Physics

1. **(d)** For Pendulum time period $(T)=2\pi\sqrt{\frac{\ell}{g}}$ in space g (effective) $=0, \therefore T=\frac{1}{0}=\infty$
2. **(a)** By Doppler shift of sound as source and listener are approaching, apparent frequency is more than true frequency
3. **(d)** Gas molecules collide with each other and also with walls of container.
4. **(d)** Excess pressure $[\Delta P]=\frac{4T}{R}$

$$\frac{\Delta P_1}{\Delta P_2}=\frac{R_2}{R_1}=\frac{3}{1}$$

$$\frac{\text{Volume }(V_1)}{\text{Volume }(V_2)}=\frac{\frac{4}{3}\pi R_1^3}{\frac{4}{3}\pi R_2^3}=\left(\frac{R_1}{R_2}\right)^3=\left(\frac{1}{3}\right)^3=\frac{1}{27}$$

5. **(d)** adiabatic
6. **(b)** When $C_1 \& C_2$ are in parallel,

$C_p=C_1+C_2$

When $C_1 \& C_2$ are in series,

$$C_s=\frac{C_1C_2}{C_1+C_2}$$

Given that (Energy in parallel) = (Energy series)

$$\frac{1}{2}C_PV_P^2=\frac{1}{2}C_sV_s^2$$

$$\left(\frac{V_P}{V_s}\right)^2=\frac{C_s}{C_P}=\frac{C_1C_2}{(C_1+C_2)^2}$$

Let $C_1=X$ and $C_2=2X$

$$\left[\frac{V_p}{V_s}\right]^2=\frac{X(2X)}{(3X)^2}=\frac{2}{9}$$

$V_p:V_s=\sqrt{2}:3$

7. **(a)** $\frac{1}{\lambda}=RZ^2\left[\frac{1}{n_1^2}-\frac{1}{n_2^2}\right]$

here $n_1=1$ and $n_2=2$

$$\frac{1}{\lambda}\propto z^2$$

For helium $(He^{++}) Z_{He}=2$ and for lithium (hi^{++})

$Z_u=3$

$$\frac{\lambda_{He}}{\lambda_{Li}}=\left(\frac{Z_u}{Z_{H0}}\right)^2=\left(\frac{3}{2}\right)^2=\frac{9}{4}$$

8. **(c)** $\phi=(6t^2+7t+1)$mWb

$$\text{emf }=\frac{d\phi}{dt}=(12t+7)\text{m } \ldots\ldots(\text{ By Lenz Law })$$

Put $t=1$ second

emf $=12(1)+7=19$Mv

9. **(c)** Power max $=\frac{E_0}{\sqrt{2}}\times\frac{I_0}{\sqrt{2}}=\frac{E_0I_0}{2}$

But $I_0=\frac{E_0}{R}$

$P_{\max}=\frac{E_0^2}{2R}$, here $E_0=NABW$

$P_{\max}=\left(\frac{NABW}{2R}\right)^2$ on solving,

$P_{\max}=12W$

10. **(b)** Force by magnetic force = C.P. f

$$qVB=\frac{mV^2}{r}$$

$r = \frac{mV}{qB}$

if V = Potential drop; then

(E) = eV

also kinetic energy $(E) = \frac{P^2}{Zm}$

$\frac{P^2}{Zm} = eV$

$P = \sqrt{ZmeV}$

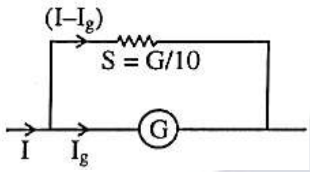
radius $(r) = \frac{mV}{qB} = \frac{P}{qB} = \frac{\sqrt{ZmeV}}{qB}$

as q = e

radius $(r) == \sqrt{\frac{ZmeV}{e^2\, B^2}} = \sqrt{\frac{ZmV}{eB^2}}$

11. (b) 17.5

12. (c) Shunt $(s) = \frac{10}{100} G = \frac{G}{10}$



To find $= I_g/I$ as G is II^{nd} to s (Potential of G) = (Potential of S)

$I_g G = (I - I_g)\frac{G}{10}$

$10 I_g = I - I_g$

$11 I_g = I$

$\frac{I_g}{I} = \frac{1}{11}$

13. (a) By equation of continuity $A_1 V_1 = A_2 V_2$

∴ at narrowest part, velocity is maximum by Bernoulli theorem

$P_0 + \frac{1}{2}\rho v^2 + \rho g h =$ constant

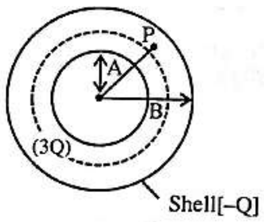
So if velocity is maximum, then pressures energy (P_0) is minimum.

14. (b) Co-efficient of restitution $(e) = \sqrt{\frac{h_2}{h_1}}$

$0.6 = \sqrt{\frac{h_2}{1}}$

Squaring both sides, $0.36 = h_2$

15. (c) draw Gaussian surface



$\oint \bar{E} \cdot d\bar{s} = \frac{Q_{enclosed}}{\epsilon_0}$

$E \oint d\bar{s} = \frac{3Q}{\epsilon_0}$

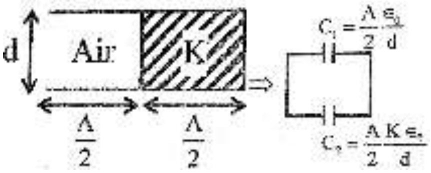
$E[4\pi R^2] = \frac{3Q}{\epsilon_0}$

here $A < R < B$

$E = \dfrac{3Q}{4\pi\epsilon_0 R^2}$

16. **(b)** Initially $C_{sit} = \frac{A\epsilon_0}{d} = 2.5\mu$ F .....(1)

on inserting dielectric



$C_{\text{effective}} = C_1 + C_2 = \dfrac{A}{2}\dfrac{\epsilon_0}{d}(1+K)$

$(5\mu F) = \left[\dfrac{A\,\epsilon_0}{d}\right] \times \dfrac{1}{2}[1+K]$

$(5) = (2.5) \times \dfrac{1}{2}[1+K]$

From equation

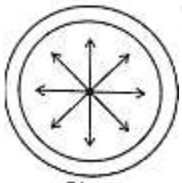
$\dfrac{10}{2.5} = 1 + \mathrm{K}$

$4 = 1 + \mathrm{K}$

$\mathrm{K} = 4 - 1 = 3$

17. **(b)** AND

18. **(a)**



Ring

net force on m = 0

19. **(a)** By Malus law $I' = I_0 \cos^2\theta$ here $\theta = 45^\circ$

$I' = I_0 \cos^2 45^\circ = \dfrac{I_0}{2}$

20. **(d)** Case-1 PV = n, RT ....(1)

Case-2 $(\cancel{2}\mathrm{P})\left(\dfrac{\mathrm{V}}{4}\right) = \mathrm{n_2 R}(\cancel{2}\,\mathrm{T})$

$\dfrac{\mathrm{PV}}{4} = \mathrm{n_2RT}$ ....(2)

Equation 1 ÷ Equation 2

$\dfrac{4}{1} = \dfrac{\mathrm{P_1}}{\mathrm{P_2}}$

21. **(c)** Suddenly compressed

Adiabatic process

$P_1 P_1 V_1^Y = P_2 V_2^Y$

Given $\mathrm{V_2} = \frac{\mathrm{V_1}}{4}, \mathrm{P_2} = ?$

$\left(\dfrac{\mathrm{V_1}}{\mathrm{V_2}}\right)^{\mathrm{Y}} \mathrm{P_1} = \mathrm{P_2}$

$(4)^{\mathrm{Y}}\mathrm{P_1} = \mathrm{P_2}$

$P_2 = (4)^{\frac{3}{2}} P_1 = (2)^3 P_1 = 8P_1$

22. **(d)** $\frac{1}{\lambda} = \mathrm{R}\left[\frac{1}{\mathrm{n}_1^2} - \frac{1}{\mathrm{n}_2^2}\right]$

$= R\left[\frac{1}{(2)^2}-\frac{1}{(3)^2}\right] = R\left[\frac{1}{4}-\frac{1}{9}\right]$

$\frac{1}{\lambda} = R\left[\frac{5}{36}\right]$

As $c = f\lambda$

$f = \frac{c}{\lambda} = RC\left(\frac{5}{36}\right)$

23. (d) $g_d(\text{ at depth }) = \left[1-\frac{d}{R}\right]$

Given $d = \frac{R}{2}; \frac{d}{R} = \frac{1}{2}$

$g_d = g\left[1-\frac{1}{2}\right] = \frac{g}{2} \therefore mg_d = \frac{mg}{2}$

$(\text{ wt. at depth }) = \left(\frac{\text{wt.on surface}}{2}\right)$

$= \frac{300}{2} = 150 \text{ N}$

24. (c) $1+\frac{2}{x}$

25. (c) Force between 2 wires will be balanced by wt. of wire

$\frac{\mu_0}{2\pi} = \frac{I_1 I_2}{h} = mg$

here $I_1 = I_2 = 25$ A; $\ell = 1$ metre;

mass$(m) = 2.5 \times 10^{-3}$ kg

On solving we get h = 5 mm.

