Physics

1. **(b)** For no collision, the speed of car A should be reduced to v_B before the cars meet, i.e. final relative velocity of car A with respect to car B is zero, i.e.

$v_{relative} = 0$

Here, initial relative velocity, $u_r = v_A - v_B$

Relative acceleration, $a_r = -a - 0 = -a$

Let relative displacement be s_r.

Thus, from third equation of motion, we get

$v_{relative}^2 = u_r^2 + 2a_r s_r = (v_A - v_B)^2 - 2as_r$

$$\Rightarrow s_r = \frac{(v_A - v_B)^2}{2a}$$

For no collision, $s \leq s_r$

i.e., $s \leq \frac{(v_A - v_B)^2}{2a}$

2. **(d)** Given, acceleration, $a = (6t + 5) m/s^2$

$$a = \frac{dv}{dt} = 6t + 5$$

$\Rightarrow dv = (6t + 5)dt$

$\Rightarrow \int dv = \int (6t + 5)dt$

$\Rightarrow v = 3t^2 + 5t + c$

where, c is constant of integration.

When $t = 0, v = 0$, so $c = 0$

Therefore, $v = 3t^2 + 5t$

$$\Rightarrow ds = (3t^2 + 5t)dt \left[\because v = \frac{ds}{dt}\right]$$

From 0 to 2 s , we have

$\int_0^s ds = \int_0^2 (3t^2 + 5t)dt$

$$s = \left(t^3 + \frac{5}{2}t^2\right)_0^2 = 8 + 10 = 18 \text{ m}$$

3. **(a)** The distance travelled in nth second is given by

$$s = u + \frac{1}{2}a(2n - 1)$$

$s = \frac{g}{2}(2n-1)$[here , $u = 0, a = g$]

According to the question,

$\frac{11}{36}h = \frac{9.8}{2}(2n-1)$ ...... (i)

From second equation of motion,

$$h = \frac{1}{2}gn^2$$

[$\because u = 0$] ... (ii)

From Eqs. (i) and (ii), we have

$\frac{11}{36} \times \frac{9.8}{2} \times n^2 = \frac{9.8}{2}(2n-1)$ [here, $g = 9.8\ m/s^2$]

$$\Rightarrow 2n - 1 = \frac{11}{36}n^2$$

$$\Rightarrow 11n^2 - 72n + 36 = 0$$

$$\Rightarrow 11n^2 - 66n - 6n + 36 = 0$$

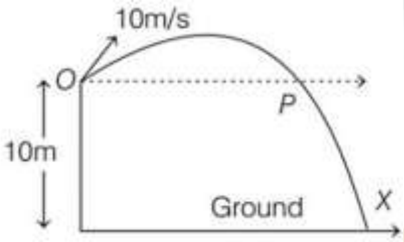
$$\Rightarrow 11n(n-6) - 6(n-6) = 0$$

$$\Rightarrow n = 6$$

(rejecting fractional values).

Therefore, $h = \frac{1}{2} \times 10 \times 6 \times 6 = 180\ m$

4. (b)



The ball will be at a point P when it is at a height of 10 m from the ground So, we have to find the distance OP, which can be calculated directly by considering it as a projectile on a levelled plane OX

Therefore, maximum range, $OP = R = \frac{u^2 \sin 2\theta}{g}$

$$= \frac{10^2 \times \sin(2 \times 30°)}{10} = \frac{10\sqrt{3}}{2} = 5\sqrt{3} = 86\ m$$

5. (b) The maximum tension in the string will be at lowest point i.e.,

$$T_{max} = \frac{mv_1{}^2}{I} + mg$$

and minimum tension in the string will be the highest point i.e., $T_{min} = \frac{mv_2^2}{l} - mg$

Therefore, $\frac{T_{max}}{T_{min}} = \frac{\frac{mv_1{}^2}{l}+mg}{\frac{mv_2{}^2}{l}-mg} = 4$

$\Rightarrow \frac{v_1{}^2+gl}{v_2{}^2-gl} = 4$ ...... (i)

As we know, $v_1{}^2 = v_2{}^2 + 4gl$ .... (ii)

So, from Eqs. (i) and (ii), we get

$v_2^2 + 4gl + gl = 4v_2^2 - 4gl$

$\Rightarrow 3v_2^2 = 9gl$

$v_2^2 = 3gl = 3 \times 10 \times \frac{10}{3}$

$\Rightarrow v_2{}^2 = 100 \Rightarrow v_2 = 10\ m/s$

6. (a) Let 2 kg mass be placed at x = 0, therefore 4 kg mass will be situated at x = 9.

Therefore,

$$x_{COM} = \frac{m_1x_1 + m_2x_2}{m_1 + m_2}$$

$$= \frac{0 + 4 \times 9}{2 + 4} = \frac{36}{6} = 6\ m$$

Thus, centre of mass will be situated at 6 m from 2 kg mass.

7. (d) The rms velocity of gas molecule,

$$v_{rms} = \sqrt{\frac{3RT}{m}}$$

As, $K = \frac{1}{2}mv_{rms}{}^2$

$$= \frac{1}{2}\ m\left(\frac{3RT}{m}\right) = \frac{3}{2}RT$$

$\Rightarrow K = \frac{3}{2}RT$ .... (i)

From ideal gas equation for one mole,

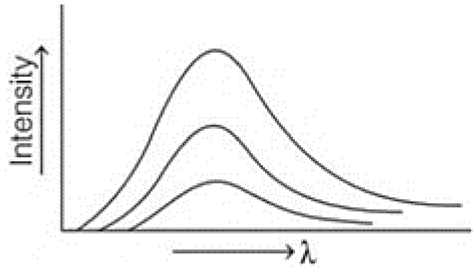
$pV = RT$ ..... (i)

From Eqs. (i) and (ii), we get,

$$p = \frac{2}{3}\frac{K}{V}$$

Hence, pressure is $\left(\frac{2}{3}\right)$ rd of kinetic energy per unit volume of gas.

8. (c) The intensity curve at different wavelength for different temperature is given below



A black body emits all wavelengths.

According to the graph, intensity is less for longer wavelength and more for shorter wavelength.

9. (c) Given, $\gamma = 1.5$

rms velocity of gas molecule,

$$v_{rms} = \sqrt{\frac{3RT}{m}} \Rightarrow T \propto v^2$$

$$\Rightarrow \frac{T_2}{T_1} = \frac{{v_2}^2}{{v_1}^2}$$

Here, $v_2 = \frac{v_1}{2}$

$$\therefore \frac{T_2}{T_1} = \frac{(v_1/2)^2}{{v_1}^2} = \frac{1}{4} \quad \ldots\ldots \text{(i)}$$

For adiabatic process,

$TV^{\gamma-1} = \text{constant}$

$$\Rightarrow \left(\frac{V_2}{V_1}\right)^{\gamma-1} = \left(\frac{T_1}{T_2}\right)$$

$$\Rightarrow \frac{V_2}{V_1} = \left(\frac{T_1}{T_2}\right)^{\frac{1}{\gamma-1}} = (4)^{\frac{1}{1.5-1}} \text{ [using Eq. (i)]}$$

$$= (4)^2 = 16$$

Hence, gas has to be expanded to 16 times.

10. (b) According to first law of thermodynamics,

$\Delta U = \Delta Q - dW$

It is given that, $dW = 0$ and $\Delta Q < 0$

Thus, $\Delta U = C_V dT = \Delta Q$ is negative

Since, C_V is specific heat, which remains constant, the temperature will decrease.

11. (b) As, we know in SHM,

$v^2 = \omega^2(a^2 - x^2)$

$\Rightarrow {v_1}^2 = \omega^2(a^2 - {x_1}^2) \ldots\ldots$ (i)

and ${v_2}^2 = \omega^2(a^2 - {x_2}^2)$ …. (ii)

Subtracting Eq. (i) from Eq. (ii), we get

${v_2}^2 - {v_1}^2 = \omega^2({x_1}^2 - {x_2}^2)$

$= \frac{4\pi^2}{T^2}({x_1}^2 - {x_2}^2)\left(\because \omega = \frac{2\pi}{T}\right)$

$\Rightarrow T = 2\pi\sqrt{\frac{{x_1}^2 - {x_2}^2}{{v_2}^2 - {v_1}^2}}$

12. (b) Given, mass, $m = 0.25$ kg

Force constant, $k = 400\ \mathrm{N/m}$

Amplitude of oscillations, $A = 4\ \mathrm{cm} = 0.04\ \mathrm{m}$

Angular frequency, $\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{400}{0.25}}$

$= \sqrt{1600} = 40\mathrm{rad/s}$

Velocity at equilibrium position $= \omega A$

$= 40 \times 0.04 = 1.6\ \mathrm{m/s}$

13. (c) Given,

$v = 360\ \mathrm{ms^{-1}}$

$\nu = 500\ \mathrm{Hz}$

$\Delta\varphi = 60^\circ = \frac{\pi}{3}$

Since, velocity of a wave, $v = \nu\lambda$

$\Rightarrow \lambda = \frac{v}{\nu} = \frac{360}{500} = 0.72\ \mathrm{m}$

As, phase difference, $\Delta\varphi = \frac{2\pi}{\lambda} \times$ path difference (Δx)

$\Rightarrow \Delta x = \frac{\lambda}{2\pi} \times \Delta\varphi$

$= \frac{0.72}{2\pi} \times \frac{\Pi}{3} = 0.12\ \mathrm{m}$

14. (b) Given, $\lambda_P = 100\ \mathrm{mm} = 0.10\ \mathrm{m}$

$\lambda_Q = 0.25$ m

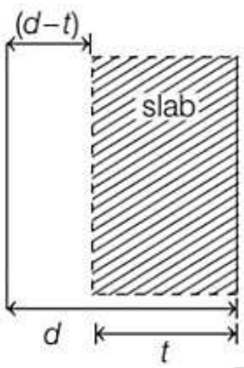
$v_P = 80 cms^{-1} = 0.80\ ms^{-1}$

Since, frequency of wave remains same in the two media,

$$\frac{v_P}{\lambda_P} = \frac{v_Q}{\lambda_Q}\left(\because v = \frac{v}{\lambda}\right)$$

$$\Rightarrow v_Q = \frac{\lambda_Q}{\lambda_P} \times v_P = \frac{0.25}{0.10} \times 0.80 = 2\ ms^{-1}$$

15. (c) The given situation can be shown as



The capacitance of a parallel plate capacitor is given by

$$C = \frac{k\varepsilon_0 A}{d}$$

The above circuit can be considered to be combination of two series capacitors as

$$\therefore \frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{(d-t)}{\varepsilon_0\ A} + \frac{t}{k\varepsilon_0\ A}$$

$$= \frac{k(d-t)+t}{k\varepsilon_0\ A} = \frac{d-t+\frac{t}{k}}{\varepsilon_0\ A} = \frac{d-t\left(1-\frac{1}{k}\right)}{\varepsilon_0\ A}$$

$$\Rightarrow C = \frac{\varepsilon_0\ A}{d-t\left(1-\frac{1}{k}\right)}$$

16. (d) At balanced condition

$$\frac{P}{Q} = \frac{S}{R} \Rightarrow \frac{22}{200} = \frac{30}{300}$$

So, it can be balanced again by increasing the resistance S by 3Ω or by increasing Q by 20Ω.

