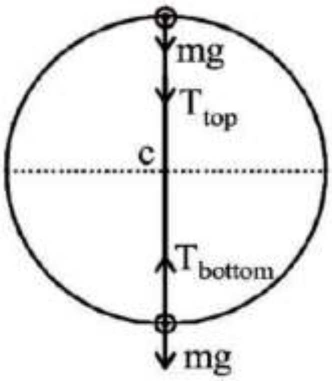
Physics

1. (b) $T_{top} = \frac{mv^2}{r} - mg$ .... (i)



$T_{bottom} = \frac{mv^2}{r} + mg$ ..... (ii)

Solving (i) and (ii) we get:

$$\frac{T_{top}}{T_{bottom}} = \frac{v^2 - rg}{v^2 + rg} = \frac{79}{81}$$

2. (a) $\varepsilon = -M\frac{di}{dt} = -M\frac{d}{dt}(5\sin 200\pi t)$

$= -M \times 5 \times 200\pi\cos(200\pi t)$

$|\varepsilon|_{max} = 10\pi$ volt

3. (c) Given, radius of earth, $R_e = 6371$ km and radius of orbit around the sun,

$R_0 = 149 \times 10^6$ km

Now, the ratio of the diameters of the earth and orbit around the sun can be expressed as

$$\frac{D_0}{D_e} = \frac{2R_0}{2R_e} = \frac{2 \times 14.9 \times 10^6}{2 \times 6371} = 2.33 \times 10^4$$

From above ratio, it can be concluded that the order of magnitude of the diameter of the orbit is greater than that of earth by multiple of 10^4.

4. (d) $v = n\lambda$ [for open pipe in fundamental

mode $\lambda = 2l$, where l is the length of the pipe]

So, $v = n_1(2l_1) = n_2(2l_2) = n_3 2(l_1 + l_2)$

$\Rightarrow \frac{1}{n_3} = \frac{1}{n_1} + \frac{1}{n_2} \Rightarrow n_3 = \frac{n_1 n_2}{n_1 + n_2}$

5. (a)

$C_p - C_v = R$ ... . (i)

$\frac{C_p}{C_v} = \gamma$ ..... (ii)

Solving (i) and (ii)
We get: $C_p = \frac{R\gamma}{\gamma-1}$

6. (a) $z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega c}\right)^2}$

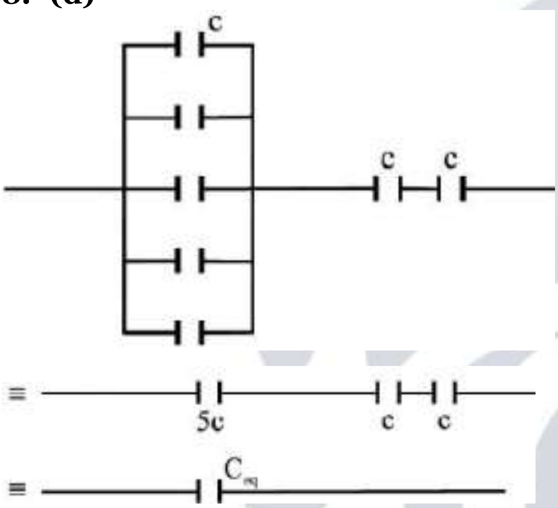
Putting $R = 300\Omega, L = 0.9H, C = 2 \times 10^{-6}$ F and $\omega = 1000 rad/s$

We get : $z = 500\Omega$

7. (b) in a simple harmonic oscillator, the energy oscillates between kinetic energy of the mass $K = \frac{1}{2}mv^2$ and potential energy $U = \frac{1}{2}kx^2$ stored in the spring. In the SHM of the mass and spring system, there are no dissipative forces, so the total energy is the sum of the potential energy and kinetic energy

The Kinetic energy changes depending on the velocity also potential energy changes depending on position but the sum of both remain constant.

8. (d)



$= C_{eq} = \frac{1}{\left(\frac{1}{5c} + \frac{1}{c} + \frac{1}{c}\right)} = \frac{5c}{1+5+5}$

$= \frac{5c}{11} = \frac{5 \times 2\mu F}{11} = \frac{10}{11}\mu F$

9. (c) $q \vee B = \frac{mv^2}{r}$ ..... (i)

$mvr = \frac{nh}{2\pi}$ .... (ii)

$K.E. = \frac{1}{2}mv^2$ .... (iii)

Solving (i), (ii) and (iii) we get:

$K.E. = \frac{1}{2}rqvB$

$= \frac{1}{2} \times qB \times \frac{nh}{2\pi m} = \frac{nhqB}{4\pi m}$

10. (c)

11. (b) $Y \alpha \frac{1}{\Delta l}$

12. (c) work done by gravitational force do not depend upon the path, it only depend upon the initial & final position of the object. Initial & final position of the object of all 3 path is same So, work done is same.

13. (c) $i = \frac{3v-1v}{100\Omega} = \frac{2}{100}$ A = 20 mA

14. (c) Particle velocity

$$v_l = \frac{dY}{dt} = a2\pi b\cos 2\pi(bt - cx)$$

So, $V_1 max = a \times 2\pi \times b = 2\pi ab$
wave velocity $v_2 = \frac{\omega}{K} = \frac{2\pi b}{2\pi c} = \frac{b}{c}$
Now, $\frac{v_1 max}{v_2} = \frac{2\pi ab}{b/c} \Rightarrow 2 = 2\pi ac$

$$\Rightarrow c = \frac{1}{\pi a}$$

15. (d) For fundamental mode let time period be T, then

$$\text{So, } t = \frac{T}{4}$$
$$\Rightarrow T = 4t$$
$$\Rightarrow \frac{1}{T} = (4t)^{-1}$$
$$\Rightarrow v = (4t)^{-1}$$

16. (c) R_{big} single drop $= 2^{\frac{1}{3}} r_{small\ drop}$ $U = T \times A$

So, $\frac{U_{initially}}{U_{finally}} = \frac{2\times T\times 4\pi r^2}{T\times 4\pi R^2}$

$$= \frac{2r^2}{(2^{1/3}r)^2} = 2^{\left(1-\frac{2}{3}\right)} = 2^{1/3}:1$$

17. (d) $I = \frac{2}{5}mr^2$

$\Rightarrow\ I \propto r^2\ \Rightarrow\ I_1:I_2$

$= r_1^2:(2r_1)^2 = 1:4$ [as $r_2 = 2r_1$]

18. (b) $R = \frac{V}{I}$ and for conductor R increases with increase in temperature.
19. (c) $r^2 = r_1^2 + r_2^2$

$r = \sqrt{3^2 + 4^2}$

$= 5$ cm

20. **(c)** Parallel currents in the same direction attract each other.
21. **(b)** The ionosphere is the ionized part of the Earth's upper atmosphere. Within the ionosphere there are several different ionospheric regions which affect the propagation of radio signals in different ways - the D layer, E layer, F layer which splits into F_1 and F_2 layers. During the day, the D and E layers become much more heavily ionized, which develops an additional weaker region of ionization known as the F_1 layer. The F_2 layer persists by day and night and is the main region responsible for the refraction and reflection of radio waves. At night the E layer weakens because the primary source of ionization is no longer present. After sunset an increase in the height of the E layer maximum increases the range to which radio waves can travel by reflection from the layer.
22. (c) as $y = 0.05\sin(x + 15t)$

so, $v = \frac{\omega}{K} = \frac{15}{1}$

Now $v = \sqrt{\frac{F}{\mu}}$

$\Rightarrow F = v^2\mu = (15)^2 \times (10^{-3}) = 0.225$ N

[Here F = tension force and $\mu = 10^{-3}$ kg/m]

23.(a) $\frac{mv^2}{r} = \frac{GmM}{r^2}$

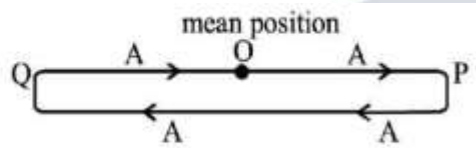
$\Rightarrow v^2 = \frac{GM}{r} \Rightarrow v = \sqrt{\frac{GM}{r}}$

Now, Kinetic energy $= \frac{1}{2}mv^2$

$\frac{1}{2}m\frac{GM}{r} = \frac{1}{2}\frac{mGM}{(3R+R)} = \frac{1}{8}mgR$

(As $g = \frac{GM}{R^2}$]

