PHYSICS

1. **(d)** Given: $P = \frac{x^3 y}{z^2}$

Percentage error is given by,

$\frac{\Delta P}{P} = \frac{3\Delta x}{x} + \frac{\Delta y}{y} + \frac{2\Delta z}{z}$

$= (3 \times 0.6) + 3 + (2 \times 1.3)$

$= 1.8 + 3 + 2.6 = 7.4$

$\frac{\Delta P}{P}\% = 7.4\%$

2. **(d)** $W = q\Delta V$

$W = q(V_B - V_A)$

$W = q[V - (-5)]$

$\frac{W}{q} = V + 5$

$V = \frac{W}{q} - 5$

3. **(c)** Using the law of conservation of energy,

$\frac{1}{2}mv_1^2 = \frac{1}{2}mv_2^2 + 2mgL$

$\therefore\ v_1^2 = v_2^2 + 4gL \quad \ldots(i)$

Maximum tension, $T_{max} = \frac{mv_1^2}{L} + mg$

Minimum tension, $T_{min} = \frac{mv_2^2}{L} - mg$

$\therefore\ \frac{T_{max}}{T_{min}} = \frac{\frac{v_1^2}{L} + g}{\frac{v_2^2}{L} - g}$

$\therefore\ \frac{T_{max}}{T_{min}} = \frac{v_1^2 + gL}{v_2^2 - gL}$

$\therefore\ \frac{T_{max}}{T_{min}} = \frac{v_2^2 + 5gL}{v_2^2 - gL}$ ....[From (i)]

$\therefore\ 3 = \frac{v_2^2 + 5gL}{v_2^2 - gL}$

$\therefore\ 3v_2^2 - 3gL = v_2^2 + 5gL$

$\therefore\ 2v_2^2 = 8gL$

$v_2^2 = 4rg = 4gL$

$\therefore\ v_2 = \sqrt{4gL}$

4. **(c)** Shunt resistance, $S = \frac{G}{\left(\frac{I}{I_g} - 1\right)}$

Given that, $I_g = \frac{5}{100}I$

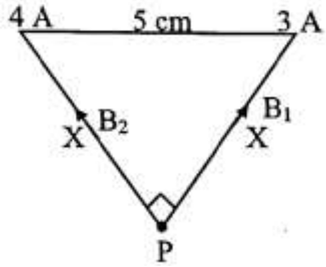
$S = \left(\frac{G}{\frac{100}{5} - 1}\right) = \frac{G}{19}\Omega$

5. **(d)** $\vec{\tau} = \vec{r} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & -3 & 0 \\ 5 & -10 & 0 \end{vmatrix}$

$= \hat{i}(0 - 0) - \hat{j}(0 - 0) + \hat{k}(-40 + 15)$

$= -25\hat{k}\ N - m$

6. **(d)**



Magnetic field produced by two wires

$B_1 = \frac{\mu_0 I_1}{2\pi X}$ and $B_2 = \frac{\mu_0 I_2}{2\pi X}$

From Figure,

$B_{net} = \sqrt{B_1^2 + B_2^2} = \frac{\mu_0}{2\pi X}\sqrt{I_1^2 + I_2^2}$

Also, using Pythagoras theorem, $2X^2 = 5 \times 5$

$\therefore\ X = \frac{5}{\sqrt{2}}$ cm

$$B_{net} = \frac{4\pi \times 10^{-7}}{2\pi \times \frac{5}{\sqrt{2}} \times 10^{-2}}\sqrt{4^2 + 3^2} = 2\sqrt{2} \times 10^{-5}\ \text{T}$$

7. **(d)** $E = eA\sigma T^4$

But for sphere, $A = 4\pi R^2$

$\therefore\ E = e(4\pi R^2)\sigma T^4$

$\therefore\ \frac{E_1}{E_2} = \frac{R_1^2 T_1^4}{R_2^2 T_2^4}$

$\therefore\ \frac{E}{E_2} = \frac{R^2 T^4}{(2R)^2 (2T)^4}$

$\therefore\ E_2 = 64E$

8. **(b)** $R = 80\Omega$

$\therefore\ |X_L - X_C| = |70 - 130| = 60\Omega$

$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{80^2 + 60^2} = 100$

$\therefore$ Power factor, $\cos\phi = \frac{R}{Z} = \frac{80}{100} = 0.8$

9. **(a)** We know,

$T = 2\pi\sqrt{\frac{l}{g}} = \sqrt{3}$ s

$g_2 = g + \frac{g}{3} = \frac{4g}{3}$ ... (lift accelerating upwards)