17. (b) According to the question,

Magnetic field at centre of coil A = Magnetic field at centre of coil B

$$\frac{\mu_0 I_1}{2(2r)} = \frac{\mu_0 I_2}{2r}$$

$\Rightarrow \frac{l_1}{l_2} = 2$ .... (i)

We know, $R = \rho\left(\frac{l}{A}\right)$, where ρ is resistivity, I is length

and A is area of cross-section.

$$\Rightarrow l_1 = \frac{V_1}{R_1} = \frac{V_1}{\rho\left(\frac{l_1}{A}\right)}$$

$\Rightarrow \frac{V_1}{l_1} = \rho \cdot \frac{l_1}{A}$ ..... (ii)

and $l_2 = \frac{V_2}{R_2} = \frac{V_2}{\rho\cdot\left(\frac{l_2}{A}\right)}$

$\Rightarrow \frac{V_2}{l_2} = \rho \cdot \frac{l_2}{A}$ ..... (iii)

From Eqs. (ii) and (iii), we get

$$\frac{V_1}{l_1} \times \frac{l_2}{V_2} = \frac{l_1}{l_2} = \frac{2\pi r_1}{2\pi r_2}$$

$$\Rightarrow \frac{V_1}{V_2} \cdot \frac{l_2}{l_1} = \frac{r_1}{r_2}$$

$$\Rightarrow \frac{V_1}{V_2} \cdot \frac{l_2}{l_1} = \frac{2r}{r}$$

$\Rightarrow \frac{V_1}{V_2} = 2\frac{2}{l_2} = 2 \times 2$ [from Eq. (i)]

$$\Rightarrow \frac{V_1}{V_2} = \frac{4}{1} = 4:1$$

18. (c) In given case, the magnetic moment of the pieces get halved

i.e., $M_1{}' = M_2{}' = \frac{M_1}{2}$

The magnetic moment of given arrangement,

$$M_2 = \sqrt{(M_1)^2 + (M_2)^2}$$

$$= \sqrt{\left(\frac{M_1}{2}\right)^2 + \left(\frac{M_1}{2}\right)^2} = \frac{M_1}{\sqrt{2}}$$

$$\Rightarrow \frac{M_1}{M_2} = \sqrt{2}$$

19. (d) We know that, wavelength of spectrum in hydrogen atom is given as

$$\frac{1}{\lambda} = R\left(\frac{1}{{n_1}^2} - \frac{1}{{n_2}^2}\right)$$

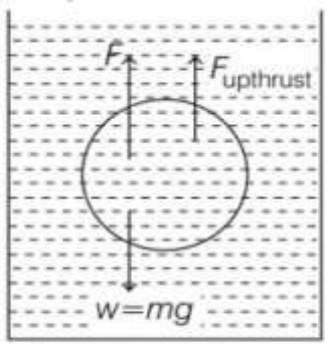
For the series limit of Balmer series, $n_1 = 2$ and $n_2 = \infty$

$$\therefore \frac{1}{\lambda} = R\left[\frac{1}{2^2} - \frac{1}{\infty^2}\right]$$

$$\Rightarrow \frac{1}{\lambda} = \frac{R}{4} \Rightarrow \lambda = \frac{4}{R} \Rightarrow \frac{C}{f} = \frac{4}{R} [\because c = f\lambda]$$

$$\Rightarrow f = \frac{Rc}{4}$$

20. **(b)** The given situation is shown below



When ball moves with terminal velocity (v_t), then

$$F_{viscous} + F_{upthrust} = W$$

$$Kv_t^2 + V\rho_2 g = mg$$

$$\Rightarrow Kv_t^2 + V\rho_2\, g = V\rho_1\, g$$

$$\Rightarrow Kv_t^2 = V(\rho_1 - \rho_2)g$$

$$v_t = \sqrt{\frac{V(\rho_1 - \rho_2)g}{K}}$$

21. (a) Given, charge on particle, $q = 3e$

Mass, $m = 2\text{ m}$
Force on charged particle in electric field,

$$F = qE = 3eE$$

$\therefore$ Acceleration imparted to charged particle,

$$a = \frac{F}{m} = \frac{3eE}{2\text{ m}}$$

22. (d) According to given situation, truth table can be given as

Input		Output	
A	B	AB	$(Y) = \overline{(AB)}$
0	0	0	1

0	1	0	1
1	0	0	1
1	1	1	0

Hence the given logic gate is NAND gate

23.(c) Given, wavelength of used light, $\lambda_1 = 6000\text{Å}$

Initial angular separation, $\beta_1 = \beta$

Final angular separation, $\beta_2 = \beta - 30\%$ of β

$\beta_2 = \beta - 0.3\beta = 0.7\beta$

We know that, $\beta \propto \lambda$

$11 \Rightarrow \frac{\beta_1}{\beta_2} = \frac{\lambda_1}{\lambda_2}$

$\Rightarrow \lambda_2 = \frac{\beta_2 \times \lambda_1}{\beta_1} = \frac{0.7\beta \times 6000}{\beta} = 4200\text{Å}$

24.(c) Given, $V = 50\text{ V},\ V_L = 90\text{ V},\ V_C = 60\text{ V}$

In $L - C - R$ circuit,

$\therefore V = \sqrt{{V_R}^2 + (V_L - V_C)^2}$

$\Rightarrow V^2 = V_R^2 + (V_L - V_C)^2$

$50^2 = {V_R}^2 + (90 - 60)^2$

$\Rightarrow 2500 = {V_R}^2 + 900$

$\Rightarrow {V_R}^2 = 1600$

$\Rightarrow V_R = \sqrt{1600} = 40\text{ V}$

25.(c) Given, $r = 20\text{ cm} = 0.2\text{ m}, t = 0.5\text{ s}, v = 4t$ and $m = 5\text{ kg}$

Radial acceleration, $a_r = \frac{v^2}{r} = \frac{(4t)^2}{0.2} = \frac{16t^2}{0.2} = 80t^2$

$= 80 \times (0.5)^2$

At, $= 20\text{ ms}^{-2}$

Tangential acceleration of particle,

$a_t = \frac{dv}{dt} = \frac{d}{dt}(4t) = 4\text{ ms}^{-2}$

$\therefore$ Net acceleration,

$a_n = \sqrt{a_r^2 + a_t^2} = \sqrt{(20)^2 + (4)^2} = 4\sqrt{26}\ ms^{-2}$

So, net force, $F_n = ma_n = 5 \times 4\sqrt{26}$

$= 20\sqrt{26}$ N

26.(a) Given, $r = 60$ cm $= 0.6$ m

$m = 1$ kg

$\omega_1 = 3rev/s = 2\pi \times 3rad/s$

$\omega_2 = 1rev/s = 2\pi \times 1rad/s$

$t = 30$ s

Torque, $t = I\alpha = I\frac{d\omega}{dt} = mr^2\frac{d\omega}{dt}$

$= 1 \times (0.6)^2 \times \frac{2\Pi(3) - 2\Pi(1)}{30}$

$= 0.15$ N − m

27. (b) The coefficient of mutual induction is given by

$M = \frac{\mu_0 N_1 N_2 A}{l}$ .... (i)

where, μ_0 is the permeability of free space, N_1 is the number of turns in primary coil, N_2 is the number of turns in secondary coil, A is the common area of cross-section and l is the length of coils.

Thus, for toroid the Eq. (i) is given as

$$M = \frac{\mu_0 N \cdot N \cdot \pi R^2}{2\pi r} = \frac{\mu_0 N^2 R^2}{2r} \begin{bmatrix} \because N_1 = & N_2 = N \\ A = \pi R^2 & l = 2\pi r \end{bmatrix}$$

where, R is the major radius and r is the minor radius.

28.(a) Since, the voltage across inductor and capacitor is same, so they are in resonance i.e., $X_L = X_C$

The impedance of circuit,

$Z = \sqrt{R^2 + (X_L - X_C)^2} = R$

∴ Voltage across, $R = 220$ V

By Ohm's law, V = I R

$\Rightarrow I = \frac{V}{R} = \frac{220}{100} = 2.2$ A

Hence, ammeter reading is 2.2 A and voltmeter reading is 220 V.

29.(b) For electron, $\lambda_e = \frac{h}{\sqrt{2mE_e}} \Rightarrow E_e = \frac{h^2}{(2\ m)\lambda_e{}^2}$

For photon, $\lambda_p = \frac{hc}{E_p}$

$$\Rightarrow E_p = \frac{hc}{\lambda_p} = \frac{hc}{2\lambda_e} (\because \lambda_p = 2\lambda_e)$$

$$\Rightarrow \frac{E_p}{E_c} = \frac{hc}{2\lambda_e} \times \frac{2\ m\lambda_e{}^2}{h^2} = mc \times \frac{\lambda_e}{h}$$

Also, $\lambda_e = \frac{h}{mv_e} \Rightarrow \frac{\lambda_e}{h} = \frac{1}{mv_e}$

$$\Rightarrow \frac{E_p}{E_e} = mc \times \frac{1}{mv_c} = \frac{c}{v_e}$$

$$= \frac{c}{c/100} \left(\because v_e = \frac{c}{100}\right)$$

$$= 100 = 10^2$$

30. **(d)** If A_1 and A_2 are the mass number of two parts.

