Physics

1. **(c)**
2. **(c)** Given that, $\alpha = 4\mathrm{rad/s^2}$
   Centripetal (radial) acceleration, $a_r = r\omega^2$
   Tangential acceleration, $a_t = r\alpha$
   If $a_r = a_t$, then $r\omega^2 = r\alpha$
   $\omega^2 = \alpha = 4$
   $\omega = \sqrt{4} = 2\mathrm{rad/s}$

But, $\omega = \omega_0 + \alpha t = 0 + \alpha t = \alpha t$

$2 = 4t$

$t = \frac{1}{2}\ \mathrm{s}$

3. **(d)** The output of the NAND gate will be $\overline{A \cdot B}$. The output of the NOR gate will be $\overline{\overline{A \cdot B} + \overline{A \cdot B}} = A \cdot B$
   The output of the NOT gate will be $\overline{A.B}$ Thus, the given network is equivalent to a NAND gate.
4. **(b)** Two coherent sources of intensities I_1 and I_2 produce,
   maximum intensity, $I_{max} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2$ and minimum intensity, $I_{min} = \left(\sqrt{I_1} - \sqrt{I_2}\right)^2$ in an interference pattern.

$$\frac{I_{max}}{I_{min}} = \frac{\left(\sqrt{I_1} + \sqrt{I_2}\right)^2}{\left(\sqrt{I_1} - \sqrt{I_2}\right)^2} = \frac{I_1 + I_2 + 2\sqrt{I_1 I_2}}{I_1 + I_2 - 2\sqrt{I_1 I_2}}$$

Given $I_1 = 2I_2$

$$\frac{I_{max}}{I_{min}} = \frac{3I_2 + 2\sqrt{2}I_2}{3I_2 - 2\sqrt{2}I_2} = \frac{3 + 2\sqrt{2}}{3 - 2\sqrt{2}} = 34$$

5. **(c)**

$$P_1 V_1 = \frac{m_1}{M_1} RT_1 \ldots\ldots (i)$$

$$P_2 V_2 = \frac{m_2}{M_2} RT_2 \ldots\ldots (ii)$$

Dividing (i) by (ii)

$$\frac{T_1}{T_2} = \frac{M_1}{M_2} = \frac{32}{2} = \frac{16}{1}$$

6. **(d)**

Given: $\frac{g'}{g} = \frac{1}{n}$

$$\frac{1}{n} = \frac{R^2}{(R+h)^2} \Rightarrow \frac{R}{R+h} = \frac{1}{\sqrt{n}}$$

$\sqrt{n}R = R + h$

$R(\sqrt{n} - 1) = h$

7. **(d)** Using Bernoulli's equation,

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$$

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$$

... (given horizontal pipe)

Substituting the given values,

$$P + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho (3v)^2$$

$$P_2 = P + \left(\frac{1}{2}\rho v^2 - \frac{9}{2}\rho v^2\right) = P - 4\rho v^2$$

8. **(d)**

$I=100\sin 200\pi t$

Peak value $I_0=100$ A

When $I=100$ A, we have

$100=100\sin 200\pi t$

$\sin 200\pi t=1$

$200\pi t=\frac{\pi}{2}$

$t=\frac{1}{400}$ s

9. (b)

$W=q\Delta V$

$W=q(V_1-V_A)$

$W=3[V_1-(-10)]$

$\frac{90}{3}=V_1+10$

$V_1=20$ V

10. (c)

$\omega^2=\frac{k}{m}, r=\frac{b}{2m}$

Angular frequency,

$\omega' \;=\sqrt{(\omega^2-r^2)}$

$=\sqrt{\frac{k}{m}-\left(\frac{b}{2m}\right)^2}$

11. (c) $U=\frac{1}{2}LI^2=\frac{1}{2}\times(5\times10^{-6})\times 2^2=10\mu$ J

12. (a)

$(M+m)g-T=(M+m)a$

$T-Mg=Ma$

Adding the above equations,

$(M+m)g-Mg=(2M+m)a$

$a=\frac{mg}{(2M+m)}$

13. (a)

$r_n\propto n^2$

$\frac{r_n}{r_0}=\left(\frac{n}{1}\right)^2=(4)^2=16$

$r_4=16\times(5.3\times10^{-11})=8.48$Å

14. (b)

$P_1+P_2=9D$ ... (i)

$P_1+P_2-dP_1P_2=\frac{27}{5}$ ..... (ii)

where $d=20$ cm $=0.2$ m

Substituting value of (P_1+P_2) in equation (ii) we get

$9-0.2P_1P_2=\frac{27}{5}$

$0.2P_1P_2=9-\frac{27}{5}=\frac{18}{5}$

$P_1P_2=\frac{18}{0.2\times5}=18$

Since $P_1+P_2=9$ and $P_1P_2=18$

$P_1=3$ and $P_2=6$

15. (a)

$y_1 \quad = a_1 \sin\left(\frac{2\pi x}{\lambda} - \omega t\right) \quad \ldots.(i)$

$y_2 \quad = a_2 \cos\left(\frac{2\pi x}{\lambda} - \omega t + \phi\right)$

$\quad\quad = a_2 \sin\left(\frac{2\pi x}{\lambda} - \omega t + \phi + \frac{\pi}{2}\right) \ldots\ldots(ii)$

From (ii) and (i)

Phase difference $= \phi + \frac{\pi}{2}$

16. **(b)** Outside pressure = 1 atm

Pressure inside soap bubble A = 1.01 atm

Pressure inside soap bubble B = 1.02 atm

Excess pressures will be

$\Delta P_A = 1.01 - 1 = 0.01$ atm and

$\Delta P_B = 1.02 - 1 = 0.02$ atm

Now, $\Delta P \propto \frac{1}{r} \Rightarrow r \propto \frac{1}{\Delta P}$

$\frac{r_A}{r_B} = \frac{\Delta P_B}{\Delta P_A} = \frac{0.02}{0.01} = \frac{2}{1}$

Now, $V = \frac{4}{3}\pi r^3$

$\Rightarrow V \propto r^3$

$\frac{V_A}{V_B} = \left(\frac{r_A}{r_B}\right)^3 = \left(\frac{2}{1}\right)^3 = \frac{8}{1}$

17. **(b)** As the angular velocity is constant, the linear velocity changes from 0 to $R\omega$ as we move from the axle to the rim.

Average velocity $v_{avg} = \frac{R\omega}{2}$

We know motional e.m.f, $E_{mot} = Blv$

$E_{mot} \quad = \frac{B \times R \times R\omega}{2} = \frac{BR^2 \times 2\pi F}{2}$

$\quad = B\pi FR^2$

18. **(a)** For a simple pendulum,

$T = 2\pi\sqrt{\frac{l}{g}} \Rightarrow T \propto \frac{1}{g}$

When the point of suspension is moved vertically upwards with acceleration a, g becomes g + a. Hence, T decreases.

19. **(c)** From Einstein's equation,

$h\nu = eV_0 + h\nu_0$

$\frac{hc}{\lambda} - \frac{hc}{\lambda_0} = eV_0$

case (i) $\lambda = \lambda; V_0 = V$

$\frac{hc}{\lambda} - \frac{hc}{\lambda_0} = eV \quad \ldots\ldots (i)$

case (ii) $\lambda = 3\lambda; V_0 = \frac{V}{6}$

$\frac{hc}{3\lambda} - \frac{hc}{\lambda_0} = \frac{eV}{6} \quad \ldots\ldots. (ii)$

dividing equation (i) by equation (ii)

$$\frac{\left(\frac{hc}{\lambda}-\frac{hc}{\lambda_0}\right)}{\left(\frac{hc}{3\lambda}-\frac{hc}{\lambda_0}\right)}=6$$

$$\frac{1}{\lambda}-\frac{1}{\lambda_0}=6\left(\frac{1}{3\lambda}-\frac{1}{\lambda_0}\right)$$

$$\frac{1}{\lambda}-\frac{1}{\lambda_0}=\frac{2}{\lambda}-\frac{6}{\lambda_0}$$

$$\frac{-1}{\lambda_0}+\frac{6}{\lambda_0}=\frac{2}{\lambda}-\frac{1}{\lambda}$$

$$\frac{5}{\lambda_0}=\frac{1}{\lambda}\Rightarrow\lambda_0=5\lambda$$

20. (c)

$$A=\pi r^2\Rightarrow r=\sqrt{\frac{A}{\pi}}$$

$$B=\frac{\mu_0 I}{2r}=\frac{\mu_0 I}{2\sqrt{\frac{A}{\pi}}}$$

$$I=\frac{2B}{\mu_0}\sqrt{\frac{A}{\pi}}$$

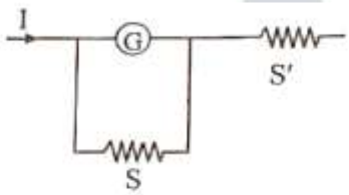
Magnetic moment $m = IA$

$$IA=\frac{2\,B}{\mu_0}\sqrt{\frac{A}{\pi}}\times A=\frac{2BA^{3/2}}{\mu_0\pi^{1/2}}$$

21. (c) The adiabatic curve is steeper (has a greater slope).This steepness reflects the rapid change in pressure with volume during an adiabatic process, due to the absence of heat transfer which allows the gas to expand or contract more quickly in response to changes in volume.

In the given P-V graph, adiabatic process is shown by region AD and BC only as their slope is greater.

22. (a)



$$\text{Now, } G=\left(\frac{GS}{S+G}\right)+S'$$

$$G-\frac{GS}{S+G}=S'$$

$$S'=\frac{G^2}{S+G}$$

23. (c)

$$V=100\sin(100t)V$$

$$I=100\sin\left(100t+\frac{\pi}{3}\right)mA$$

Comparing these equations with the standard forms, $V = V_0\sin\omega t$ and $I = I_0\sin\omega t$ we get,

$V_0 = 100\text{ V}, I_0 = \quad 100 \times 10^{-3}\text{ A and } \phi = \frac{\pi}{3}$

$P = V_{rms} I_{rms} \cos\phi \quad = \frac{V_0}{\sqrt{2}} \times \frac{I_0}{\sqrt{2}} \cos\frac{\pi}{3}$

$= \frac{100}{\sqrt{2}} \times \frac{100 \times 10^{-3}}{\sqrt{2}} \times \frac{1}{2} = 2.5\text{ W}$

24. (b) The moment of inertia of the annular ring is

$I \quad = \frac{1}{2} M(R_1^2 + R_2^2)$

$= \frac{10(100+25)}{2} = 625\text{kgm}^2$

25. (c)

$L_2 - L_1 = L_1 \alpha (t_2 - t_1)$

$x \times 10^{-5} = 10 \times 1.2 \times 10^{-5} \times (45 - 17)$

$x \times 10^{-5} = 10 \times 1.2 \times 10^{-5} \times 28$

$x = 336$

26. (b) Let n be frequency of tuning fork.

