Physics

1. (a) $\delta = \frac{W\ell^3}{3YI}$, where W = load, ℓ = length of beam and I is geometrical moment of inertia for rectangular beam,

$I = \frac{bd^3}{12}$ where b = breadth and d = depth

For square beam $b = d \therefore I_1 = \frac{b^4}{12}$

For a beam of circular cross-section, $I_2 = \left(\frac{\pi r^4}{4}\right) \therefore \delta_1 = \frac{W\ell^3 \times 12}{3Yb^4} = \frac{4W\ell^3}{Yb^4}$

(for sq. cross-section)

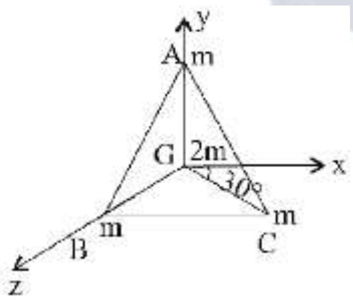
and $\delta_2 = \frac{W\ell^3}{3Y(\pi r^4/4)} = \frac{4W\ell^3}{3Y(\pi r^4)}$

(for circular cross-section)

Now $\frac{\delta_1}{\delta_2} = \frac{3\pi r^4}{b^4} = \frac{3\pi r^4}{(\pi r^2)^2} = \frac{3}{\pi}$

($\because b^2 = \pi r^2$ i.e., they have same cross-sectional area)

2. (c)



$$F_{GA} = \frac{Gm(2\,m)}{1}\hat{j}$$
$$F_{GB} = \frac{Gm(2m)}{1}(-\hat{i}\cos 30^\circ - \hat{j}\sin 30^\circ)$$
$$F_{GC} = \frac{Gm(2m)}{1}(\hat{i}\cos 30^\circ - \hat{j}\sin 30^\circ)$$

$\therefore$ Resultant force on (2 m) is F_R

$= F_{GA} + F_{GB} + F_{GC}$

$= 2Gm^2\hat{j} + 2Gm^2\hat{i}(-\cos 30^\circ + \cos 30^\circ)$

$+2Gm^2\hat{j}(-\sin 30^\circ - \sin 30^\circ)$

$= 2Gm^2\hat{j} \cdot 2Gm^2\hat{j}(-2 * 1/2)$

$= 2Gm^2\hat{j} - 2Gm^2\hat{j} = 0.$

3. (b) Reverse resistance

$$= \frac{\Delta V}{\Delta I} = \frac{1}{0.5 \times 10^{-6}} = 2 \times 10^6 \Omega$$

4. (b)

$$: v_1 = \frac{d}{dt}(y_1) = (0.1 \times 100\pi)\cos\left(100\pi t + \frac{\pi}{3}\right)$$

$$v_2 = \frac{d}{dt}(y_2) = (-0.1 \times \pi)\sin \pi t$$

$$= (0.1 \times \pi)\cos\left(\pi t + \frac{\pi}{2}\right)$$

$$\therefore\ \Delta\phi = \frac{\pi}{3} - \frac{\pi}{2} = -\frac{\pi}{6}$$

5. **(a)** $L_0 = 60$ cm $v_0 = 256$ Hz.

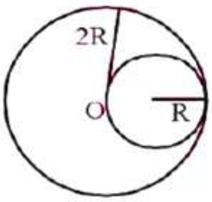
$$v = \frac{1}{2L}\sqrt{\frac{T}{m}}\ \therefore\ v \propto \frac{1}{L}$$

$$\frac{v_1}{v_0} = \frac{L_0}{L_1} \Rightarrow v_1 = v_0\frac{L_0}{L_1} = 256 \times \frac{60}{15} = 1024 \text{ Hz.}$$

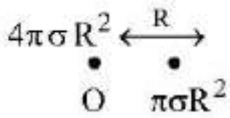
6. **(c)** $m \times 10 = 2 \times 3 \times 10^{-2} \times \frac{10}{100}$ or $m = 6 \times 10^{-4}$ kg $= 6 \times 10^{-4} \times 10^3$ g $= 0.6$ g

7. **(b)** Let the mass per unit area be σ. Then the mass of the complete dise

$= \sigma[\pi(2R)^2] = 4\pi\sigma R^2$



The mass of the removed disc $= \sigma(\pi R^2) = \pi\sigma R^2$ Let us consider the above situation to be a complete dise of radius 2 R on which a dise of radius R of negative mass is superimposed. Let O be the origin. Then the above figure can be redrawn keeping in mind the concept of centre of mass as:

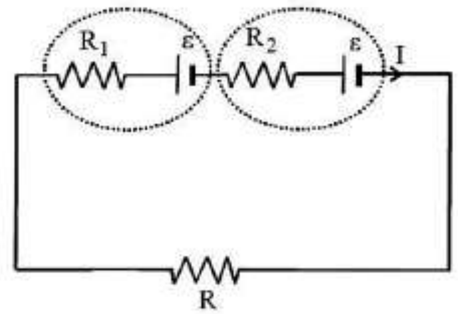


$$X_{c.m} = \frac{(4\pi\sigma R^2) \times 0 + (-\pi\sigma R^2)R}{4\pi\sigma R^2 - \pi\sigma R^2}$$

$$\therefore\ x_{c.m} = \frac{-\pi\sigma R^2 \times R}{3\pi\sigma R^2}$$

$$\therefore\ x_{c.m} = -\frac{R}{3} \Rightarrow \alpha = \frac{1}{3}$$

8. **(c)** $I = \frac{2\varepsilon}{R+R_1+R_2}$



Pot. difference across second cell

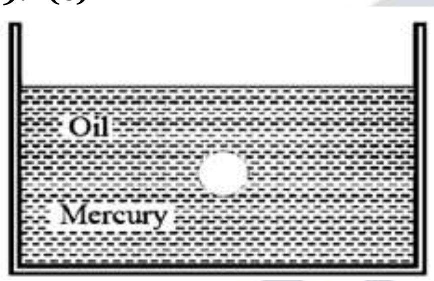
$= V = \varepsilon - IR_2 = 0$

$\varepsilon = \frac{2\varepsilon}{R + R_1 + R_2} \cdot R_2 = 0$

$R + R_1 + R_2 - 2R_2 = 0$

$R + R_1 - R_2 = 0 \ \therefore R = R_2 - R_1$

9. (c)



As the sphere floats in the liquid. Therefore, its weight will be equal to the up-thrust force on it

Weight of sphere $= \frac{4}{3}\pi R^3 \rho g$ …. (i)

Up thrust due to oil and mercury

$= \frac{2}{3}\pi R^3 \times \sigma_{oil} g + \frac{2}{3}\pi R^3 \sigma_{Hg} g$ ……. (ii)

Equating (i) and (ii)

$\frac{4}{3}\pi R^3 \rho g = \frac{2}{3}\pi R^3 0.8g + \frac{2}{3}\pi R^3 + 13.6g$

$\Rightarrow 2\rho = 0.8 + 13.6 = 14.4 \Rightarrow \rho = 7.2$

10. (a) $B = \frac{\mu_0 I}{2r} \times \frac{\theta}{2\pi} = \frac{\mu_0 I\theta}{4\pi r}$

11. (c) $P_c = \frac{P_t}{1+\frac{m_a^2}{2}} = \frac{12}{1+\frac{(0.5)^2}{2}} = \frac{12}{1.25} = 9.6 \text{ kW}$

12. (d) $v' = v\left(\frac{v+v_D}{v-v_S}\right)$

Here, $v = 600 \text{ Hz}, v_D = 15 \text{ m/s}$

$v_s = 20 \text{ m/s}, v = 340 \text{ m/s}$

$\therefore\ v' = 600\left(\frac{355}{320}\right) \approx 666$ Hz

13. (a) Potential at the given point = Potential at the point due to the shell + Potential due to the particle

$$= -\frac{GM}{a} - \frac{2GM}{a} = -\frac{3GM}{a}$$

14. (a) In $x = A\cos\omega t$, the particle starts oscillating from extreme position. So, at $t = 0$, its potential energy is maximum.