The radius of nucleus is given by

$R = R_0(A)^{1/3}$

So, $R_1 = R_0(A_1)^{1/3}$ and $R_2 = R_0(A_2)^{1/3}$

$$\Rightarrow \frac{R_1}{R_2} = \left(\frac{A_1}{A_2}\right)^{1/3}$$

$$\Rightarrow \frac{A_1}{A_2} = \left(\frac{R_1}{R_2}\right)^3 = \left(\frac{1}{2}\right)^3 = \frac{1}{8}$$

$\therefore$ Ratio of masses, $\frac{m_1}{m_2} = \frac{A_1}{A_2} = \frac{1}{8}$

From conservation of momentum,

$\Rightarrow m_1v_1 = m_2v_2$
$\Rightarrow \frac{v_1}{v_2} = \frac{m_2}{m_1} = \frac{8}{1}$ or 8: 1

31. **(b)** Given, voltage gain = 40, $R_{in} = 100\Omega$,

$R_{out} = 400\Omega$

Since, voltage gain $= \beta\left(\frac{R_{out}}{R_{in}}\right)$

$\Rightarrow \beta =$ Voltage gain $\times \frac{R_{in}}{R_{out}}$

$$= 40 \times \frac{100}{400} = 10$$

Power gain $= \beta \times$ voltage gain

$= 10 \times 40 = 400$

32.(d) Given, mass of both particle = m

Charge of particle, A = +q

Charge of particle, B = +4q

Potential difference = V

Kinetic energy is given by

$K = \frac{1}{2}mv^2 \ldots. (i)$

This energy is equal to electrostatic potential energy is

$K = V \times Q \cdots\cdots (ii)$

From Eqs. (i) and (ii), we have

$$\frac{1}{2}mv^2 = V \times Q$$

For particle A,

$qV = \frac{1}{2}mv_A^2 \ldots (iii)$

For particle B,

$4qV = \frac{1}{2}m{v_B}^2 \ldots. (iv)$

Dividing Eq. (iii) by Eq. (iv), we get

$$\frac{1}{4} = \frac{{v_A}^2}{{v_B}^2}$$

$$\Rightarrow \frac{v_A}{v_B} = \frac{1}{2}$$

Hence, the ratio of their speed $\frac{v_A}{v_B} = \frac{1}{2}$

33.(d) $E = -\frac{\partial V}{\partial x}\hat{i} - \frac{\partial V}{\partial y}\hat{j} - \frac{\partial V}{\partial z}\hat{k}$

$$\Rightarrow E_x = -\frac{\partial V}{\partial x} = -\frac{d}{dx}\left[\frac{20}{x^2 - 4}\right] = \frac{40x}{(x^2 - 4)^2}$$

$\Rightarrow E_x$ at $x = 4\mu m = \frac{10}{9}\ V/\mu m$

and is along + ve x-direction

34.(c) Given, length of wire, AB = L

Resistance of wire, AB = 12r

emf of cell D = ε, internal resistance of D = r

emf of cell C = $\frac{\varepsilon}{2}$, internal resistance of C = 3r

Current in potentiometer wire (i) = $\frac{\text{Total emf}}{\text{Total resistance}}$

$$i = \frac{\varepsilon}{r + 12r} = \frac{\varepsilon}{13r}$$

Potential drop across the balance length AJ of potentiometer wire is $V_{AJ} = i \times R_{AJ}$

$\Rightarrow V_{AJ} = i$ (resistance per unit length × length AJ)

$$V_{AJ} = i\left(\frac{12r}{L} \times x\right)$$

where, x is the balance length AJ.

As null point occurs at J, so potential drop across balance length,

AJ = emf of cell C

$$V_{AJ} = \frac{\varepsilon}{2} \Rightarrow i\left(\frac{12r}{L} \times x\right) = \frac{\varepsilon}{2}$$

$$\Rightarrow \frac{\varepsilon}{13r} \times \frac{12r}{L} \times x = \frac{\varepsilon}{2}$$

$$\Rightarrow x = \frac{13}{24} L$$

35.(c) For CE n − p − n transistor, DC current gain,

$$B_{DC} = \frac{I_C}{I_B}$$

At saturation state, V_{CE} becomes zero,

$$\therefore V_{CC} - I_C R_C = 0$$

$$I_C \approx \frac{V_{CC}}{R_C} = \frac{20}{4000} = \frac{1}{200}\text{ A}$$

Hence, saturation base current,

$$I_B = \frac{I_C}{\beta_{DC}} = \frac{1}{200 \times 300} = \frac{1}{60000}\text{ A} = 16.66\mu\text{A}$$

36.(a) According to deduction of Kepler's third law and with the help of Newton's law, the law of period is given by

$$\text{or } T^2 = \frac{4\pi^2}{GM} r^3$$

$$\Rightarrow T = 2\pi\sqrt{\frac{r^3}{GM}}$$

where, r is the radius of orbit and M is mass of the planet or star.

As satellite is very close of the planet, thus r = R

Also, mass (M) = density (ρ) × volume (V)

$= \rho \times \frac{4}{3}\Pi R^3$

$$\therefore T = 2\pi\sqrt{\frac{R^3}{G \times \frac{4}{3}\Pi R^3}} = \sqrt{\frac{3\pi}{\rho G}}$$

37. (c) According to Kepler's third law, time period

$$T^2 = \frac{4\Pi^2}{GM}a^3$$

where, a is the semi-major axis,

$$\Rightarrow a = \left[\frac{76 \times 86400 \times 365 \times 6.67 \times 2 \times 10^{30}}{4 \times 3.14 \times 3.14}\right]^{1/3}$$

$= 2.7 \times 10^{12}$ m

Also, in case of ellipse,

2a = perihelion + aphelion

⇒ Aphelion = 2a − perihelion

$= 2 \times 2.7 \times 10^{12} - 8.9 \times 10^{10}$

$\approx 5.3 \times 10^{12}$ m

38.(a) From the principle of calorimetry,

$m_1 s_1 \Delta T_1 = m_2 s_2 \Delta T_2$

$540 \times S_w(80 - T) = 540 \times \frac{S_w}{2} \times (T - 0)$ where, s_w is specific heat of water.

$$\Rightarrow T = \frac{160}{3}\ ^\circ C = 53.3^\circ C$$

39.(d) According to Newton's law of cooling,

$$\frac{T_1 - T_2}{2} = K\left(\frac{T_1 + T_2}{2} - T_0\right)$$

$$\frac{365 - 361}{2} = K\left[\frac{365 + 361}{2} - 293\right]$$

$$K = \frac{1}{35}$$

$$\Rightarrow t = \frac{14}{10}\text{min} = 84 \text{ s}$$

40. **(b)** Molecular weight of the mixture,

$$M_{mix} = \frac{n_1M_1 + n_2M_2}{n_1 + n_2}$$

$$= \frac{1\times 4 + 2\times 32}{1+2} = \frac{68}{3}\times 10^{-3}\text{kgmol}^{-1}$$

For helium, $C_{V_1} = \frac{3}{2}R$

For oxygen, $C_{V_2} = \frac{5}{2}R$

$$(C_V)_{mix} = \frac{n_1C_{V_1} + n_2C_{V_2}}{n_1 + n_2}$$

$$= \frac{1 + \frac{3R}{2} + 2\times\frac{5R}{2}}{1+2} = \frac{13R}{6}$$

Now, $\left(C_p\right)_{mix} = (C_V)_{mix} + R$

$$= \frac{13R}{6} + R = \frac{19}{6}R$$

$$r_{mix} = \frac{\left(C_p\right)_{mix}}{(C_V)_{mix}} = \frac{19}{13}$$

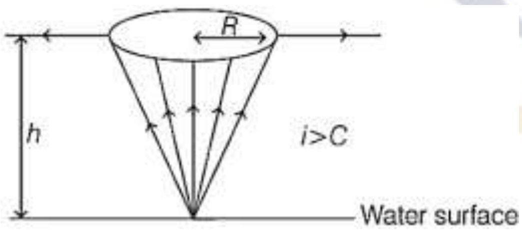
Speed of sound in the mixture, $v = \sqrt{\frac{r_{mix}\times RT}{M_{mix}}}$

$$= \sqrt{\frac{19}{13}\times\frac{8.31\times 300}{\frac{68}{3}\times 10^{-3}}}$$

$\approx 401\ \text{ms}^{-1}$

41. **(b)** The law of reflection is true for all points of the remaining part of the mirror, so the image will be that of the whole object. However, as the area of the reflecting surface has been reduced, hence the intensity of the image will be low.

42. **(b)** The given situation is shown below



From the figure we can see the light from the LED will not emerge out of the water, if at the edge of the disc, the incidence angle is greater than critical angle.

i.e., $i > C$

or $\sin i > \sin C$

Now, if R is the radius of disc and λ is the depth of the LED, then

$\sin i = \frac{R}{\sqrt{R^2+h^2}}$ and $\sin C = \frac{1}{\mu}$

From Eq (i), we have

$$\frac{R}{\sqrt{R^2+h^2}} > \frac{1}{\mu} \Rightarrow R > \frac{h}{\sqrt{\mu-1}}$$

43.(c) The angle between any two lines is equal to the angle between their perpendiculars.

$\therefore$ i = 30°

From Snell's law, we have

$$\frac{1}{1.5} = \frac{\sin 30°}{\sin r}$$

$\Rightarrow \sin r = 0.75$ or $r = 48.6°$

Therefore, $\theta = r - i$

$= 48.6° - 30° = 18.6°$

$\therefore$ Required angle between two emergent rays

$= 2 \times 18.6$

$= 37.2°C \approx 37°$

44.(a) From the principle of superposition, we have

$$F_{net} = \frac{1}{4\Pi\varepsilon_0}\frac{Q \times q}{\left(\frac{d}{2}\right)^2} + \frac{1}{4\Pi\varepsilon_0}\frac{Q \times Q}{(d)^2}$$

Since, $F_{net} = 0$

$$\Rightarrow \frac{1}{4\Pi\varepsilon_0}\frac{Q \times q}{\left(\frac{d}{2}\right)^2} = -\frac{1}{4\Pi\varepsilon_0}\frac{Q \times Q}{d^2} \Rightarrow q = -\frac{Q}{4}$$

45.(d) Potential drop across silicon diode in forward bias is around 0.7 V.

In the given circuit, potential drop across 300Ω resistor is

$$\Delta V = IR$$
$$\Rightarrow I = \frac{\Delta V}{R} = \frac{5-0.7}{300} = 0.01433\ A$$
$$\text{or } I = 14.33\ mA$$

46.(d) We know that, fringe width,

$$\beta = \frac{D\lambda}{d}$$

$\Rightarrow \beta \propto D$

Hence, graph (d) is the correct

47.(d) According to Bohr's model, the kinetic energy of moving electron in nth orbit is given as

$K = \frac{Rhc}{n^2}$ ... (i)

where, R = Rydberg constant, h = Planck's constant and c = speed of light.

Similarly, potential energy of electron moving in nth orbit,

$P = -2\frac{Rhc}{n^2}$.... (ii)

From Eqs. (i) and (ii), we have

$P = -2K$

Total energy of electron moving in nth orbit,

$E = K + P = K - 2K$

$E = -K$

$\Rightarrow K = -E = -(-3.4eV)$ (given $E = -3.4eV$)

$K = 3.4eV$

From Eq. (iii), we have

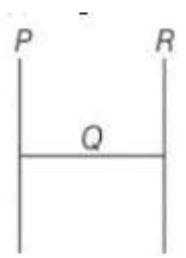
$P = -2K = -2(3.4) = -6.8eV$

$\because P - K = -6.8 - 3.4 = -10.2eV$

and $K - P = 3.4 - (-6.8) = 10.2eV$

48. **(b)** In most of liquids, the cause of dynamic viscosity is the intermolecular force of attraction or cohesive forces and on increasing the temperature, molecules of liquids try to move away from each other which in turn reduces the cohesive force and hence dynamic viscosity decreases with increase in temperature in case of liquid

49.(c) The given situation can be shown as



So, moment of inertia of the system about P,

$I = I_P + I_Q + IR$

$= 0 + \left(\frac{MI^2}{12} + \frac{MI^2}{4}\right) + MI^2 = \frac{4}{3}MI^2$

50.(b) Let initial radius be r_1.

Surface area of 1000 small water drops $= 1000 \times 4\pi {r_1}^2$

Volume of 1000 small drops $= 1000 \times \frac{4}{3}\pi r_1{}^3$

Let final radius be r_2.