Let n_1, n_2 be frequency of wire at tension T_1, T_2 respectively.

$n \propto \sqrt{T} \quad \ldots\ldots \text{ (i)}$

$n_1 = n - 6 \ldots\text{. (ii)}$

$n_2 = n + 6 \quad \ldots\ldots\text{ (iii)}$

$\frac{n_1}{n_2} = \sqrt{\frac{T_1}{T_2}} = \sqrt{\frac{225}{256}} = \frac{15}{16}$

$\frac{n-6}{n+6} = \frac{15}{16} \quad \ldots\text{(form (i)(ii)(iii)}$

$16n - 96 = 15n + 90$

$n = 186\text{ Hz}$

27. (b)

$E \propto \frac{Z^2}{n^2}$

$\frac{E_H}{E_{He}} = \frac{(Z^2)_H}{(n)_H^2} \times \frac{(n)_{He}^2}{(Z^2)_{He}}$

$= \frac{1}{3^2} \times \frac{5^2}{2^2} = \frac{25}{36}$

$E_{He} = \frac{36}{25} E_H = \frac{36}{25} E$

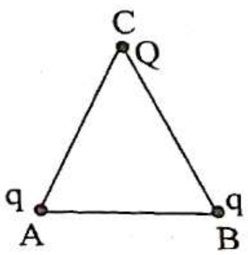
28. (b) We know,

Mean free path $\lambda = \frac{k_B T}{\sqrt{2}\pi d^2 P} \Rightarrow \lambda \propto T$

$\frac{1500\text{ d}}{\lambda} = \frac{273}{373}$

$\lambda = 1500\text{ d} \times \frac{373}{273}$

29. (d)



Potential energy $U = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{a}$

Potential energy at A due to $B = \frac{q^2}{4\pi\varepsilon_0 a}$

Potential energy at B due to $C = \frac{\mathrm{Qq}}{4\pi\varepsilon_0 \mathrm{a}}$

Potential energy at C due to A $= \frac{\mathrm{Qq}}{4\pi\varepsilon_0 \mathrm{a}}$

Total P.E $= \frac{q}{4\pi\varepsilon_0 a}(q + Q + Q) = 0$

$2Q + q = 0 \Rightarrow Q = -\frac{q}{2}$

30. (d) For an adiabatic process,

$$\mathrm{W} = \frac{\mathrm{nR}(\mathrm{T_i} - \mathrm{T_f})}{\gamma - 1}$$

$$6R = \frac{R(T - T_f)}{\left(\frac{5}{3} - 1\right)} \ldots.$$

$$T_f = (T - 4)K$$

31. (c) Both the blue and red light have different wavelengths and for interference fringes to be seen, we require coherent light sources.

32. (a) Using Bernoulli's equation,

$$\mathrm{P_1} + \frac{1}{2}\rho v_1^2 + \rho g h_1 = \mathrm{P_2} + \frac{1}{2}\rho v_2^2 + \rho g h_2$$

$$\mathrm{P_1} + \frac{1}{2}\rho v_1^2 = \mathrm{P_2} + \frac{1}{2}\rho v_2^2$$

$$\mathrm{P_1} - \mathrm{P_2} = \frac{1}{2}(\rho v_2^2 - \rho v_1^2)$$

$$v_2^2 - v_1^2 = \frac{2 \times 3}{1.25 \times 10^3} = 4.8 \times 10^{-3} \quad \text{.....(i)}$$

From equation of continuity,

$$\mathrm{A_1 v_1} = \mathrm{A_2 v_2}$$

$$\frac{v_1}{v_2} = \frac{\mathrm{A_2}}{\mathrm{A_1}} = \frac{5}{10} = 0.5$$

$$v_1 = 0.5 v_2 \quad \ldots\ldots \text{(ii)}$$

Substituting (ii) into (i),

$0.75 v_2^2 = 4.8 \times 10^{-3} \Rightarrow v_2 = 0.08\ \mathrm{m/s}$

Rate of flow of glycerine $= \mathrm{A_2 v_2}$

$= 5 \times 10^{-4} \times 0.08$

$= 4 \times 10^{-5}\ \mathrm{m^3/s}$

33. (a)

$$\mathrm{I_e} = \frac{\mathrm{n_e} \times \mathrm{e}}{\mathrm{t}} \text{ and } \mathrm{I_c} = \frac{\mathrm{n_c} \times \mathrm{e}}{\mathrm{t}}$$

From the data given,

$$\mathrm{I_c} = \frac{(98/100)\mathrm{n_e} \times \mathrm{e}}{\mathrm{t}} = \frac{98}{100}\mathrm{I_e}$$

$$\alpha = \frac{\mathrm{I_c}}{\mathrm{I_c}} = \frac{98\mathrm{I_e}}{100\mathrm{I_e}} = 0.98$$

$$\beta = \frac{\alpha}{1 - \alpha} = \frac{0.98}{1 - 0.98} = \frac{0.98}{0.02} = 49$$

34. (c) Given: $\phi = 1.6\pi$ and $\lambda = \frac{\mathrm{V}}{\mathrm{f}}$

We know,

Phase difference $\phi = \frac{2\pi x}{\lambda}$

$$1.6\pi = \frac{2\pi x f}{330} \Rightarrow f = \frac{1.6 \times 330}{2 \times 40 \times 0.01}$$

$f = 660$ Hz

35. **(d)** For a body performing U.C.M, $a = \omega^2 x$ and

$\omega = 2\pi f$

$x = \frac{d}{2} = .5$ m

$a = 4\pi^2 f^2 \times .5 \Rightarrow a = 4\pi^2 (4)^2 \times .5$

$a = 32\pi^2$

36. **(d)**

$$\text{Electric flux } \phi = \frac{q}{\varepsilon_0} = \frac{4\pi\pi^2\sigma}{\varepsilon_0}$$
$$= \frac{4 \times 3.14 \times (7 \times 10^{-2})^2 \times 40 \times 10^{-6}}{8.85 \times 10^{-12}}$$
$$= \frac{4 \times 3.14 \times 49 \times 10^{-4} \times 4 \times 10^{-5}}{8.85 \times 10^{-12}}$$
$$= \frac{4 \times 3.14 \times 49 \times 4 \times 10^{3}}{8.85}$$
$$= 278 \times 10^3 \approx 280 \times 10^3 \text{ Wb}$$
$$= 280 \text{kWb}$$

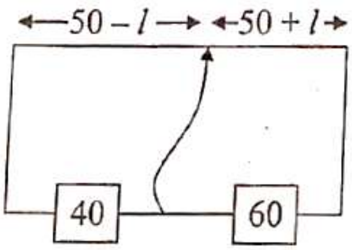
37. **(c)**

38. **(a)**

$$i = \sqrt{2} \text{ A}$$
$$B_{net} = \sqrt{B_1^2 + B_2^2}$$
$$= \sqrt{\left(\frac{\mu_0 i}{2R}\right)^2 + \left(\frac{\mu_0 i}{2R}\right)^2} = \sqrt{2\left(\frac{\mu_0 i}{2R}\right)^2}$$
$$= \sqrt{2\frac{\mu_0^2 i^2}{4R^2}} = \sqrt{\frac{1}{2}\left(\frac{\mu_0 i}{R}\right)^2}$$
$$= \frac{1}{\sqrt{2}}\frac{\mu_0 i}{R} = \frac{1}{\sqrt{2}} \times \frac{\mu_0\sqrt{2}}{1} = \mu_0$$

39. **(c)**

40. **(b)**



From the given data,

$$\frac{40}{60} = \frac{50 - l}{50 + l}$$
$$2000 + 40l = 3000 - 60l$$
$$100l = 1000$$
$$l = 10 \text{ cm}$$

41. **(c)** For $r > R, F_1 = \frac{GM}{r_1^2}$ For $r < R, F_2 = \frac{GMr_2}{R^3}$ $\frac{F_1}{F_2} = \frac{GM}{r_1^2} \times \frac{R^3}{GMr_2} = \frac{R^3}{r_1^2 r_2}$

42. **(b)**

We know $T = 2\pi\sqrt{\frac{m}{k}}$

When 500 g is removed, $m = (100 + 300)g = 0.4$ kg

$$T = 2\pi\sqrt{\frac{0.4}{k}} = 3\text{ s}$$

$$\frac{2\pi}{\sqrt{k}} = \frac{3}{\sqrt{0.4}} \quad \dots (i)$$

Similarly when 300 g is removed, $m' = 100\text{ g} = 0.1\text{ kg}$

$$T' = 2\pi\sqrt{\frac{0.1}{k}} = x$$

$$\frac{2\pi}{\sqrt{k}} = \frac{x}{\sqrt{0.1}}$$

$$\frac{3}{\sqrt{0.4}} = \frac{x}{\sqrt{0.1}} \quad . \quad \dots (\text{form}(i)$$

$$x = T' = \frac{3}{2} = 1.5\text{ s}$$

43. (d) Given: $V = ar^2 + b$

$$E = -\frac{dV}{dr}$$

$$= -\frac{d}{dr}(ar^2 + b)$$

$$E = -2ar \quad \dots (i)$$

By Gauss' law,

$$\vec{E} \cdot \vec{S} = \frac{q}{\varepsilon_0}$$

$$4\pi r^2 E = \frac{q}{\varepsilon_0} \dots (ii)$$

Total charge (q) enclosed $= \frac{4p}{3}\pi r^3$

... (ρ = charge density of ball)

From (i) and (ii)

$$4\pi r^2 \times (-2ar) = \frac{4\rho}{3\varepsilon_0}\pi r^3$$

$$-2a = \frac{\rho}{3\varepsilon_0}$$

$$\rho = -6a\varepsilon_0$$

44. (c) Maximum energy in capacitor = Maximum energy in inductor

$$\frac{1}{2}CV^2 = \frac{1}{2}LI^2$$

$$I^2 = \frac{C}{V}V^2$$

$$I = V\sqrt{\frac{C}{L}}$$

45. (b)

46. (b)

For A, $E_{A_{max}} = h\nu - \phi_A$

For B, $E_{B_{max}} = h(2\nu) - \phi_B$

$$\frac{E_{A_{max}}}{E_{B_{max}}} = \frac{h\nu - \phi_A}{2h\nu - \phi_B}$$

As $\frac{\phi_A}{\phi_B} = \frac{1}{2} \Rightarrow \phi_B = 2\phi_A$

$$\frac{E_{A_{max}}}{E_{B_{max}}} = \frac{\frac{h\nu - \phi_A}{\phi_A}}{\frac{2h\nu - \phi_B}{\phi_A}} = \frac{\frac{h\nu}{\phi_A} - 1}{\frac{2h\nu}{\phi_A} - 2}$$

$$= \frac{h\nu - \phi_A}{\phi_A} \times \frac{\phi_A}{2(h\nu - \phi_A)} = \frac{1}{2}$$

47. (b) According to mathematical equation R.P. $\propto \frac{1}{\lambda}$, telescopes being used in sunlight to observe celestial objects, practically there is no control on wavelength incident (λ). Hence, using objective lens of large aperture is effective way to increase R.P. of telescope.