26. (d) Angular Momentum $[L] = n = \frac{h}{2\pi}$

$rp = \frac{nh}{2\pi}$

$\frac{2\pi r}{n} = \frac{h}{P} =$ De Broglie wavelength (λ)

27. (c) Under isothermal condition, surface energy is conserved.

$U_1 + U_2 = U_{TOTAL}$

$4\pi r_1^2\, T + 4\pi r_2^2\, T = 4\pi R^2\, T$

$r_1^2 + r_2^2 = R^2$

28. (b) For pipe open at both ends, frequency of 1st over tone (n_1)

$\Rightarrow (P+1)\frac{V}{2\,L_0} = \frac{2\,V_0}{2\,L_0} = \frac{V_0}{L_0}$

For pipe open at end, frequency of 1^{st} overtone

$(n_1') = (2P+1)\frac{V}{4\,L_c} \Rightarrow \frac{3}{4}\frac{V_e}{L_c}$

Given $n_1 = n_1'$

$\frac{V_0}{L_0} = \frac{3}{4}\frac{V_c}{L_c}, L_0 = \frac{4}{3}L_c\left(\frac{V_0}{V_c}\right)$

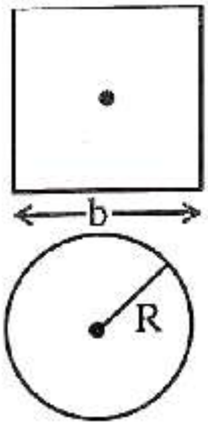
Now $V_0 = \sqrt{\frac{E}{\rho_0}}\; V_c = \sqrt{\frac{E}{\rho_c}}$

$\frac{V_0}{V_c} = \sqrt{\frac{\rho_c}{\rho_0}}$

$L_0 = \frac{4}{3}L_c\sqrt{\frac{\rho_c}{\rho_0}} = \frac{4}{3}L_1\sqrt{\frac{\rho_1}{\rho_2}}$

29.(b) For square Lamina.

$I_1 = \frac{Mb^2}{6}$



For disc $I_2 = \frac{MR^2}{2}$

Given that $\mathbf{I_1} = \mathbf{I_2}$

$\therefore \frac{mb^2}{6} = \frac{mR^2}{2}$

$\frac{b^2}{R^2} = \frac{3}{1} \therefore \frac{b}{R} = \sqrt{3}:1$

30. **(d)** chromatic aberration

31. **(a)** $[\text{ Kinetic energy }]_{\text{Toul}} = \frac{1}{2}mv^2\left[1+\frac{K^2}{R^2}\right]$

$[\text{ Kinetic energy }]_{\text{Rotational}} = \frac{1}{2}Iw^2$

$= \frac{1}{2}\frac{MK^2v^2}{R^2}$

$\frac{KE_{TOTAL}}{KR_{rotational}} = \frac{\left(1+\frac{K^2}{R^2}\right)}{\frac{K^2}{R^2}}$

For solid sphere, $K = \sqrt{\frac{2}{5}}R \therefore \frac{K^2}{R^2} = \frac{2}{5}$

$\frac{KE_{TOTAL}}{KE_{rotational}} = \frac{1+\frac{2}{5}}{\frac{2}{5}} \Rightarrow \frac{\frac{7}{5}}{\frac{2}{5}} = \frac{7}{2}$

32. **(b)** Velocity (v) $= \sqrt{2gh}$

$= \sqrt{2 \times 10 \times 10} \Rightarrow 10\sqrt{2}$

Emf (e) = Blv

$\Rightarrow (2 \times 10^{-5}) \times (2500)(10\sqrt{2})$

$\text{Emf} \Rightarrow 5 \times 10^{4-5} \times \sqrt{2} = 0.5\sqrt{2}$

$\therefore$ current $[I] = \frac{emf}{R} = \frac{0.5\sqrt{2}}{25\sqrt{2}} = \frac{1}{50} = 0.02$ A

33. **(d)** Maximum velocity (At MP) = v = Aw

$w = \frac{2\pi}{T}; w' = \frac{2\pi}{(T/3)} \Rightarrow 3\left[\frac{2\pi}{T}\right] = 3w$

$A' = 2$ A

$\therefore$ new maximum velocity (v′)

$= A'W' = (2A)3w' = 6v$

34. **(b)** Mass = volume × density

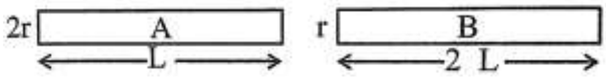
$m = \pi R^2 td \therefore R^2 = \frac{m}{\pi td}$

Moment of inertia (I) $= \frac{MR^2}{2}$

$I = \frac{M^2}{2\pi t d}; I \propto \frac{1}{d}$

$\because \frac{I_1}{I_2} = \frac{d_2}{d_1}$

35. (c) $\Delta V_A = V_{iA}[1 + Y\Delta t]$



$\Delta V_B = V_{iB}[1 + Y\Delta t]$

$\frac{\Delta V_A}{\Delta V_B} = \frac{V_{iA}}{V_{iB}} = \frac{L\pi(2r)^2}{2L\pi r^2}$

$= \frac{4}{2} = \frac{2}{1}$

36. (d) $y = \frac{1}{\sqrt{a}} \sin \omega t + \frac{1}{\sqrt{b}} \cos \omega t$

$R = \sqrt{A_1^2 + A_2^2 + 0} = \sqrt{\frac{1}{a} + \frac{1}{b}}$

$R = \sqrt{\frac{b + a}{ab}}$

37. (c) (Kinetic energy) $= h\nu - \phi_0$

$\therefore KE_1 = h\nu_1 - \phi_0$

$KE_2 = h\nu_2 - \phi_0$

here ϕ_0 = work function $= h\nu_0$

where ν_0 = threshold frequency

As $\frac{KE_1}{KE_2} = \frac{1}{X}$ (Given)

$\therefore \frac{KE_1}{KE_2} = \frac{1}{X} = \frac{h\nu_1 - h\nu_0}{h\nu_2 - h\nu_0}$

$\therefore \frac{1}{X} = \frac{\nu_1 - \nu_0}{\nu_2 - \nu_0}$ $\therefore \nu_2 - \nu_0 = X\nu_1 - X\nu_0$

$\therefore X\nu_0 - \nu_0 = X\nu_1 - \nu_2$

$\therefore \nu_0(X - 1) = X\nu_1 - \nu_2$ $\therefore \nu_0 = \frac{X\nu_1 - \nu_2}{X - 1}$

38. (c) For electromagnets (Soft iron); their must be high retentively and low co-reactivity.

39. (b) For intrinsic semiconductor $n_e = n_h$

for P-type $n_h > n_c$

for N-type $n_e > n_h$

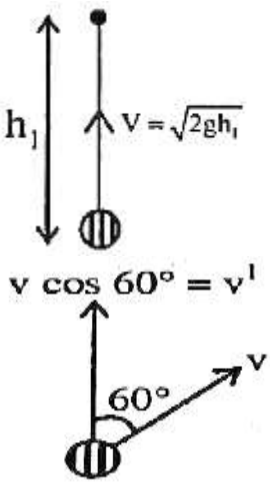
40. (c) Self inductance $[L] = \frac{\mu_0 N^2 A}{\ell}$

$L \propto$ area(A)

41. (c) At resonance, impedance is minimum and power is maximum.