$\therefore\ T_2 = 2\pi\sqrt{\frac{3l}{4g}} = \frac{\sqrt{3}}{2}\sqrt{3}$ ... $(\because\ g_2 = 4g/3)$

$\therefore\ T_2 = 1.5$ s

10. **(b)** When the three inductors are connected in series, their equivalent inductance (L_e) is given by,

$L_e = L_1 + L_2 + L_3$

$= L + L + L = 3L$

$\therefore\ L_e = 3L$

The energy stored $U = \frac{1}{2}LI^2 = \frac{3LI^2}{2}$

11. **(a)** Using Rydberg's formula,

$\frac{1}{\lambda} = R\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$

$\frac{1}{\lambda_1} = R\left[\frac{1}{2^2} - \frac{1}{3^2}\right]$. ... $(\because\ n_1 = 2$ and $n_2 = 3$ for Balmer series)

$\frac{1}{\lambda_1} = \frac{5R}{36}$

$\frac{1}{\lambda_2} = R\left[\frac{1}{9} - \frac{1}{16}\right]$ ... ($\because n_1 = 3$ and $n_2 = 4$ for Paschen series)

$\frac{1}{\lambda_2} = \frac{7R}{144}$

$\therefore \frac{1}{\lambda_1} \times \frac{\lambda_2}{1} = \frac{5R}{36} \times \frac{144}{7R}$

$\therefore \frac{\lambda_2}{\lambda_1} = \frac{20}{7}$

$= \frac{f_1}{f_2}$ ... $\left(\because f = \frac{c}{\lambda}\right)$

12. (b) $L = \frac{nh}{2\pi}$ and $I = \frac{q}{T} = \frac{e\omega}{2\pi}$

Now, $M = IA$

$\therefore M = \frac{e\omega}{2\pi}\pi R^2 = \frac{e\omega R^2}{2}$

$\therefore M = \frac{e}{2}R^2 \times \frac{nh}{2\pi m R^2} = \frac{enh}{4\pi m}$

$\therefore \frac{L}{M} = \frac{4\pi m}{enh} \times \frac{nh}{2\pi} = \frac{2m}{e}$

13. (b) Given: $V_2 = 2V_1, \gamma = \frac{3}{2}$

For gas y subjected to adiabatic process,

$P_1V_1^\gamma = P_2V_2^\gamma$

$\therefore \frac{P_1}{P_2} = \left(\frac{V_2}{V_1}\right)^\gamma = (2)^{\frac{3}{2}} = 2\sqrt{2}$

$\therefore P_1 = 2\sqrt{2}P_2$

For gas Z subjected to isobaric process,

$P_1 = P_2$ or $\frac{P_1}{P_2} = 1$

For gas X subjected to isothermal process,

$P_1 V_1 = P_2 V_2$

$\frac{P_1}{P_2} = \frac{V_2}{V_1} = 2$ or $P_1 = 2P_2$

$\therefore$ Ratio of initial pressures is $2 : 2\sqrt{2} : 1$

14. (b) $W = \frac{1}{2}LI^2$

If $I = 1$ A

$\therefore L = 2W$

15. (b) Apparent frequency for source moving towards a stationary observer is given by,

$n' = n\left[\frac{v}{v - v_s}\right]$

As the source moves towards the observer, frequency increases, hence wavelength decreases.