Since, initial volume = final volume

$$1000 \times \frac{4}{3}\pi r_1^3 = \frac{4}{3}\pi r_2^3$$

$$\Rightarrow 1000 r_1^3 = r_2^3 \Rightarrow \frac{r_2}{r_1} = 10$$

Let us assume surface tension remain constant, i.e. S.

$$= \frac{S \times 4\pi r_2^2}{S \times 4\pi r_1^2 \times 1000}$$

$$= \left(\frac{r_2}{r_1}\right)^2 \times \frac{1}{1000} = \frac{1}{10}$$

Chemistry

51. (c) Last element of lanthanoid series is Lu i.e. leutetium with atomic number 71 and mass number 175.5. It is a silver white metal, which resists corrosion in dry air, but not in moist air.

52. (d) Equivalent mass of copper

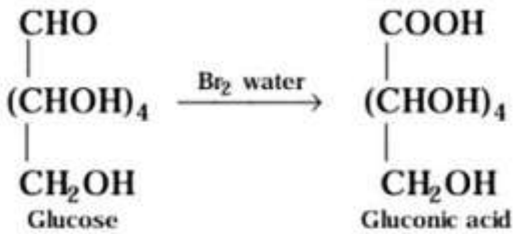
$$= \frac{\text{Atomic mass}}{\text{Valency}} = \frac{63.5}{2} = 31.75$$

Amount of copper deposited by 0.05 F

$= 0.05 \times 3175 = 1.5875 \sim \text{eq } 1.59$ g

53. (c)

54. (b) Glucose gets oxidised to six carbon. Carboxylic acid (gluconic acid) on reaction with a mild oxidising agent like bromine water. This indicates that the carbonyl group is present as an aldehyde group.



55. (b) For density, $\rho = \frac{Z \times M}{a^3 \times N_A}$

Edge length, $a = 4\text{Å} = 4 \times 10^{-8}$ cm

For bcc structure, Z = 2

$$\rho = \frac{2 \times 39}{(4 \times 10^{-8})^3 \times 6.022 \times 10^{23}}$$
$$= 0.202 \times 10^{24} \times 10^{-23} \text{gcm}^{-3}$$
$$= 0.202 \times 10 \text{gcm}^{-3} = 2.02 \text{gcm}^{-3}$$

56. (b) The formula used to determine the molar mass of solute from depression in freezing point is

$$M_2 = \frac{1000 \times K_f \times W_2}{\Delta T_f \times W_1}$$

where, W_2 = weight of solute

W_1 = weight of solvent

& K_f = molal depression constant

57. (b)

$T_1 = 273$ K
$T_2 = 373$ K
$V_1 = 10$ L
$V_2 = ?$

From Charles law,

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$
$$\& \ V_2 = 373 \times \frac{10}{273} = 13.66 \text{ L} = 13.66 \text{ d m}^3$$

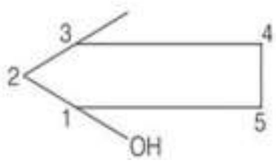
58. (c) $T_1 = 50$ K for H_2

$T_2 = 800$ K for O_2

$$u_{ms} = \sqrt{\frac{3RT}{M}}$$

$$\Rightarrow \frac{u_{H_2}}{u_{O_2}} = \sqrt{\frac{T_1}{M_1} \times \frac{M_2}{T_2}} = \sqrt{\frac{50}{2} \times \frac{32}{800}} = 1$$

59. (c) The correct IUPAC name of the given compound is 3 -methylcyclopentanol.



60. **(c)** Given, C = 42.8%, H = 7.2% and N = 50%

Element	%	Moles	Simplest ratio	Whole number

C	42.8	$\frac{42.8}{12}=3.56$	1	1
H	7.2	$\frac{7.2}{1}=7.2$	2	2
N	50	$\frac{50}{14}=3.57$	1	1

$\therefore$ Empirical formula $=CH_2N$

Empirical formula mass $=28$

Now, number of moles $=\frac{\text{volume given at STP}}{22400\text{ mL}}$

$=\frac{50}{22400}=2.23\times10^{-3}$

Also, number of moles $=\frac{\text{weight}}{\text{molecular weight}}$

$\Rightarrow$ Molecular weight $=\frac{1}{2.23\times10^{-3}}=448$

$\therefore n=\frac{\text{molecular weight}}{\text{empirical weight}}=\frac{448}{28}=16$

Hence, molecular formula $=(\text{empirical formula})_n$

$=(CH_2N)_{16}=C_{16}H_{32}N_{16}$

61. (b) $w=12$ g of $BaCl_2$, $W=100$ g of solution.

For 50 g of solution, $w=6$ g of $BaCl_2$

$W=50$ g of solution

$\therefore w_{BaCl_2\cdot 2H_2O}=\left(\frac{6\times244}{208}\right)\text{g}=7.038\text{ g}$

$w_{H_2O}=(50-7.038)\text{g}=42.96\text{ g}\sim\text{eq } 42.9\text{ g}$

62.(d) $N_2+3H_2\rightarrow 2NH_3$

$\Delta_fH(NH_3)=$ Bond energy of reactant - Bond energy of product

$=\left(\frac{1}{2}\times\text{BE of N}\equiv\text{N bond}+\frac{3}{2}\text{BE of H}-\text{H bond}\right)-(3\text{BE of N}-\text{H bond})$

$=\left(\frac{1}{2}\times941+\frac{3}{2}\times436\right)-(3\times389)$

$=470.5+654-1167=-42.5\text{ kJ/mol}$

63.(b) Standard enthalpy of formation is the change in enthalpy that accompanies the formation of one mole of a compound from its elements, with all substances in their standard states, it is also called as 'standard heat of formation'.

64.(d) Conductivity $(\sigma)=\frac{1}{\text{Resistivity }(\rho)}$

In SI unit, $\sigma=\frac{1}{\text{ohmm}}=\text{Sm}^{-1}$.

65.(d) For first order reaction,

$$t = \frac{2.303}{k}\log\frac{a}{a-x}$$

where, t = time, k = rate constant

a = initial concentration

a − x = remaining concentration

$$\Rightarrow k = \frac{2.303}{23.03}\log\frac{0.08}{0.02} = \frac{1}{10}\log 4$$

$$= \frac{0.6020}{10} = 0.06020\ \text{min}^{-1}$$

66.(d) Among the given statements, (d) is not correct regarding the rate $= k[H_2][I_2]$ because overall order of the reaction is 2 not 1.

67.(a) Given number of atoms per unit cell, Z = 1

Molecular weight, M = 209 g/mol

Avogadro number, $N_A = 6.022 \times 10^{23}$

$V = a^3 = (3.36 \times 10^{-8})^3\ \text{cm}^3 [\because Z = 1]$

We know that

$$d = \frac{ZM}{N_A V} = \frac{1 \times 209}{6.022 \times 10^{23} \times (3.36 \times 10^{-8})^3}$$

$= 9.15\ \text{g/cm}^3$

68. **(b)** Given $p^\circ = 17.53, p_s = 17.22$ and $W = 17.10$

$$\frac{p^\circ - p_s}{p^\circ} = \frac{n}{n+N} = \frac{\frac{w}{m}}{\frac{w}{m} + \frac{W}{M}}$$

$$\because \frac{w}{m} < \frac{W}{M}$$

$$\therefore \frac{p^\circ - p_s}{p^\circ} = \frac{w/m}{W/M} = \frac{w}{m} \times \frac{M}{W}$$

$$\frac{17.53 - 17.22}{17.53} = \frac{17.10}{m} \times \frac{18}{100}$$

$$\Rightarrow m = \frac{17.10 \times 18 \times 17.53}{0.31 \times 100} = 174.05$$

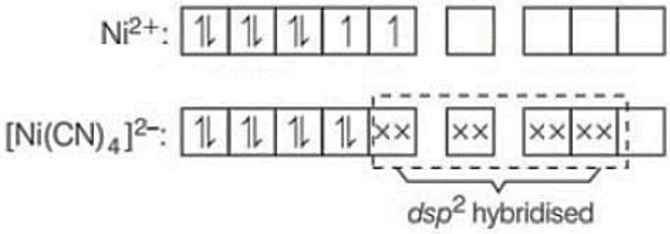
174 is nearest to the molecular weight of glucose ($C_6H_{12}O_6$), thus the substance X can be glucose.

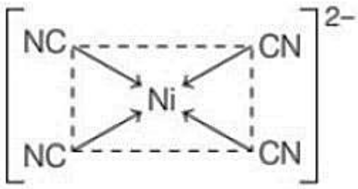
69.(b) Desorption is the process of removing an adsorbed substance from the surface of adsorbent, so that more adsorbate can occupy surface of adsorbent.

70.(a) Starch is an example of macromolecular colloid In this type of colloid, the particles are themselves large molecules which on dissolution form size in the colloidal range.

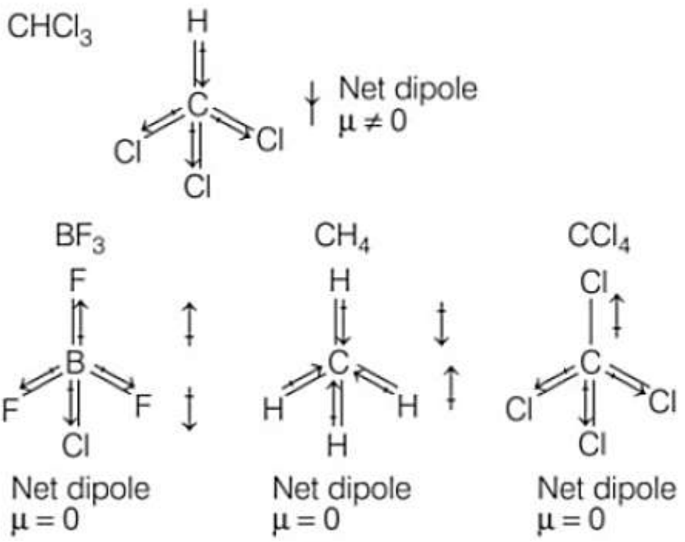
71. (a) Hybridisation of $[Ni(CN)_4]^{2-}$ is dsp^2.

$Ni = 1s^2, 2s^2, 2p^6, 3s^2 3v^6 3d^8 4s^2$
$Ni^{2+} = 1s^2, 2s^2, 2p^6, 3s^2 3p^6 3d^8, 4s^0 4p^0$



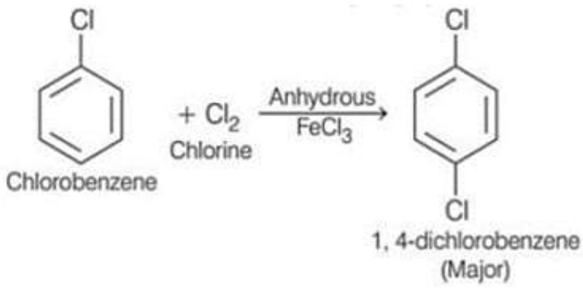


72. (a) The IUPAC name of the coordination compound $[Co(H_2O)_2(NH_3)_4]Cl_3$ is tetraamminediaquacobalt (III) chloride.
73. (c) Mn exhibits the maximum number of oxidation states. $Mn(Z = 25) = [Ar]3d^5 4s^2$. It shows $+2, +3, +4, +5, +6$ and $+7$ oxidation states.
74. (b)
75. (c)



Among the given compound $CHCl_3$ is having dipole moment.