24.(d)



in one time period total distance travelled = A + A + A + A = 4 A

[as in each quarter starting from mean position it travelles A distance as shown]

25.(d) $i(Rg + R) = 50$

$iRg + iR = 50$

$\Rightarrow R = \frac{50}{10 \times 10^{-3}} - 100 = 4900\Omega$

26. (c)

27.(a) Given circuit forms wheat stone bridge:

so, $i_1(5+1)\Omega = i_2(50+10)\Omega$

$\Rightarrow i_1 = 10i_2$ ….. (i)

also $i_1 + i_2 = 1.1$ A …. (ii)

Solving (i) and (ii) we get i_1 = IA which passes through 1Ω resistor

28.(a) $\frac{1}{\lambda} = K\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$

$\Rightarrow \frac{1}{\lambda_1} = K\left(\frac{1}{2^2} - \frac{1}{3^2}\right)$

[for first line of Balmer series

$n_1 = 2$ and $n_2 = 3]$
also $\frac{1}{\lambda_2} = K\left(\frac{1}{2^2} - \frac{1}{4^2}\right)$

[for second line in Balmer series

$n_1 = 2$ and $n_2 = 4]$

so, $\frac{\lambda_2}{\lambda_1} = \frac{20}{27} \Rightarrow \lambda_2 = \frac{20}{27}\lambda_1 = \frac{20}{27}\lambda$

29.(d) Work done in streatching a wire

$$= \frac{1}{2} \times \text{ stress } \times \text{ strain } \times \text{ volume}$$
$$= \frac{1}{2} \times Y \times (\Delta l)^2 \times \frac{\pi r^2}{l}$$
$$\text{so, } \frac{w_2}{w_1} = \left(\frac{r_2}{r_1}\right)^2 \times \left(\frac{l_1}{l_2}\right)$$
$$\Rightarrow w_2 = 8w_1 = 16 \text{ J}$$

30. (d)
31. (d) Resolving power $\propto$, d Resolving power of a telescope is proportional to the diameter.
32. (d)
33. (c)
34.(a) $I = \frac{3MR^2}{2} = \frac{3}{8\pi^2}.Q.\ L^{\ 3}$
35.(a) Torque = Force × distance

Moment of force = Force × distance

So, Moment of force and torque have same dimension.

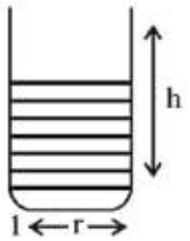
36.(b) $\beta_{dc} = \frac{I_C}{I_B}$

$$= \frac{\text{Collector current}}{\text{Base current}}$$

37.(d) $\Delta T = \frac{1}{2}\ T\alpha\theta$

$$= \frac{1}{2} \times 9 \times 10^{-7} \times 10 \times 0.5$$

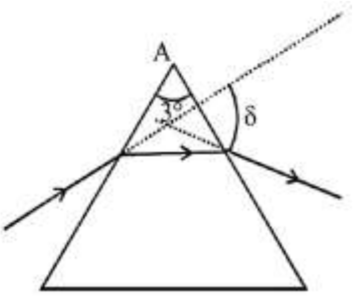
38.(d)



Height in the capillary

$$h = \frac{2\ s\cos\theta}{\rho g r} \quad \therefore\ h \propto \frac{1}{r}$$

39. (b)



Angular deviation $\delta = (\mu - 1)A$

$\therefore 1 = (\mu - 1)3$

$\therefore \mu = 1 + \frac{1}{3}$

$= 1.33$

40. **(b)** Using conservation of energy

Total mechanical energy at surface = total mechanical energy at height h Using this, we have

$$\begin{aligned} \frac{1}{R} - \frac{1}{r} &= \frac{1}{4R} \\ \therefore \quad \frac{1}{r} &= \frac{1}{R} - \frac{1}{4R} \\ &= \frac{1}{R} \cdot \frac{3}{4} \\ \therefore \quad r &= 4/3R \\ \therefore \quad h &= R/3 \end{aligned}$$

41. **(d)** In a biprism experiment, wavelength of the light is given as

$\lambda = \frac{d\beta}{D}$ ... (i)

where, β is the fringe width, dis the distance between the two sources and D is the distance between the source and eyepeice.
Given, $D = 1.2$ m.

$$d = 0.84 \text{ mm} = 0.84 \times 10^{-3} \text{ m and}$$
$$\beta = \frac{2.799 \times 10^{-2}}{30} = 9.33 \times 10^{-4}$$

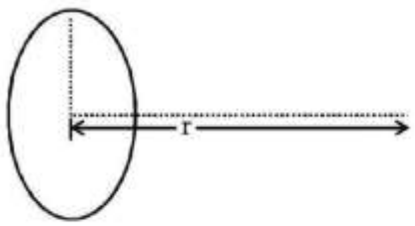
Substituting these values in Eq. (i), we get

$$\lambda = \frac{0.84 \times 10^{-3} \times 9.33 \times 10^{-4}}{1.2}$$
$$= 6.531 \times 10^{-7} \text{ m} = 6531 \text{ A}$$

42.(b) $K_{max} = hv - \varphi$

$\therefore K_2 = 2\ hv - Wn = k + hv$

43.(c) From Doppler's effect. We know that frequency increases.
44.(b)



$B_{axis} = \frac{\mu_0}{4\pi}\frac{2}{r^3}NIA$

45.(c) Terminal velocity $v \propto \rho_s - \rho_l$

$\therefore \frac{v_2}{v_1} = \frac{e_2 - \sigma}{e_1 - \sigma}$

$\therefore v_2 = \left[\frac{e_2 - \sigma}{e_1 - \sigma}\right]v$

46.(c) $\alpha = \frac{Q_2}{Q_1 - Q_2}$

$\therefore \frac{1}{\alpha} = \frac{Q_1 - Q_2}{Q_2}$

$= \frac{Q_1}{Q_2} - 1$

$\therefore \frac{Q_1}{Q_2} = 1 + \frac{1}{\alpha}$

$= \frac{\alpha + 1}{\alpha}$

$\therefore Q_2 = \frac{\alpha}{\alpha + 1} \cdot Q_1$

47.(a) $F = m\omega^2 r$

$= mr\frac{4\pi^2}{T^2}$

$\therefore \sqrt{F} = \sqrt{mr} \cdot \frac{2\pi}{T}$

48. (b) Gyromagnetic Ratio of a particle or system is the ratio of its magnetic moment to its angular momentum

Its S. I unit is $kg^{-1} \times sec \times$ Ampere

Hence it has Dimensions $[L^0M^{-1}T^1I]$

49.(b) $K_{max} = \frac{hc}{\lambda} - \varphi$

$\therefore Vs = \frac{1}{2}\frac{mv^2}{e} = \frac{v^2}{2\frac{e}{m}}$

50.(b) Focal length of new lens = 2 × focal length of convex lens.

Chemistry

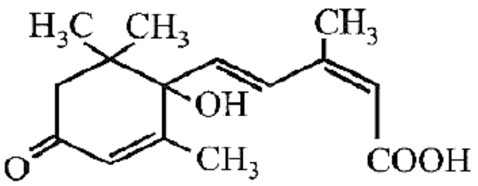
51. (d) Wolframite is magnetic in nature whereas stannic oxide is non-magnetic in nature. Hence, they can be separated by magnetic seperation method.

52. (d)

$$\overset{+7}{Mn}O_4^- + Br^- \longrightarrow \overset{+4}{Mn}O_2 + \overset{+5}{Br}O_3^-$$

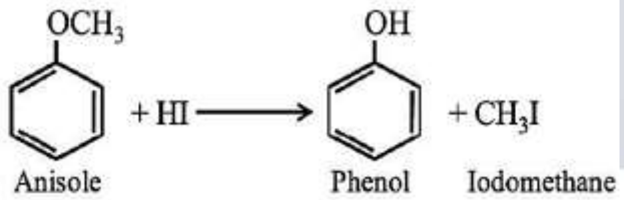
Hence (d) is correct option.

53. (a) Abscisic acid (molecular formula $(C_{15}H_{20}O_4)$ composed of three isoprene residues and having a cyclohexene ring with keto and one hydroxyl group and a side chain with terminal carboxylic group in its structure.