15. (c) The electron moves with constant velocity without deflection. Hence, force due to magnetic field is equal and opposite to force due to electric field.

$$qvB = qE \Rightarrow v = \frac{E}{B} = \frac{20}{0.5} = 40 \text{ m/s}$$

16. (a) $T = 2\pi\sqrt{\left(\frac{I}{MB_H}\right)}$

$$T' = 2\pi\sqrt{\left(\frac{I}{4MB_H}\right)} = \frac{1}{2}\left[2\pi\sqrt{\frac{I}{(MB_H)}}\right]$$

$$= \frac{1}{2} \times 2 = 1 \text{ second.}$$

17. (b) Current in the circuit,

$$I = \frac{\varepsilon}{R + r}$$

Potential difference across R,

$$V = IR = \left(\frac{\varepsilon}{R + r}\right)R = \frac{\varepsilon}{1 + \frac{r}{R}}$$

When $R = 0, V = 0$

$R = \infty, V = \varepsilon$

18. (d) $\lambda_{IR} > \lambda_{UV}$ also, wavelength of emitted radiation $\lambda \propto \frac{1}{\Delta E}$.

19. (d) For stress to be equal, $\frac{T_1}{A_1} = \frac{T_2}{A_2}$

$$\therefore\ \frac{T_1}{T_2} = \frac{A_1}{A_2} = \frac{1}{2}$$

20. **(b)** $|\vec{A} + \vec{B}| = \sqrt{A^2 + B^2 + 2AB\cos 120^\circ}$

$$= \sqrt{A^2 + B^2 + \cancel{2}AB\left(\frac{-1}{\cancel{2}}\right)}\left(\cos 120^\circ = -\frac{1}{2}\right)$$

$$= \sqrt{A^2 + B^2 - A(A)} = \sqrt{B^2} = B\ (\because A = B)$$

21. (d) It is a one of Fraunh offer diffraction from single slit. so for bright fringe where a is the width of slit

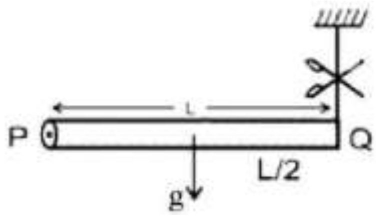
$a \sin\theta = (2n+1)\frac{\lambda}{2}$

$\lambda = \frac{2a\sin\theta}{2n+1} = \frac{2\times1.2\times10^{-5}\times0.0906}{2\times1+1} = 7248\text{Å}.$

22.(c) $\mu = \frac{F}{R} = \frac{mg\sin\alpha}{mg\cos\alpha} = \tan\alpha$

23.(d) $\frac{H_1}{H_2} = \frac{u^2\sin^2\theta/2g}{u^2\sin^2(90^\circ-\theta)/2g} = \tan^2\theta$

24.(d)



angular acceleration $\alpha = \frac{3g}{2L}$

25.(c) We know that $\frac{Q^2}{2C}$ is energy of capacitor, so it represent the dimension of energy = $[ML^2T^{-2}]$.

26.(c) Power gain = voltage gain × current gain

$$-V_G \cdot I_G = \frac{V_0}{V_i}\cdot\frac{I_0}{I_i}$$
$$= \frac{V_0^2}{V_i^2}\cdot\frac{R_i}{R_0} = 50\times50\times\frac{100}{200} = 1250$$

27.(c) Due to volume expansion of both mercury and flask, the change in volume of mercury relative to flask is given by

$$\Delta V = V_0[\gamma_L - \gamma_g]\Delta\theta = V[\gamma_L - 3\alpha_g]\Delta\theta$$
$$= 50[180\times10^{-6} - 3\times9\times10^{-6}](38-18)$$
$$= 0.153\text{cc}$$

28.(a) With the increase in temperature, the surface tension of liquid decreases and angle of contact also decreases.

29.(b) $i = \frac{A+\delta_m}{2} = \frac{60+30}{2} = 45^\circ$

30. (c) $\frac{(v_e)_p}{(v_e)_e} = \frac{\sqrt{\frac{2GM_p}{R_p}}}{\sqrt{\frac{2GM_e}{R_e}}} = \sqrt{\frac{M_p}{M_e}\times\frac{R_e}{R_p}}$

$$= \sqrt{\frac{10M_e}{M_e}\times\frac{R_e}{R_e/10}} = 10$$

$\therefore\ (v_e)_p = 10\times(v_e)_e = 10\times11 = 110\ \text{km/s}$

31. (b) Fringe width $\beta = \frac{\lambda D}{d}$

32.(b) According to Einsten's photoelectric effect, the K.E. of the radiated electrons

$K.E_{max} = E - W$

$\frac{1}{2}mv_1^2 = (1 - 0.5)eV = 0.5eV$

$\frac{1}{2}mv_2^2 = (2.5 - 0.5)eV = 2eV$

$\frac{v_1}{v_2} = \sqrt{\frac{0.5}{2}} = \frac{1}{\sqrt{4}} = \frac{1}{2}$

33.(a) $\frac{E_0}{B_0} = c \cdot$ also $k = \frac{2\pi}{\lambda}$ and $\omega = 2\pi v$

These relation gives $E_0 k - B_0 \omega$

34.(b) Here, $R_g = 100\Omega; I_g = 10^{-5}$ A; $I = 1$ A; $S = ?$ $S = \frac{I_g R_g}{I - I_g} = \frac{10^{-5} \times 100}{1 - 10^{-5}} = 10^{-3}\Omega$ in parallel

35.(b) Terminal velocity, $v_0 = \frac{2I^2(\rho - \rho_0)g}{9\eta}$

$$= \frac{2 \times (2 \times 10^{-3})^2 \times (8 - 1.3) \times 10^3 \times 9.8}{9 \times 0.83}$$

$= 0.07\ ms^{-1}$

36.(d) $T_2 = 7°C = (7 + 273) = 280$ K

$\eta = 1 - \frac{T_2}{T_1} \Rightarrow \frac{T_2}{T_1} = 1 - \eta$

$= 1 - \frac{50}{100} = \frac{50}{100} = \frac{1}{2}$

$\therefore\ T_1 = 2 \times T_2 = 2 \times 280 = 560$ K

New efficiency, $\eta' = 70\%$

$\therefore\ \frac{T_2}{T_1} = 1 - \eta' = 1 - \frac{70}{100} = \frac{30}{100} = \frac{3}{10}$

$\therefore\ T_1' = \frac{10}{3} \times 280 = \frac{2800}{3} = 933.3$ K

$\therefore$ Increase in the temperature of high temp. reservoir $= 933.3 - 560 = 373.3$ K $= 380$ K

37.(c) As m would slip in vertically downward direction, then $mg = \mu N$

$\Rightarrow\ N = \frac{mg}{\mu} = \frac{100}{0.5} = 200$ Newton

Same normal force would accelerated M,

thus $a_M = \frac{200}{50} = 4\ m/s^2$

Taking m + M as system

$F = (m + M)4 = 240$ N

38.(c) $\frac{\Delta P}{P} \times 100 = \frac{\Delta F}{F} \times 100 + 2\frac{\Delta \ell}{\ell} \times 100$

$= 6\% + 2 \times 3\%$

39.(a) Here, $u = 0; a = \frac{eE}{m}; v = ?; t = t$

$\therefore\ v = u + at = 0 + \frac{eE}{m}t$

de-Broglie wavelength,

$$\lambda = \frac{h}{mv} = \frac{h}{m(eEt/m)} = \frac{h}{eEt}$$

Rate of change of de-Broglie wavelength

$$\frac{d\lambda}{dt} = \frac{h}{eE}\left(-\frac{1}{t^2}\right) = \frac{-h}{eEt^2}$$

40. **(b)** $R_1 = 60\ \text{cm}, R_2 = \propto, \mu = 1.6$

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$
$$\frac{1}{f} = (1.6 - 1)\left(\frac{1}{60}\right) \Rightarrow f = 100\ \text{cm}$$

41. (c) The tension T_1 at the topmost point is given by

$$T_1 = \frac{m_1^2}{20} - mg$$

Centrifugal force acting outward while weight acting downward.