48. (b) The energy stored in the inductance is $U = \frac{1}{2}LI^2$

$$U = \frac{1}{2} \times 1 \times 1^2 = 0.5\text{ J}$$

This energy must be transferred to the capacitor.

Energy stored by a capacitor $U = \frac{1}{2}CV^2$

$$\frac{1}{2}CV^2 = \frac{1}{2}LI^2$$

$$C = L\left(\frac{I}{V}\right)^2 = 1\left(\frac{1}{500}\right)^2 = 4\mu\text{ F}$$

49. (b) Frequency of n^{th} harmonic of a closed pipe

$$f_n = \frac{nV}{4L}$$

Frequency of different modes of vibration in a closed pipe $f_{n=}\frac{(2n-1)V}{4L_1}$

Frequency of 2^{nd} overtone, $f_3 = \frac{5V}{4L_1}(n = 3)$

In a closed pipe, the number of nodes and antinodes are the same. Hence, there will be 3 nodes and 3 antinodes.

50. (a)

$$T = 2\pi\sqrt{\frac{I}{MB}} \Rightarrow T \propto \frac{1}{\sqrt{B}}$$

Given $n_1 = 30\text{osc./min} \Rightarrow T_1 = 2\text{ s}$

$$\frac{T_2}{T_1} = \frac{\sqrt{B_1}}{\sqrt{B_2}}$$

Now, $B_2 = B_1 + 2B_1 = 3B_1$

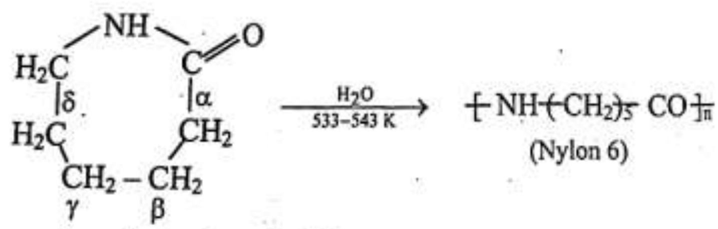
$$\frac{T_2}{T_1} = \sqrt{\frac{B_1}{3B_1}} = \frac{1}{\sqrt{3}} \Rightarrow T_2 = \frac{1}{\sqrt{3}} \times 2 = \frac{2}{\sqrt{3}}s$$

Chemistry

51. (c)

52. (b) In bcc unit cell, $\left(\frac{1}{8} \times 8\right) + 1 = 2$

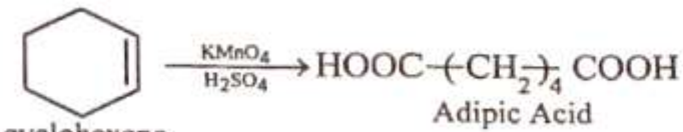
53. (c)



ε-Caprolactam (n molecules)

54. (a)

55. (d)



cyclohexene

56. (a) Cell representation:

$Pb_{(s)}|Pb^{++}_{(1M)}||Ag^{+}_{(10M)}|Ag_{(s)}$

Reaction: $Pb_{(s)} + 2Ag^{+}_{(10M)} \rightarrow Pb^{++}_{(1M)} + 2Ag_{(s)}$

Nernst equation at 25°C :

$$E_{cell} = E^{o}_{cell} - \frac{0.0592}{2}\log_{10}\frac{[Pb^{++}]}{[Ag^{+}]^{2}}$$

$$E_{cell} = E^{o}_{cell} - \frac{0.0592}{2}\log_{10}\frac{1}{100}$$

$$E_{cell} = E^{o}_{cell} - \frac{0.0592}{2}\log_{10}10^{-2}$$

$$E_{cell} = E^{o}_{cell} - \frac{0.0592}{2}\times(-2)$$

$$E_{cell} = (E^{o}_{ccll} + 0.0592)V$$

57. (d)

Swartz reaction:

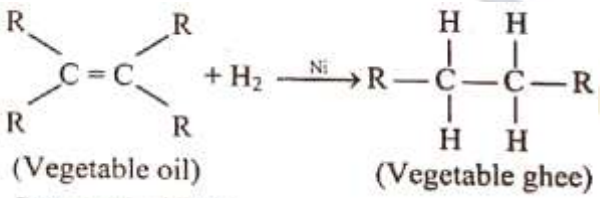
$R-Br + AgF \rightarrow R-F + AgBr$

(Alkyl fluorides)

58. (b) According to Lewis theory the bond order is equal to the number of bonds between the two atoms in a molecule.

$C \equiv O$ has bond order 3.

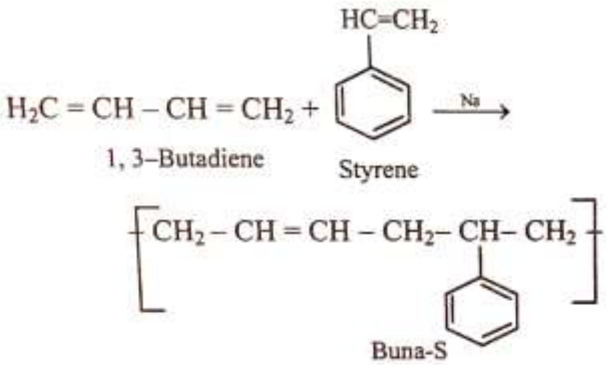
59. (d)



Reactant = liquid

Catalyst = solid

60. (b)



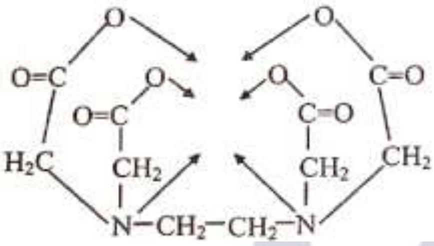
61. **(a)**
62. **(b)**

$$CH_3NH_2 \xrightarrow[\Delta]{CH_3I} (CH_3)_2NH \xrightarrow[\Delta]{CH_3I}$$
$$(CH_3)_3 N \xrightarrow[CH_3I]{\Delta} (CH_3)_4 N^+I^-$$

63. **(d)**

$$kt = \ln\frac{[A_0]}{A}$$
$$0.02303 \text{ hour }^{-1} \times t = \ln\frac{100}{20}$$
$$t = 69.88 \text{ hour } \approx 70 \text{ hour}$$

64. **(c)**



(EDTA)

65. **(d)** Let us consider the mass of each gas $= 1$ g Order of molecular weight of the given gases: $H_2 < N_2 < O_2 < Cl_2$
The order of no. of moles: $H_2 > N_2 > O_2 > Cl_2$ From ideal gas equation, $n \propto V$
Volume occupied by the gases is as follows:

$H_2 > N_2 > O_2 > Cl_2$
At constant temperature, (Boyle's law)

$$P \propto \frac{1}{V}$$

Since volume occupied by Cl_2 gas is the least, Cl_2 gas exerts the highest pressure.

66. **(b)**

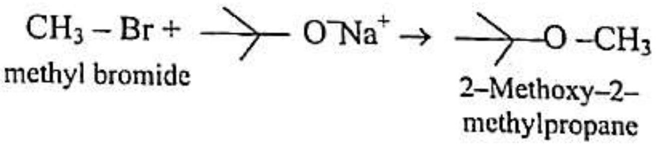
$$\rho = \frac{n \times M}{a^3 \times N_A}$$
$$a^3 = \frac{n \times M}{\rho \times N_A} = \frac{4 \times 27 \text{ g mol}^{-1}}{16 \times 10^{23} \text{ g cm}^{-3} \text{ mol}^{-1}}$$
$$V = a^3 = 6.75 \times 10^{-23} \text{ cm}^3$$

67. **(b)**

$$pH = pKa + \log\frac{[\text{ Salt }]}{[\text{ Acid }]}$$
$$7.2 = 6.2 + \log\frac{[\text{ Salt }]}{[\text{ Acid }]}$$
$$1 = \log\frac{[\text{ Salt }]}{[\text{ Acid }]} = \log_{10} 10$$
$$\frac{[\text{ Salt }]}{[\text{ Acid }]} = 10$$

68. **(d)** Based on the extent of the intermolecular forces the observed order of boiling points of isomeric amines is: primary amine > secondary amine > tertiary amine.
Hence, n-butyl amine has the highest boiling point.

69. **(d)**



70. **(c)** The decomposition of NaCl into $Na_{(s)}$ and $Cl_{2(g)}$ is not spontaneous as cell potential for the process is negative.

71. **(d)** As the number of unpaired electrons in 3d orbitals increases, the number of oxidation states shown by the element also increases.
$Zn = [Ar]3\,d^{10}4\,s^2$ shows only +2 oxidation state.