76. (c) 1,4 -dichlorobenzene

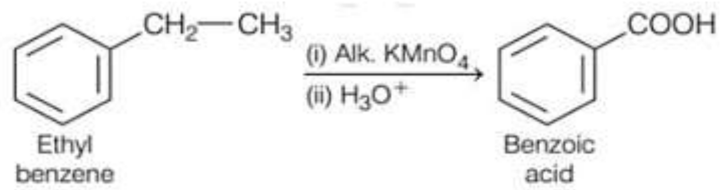


77. (d) IF_3 is a yellow powder whereas ClF_3, ClF and IF_7 is a colourless gas.
78. (d) $^{14}_{6}C$ and $^{16}_{8}O$ have same number of neutrons, i.e. 8. Therefore, they are the examples of isotones.

79. (b) Balmer series of transitions in the spectrum of hydrogen atom fall in visible region. Lyman series falls in ultraviolet region, while Paschen and Brackett series fall in infrared region.

80. (a) Oxygen is the most reactive among the other elements of group 16. As we move down the group, the reactivity decreases because there is an increase in the atomic radius down the group and hence, the effective nuclear charge decreases which leads to decrease in chemical reactivity. So, the reactivity order will be O > S > Se > Te > Po.

81. (b)



82. (a) An electron withdrawing group (EWG) is a group that reduces electron density in a molecule through the carbon atom it is bonded to. Here - COOH is one such example of electron withdrawing group.

83. (a) $C_6H_5NH_2$ and $C_6H_5NHCH_3$ are less basic than C_2H_5 NH_2 and $(C_2H_5)_2NH$ it is due to the delocalisation of lone pair of electrons of N -atom over the benzene ring.

Next, $C_6H_5NHCH_3$ is little more basic than $C_6H_5NH_2$ it is due to the +-effect of the CH_3 group. Among C_2H_5 N_2 and $(C_2H_5)_2NH$, $(C_2H_5)_2NH$ is more basic than $C_2H_5NH_2$ due to greater +1 -effect of two C_2H_5 group.

By combining above facts, the relative basic strength of given amines decreases as:

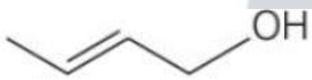
$(C_2H_5)_2NH > C_2H_5NH_2 > C_2H_5NHCH_3 > C_6H_5NH_2$

As a stronger base has a lower pK_b value, therefore, pKb_b values decreases in the reverse order.

$(C_6H_5)NH_2 > C_6H_5NHCH_3 > C_2H_5NH_2 > (C_2H_5)_2NH$

84. (b)

85. (c) The bond line structure of crotonyl alcohol is



86. (c) p-nitrophenol has highest melting point due to the presence of intermolecular hydrogen bonding, which is not present in all other given compounds.

87. (c) Cyclohexene on oxidation with $KMnO_4$ in presence of dil. H_2SO_4 will form adipic acid, in which oxidative cleavage takes place. The double bond is broken to which oxygen atoms are going to be added forming a carboxylic acid group at each. Thus, the cyclic structure is broken forming adipic acid

88. (b) Water is amphoteric in nature i.e. it can acts as an acid as well as a base. It acts as an acid with NH_3 and a base with H_2 S.

$H_2O + NH_3 \rightleftharpoons OH^- + NH_4^+$

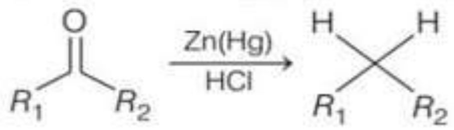
$H_2O + H_2S \rightleftharpoons H_3O^+ + HS^-$

The auto protolysis (self ionisation) of water takes place as follows

$H_2O + H_2O \rightleftharpoons H_3O^+ + OH^-$

acid base

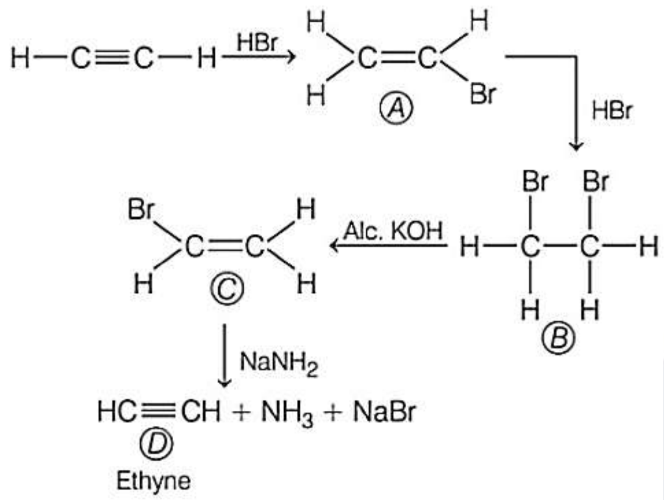
89. (b) The reduction of carbonyl groups (in aldehydes and ketones) to methylene groups with zinc amalgam and hydrochloric acid is known as the Clemmension reduction reaction.



90. (d) Bio-plastics or bio-diesel is a product of green chemistry so, other options (a, b, c) are incorrect.

91. (d) In the Wurtz reaction, alkyl halides react with sodium in dry ether to give hydrocarbon contain double the number of carbon atoms present in the halide

92. (c)



93.(a) Polyvinyl chloride or PVC is used in making floor tiles. Flexible PVC is made flexible and strong to prepare floor tiles.

94.(b) Bakelite is thermosetting polymer as it contains cross links, or heavily branched polymer chains. It gets hardened on heating and cannot be softened again

95.(b) SO_2 turns lime water milky

SO_2 in aqueous solution gives H_2SO_3.

$H_2SO_3 \rightleftharpoons H^+ + HSO_3^-$

CO_2 also turns lime water milky.

CO_2 in aqueous solution gives H_2CO_3.

$H_2CO_3 \rightleftharpoons H^+ + HCO_3^-$

H_2SO_3 is stronger acid than H_2CO_3. Thus, ionisation of H_2SO_3 is larger than H_2CO_3. Thus, pH of H_2SO_3 is lower than that of H_2SO_2.

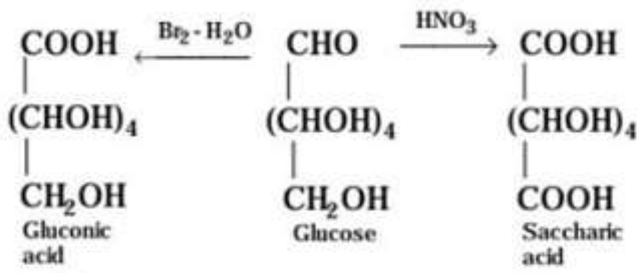
96.(c) The equilibrium reaction of AgBr is

$AgBr(s) \rightleftharpoons Ag^+(aq) + Br^-(aq)$

Molar solubility (S) of AgBr $= \sqrt{4.9 \times 10^{-13}}$

$= 7.0 \times 10^{-7}$ mol d m^{-3}

97.(b)



98. **(a)** The solubility of fluorides of alkaline earth metals decreases down the group due to increase in hydration enthalpy. Thus, the order of solubility of the given compounds is $BeF_2 > MgF_2 > SrF_2$ Hence, BeF_2 has maximum solubility.

99.(c)

$$\underset{+6}{Cr_2C_7^{2-}} + \underset{+1}{H_2O} \longrightarrow \underset{+6}{2CrO_4^{2-}} + \underset{+1}{2H^+}$$

The oxidation number of Cr in $Cr_2O_7{}^{2-}$ and $CrO_4{}^{2-}$ is same, i.e. 6. Thus, oxidation of Cr has neither decreased nor increased

100. **(a)** Abstraction of H -atom, attached to carbonyl group in an aldehyde, is much easier than the abstraction from bulkier alkyl group attached to carbonyl group in a ketone.

Mathematics

101. **(a)** Using sine rule

$$\frac{a}{\sin A} = \frac{b}{\sin B} = K$$

We have, $\frac{\cos A}{a} = \frac{\cos B}{b}$

$$\Rightarrow \frac{\cos A}{K\sin A} = \frac{\cos B}{K\sin B}$$

$\Rightarrow K\sin B\cos A = K\sin A\cos B$

$\Rightarrow \cos A\sin B - \cos B\sin A = 0$

$\Rightarrow \sin(A - B) = 0$

$A = B$

$\therefore$ Triangle in an isosceles triangle.

102. **(a)** Given, $\tan\theta = \frac{4}{5}$

$\therefore \sin\theta = \frac{4}{\sqrt{41}}$ and $\cos\theta = \frac{5}{\sqrt{41}}$

Now, $\frac{5\sin\theta - 3\cos\theta}{\sin\theta + 2\cos\theta} = \frac{5\times\frac{4}{\sqrt{41}} - 3\times\frac{5}{\sqrt{41}}}{\frac{4}{\sqrt{41}} + 2\times\frac{5}{\sqrt{41}}} = \frac{5}{14}$

103. **(c)** $\cos^{-1}x > \sin^{-1}x$[where $x \in [-1,1]$]

$\because \cos^{-1} x = \frac{\pi}{2} - \sin^{-1} x$

$\therefore \frac{\pi}{2} - \sin^{-1} x > \sin^{-1} x \Rightarrow 2\sin^{-1} x < \frac{\pi}{2}$

$\Rightarrow \sin^{-1} x < \frac{\pi}{4} \Rightarrow x < \frac{1}{\sqrt{2}}$

$\because -1 \leq x \leq 1$

$-1 \leq x < \frac{1}{\sqrt{2}}$

104. **(a)** Let $3\,A = A + 2\,A$

$\tan 3\,A = \tan(A + 2\,A)$

$\tan 3\,A = \frac{\tan A + \tan 2\,A}{1 - \tan A \cdot \tan 2\,A}$

$\Rightarrow \tan A + \tan 2\,A = \tan 3\,A - \tan 3\,A \tan 2\,A \cdot \tan A$

$\Rightarrow \tan 3\,A - \tan 2\,A - \tan A = \tan 3\,A \cdot \tan 2\,A \cdot \tan A$

105. **(c)** Given lines are $ax + by = c, bx + cy = a$ and $cx + ay = b$

On adding the given three equations, we get

$ax + by + bx + cy + cx + ay = a + b + c$

$\Rightarrow (a + b + c)x + (a + b + c)y = (a + b + c)$

On comparing with $0x + 0y = 0$ for collinearity, we get

$a + b + c = 0$

106. **(b)** The equation of line in new position is

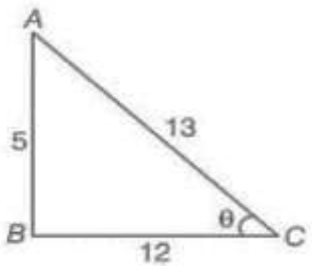
$y - 0 = \tan 15^\circ (x - 2)$

$\Rightarrow y = (2 - \sqrt{3})(x - 2)$

$\Rightarrow (2 - \sqrt{3})x - y - 4 + 2\sqrt{3} = 0$

107. **(d)** Given, $\tan x = \frac{5}{12}$ and x lies in III quadrant.