$\therefore$ Brightness of bulb is maximum at resonance. If frequency is not equal to resonating frequency then impedance is more and brightness of bulb is less.

42. (a)



$h_1 = \frac{v^2}{2\,g}$

$PE_1 = mgh_1 = \frac{mv^2}{2}$

$V' = \sqrt{2gh_2}$

$v\cos 60^\circ = \sqrt{2g_2}$

$\frac{v}{2} = \sqrt{2gh_2}$

Squaring both sides

$\frac{v^2}{4} = 2gh_2 \;\therefore\; h_2 = \frac{v^2}{8\,g}$

$PE_2 = mgh_2 = mg\frac{v^2}{8\,g} = \frac{mv^2}{8}$

$\frac{PE_1}{PE_2} = \frac{\frac{mv^2}{2}}{\frac{mv^2}{8}} \Rightarrow \frac{4}{1}$

43. (c)

$f_{eff} = \ell_1 + \ell_2 + \ell_3$ ......(1)

As $v = f\lambda \;\therefore\; f = \frac{v}{\lambda}$

here $\ell = \frac{\lambda}{2}$

$\therefore \lambda = 2\ell \;\therefore\; f = \frac{v}{\lambda} = \frac{v}{2\ell}$

$\ell = \frac{v}{2f}$ ....(2)

Put Equation 2 in Equation 1

$\frac{v}{2f_{eff}} = \frac{v}{2f_1} + \frac{v}{2f_2} + \frac{v}{2f_3}$

$\therefore \frac{1}{f_{eff}} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_3}$

44. (a) Condition for 1" minima

Path difference $(a\sin\theta) = n\lambda($ here $n = 1)$

$a\sin 30^\circ = 1 \times 5400$ ....(1)

Condition for 1^{st} maxima;

$a\sin\theta = (2n+1)\frac{\lambda}{2}($ here $n = 1)$

$a\sin\theta = \frac{3}{2}\lambda = \frac{3}{2}[5400]$ ....(2)

Equation 2 ÷ Equation 1

$$\frac{\sin\theta}{\frac{1}{2}} = \frac{3}{2}; \therefore \sin\theta = \frac{3}{4}$$

$$\theta = \sin^{-1}\left(\frac{3}{4}\right)$$

45. **(d)** Intensity $\propto$ area $\propto$ (slit width) ^{2}

$\therefore$ if slit width is doubled then intensity increases four times

Angular / Width $(\theta) \Rightarrow \frac{\lambda}{\mathrm{d}}$

$\therefore \theta \propto \frac{1}{\mathrm{d}}$ here, d = slit width

if d becomes double than θ is halved.

46. **(a)** Potential energy $(\mathrm{V_0}) = \frac{1}{2}\frac{\mathrm{Q}^2}{\mathrm{C}}$

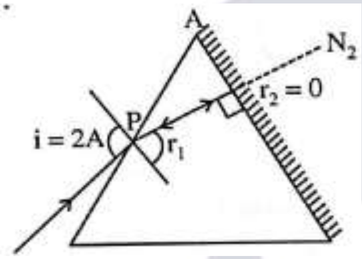
On inserting di-electric slab, new potential energy $(v^1) = \frac{1}{2}\frac{Q^2}{KC} = \frac{V_0}{K}$

47. **(b)** Power consumed $= \frac{\mathrm{e_0 i_0}}{2}\cos 90° = 0$

48. **(b)** Position of null point is independent of increase or decrease in cross-sectional area of wire.

49. **(d)** Time period $(T) = 2\pi\sqrt{\frac{\ell}{g}}$ here ℓ. s distance from pt. of suspension to centre of mass [C.O.M] Initially as water leaks from hole C.O.M. shifts down so ℓ increases and T also increase. when entire water leaks out, C.O.M. again comes to center of sphere i.e. ℓ decreases and T also decreases.

50. **(c)**



As light retraces back from same path it must strike normally on silvered surface ($r_2 = 0$)

As $r_1 + r_2 = A$; $\therefore r_1 + 0 = A$

By snell's law at P; $\frac{\sin i}{\sin r} = \mu$

$\therefore \frac{\sin 2\mathrm{A}}{\sin \mathrm{A}} = \mu$

$2\sin \mathrm{A}\cos \mathrm{A}$

$\therefore \mu = 2\cos A$

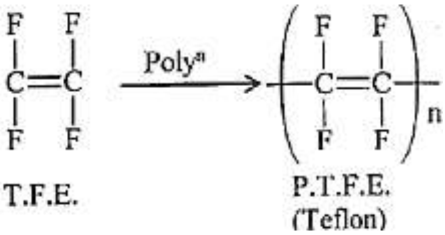
Chemistry

51. **(b)** vanadium pentaoxide

52. **(a)** G.E.C $\mathrm{ns^2np^4}$ is for group 16 it belongs to group 16

53. **(d)** Ag $\Rightarrow$ Fcc $\Rightarrow$ P.E is 74%

54. **(d)**



55. **(b)** SF_6

S contains $6e^-$ in valence shell & all are bounded to 6 Fluorine atom

Hence, no L. P of e^-

56. **(b)** Slope of graph rate v/s concentration of first order reaction is K .

57. (d) H_2Te exhibits highest acidic nature.

58. (d) Hexa-1, 4-diyne

No of moles of sp^2 hybrid carbon = 0

$$\underset{sp}{\overset{}{H-C}}\equiv\underset{sp}{C}-\underset{sp^3}{CH_2}-\underset{sp}{C}\equiv\underset{sp}{C}-\underset{sp^3}{CH_3}$$

59. (b) Mass in gram of 1 atom of an element if its atomic mass is 10μ is 1.66056×10^{-23} g

60. (a) $I = \frac{\omega \times F}{t_{(s)} \times \text{moleratio} \times \text{molar mass}}$

$= \frac{4.8 \times 96500 \times 2}{30 \times 60 \times 63}$

$= 8.16$ g

61. (a) $Sn_{(s)}|Sn^{2+}||Ag^{+}|Ag_{(s)}(E^{\circ}_{cell} = 0.90)$

$\underset{AN[OX]}{\overset{1M}{\longleftarrow}}$ $\underset{CAT(RED)}{\overset{IM}{\longrightarrow}}$

At Anode $Sn \rightarrow Sn^{+2} + 2e^{-}(ox)$

At cathode $2Ag^{+} + 2e^{-} \rightarrow 2Ag(\text{ Red })$

n = 2

$\Delta G^{\circ} = -nFEE^{\circ}_{cell}$

$= -2 \times 96500 \times 0.90$ V

$= -173700$ J $= -173.700$KJ

62. (d) $\bar{\upsilon} = R_H\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$

$= 109677\left[\frac{1}{1^2} - \frac{1}{2^2}\right]$

$= 109677 \times \frac{3}{4} = 82257.8\ cm^{-1}$

63. (c) Phenol + conc. $HNO_3 \rightarrow$ Picric acid (2, 4, 6-Trinitro phenol)

64. (a) Structure of material surface

65. (c) $pH = pKa + \log_{10}\frac{[A^-]}{[HA]}$

$pH = 4.7 + \log_{10}\frac{0.2}{0.1}$

$= 4.7 + \log_{10} 2$

$= 4.7 + 0.3010 = 5.0$

66. (b) Benzyllic carbon is sp^3 hybridized carbon next to benzene ring

67. (d) $\Delta G = \Delta H - T.\Delta S$

$= 7 - 300 \times 24.8 \times 10^{-3}$

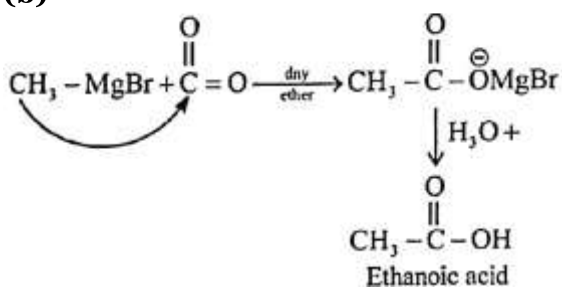
$= 7 - 7.44 = -0.44KJ\ mol^{-1}$

68. (d) $R - X + K - O - N = O \rightarrow R - O - NO + Kx$

69. (c) $M_2 = \frac{Kb \times w_2 \times 1000}{\Delta\ Tb \times w_1 \text{ in g}}$