Abscisic acid

54. (a)



55. (c) The decaffeination process includes soaking green coffee in hot water and then some of solvent or activated carbon is used to extract the caffeine. The solvent typically used are methylene dichloride or ethyl acetate.

56. (c)

$$[Co(NH_3)_5NO_2]SO_4 + BaCl_2 \rightarrow [Co(NH_3)_5NO_2]Cl_2 + \underset{\text{(White ppt)}}{BaSO_4}$$

The precipitate of barium sulphate is white in colour.

57. (b) When concentrated HCl is added to a very diluted solution of $CuSO_4$, the pale blue solution slowly turns greenish yellow on the formation of copper chloride complex.

$$\underset{\text{(Pale blue)}}{[Cu(H_2O)_6]^{2+}} + 4Cl^- \longrightarrow \underset{\text{(Yellow)}}{[CuCl_4]^{2-}} + 6H_2O$$

58. (d) Novolac polymer is used in paints.

59. (a) Given, $n = 3$ moles, $v_1 = 0.3$ L, $v_2 = 2.5$ L, $P_{ext} = 1.9$ atm

Work done in isothermal process, $w = -P_{ex}dv$

$\therefore w = -1.9 \times (2.5 - 0.3)$

$w = -4.18$ L atm

$w = -4.18 \text{ L atm} \times 101.325 \text{JL}^{-1} \text{ atm}^{-1}$

$= -423.54$ J

60. **(b)** cis-platin is used in the treatment of cancer.

61. **(c)** NaCl and KCl have same atomic ratio, similar molecular formula and similar chemical properties but different crystal structure. Thus, NaCl and KCl are not isomorphous.

62. **(b)** 2,4-dichlorophenoxy acetic acid is the active ingredient in many products as an herbicide to kill weeds on land and in the water.

63. **(c)** $CO(g)+\frac{1}{2}O_2(g) \longrightarrow CO_2(g)$

$$\Delta H=\Delta U+\Delta n_g RT$$
$$\Delta H-\Delta U=\Delta n_g RT$$
$$\Delta n_g=1-1+\frac{1}{2}=-\frac{1}{2}$$

$$\therefore \Delta H-\Delta U=-\frac{1}{2}\times 2\times 300$$
$$=-300\text{cal}$$

64. **(a)** Conductivity decreases with decrease in concentration as the number of ions per unit volume that carries the current in a solution decreases on dilution

65. **(b)**

$$\underset{\text{Acetaldoxime}}{CH_3-CH=N-OH} \xrightarrow[\text{alcohol}]{\text{Na}} \underset{(A)}{CH_3CH_2NH_2}$$

$$\xrightarrow[\text{HCl}]{\text{NaNO}_2} \underset{(B)}{CH_3CH_2OH}+H_2O+N_2$$

66. **(d)** Frenkel defect is found in AgCl because there is a large difference between the size of Ag^+ and Cl^-. Hence the cation Ag^+ occupy the interstitial site by leaving a corresponding number of normal lattice site vacant.

67. **(d)** 0.1 molar $\approx$ 0.1 mol is present in 1 L Given volume $=200$ mL ≈ 0.2 L No. of mole in 0.2 L liquid

$$=\frac{2}{1}\times 0.1$$

$$=0.02 \text{ mol of } CO_2$$

$$V=0.02\times 22.4=0.448 \text{ L}$$

68. **(d)** The given reaction is an example of Wurtz reaction which is used in preparation of alkanes.

69. **(a)** Bacteriostatic antibiotics limit the growth of bacteria by interfering with bacterial protein production, DNA replication or other aspects of bacterial cellular metabolism. This group includes tetracyclines.

70. **(a)** Alkyl halides react with silver nitrite in ethanolic solution to give nitro compounds.

$$R-X+AgNO_2 \longrightarrow RNO_2+AgI$$

71. **(b)** Bond length order for the given options is, $C-H>C-C>C-N\approx C-O$

72. **(d)** Ethyl alcohol and water, after mixing, can very easily become a homogeneous mixture, because the two liquids are miscible, soluble in all proportions. The dipoles on the ethanol and water molecules cause the formation of hydrogen bonds between the molecules.

73. **(c)** Nicol's prism is a type of polarizer, an optical device made from calcite crystal. Calcite is a carbonate mineral and the most stable polymorph of calcium carbonate.

74. **(b)** Applying Faraday's second law of electrolysis

$$\frac{\text{wt. of Cu}}{\text{wt. of Al}} = \frac{E_w \text{ of Cu}}{E_w \text{ of Al}}$$

$$E_w \text{ of Cu} = \frac{\text{Atomic wt}}{\text{n factor}} = \frac{63.5}{2}$$

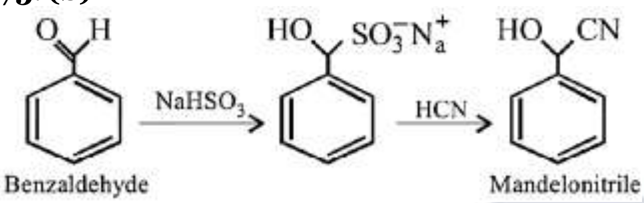
$$E_w \text{ of Al} = \frac{27}{3}$$

$$\therefore \frac{0.4 \times 63.5}{\text{wt of Al}} = \frac{31.75}{9}$$

wt of Al = 7.2 g

wt of Al in moles $= \frac{7.2}{27} = 0.27$ mol

75. (b)



76. (a) $K_2CrO_4 \longrightarrow 2\,K^+ + CrO_4^{2-}$

$K_2Cr_2O_7 \longrightarrow 2\,K^+ + Cr_2O_7^{2-}$

Both ions contain -2 charge.

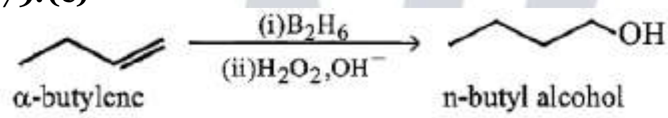
77. (d)

78. (a) Molar mass of urea (NH_2CONH_2) = 28 + 4 + 12 + 16 = 60 g/mol.

60 g urea contains 12 g C.

$\therefore$ 100 g urea contains $\frac{12}{60} \times 100 = 20\%C$

79. (c)



80. (b) $T_f = i \cdot K_f m$

Given, $T_f = T_f^\circ - T_f$

$= 0 - (-0.680)$

$= +0.680^\circ C$

$m = 0.2$

$K_f = 1.86$

Thus, $0.680 = i \times 0.2 \times 1.86$

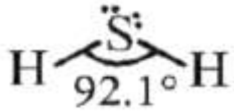
$$i = \frac{0.680}{0.2 \times 1.86} = 1.83$$

81. (c) $NaCl, NH_3, Ca(OH)_2$

$Ca(OH)_2$ is used for the regeneration of ammonia

$2NH_4Cl(aq) + Ca(OH)_2(aq/s) \rightarrow CaCl_2(aq) + 2NH_3(g) + 2H_2O(l)$

82.(b)



83.(d) mol dm^{-3} atm^{-1}

84. **(c)** Nitric oxide (pollutant) act as an homogeneous catalyst in the conversion of oxygen to ozone, since at very high concentration in air it converts into NO_2 which generate free oxygen atom.

85.(d) Bi does not exhibit allotropy.

86. **(b)** Nitrogen sesquioxide is N_2O_3.

87.(c) $\frac{-1}{2}\frac{d[SO_2]}{dt} = \frac{-d[O_2]}{dt} = \frac{1}{2}\frac{d[SO_3]}{dt}$

$$\therefore \frac{d[SO_3]}{dt} = \frac{-2\,d[O_2]}{dt}$$

88. **(a)** $\Delta s = \frac{q_{rev}}{T}$

89. **(a)** Terylene is a synthetic polymer which is formed by the interaction of ethylene glycol and terephthalic acid.