The tension T_2 at the lowest point

$$T_2 = \frac{mv_2^2}{20} + mg$$

Centrifugal force and weight (both) acting downward

$$T_2 - T_1 = \frac{mv_2^2 - mv_1^2}{20} + 2mg$$

$v_1^2 = v_2^2 - 2gh$ or $v_2^2 - v_1^2 = 2g(40) = 80g$

$$\therefore\ T_2 - T_1 = \frac{80mg}{20} + 2mg = 6mg$$

42.(d) Mutual inductance between two coil in the same plane with their centers coinciding is given by

$$M = \frac{\mu_0}{4\pi}\left(\frac{2\pi^2 R_2^2 N_1 N_2}{R_1}\right)\ \text{henry.}$$

43.(c) $[x] = [bt^2]$. Hence $[b] = [x/t^2] = km/s^2$

44.(b) $3 \times \frac{v}{4l_c} = 4 \times \frac{v}{2l_0}$ or $\frac{l_c}{l_0} = \frac{3v}{4} \times \frac{2}{4v} = \frac{3}{8}$

45.(b) $\frac{n_1+n_2}{\gamma-1} = \frac{n_1}{\gamma_1-1} + \frac{n_2}{\gamma_2-1}$

or $\frac{2}{\gamma-1} = \frac{1}{\frac{5}{3}-1} + \frac{1}{\frac{7}{5}-1}$

$\therefore\ \gamma = \frac{3}{2}$.

46.(d) At resonance $\omega L = 1/\omega C$

and $i = E/R$, So power dissipated in circuit is $P = i^2R$.

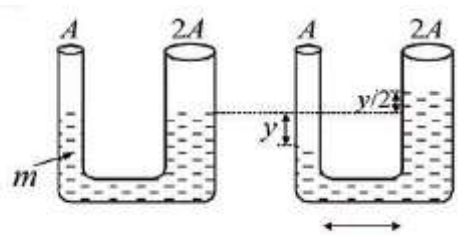
47.(a) Suppose the liquid in left side limb is displaced slightly by y, the liquid in right limb will increase by $y/2$.

The restoring force

$F = -PA$

$= -\rho g\left(\frac{3y}{2}\right) \times 2A = 3\rho gA(-y)$.

$a = \frac{F}{m} = 3\rho gA(-y)/m$



On comparing with, $a = -\omega^2 y$, we get

$\omega = \sqrt{\frac{3\rho gA}{m}}$ and $T = 2\pi\sqrt{\frac{m}{3\rho gA}}$

48. **(d)** Let ' n ' such capacitors are in series and such ' m ' such branch are in parallel.

$\therefore 250 \times n = 1000\ \ \therefore n = 4$ .... (i)

Also $\frac{8}{n} \times m = 16 \Rightarrow m = \frac{16\times n}{8} = 8$ ...... (ii)

$\therefore$ No. of capacitor $= 8 \times 4 = 32$

49.(a) Distance of closest approach

$r_0 = \frac{Ze(2e)}{4\pi\varepsilon_0\left(\frac{1}{2}mv^2\right)}$

Energy, $E = 5 \times 10^6 \times 1.6 \times 10^{-19}$ J

$\therefore r_0 = \frac{9 \times 10^9 \times (92 \times 1.6 \times 10^{-19})(2 \times 1.6 \times 10^{-19})}{5 \times 10^6 \times 1.6 \times 10^{-19}}$

$\Rightarrow r = 5.2 \times 10^{-14}$ m $= 5.3 \times 10^{-12}$ cm

50.(d) $\tau = MB\sin\theta \Rightarrow \tau_{max} = NiAB, [\theta = 90°]$

Chemistry

51. (a) $4KO_2 + 2CO_2 \rightarrow 2\ K_2CO_3 + 3O_2$.

KO_2 is used as an oxidising agent. It is used as air purifier in space capsules. Submarines and breathing masks as it produces oxygen and remove carbon dioxide.

52. (a) Bactericidal are the drugs that kills bacteria. Ofloxacin works by stopping the growth of bacteria. This antibiotic treats only bacterial infections.

53. (b) $\Delta U = q + w$

$= 10 \times 1000 - 2 \times (20) \times 101.3 = 5948\ J$

54. (a) $m = \frac{E \times i \times t}{96500}; 2 = \frac{31.75 \times .5 \times t}{96500},$

$\therefore\ t = 12157.48 sec$

55. (d) Aldol condensation is given by only those aldehydes or ketones which have α-hydrogen atom on a saturated carbon; $\alpha - H$ present on unsaturated carbon atom cannot be easily removed by a base.

56. (d) For a fcc unit cell

$$r = \frac{\sqrt{2}a}{4}$$

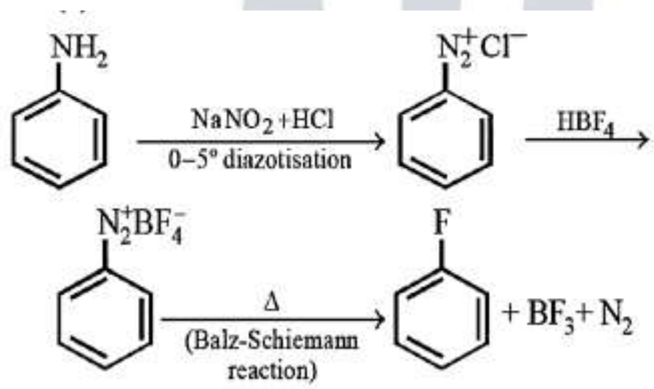
$$a = \frac{4r}{\sqrt{2}} = 2\sqrt{2} \times 0.14 = 0.39 \approx 0.4\ nm$$

57. (b) $PV = RT, PV = \frac{w}{M}RT,$

$$20P = \frac{120}{40} \times .0821 \times 400$$

or $P = 4.92$ atm

58. (d)



59. (b) Benzoic acid is used as preservative as sodium benzoate.

60. (a) Only 1° aromatic amines undergo coupling reactions to form a dye.

61. (c) $H_2^+ : \sigma 1s^1\ \therefore\ B.O. = \frac{1}{2}(1-0) = \frac{1}{2}$

$H_2^- : \sigma 1s^2 \sigma^* 1s^1\ \therefore B.O. = \frac{1}{2}(2-1) = \frac{1}{2}$

Even though the bond order of H_2^+ and H_2^- are equal but H_2^+ is more stable than H_2^- as in the latter, one electron is present in the antibonding $(\sigma * 1\ s)$ orbital of higher energy.

62.(b) Moles of glucose $= \frac{18}{180} = 0.1$

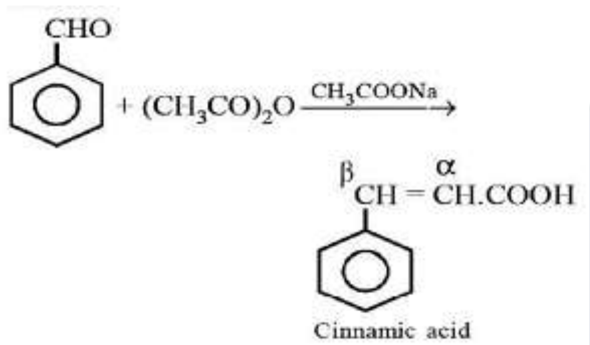
Moles of water $= \frac{178.2}{18} = 9.9$

Total moles $= 0.1 + 9.9 = 10$

p_{H_2O} = Mole fraction × total pressure

$$= \frac{9.9}{10} \times 760$$
$$= 752.4 \text{Torr}$$

63.(c) CO_2, SiO_2 are acidic, CaO is basic and SnO_2 is amphoteric.
64.(a) Since concentration of ions is the same hence $E_{cell} = E^{\circ}_{cell}$
65.(b) Benzaldehyde forms cinnamic acid as follows.



