\vec{r}\cdot(2\hat{i}-p\hat{j}-\hat{k})-5=0$

Since angle between them is $\frac{\pi}{3}$.

$$\therefore \cos\theta=\left|\frac{\vec{n}_1\cdot\vec{n}_2}{|\vec{n}_1|\cdot|\vec{n}_2|}\right|$$

$$\Rightarrow \cos\frac{\pi}{3}=\frac{(p\hat{i}-\hat{j}+2\hat{k})\cdot(2\hat{i}-p\hat{j}-\hat{k})}{\sqrt{(p)^2+(-1)^2+(2)^2}\sqrt{(2)^2+(-p)^2+(-1)^2}}$$

$$\Rightarrow \frac{1}{2}=\frac{2p+p-2}{\left(\sqrt{p^2+5}\right)\left(\sqrt{p^2+5}\right)} \Rightarrow \frac{1}{2}=\frac{3p-2}{(p^2+5)}$$

$$\Rightarrow p^2+5=6p-4 \Rightarrow p^2-6p+9=0$$

$$\Rightarrow (p-3)^2=0 \Rightarrow p=3$$

147. **(d)** Equation of parabola whose axis is parallel to X axis and latus rectum is 4 a

$(y-k)^2 = 4a(x-h)$

(where h&k are arbitrary constants)

differentiating b/s, we get

$2(y-k)y' = 4ax$

again differentiating b/s, we get.

$(y-k)y'' + y'^2 = 2a$

$\Rightarrow \frac{2ax}{y'} \cdot y'' + y'^2 = 2a$ from (i) $\Rightarrow 2axy'' + y'^3 = 2ay'$

$\Rightarrow$ order 2

148. **(a)** Points on the given lines are respectively $(1,-1,1)$ and $(3,k,0)$ and their direction ratios are respectively 2,3,4 and 1,2,1

Since lines intersect, then lines are coplanar

$$\therefore \begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$$

$$\therefore \begin{vmatrix} 2 & k+1 & -1 \\ 2 & 3 & 4 \\ 1 & 2 & 1 \end{vmatrix} = 0$$

$\therefore 2(-5) - (k+1)(-2) - 1(1) = 0$

$-11 + 2k + 2 = 0$

$k = \frac{9}{2}$

149. **(c)** Let $\cos\alpha, \cos\beta \& \cos\gamma$ are the direction cosines of the line. We know that

$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$\Rightarrow (\cos 120°)^2 + \cos^2\beta + (\cos 60°)^2 = 1$

$\Rightarrow \left(-\frac{1}{2}\right)^2 + \cos^2\beta + \left(\frac{1}{2}\right)^2 = 1$

$\Rightarrow \cos^2\beta = 1 - \frac{1}{2} = \frac{1}{2} \Rightarrow \cos\beta = \pm\frac{1}{\sqrt{2}}$

$\Rightarrow \beta = 135°$

150. **(c)** We have $L \equiv (2,-1)$ and $M \equiv (1,2)$ and is divided by N in ratio 2: 1 externally.

$\therefore N \equiv \frac{(2)(1)-(2)(1)}{2-1}, \frac{(2)(2)-(1)(-1)}{2-1}$

i.e. $N \equiv \left(0, \frac{5}{1}\right)$ i.e. $N \equiv (0,5)$

$\therefore$ position vector of point N is $5\vec{b}$