72. **(a)**

73. **(c)**

In FCC $,\sqrt{2}a = 4r$

$$r = \frac{a}{2\sqrt{2}}$$
$$a = 2\times\sqrt{2}\times 139\text{pm} = 3.93\times10^{-8}\text{ cm}$$

74. **(a)**

75. **(b)** For a basic buffer,

$$\begin{aligned} pOH &= pK_b + \log\frac{[\text{ Salt }]}{[\text{ Base }]} \\ &= 4.730 + \log\left(\frac{0.5}{0.4}\right) \\ &= 4.83 \end{aligned}$$

76. **(c)**

$$\begin{aligned} \Delta U &= q + w \\ &= 605 + (-380) \\ &= 225\text{ J} \end{aligned}$$

77. **(a)**

78. **(c)**

$$\begin{aligned} \text{Conductivity} &= \frac{\text{cell constant}}{\text{Resistance}} \\ &= \frac{0.33\text{ cm}^{-1}}{30\Omega} \\ &= 0.011\Omega^{-1}\text{ cm}^{-1} \end{aligned}$$

79. **(a)** Zn belongs to period 4 and group 12 of the periodic table.
periodic table.
$Zn(\text{ ground state }) = [Ar]3\,d^{10}4\,s^2$
$Zn^{2+}(\text{ excited state }) = [Ar]3\,d^{10}$

80. **(a)**

$$CH_3-CH_2-Br + CH_3-CH_2-CH_2-Br + 2Na \xrightarrow{\text{dry ether}} CH_3(CH_2)_2CH_3 + CH_3(CH_2)_3CH_3 + CH_3(CH_2)_4CH_3$$

Bromoethane, Bromopropane → (Butane), (Pentane), (Hexane)

81. (b) Let oxidation number of P in $PF_6^- = x$

$x + (6 \times -1) = -1$

$x = +5$

Let oxidation number of V in $V_2O_7^{4-} = y$

$2y + (-2 \times 7) = -4$

$y = +5$

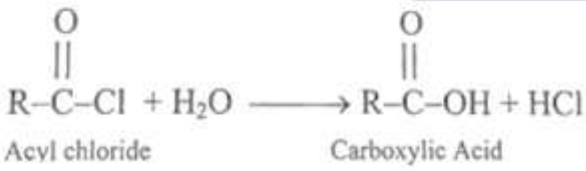
82. (c)

$\Delta T_f = K_f m$

$$m = \frac{\Delta T_f}{K_f} = \frac{0.93\ \text{K}}{1.86\ \text{K kg mol}^{-1}} = 0.5\ \text{mol kg}^{-1}$$

83. (a) The pH of a solution depends on the hydrolysis of a salt. Except NH_4CN, all the other salts are products of strong acids HCl or HNO_3. Hence, NH_4CN turns red litmus blue as HCN is a weak acid.

84. (c)



85. (a)

$$\text{Rate of reaction} = \frac{1}{2}\frac{d[N_2O_5]}{dt} = \frac{d[O_2]}{dt}$$

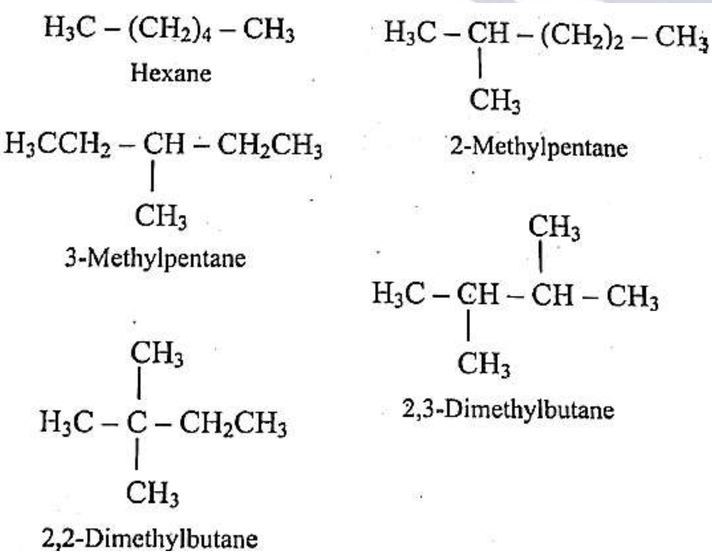
$$\frac{d[O_2]}{dt} = \frac{0.02}{2} = 0.01\ \text{mol dm}^{-3}\ \text{s}^{-1}$$

86. (a) $[Co(NH_3)_5H_2O]Cl_3$

NH_3, H_2O = Neutral ligands

SCN^-, en, NO_2^-, CN^- = anionic ligands.

87. (d) An alkane will exhibit five structural isomers when there are 6 moles of carbon atoms present in it. The structural isomers of alkane C_6H_{14} are as follows:



88. (b)

$CH_3MgBr \xrightarrow{CdCl_2} (CH_3)_2Cd + 2Mg(Cl)Br$

$$\underset{(A)}{(CH_3)_2Cd} \xrightarrow{2CH_3COCl} 2CH_3-\overset{\overset{\large O}{||}}{C}-CH_3$$

(B)

(Propanone)

89. (a) Arrhenius equation $k = Ae^{-E_a/RT}$ shows that, as the temperature rises, $\frac{E_a}{RT}$ decreases. This causes an increase in $e^{-E_a/RT}$ which increases k and the rate of reaction.

90. (d)

91. (d)

$$E_4 = -2.18 \times 10^{-18} \frac{(1)^2}{(4)^2} \text{ J (For H atom, Z = 1)}$$

$$= -0.136 \times 10^{-18} \text{ J}$$

92. (d)

93. (c)

$Ne = 10 = 1s^2 2s^2 2p^6$

$O^{2-} = 10 = 1s^2 2s^2 2p^6$

$Na^+ = 10 = 1s^2 2s^2 2p^6$

$Na = 11 = 1s^2 2s^2 2p^6 3s^1$

94. (d)  OH (tert-butyl alcohol)

95. (a) ns^2np^5 is general E.C. of halogens I has outer E.C. as ns^2np^5

96. (a)

Number of moles $= 5 = \frac{\text{Given mass (g)}}{\text{molar mass}}$

For acetic acid, $M = 60 \text{ g mol}^{-1}$.

Mass in kg $= \frac{5 \text{ mol} \times 60 \text{ g mol}^{-1}}{1000} = 0.3$ kg

97. (c) Relative lowering in vapour pressure

$$\frac{\Delta P}{P^\circ} = \frac{P^\circ - P}{P^\circ} = x_2 = \frac{W_2 M_1}{W_1 M_2}$$

$$\frac{640 - P}{640} = \frac{2 \times 78}{50 \times 64}$$

$640 - P = 640 \times 0.048$

$P = 640 - 31.2$

$P = 608.8$ mm Hg

98. (b)

$W = -P\Delta V$

$= -2 \times 10^5 \text{ N m}^{-2} \times 200 \times 10^{-6} \text{ m}^3$

$= -40.0 \text{ J} \approx -40.53 \text{ J}$

99. (b)

$$H_2C = CH - \underset{\underset{CH_3}{|}}{CH} - OH \Rightarrow 2^\circ \text{ Allylic alcohol}$$

$CH_2 = CH - CH_2 - OH \quad \Rightarrow 1^\circ$ Allylic alcohol

$CH_2 = CH - C(CH_3)_2 - OH \quad \Rightarrow 3^\circ$ Allylic alcohol

$CH_3 - CH = CH - CH_2 - CH_2 \quad -OH$

$\Rightarrow$ Not allylic alcohol

100. **(d)** Copper pyrite $= CuFeS_2$

Mathematics

101. **(c)** Given cartesian equation of line is

$4x-3z+5=0, y=2$

$\Rightarrow 4x=3z-5, y=2$

$\Rightarrow 4x=3\left(z-\frac{5}{3}\right), y=2$

$\Rightarrow \frac{x}{3}=\frac{z-\frac{5}{3}}{4}, y=2$

The given line passes $\left(0,2,\frac{5}{3}\right)$ and the direction ratios are proportional to 3,0,4.

The vector equation is

$\bar{r}=\left(2\hat{j}+\frac{5}{3}\hat{k}\right)+\lambda(3\hat{i}+4\hat{k})$

102. **(c)** There are 3 books on physics, 2 books on Chemistry and 4 books on Mathematics.

All Physics books and Mathematics books are kept together.

All Physics books will be considered as 1 unit and all Mathematics books are also considered as 1 unit.

Physics, Mathematics and 2 Chemistry books can be arranged in 4 ! ways.

Also, 3 Physics and 4 Mathematics books can be arranged themselves in $3! \times 4!$ ways

Total number of arrangements $= 4! \times 3! \times 4!$

$= 3456$

103. **(d)**

$$\begin{aligned}
\text{L.H.L.} &= \lim_{x\to 0^-}\frac{x}{|x|+x^2}\\
&= \lim_{x\to 0^-}\frac{x}{-x+x^2}\\
&= \lim_{x\to 0^-}\frac{1}{x-1}=-1\\
\text{R.H.L.} &= \lim_{x\to 0^+}\frac{x}{|x|+x^2}\\
&= \lim_{x\to 0^+}\frac{x}{x+x^2}\\
&= \lim_{x\to 0^+}\frac{1}{1+x}=1\\
&= \text{L.H.L.} \neq \text{R.H.L.}
\end{aligned}$$

Limit does not exist.

104. **(a)**

$\bar{a}=2\hat{i}+\hat{j}-2\hat{k}$ and $\bar{b}=\hat{i}+\hat{j}$

$|a|=\sqrt{4+1+4}=3$

$$\bar{a}\times\bar{b}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 1 & -2\\ 1 & 1 & 0\end{vmatrix}=2\hat{i}-2\hat{j}+\hat{k}$$

$|\bar{a}\times\bar{b}|=\sqrt{4+4+1}=3$

Angle between $\bar{c}$ and $\bar{a}\times\bar{b}$ is $\frac{\pi}{6}$ ... [Given]

$\sin\frac{\pi}{6}=\frac{|(\bar{a}\times\bar{b})\times\bar{c}|}{|\bar{a}\times\bar{b}||\bar{c}|}$

$\Rightarrow \frac{1}{2}=\frac{3}{3\times|\bar{c}|}$

$\Rightarrow |\bar{c}|=2$

Now, $|\bar{c}-\bar{a}|=3$

$\Rightarrow |\bar{c}|^2+|\bar{a}|^2-2\bar{a}\cdot\bar{c}=9$

$\Rightarrow 4+9-2\bar{a}\cdot\bar{c}=9$

$\Rightarrow \bar{a}\cdot\bar{c}=2$

105. (d)

$\cos(2\cos^{-1}x + \sin^{-1}x)$

$= \cos[(\sin^{-1}x + \cos^{-1}x) + \cos^{-1}x]$

$= \cos\left(\frac{\pi}{2} + \cos^{-1}x\right)$

$= -\sin(\cos^{-1}x)$

$= -\sin\left(\sin^{-1}\sqrt{(1-x^2)}\right)$

$= -\sqrt{1-x^2}$

$= -\sqrt{1-\left(\frac{1}{5}\right)^2}$

$= -\sqrt{\frac{24}{25}} = -\frac{2\sqrt{6}}{5}$

106. (b)

$\tan^{-1}(2x) + \tan^{-1}(3x) = \frac{\pi}{4}$

$\Rightarrow \tan^{-1}\left[\frac{2x+3x}{1-(2x)(3x)}\right] = \frac{\pi}{4}$

$\Rightarrow \frac{5x}{1-6x^2} = \tan\frac{\pi}{4} = 1$

$\Rightarrow 6x^2 + 5x - 1 = 0$

$\Rightarrow (x+1)(6x-1) = 0$

$\Rightarrow x = -1 \text{ or } x = \frac{1}{6}$

But, $x \geq 0$

$x = \frac{1}{6}$

A is a singleton set.