66.(a) $[Fe(H_2O)_5NO]SO_4$
Let O.N. of Fe be x then,

$$1 \times (x) + 5 \times (0) + 1 \times (+1) + 1 \times (-2) = 0$$
$$\therefore \quad x = +1$$

67.(b)

$$C_4H_8 \xleftarrow[(-H_2O)]{\text{Conc.} H_2SO_4} C_4H_{10}O \xrightarrow{\text{Oxidation}} C_4H_8O\ (R-COCH_3)$$

Thus C_4H_8O should be $CH_3CH_2COCH_3$, hence $C_4H_{10}O$ should be $CH_3CH_2CHOHCH_3$

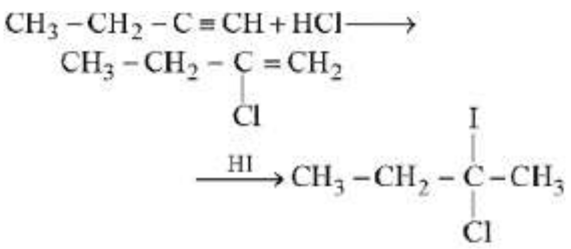
68. **(b)** The required equation is

$$2KMnO_4 + 3H_2SO_4 \longrightarrow$$
$$K_2SO_4 + 2MnSO_4 + 3H_2O + \underset{\text{nascent oxygen}}{5[O]}$$
$$2Fe(C_2O_4) + 3H_2SO_4 + 3[O] \longrightarrow$$
$$Fe_2(SO_4)_3 + 2CO_2 + 3H_2O$$

O required for 1 mol. of $Fe(C_2O_4)$ is 1.5,5O are obtained from 2 moles of $KMnO_4$

$\therefore$ 1.5[O] will be obtained from $= \frac{2}{5} \times 1.5 = 0.6$ moles of $KMnO_4$.

69.(c) This reaction occurs according to Markownikoff's rule which states that when an unsymmetrical alkene undergo hydrohalogenation, the negative part goes to that C -atom which contain lesser no. of H -atom.



70.(a) $\frac{0.80-0.6}{0.80} = x_B; x_B = 0.25$
71. (c) MgO being high melting does not catch fire and hence protects the cooker against fire.
72.(d) $K_2Cr_2O_7$ + conc. HCl $\rightarrow Cl_2$
73.(a) Maximum lowering of vapour pressure will be given by the substance which give maximum number of particles in solution.
74.(d) Positive charge ↑; coagulating power ↑ so, $MgCl_2$ will be most effective.
75.(a) No compound of Ar has yet been reported with F_2
76.(b) Oxygen being more electronegative, will be best oxidising agent among given options.
77. **(c)** $l_{1/2}$ is independent of initial concentration. $\therefore\ t_{1/2} \propto a^0$
78.(b) $\Delta G = \Delta H - T\Delta S$

$= -382.64 + (298 \times 145.6 \times 10^{-3})$
$= -339.3 \text{ kJ mol}^{-1}$

79.(c) Polythene is a linear polymer.
80. **(b)** Since silica is acidic impurity the flux must be basic.

$CaO + SiO_2 \rightarrow CaSiO_3$

81.(d) $A \longrightarrow B, \Delta H = -10 \text{ kJ mol}^{-1}$

It is an exothermic reaction,

$E_{a(b)} = E_{a(A)} - (\Delta H)$
$= 50 - (-10) = 60 \text{ kJ}$

82.(b) $Cl^-$is oxidised to Cl_2 at anode.
83.(b) Density = 1.17 g/cc (Given)

As $d = \frac{\text{Mass}}{\text{Volume}}$

Volume = 1cc $\therefore$ Mass = d = 1.17 g

Molarity $= \frac{\text{No. of moles}}{\text{Volume in litre}} = \frac{1.17\times1000}{36.5\times1}$

$= \frac{1170}{36.5} = 32.05\text{M}$

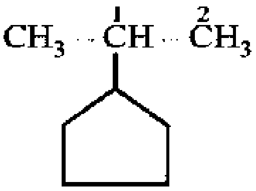
84. **(d)** $LiHCO_3$ is unstable and exists only in solution.
85.(d) P_2O_5 have great affinity for water, so the final product will be orthophosphoric acid.

$$P_4O_{10} \xrightarrow{2H_2O} \underset{\text{Metaphosphoric acid}}{4HPO_3}$$

$$\downarrow 2H_2O$$

$$\underset{\text{orthophosphoric acid}}{4H_3PO_4} \xleftarrow{2H_2O} \underset{\text{Pyrophosphoric acid}}{2H_4P_2O_7}$$

86. **(c)** The cyclic portion contains more C -atoms than acyclic portion. Hence it is derivative of cyclopentane



1-(1-methyl)ethyl cyclopentane

87. (a)

$$2RCOOK \xrightarrow[\text{oxidation}]{\text{Electrolytic}} \underset{\text{Anode}}{2RCOO^-} + \underset{\text{Cathode}}{2K^+}$$

At anode $\quad 2RCOO^- \rightarrow 2RCOO^{\bullet} + 2e^-$

$\quad 2RCOO^{\bullet} \rightarrow R—R + 2CO_2$

At cathode $\quad 2K^+ + 2e^- \rightarrow 2K$

$\quad 2K + H_2O \rightarrow 2KOH + H_2 \uparrow$

88. **(c)** The lesser the electronegativity of halogen in NX_3 the more is the basic character. N can donate more electrons in that case.

89. **(d)** Both drugs are used as antacids.

90. **(d)** Since sucrose is dextrorotatory while hydrolysis product of sucrose, having equimolar mixture of glucose and fructose, is laevorotatory. Hence the hydrolysed product of sucrose is known as invert sugar and the hydrolysis of sucrose is known as inversion.

91. (a) This method is very useful for removing all the oxygen and nitrogen present in the from of impurity in certain metals like Zr & Ti .

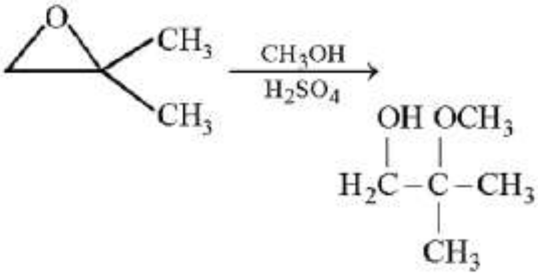
$$Ti + 2I_2 \xrightarrow{523\text{ K}} \underset{\substack{\text{Volatile}\\\text{stable compound}}}{TiI_4} \xrightarrow[\text{Pure metal}]{1700\text{ K}} Ti + 2I_2$$

92.(c) $K_2Cr_2O_7 + 3SO_2 + 4H_2SO_4 \rightarrow K_2SO_4 + Cr_2(SO_4)_3 + 3SO_3 + 4H_2O$

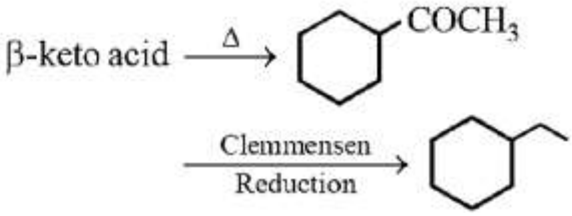
O.N.of chrominum changes from +6 to +3

93.(b) High density polythene is used for manufacturing of buckets, dustbins, pipes etc.

94.(a)



95.(b)



96.(c) $[PtCl_2(NH_3)_4]Br_2$ and $[PtBr_2(NH_3)_4]Cl_2$ are ionisation isomers

97.(b) In lanthanides, there is poorer shielding of 5d electrons by 4f electrons resulting in greater attraction of the nucleus over 5d electrons and contraction of the atomic radii.