90. **(b)** Copper

91. (b) The decomposition of ammonia on platinum surface is a zero- order reaction

92.(b) Resistance (R) $= 2.5 \times 10^3$ ohm

$$\text{Conductivity } (\kappa) = \frac{\text{Cell constant}}{\text{Resistance}}$$

$$\text{Conductivity } (\kappa) = \frac{1.25\ cm^{-1}}{2.5 \times 10^3 ohm}$$

$$= 5 \times 10^{-4} ohm^{-1}\ cm^{-1}$$

$$\text{Molar conductivity } (\Lambda_m) = \frac{\kappa}{C} \times 1000$$

$$\Lambda_m = \frac{5 \times 10^{-4} ohm^{-1}\ cm^{-1}}{0.1\ mol\ cm^{-3}} \times 1000$$

$$\Lambda_m = 5\ ohm^{-1}\ cm^2\ mol^{-1}$$

93. (a)

$$Ba(NO_3)_2 \longrightarrow Ba^{2+} + 2NO_3^-$$

n mol	0	0
$n - n\alpha$	$n\alpha$	$2n\alpha$

Total moles of particles

$$= n - n\alpha + n\alpha + 2n\alpha = n(1 + 2\alpha)$$

$$\text{Vant Haff factor (i)} = \frac{n(1+2\alpha)}{n}$$

$$2.74 - \frac{n(1+2\alpha)}{n} - 1 + 2\alpha$$

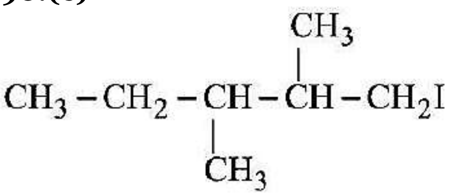
$$\alpha = \frac{2.74 - 1}{2} = 0.87$$

94.(d) Ionic hydrides of S-block elements, in molten state, liberate dihydrogen gas at anode on electrolysis.

$$2H^-(\text{melt}) \xrightarrow{\text{anode}} H_2(g) + 2e^-$$

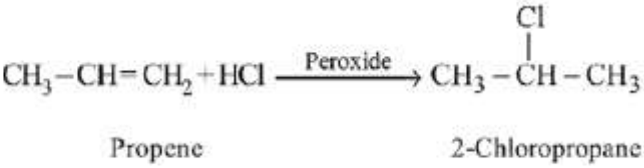
95.(c) Phosphine is formed by heating white phosphorous with conc. NaOH solution.

96.(c)



1-Iodo-2, 3-dimethylpentane

97.(c)



98. **(b)** As we move down the group, M-H bond dissociation enthalpy of hydrides decreases. Therefore, reducing property of metal hydrides increases in the order as follows,

$PH_3 < AsH_3 < SbH_3 < BiH_3$

99.(d) Potassium sulphite is not an antiseptic.

100. **(a)** Secondary structure of protein refers to the shape in which a long polypeptide chain can exist. They are found to exist in two different types of structures viz. α-helix and β-pleated sheet structure.

Mathematics

101. **(d)** $\because P(x_1, y_1)$ lie on $x^2 - y^2 = a^2$.

then; $x_1^2 - y_1^2 = a^2$

$\Rightarrow x_1^2 - a^2 = y_1^2$ ..... (i)

$\because x^2 - y^2 = a^2$ is an equation of rectangular hyperbola.

$\therefore e = \sqrt{2}$

$SP = ex_1 - a = \sqrt{2}x_1 - a$

$S'P = ex_1 + a = \sqrt{2}x_1 - a$

$\therefore SP.\ S'P = e^2x_1^2 - a^2 = 2x_1^2 - a^2$

$= x_1^2 + x_1^2 - a^2 = x_1^2 + y_1^2$ (from (i))

102. **(a)** $f(x) = \cos^{-1}\left[\frac{1-(\log x)^2}{1+(\log x)^2}\right]$

Let $1 + (\log x)^2 = u$

$\Rightarrow 1 - (\log x)^2 = 2 - u$

$\Rightarrow f(u) = \cos^{-1}\left(\frac{2-u}{u}\right) = \cos^{-1}\left(\frac{2}{u} - 1\right)$

$$\Rightarrow f'(u) = \left(\frac{\left(\frac{2}{u^2}\right)}{\sqrt{1-\left(\frac{2}{u}-1\right)^2}}\right) = \frac{1}{u\sqrt{u-1}}$$

$$\Rightarrow f'(x) = \frac{1}{(1+((\log x)^2)\sqrt{(\log x)^2}}$$

$$= \frac{1}{\log x(1+(\log x)^2)}$$

$$\Rightarrow f'(e) = \frac{1}{\log e(1+(\log e)^2)} = \frac{1}{2}$$

103. **(b)** Equation of family of circles whose radius is 4 is:

$(x-a)^2 + (y-b)^2 = 16$ ……… (i)

(where a& b are arbitrary constant)

Differentiating we get:

$2(x-a) + 2(y-b)y_1$ ……. (ii)

$\left(y_1 = \frac{dy}{dx}\right)$

Again, differentiating we get:

$$1 + y_1 \cdot y_1 + (y-b)y_2 = 0 \left(y_2 = \frac{d^2y}{dx^2}\right)$$

$$\Rightarrow 1 + y_1^2 + (y-b)y_2 = 0$$

$$\Rightarrow (y-b)y_2 = -(1+y_1^2)$$

$\Rightarrow y - b = -\frac{(1+y_1^2)}{y_2}$ ….. (iii)

from (ii) we get:

$x - a = -(y-b)y_1$

∴ from (i), we get:

$$(y-b)^2 y_1^2 + (y-b)^2 = 16$$

$$\Rightarrow (y-b)^2 - (1+y_1^2) = 16$$

$\Rightarrow \left(\frac{(1+y_1^2)^2}{y_2^2}\right)(1+y_1^2) = 16$ (form (iii)

$$\Rightarrow (1+y_1^2)^3 = 16y_2^2$$

$$\Rightarrow \left[1+\left(\frac{dy}{dx}\right)^2\right]^3 = 16\left[\frac{d^2y}{dx^2}\right]^2$$

$\therefore$ Order = 2 & degree = 2

104. **(a)** $A = \begin{bmatrix} x & 1 \\ 1 & 0 \end{bmatrix}$

$|A| = 0 - 1 = -1$

$\therefore A^{-1} = -1\begin{bmatrix} 0 & -1 \\ -1 & x \end{bmatrix}$

$\because = A = A^{-1} \Rightarrow x = 0$

105. **(c)** (a) $\lim_{n\to 0} f(x) = \lim_{n\to 0}(1+2x)^{1/x} = e^2$

$\& f(0) = e^2$

$\therefore$ Continuous at $x = 0$

(b) $\lim_{n\to 0} f(x) = \lim_{n\to 0}(\sin x - \cos x) = 0 - 1 = -1$

$\& f(0) = -1$

$\therefore$ Continuous at $x = 0$

(c) $\lim_{n\to 0} f(x) = \lim_{n\to 0} \frac{e^{1/x}-1}{e^{1/x}+1}$

$$= \lim_{n\to 0} \frac{e^{1/x}\left[1 - \frac{1}{e^{1/x}}\right]}{e^{1/x}\left[1 + \frac{1}{e^{1/x}}\right]}$$

$$= \frac{(1-0)}{(1+0)} = 1 \& f(0) = -1$$

$\therefore$ not continuous at $x = 0$

(d) $\lim_{n\to 0} f(x) = \lim_{n\to 0} \frac{e^{5x}-e^{2x}}{\sin 3x}$ (0/0)

is using L' Hospital's rule:

$$= \lim_{n\to 0} \frac{5e^{5x} - 2e^{2x}}{3\cos x} = \frac{5(1) - 2(1)}{3} = \frac{3}{3} = 1$$

$\therefore \& f(0) = 1$

$\therefore$ Continuous at $x = 0$

106. **(d)** Let p be the probability that a child labour case that is reported to the police station is solved. It is given to us that $p = \frac{1}{4}$.

Let X denote the number of solved cases amongst the 6 cases of child labour that are reported to the police station.

We note that X follows the binomial distribution with parameters $n = 6$ and

$$p = \frac{1}{4}$$

We are required to calculate $P(X \geq 5) = P(X = 5) + P(X = 6)$.

By applying the formula for the binomial distribution we get,

$P(X=5)+P(X=6)=\binom{6}{5}(1/4)^5(3/4)+\binom{6}{6}(1/4)^6=\frac{19}{4096}$

So, the required probability is equal to $\frac{19}{4096}$.