$= \frac{2.5 \times 3.5 \times 1000}{0.35 \times 100} = 250gml^{-1}$

70. (b)



71. (a) $1H_{2(g)} + 1Br_{2(g)} \rightarrow 2HBr_{(g)}$

$\Delta H = \Delta U + \Delta n(g)RT$

$n_R = 2, n_p = 2, \Delta n = n_p - n_R = 0$

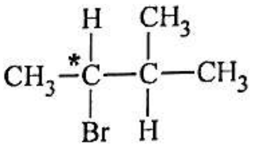
$\Delta H = \Delta U$

72. (b) Propane

In wurtz reaction symmetrical alkane are produced

73. (b) Hydrogen

74. (c) 2-bromo-3-methylbutane



For O.A. atleast one carbon should be Chiral

75. (d) $C_1 \& C_5$

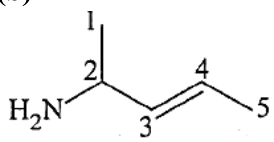
76. (b) $= \sqrt{n(n+2)}$

$Cr^{+2}: [Ar]45^0 3\,d^4$

$n = 4$

$\mu = \sqrt{4(4+2)} = 4.90$ B.M

77. (b)



Pent-3-en-2-amine

78. (a) 6.022×10^{23}

79. (a) Pauli's exclusion principle

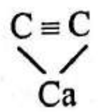
80. (c) $R \propto [A]^2[B]^1$

$R = K[A]^2[B]^1$

$$K = \frac{R}{[A]^2[B]^1} = \frac{1.8 \times 10^{-2}}{(0.2)^2(0.1)^1}$$

$$= \frac{1.8 \times 10^{-2}}{0.04 \times 0.1} = \frac{18}{4} = 4.5$$

81. (b) $K_2C_2O_4$



$C = -1$

$2(1) + 2x + 4(-2) = 0$

$2 + 2x - 8 = 0$

$2x = +6$

$x = +3$

82. (c) 4n

83. (d) Melting point of group - 1

Element increase down the group

84. (b) $W = -200$ J, $\Delta U = 432$ J

$\Delta H = \Delta U + P\Delta V$

$= \Delta U + (-w)$

$= 432 - 200 = 232$ J

85. (b) Molarity

86. (d) $_{21}sc = [Ar]4\,s^2 3\,d^1$

sc^{+2}; [ar]4 s°3 d^1
sc^{+3}; [Ar]4 s°3 d°

87. (c) According to Charles law

$$\frac{V_1}{T_1}=\frac{V_2}{T_2} \therefore T_2=\frac{T_1\times V_2}{V_1}$$

$$=\frac{273\times 224}{22.4}=2730$$

88. (c)

89. (c) $CuSO_4$ pH < 7

90. (a) $K=\frac{2.303}{60}\log_{10}\frac{100}{x}$

$$0.02303=\frac{2.303}{60}\log\left(\frac{100}{x}\right)$$

$$\frac{2.303}{100}=\frac{2.303}{60}\log\left(\frac{100}{x}\right)$$

$$0.6=\log\left(\frac{100}{x}\right)$$

$$\log 4=\log\left(\frac{100}{x}\right)$$

$$x=\frac{100}{4}=25\%$$

91. (c) FCC

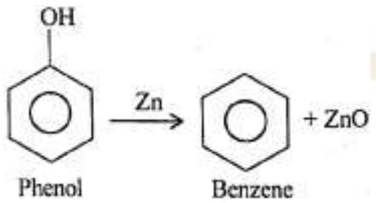
$A\Rightarrow$ corner $=\frac{1}{8}\times 8=1$

$B\Rightarrow$ face $=\frac{1}{2}\times 6=3$

Ratio of A: B = 1: 3
Formula = AB_3

92. (a) $Ksp = 108[\,s]^5$
$= 108(1\times 10^{-3})^5$
$= 108\times 1\times 10^{-15} = 1.08\times 10^{-13}$
$Ksp = 1.08\times 10^{-13}$

93.(c)



94. (d) $\frac{\pi_1}{\pi_2}=\frac{C_1RT}{C_2RT} \therefore \frac{\pi_1}{\pi_2}=\frac{C_1}{C_2}$

$$\frac{4.9}{1.5}=\frac{0.2}{C_2}$$

$C_2 = 0.06M$

95. (a) +52.7°

96. (a) P.A.N
Polyacrylonitrile

97. (a) NH_3 is neutral ligand

98. (b) Addition reaction

99.(a) n

100. **(b)** N-Methyl aniline

Mathematics

101. **(a)** Let $\vec{a} = \hat{\imath} + \hat{\jmath} + \hat{k}$

$\vec{b} = 2\hat{\imath} + 4\hat{\jmath} - 5\hat{k}$

$\vec{c} = \lambda\hat{\imath} + 2\hat{\jmath} + 3\hat{k}$

$(\vec{b} + \vec{c}) = (2+\lambda)\hat{\imath} + (4+2)\hat{\jmath} + (-5+3)\hat{k}$

$= (2+\lambda)\hat{\imath} + 6\hat{\jmath} - 2\hat{k}$

Let $\vec{r}$ be unit vector along $(\vec{b} + \vec{c})$

$$\hat{r} = \frac{1}{|\vec{b} + \vec{c}|} \times (\vec{b} + \vec{c})$$

$$\hat{r} = \frac{1}{\sqrt{(2+\lambda)^2 + 36 + 4}} \times [(2+\lambda)\hat{\imath} + 6\hat{\jmath} - 2\hat{k}]$$

$$\hat{r} = \frac{1}{\sqrt{(\lambda^2 + 4\lambda + 44}} \times [(2+\lambda)\hat{\imath} + 6\hat{\jmath} - 2\hat{k}]$$

Now $\vec{a} \cdot \vec{r} = 1$ (given)

$$(\hat{\imath} + \hat{\jmath} + \hat{k}) \cdot \left(\frac{1}{\sqrt{\lambda^2 + 4\lambda + 44}} \times (2+\lambda)\hat{\imath} + 6\hat{\jmath} - 2\hat{k}\right) = 1$$

$(\hat{\imath} + \hat{\jmath} + \hat{k}) \cdot (2+\lambda)\hat{y} + 6\hat{\jmath} - 2\hat{k}) = \sqrt{\lambda^2 + 4\lambda + 44}$

$(\lambda + 2) + 6 + 1 \cdot (-2) = \sqrt{\lambda^2 + 4\lambda + 44}$

$\lambda + 2 + 6 - 2 = \sqrt{\lambda^2 + 4\lambda + 44}$

$(\lambda + 6)^2 = \lambda^2 + 4\lambda + 44$

$\lambda^2 + 12\lambda + 36 = \lambda^2 + 4\lambda + 44$

$8\lambda = 8$

Hence, the value of λ is 1 .

102. **(b)** $x^2 + y^2 = t + \frac{1}{t}$

Squaring both sides we get

$$x^4 + y^4 + 2x^2y^2 = t^2 + \frac{1}{t^2} + 2$$

$$t^2 + \frac{1}{t^2} + 2x^2y^2 = t^2 + \frac{1}{t^2} + 2$$

$2x^2y^2 = 2 \therefore x^2y^2 = 1$

$$x^2 \cdot 2y\frac{dy}{dx} + y^2(2x) = 0$$

$$\frac{dy}{dx} = -\frac{y}{x}$$

103. **(b)** $\sim [(p \wedge q) \to (\sim p \vee r)]$

$= (p \wedge q) \wedge \sim (\sim p \vee r)$

$= (p \wedge q) \wedge (p \wedge \sim r)$

$= p \wedge (q \wedge \sim r)$

104. **(d)** Equation of line is

$\vec{r} = (2\hat{\imath} - 3\hat{\jmath} - 4\hat{k}) + \lambda(6\hat{\imath} + 3\hat{\jmath} - 4\hat{k})$

$\vec{\alpha} = \hat{\imath} - 2\hat{\jmath} - 6\hat{k}$

$\vec{a} = 2\hat{\imath} - 3\hat{\jmath} - 4\hat{k}$

$\vec{b} = 6\hat{\imath} + 3\hat{\jmath} - 4\hat{k} \therefore |\vec{b}| = \sqrt{61}$