98. **(a)** SARAN, a polymer of vinyl chloride $(CH_2 = CHCl)$ and vinylidene chloride, is used for making synthetic hair wigs.

99.(a) Wilkinson's catalyst in $[RhCl(PPh_3)_3]$, redviolet in colour and has square planar structure. It is used for selective hydrogenation of organic molecules at room temperature and pressure. $TiCl_4 + (C_2H_5)_3Al$ is Zeigler Natta catalyst. $(C_2H_5)_4$ Pb is an anti-knocking agent. cis-platin is used as an anti-cancer agent.

100. **(a)** Work done during adiabatic expansion

$$= C_v(T_2 - T_1)$$

or $-3000 = 20(T_2 - 300) \Rightarrow T_2 = 150\text{ K}$

Mathematics

101. **(c)** We define the following events:

A_1: He know the answer

A_2: He does not know the answer

E: He gets the correct answer

Then, $P(A_1) = 9/10, P(A_2) = 1 - 9/10 = 1/10, P(E/A_1) = 1, P(E/A_2) = 1/4$.

Therefore, the required probability is

$$P(A_2/E) = \frac{P(A_2)P(E/A_2)}{P(A_1)P(E/A_1) + P(A_2)P(E/A_2)}$$

$$= \frac{\frac{1}{10} \times \frac{1}{4}}{\frac{9}{10} \times 1 + \frac{1}{10} \times \frac{1}{4}} = \frac{1}{37}.$$

102. **(b)** $\frac{\pi}{2} < x < \pi$ then

$$\sqrt{\frac{1+\cos 2x}{2}} = |\cos x| = -\cos x$$

$$\therefore \int x\sqrt{\frac{1+\cos 2x}{2}}\,dx = -\int x\cos x\,dx$$

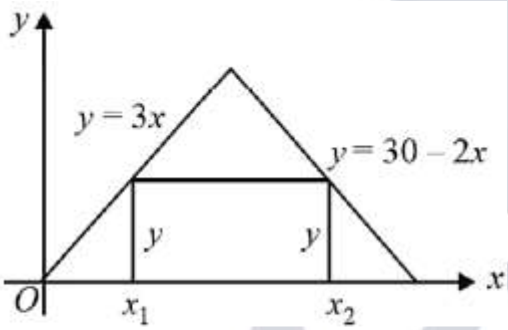
$$= -[x\sin x + \cos x] + c$$

103. **(c)** $A = (x_2 - x_1)y$

$y = 3x_1$ and $y = 30 - 2x_2$

$$A(y) = \left(\frac{30-y}{2} - \frac{y}{3}\right)y$$

$$6A(y) = (90 - 3y - 2y)y = 90y - 5y^2$$
$$6A'(y) = 90 - 10y = 0$$
$$\Rightarrow y = 9; A''(y) = -10 < 0$$
$$x_1 = 3; x_2 = 21/2$$
$$\Rightarrow A_{max} = \left(\frac{21}{2} - 3\right)9 = \frac{15 \times 9}{2} = \frac{135}{2}$$



104. **(d)** We have $f(x) = 4x^3 - 7, x \in \mathbf{R}$.

f is one-one. Let $x_1, x_2 \in R$ and $f(x_1) = f(x_2)$.

$$\Rightarrow 4x_1^3 - 7 = 4x_2^3 - 7 \Rightarrow 4x_1^3 = 4x_2^3$$

$$\Rightarrow x_1^3 = x_2^3 \Rightarrow x_1^3 - x_2^3 = 0$$
$$\Rightarrow (x_1 - x_2)(x_1^2 + x_1x_2 + x_2^2) = 0$$
$$\Rightarrow (x_1 - x_2)\left[\left(x_1 + \frac{x_2}{2}\right)^2 + \frac{3x_2^2}{4}\right] = 0$$

$\Rightarrow x_1 - x_2 = 0$, because the other factor is non zero.

$\Rightarrow x_1 = x_2$ $\therefore$ f is one-one.

f is onto. Let $k \in R$ any real number.

$$f(x) = k \Rightarrow 4x^3 - 7 = k \Rightarrow x = \left(\frac{k+7}{4}\right)^{1/3}$$

Now $\left(\frac{k+7}{4}\right)^{1/3} \in \mathbf{R}$, because $k \in R$ and $f\left[\left(\frac{k+7}{4}\right)^{1/3}\right] = 4\left[\left(\frac{k+7}{4}\right)^{1/3}\right]^3 - 7$

$$= 4\left(\frac{k+7}{4}\right) - 7 = k$$

$\therefore$ k is the image of $\left(\frac{k+7}{4}\right)^{1/3}$
$\therefore$ f is onto.
$\therefore$ f is a bijective function.

105. **(a)** $\sim((\sim p)\wedge q) \equiv \sim(\sim p) \vee \sim q \equiv p \vee (\sim q)$

106. **(c)** $I = \int_1^2 [x^2]dx - \int_1^2 [x]^2 dx$

$$= \int_1^{\sqrt{2}} dx + \int_{\sqrt{2}}^{\sqrt{3}} 2dx + \int_{\sqrt{3}}^{2} 3dx - \int_1^2 1dx$$
$$= 4 - \sqrt{2} - \sqrt{3}$$

107. **(c)** $x(1+y^2)^{1/2}dx + y(1+x^2)^{1/2}dy = 0$

$$\Rightarrow \frac{xdx}{(1+x^2)^{1/2}} + \frac{ydy}{(1+y^2)^{1/2}} = 0$$

Integrating we get

$$2\sqrt{1+x^2} + 2\sqrt{1+y^2} = 2c \text{ or}$$
$$(1+x^2)^{1/2} + (1+y^2)^{1/2} = c$$

108. **(c)** $A^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I_3$

109. **(b)** Let $D(x, y, z)$ be the required point, Then, the mid-point of diagonal BD is

$$\left(\frac{x+1}{2}, \frac{y+2}{2}, \frac{z-4}{2}\right)$$

Also, the mid-point of diagonal AC is

$\left(\frac{3-1}{2}, \frac{-1+1}{2}, \frac{2+2}{2}\right)$ i.e., $(1,0,2)$.

But, the mid-points of the diagonals of a parallelogram always coincide.

$\therefore \frac{x+1}{2} = 1, \frac{y+2}{2} = 0$ and $\frac{z-4}{2} = 2$

So, $x = 1, y = -2$, and $z = 8$.

Hence, the required point is $D(1, -2, 8)$.

110. **(d)** $\int \frac{\sin x + \cos x}{\sqrt{1-\sin 2x}} dx$

$$= \pm\int \frac{\sin x + \cos x}{\sin x - \cos x} dx = \pm\log(\sin x - \cos x) + c$$

111. **(c)** Any plane containing $\frac{x+1}{-3} = \frac{y-3}{2}$

$= \frac{z+2}{1}$ is

$a(x+1) + b(y-3) + c(z+2) = 0$ ….. (i)

where $-3a + 2b + c = 0$ ……. (ii)

If the plane passes through $(0,7,-7)$.