107. **(d)** $S_n=\frac{4^n-3^n}{3^n}$ $S_1=\frac{4-3}{3}=\frac{1}{3}$

$$S_2=\frac{4^2-3^2}{3^2}=\frac{16-9}{9}=\frac{7}{9}$$

$$\therefore t_2=S_2-S_1=\frac{7}{9}-\frac{1}{3}=\frac{7-3}{9}=\frac{4}{9}$$

108. **(a)** Given curves are $y=2x-x^2$

$\&y=x$

From the above equations we get,

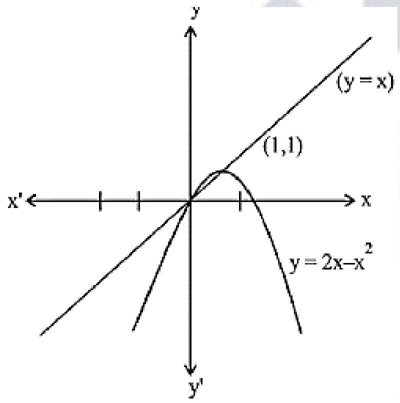
$x=2x-x^2$

$\Rightarrow x^2-x=0$

$\Rightarrow x(x-1)=0\Rightarrow x=0,1$

$\Rightarrow y=0,1$ (respectively)

$\therefore$ intersecting points of the two curves are (0,0) and (1,1)



$\therefore$ required area $=\int_0^1(2x-x^2)dx-\int_0^1 x\cdot dx$

$$=\left[\frac{2x^2}{2}-\frac{x^3}{3}\right]_0^1-\left[\frac{x^2}{2}\right]_0^1$$

$$=\left[1-\frac{1}{3}\right]-\left[\frac{1}{2}\right]$$

$=\frac{2}{3}-\frac{1}{2}=\frac{1}{6}$ sq. units.

109. **(c)**
110. **(b)**

p	q	r	$\sim r$	$\sim p$	$p \wedge q$	$\sim p \wedge q$	$\sim r \vee (p \wedge q)$	$(p \wedge q) \wedge [\sim r \vee (p \wedge q)]$	$(p \wedge q) \wedge [\sim r \vee (p \wedge q)] \vee (\sim p \wedge q)$
T	T	T	F	F	T	F	T	T	T
T	T	F	T	F	T	F	T	T	T
T	F	T	F	F	F	F	F	F	F
T	F	F	T	F	F	F	T	F	F
F	T	T	F	T	F	T	F	F	T
F	T	F	T	T	F	T	T	F	T
F	F	T	F	T	F	F	F	F	F
F	F	F	T	T	F	F	T	F	F

$\therefore (p \wedge q) \wedge [\sim r \vee (p \wedge q)] \vee (\sim p \wedge q) \equiv q$

111. **(c)** Let S be the sample space

Then $n(S)$ = number of ways of drawing 2 balls out of $(6 + 4) = 10$

$$\Rightarrow {}^{10}C_2 = \frac{10 \times 9}{2 \times 1} = 45$$

Let E = event of getting both balls of same color

Then, $n(E)$ = no of ways (2 balls out of six) or (2 balls out of 4)

$$= {}^6C_2 + {}^4C_2$$
$$= \frac{6 \times 5}{2 \times 1} + \frac{4 \times 3}{2 \times 1} = 15 + 6 = 21$$

$$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{21}{45} = \frac{7}{15}$$

112. **(b)**

113. **(a)** Radius increases at the rate of 5 cm/sec.

$\therefore$ radius after 2 seconds = 10 cm.

Now, Area (A) $= \pi r^2$ (r = radius)

$$\Rightarrow \frac{dA}{dt} = 2\pi r \cdot \frac{dr}{dt}$$

$\therefore$ after 2 seconds.

$$\frac{dA}{dt} = 2\pi(10)(5) = 100\pi cm^2/sec$$

114. **(a)** $f(x) = 3x - 2$ and $g(x) = x^2$.

$\Rightarrow f[g(x)] = 3(x^2) - 2 = 3x^2 - 2$

115. **(c)** " q only if p " is not equivalent " $p \rightarrow q$ ".

116. **(b)** $\int_{-3}^{3}(ax^5 + bx^3 + cx + k)dx$

$$= \left[\frac{ax^6}{6} + \frac{bx^4}{4} + \frac{cx^2}{2} + kx\right]_{-3}^{3}$$

$$= \left[\frac{a(3)^6}{6} + \frac{b(3)^4}{4} + \frac{c(3)^2}{2} + k(3)\right]$$

$$-\left[\frac{a(-3)^6}{6} + \frac{b(-3)^4}{4} + \frac{c(-3)^2}{2} + k(-3)\right]$$

$$= \frac{3^6a}{6} + \frac{3^4b}{4} + \frac{a}{2}c + 3k$$

$$-\frac{3^6a}{6} - \frac{3^4b}{4} - \frac{a}{2}c + 3k$$

$= 6k.$

$\therefore$ given integral depends only on k.

117. **(d)** Equation of all circles having centre at $(-1,2)$ is:

$(x - (-1))^2 + (y - 2)^2 = r^2$ (r = radius) $\Rightarrow (x + 1)^2 + (y - 2)^2 = r^2$

$\Rightarrow x^2 + 1 + 2x + y^2 + 4 - 4y = r^2.$

$\Rightarrow x^2 + y^2 + 2x - 4y + 5 - r^2 = 0.$

$\Rightarrow x^2 + y^2 + 2x - 4y + c = 0$

where $(c = 5 - r^2)$.

Above equation is the required solution.

118. **(d)** $\because (A - 2I)(A - 4I) = 0$

$\Rightarrow A^2 - 4A - 2A + 8I = 0$

$\Rightarrow A^2 - 6A + 8I = 0$

Multiply A^{-1} both sides we get:

$A^{-1} \cdot A \cdot A - 6A^{-1} \cdot A + 8A^{-1} \cdot I = A^{-1}.0$

$\Rightarrow IA - 6I + 8A^{-1} = 0$

$\Rightarrow A - 6I + 8A^{-1} = 0$

$\Rightarrow A + 8A^{-1} = 6I.$

119. **(d)** Here; $\frac{7+p+q+1}{3} = 3 \Rightarrow p + q = 1$ ... (i)

$\frac{-8+q+5p}{3} = -5 \Rightarrow 5p + q = -7$ ... (ii)

and $\frac{1+5+0}{3} = r \Rightarrow r = 2$

Subtract (ii) form (i), we get:

$p + q - 5p - q = 1 + 7$

$\Rightarrow -4p = 8 \Rightarrow p = -2$

from (1) we get,

$-2 + q = 1 \Rightarrow q = 3.$

$\therefore p = -2, q = 3 \& r = 2.$

120. **(b)** let $I = \int \frac{1}{(x^2+1)^2} dx$

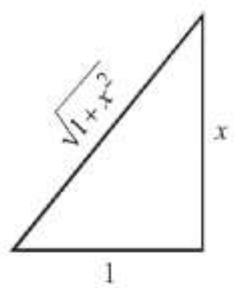
Put $x = \tan\theta \Rightarrow dx = \sec^2\theta d\theta.$

$$\Rightarrow I = \int \frac{\sec^2\theta d\theta}{(\tan^2\theta + 1)^2} = \int \frac{\sec^2\theta}{\sec^4\theta} d\theta$$

$$\Rightarrow I = \int \cos^2\theta d\theta = \frac{1}{2}\int (\cos 2\theta + 1) d\theta$$

$\Rightarrow I = \frac{1}{4}\sin 2\theta + \frac{\theta}{2} + c.$ ..... (i)

$\because \tan\theta = x$



$\Rightarrow \sin\theta = \frac{x}{\sqrt{1+x^2}} \& \cos\theta = \frac{1}{\sqrt{1+x^2}}.$

$\Rightarrow \sin 2\theta = 2\sin\theta\cos\theta = \frac{2x}{(1+x^2)}.$

$\therefore I = \frac{1}{2} \cdot \frac{x}{(1+x^2)} + \frac{1}{2}\tan^{-1} x + c.$

121. **(d)** $\because \theta = \frac{17\pi}{3} = 6\pi - \frac{\pi}{3}$.

$$\therefore \tan\theta - \cot\theta = \tan\left(6\pi - \frac{\pi}{3}\right) - \cot\left(6\pi - \frac{\pi}{3}\right)$$
$$= -\tan\frac{\pi}{3} + \cot\frac{\pi}{3}.$$
$$= -\sqrt{3} + \frac{1}{\sqrt{3}} = \frac{-3+1}{\sqrt{3}} = \frac{-2}{\sqrt{3}}$$

122. **(c)** Let $y = \log_{e^2}(\log x)$

$$= \frac{\log(\log x)}{\log e^2} = \frac{\log(\log x)}{2}$$
$$\Rightarrow \frac{dy}{dx} = \frac{1}{2}\cdot\frac{1}{\log x}\frac{d}{dx}(\log x)$$
$$= \frac{1}{2\log x}\cdot\frac{1}{x} = \frac{1}{2x\log x} = \frac{1}{x\log x^2}$$

123. **(d)** $\because \cos A - \frac{\sin B}{\sin C}$

$\Rightarrow \cos A \sin C = \sin B.$

$\Rightarrow \cos A \sin C = \sin(\pi - (A + C))$

$(\because A + B + C = \pi)$.