$\bar{\alpha} - \vec{a} = -\hat{\imath} + \hat{\jmath} - 2\hat{k}$

$|\vec{\alpha} - \vec{a}| = \sqrt{6}$

$(\vec{\alpha} - \vec{a}) \cdot \vec{b} = -6 + 3 + 8 = 5$

Distance $= \sqrt{|\vec{\alpha}-\vec{a}|^2-\left[\frac{\bar{\alpha}-\bar{a}}{|\bar{b}|}\right]^2}$

$= \sqrt{6-\frac{25}{61}} = \sqrt{\frac{341}{61}}$

105. **(c)** Let $u = f(\sec x)$

$\frac{du}{dx} = f'(\sec x)\cdot \sec x \tan x$

Let $v = g(\tan x)$

$\frac{dv}{dx} = g'(\tan x)\cdot \sec^2 x$

$\frac{du}{dv} = \frac{f'(\sec x)\cdot \sec x \tan x}{g'(\tan x)\cdot \sec^2 x}$

$\frac{du}{dv} = \frac{f'(\sec x)\tan x}{g'(\tan x)\sec x}$

at $x = \pi/4$

$\frac{du}{dv} = \frac{f'(\sqrt{2})}{g'(1)\cdot\sqrt{2}} = \frac{4}{2\sqrt{2}} = \sqrt{2}$

106. **(b)** $l = \int \frac{\sin x+\sin^3 x}{\cos 2x}dx = \int \frac{\sin x(2-\cos^2 x)}{(2\cos^2 x-1)}dx$

Put $\cos x = t \Rightarrow dx = -\sin x dx$

$\therefore l = \int \frac{t^2-2}{2t^2-1}dt = \frac{1}{2}\int \frac{2t^2-4}{2t^2-1}dt$

$= \frac{1}{2}\int dt - \frac{3}{2}\int \frac{dt}{2t^2-1}$

$= \frac{1}{2}t - \frac{3}{2\sqrt{2}} \times \frac{1}{2}\ln\left|\frac{\sqrt{2}t-1}{\sqrt{2}t+2}\right| + C$

So, $P = 1/2, Q = \frac{-3}{4\sqrt{2}}$ and

$f(x) = \frac{\sqrt{2}\cos x - 1}{\sqrt{2}\cos x + 1}$

or $P = 1/2, Q = \frac{3}{4\sqrt{2}}$ and

$f(x) = \frac{\sqrt{2}\cos x + 1}{\sqrt{2}\cos x - 1}$

107. **(a)** Volume of parallelepiped is $[\vec{a} \quad \vec{b} \quad \vec{c}]$

$V = \begin{vmatrix} 1 & \alpha & 1 \\ 0 & 1 & \alpha \\ \alpha & 0 & 1 \end{vmatrix}$

$= 1-\alpha(-\alpha^2)-\alpha \ = 1+\alpha^3-\alpha$

Differentiating w.r.t. α, we get

$\frac{dV}{d\alpha} = 3\alpha^2 - 1$

Let $\frac{dV}{d\alpha} = 0$

$3\alpha^2 - 1 = 0$

At $\alpha = \frac{d^2 V}{d^2} = 6\alpha$

$\alpha = \pm\frac{1}{\sqrt{3}}$,

$\frac{d^2 V}{d\alpha^2} = \frac{-6}{\sqrt{3}} < 0$

V is maximum at $\alpha = \frac{-1}{\sqrt{3}}$

108. **(a)** The region lies towards non - origin side of $3x + 4y \geq 12$

Also, it lies on left side of $y - x \geq 0$& origin side of $y \leq 3$

109. **(c)** $\cos^2 48° - \sin^2 12°$

$= \cos(48° + 12°) \cdot \cos(48° - 12°)$

$= \cos 60° \cdot \cos 36°$

$= \frac{1}{2}[1 - 2\sin^2 18°]$

$= \frac{1}{2}\left[1 - 2\frac{(\sqrt{5}-1)^2}{16}\right]$

$= \frac{1}{2}\left[1 - \frac{(5 - 2\sqrt{5} + 1)}{8}\right]$

$= \frac{1}{2}\left[\frac{2 + 2\sqrt{5}}{8}\right] = \frac{\sqrt{5}+1}{8}$

110. **(b)** $z_1 = 4i^{40} - 5i^{35} + 6i^{17} + 2$

$z_1 = 4 + 5i + 6i + 2$

$= 6 + 11\text{i}$

$\text{z}_2 = -1 + \text{i}$

$\text{z}_1 + \text{z}_2 = 5 + 12\text{i}$

$|z_1 + z_2| = \sqrt{25 + 144} = 13$

111. **(a)** $\text{a} = 1, \text{h} = 2, \text{b} = \text{p}, \text{g} = \frac{3}{2}$

$f = \frac{q}{2}, c = -4$

For pair of parallel lines

$\text{h}^2 - \text{ab} = 0$

$4 - \text{p} = 0 \ \therefore \text{p} = 4$

For pair of lines

$$\begin{vmatrix} 1 & 2 & 3/2 \\ 2 & 4 & q/2 \\ 3/2 & q/2 & -4 \end{vmatrix} = 0$$

$1\left(-16 - \frac{q^2}{4}\right) - 2\left(-8 - \frac{3q}{4}\right) + \frac{3}{2}(q - 6) = 0$

$-16 - \frac{\text{q}^2}{4} + 16 + \frac{3\text{q}}{2} + \frac{3\text{q}}{2} - 9 = 0$

$\frac{-q^2}{4} + 3q - 9 = 0$

$\text{q}^2 - 12\text{q} + 36 = 0$

$(q - 6)^2 = 0$

$q = 6$

112. **(a)** $\cos^2 45° + \cos^2 60° + \cos^2 \gamma = 1$

$\frac{1}{2} + \frac{1}{4} + \cos^2 \gamma = 1$

$\frac{3}{4} + \cos^2 \gamma = 1$

$\cos^2 \gamma = \frac{1}{4}$

$\cos \gamma = \frac{1}{2}$

$\hat{\text{n}} = \frac{1}{\sqrt{2}}\hat{\text{i}} + \frac{1}{2}\hat{\text{j}} + \frac{1}{2}\hat{\text{k}}$

$\vec{n} = \sqrt{2}\hat{i} + \hat{j} + \hat{k}$
$\vec{a} = -\sqrt{2}\hat{i} + \hat{j} + \hat{k}$
Equation of plane is
$\vec{r}.\vec{n} = \vec{a}.\vec{n}$
$\vec{r} \cdot (\sqrt{2}\hat{\imath} + \hat{\jmath} + \hat{k}) = (-\sqrt{2}\hat{\imath} + \hat{\jmath} + \hat{k})(\sqrt{2}\hat{\imath} + \hat{\jmath} + \hat{k})$
$\vec{r} \cdot (\sqrt{2}\hat{i} + \hat{j} + \hat{k}) = -2 + 1 + 1$
$\vec{r}.(\sqrt{2}\hat{i} + \hat{j} + \hat{k}) = 0$
$\therefore \sqrt{2}x + y + z = 0$

113. **(b)** $\cot\left(\sum_{n=1}^{23} \cot^{-1}(1 + \sum_{k=1}^{n} 2k)\right)$
$= \cot\left(\sum_{n=1}^{23} \cot^{-1}\left(1 + 2 \times \frac{n(n+1)}{2}\right)\right)$
$= \cot\left(\sum_{n=1}^{23} \cot^{-1}(n^2 + n + 1)\right)$
$= \cot\left(\sum_{n=1}^{23} \tan^{-1}\left(\frac{n+1-n}{1+n(n+1)}\right)\right)$
$= \cot(\sum_{n=1}^{23} \tan^{-1}(n+1) - \tan^{-1} n)$
$= \cot(\tan^{-1} 24 - \tan^{-1} 1)$
$= \cot\left(\tan^{-1}\left(\frac{23}{25}\right)\right)$
$= \cot\left(\cot^{-1}\left(\frac{25}{23}\right)\right) = \frac{25}{23}$