16. (c) $h_P r_P = h_Q r_Q$

$\therefore h_Q = \frac{h_P r_P}{r_Q} = \frac{2.4 \times r_P}{0.8 r_P} = 3$ cm

17. (b) We know

Fourth overtone of a closed pipe is

$f_c = \frac{(2n+1)V}{4l_c} = \frac{9V}{4l_c}$

Fifth overtone of an open pipe is given by

$f_o = \frac{(n+1)V}{2l_o} = \frac{3V}{l_o}$

Given: $f_c = f_o$

$\Rightarrow \frac{9\text{ V}}{4l_c} = \frac{3\text{ V}}{l_o}$

$\therefore \frac{l_c}{l_o} = \frac{3}{4}$

18. **(a)** Initially, $C_1 = \frac{A\varepsilon_0}{d}$

After inserting dielectric,

$C_2 = \frac{A\varepsilon_0}{d - t + \frac{t}{k}}$

$= \frac{A\varepsilon_0}{d - t + \frac{t}{3}}$

Given: $C_2 = C_1 + 50\%$ of $C_1 = C_1 + \frac{1}{2}C_1 = \frac{3}{2}C_1$

$\therefore \frac{A\varepsilon_0}{\frac{3\text{ d} - 3t + t}{3}} = \frac{3}{2}\frac{A\varepsilon_0}{d}$

$\therefore \frac{3}{3d - 2t} = \frac{3}{2d}$

$\therefore 2\text{ d} = 3\text{ d} - 2t$

$2t = d$

$\therefore t = \frac{d}{2}$

19. **(a)** $\frac{Gm^2}{(2r)^2} = \frac{mv^2}{r}$ or $\frac{Gm}{4r} = v^2$

$\therefore v = \sqrt{\frac{Gm}{4r}}$

20. **(d)**

21. **(c)** In reverse biased condition in a p − n junction, the width of the depletion region increases which leads to an increase in the electric field strength. Thus, the correct answer is electric field strength is maximum in the depletion layer.

22. **(b)** Length of first rod at temp. $t = l_1 + l_1\alpha_1\Delta t$

Length of second rod at temp. $t = l_2 + l_2\alpha_2\Delta t$

Length of second rod - length of first rod $= (l_2 - l_1) + (l_2\alpha_2 - l_1\alpha_1)\Delta t$

Difference in length is independent of temperature.

$\therefore$ Coefficient of Δt must be zero.

$\therefore l_2\alpha_2 - l_1\alpha_1 = 0$

$\therefore \frac{l_1}{l_2} = \frac{\alpha_2}{\alpha_1}$

23. **(d)** Mutual inductance of two concentric coplanar circular coils,

$M = \frac{\mu_0\pi N_1 N_2R_2^2}{2R_1}$

$\therefore M \propto \frac{R_2^2}{R_1}$

24. **(a)** Given: T = 0.1 s

We know

$T = 2\pi\sqrt{\frac{m}{K}}$

Kx = mg, where x is the amplitude. ... ($\because$ F = ma)

$\therefore \frac{m}{K} = \frac{x}{g} \Rightarrow T = 2\pi\sqrt{\frac{x}{g}}$

$\therefore 0.1 = 2\pi\sqrt{\frac{x}{g}}$

$\therefore \frac{1}{100} = 4\pi^2\frac{X}{g}$

$\therefore x = \frac{g}{400\pi^2} = \frac{10}{400\pi^2} = \frac{1}{40\pi^2}$

$\therefore V_{max} = x\omega = x \times 2\pi f = \frac{1}{40\pi^2} \times 2\pi \times 10$

$= \frac{1}{2\pi}$ m/s $\quad \ldots\left(\because f = \frac{1}{T}\right)$

25. (b) For thin prism, $\delta = (\mu - 1)A$

Given, $\mu_1 = \frac{\mu_{glass}}{\mu_{air}} = \frac{3}{2}$ and $\frac{\mu_{water}}{\mu_{air}} = \frac{4}{3}$

$\therefore \mu_2 = \frac{\mu_{glass}}{\mu_{water}} = \frac{3/2}{4/3} = \frac{9}{8}$

$\therefore \frac{\delta_1}{\delta_2} = \frac{\mu_1 - 1}{\mu_2 - 1} = \frac{3/2 - 1}{9/8 - 1} = \frac{4}{1}$

$\therefore \frac{\delta_2}{\delta_1} = \frac{1}{4}$

26. (c) Current gain $\beta = \frac{\alpha}{1-\alpha}$

Given that, $\alpha = \frac{I_c}{I_E} = 0.95$

$\therefore \beta = \frac{0.95}{1 - 0.95} = 19$

27. (c) As the string is vibrating in three segments, therefore:

$l = \frac{3\lambda}{2}$

$\lambda = \frac{2l}{3} = \frac{2(0.9)}{3} = 0.6$ m

Now, $v = \sqrt{\frac{T}{m}}$

$\therefore v = \sqrt{\frac{40}{0.1}} = 20$ m/s

$n = \frac{v}{\lambda} = \frac{20}{0.6} = \frac{100}{3}$ Hz

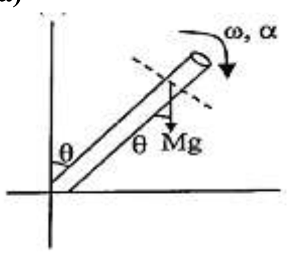
Now, amplitude of particle velocity

$= \left(\frac{dy}{dt}\right)_{max} = (a_{max})\omega$

$= (a_{max})(2\pi n) = 0.3 \times 10^{-2} \times 2\pi \times \frac{100}{3}$

$= \frac{\pi}{5}$ m/s

28. (a)



Torque at angle θ,

$\tau = Mg\sin\theta\frac{L}{2}$

Also , we know , $\tau = I\alpha$

$\Rightarrow\ I\alpha = mg\sin\theta\frac{L}{2}$

∵ For a thin uniform rod, the moment of inertia about an axis perpendicular to its length and passing through its one end is $\frac{ML^2}{3}$, the above equation becomes:

$\Rightarrow \frac{ML^2}{3}\cdot\alpha = Mg\sin\theta\frac{L}{2}$

$\Rightarrow \frac{L\alpha}{2} = g\frac{\sin\theta}{2}$

$\Rightarrow \alpha = \frac{3\ g\sin\theta}{2\ L}$

29.(c) $C_p - C_v = R$

Dividing both the sides by C_P,

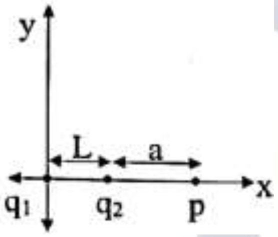
$\therefore\ 1 - \frac{1}{\gamma} = \frac{R}{C_P} \cdots \left(\because \frac{C_p}{C_v} = \gamma\right)$

$\therefore\ C_P = \frac{R\gamma}{\gamma - 1}$

30. **(a)** Let ' a ' be the distance between q_2 and p From figure,

Distance between q_1 and $p = L + a$

For proton to be in equilibrium, it should be placed at point where electric field due to q_1 and q_2 is zero.



$E_1 + E_2 = 0$

$\therefore\ \frac{kq_1}{r_1^2} + \frac{kq_2}{r_2^2} = 0$

$\therefore\ \frac{+6q}{(L+a)^2} + \frac{(-3q)}{a^2} = 0$

$\therefore\ \frac{6q}{(L+a)^2} = \frac{3q}{a^2}$

$\therefore\ \frac{2}{(L+a)^2} = \frac{1}{a^2}$

$\therefore\ \frac{\sqrt{2}}{L+a} = \frac{1}{a}$ ....(Taking sq. root)

$\therefore\ \sqrt{2}a = L + a$

$\Rightarrow\ \sqrt{2}a - a = L$

$\Rightarrow\ a = \frac{L}{\sqrt{2}-1}$

$\therefore$ Distance between q_1 and $p = L + a$

$= L + \frac{L}{\sqrt{2}-1} = \frac{(\sqrt{2}-1)L+L}{\sqrt{2}-1} = \frac{\sqrt{2}\,L}{\sqrt{2}-1}$

$= \frac{\sqrt{2}\,L}{\sqrt{2}-1}$

31. **(a)** The angular displacement after 24 rotations,

$\theta = 2\pi \times 24$

By using equation $\omega^2 = \omega_0^2 - 2\alpha\theta$

$\left(\frac{\omega_0}{3}\right)^2 = \omega_0^2 - 2\alpha(2\pi \times 24)$

$\therefore\ \alpha = \left(\frac{8}{9}\right)\frac{\omega_0^2}{4\pi\times 24} \quad \ldots(\text{i})$