$\therefore\ a+4b-5c=0 \quad \ldots\ldots \text{(iii)}$

From Eqs. (ii) and (iii),

$$\frac{a}{-10-4}=\frac{b}{1-15}=\frac{c}{-12-2}$$
$$\Rightarrow \frac{a}{1}=\frac{b}{1}=\frac{c}{1}$$

Therefore, the plane (i) becomes

$$(x+1)+(y-3)+(z+2)=0$$
$$\Rightarrow x+y+z=0$$

112. **(a)** The equations of line BC are (using two -point form)

$\frac{x-1}{5-1}=\frac{y-4}{4-4}=\frac{z-6}{4-6}$ i.e.,

$$\frac{x-1}{2}=\frac{y-4}{0}=\frac{z-6}{-1}$$

Any point on this line is $(2t+1,4,-t+6)$. If this is the foot of perpendicular from A on the line BC, then d.n. of this perpendicular are

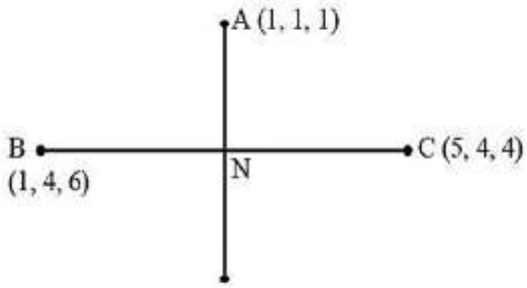
$< 2t+1-1, 4-1, -t+6-1 >$ i.e. $< 2t, 3, -t+5 >$

Using condition of perpendicularity, we have

$(2t)\,(2)+3\times 0+(-t+5)(-1)=0$

$\Rightarrow 5t-5=0 \Rightarrow t=1.$

$\therefore$ Required foot of perpendicular is $(2+1,4,-1+6)=(3,4,5)$.



113. **(b)** $(p \wedge \sim q) \wedge (\sim p \wedge q) = (p \wedge \sim q) \wedge (\sim q \wedge q)$

$= f \wedge f = f$

(By using associative laws and commutative laws)

$\therefore\ (p \wedge \sim q) \wedge (\sim p \wedge q)$ is a contradiction.

114. **(b)** $2^m = 2^n + 56$

$\Rightarrow 2^m - 2^n = 64 - 8 = 2^6 - 2^3$

115. **(d)** For $x < 1, f(x) = \frac{x^2-1}{x^2+2x-3} = \frac{x+1}{x+3}$

$\therefore \lim_{x\to1^-} f(x) = \frac{1}{2}$

For $x > 1, f(x) = \frac{x^2-1}{x^2-2x+1} = \frac{x+1}{x-1}$

$\therefore \lim f(x) = \infty \; x \to 1^+$

$\therefore$ The function is not continuous at $x = 1$.

116. **(a)** Equation of the line through $(1,-2,3)$ parallel

to the line $\frac{x}{2} = \frac{y}{3} = \frac{z-1}{-6}$ is

$\frac{x-1}{2} = \frac{y+2}{3} = \frac{z-3}{-6} = r$ (say) ...... (i)

Then, any point on Eq. (i) is

$(2r+1, 3r-2, -6r+3)$.

If this point lies on the plane $x - y + z = 5$, then

$(2r+1) - (3r-2) + (-6r+3) = 5$

$\Rightarrow -7r + 6 = 5 \Rightarrow r = \frac{1}{7}$

Since, the point is $\left(\frac{9}{7}, -\frac{11}{7}, \frac{15}{7}\right)$.

Distance between $(1,-2,3)$ and $\left(\frac{9}{7}, \frac{11}{7}, \frac{15}{7}\right)$

$$= \sqrt{\left(\frac{4}{49} + \frac{9}{49} + \frac{36}{49}\right)} = \sqrt{\left(\frac{49}{49}\right)} = 1$$

117. **(a)**

$$\int \frac{x + \sin x}{1 + \cos x} dx = \int \frac{x + 2\sin\frac{x}{2}\cos\frac{x}{2}}{2\cos^2\frac{x}{2}} dx$$

$$= \int \left[\frac{x}{2}\sec^2\frac{x}{2} + \tan\frac{x}{2}\right] dx$$

$$= \int x\frac{1}{2}\sec^2\frac{x}{2} dx + \int \tan\frac{x}{2} dx$$

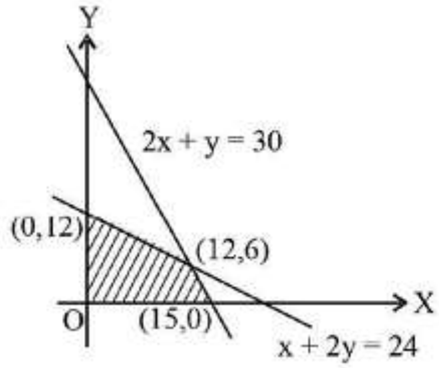
$$= x\tan\frac{x}{2} - \int \tan\frac{x}{2} dx + \int \tan\frac{x}{2} dx + C$$

$$= x\tan\frac{x}{2} + C$$

118. **(b)** Here, $2x + y \leq 30, x + 2y \leq 24, x, y \geq 0$

The shaded region represents the feasible region, hence z = 6x | 8y. Obviously, it is maximum at (12,6).

Hence $z = 12 \times 6 + 8 \times 6 = 120$



119. **(b)** In the interval $\frac{\pi}{3}$ to $\frac{\pi}{2}$, $[x] = 1$

$$\therefore I = \int_{\pi/3}^{\pi/2} x\sin(\pi - x)dx = \int_{\pi/3}^{\pi/2} x\sin x dx$$

$$= [-x\cos x + \sin x]_{\pi/3}^{\pi/2} = 1 - \frac{\sqrt{3}}{2} + \frac{\pi}{6}$$

120. **(b)** We write the given equation as

$$\tan\theta + \tan 4\theta = -\tan 7\theta(1 - \tan\theta\tan 4\theta)$$
$$\Rightarrow \tan(\theta + 4\theta) = -\tan 7\theta \Rightarrow \tan 5\theta = \tan(-7\theta)$$
$$\therefore 5\theta = n\pi + (-7\theta) \text{ or } 12\theta = n\pi$$
$$\therefore \theta = n\pi/12, n \in I$$

121. **(d)** We can write the given expression

$$= \{\hat{\imath}\cdot(\vec{p}\times\vec{q})\}\hat{\imath} + \{\hat{\jmath}\cdot(\vec{p}\times\vec{q})\}\hat{\jmath} + \{\hat{k}\cdot(\vec{p}\times\vec{q})\}\hat{k}$$

$$= \vec{p}\times\vec{q}$$

Since for any vector $\vec{a}$,

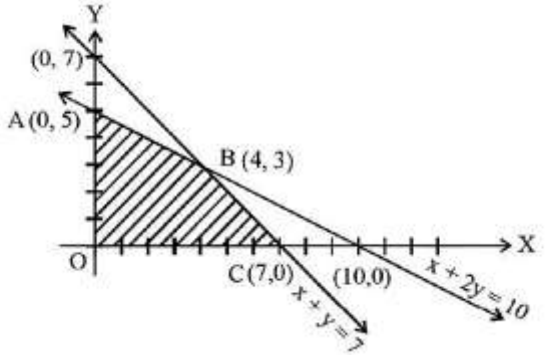
$$\vec{a} = (\vec{a}\cdot\hat{\imath})\hat{\imath} + (\vec{a}\cdot\hat{\jmath})\hat{\jmath} + (\vec{a}\cdot\hat{k})\hat{k}$$

122. **(a)** $f(\theta) = \sin\theta(\sin\theta + \sin 3\theta)$

$$= \sin\theta(2\sin 2\theta\cos\theta)$$

$$= (2\sin\theta\cos\theta)\sin 2\theta = (\sin 2\theta)^2 \geq 0 \forall \theta \in R$$

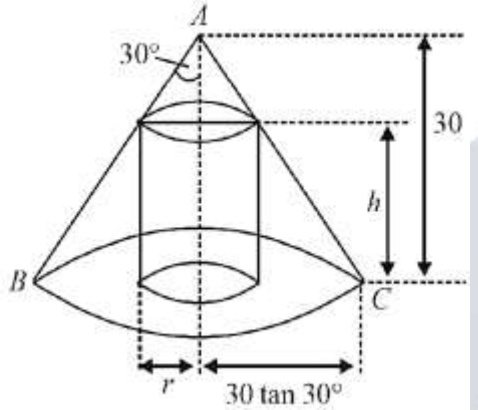
123. **(c)** Change the inequalities into equations and draw the graph of lines, thus we get the required feasible region as shown below.



The region bounded by the vertices $A(0,5), B(4,3), C(7,0)$.