$\Rightarrow \cos A \sin C = \sin(A + C)$

$\Rightarrow \cos A \sin C = \sin A \cos C + \cos A \sin C$

$\Rightarrow \sin A \cos C = 0$

$\Rightarrow$ Either $\sin A = 0$ or $\cos C = 0$.

For $\sin A = 0$, $A = 0°$ (not possible)

For $\cos C = 0$, $C = 90°$

$\therefore \triangle ABC$ is right angled triangle.

124. **(b)** Let a is the first term & r is the common ratio.

$\therefore p = ar^{m+n-1} \& q = ar^{m-n-1}$

$\Rightarrow pq = a^2 r^{m+n-1} r^{m-n-1}$

$\Rightarrow pq = a^2 r^{2m-2} = (ar^{m-1})^2$

$\Rightarrow \sqrt{pq} = ar^{m-1} = m^{\text{th}}$ term.

125. **(a)**

$X = x$	1	2	3	4	5	6
$P(X = x)$	k	3k	5k	7k	8k	k

$\because \sum_{x=1}^{6} P(X) = 1$

$\Rightarrow k + 3k + 5k + 7k + 8k + k = 1 \Rightarrow k = \dfrac{1}{25}$

$\therefore P(2 \le x < 5) = P(2) + P(3) + P(4)$

$= 3k + 5k + 7k = 15 \times \frac{1}{25} = \frac{3}{5}$.

126. **(d)** $\because y = \log_e x \Rightarrow \frac{dy}{dx} = \frac{1}{x}$.

$\Rightarrow \left.\dfrac{dy}{dx}\right)_{(1,0)} = 1$

$\therefore$ equation of normal is:

$(y - 0) = -1(x - 1)$

$\Rightarrow y = -x + 1 \Rightarrow x + y = 1.$

127. **(b)** $\because \sin x \cos x = \frac{1}{4} \Rightarrow 2\sin x \cos x = \frac{1}{2}$

$\Rightarrow \sin 2x = \frac{1}{2} P\ 2x = n\pi + (-1)^n \frac{\pi}{6}, n \in I.$

For $n = 0, x = \frac{\pi}{12}$.

For $n = 1, x = \frac{5\pi}{12}$.

$\therefore$ x has only 2 values is $\left(0, \frac{\pi}{2}\right)$.

128. **(c)** $\because \vec{a} + \vec{b}, \vec{b} + \vec{c}$ and $\vec{c} + \vec{a}$ are coterminous edges of a parallelopiped.

Then, its volume (v) $= [\vec{a} + \vec{b} \quad \vec{b} + \vec{c} \quad \vec{c} + \vec{a}]$

We know, scalar triple product

$[\vec{a}\vec{b}\vec{c}] = \vec{a} \cdot (\vec{b} \times \vec{c}) \equiv (\vec{a} \times \vec{b}) \cdot \vec{c}$

Consider $[\vec{a} + \vec{b} \quad \vec{b} + \vec{c} \quad \vec{c} + \vec{a}]$

$= (\vec{a} + \vec{b}) \cdot \{(\vec{b} + \vec{c}) \times (\vec{c} + \vec{a})\}$

$= (\vec{a} + \vec{b}).\{(\vec{b} \times \vec{c}) + (\vec{b} \times \vec{a})$

$+(\vec{c} \times \vec{c}) + (\vec{c} \times \vec{a})\}$

$= (\vec{a} + \vec{b}) \cdot \{(\vec{b} \times \vec{c}) + (\vec{b} \times \vec{a}) + (\vec{c} \times \vec{a})\}$

$(\because \vec{c} \times \vec{c} = 0)$

$= \vec{a} \cdot (\vec{b} \times \vec{c}) + \vec{a} \cdot (\vec{b} \times \vec{a}) + \vec{a} \cdot (\vec{c} \times \vec{a})$

$+\vec{b} \cdot (\vec{b} \times \vec{c}) + \vec{b} \cdot (\vec{b} \times \vec{a}) + \vec{b} \cdot (\vec{c} \times \vec{a})$

$= [\vec{a}\vec{b}\vec{c}] + [\vec{a}\vec{b}\vec{a}] + [\vec{a}\vec{c}\vec{a}] + [\vec{b}\vec{b}\vec{c}]$

$+[\vec{b}\vec{b}\vec{a}] + [\vec{b}\vec{c}\vec{a}]$

$= [\vec{a}\ \ \vec{b}\ \ \vec{c}] + [\vec{b}\ \ \vec{c}\ \ \vec{a}] = 2[\vec{a}\ \ \vec{b}\ \ \vec{c}]$

129. **(a)** By definition, $F(x) = P(X \le x)$.
Therefore, $P(27 \le X \le 33) = P(X \le 33) - P(X \le 27)$

$$= F(33) - F(27)$$
$$= \frac{33-25}{10} - \frac{27-25}{10}$$
$$= \frac{6}{10} = \frac{3}{5}$$

That is, $P(27 \le X \le 33) = \frac{3}{5}$.

130. **(a)** Equations of lines are:

$(x-0) = (1+\sqrt{2})(y-0) \& (x-0)$

$= \left(\frac{1}{1+\sqrt{2}}\right)(y-0)$

or $x = y(1+\sqrt{2}) \& x = \frac{y}{1+\sqrt{2}} \times \frac{1-\sqrt{2}}{1-\sqrt{2}}$

or $x - y(1+\sqrt{2}) = 0 \& x + y(1-\sqrt{2}) = 0$

$\therefore$ joint equation is:

$[x - y(1+\sqrt{2})][x + y(1-\sqrt{2})] = 0$

$\Rightarrow x^2 + xy(1-\sqrt{2}) - xy(1+\sqrt{2})$

$-y^2\left(1 - (\sqrt{2})^2\right) = 0$

$\Rightarrow x^2 + xy - xy\sqrt{2} - xy$

$-xy\sqrt{2} - y^2(-1) = 0$

$\Rightarrow x^2 - 2\sqrt{2}xy + y^2 = 0.$

131. **(d)** Angle between the lines:

$\frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1}$

and $\frac{x-x_2}{a_2} = \frac{y-y_2}{b_2} = \frac{z-z_2}{c_2}$.

is: $\cos\theta = \left|\frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}\right|$

$\therefore$ angle between two given lines is:

$$\cos q = \left|\frac{(2)(1)+(-2)(2)+(1)(2)}{\sqrt{4+4+1}\sqrt{1+4+4}}\right|$$

$$= \left|\frac{2-4+2}{9}\right|$$

$\Rightarrow \cos\theta = 0 \Rightarrow \theta = 90^\circ.$

132. **(c)** Here the given three points $P(6,-1,2)$, $Q(8,-7,2\lambda)$ and $R(5,2,4)$ are collinear. we know that if three points (x_1,y_1,z_1), (x_2,y_2,z_2) and (x_3,y_3,z_3) are collinear, then

$\frac{x_1-x_2}{x_2-x_3} = \frac{y_1-y_2}{y_2-y_3} = \frac{z_1-z_2}{z_2-z_3}.$

$$\therefore \frac{6-8}{8-5} = \frac{-1+7}{-7-2} = \frac{2-2\lambda}{2\lambda-4}$$

$$\Rightarrow \frac{-2}{3} = \frac{2-2\lambda}{2\lambda-4} \Rightarrow -4\lambda+8 = 6-6\lambda$$