107. (b) Given equation of pair of lines is

$x^2 + 2hxy + 2y^2 = 0$

$A = 1, B = 2, H = h$

Let the slopes of the lines be m_1, m_2.

$m_1 + m_2 = \frac{-2h}{2}$, $m_1 m_2 = \frac{1}{2}$

Given $\frac{m_1}{m_2} = \frac{1}{2}$

$\Rightarrow m_1 = \frac{1}{2} m_2$

$\Rightarrow \frac{1}{2} m_2 \times m_2 = \frac{1}{2}$

$\Rightarrow m_2^2 = 1$

$\Rightarrow m_2 = \pm 1$

$m_1 = \pm\frac{1}{2}$

Now, $m_1 + m_2 = \frac{-2h}{2}$

$\therefore$ when $m_1 = \frac{1}{2}, m_2 = 1$

$m_1 + m_2 = \frac{-2h}{2}$

$\therefore$ $h = \frac{-3}{2}$

When $m_1 = \frac{-1}{2}, m_2 = -1$

$\therefore$ $m_1 + m_2 = \frac{-2h}{2}$

$\frac{-1}{2} - 1 = -\frac{2h}{2}$

$\therefore$ $h = \frac{3}{2}$

108. **(a)** $y = e^{4x}\left(\frac{x-4}{x+3}\right)^{\frac{3}{4}}$

Taking log on both sides,

$$\log y = \log\left[e^{4x}\left(\frac{x-4}{x+3}\right)^{\frac{3}{4}}\right]$$

$$\log y = \log e^{4x} + \log\left(\frac{x-4}{x+3}\right)^{\frac{3}{4}}$$

$$\log y = 4x\log e + \frac{3}{4}\log\left(\frac{x-4}{x+3}\right)$$

$$\log y = 4x + \frac{3}{4}\log\left(\frac{x-4}{x+3}\right)$$

Differentiating w.r.to x on both sides,

$$\frac{1}{y}\frac{dy}{dx} = 4 + \frac{3}{4} \times \frac{1}{\frac{(x-4)}{(x+3)}} \cdot \frac{d}{dx}\left(\frac{x-4}{x+3}\right)$$

$$\frac{1}{y}\frac{dy}{dx} = 4 + \frac{3}{4}\left(\frac{x+3}{x-4}\right) \times \left(\frac{(x+3)-(x-4)}{(x+3)^2}\right)$$

$$\frac{1}{y}\frac{dy}{dx} = 4 + \frac{3}{4}\left(\frac{1}{x-4}\right) \times \left(\frac{x+3-x+4}{(x+3)}\right)$$

$$\frac{1}{y}\frac{dy}{dx} = 4 + \frac{3}{4} \times \left(\frac{1}{x-4}\right) \times \left(\frac{7}{x+3}\right)$$

$$\frac{dy}{dx} = y\left[4 + \frac{21}{4(x-4)(x+3)}\right]$$

109. **(c)**

Let $I = \int 3^{3^x} \cdot 3^x\, dx$

Let $3^x = t$

$3^x \log 3\, dx = dt$

$$3^x\, dx = \frac{1}{\log 3} dt$$

$$I = \int 3^t \cdot \frac{1}{\log 3} \cdot dt$$

$$= \frac{1}{\log 3} \int 3^t dt$$

$$= \frac{1}{\log 3} \times \frac{3^t}{\log 3} + c$$

$$I = \frac{3^{3^x}}{(\log 3)^2} + c$$

110. (a)

$$f(x) = \frac{1 + \cos \pi x}{\pi(1 - x)^2}$$

$$f(1) = \lim_{x \to 1} f(x)$$

$$= \lim_{x \to 1} \frac{1 + \cos \pi x}{\pi(1 - x)^2}$$

Put $1 - x = h$

$\Rightarrow 1 - h = x$

As $x \to 1$, $h \to 0$

$$\therefore\ f(1) = \lim_{h \to 0} \frac{1 + \cos \pi(1 - h)}{\pi h^2}$$

$$= \lim_{h \to 0} \frac{1 + \cos(\pi - \pi h)}{\pi h^2}$$

$$= \lim_{h \to 0} \frac{1 - \cos \pi h}{\pi h^2}$$

$$= \lim_{h \to 0} \frac{2\sin^2 \frac{\pi h}{2}}{\pi h^2}$$

$$= 2\lim_{h \to 0} \frac{\sin^2 \frac{\pi h}{2}}{\frac{\pi^2 h^2}{4} \times 4} \cdot \pi$$

$$= \frac{2\pi}{4} \lim_{h \to 0} \left(\frac{\sin \frac{\pi h}{2}}{\frac{\pi h}{2}} \right)^2$$

$$\therefore\ f(1) = \frac{\pi}{2}$$

111. (a)

$f(x) = 3\sin x - 4\sin^3 x$

$\Rightarrow f(x) = \sin 3x$

$\Rightarrow f'(x) = 3\cos 3x$

For $f(x)$ to be increasing,

$f'(x) \geq 0$

$\Rightarrow 3\cos 3x \geq 0$

$\Rightarrow \cos 3x \geq 0$

$$\Rightarrow \frac{-\pi}{2} \leq 3x \leq \frac{\pi}{2}$$

$$\Rightarrow \frac{-\pi}{6} \leq x \leq \frac{\pi}{6}$$

largest length in which f(x) is increasing

$= \frac{\pi}{6} - \left(\frac{-\pi}{6}\right) = \frac{\pi}{3}$

112. **(a)**

$\bar{a}[(\bar{b} \times \bar{c}) \times (\bar{a} + \bar{b} + \bar{c})]$

$= \bar{a}[(\bar{b} \times \bar{a} + \bar{b} \times \bar{b} + \bar{b} \times \bar{c} + \bar{c} \times \bar{a} + \bar{c} \times \bar{b} + \bar{c} \times \bar{c})]$

$= \bar{a}(\bar{b} \times \bar{a}) + \bar{a}(\bar{b} \times \bar{c}) + \bar{a}(\bar{c} \times \bar{a}) + \bar{a}(\bar{c} \times \bar{b})$

$= [\bar{a} \quad \bar{b} \quad \bar{c}] + [\bar{a} \quad \bar{c} \quad \bar{b}]$

$= [\bar{a} \quad \bar{b} \quad \bar{c}] - [\bar{a} \quad \bar{b} \quad \bar{c}]$

$= 0$

113. **(d)** Volume of the parallelopiped formed by vectors is i.e., $V = \begin{vmatrix} 1 & m & 1 \\ 0 & 1 & m \\ m & 0 & 1 \end{vmatrix} = 1 - m + m^3$

$\therefore \frac{dV}{dm} = -1 + 3m^2, \frac{d^2V}{da^2} = 6m$

For max. or min. of V, $\frac{dV}{dm} = 0$

$\therefore m^2 = \frac{1}{3}$

$\therefore m = \frac{1}{\sqrt{3}}$

$\frac{d^2V}{d^2} = 6m > 0$ for $m = \frac{1}{\sqrt{3}}$

$\therefore$ V is minimum for $m = \frac{1}{\sqrt{3}}$

114. **(d)** $\bar{a} = \hat{i} - \hat{j} + 2\hat{k}, \bar{b} = 2\hat{i} + 4\hat{j} + \hat{k}$ and $\bar{c} = m\hat{i} + \hat{j} + n\hat{k}$

$\bar{a}$ and $\bar{c}$ are orthogonal

$\bar{a} \cdot \bar{c} = 0$

$m(1) + (1)(-1) + n(2) = 0$

$m - 1 + 2n = 0$

$m + 2n = 1 \quad \ldots\ldots \text{(i)}$

Also, $\bar{b}$ and $\bar{c}$ are orthogonal

$\bar{b} \cdot \bar{c} = 0$

$2(m) + 4(1) + n(1) = 0$

$2m + n = -4 \quad \ldots\ldots \text{(ii)}$

Solving (i) and (ii), we get

$m = -3, n = 2$

$(m, n) = (-3, 2)$

115. **(c)**

Let $I = \int \log(1+x)^{1+x}\,dx$

$I = \int (1+x)\cdot\log(1+x)dx$

Put $1 + x = t$

$dx = dt$

$I = \int t\cdot\log t\cdot dt$

$= \log t\int tdt - \int\left(\frac{d}{dt}\log t\cdot\int t\,dt\right)dt$

$= \log t\cdot\frac{t^2}{2} - \int\left(\frac{1}{t}\times\frac{t^2}{2}\right)dt$

$= \log t\cdot\frac{t^2}{2} - \frac{1}{2}\int t\,dt$

$= \log t\frac{t^2}{2} - \frac{t^2}{4} + c$

$= t^2\frac{\log t}{2} - \frac{t^2}{4} + c$

$= \frac{t^2}{2}\left[\log t - \frac{1}{2}\right] + c$

$I = \frac{(1+x)^2}{2}\left[\log(1+x) - \frac{1}{2}\right] + c$

116. (a)

$I = \int\left(\frac{x+2}{x+4}\right)^2 e^x\,dx$

$= \int\left(\frac{x^2+4x+4}{(x+4)^2}\right)e^x\,dx$

$= \int e^x\left[\frac{x(x+4)}{(x+4)^2} + \frac{4}{(x+4)^2}\right]dx$

$= \int e^x\left[\frac{x}{(x+4)} + \frac{4}{(x+4)^2}\right]dx$

$= e^x\left(\frac{x}{x+4}\right) + c$

$\ldots\cdot\left[\int (f(x) + f'(x)e^x\,dx = e^x f(x) + c)\right]$

117. (a)

$\frac{1}{b+c} + \frac{1}{c+a} = \frac{3}{a+b+c}$

$\frac{a+b+c}{b+c} + \frac{a+b+c}{c+a} = \frac{3(a+b+c)}{a+b+c}$

$\frac{a}{b+c} + 1 + \frac{b}{c+a} + 1 = 3$

$\Rightarrow \frac{a}{b+c} + \frac{b}{c+a} = 1$

$\Rightarrow a(c+a) + b(b+c) = (b+c)(c+a)$

$\Rightarrow ac + a^2 + b^2 + bc = bc + ab + c^2 + ac$

$\Rightarrow a^2 + b^2 - c^2 = ab \quad \ldots.(i)$

By cosine Rule,

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab} \ldots.(\text{form(i)})$$

$$\cos C = \frac{ab}{2ab}$$

$$\Rightarrow \cos C = \frac{1}{2}$$

$$\Rightarrow C = \frac{\pi}{3}$$

118. (b)

$$z = \frac{(1+i)^2}{a-i}$$

$$= \frac{2i}{a-i}$$

$$= \frac{2i(a+i)}{a^2+1}$$

$$= \frac{-2+2ai}{a^2+1}$$

$$\therefore \ |z| = \sqrt{\frac{4+4a^2}{(a^2+1)^2}}$$

$$\Rightarrow \sqrt{\frac{2}{5}} = \frac{2}{\sqrt{a^2+1}}$$

$$\Rightarrow \ a = 3$$

$$\therefore \ z = \frac{-2+2(3)i}{3^2+1} = \frac{-2+6i}{10}$$

$$\Rightarrow z = \frac{-1+3i}{5}$$

$$\Rightarrow \bar{z} = \frac{-1}{5} - \frac{3i}{5}$$

119. (c)

(i) Probability that the first ball is black and second is red.