$\therefore \sin x = \frac{-5}{13}$ and $\cos x = \frac{-12}{13}$



Now, $\cos x = 2\cos^2\frac{x}{2} - 1$

$\Rightarrow \cos^2\frac{x}{2} = \frac{1}{2}(\cos x + 1)$

$= \frac{1}{2}\left(\frac{-12}{13} + 1\right) = \frac{1}{2}\left(\frac{1}{13}\right) = \frac{1}{26}$

$\therefore \cos\frac{x}{2} = \sqrt{\frac{1}{26}}$

108. **(d)** Since, $\frac{\sqrt{S_1}}{\sqrt{S_2}} = \frac{2}{3}$

$\therefore \frac{\sqrt{x_1{}^2 + y_1{}^2 + 4x_1 + 3}}{\sqrt{x_1{}^2 + y_1{}^2 - 6x_1 + 5}} = \frac{2}{3}$

$\Rightarrow 9x_1{}^2 + 9y_1{}^2 + 36x_1 + 27 - 4x_1{}^2 - 4y_1{}^2 + 24x_1 - 20 = 0$

$\Rightarrow 5x_1{}^2 + 5y_1{}^2 + 60x_1 + 7 = 0$

$\therefore$ Locus of point (x, y) is

$5x^2 + 5y^2 + 60x + 7 = 0$

109. **(d)** The centres of given circles are $C_1(-3,-3), C_2(6,6)$ and radii are

$r_1 = \sqrt{9+9+0} = 3\sqrt{2},\ r_2 = \sqrt{36+36+0} = 6\sqrt{2}$, respectively.

Now, $C_1C_2 = \sqrt{(6+3)^2 + (6+3)^2} = 9\sqrt{2}$

and $r_1 + r_2 = 3\sqrt{2} + 6\sqrt{2} = 9\sqrt{2}$

Here, $C_1C_2 = r_1 + r_2$

So, both circles touch each other externally.

110. **(d)** $\bar{x} = 5$, variance $= \frac{1}{n}\Sigma x_i{}^2 - (\bar{x})^2$

$\Rightarrow\ 0 = \frac{1}{n}\cdot 400 - 25$

$\Rightarrow n = \frac{400}{25} = 16$

111. **(d)** Variance of first n natural numbers

$$= \frac{\Sigma n^2}{n} - \left(\frac{\Sigma\ n}{n}\right)^2$$

$$= \frac{n(n+1)(2n+1)}{6n} - \left(\frac{n(n+1)}{2n}\right)^2$$

$$= (n+1)\left[\frac{2n+1}{6} - \frac{(n+1)}{4}\right]$$

$$= \frac{(n+1)}{12}[4n+2-3n-3]$$

$$= \frac{(n+1)}{12} \times (n-1) = \frac{n^2-1}{12}$$

112. **(c)** Total number of cases $= {}^{50}C_1 = 50$

Let A be the event of selecting ticket with sum of digits ' 8 '.

Favourable cases to A are $\{08, 17, 26, 35, 44\}$.

Let B be the event of selecting ticket with product of its digits ' 7 '.

Favourable cases to B is only $\{17\}$.

Now, $P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(a)} = \frac{1/50}{5/50} = \frac{1}{5}$

113. **(c)** Given, probability of winning a test match, $P(W) = \frac{1}{2}$

Probability of lossing a match, $P(L) = 1 - \frac{1}{2} = \frac{1}{2}$

Probability that India's second win occurs at the third day

$= P(L) \cdot P(W) \cdot P(W) + P(W) \cdot P(L) \cdot P(W)$

$$= \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} + \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{8} + \frac{1}{8} = \frac{1}{4}$$

114. **(d)** (I) p only if q. This says that number is a multiple of 9 only, if it is a multiple of 3.

(II) q is a necessary condition for p.This says that when a number is a multiple of 9, it is necessarily a multiple of 3

(III) $\sim$ q implies $\sim$ p. This says that if a number is not a multiple of 3, then it is not a multiple of 9.

115. **(a)** Let p: 7 is greater than 4

and q: 6 is less than 7.

Then, the given statement is disjunction p v q.

Here, $\sim$ p: 7 is not greater than 4.

and $\sim$ q: 6 is not less than 7.

$\therefore \sim (p \vee q)$: 7 is not greater than 4 and 6 is not less than 7.

116. **(a)** Given, $A(\text{ad}\, jA) = \begin{vmatrix} 20 & 0 \\ 0 & 20 \end{vmatrix}$

$20\begin{vmatrix}1 & 0\\0 & 1\end{vmatrix} = 20/$

We know that

$A^{-1} = \frac{1}{|A|}\text{ad jA}$

$\Rightarrow A(\text{adj}\,A) = |A|I$

$\Rightarrow 20I = |A|I$

$\Rightarrow |A| = 20$

117. (a)

Given, $A = \begin{bmatrix}1 & -1\\2 & 3\end{bmatrix}$

$\therefore A^2 = A\cdot A = \begin{bmatrix}1 & -1\\2 & 3\end{bmatrix}\begin{bmatrix}1 & -1\\2 & 3\end{bmatrix}$

$= \begin{bmatrix}1\times1+(-1)\times2 & 1\times(-1)+(-1)\times3\\2\times1+3\times2 & 2\times(-1)+3\times3\end{bmatrix}$

$= \begin{bmatrix}1-2 & -1-3\\2+6 & -2+9\end{bmatrix} = \begin{bmatrix}-1 & -4\\8 & 7\end{bmatrix}$

Now, $A^2 - 4A + 5I$

$= \begin{bmatrix}-1 & -4\\8 & 7\end{bmatrix} - 4\begin{bmatrix}1 & -1\\2 & 3\end{bmatrix} + 5\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$

$= \begin{bmatrix}-1 & -4\\8 & 7\end{bmatrix} - \begin{bmatrix}4 & -4\\8 & 12\end{bmatrix} + \begin{bmatrix}5 & 0\\0 & 5\end{bmatrix}$

$= \begin{bmatrix}-1-4+5 & -4+4+0\\8-8+0 & 7-12+5\end{bmatrix} = \begin{bmatrix}0 & 0\\0 & 0\end{bmatrix} = 0$

118. (d) Given, $f(x) = \begin{cases}\frac{3\sin\pi x}{5x} & x\neq0\\2k & x=0\end{cases}$

Now, $\lim_{x\to0}f(x) = \lim_{x\to0}\left(\frac{3\sin\pi x}{5x}\right)$

$= \frac{3}{5}\lim_{x\to0}\left(\sin\frac{\pi x}{\pi x}\right)\times\pi = \frac{3}{5}\times1\times\pi = \frac{3}{5}\Pi$

Also, $f(0) = 2k$

Since, $f(x)$ is continuous at $x = 0$.

$\therefore f(0) = \lim_{x\to0}f(x)$

$\Rightarrow 2k = \frac{3}{5}\Pi \Rightarrow k = \frac{3\Pi}{10}$

119. (b) We have, $f(x) = \begin{cases}\frac{\sin^3(\sqrt{3})\cdot\log(1+3x)}{(\tan^{-1}\sqrt{x})^2(e^{5\sqrt{3}}-1)x} & x\neq0\\a & x=0.\end{cases}$

For continuity in $[0,1], f(0)=\lim_{x\to 0} f(x)$ otherwise it is discontinuous.

$$\therefore a=\lim_{x\to 0}\frac{\sin^3(\sqrt{x})\cdot\log(1+3x)}{x\left(\tan^{-1}\sqrt{x}\right)^2\cdot\left(e^{5\sqrt{x}}-1\right)}$$

$$=\lim_{x\to 0}\left[\frac{3}{5}\cdot\frac{\sin^3\sqrt{x}}{(\sqrt{x})^3}\cdot\frac{(\sqrt{x})^3}{\left(\tan^{-1}\sqrt{x}\right)^3}\right.$$

$$\left.\&\times\frac{\log(1+3x)}{3x}\cdot\frac{5\sqrt{x}}{3^{5\sqrt{x}}-1}\right]$$

$$=\frac{3}{5}\lim_{x\to 0}\frac{\sin^3}{(\sqrt{x})^3}\cdot\frac{(\sqrt{x})^3}{\tan^{-1}\sqrt{x}}$$

$$\therefore a=\frac{3}{5}$$

120. **(c)** The general equation of an parabola having vertex at the origin and axis along positive Y -axis is $x^2=4\,ay\ldots(i)$

On differentiating Eq. (i), we get

$$2x=4a\frac{dy}{dx}$$

$$\Rightarrow\frac{dy}{dx}=\frac{x}{2a}\Rightarrow 2a=\frac{x}{dy/dx}$$

Putting value of 2 a in Eq. (i), we get

$$x^2=2\left(\frac{x}{dy/dx}\right)y\Rightarrow x\frac{dy}{dx}=2y$$

121. **(d)** Given equation is $\sqrt{\frac{dy}{dx}}-4\frac{dy}{dx}-7x=0$

$$\Rightarrow\frac{dy}{dx}=16\left(\frac{dy}{dx}\right)^2+49x^2+56x\frac{dy}{dx}$$

Obviously, it is of first order and second - degree differential equation.