114. **(a)** Let no. of members present of the meeting be " n ".
${}^{n}c_2 = 45$
$\frac{n(n-1)}{2} = 45$
$n(n-1) = 90 = 10 \times 9 = n = 10$

115. **(d)** $a = 1000\ h = -2$
$f(x) = \log_{10}^{x}$
$f(a) = \log_{10}{}^{1000} = 3$
$f'(x) = \frac{1}{x \cdot \log_{c}^{10}} = \frac{0.4343}{x}$
$f'(a) = \frac{0.4343}{1000} = 0.0004343$
$f'(a+h) = f(a) + h.f'(a)$
$= 3 - (2 \times 0.0004343)$
$= 3 - (0.0008686)$
= 2.9991314

116. **(a)** Perpendicular distance of plane from origin
$= \sqrt{4^2 + (-2)^2 + 5^2}$
$= \sqrt{16 + 4 + 25}$
$P = \sqrt{45}$
Also, $\ell p = 4, mp = 2, np = 5$
$\ell = \frac{4}{\sqrt{45}}, m = \frac{-2}{\sqrt{45}}, n = \frac{5}{\sqrt{45}}$
Equation of plane is $\ell x + my + nz = P$

$$\frac{4}{\sqrt{45}}x-\frac{2}{\sqrt{45}}y+\frac{5}{\sqrt{45}}z=\sqrt{45}$$

$4x-2y+5z=45$

117. **(b)** $x^2+y^2=1$

$$2x\frac{dx}{dt}+2y\frac{dy}{dt}=0$$

$$x\frac{dx}{dt}+y\frac{dy}{dt}=0$$

$$\frac{1}{2}\frac{dx}{dt}+\frac{\sqrt{3}}{2}(-3)=0$$

$$\frac{1}{2}\frac{dx}{dt}=\frac{3\sqrt{3}}{2}$$

$\frac{dx}{dt}=3\sqrt{3}$ unit /sec

118. **(c)** $[(\bar{a}+2\bar{b}+3\bar{c})\times(\bar{b}+2\bar{c}+3\bar{a})][\bar{c}+2\bar{a}+3\bar{b}]=54$

$[(\bar{a}\times\bar{b})+2(\bar{a}\times\bar{c})+4(\bar{b}\times\bar{c})+6(\bar{b}\times\bar{a})+3(\bar{c}\times\bar{b})+9(\bar{c}\times\bar{a})]$

$.(\bar{c}+2\bar{a}+3\bar{b})=54$

$[-5(\bar{a}\times\bar{b})+(\bar{b}\times\bar{c})+7(\bar{c}\times\bar{a})]\cdot(\bar{c}+2\bar{a}+3\bar{b})=54$

$-5[\overline{abc}]+2[\overline{bca}]+21[\overline{cab}]=54$

$-5[\overline{abc}]+2[\overline{abc}]+21[\overline{abc}]=54$

$18(\overline{abc})=54$

$(\overline{abc})=3$

119. **(c)** $\cos^{-1}x-\cos^{-1}\frac{y}{3}=\alpha$

$$\cos^{-1}\left[x\frac{y}{3}+\sqrt{1-x^2}\sqrt{1-\frac{y^2}{9}}\right]=\alpha$$

$$\cos^{-1}\left[\frac{xy+\sqrt{1-x^2}\sqrt{9-y^2}}{3}\right]=\alpha$$

$$\frac{xy+\sqrt{1-x^2}\sqrt{9-y^2}}{3}=\cos\alpha$$

$$xy+\sqrt{1-x^2}\sqrt{9-y^2}=3\cos\alpha$$

$$\sqrt{1-x^2}\sqrt{9-y^2}=3\cos\alpha-xy$$

Squaring both sides

$(1-x^2)(9-y^2)=(3\cos\alpha-xy)^2$

$9-y^2-9x^2+x^2y^2$

$=9\cos^2\alpha-6xy\cos\alpha+x^2y^2$

$9x^2+y^2-9=-9\cos^2\alpha+6xy\cos\alpha$

$9x^2-6xy\cos\alpha+y^2=-9\cos^2\alpha+9$

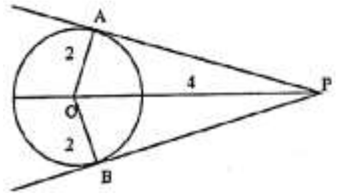
$=9(1-\cos^2\alpha)$

$=9\sin^2\alpha$

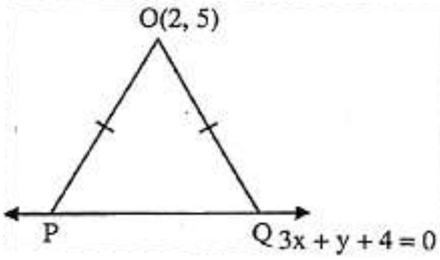
120. **(c)** According to given conditions, $OP=4, OA=OB=2$ and $AP=BP=2\sqrt{3}$ (pythagorus theorem).

area $POAB=2\times\frac{1}{2}\times2\times2\sqrt{3}=4\sqrt{3}$

This is the required solution.



121. **(d)**



Let OP measured along line L_1 & OQ measured along line L_2.

Slope of $OP = \frac{3}{4}$

Slope of $OQ = m$

As $OP = OQ$

$\therefore \angle OPQ = \angle OQP$

$$\left|\frac{3/4+3}{1-9/4}\right| = \left|\frac{m+3}{1-3m}\right| \Rightarrow 3 = \left|\frac{m+3}{1-3m}\right|$$

$$\therefore \frac{m+3}{1-3m} = \pm 3$$

$\frac{m+3}{1-3m} = +3$ or $\frac{m+3}{1-3m} = -3$

$m + 3 = 3 - 9\,m$ or $m + 3 = -3 + 9\,m$

$10\,m = 0\;8\,m = 6$

$m = 0$

$$m = \frac{3}{4}$$

Which is slope of OP .

Slope of $OQ = 0$

122. **(b)** $f(x) = \frac{3x-2}{5x+3}$

$$f[f(x)] = \frac{3\left(\frac{3x-2}{5x+3}\right)-2}{5\left(\frac{3x-2}{5x+3}\right)+3}$$

$$= \frac{9x-6-10x-6}{15x-10+15x+9} = \frac{-x-12}{30x-1}$$

$$f[f(1)] = \frac{-1-12}{30-1} = \frac{-13}{29}$$

123. **(a)** $A = \begin{bmatrix} 2 & -1 \\ -1 & 3 \end{bmatrix}$

$$A^2 = \begin{bmatrix} 2 & -1 \\ -1 & 3 \end{bmatrix}\begin{bmatrix} 2 & -1 \\ -1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & -5 \\ -5 & 10 \end{bmatrix}$$

$$2\,A^2 + 5\,A = \begin{bmatrix} 10 & -10 \\ -10 & 20 \end{bmatrix} + \begin{bmatrix} 10 & -5 \\ -5 & 15 \end{bmatrix}$$

$$= \begin{bmatrix} 20 & -15 \\ -15 & 35 \end{bmatrix}$$

$$(2\,A^2 + 5\,A)^{-1} = \frac{1}{475}\begin{bmatrix} 35 & 15 \\ 15 & 20 \end{bmatrix}$$

$$= \frac{1}{95}\begin{bmatrix} 7 & 3 \\ 3 & 4 \end{bmatrix}$$

124. **(b)** $2p(x=3)=3p(x=2)$

$2.\ {}^4C_3p^3q = 3.\ {}^4C_2p^2q^2$

$2 \times 4p = 3 \times 6 \times q$

$8p = 18q$

$4p = 9q$

$4p = 9(1-p)$

$4p = 9 - 9p$

$13p = 9$

$p = \frac{9}{13} \therefore q = \frac{4}{13}$

$\text{Variance} = npq = 4 \times \frac{9}{13} \times \frac{4}{13} = \frac{144}{169}$