Total number of black balls $= 6$

Total number of red balls $= 4$

Probability of getting black ball in first draw $= \frac{6}{10}$.

Now, number of black balls $= 9$ and

Total number of balls $= 13$

$\therefore$ Probability of getting red ball in second draw $= \frac{4}{13}$.

(ii) Probability that both the balls are red.

Probability of getting red ball in first draw $= \frac{4}{10}$.

Now for second draw,

Number of red balls $= 7$ and

Total number of balls $= 13$.

$\therefore$ Probability of getting red ball in second draw $= \frac{7}{13}$

$\therefore$ Total probability of drawing red ball

$$= \left(\frac{6}{10} \times \frac{4}{13}\right) + \left(\frac{4}{10} \times \frac{7}{13}\right)$$

$$= \frac{24+28}{130}$$

$$= \frac{26}{65}$$

120. (b)

$f(x) = \sin|x| - |x| + 2(x-\pi)\cos|x|$

$f(x) = \sin x - x + 2(x-\pi)\cos x, x \geq 0$

$= -\sin x + x + 2(x-\pi)\cos x, x < 0$

Now,

$f'(x) = \cos x - 1 - 2(x-\pi)\sin x + 2\cos x, x \geq 0$

$= -\cos x + 1 - 2(x+\pi)\sin x + 2\cos x, x < 0$

$f(0^+) = \cos 0 - 1 - 2(0-\pi)\sin 0 + 2\cos 0 = 2$

$f(0^-) = -\cos 0 + 1 - 2(0-\pi)\sin 0 + 2\cos 0 = 2$

$f'(0^+) = f'(0^-) = 2$

f(x) is differentiable at $x = 0$

f(x) is differentiable everywhere.

$k = \phi$

121. (c)

Given,

$f(x) = f(a-x)$

$g(x) + g(a-x) = 4$

$$\text{Let } I = \int_0^a f(x)g(x)dx$$

$$= \int_0^a f(a-x)\cdot g(a-x)dx$$

$$= \int_0^a f(x)\cdot[(4-g(x)]dx$$

$$= 4\int_0^a f(x)dx - \int_0^a f(x)g(x)dx$$

$$I = 4\int_0^2 f(x)dx - I$$

$$2I = 4\cdot\int_0^a f(x)dx$$

$$\therefore\ I = 2\int_0^a f(x)dx$$

122. (c)

$\sqrt{3}\sec x + 2 = 0$

$$\Rightarrow \sec x = \frac{-2}{\sqrt{3}}$$

$$\Rightarrow \cos x = \frac{-\sqrt{3}}{2}$$

$$\Rightarrow \cos x = \cos\left(\pi - \frac{\pi}{6}\right) = \cos\frac{5\pi}{6}$$

$$\text{and } \cos x = \cos\left(\pi + \frac{\pi}{6}\right) = \cos\frac{7\pi}{6}$$

The principal solutions are $\frac{5\pi}{6}$ and $\frac{7\pi}{6}$

123. (c)

$$\tan^{-1}\sqrt{x(x+1)} + \sin^{-1}\sqrt{x^2+x+1} = \frac{\pi}{2}$$

$\tan^{-1}\sqrt{x(x+1)}$ is defined when

$x(x+1) \geq 0 \quad \ldots..(i)$

$\sin^{-1}\sqrt{x^2+x+1}$ is defined when

$x(x+1) + 1 \leq 1$ or $x(x+1) \leq 0 \ldots.(ii)$

From (i) and (ii),

$x(x+1) = 0$ or $x = 0$ and -1

Hence, number of solutions is 2.

124. **(b)** $(\bar{a}+\lambda\bar{b}) = (2-\lambda)\hat{i} + (2+2\lambda)\hat{j} + (3+\lambda)\hat{k}$

Since $\bar{a}+\lambda\bar{b}$ is perpendicular to $\bar{c}$

$$(\bar{a}+\lambda\bar{b})\cdot\bar{c} = 0$$
$$\Rightarrow (2-\lambda)(3) + (2+2\lambda)(1) + (3+\lambda)(0) = 0$$
$$\Rightarrow 6-3\lambda+2+2\lambda = 0$$
$$\Rightarrow \lambda = 8$$

125. **(d)** Given differential equation is

$\frac{dy}{dx} = (x-y)^2$ …. (i)

Let $x-y = t$

$$1-\frac{dy}{dx} = \frac{dt}{dx}$$
$$\Rightarrow \frac{dy}{dx} = 1-\frac{dt}{dx}$$

From (i)

$$1-\frac{dt}{dx} = t^2$$
$$1-t^2 = \frac{dt}{dx}$$
$$dx = \frac{1}{1-t^2}dt$$

Integrating on both sides, we get

$$\int dx = \int \frac{1}{1-t^2}dt$$
$$x = \frac{1}{2}\log\left|\frac{1+t}{1-t}\right| + c$$
$$x = \frac{1}{2}\log\left|\frac{1+x-y}{1-(x-y)}\right| + c$$
$$x = \frac{1}{2}\log\left|\frac{1+x-y}{1-x+y}\right| + c$$

But $y(1) = 1$

$c = 1$

$$x = \frac{1}{2}\log\left|\frac{1+x-y}{1-x+y}\right| + 1$$
$$x-1 = \frac{1}{2}\log\left|\frac{1+x-y}{1-x+y}\right|$$
$$2(x-1) = \log\left|\frac{1+x-y}{1-x+y}\right|$$
$$\Rightarrow -\log\left|\frac{1-x+y}{1+x-y}\right| = 2(x-1)$$

126. **(b)**

$$f(x) = \bar{a}\cdot(\bar{b}\times\bar{c})$$
$$= \begin{vmatrix} x & -2 & 3 \\ -2 & x & -1 \\ 7 & -2 & x \end{vmatrix}$$
$$= x(x^2-2) + 2(-2x+7) + 3(4-7x)$$

$\therefore$ $f(x) = x^3 - 27x + 26$

Now, $f'(x) = 0$

$\Rightarrow 3x^2 - 27 = 0$

$\Rightarrow x^2 = 9$

$\Rightarrow x = \pm 3$

$f''(x) = 6x$

$\Rightarrow f''(x) = 18 > 0$

$\therefore$ f(x) has local minimum at $x = 3$.

$\bar{a} = 3\hat{\imath} - 2\hat{\jmath} + 3\hat{k}$

$\bar{b} = -2\hat{\imath} + 3\hat{\jmath} - \hat{k}$

$$\begin{aligned}\vec{a}\cdot\bar{b} &= (3\hat{\imath} - 2\hat{\jmath} + 3\hat{k})\cdot(-2\hat{\imath} + 3\hat{\jmath} - \hat{k})\\ &= 3(-2) + (-2)(3) + 3(-1)\\ &= -15\end{aligned}$$

127. **(a)**

$$\bar{a}\times(\bar{b}\times\bar{c}) = \frac{\sqrt{3}}{2}(\bar{b}+\bar{c})$$

$$\Rightarrow (\bar{a}\cdot\bar{c})\bar{b} - (\bar{a}\cdot\bar{b})\bar{c} = \left(\frac{\sqrt{3}}{2}\right)\bar{b} + \left(\frac{\sqrt{3}}{2}\right)\bar{c}$$

$$\Rightarrow \bar{a}\cdot\bar{c} = \frac{\sqrt{3}}{2} \text{ and } \bar{a}\cdot\bar{b} = \frac{-\sqrt{3}}{2}$$

$$\Rightarrow |\bar{a}|\bar{b}\Big|\cos\theta = \frac{-\sqrt{3}}{2}$$

$$\Rightarrow \cos\theta = \frac{-\sqrt{3}}{2} = \cos\frac{5\pi}{6}$$

$$\Rightarrow \theta = \frac{5\pi}{6}$$

128. **(a)** Given: $f(x) = \frac{a-x}{a+x}$

$\because f(f(x)) = x$

$$\Rightarrow \frac{a - f(x)}{a + f(x)} = x$$

$$\Rightarrow \frac{a - \left(\frac{a-x}{a+x}\right)}{a + \left(\frac{a-x}{a+x}\right)} = x$$

$$\Rightarrow \frac{a^2 + ax - a + x}{a^2 + ax + a - x} = x$$

$$\Rightarrow (a^2 - a) + (a+1)x = (a^2 + a)x + (a-1)x^2$$

$$\Rightarrow (a-1)x^2 + (a^2-1)x - a^2 + a = 0$$

$$\Rightarrow (a-1)[x^2 + (a+1)x - a] = 0$$

This is possible when $a = 1$

$$f(x) = \frac{1-x}{1+x}$$

$$f\left(\frac{-1}{5}\right) = \frac{1 - \left(\frac{-1}{5}\right)}{1 + \left(\frac{-1}{5}\right)}$$

$$= \frac{1 + \frac{1}{5}}{1 - \frac{1}{5}}$$

$$= \frac{\frac{6}{5}}{\frac{4}{5}} = \frac{6}{4} = 1.5$$

129. **(c)**

p	q	r	$q \wedge r$	$\sim(q \wedge r)$	$q \vee \sim(q \wedge r)$
T	T	T	T	F	T
T	T	F	F	T	T
T	F	T	F	T	T
T	F	F	F	T	T
F	T	T	T	F	T
F	T	F	F	T	F
F	F	T	F	T	T
F	F	F	F	T	T

From above table, $p \vee \sim(q \wedge r)$ is false, if $p = F, \mathrm{q} = \mathrm{T}, \mathrm{r} = \mathrm{T}$