The objective function is maximum at $C(7,0)$ and $\text{Max} z = 5 \times 7 + 2 \times 0 = 35$.

124. (a)



From geometry, we have $\frac{r}{30 \tan 30^\circ} = \frac{30-h}{30}$

or $h = 30 - \sqrt{3}r$

Now, the volume of cylinder,

$V = \pi r^2 h = \pi r^2(30 - \sqrt{3}r)$

Now, let $\frac{dV}{dr} = 0$ or $\pi\left(60r - 3\sqrt{3}r^2\right) = 0$

or $r = \frac{20}{\sqrt{3}}$

Hence, $V_{max} = \pi\left(\frac{20}{\sqrt{3}}\right)^2\left(30 - \sqrt{3}\frac{20}{\sqrt{3}}\right) = \pi\frac{400}{3} \times 10 = \frac{4000\pi}{3}$

125. (a) Any plane through the given line

$2x - y + 3z + 1 + \lambda(x + y + z + 3) = 0$

(From $S + \lambda S' = 0$)

If this plane is parallel to the line $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$, then the normal to the plane is also perpendicular to the above line.

$\therefore\ (2+\lambda)1+(\lambda-1)2+(3+\lambda)3=0$

$\Rightarrow\ (\because l_1l_2+m_1m_2+n_1n_2=0)$

$\Rightarrow\ \lambda=-\dfrac{3}{2}$

and the required plane is $x-5y+3z-7=0$

and the required plane is $x-5y+3z-7=0$.

126. **(c)** $|\vec{a}|=|\vec{b}|=|\vec{c}|=1$

$\vec{a}\cdot\vec{b}=(1)(1)\cos\theta=\cos\theta$ and

$\vec{c}\cdot\vec{a}=\cos\theta, \vec{b}\cdot\vec{c}=\cos\theta$

$$[\vec{a}\vec{b}\vec{c}]^2=\begin{vmatrix}\vec{a}\cdot\vec{a} & \vec{a}\cdot\vec{b} & \vec{a}\cdot\vec{c}\\ \vec{b}\cdot\vec{a} & \vec{b}\cdot\vec{b} & \vec{b}\cdot\vec{c}\\ \vec{c}\cdot\vec{a} & \vec{c}\cdot\vec{b} & \vec{c}\cdot\vec{c}\end{vmatrix}=\begin{vmatrix}1 & \cos\theta & \cos\theta\\ \cos\theta & 1 & \cos\theta\\ \cos\theta & \cos\theta & 1\end{vmatrix}$$

Operate $R_1\to R_1+R_2+R_3$

$$=(1+2\cos\theta)\begin{vmatrix}1 & 1 & 1\\ \cos\theta & 1 & \cos\theta\\ \cos\theta & \cos\theta & 1\end{vmatrix}$$

Operate $C_2\to C_2-C_1; C_3\to C_3-C_1$

$$=(1+2\cos\theta)\begin{vmatrix}1 & 0 & 0\\ \cos\theta & 1-\cos\theta & 0\\ \cos\theta & 0 & 1-\cos\theta\end{vmatrix}$$

$=(1+2\cos\theta)(1-\cos\theta)^2$

$\therefore [\vec{a}\vec{b}\vec{c}]=(1-\cos\theta)\sqrt{1+2\cos\theta}$

127. **(a)** $\sin 2x-\sin 4x+\sin 6x=0$

$\Rightarrow (\sin 2x+\sin 6x)-\sin 4x=0$

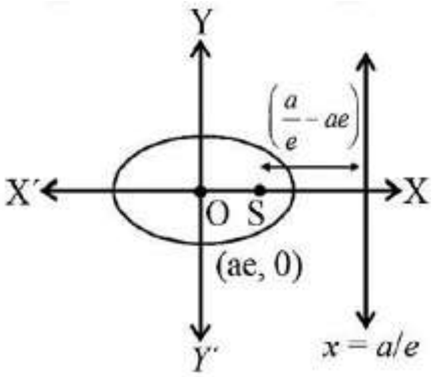
$\Rightarrow 2\sin 4x\cos 2x-\sin 4x=0$

$\sin 4x=0\Rightarrow 4x=n\pi\Rightarrow x=\dfrac{\pi}{4}$

$2\cos 2x-1=0\Rightarrow \cos 2x=\dfrac{1}{2}$

$\Rightarrow 2x=2n\pi\pm\frac{\pi}{3}$ or $x=n\pi\pm\frac{\pi}{6}$

128. **(a)** Perpendicular distance of directrix from focus $=\frac{a}{e}-ae=4\Rightarrow a\left(2-\frac{1}{2}\right)=4\Rightarrow a=\frac{8}{3}$



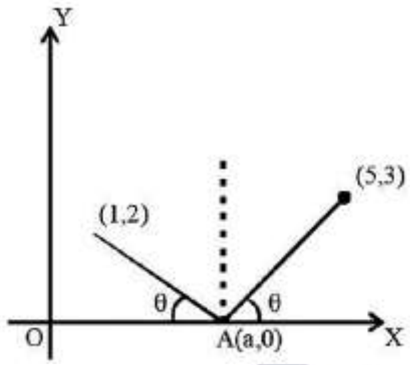
$\therefore$ Semi major axis $= 8/3$

129. **(c)** For any point $P(x, y)$ that is equidistant from the given line, we have

$x + y - \sqrt{2} = -(x + y - 2\sqrt{2})$

or $2x + 2y - 3\sqrt{2} = 0$

130. **(a)** Let the co-ordinates of A be (a, 0). Then the slope of the reflected ray is



$\frac{3-0}{5-a} = \tan\theta$ (say)……… (1)

Then the slope of the incident ray

$= \frac{2-0}{1-a} = \tan(\pi - \theta)$ … (2)

from (1) and (2) $\because \tan\theta + \tan(\pi - \theta) = 0$

$$\Rightarrow \frac{3}{5-a} + \frac{2}{1-a} = 0 \Rightarrow 3 - 3a + 10 - 2a = 0$$

$$\Rightarrow a = \frac{13}{5}$$

Thus, the co-ordinates of A are $\left(\frac{13}{5}, 0\right)$.

131. **(c)** Let $x^2 = \cos 2\theta \Rightarrow 0 \le 2\theta < \frac{\pi}{2} (\because x^2 > 0)$

The expression $= \tan^{-1}\left(\frac{\sqrt{2}\cos\theta + \sqrt{2}\sin\theta}{\sqrt{2}\cos\theta - \sqrt{2}\sin\theta}\right)$

$$= \tan^{-1}\left(\frac{1 + \tan\theta}{1 - \tan\theta}\right) = \frac{\pi}{4} + \theta = \frac{\pi}{4} + \frac{1}{2}\cos^{-1}(x^2)$$

132. **(d)** $y = x^{x^2}, \ln y = x^2 \ln x$

$$\frac{1}{y}\frac{dy}{dx} = 2x\ln x + x^2 \cdot \frac{1}{x} = x(1 + 2\ln x)$$

$$\frac{dy}{dx} = x^{x^2} \cdot x(1 + 2\ln x) = x^{x^2+1}(1 + 2\ln x)$$

133. **(b)** $A + B = 180^\circ - C = 90^\circ$

$a = 2R\sin A, b = 2R\sin B, c = 2R\sin C$

$$\therefore \frac{a^2 - b^2}{a^2 + b^2} = \frac{\sin^2 A - \sin^2 B}{\sin^2 A + \sin^2 B}$$

$$= \frac{\sin(A + B)\sin(A - B)}{\sin^2 A + \sin^2(90^\circ - A)} \ [\because A + B = 90^\circ]$$

$$= \frac{\sin 90^\circ \sin(A - B)}{\sin^2 A + \cos^2 A} = \sin(A - B)$$

134. **(b)** From geometry, the sum of all internal angles $= (n - 2) \times 180^\circ$

where n is the number of sides of the polygon.