$\Rightarrow 2\lambda = -2 \Rightarrow \lambda = -1.$

133. **(a)** $\sim(p \rightarrow \sim q) = p \wedge \sim(\sim q) = p \wedge q$

134. **(d)** $x^2 - 5|x| + 6 = 0.$

If $x < 0$, then $|x| = -x$

$\therefore x^2 + 5x + 6 = 0$

$\Rightarrow x^2 + 3x + 2x + 6 = 0$

$\Rightarrow x(x+3) + 2(x+3) = 0$

$\Rightarrow (x+3)(x+2) = 0$

$\Rightarrow x = -3, -2.$

If $x > 0$, then $|x| = x$

$\therefore x^2 - 5x + 6 = 0$

$\Rightarrow x^2 - 3x - 2x + 6 = 0$

$\Rightarrow x(x-3) - 2(x-3) = 0$

$\Rightarrow (x-2)(x-3) = 0$

$\Rightarrow x = 2,3.$

$\therefore n(A) = 4.$

135. **(b)** $\lim_{x\to0} f(x) = \lim_{x\to0} \frac{\log(1+ax)-\log(1-bx)}{x}\left(\frac{0}{0}\right)$

$= \lim_{x\to0} \frac{\frac{a}{1+ax}+\frac{b}{1-bx}}{1}$ (Using L' Hospital's Rule)

$= \frac{a}{1+0} + \frac{b}{1-0} = a + b.$

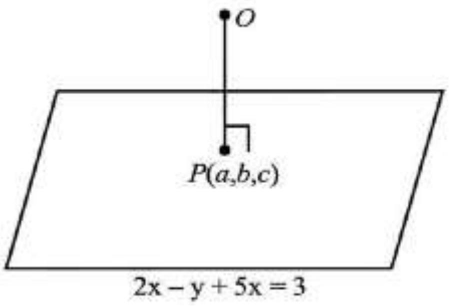
$\because$ f(x) is continuous at x = 0.

$\therefore f(0) = \lim_{x \to 0} f(x) = a + b.$

136. **(d)** let the co-ordinates of foot of perpendicular from the orgin (o) to the plane $2x - y + 5x = 3$ is $P(a, b, c)$.

$\therefore$ direction ratios of OP are $\langle a, b, c \rangle$

which is also the direction ratios of normal to the given plane.



$\therefore \frac{a}{2} = \frac{b}{-1} = \frac{c}{5} = k.$

$\Rightarrow a = 2k, b = -k, c = 5k.$

$\because$ P(a, b, c) passes the given plane

$\therefore 2(2k) - (-k) + 5(5k) = 3$

$\Rightarrow 4k + k + 25k = 3$

$$\Rightarrow k = \frac{3}{30} = \frac{1}{10}$$

$\therefore a = \frac{2}{10} = \frac{1}{5}; b = -\frac{1}{10}$ and $c = \frac{5}{10} = \frac{1}{2}.$

137. **(a)** Let $I = \int \frac{\sqrt{x^2 - a^2}}{x} dx$

Put $x = a\sec\theta \Rightarrow dx = a\sec\theta\tan\theta d\theta$

$$\Rightarrow I = \int \frac{\sqrt{a^2(\sec^2\theta - 1)}}{a\sec\theta} \cdot a\sec\theta\tan\theta d\theta$$

$= \int a\tan^2\theta \cdot d\theta = a\int (\sec^2\theta - 1)d\theta$

$= a(\tan\theta - \theta) + c \left(\because \int \sec^2 x dx = \tan x\right).$

$= a\sqrt{ser^2\theta - a} - a\theta + c$

$= a\sqrt{\left(\frac{x^2}{a^2}\right)} - a - a\sec^{-1}\left(\frac{x}{a}\right) + c.$

$\left(\because \sec\theta = \frac{x}{a}\right)$

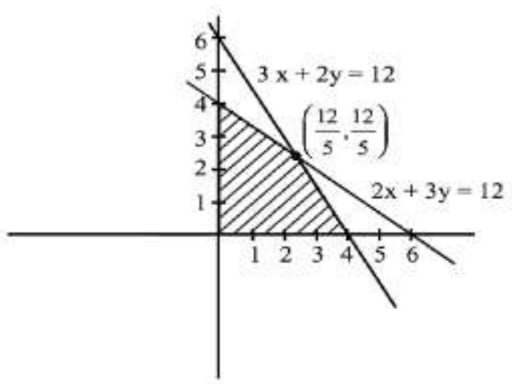
$= \sqrt{x^2 - a^2} - a\sec - 1\left(\frac{x}{a}\right) + c.$

$= \sqrt{x^2 - a^2} - a\cos - 1\left(\frac{a}{x}\right) + c.$

138. (d) $\because 3x + 2y£12 \text{ or } \frac{x}{4} + \frac{y}{6}£1$

$2x + 3y£12 \text{ or } \frac{x}{6} + \frac{y}{4}£1.$

and $x^3 0, y^3 0$



$\therefore$ Corner points are: $(0,0), (0,4), (4,0)$ and $\left(\frac{12}{5}, \frac{12}{5}\right)$.

$\because z = 9x + 11y.$

$At(0,0), z = 0.$

$At(0,4), z = 44. \ At(4,0), z = 36.$

$At\left(\frac{12}{5}, \frac{12}{5}\right), z = \frac{108+132}{5} = \frac{240}{5} = 48.$

$\therefore$ maximum value of z is 48.

139. (c) Let $I = \int_0^4 \frac{1}{1+\sqrt{x}} dx$

put $u = \sqrt{x} \Rightarrow u^2 = x \Rightarrow 2udu = dx$. when $x = 0, u = 0$ & when $x = 4, u = 2$.

$\Rightarrow I = \int_0^2 \frac{1}{1+u} 2udu.$

$$= 2\int_0^2 \frac{u}{1+u} du$$

put $l + u = w \Rightarrow du = dw.$

when $u = 0, w = 1$& when $u = 2, w = 3$

$\Rightarrow I = 2\int_0^3 \frac{w-1}{w} dw = 2\int_1^3 \left(1 - \frac{1}{w}\right) dw$

$= 2[w - \log w]_1^3$

$= 2[3 - \log 3] - 2[1 - \log 1]$

$= 6 - 2\log 3 - 2 = 4 - 2\log 3$

$= 4\log e - \log 3^2 = \log e^4 - \log 9 = \log\left(\frac{e^4}{9}\right)$

140. **(c)** $\because \sin^2 q = \frac{1}{2}$

$\Rightarrow \sin^2\theta = \left(\frac{1}{\sqrt{2}}\right)^2$

$\Rightarrow \sin^2\theta = \sin^2\left(\frac{\pi}{4}\right).$

$\Rightarrow \theta = n\pi \pm \frac{\pi}{4}, n \in I.$

$\therefore$ in $[0, \pi]$, there are only two

solutions i.e.; $\frac{\pi}{4}$ and $\frac{3\pi}{4}$.

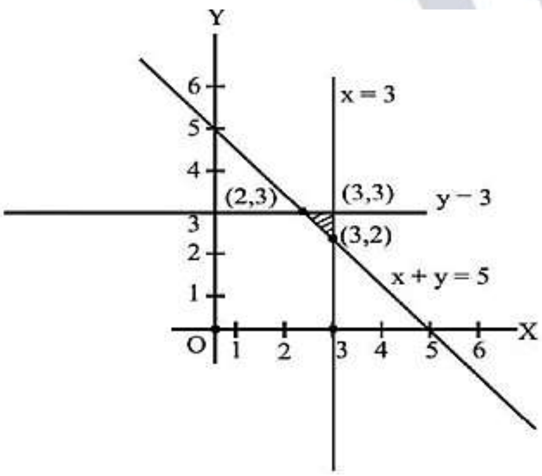
141. **(c)**

$$\begin{aligned}
&[\vec{p} + \vec{q} - \vec{r}\vec{p} - \vec{q}\vec{q} - \vec{r}] \\
&= (\vec{p} + \vec{q} - \vec{r}) \times (\vec{p} - \vec{q}) \cdot (\vec{q} - \vec{r}) \\
&= (-2\vec{p} \times \vec{q} - \vec{r} \times \vec{p} + \vec{r} \times \vec{q}) \cdot (\vec{q} - \vec{r}) \\
&= \vec{p} \times \vec{q} \cdot \vec{r} \\
&= [\vec{p}\vec{q}\vec{r}]
\end{aligned}$$

142. **(b)** $\cos\theta = \sqrt{2}$ has no solution, since value of $\cos\theta$ lies in $[-1,1]$

143. **(a)** $z = 10x + 25y$ subject to:

$x + y \geq 5; x \leq 3; y \leq 3; x \geq 0; y \geq 0$



$\therefore$ Corner points of the bounded region are:

$(3,2), (2,3) \& (3,3)$

$z = 10x + 25y.$

$At(3,2), z = 30 + 50 = 80$ (Minimum).