130. **(b)** Let $\left(m_i, \frac{1}{m_i}\right)$ lie on the circle $x^2 + y^2 + 2\,gx + 2fy + c = 0$

where i = 1,2,3,4

$$(m_i)^2 + \left(\frac{1}{m_i}\right)^2 + 2\mathrm{gm_i} + \frac{2\mathrm{f}}{m_i} + \mathrm{c} = 0$$

$$\Rightarrow \mathrm{m_i^2} + \frac{1}{\mathrm{m_i^2}} + 2\mathrm{gm_i} + \frac{2\mathrm{f}}{\mathrm{m_i}} + \mathrm{c} = 0$$

$$\Rightarrow \mathrm{m_i^4} + 1 + 2\mathrm{gm_i^3} + 2\mathrm{fm_i} + \mathrm{cm_i^2} + 2 = 0$$

$$\Rightarrow \mathrm{m_i^4} + 2\mathrm{gm_i^3} + \mathrm{cm_i^2} + 2\mathrm{fm_i} + 1 = 0$$

m_1, m_2, m_3, m_4 are roots of the above equation product of roots $= \frac{\mathrm{e}}{\mathrm{a}} = \frac{1}{1} = 1$

$\mathrm{m_1\ m_2\ m_3\ m_4} = 1$

131. **(d)** Given equations of lines

$x + (\mathrm{a} - 1)y = 1$

$2x + \mathrm{a}^2 y = 1$

Slope of $x + (a-1)y = 1$ is $\frac{-1}{a-1}$ and slope of $2x + a^2 y = 1$ is $\frac{-2}{a^2}$

Given lines are perpendicular

Product of slope $= -1$

$$\frac{-1}{(a-1)} \times \frac{-2}{a^2} = -1$$

$$\Rightarrow \frac{2}{a^2(a-1)} = -1$$

$\Rightarrow a^3 - a^2 = -2$

$\Rightarrow a^3 - a^2 + 2 = 0$

$\Rightarrow (a+1)(a^2 - 2a + 2) = 0$

$\Rightarrow a = -1, a^2 - 2a + 2 = 0$

$\Rightarrow a^2 - 2a + 2 \neq 0 \ldots [\because a \in R - \{0,1\}]$

$a = -1$

$\therefore$ Equations will be

$x - 2y = 1$

$2x + y = 1$

$\therefore$ Point of intersection is $\left(\frac{3}{5}, \frac{-1}{5}\right)$

Distance of point of intersection from origin

$$= \sqrt{\left(0-\frac{3}{5}\right)^2+\left(0-\left(\frac{-1}{5}\right)\right)^2}$$

$$= \sqrt{\frac{9}{25}+\frac{1}{25}}$$

$$= \sqrt{\frac{10}{25}} = \sqrt{\frac{2}{5}}$$

132. (d)

$$x\frac{\mathrm{d}y}{\mathrm{d}x} = y(\log y - \log x + 1)$$

$$\Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{y}{x}\left[\log\left(\frac{y}{x}\right)+1\right] \ldots (i)$$

Put $\mathrm{v} = \frac{y}{x}$

$\therefore \quad y = vx$

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \mathrm{v} + x\frac{\mathrm{dv}}{\mathrm{d}x}$$

From (i)

$$\therefore \; v + x\frac{dv}{dx} = v(\log v + 1)$$

$$v + x\frac{dv}{dx} = v\log v + v$$

$$x\frac{dv}{dx} = v\log v$$

$$\frac{1}{v\log v}dv = \frac{dx}{x}$$

Integrating on both sides, we get

$$\int \frac{1}{v\log v}dv = \int \frac{dx}{x}$$

$$\log(\log v) = \log x + c_1$$

$\log(\log v) = \log x + \log c$ where, $c_1 = \log c$

$$\Rightarrow \log(\log v) = \log(xc)$$

$$\Rightarrow \log v = xc$$

$$\Rightarrow \log\left(\frac{y}{x}\right) = cx$$

133. (d) Let θ be the temperature of ball at any time ' t '.

$$\therefore \quad \frac{\mathrm{d}\theta}{\mathrm{dt}} \propto (\theta - 20)$$

$$\Rightarrow \frac{\mathrm{d}\theta}{\mathrm{dt}} = -\mathrm{k}(\theta - 20), \mathrm{k} > 0$$

Integrating on both sides, we get

$\log|\theta - 20| = -\mathrm{kt} + \mathrm{c}$

when $\mathrm{t} = 0, \theta = 80^\circ$

$\therefore \quad c = \log 60$

$\therefore \quad \log|\theta - 20| = -kt + \log 60 \cdots\cdots (i)$

When $t = 5, \theta = 60°$

$\therefore \quad \log 40 = -5k + \log 60$

$\Rightarrow 5k = \log 60 - \log 40$

$\Rightarrow 5k = \log\left(\frac{3}{2}\right)$

$\Rightarrow k = \frac{1}{5}\log\left(\frac{3}{2}\right)$

$\therefore \quad \log|\theta - 20| = \frac{-1}{5}\log\left(\frac{3}{2}\right)t + \log 60 \ \ (form(i)$

when $t = 20$

$\Rightarrow \log|\theta - 20| = \frac{-1}{5}\log\left(\frac{3}{2}\right)20 + \log 60$

$\Rightarrow \log|\theta - 20| = -4\log\left(\frac{3}{2}\right) + \log 60$

$\Rightarrow \log|\theta - 20| = \log\left(\frac{2}{3}\right)^4 + \log 60$

$\Rightarrow \quad \log|\theta - 20| = \log\left(\frac{16 \times 60}{81}\right)$

$\Rightarrow \quad \log|\theta - 20| = \log(11.85)$

$\Rightarrow \quad \theta - 20 = 11.85$

$\Rightarrow \quad \theta = 11.85 + 20$

$\Rightarrow \quad \theta = 31.85$

The temperature of ball after 20 minutes is 31.85°C

134. **(b)** Given that

$P(X = x) = k(x + 1)5^{-x}$, where $X = 0,1,2,3, ...$(i)

Since, $\sum_{x=0}^{\infty} P(X = x) = 1$

$\Rightarrow k\sum_{x=0}^{\infty} (x + 1)5^{-x} = 1$

$\Rightarrow k[1 + 2(5)^{-1} + 3(5)^{-2} + 4(5)^{-3} + \cdots] = 1$

$\Rightarrow k\left[1 + 2\left(\frac{1}{5}\right) + 3\left(\frac{1}{5}\right)^2 + 4\left(\frac{1}{5}\right)^3 + \cdots\right] = 1$

$\Rightarrow k \times \left[\frac{1}{1 - \frac{1}{5}} + \frac{1 \times \frac{1}{5}}{\left(1 - \frac{1}{5}\right)^2}\right] = 1$

$\Rightarrow k \times \frac{25}{16} = 1$

$\Rightarrow k = \frac{16}{25}$

$P(X = 0) = \frac{16}{25}(0 + 1)\left(\frac{1}{5}\right)^0 \ (form(i)$

$= \frac{16}{25}$

135. **(c)** Equation of plane passing through (3,1,1), (1,2,3) and (−1,4,2) is

$$\begin{vmatrix} x-3 & y-1 & z-1 \\ 1-3 & 2-1 & 3-1 \\ -1-3 & 4-1 & 2-1 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} x-3 & y-1 & z-1 \\ -2 & 1 & 2 \\ -4 & 3 & 1 \end{vmatrix} = 0$$

$\Rightarrow (x-3)(1-6)-(y-1)(-2+8)$
$+(z-1)(-6+4)=0$
$\Rightarrow -5(x-3)-6(y-1)-2(z-1)=0$
$\Rightarrow -5x+15-6y+6-2z+2=0$
$\Rightarrow -5x-6y-2z+23=0$
$\Rightarrow 5x+6y+2z-23=0$

136. **(d)** Let a, b, c be the direction ratios of the required line.
Since the line is perpendicular to the lines with d.r.s. 1,2,3 and −3,2,5.
$a+2b+3c=0$ ….(i)
and $-3a+2b+5c=0$ ….. (ii)
$\Rightarrow \frac{a}{2}=\frac{b}{-7}=\frac{c}{4}$ ...[From (i) and (ii)]
Equation of the required line is $\frac{x+1}{2}=\frac{y-3}{-7}=\frac{z+2}{4}$

137. **(d)**
$y = a\sin x + b\cos x$
Differentiating w. r. t. x, we get
$\frac{dy}{dx} = a\cos x - b\sin x$
$\therefore$ Now,

$y^2+\left(\frac{dy}{dx}\right)^2$
$=(a\sin x+b\cos x)^2+(a\cos x-b\sin x)^2$
$=a^2\sin^2 x+b^2\cos^2 x+2ab\sin x\cos x+a^2\cos^2 x$
$+b^2\sin^2 x-2ab\sin x\cdot\cos x$
$=a^2(\sin^2 x+\cos^2 x)+b^2(\sin^2 x+\cos^2 x)$
$=a^2+b^2$

$\therefore \quad y^2+\left(\frac{dy}{dx}\right)^2=a^2+b^2$
Here, a, b are constants.
$\therefore\ y^2+\left(\frac{dy}{dx}\right)^2$ is also a constant.

138. **(d)** Since f(x) satisfies the Rolle's theorem, $f(1)=f(3)$
$\therefore \quad 1+b+a+5=27+9b+3a+5$
$\Rightarrow 2a+8b=-26$
$\Rightarrow a+4b=-13$ ….(i)
$f(x)=x^3+bx^2+ax+5$
$\therefore \quad f'(x)=3x^2+2bx+a$
Now, $f'(c)=0$

$\Rightarrow f'\left(2+\frac{1}{\sqrt{3}}\right)=0$

$\Rightarrow 3\left(2+\frac{1}{\sqrt{3}}\right)^2+2b\left(2+\frac{1}{\sqrt{3}}\right)+a=0$

$\Rightarrow 3\left(4+\frac{4}{\sqrt{3}}+\frac{1}{3}\right)+4b+\frac{2b}{\sqrt{3}}+a=0$

$\Rightarrow a+4b+\frac{2b+12}{\sqrt{3}}+13=0$

$\Rightarrow -13+\frac{2b+12}{\sqrt{3}}+13=0$ (form(i))

$\Rightarrow \dfrac{2b+12}{\sqrt{3}}=0$

$\Rightarrow b=-6$

Substituting $b=-6$ in (i), we get $a=11$

139. (c)

Let X : be the number of defective bulbs Possible values of X is 1 .

Here,

$n=$ number of bulbs picked $=10$.