125. **(d)** $f(f(f(x)) + (f(x))^2$

$f'(f(f(x)) \cdot f'(f(x)) \cdot f'(x) + 2f(x)f'(x)$

At $x = 1$

$= 3 \times 3 \times 3 + 2\ (1)\ (3)$

$= 27 + 6 = 33$

126. **(a)** $f(x) = \sqrt{25-x^2}$

$f'(x) = \frac{-x}{\sqrt{25-x^2}}$

Acc. to L.M.V.T

$f'(c) = \frac{f(b)-f(a)}{b-a}$

$\frac{-c}{\sqrt{25-c^2}} = \frac{-2\sqrt{6}}{4}$

Squaring both sides

$\frac{c^2}{25-c^2} = \frac{24}{16}$

$2c^2 = 75 - 3c^2$

$5c^2 = 75$

$c^2 = 15$

$c = \sqrt{15}$

127. **(a)** Let $u = \sqrt{x^2+16}$

$v = \frac{x}{x-1}$

$\frac{du}{dx} = \frac{x}{\sqrt{x^2+16}}$

$\frac{dv}{dx} = \frac{(x-1)-x}{(x-1)^2} = \frac{-1}{(x-1)^2}$

$\frac{du}{dv} = \frac{-x(x-1)^2}{\sqrt{x^2+16}}$

At $x = 5$

$\frac{du}{dv} = \frac{-5(16)}{\sqrt{41}} = \frac{-80}{\sqrt{41}}$

128. **(d)** $y = \frac{8-x}{2}$

$f(x) = xy$

$= x\frac{(8-x)}{2}$

$= \frac{1}{2}[8x - x^2]$

$f'(x)=\frac{1}{2}[8-2x]$

$=4-x$

$f'(x)=0$

$x=4$

Where $f''(x)=-1$, f(x) is maximum at $x=4$

$f(4)=4\times 2=8$

129. **(b)** $\frac{{}^{n}c_0+{}^{n}c_1+{}^{n}c_2+\cdots+{}^{n}c_n}{32}=1$

$2^n=32$

$n=5$

$p(x\le 2)=\frac{{}^5C_0+{}^5C_1+{}^5C_2}{32}$

$=\frac{1+5+10}{32}=\frac{16}{32}=\frac{1}{2}$

130. **(d)** Let $16^{\sin^2}x=m$

$m^2+16=10\text{ m}$

$m^2-10\text{ m}+16=0$

$m=8$ or $m=2$

$16\sin^2 x=8$ or $16\sin^2 x=2$

$2^4\sin^2 x=2^3, 2^4\sin^2 x=2$

$4\sin^2 x=3\ 4\sin^2 x=1$

$\sin^2 x=\frac{3}{4}\ \sin^2 x=\frac{1}{4}$

No. of solutions of 4 in each case.

Total no. of solutions = 8

131. **(a)** $6x-2=3y+1=2x-2$

$\frac{x-1/3}{1/6}=\frac{y+1/3}{1/3}=\frac{x-1}{1/2}$

Vector equation is

$\bar{r}=\left(\frac{1}{3}\hat{\imath}-\frac{1}{3}\hat{\jmath}+\hat{k}\right)+\lambda\left(\frac{1}{6}\hat{\imath}+\frac{1}{3}\hat{\jmath}+\frac{1}{2}\hat{k}\right)$

$\bar{r}=\left(\frac{1}{3}\hat{\imath}-\frac{1}{3}\hat{\jmath}+\hat{k}\right)+\lambda(\hat{\imath}+2\hat{\jmath}+3\hat{k})$

132. **(a)** $P(A)=\frac{4}{10}$

$P(A')=\frac{6}{10}$

$P(B')=\frac{7}{10}\ P(B)=\frac{3}{10}$

According to given condition.

$P(A)+P(B)+P(C)=1$

$\frac{4}{10}+\frac{3}{10}+P(C)=1$

$\frac{7}{10}+P(C)=1$

$P(C)=1-\frac{7}{10}=\frac{3}{10}$

$P(C')=\frac{7}{10}\ \therefore$ Odds against $C=\frac{7}{3}$

133. **(b)** $S=\{(1,1,1)(1,1,0),(1,0,1)\},(1,0,0),(0,1,1)(0,1,0)(0,0,1)(0,0,0)\}$

$n(s)=8$

x 0 1 2 3

$$P(x) \quad \frac{1}{8} \quad \frac{3}{8} \quad \frac{3}{8} \quad \frac{1}{8}$$

Means $= E(x) = 0 + \frac{3}{8} + \frac{6}{8} + \frac{3}{8} = \frac{12}{8} = \frac{3}{2}$

Variance $= \left(0 + \frac{3}{8} + \frac{12}{8} + \frac{9}{8}\right) - \frac{9}{4}$

$= \frac{24}{8} - \frac{9}{4} = \frac{6}{8} = \frac{3}{4} = 0.75$

134. **(c)** Let $f(x) = e^x$

$g(x) = x^2 - e^x$

$\int_0^1 e^x(x^2 - e^x)dx$

$\int_0^1 x^2e^x - \int_0^1 e^{2x}dx$

$[x^2e^x - 2xe^x + 2e^x]_0^1 - \left[\frac{e^{2x}}{2}\right]_0^1$

$[(e - 2e + 2e) - 2] - \left[\frac{e^2}{2} - \frac{1}{2}\right]$

$e - 2 - \frac{e^2}{2} + \frac{1}{2}$

$e - \frac{e^2}{2} - \frac{3}{2}$

135. **(d)** $\int \frac{x+1}{x(1+xe^x)^2}dx$

$\int \frac{e^x(x+1)}{xe^x(1+xe^x)^2}dx$

Let $xe^x = t$

$e^x(x+1)dx = dt$

$\int \frac{1}{t(1+t)^2}dt$

$\int \frac{1+t-t}{t(1+t)^2}dt$

$\int \frac{1}{t(1+t)}dt - \int \frac{1}{(1+t)^2}dt$

$\int \frac{1+t-t}{t(1+t)}dt + \frac{1}{(1+t)} + c$

$\int_t^1 dt - \int \frac{1}{1+t} + \frac{1}{1+t} + c$

$\log t - \log(1+t) + \frac{1}{1+t} + c$

$\log\left|\frac{xe^x}{1+xe^x}\right| + \frac{1}{1+xe^x} + c$

136. **(b)** $\int \frac{1}{\sin(x-\alpha)\sin x}dx$

$\frac{1}{\sin(\alpha)}\int \frac{\sin(\alpha)}{\sin(x-\alpha)\sin x}dx$

$\frac{1}{\sin(\alpha)}\int \frac{\sin[x-(x-\alpha)]}{\sin(x-\alpha)\sin x}dx$

$\operatorname{cosec}\alpha\int \frac{\sin x\cos(x-\alpha) - \cos x\sin(x-\alpha)}{\sin(x-\alpha)\sin x}dx$

$\operatorname{cosec}\alpha\int (\cot(x-\alpha) - \cot x)dx$

$\operatorname{cosec}\alpha[\log(\sin(x-\alpha) - \log\sin x] + c$

$\operatorname{cosec}\alpha\left[\log\left(\sin\frac{x-\alpha}{\sin x}\right)\right]+c$

$\operatorname{cosec}\alpha[\log\sin(x-\alpha)\cdot\operatorname{cosec}x]+c$

137. **(d)** Let $\mathrm{b}=\sqrt{3}+1$

$c=\sqrt{3}-1$

$\mathrm{A}=60^{\circ}$

By Napier's analogy.