$At(2,3), z = 20 + 75 = 95$

$At(3,3), z = 30 + 75 = 105.$

144. **(b)** $f(x) = 3x^3 - 9x^2 - 27x + 15.$

$f'(x) = 9x^2 - 18x - 27$

For maxima or minima:

$f'(x) = 0 \Rightarrow 9x^2 - 18x - 27 = 0.$

$\Rightarrow x^2 - 2x - 3 = 0$

$\Rightarrow x^2 - 3x + x - 3 = 0$

$\Rightarrow x(x - 3) + 1(x - 3) = 0$

$\Rightarrow x = -1,3.$

$f''(x) = 18x - 18.$

$f''(-1) = -18 - 18 = -36 < 0$

$f''(3) = 18(3) - 18 = 36 > 0.$

$\therefore$ f(x) has maximum value at $x = -1$.

& max. value $= 3(-1)^3 - 9(-1)^2 - 27(-1) + 15$

$= -3 - 9 + 27 + 15 = 30.$

145. **(a)** We know that equation of plane passing through a point with position vector $\vec{a}$ and normal to the vector $\vec{r}$ is:

$(\vec{r} - \vec{a}) \cdot n = 0$

$\because$ the plane passes through $(-1,2,1)$

$\therefore \vec{a} = -\hat{\imath} + 2\hat{\jmath} + \hat{k}$

Also, plane is perpendicular to the line containing $(-3,1,2)$ and $(2,3,4)$

$\therefore \vec{n} = 5\hat{\imath} + 2\hat{\jmath} + 2\hat{k}$

$\therefore$ required equation is:

$[\vec{r} - (-\hat{\imath} + 2\hat{\jmath} + \hat{k})] \cdot (5\hat{\imath} + 2\hat{\jmath} + 2\hat{k}) = 0$

$\Rightarrow \vec{r} \cdot (5\hat{\imath} + 2\hat{\jmath} + 2\hat{k})$

$-[(-\hat{\imath} + 2\hat{\jmath} + \hat{k}) \cdot (5\hat{\imath} + 2\hat{\jmath} + 2\hat{k})] = 0$

$\Rightarrow \vec{r} \cdot (5\hat{\imath} + 2\hat{\jmath} + 2\hat{k}) - [-5 + 4 + 2] = 0$

$\Rightarrow \vec{r} \cdot (5\hat{\imath} + 2\hat{\jmath} + 2\hat{k}) = 1$

146. **(b)** For $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

Length of transverse axis $= 2a = 6 \Rightarrow a = 3$ and length of latus rectum $= \frac{2b^2}{a} = \frac{8}{3}$

$$\Rightarrow \frac{2b^2}{3} = \frac{8}{3} \Rightarrow b^2 = 4$$

$\therefore$ equation of hyperbola is:

$$\frac{x^2}{9} - \frac{y^2}{4} = 1$$

$$\Rightarrow 4x^2 - 9y^2 = 36$$

147. **(b)** $\because \tan^{-1} + \tan^{-1} y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)$, if $xy < 1$.

$$\therefore \tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{5} + \tan^{-1}\frac{1}{7} + \tan^{-1}\frac{1}{8}$$

$$= \tan^{-1}\left[\frac{\frac{1}{3}+\frac{1}{5}}{1-\frac{1}{15}}\right] + \tan^{-1}\left[\frac{\frac{1}{7}+\frac{1}{8}}{1-\frac{1}{56}}\right]$$

$$= \tan^{-1}\left[\frac{\frac{5+3}{15}}{\frac{14}{15}}\right] + \tan^{-1}\left[\frac{\frac{7+8}{56}}{\frac{55}{56}}\right]$$

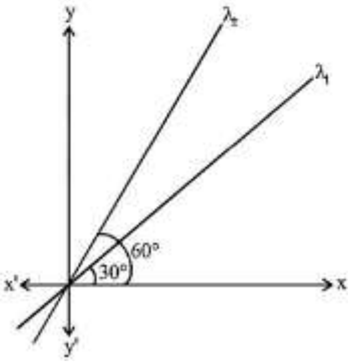
$$= \tan^{-1}\frac{8}{14} + \tan^{-1}\frac{15}{55}$$

$$= \tan^{-1}\frac{4}{7} + \tan^{-1}\frac{3}{11}$$

$$= \tan^{-1}\left[\frac{\frac{4}{7}+\frac{3}{11}}{1-\frac{12}{77}}\right] = \tan^{-1}\left[\frac{\frac{44+21}{77}}{\frac{65}{77}}\right] = \tan^{-1}\left(\frac{65}{65}\right)$$

$$= \tan^{-1}(1) = \frac{\pi}{4}$$

148. **(a)** Let ℓ_1 and ℓ_2 are the two lines, which trisects the first quadrant (as shown in the figure)



slope of $\ell_1 = \tan 30° = \frac{1}{\sqrt{3}}$

and slope of $\ell_2 = \tan 60° = \sqrt{3}$

$\therefore$ equation of ℓ_i is:

$x = \frac{y}{\sqrt{3}}$ ($\because$ ℓ_i passes through centre)

& equation of ℓ_2 is:

$x = \sqrt{3}y$ ($\because$ ℓ_2 passes through centre)

$\therefore$ joint equation is:

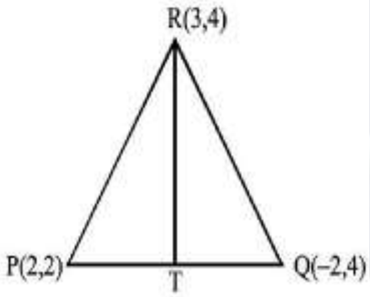
$$\left(x - \frac{y}{\sqrt{3}}\right)(x - \sqrt{3}y) = 0$$

$$x^2 - \sqrt{3}xy - \frac{xy}{\sqrt{3}} + y^2 = 0$$

$$\Rightarrow \frac{\sqrt{3}x^2 - 3xy - xy + \sqrt{3}y^2}{\sqrt{3}} = 0$$

$$\Rightarrow \sqrt{3}x^2 - 4xy + \sqrt{3}y^2 = 0$$

149. **(b)** From figure it is clear that T is the mid- point of PQ



$\therefore$ co-ordinates of T $\equiv \left(\frac{2-2}{2}, \frac{2+4}{2}\right)$

$\equiv (0,3)$

Equation of RT is $(y - 4) = \left(\frac{3-4}{0-3}\right)(x - 3)$

or $(y - 4) = \frac{1}{3}(x - 3)$ or $3y - 12 = x - 3$

or $x - 3y + 9 = 0$.

150. **(a)** $x = \sqrt{a^{\sin^{-1} t}}$

$$\Rightarrow \frac{dx}{dt} = \frac{1}{2\sqrt{a^{\sin^{-1} t}}} \cdot \frac{a^{\sin^{-1} t}\log a}{\sqrt{1-t^2}} = \frac{\log a}{2} \frac{\sqrt{a^{\sin^{-1} t}}}{\sqrt{1-t^2}}.$$

and $y = \sqrt{a^{\cos^{-1} t}}$

$$\Rightarrow \frac{dy}{dt} = \frac{1}{2\sqrt{a^{\cos^{-1} t}}} \cdot \frac{a^{\cos^{-1} t}\log a}{(-\sqrt{1-t^2})} = \frac{-\log a}{2} \frac{\sqrt{a^{\cos^{-1} t}}}{\sqrt{1-t^2}}.$$

$$\therefore \frac{dy}{dt} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{-\log a}{2} \frac{y}{\sqrt{1-t^2}} \times \frac{2}{\log a} \frac{\sqrt{1-t^2}}{x} = \frac{-y}{x}.$$