$$\therefore \frac{n}{2}[2 \times 120^\circ + (n - 1) \times 5^\circ] = (n - 2) \times 180^\circ$$

$$\Rightarrow n^2 - 25n + 144 = 0 \Rightarrow (n - 16)(n - 9) = 0$$

If $n = 16$ then the 16^{th} internal angle $= 120^\circ + (16 - 1) \times 5^\circ = 195^\circ > 180^\circ$

$\therefore n \neq 16$. Hence $n = 9$

135. **(a)** $0.4096 = {}^5C_1 pq^4$

$0.2048 = {}^5C_2 p^2 q^3$ where $p + q = 1$

$$\Rightarrow p = \frac{1}{5}, q = \frac{4}{5}$$

Mean $= np = 1$

136. **(b)** $y = \sin^{-1}\frac{2x}{1+x^2} = \pi - 2\tan^{-1} x$, for $x > 1$

or $\frac{dy}{dx} = -\frac{2}{1+x^2}$

or $\left(\frac{dy}{dx}\right)_{x=\sqrt{3}} = -\frac{2}{1+3} = -\frac{1}{2}$

Also, when $x = \sqrt{3}, y = \pi - 2 \times \frac{\pi}{3} = \frac{\pi}{3}$

Hence, equation of tangent is

$y - \frac{\pi}{3} = -\frac{1}{2}(x - \sqrt{3})$.

137. **(c)** Desired probability

$= P(A) + P(B) - 2P(A \cap B)$

and $P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)} = \frac{1}{2}$

138. **(a)** Put $\left(1 + \frac{r}{\sqrt{2}}, 2 + \frac{r}{\sqrt{2}}\right)$ in $x^2 + 4xy + y^2 = 0$, we get $3r^2 + 9\sqrt{2}r + 13 = 0$ for which product of the roots is $13/3$.

139. **(b)** $ay + x^2 = 7$, and $x^3 = y$ cuts orthogonally. Now

$\left(\frac{dy}{dx}\right) = -\frac{2x}{a}$ and $\left(\frac{dy}{dx}\right) = 3x^2$

or $\left[\left(-\frac{2x}{a}\right)(3x^2)\right]_{(1,1)} = -1$

or $-\frac{2}{a} \times 3 = -1$ or $a = 6$.

140. **(a)** $\cos[2\cos^{-1} x + \sin^{-1} x]$

$= \cos[\cos^{-1} x + \cos^{-1} x + \sin^{-1} x]$

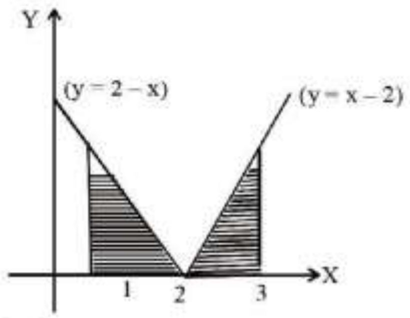
$= \cos[\cos^{-1} x + \pi/2] = -\sin \cos^{-1} x$

$= -\sin \sin^{-1} \sqrt{1 - x^2} = -\sqrt{1 - x^2}$

$$= -\sqrt{1 - \left(\frac{1}{5}\right)^2} = -\sqrt{\frac{24}{25}} = -\frac{2\sqrt{6}}{5}$$

141. **(d)** Since $\sim (p \Rightarrow q) \equiv p \wedge \sim q \sim (\sim p \Rightarrow q) = \sim p \wedge \sim q$

142. **(d)** The required area is shown by shaded region



Required Area

$$A = \int_1^3 |x - 2| dx = 2\int_2^3 (x - 2) dx$$

$$= 2\left[\frac{x^2}{2} - 2x\right]_2^3 = 1$$

143. **(c)** Given, $\frac{dy}{dx} = \frac{y}{x} - \cos^2\left(\frac{y}{x}\right)$

Putting $y = vx$ so that $\frac{dy}{dx} = v + x\frac{dv}{dx}$

We get, $v + x\frac{dv}{dx} = v - \cos^2 v$

$\Rightarrow \frac{dv}{\cos^2 v} = -\frac{dx}{x} \Rightarrow \sec^2 v dv = -\frac{dx}{x}$

Integrating, we get, $\tan v = -\ln x + \ln c$

$\tan\left(\frac{y}{x}\right) = -\ln x + \ln c$

This passes through $\left(1, \frac{\pi}{4}\right) \Rightarrow \ln c = 1 \therefore y = x\tan^{-1}\left(\log\frac{e}{x}\right)$

144. **(a)** Putting $n = 99$ and $p = \frac{1}{2}$, we have $(n+1)p = (100)\left(\frac{1}{2}\right) = 50$

so that the maximum value of $P(X = r)$ occurs at $r = (n+1)p = 50$ and at $r = (n+1)p - 1 = 49$

145. **(a)** Let the progression be $a, a + d, a + 2d$,

Then $x_4 = 3x_1 \Rightarrow a + 3d = 3a \Rightarrow 3d = 2a \ldots.$ (i)

Again $x_7 = 2x_3 + 1$

$\Rightarrow a + 6d = 2(a + 2d) + 1 \Rightarrow 2d = a + 1 \ldots.$ (ii)

Solving (i) and (ii) we get

$a = 3, d = 2$

146. **(c)** The given equation reduces to

$\frac{(x-1)^2}{9} - \frac{y^2}{3} = 1$. Thus $a^2 = 9,\ b^2 = 3$

Using $b^2 = a^2(e^2 - 1)$, we get

$3 = 9(e^2 - 1) \Rightarrow e = \frac{2}{\sqrt{3}}$.

147. **(c)** $2yy_1 = 2c \Rightarrow c = yy_1$

Eliminating c, we get, $y^2 = 2yy_1\left(x + \sqrt{yy_1}\right)$

or $(y^2 - 2xy_1)^2 = 4y^3y_1^3$

It involves only lst order derivative, its degree is 3 as y_1^3 is there.

148. **(a)** $f\{f[f(x)]\} = f\left[f\left(\frac{1}{1-x}\right)\right]$

$= f\left(\frac{1}{1 - \frac{1}{1-x}}\right) = f\left(\frac{x-1}{x}\right)$

$\therefore$ f(x) is not defined for $x = 1$; $f\left(\frac{1}{1-x}\right)$ is not defined for $x = 0$.

$\therefore$ $f\{f[f(x)]\}$ is discontinuous at $x = 0$ and 1 i.e., there are two points of discontinuity.

149. **(c)** It is given that $x = a(\cos t + t\sin t)$ and $y = (\sin t - t\cos t)$. Therefore,

$$\frac{dx}{dt} = a[-\sin t + \sin t + t\cos t] = at\cos t$$
$$\frac{dy}{dt} = a[\cos t - \{\cos t - t\sin t\}] = at\sin t$$
$$\therefore \frac{dy}{dx} = \frac{\left(\frac{dy}{dt}\right)}{\left(\frac{dx}{dt}\right)} = \frac{at\sin t}{at\cos t} = \tan t$$

Then, $\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right)$

$$= \frac{d}{dx}(\tan t)$$
$$= \frac{d}{dt}(\tan t)\frac{dt}{dx}$$
$$= \sec^2 t \cdot \frac{dt}{dx} = \sec^2 t \cdot \frac{1}{at\cos t}$$
$$= \frac{\sec^3 t}{at}$$

150. **(a)** The system is $0x_1 + x_2 - x_3 = 1$

$$-x_1 + 0x_2 + 2x_3 = 2$$
$$x_1 - 2x_2 + 0x_3 = 3$$

$$\Rightarrow \begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & 2 \\ 1 & -2 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \text{ or } AX = B$$

Clearly $|A| = 0$

Now $\text{Adj } A = \begin{bmatrix} 4 & 2 & 2 \\ 2 & 1 & 1 \\ 2 & 1 & 1 \end{bmatrix}$

$\therefore$ (Adj A)B $\neq 0 \Rightarrow$ system is inconsistent