122. **(a)** Given equation can be rewritten as

$$\left(1+\frac{1}{y}\right)dy=-e^x(\cos^2x-\sin 2x)dx$$

On integrating both sides, we get

$$y+\log y=-e^x\cos^2x+\int e^x\sin 2x dx$$

$$-\int e^x\sin 2x dx+C$$

$$\Rightarrow y+\log y=-e^x\cos^2x+C$$

At $x = 0$ and $y = 1$,

$1 + 0 = -e^0 \cos 0 + C$

$C = 2$ [given]

$\therefore$ Required solutions is

$y + \log y = -e^x \cos^2 x + 2$

$\Rightarrow y + \log y + e^x \cos^2 x = 2$

123. (c)

$$\int_{-1}^{1} \frac{17x^5 - x^4 + 29x^3 - 31x + 1}{x^2 + 1} dx$$
$$= \int_{-1}^{1} \frac{17x^5 + 29x^3 - 31x}{x^2 + 1}_{\text{Odd function}} dx - \int_{-1}^{1} \frac{x^4 - 1}{x^2 + 1}_{\text{Even function}} dx$$

$$= 0 - 2\int_{0}^{1} \frac{(x^2 - 1)(x^2 + 1)}{(x^2 + 1)} dx = -2\left[\left(\frac{x^3}{3} - x\right)\right]_0^1 = \frac{4}{3}$$

124. (b) Given word is 'HAVANA' (3A, 1H, 1N, 1V)

Total number of ways of arranging the given word

$= \frac{6!}{3!} = 120$

Total number of words in which N, V together

$= \frac{5!}{3!} \times 2! = 40$

$\therefore$ Required number of ways $= 120 - 40 = 80$

125. (c) 3 consonants can be selected from 7 consonants $= {}^7C_3$ ways

2 vowels can be selected from 4 vowels $= {}^4C_2$ ways

$\therefore$ Required number of words $= {}^7C_3 \times {}^4C_2 \times 5!$

[selected 5 letters can be arranged in 5! ways, to get a different word]

$= 35 \times 6 \times 120 = 25200$

126. (a) Given $f(x) = \sin^2 x + \sin^2\left(x + \frac{\pi}{3}\right) + \cos x$

$$\cos\left(x+\frac{\pi}{3}\right)$$
$$= \sin^2 x + \left[\sin x\cos\frac{\pi}{3} + \cos x\sin\frac{\pi}{3}\right]^3$$
$$+\cos x\left[\cos x\cos\frac{\pi}{3} - \sin x\sin\frac{\pi}{3}\right]$$
$$= \sin^2 x + \left[\frac{\sin x}{2} + \cos x\cdot\frac{\sqrt{3}}{2}\right]^2$$
$$\therefore = \cos x\left[\frac{\cos x}{2} - \sin x\cdot\frac{\sqrt{3}}{2}\right]$$
$$= \sin^2 x + \frac{\sin^2 x}{4} + \frac{3\cos 2}{4} + \sin x\cos x\cdot\frac{\sqrt{3}}{2}$$
$$= \frac{5\sin^2 x}{4} + \frac{5\cos^2 x}{4} = \frac{5}{4}$$
$$\Rightarrow (x) = g[f(x)] = g\left(\frac{5}{4}\right) = 1$$

127. **(c)** Let $y = \frac{x}{1+x^2} \Rightarrow x^2y - x + y = 0$

For x to be real, $1 - 4y^2 \geq 0$ (Discriminant $= 1 - 4y^2$}

$\Rightarrow 4y^2 - 1 \leq 0$

$\Rightarrow (2y-1)(2y+1) \leq 0$

$\Rightarrow -\frac{1}{2} \leq y \leq \frac{1}{2}$

$\therefore y = f(x) \in \left[-\frac{1}{2}, \frac{1}{2}\right]$

128. **(c)** Given, $f(x) = \sin^{-1}\left[\log_2\left(\frac{x}{2}\right)\right]$

$\Rightarrow -1 \leq \log_2\left(\frac{x}{2}\right) \leq 1 \Rightarrow 2^{-1} \leq \frac{x}{2} \leq 2^1$

$\Rightarrow \frac{1}{2} \leq \frac{x}{2} \leq 2 \Rightarrow 1 \leq x \leq 4$

129. **(d)** We have, $f(x) = \frac{k}{2^x}, x = 0,1,2,3,4$

Since, f(x) is a probability distribution of a random variable X , therefore we have

$\sum_{x=0}^{4} f(x) = 1 \Rightarrow \sum_{x=0}^{4}\left(\frac{k}{2^x}\right) = 1$

$\Rightarrow k\sum_{x=0}^{4}\frac{1}{2^x} = 1$

$\Rightarrow k\left(1 + \frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \frac{1}{2^4}\right) = 1$

$\Rightarrow k\left(\frac{16+8+4+2+1}{2^4}\right) = 1$

$$\Rightarrow k \times \left(\frac{31}{16}\right) = 1$$

$$\therefore k = \frac{16}{31}$$

130. (d)

$$\lim_{x\to 0} \frac{e^{x^2} - \cos x}{x^2} \left[\frac{0}{0} \text{ form}\right] \text{ [using L'Hospital's rule]}$$

$$= \lim_{x\to 0} \frac{2xe^{x^2} + \sin x}{2x} \left[\frac{0}{0} \text{ form}\right]$$

$$= \lim_{x\to 0} \frac{2e^{x^2} + 4x^2e^{x^2} + \cos x}{2}$$

$$= \frac{2+0+1}{2} = \frac{3}{2} \text{ [using L'Hospital's rule]}$$

131. (a)

$$\lim_{x\to\sqrt{2}} \frac{x^4 - 4}{x^2 + 3\sqrt{2}x - 8}$$

$$= \lim_{x\to\sqrt{2}} \frac{(x+\sqrt{2})(x-\sqrt{2})(x^2+2)}{(x-\sqrt{2})(x+4\sqrt{2})}$$

$$= \lim_{x\to\sqrt{2}} \frac{(x+\sqrt{2})(x^2+2)}{(x+4\sqrt{2})}$$

$$= \frac{(\sqrt{2}+\sqrt{2})(2+2)}{(\sqrt{2}+4\sqrt{2})} = \frac{(2\sqrt{2})(4)}{5\sqrt{2}} = \frac{8\sqrt{2}}{5\sqrt{2}} = \frac{8}{5}$$

132. (a) Let point $A = (2,3,-1)$ and point $B = (-2,-4,3)$.

Now the position vector of line joining A and B,

$AB = OB - OA$

$$= -2\hat{i} - 4\hat{j} + 3\hat{k} - 2\hat{i} - 3\hat{j} + \hat{k}$$
$$= -4\hat{i} - 7\hat{j} + 4\hat{k}$$

Again let $a = AB = -4\hat{i} - 7\hat{j} + 4\hat{k}$

and $b = 4\hat{i} - 3\hat{j} + \hat{k}$

Then, $a \cdot b = (-4\hat{i} - 7\hat{j} + 4\hat{k}) \cdot (4\hat{i} - 3\hat{j} + \hat{k})$

$= -16 + 21 + 4 = 9$

$|a| = \sqrt{(-4)^2 + (-7)^2 + (4)^2}$

$= \sqrt{16 + 49 + 16} = 9$

Now projection of b on a

$$= \frac{(a \cdot b)}{|a|} = \frac{9}{9} = 1$$

133. (b)

$a = 3\hat{i} - 2\hat{j} + \hat{k}$ ... (i)
$b = 2\hat{i} - 4\hat{j} - 3\hat{k}$ ... (ii)

Multiplying by 2 both sides, we get

$2b = 4\hat{i} - 8\hat{j} - 6\hat{k}$ ...(iii)

Subtracting Eq. (iii) from Eq. (i), we get

$$\begin{aligned} a \quad -2b &= (3\hat{i} - 2\hat{j} + \hat{k}) - (4\hat{i} - 8\hat{j} - 6\hat{k}) \\ &= -\hat{i} + 6\hat{j} + 7\hat{k} \\ &= \sqrt{(-1)^2 + (6)^2 + (7)^2} = \sqrt{86} \end{aligned}$$

134. (a) $r \cdot (2\hat{i} - 3\hat{j} + 5\hat{k}) + 2 = 0$

Equation of plane parallel to the given plane is

$-2x + y - 3z = \lambda.$ ... (i)

Since, plane (i) passes through the point $(1,4,-2)$.

Therefore, this point satisfy the given plane. Put $x = 1, y - 4$ and $z = -2$ in Eq. (i)

$-2(1) + 4 - 3(-2) = \lambda$

$-2 + 4 + 6 = \lambda$

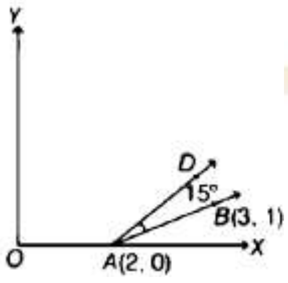
$\lambda = 8$

Put $\lambda = 8$ in Eq. (i), we get

$-2x + y - 3z = 8$

$2x - y + 3z + 8 = 0$

135. (d)



The slope of the line is $\frac{y_2 - y_1}{x_2 - x_1} = \frac{1-0}{3-2} = 1$ or $\tan 45°$ rotated anti-clockwise direction the line through $15°$ the slope of line AD in new position. New position will be $\tan 60° = \sqrt{3}$.

Therefore, the equation of the new line AD is

$y - 0 = \sqrt{3}(x - 2)$

$y = \sqrt{3}x - 2\sqrt{3}$

136. **(d)** Let $I = \int \frac{(\log x)^2}{x} dx$

Put $\log x = t \Rightarrow \frac{1}{x} d = dt$

$\therefore I = \int \frac{(\log x)^2}{x} dx = \int t^2 dt$

$= \frac{t^3}{3} + C = \frac{(\log x)^3}{3} + C$ [put $t = \log x$]

137. **(c)** Let $I = \int \frac{\tan^4 \sqrt{x} \cdot \sec^2 \sqrt{x}}{\sqrt{x}} dx$

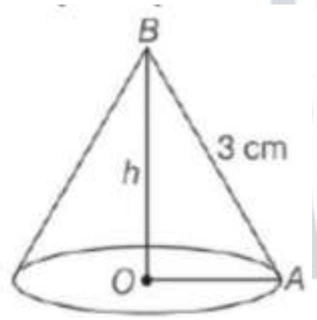
Put $\tan \sqrt{x} = t$

$\Rightarrow \frac{\sec^2 \sqrt{x}}{2\sqrt{x}} dx = dt$

$\therefore I = \int \frac{\tan^4 \sqrt{x} \cdot \sec^2 \sqrt{x}}{\sqrt{x}} dx = \int 2t^4 \, dt$

$= \frac{2t^5}{5} + C = \frac{2(\tan \sqrt{x})^5}{5} + C$

138. **(b)** Let height of a right circular cone $=$ hcm and $OA =$ rcm



Given, slant height of a right circular cone $= 3$ cm In$\triangle$ OAB, $\angle BOA = 90°$

$(OB)^2 + (OA)^2 = (AB)^2$

[apply pythogoras theorem]

$(h)^2 + (r)^2 = (3)^2$

$r = \sqrt{9 - h^2}$ ... (i)

We know that, Volume of cone

$= \frac{\pi}{3} \Pi r^2 h$

$V = \frac{\pi}{3}(9 - h^2) \times h$

[from Eq. (i), $r = \sqrt{9 - h^2}$]

$V = \frac{\pi}{3}(9h - h^3)$

$$\frac{dV}{dh} = \frac{\pi}{3}(9 - 3h^2)$$

$$\therefore \frac{dV}{dh} = 0 \Rightarrow \frac{\pi}{3}(9 - 3\,h^2) = 0$$

$$\Rightarrow \underline{h^2} = \frac{9}{3} = 3 \Rightarrow h = \sqrt{3}$$

$$\frac{d^2\,V}{d\,h^2} = \frac{-6 \times h}{3} < 0$$

So, $h = \sqrt{3}$ of the cone for maximum volume

139. **(b)** Given, function is $y = [x(x - 2)]^2 = [x^2 - 2x]^2$

On differentiating both sides w.r.t. x , we get

$$\begin{aligned}\frac{dy}{dx} &= 2(x^2 - 2x)\frac{d}{dx}(x^2 - 2x)\\ &= 2(x^2 - 2x)(2x - 2) = 4x(x - 2)(x - 1)\end{aligned}$$

On putting $\frac{dy}{dx} = 0$, we get

$4x(x - 2)(x - 1) = 0 \Rightarrow x = 0,1$ and 2

Now, we find interval in which f(x) is strictly increasing or strictly decreasing.