Let P (probability of getting defective bulb)

$$=10\%=\frac{1}{10}$$

$$q=1-p=1-\frac{1}{10}=\frac{9}{10}$$

Probability that at least one bulb is defective $=1-$ P (getting 0 defective bulb)

$$=1-P(X=0)$$

$$=1-{}^{10}C_0(p)^0(q)^{10-0}$$

$$=1-1\times1\times\left(\frac{9}{10}\right)^{10}$$

$$=1-\left(\frac{9}{10}\right)^{10}$$

140. (c) Let p: work is finished on time

q: contractor is in trouble

Given statement is

$\sim p\rightarrow q$

$\equiv p\vee q$

i.e., Either the work is finished on time or the contractor is in trouble.

141. (b) Let $\cos^{-1}\left(\frac{-3}{5}\right)=x$

$$\Rightarrow \cos x=\frac{-3}{5}$$

$$\sin x=\sqrt{1-\cos^2 x}=\sqrt{1-\left(\frac{-3}{5}\right)^2}=\frac{4}{5}$$

$$\begin{aligned}\therefore\ \sin\left(2\cos^{-1}\left(\frac{-3}{5}\right)\right)&=\sin 2x\\&=2\sin x\cdot\cos x\\&=2\times\frac{4}{5}\times\frac{-3}{5}\\&=\frac{-24}{25}\end{aligned}$$

142. (d)

$$y=\sqrt{\frac{1-\sin^{-1}x}{1+\sin^{-1}x}}$$

Differentiating w.r.t. x, we get

$$\frac{dy}{dx}=\frac{d}{dx}\left(\sqrt{\frac{1-\sin^{-1}x}{1+\sin^{-1}x}}\right)$$

$$=\frac{1}{2\sqrt{\frac{1-\sin^{-1}x}{1+\sin^{-1}x}}}\cdot\frac{d}{dx}\left(\frac{1-\sin^{-1}x}{1+\sin^{-1}x}\right)$$

$$=\frac{\sqrt{1+\sin^{-1}x}}{2\sqrt{1-\sin^{-1}x}}$$

$$\frac{(1+\sin^{-1}x)\cdot\frac{d}{dx}(1-\sin^{-1}x)-(1-\sin^{-1}x)\frac{d}{dx}(1+\sin^{-1}x)}{(1+\sin^{-1}x)^2}$$

$$=\frac{\sqrt{1+\sin^{-1}x}}{2\sqrt{1-\sin^{-1}x}}$$

$$\frac{(1+\sin^{-1}x)\times\frac{-1}{\sqrt{1-x^2}}-(1-\sin^{-1}x)\cdot\frac{1}{\sqrt{1-x^2}}}{(1+\sin^{-1}x)^2}$$

$$=\frac{\sqrt{1+\sin^{-1}x}}{2\sqrt{1-\sin^{-1}x}}\times\frac{\frac{1}{\sqrt{1-x^2}}[-1-\sin^{-1}x-1+\sin^{-1}x]}{(1+\sin^{-1}x)^2}$$

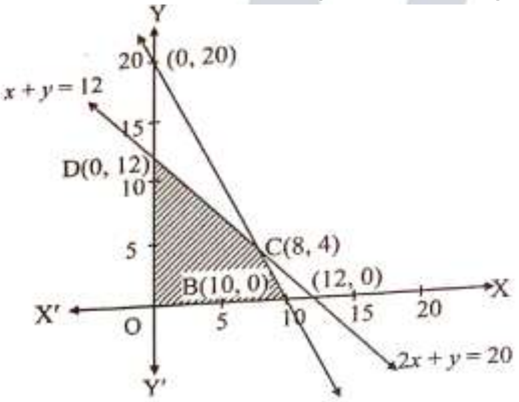
$$\frac{dy}{dx}=\frac{(-1)\sqrt{1+\sin^{-1}x}}{\sqrt{1-\sin^{-1}x}\cdot\left(\sqrt{1-x^2}\right)(1+\sin^{-1}x)^2}$$

$$\left(\frac{dy}{dx}\right)_{x=0}=\frac{-1\sqrt{1+\sin^{-1}0}}{\left(\sqrt{1-\sin^{1}0}\right)\left(\sqrt{1-0^2}\right)(1+\sin^{-1}0)^2}$$

$$=\frac{-1}{(1)(1)(1+0)}$$

$$\therefore\left(\frac{dy}{dx}\right)_{x;x=0}=-1$$

143. **(b)** The feasible region lies on origin side of $x+y=12$ and $2x+y=20$ and it is in the first quadrant.



The corner point of the feasible region are O(0,0), B(10,0), C(8,4), D(0,12).

At $O(0,0) z=10(0)+6(0)=0$

At B(10,0)z $=10(10)+6(0)=100$

At $C(8,4) z=10(8)+6(4)=104$

At $D(0,12) z=10(0)+6(12)=72$

Maximum value of z is 104 at it occurs at $C(8,4)$

144. **(c)** Line is perpendicular to normal of plane

$\Rightarrow(2\hat{\imath}-\hat{\jmath}+3\hat{k})\cdot(l\hat{\imath}+m\hat{\jmath}-\hat{k})=0$

$\Rightarrow 2l-m-3=0$ ..... (i)

$(3,-2,-4)$ lies on the plane $lx+my-z=9$

$3l - 2\text{ m} + 4 = 9$

$\Rightarrow 3l - 2\text{ m} = 5 \ldots.(\text{ii})$

Solving (i) and (ii), we get

$l = 1, \text{m} = -1$

$l^2 + \text{m}^2 = 2$

145. **(a)**

$\bar{x} = 50$, Standard deviation $= 5$

$\text{n} = 100$

Mean $\bar{x} = 50$

$\Rightarrow \frac{\sum x_\text{i}}{100} = 50$

$\Rightarrow \sum x_\text{i} = 5000 \ldots.(\text{i})$

Now, standard deviation $= \sqrt{\frac{\sum x_\text{i}^2}{\text{n}} - \left(\frac{\sum x_\text{i}}{\text{n}}\right)^2}$

$\Rightarrow 5 = \sqrt{\frac{\sum x_\text{i}^2}{100} - \left(\frac{5000}{100}\right)^2}$

$\Rightarrow 25 = \frac{\sum x_\text{i}^2}{100} - (50)^2$

$\Rightarrow \frac{\sum x_\text{i}^2}{100} = 25 + 2500$

$\Rightarrow \sum x_\text{i}^2 = 252500$

146. **(d)**

147. **(b)** Let $I = \int \frac{dx}{3 - 2\cos 2x}$

Put $\tan x = \text{t}$

$x = \tan^{-1} t$

$\therefore \quad \text{d}x = \frac{\text{dt}}{1 + \text{t}^2}$

$\cos 2x = \frac{1 - \text{t}^2}{1 + \text{t}^2}$

$\therefore \text{ I} = \int \frac{\frac{\text{dt}}{1 + \text{t}^2}}{3 - 2\left(\frac{1 - \text{t}^2}{1 + \text{t}^2}\right)}$

$= \int \frac{\text{dt}}{3(1 + \text{t}^2) - 2(1 - \text{t}^2)}$

$= \int \frac{\text{dt}}{3 + 3\text{t}^2 - 2 + 2\text{t}^2}$

$= \int \frac{\text{dt}}{(\sqrt{5}\text{t})^2 + (1)^2}$

$= \frac{1}{\sqrt{5}} \tan^{-1}(\sqrt{5}\text{t}) + \text{c}$

$\therefore \text{ I} = \frac{1}{\sqrt{5}} \tan^{-1} \sqrt{5} \tan x + \text{c}$

Comparing with $\frac{\tan^{-1} \text{f}(x)}{\sqrt{5}}$, we get

$\text{f}(x) = \sqrt{5} \tan x$

$\therefore \quad \text{f}\left(\frac{\pi}{4}\right) = \sqrt{5} \tan \frac{\pi}{4} = \sqrt{5}$

148. **(b)** Given equation of curve

$y(x-2)(x-3)=x+6$

$\Rightarrow y=\dfrac{x+6}{(x-2)(x-3)}$

$\Rightarrow y=\dfrac{x+6}{x^2-5x+6}$

Differentiating w.r.t. x, we get

$$\frac{dy}{dx}=\frac{(x^2-5x+6)(1)-(x+6)(2x-5)}{(x^2-5x+6)^2}$$

$$=\frac{x^2-5x+6-(x+6)(2x-5)}{(x^2-5x+6)^2}$$

$$\frac{dy}{dx}=\frac{-x^2-12x+36}{(x^2-5x+6)^2}$$

At Y-axis, $x=0$

$$\frac{dy}{dx}=\frac{-(0)^2-12(0)+36}{(0^2-5(0)+6)^2}$$

$$=\frac{36}{36}$$

$$=1$$

The equation of normal is

$y-1=-1(x-0)$

i.e., $x+y=1$

Option (B) i.e., $\left(\frac{1}{2},\frac{1}{2}\right)$ satisfies above equation

Normal passes through $\left(\frac{1}{2},\frac{1}{2}\right)$.

149. **(c)** Let $\bar{a}=Cx\hat{i}-6\hat{j}-3\hat{k}$ and $\bar{b}=x\hat{i}+2\hat{j}+2Cx\hat{k}$

Angle between $\bar{a}$ and $\bar{b}$ is obtuse.

$\bar{a}\cdot\bar{b}<\cos 180°$

$\bar{a}\cdot\bar{b}<0$

$Cx^2-12-6Cx<0$

$\Rightarrow Cx^2-6cx-12<0$

$\Rightarrow C<0$ and $D<0$

$\Rightarrow C<0$ and $36C^2+48C<0$

$\Rightarrow C<0$ and $3C^2+4C<0$

$\Rightarrow C<0, C(3C+4)<0$

$\Rightarrow C<0, -\dfrac{4}{3}<C<0$

$C=\left(-\dfrac{4}{3},0\right)$

150. **(d)**

$$A=\begin{bmatrix}3 & -1\\ -4 & 2\end{bmatrix}$$

$$A^{-1}=\frac{1}{6-4}\begin{bmatrix}2 & 1\\ 4 & 3\end{bmatrix}$$

$\ldots\left[\text{If } A=\begin{bmatrix}a & b\\ c & d\end{bmatrix}\text{ and } ad-bc\neq 0\text{, then}\right.$

$$\left.A^{-1}=\frac{1}{(ad-bc)}\begin{bmatrix}d & -b\\ -c & a\end{bmatrix}.\right]$$

$$=\frac{1}{2}\begin{bmatrix}2 & 1\\ 4 & 3\end{bmatrix}$$

$$A^{-1}=\begin{bmatrix}1 & \frac{1}{2}\\ & \frac{3}{2}\end{bmatrix}$$