$\tan\left(\frac{\mathrm{B}-\mathrm{C}}{2}\right)=\frac{\mathrm{b}-\mathrm{c}}{\mathrm{b}+\mathrm{c}}\cot\frac{\mathrm{A}}{2}$

$\tan\left(\frac{B-C}{2}\right)=\frac{2}{2\sqrt{3}}\cot 30$

$=\frac{1}{\sqrt{3}}\times\sqrt{3}$

$\tan\left(\frac{\mathrm{B}-\mathrm{C}}{2}\right)=1$

$\frac{\mathrm{B}-\mathrm{C}}{2}=45^{\circ}$

$\mathrm{B}-\mathrm{C}=90^{\circ}$

138. **(b)** $\lim_{x\to\pi/2}\frac{\cot x-\cos x}{(\pi-2x)^3}$

Let $x=\frac{\pi}{2}+h$

As $\mathrm{x}\to\pi/2\ \ \mathrm{h}\to 0$

$\lim_{h\to 0}\frac{\cot\left(\frac{\pi}{2}+h\right)-\cos\left(\frac{\pi}{2}+h\right)}{-8h^3}$

$\lim_{h\to 0}\frac{-\tanh-\sinh}{-8\ \mathrm{h}^3}$

$\lim_{8h\to\infty}\frac{\tanh+\sinh}{h^3}$

$\frac{1}{8}\lim_{h\to 0}\frac{\sinh(1+\cos h)}{h^3}\times\frac{1}{\cosh}$

$\frac{1}{8}\lim_{h\to 0}\frac{\sinh}{h}\times\frac{1+\cosh}{h^2}\times\frac{1}{\cosh}$

$\frac{1}{8}(1)\left(\frac{1}{2}\right)\times(1)=\frac{1}{16}$

139. **(b)** $\overline{\mathrm{g}}=\frac{\overline{\mathrm{a}}+\overline{\mathrm{b}}+\overline{\mathrm{c}}}{3}$

$3\overline{\mathrm{g}}-\overline{\mathrm{a}}-\overline{\mathrm{b}}=\overline{\mathrm{c}}$

$\bar{c}=(-3\hat{\imath}+6\hat{\jmath}+3\hat{k})-(3\hat{\imath}+\hat{\jmath}+4\hat{k})-(-4\hat{\imath}+5\hat{\jmath}-3\hat{k})$

$=-2\hat{\imath}+0\hat{\jmath}+2\hat{k}$

$C\equiv(-2,0,2)$

140. **(c)** $(1+y^2)dx-xydy=0$

$\int\frac{dx}{x}=\frac{1}{2}\int\frac{2y}{1+y^2}dy$

$\log|x|=\frac{1}{2}\log|1+y^2|+\log c$

$2\log x=\log(1+y^2)+\log c$

$\mathrm{x}^2=\mathrm{c}(1+\mathrm{y}^2)$

when $x=1y=0c=1$

$\therefore$ particular solution is $\mathrm{x}^2=1+\mathrm{y}^2$

Which is Hyperbola.

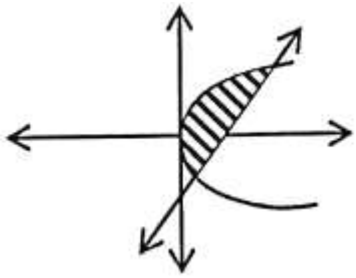
141. **(d)** $\frac{y^2}{2} = y + 4$

$y^2 = 2y + 8$

$y^2 - 2y - 8 = 0$

$(y-4)(y+2) = 0$

$y = -2$ or $y = 4$



Required area

$= -\int_{-2}^{4} \frac{y^2}{2} + \left[\int_{-2}^{4} y \cdot dy + \int_{-2}^{4} 4dy\right]$

$= -\left[\frac{y^3}{6}\right]_{-2}^{4} + \left[\left|\frac{y^2}{2}\right|_{-2}^{4} + |4y|_{-2}^{4}\right]$

$= -\left[\frac{64+8}{6}\right] + \left[\frac{16-4}{2} + 16 + 8\right]$

$= -\frac{72}{6} + \left[\frac{12}{2} + 24\right]$

$= -12 + 6 + 24$

$= 18$

142. **(d)** For $x \in -3, -2, -1, \theta, 1, 2, \dots \dots, 99$

f(x) is discontinuous.

f(x) is discontinuous for 103 values.

143. **(c)** Equation of parabola is $(x-h)^2 = 4a(y-k)$

Differentiating w.r.t.x, we get,

$\Rightarrow 2(x-h) = 4ay_1$

$\Rightarrow (x-h) = 2ay_1$

$\Rightarrow 1 = 2ay_2$

$\Rightarrow y_2 = \frac{1}{2a}$

Differentiating w.r.t. x , we get,

$\Rightarrow y_3 = 0 \Rightarrow \frac{d^3y}{dx^3} = 0$

144. **(a)** Let $\tan^{-1} x = t$

$\frac{1}{1+x^2} dx = dt$

$\int e^t \left[\left(\sec^{-1}\sqrt{1+\tan^2 t}\right)^2 + \cos^{-1}\left(\frac{1-\tan^2 t}{1+\tan^2 t}\right)\right] dt$

$\int e^t[(\sec^{-1}\sec t)^2 + \cos^{-1}(\cos t)]dt$

$\int e^t[t^2 + 2t]dt$

$= e^t \cdot t^2 + c$

$= e^{\tan^{-1}x}(\tan^{-1}x)^2 + c$

145. **(b)** $\frac{dv}{dt} \propto s \ \frac{dv}{dt} = -ks \ \frac{dv}{dt} = -k4\pi r^2$

But $v = \frac{4}{3}\pi r^3$

$$\frac{dv}{dt}=\frac{4}{3}\pi 3r^2\frac{dr}{dt}$$
$$=4\pi r^2\frac{dr}{dt}$$
$$4\pi r^2\frac{dr}{dt}=-k4\pi r^2$$
$$\frac{dr}{dt}=-k$$
$$\int dr=-\int k dt$$
$$r=-kt+c$$
when $t=0 r=3 c=3$ when $t=1, r=2$
$$2=-k+3$$
$$k=1$$
$$r=-kt+c$$
$$r=-t+3$$
$$r=3-t$$

146. **(b)** $\sin^{-1}x+\cos^{-1}y=\frac{3\pi}{10}$

$$\frac{\pi}{2}-\cos^{-1}x+\frac{\pi}{2}-\sin^{-1}y=\frac{3\pi}{10}$$
$$\pi-\frac{3\pi}{10}=\cos^{-1}x+\sin^{-1}y$$
$$\cos^{-1}x+\sin^{-1}y=\frac{7\pi}{10}$$

147. **(b)** Volume of parallelepiped

$$\begin{vmatrix}1 & 1 & \lambda\\ 1 & 1 & 3\\ 2 & 1 & 3\end{vmatrix}$$
$$=1(0)-1(-3)+\lambda(-1)$$
$$=3-\lambda=1$$
$$3-1=\lambda$$
$$\lambda=2$$

148. **(b)** a: $\sim(p\wedge\sim r)\vee(\sim q\vee s)$

$\sim(T\wedge T)\vee(F\vee F)$
$\sim(\mathrm{T})\vee(\mathrm{F})$
$F\vee F=F$
b: $(\sim q\wedge\sim r)\leftrightarrow(p\vee s)$
$(F\wedge T)\leftrightarrow(T\vee F)$
$(F)\leftrightarrow T=F$
c : $(\sim p\vee q)\rightarrow(r\wedge\sim s)$
$(F\vee T)\rightarrow(F\wedge T)$
$(T)\rightarrow(F)=F$

149. **(c)**

C.I	x_i	f_i	$f_i x_i$	$f_1 x_1^2$
$0-6$	3	2	6	18
$6-12$	9	4	36	324
$12-18$	15	6	90	1350
		12	**132**	**1692**

Variance $=\frac{\Sigma f_i x_1^2}{\Sigma f_i}-\left(\frac{\Sigma f_i x_i}{\Sigma f_i}\right)^2$

$$=\frac{1692}{12}-\left(\frac{132}{12}\right)^2$$

$= 141 - 121 = 20$

rd deviation $= \sqrt{20}$

$= 2\sqrt{5}$

150. **(a)** $\overline{\mathrm{OP}} = a\sin t + b\cos t$

$\mathrm{M}^2 = |\overline{\mathrm{OP}}|^2$

$= \left(\sin^2 \mathrm{t} + \cos^2 \mathrm{t} + 2\hat{a}.\hat{b}\sin \mathrm{t} \cdot \cos \mathrm{t}\right)$

$= [1 + \sin 2\mathrm{t}(\hat{a}.\hat{b})]$

For max OP –

$\hat{a}.\hat{b}(2 \cdot \cos 2t) = 0$

$\cos 2t = 0$

$2t = \dfrac{\pi}{2}$

$t = \dfrac{\pi}{4}$

$|OP|_{\max} = (1 + \hat{a} \cdot \hat{b})^{1/2}$

$\hat{u} = \dfrac{\overline{OP}}{|\overline{OP}|} = \dfrac{\hat{a}/\sqrt{2} + \hat{b}/\sqrt{2}}{\left|\dfrac{\hat{a}}{\sqrt{2}} + \dfrac{\hat{b}}{\sqrt{2}}\right|}$

$= \dfrac{\hat{a} + \hat{b}}{|\hat{a} + \hat{b}|}$