Interval	$\mathbf{\frac{dy}{dx} = 4x(x - 2)(x - 1)}$	Sign of $\mathbf{f'(x)}$
$(-\infty, 0)$	$(-)(-)(-)$	$-$ve
(0,1)	$(+)(-)(-)$	+ve
(1,2)	$(+)(-)(+)$	$-$ve
$(2, \infty)$	$(+)(+)(+)$	+ve

Hence, y is strictly increasing in (0,1) and $(2, \infty)$. Also, y is a polynomial function, so it continuous at $X = 0,1$ and 2 . Hence, y is increasing in $[0,1] \cup [2, \infty]$.

140. **(d)**

$$y = \tan^{-1}\sqrt{\frac{1 + \cos x}{1 - \cos x}}$$

$$y = \tan^{-1}\sqrt{\frac{2\cos^2\frac{x}{2}}{2\sin^2\frac{x}{2}}}$$

$$y = \tan^{-1}\left(\cot\frac{x}{2}\right)$$

$$y = \tan^{-1}\left[\tan\left(\frac{\pi}{2} - \frac{x}{2}\right)\right]$$

$$y=\frac{\pi}{2}-\frac{x}{2} \Rightarrow \frac{dy}{dx}=-\frac{1}{2}$$

141. **(b)** Let $u=\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$

Put $x=\tan\theta \Rightarrow \theta=\hat{\tan}^{-1}x$, then

$$\Rightarrow u=\frac{\theta}{2}=\frac{1}{2}\tan^{-1}x$$

$r\cdot\tan^{-1}(\tan\theta)=\theta]$

On differentiating both sides w.r.t. x , we get

$$\frac{du}{dx}=\frac{1}{2(1+x)^2}\left[\because \frac{d}{dx}(\tan^{-1}x)=\frac{1}{1+x^2}\right]\ldots$$

Also, let $v=\sin^{-1}\left(\frac{2x}{1+x^2}\right)$

Put $x=\tan\theta \Rightarrow \theta=\tan^{-1}x$, then we get

$$v=\sin^{-1}\left[\frac{2\tan\theta}{1+\tan^2\theta}\right]$$

$$\Rightarrow v=\sin^{-1}[\sin 2\theta]$$

$$\Rightarrow v=2\theta \Rightarrow v=2\tan^{-1}x$$

On differentiating both sides w.r.t. x, we get

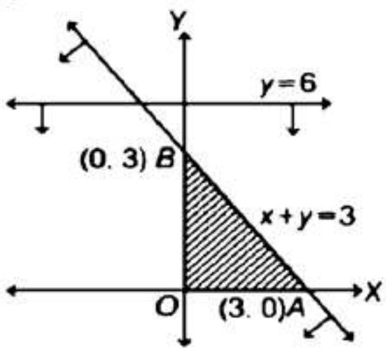
$$\frac{dv}{dx}=\frac{2}{1+x^2}$$

Now, $\frac{du}{dv}=\frac{du}{dx}\times\frac{dx}{dv}=\frac{1}{2(1-x^2)}\times\frac{(1+x^2)}{2}$

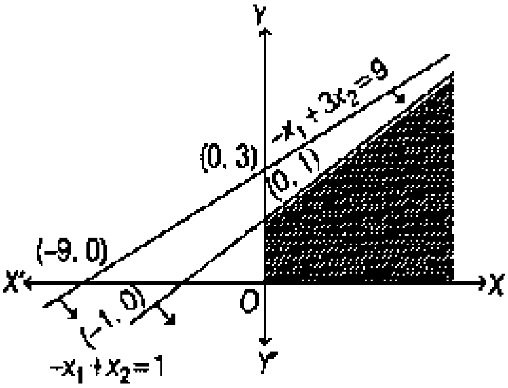
[from Eqs. (i) and (ii)]

$$\therefore \frac{du}{dv}=\frac{1}{4}$$

142. **(c)** The given region is bounded in first quadrant.



143. **(b)** It is clear from the figure that feasible space (shaded portion) is unbounded



144. (b)

Given, $z = \dfrac{(\sqrt{3}+i)^2(3i+4)^2}{(8+6i)^2}$

Now, $|z| = \left|\dfrac{(\sqrt{3}+i)^3(3i+4)^2}{(8+6i)^2}\right|$

$= \dfrac{|(\sqrt{3}+i)^3||(3i+4)^2|}{|(8+6i)^2|}\left[\because \left|\dfrac{z_1}{z_2}\right| = \left|\dfrac{z_1}{z_2}\right|\right]$

$= \dfrac{|\sqrt{3}+i|^3|3i+4|^2}{|8+6i|^2}[\because |z^n| = |z|^n]$

$= \dfrac{(\sqrt{3+1})^3(\sqrt{9+16})^2}{(\sqrt{64+36})^2}$

$= \dfrac{(2)^3(5)^2}{(10)^2} = \dfrac{10^2 \cdot 2}{(10)^2} = 2$

145. (b) Given, $\frac{3}{2+\cos\theta+i\sin\theta} = a+ib$

$\Rightarrow \dfrac{3[(2+\cos\theta)-i\sin\theta]}{(2+\cos\theta)^2+\sin^2\theta} = a+ib$

$\Rightarrow \dfrac{3[2+\cos\theta-i\sin\theta]}{5+4\cos\theta} = a+ib$

$\Rightarrow a = \frac{3(2+\cos\theta)}{5+4\cos\theta}$ and $b = -\frac{3\sin\theta}{5+4\cos\theta}$

$\therefore (a-2)^2+b^2 = \left(\dfrac{6+3\cos\theta}{5+4\cos\theta}-2\right)^2 + \dfrac{9\sin^2\theta}{(5+4\cos\theta)^2}$

$= \dfrac{(-4-5\cos\theta)^2+9\sin^2\theta}{(5+4\cos\theta)^2}$

$= \dfrac{16+25\cos^2\theta+40\cos\theta+9\sin^2\theta}{(5+4\cos\theta)^2}$

$= \dfrac{16+16\cos^2\theta+40\cos\theta+9}{(5+4\cos\theta)^2}$

$= \dfrac{(5+4\cos\theta)^2}{(5+4\cos\theta)^2} = 1$

146. (c) Let probability of defective bulb,

$p = \frac{10}{100} = \frac{1}{10} = 0.1$

and probability of non-defective bulb,

$q = 1 - 0.1 = 0.9$

Here, $n = 5$

$\therefore$ P(none is defective) $= P(X = 0)$

$= {}^5C_0 (0.1)^0 (0.9)^5$

$= 1 \times (0.9)^5 = \left(\frac{9}{10}\right)^5$

147. (b)

X	1	2	3	4	5
$P(X = x)$	K	2K	3K	2K	K

$\therefore$ Variance $= \Sigma x_i^2 p - \left(\sum x_p p\right)^2$

$= (1k + 8k + 27k + 32k + 25k)$

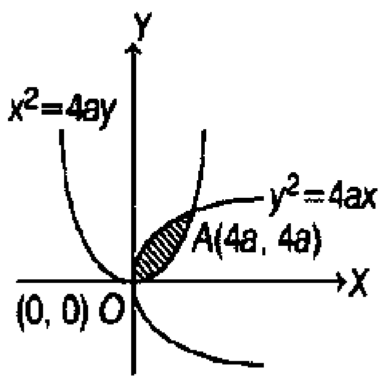
$-(k + 4k + 9k + 8k + 5k)^2$

$= (93k) - (27k)^2 = \left(93 \times \frac{1}{9}\right) - \left(27 \times \frac{1}{9}\right)^2$

$= \frac{93}{9} - 9 = \frac{93-81}{9} = \frac{12}{9} = \frac{4}{3}$

148. (a) The equations of given curves are

$y^2 = 4ax$ and $x^2 = 4ay$



On solving these equations, we get the intersection points, i.e. (0,0) and (4a, 4a).

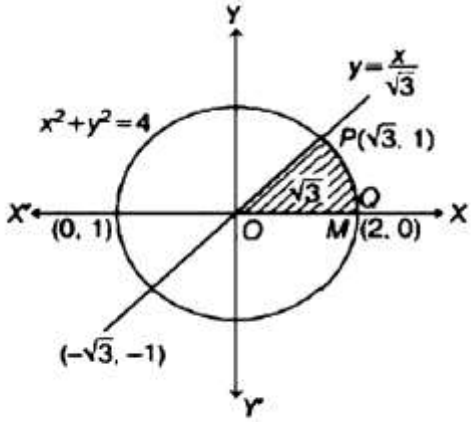
$\therefore$ Required area $= \int_0^{4a} \left(2\sqrt{a}\sqrt{x} - \frac{x^2}{4a}\right) dx$

$= 2\sqrt{a}\left[\frac{x^{3/2}}{3/2}\right]_0^{4a} - \left[\frac{x^3}{12a}\right]_0^{4a}$

$= \frac{32a^2}{3} - \frac{16a^2}{3}$

$= \frac{16a^2}{3}$

149. **(c)** The intersection points of curves $x^2+y^2=4$ and $y=\frac{x}{\sqrt{3}}$ are (0,0) and $P(\sqrt{3},1)$.



$\therefore$ Area of $\triangle$ OPM $=\frac{1}{2}\times\sqrt{3}\times 1=\frac{\sqrt{3}}{2}$

and area of curve MPQ $=\int_{\sqrt{3}}^{2}\sqrt{4-x^2}dx$

$$=\left[\frac{x}{2}\sqrt{4-x^2}+\frac{4}{2}\sin^{-1}\left(\frac{x}{2}\right)\right]_{\sqrt{3}}^{2}$$

$$=\left[0+2\left(\frac{\pi}{2}\right)-\left(\frac{\sqrt{3}}{2}+2\times\frac{\pi}{3}\right)\right]$$

$$=\left(\frac{\pi}{3}-\frac{\sqrt{3}}{2}\right)]$$

150. **(b)** Let $I=\int_{\pi/6}^{\pi/2}\frac{\operatorname{cosec}x\cdot\cot x}{1+\operatorname{cosec}^2 x}dx$

Let cosecx = t

$\Rightarrow -\operatorname{cosec}x\cot x dx=dt$

When $x=\frac{\pi}{6}$, then $t=\operatorname{cosec}\frac{\pi}{6}=2$

and when $x=\frac{\pi}{2}$, then $t=\operatorname{cosec}\frac{\pi}{2}=1$

$$\therefore I=\int_2^1-\frac{dt}{1+t^2}=-\int_2^1\frac{dt}{1+t^2}$$

$$=\int_1^2\frac{dt}{1+t^2}=[\tan^{-1}(t)]_1^2$$

$$=\tan^{-1}(2)-\tan^{-1}(1)$$

$$=\tan^{-1}\left(\frac{2-1}{1+2\times 1}\right)=\tan^{-1}\left(\frac{1}{1+2}\right)$$

$$=\tan^{-1}\left(\frac{1}{3}\right)$$