Now let fan complete total n' rotations from the starting to come to rest

$0 = \omega_0^2 - 2\alpha(2\pi n')$

$\therefore\ n' = \frac{\omega_0^2}{4\alpha\pi}$

Substituting the value of α from equation (i),

$n' = \left(\frac{\omega_0^2}{4\pi}\right)\frac{9\times 4\pi\times 24}{8\omega_0^2} = 27$ rotations

Number of rotations $= 27 - 24 = 3$

32. **(b)** $\frac{\text{Intensity of bright band}}{\text{Intensity of dark band}} = \frac{5}{1}$

But $I \propto a^2$

$\Rightarrow$ Amplitude of bright band $a_b = \sqrt{5}$ and amplitude of dark band $a_d = 1$

$\therefore$ Ratio of resultant amplitudes of bright and dark fringe will be:

$\frac{a_b}{a_d} = \frac{\sqrt{5}}{1}$

33. **(a)** $F = 98$ dyne $= 98 \times 10^{-5}$ N,

$T = 7 \times 10^{-2}\ \text{N/m}$

Now the force due to surface tension on the circular cross-section of capillary with inner radius r will be,

$F = 2\pi r T$

$\therefore\ 2\pi r = \frac{F}{T} = \frac{98 \times 10^{-5}}{7 \times 10^{-2}}$

$= 14 \times 10^{-3}$ m $= 1.4$ cm

34. **(c)** Since the energy of the emitted electrons is greater for wavelength λ_1,

$\lambda_1 < \lambda \quad \ldots(\text{i})$

$E = \frac{hc}{\lambda} - \phi_0 \quad \ldots(\text{ii})$

and $2E = \frac{hc}{\lambda_1} - \phi_0$

Dividing equation (iii) by (ii),

$E = \frac{hc}{2\lambda_1} - \frac{\phi_0}{2} \quad \ldots(\text{iv})$

Equating R.H.S. of equation (ii) and (iv), $\frac{hc}{\lambda} - \phi_0 = \frac{hc}{2\lambda_1} - \frac{\phi_0}{2}$

$\therefore\ \frac{hc}{\lambda} - \frac{hc}{2\lambda_1} = \phi_0 - \frac{\phi_0}{2} = \frac{\phi_0}{2}$

$\therefore\ \frac{hc}{\lambda} - \frac{hc}{2\lambda_1} > 0$

$\therefore\ \frac{1}{\lambda} > \frac{1}{2\lambda_1}$

$2\lambda_1 > \lambda$

$\Rightarrow\ \lambda_1 > \frac{\lambda}{2}$

35. **(c)** $C_p - C_v = R$

Dividing both the sides by Cv,

$\therefore\ 1 - \frac{1}{\gamma} = \frac{9}{7} - 1 = \frac{R}{C_v} \qquad \ldots\left(\because \frac{C_p}{C_v} = \gamma = \frac{9}{7}\right)$

$\therefore \frac{R}{C_v} = \frac{2}{7} = 0.28$

36.(c) $F \propto \frac{I_1 I_2}{d}$

$\frac{F}{F_2} = \frac{I^2}{d_1} \times \frac{d_1}{4I^2 \times 2} = \frac{1}{8}$

$F_2 = 8\ F$

37. **(a)** For elastic collision, the coefficient of restitution, $e = 1$

We know, $e = \frac{v_1 - v_2}{u_1 - u_2} = \frac{4 - v_2}{3 - 5}$

$\Rightarrow 1 = \frac{4 - v_2}{3 - 5}$

$\Rightarrow -2 = 4 - v_2$

$\Rightarrow v_2 = 4 + 2 = 6$

$\therefore v_2 = 6\ m/s$ in the same direction

38. **(a)** Since process is isothermal, there is no change in internal energy. Hence, supplied energy is used to perform work on the surroundings. Thus, gas will do positive work.

39. **(a)** Given voltage equation: $e = 100\sqrt{2}\sin(50t)$

Comparing with standard equation,

$E = E_0 \sin \omega t$

$\omega = 50 \Rightarrow 2\pi f = 50 rad/s$

$\therefore X_C = \frac{1}{2\pi f C} = \frac{1}{50 \times 2 \times 10^{-6}} = 1 \times 10^4 \Omega$

$I_0 = \frac{e_0}{X_c}$

$I_0 = \frac{100\sqrt{2}}{10^4}$

$I_0 = \sqrt{2} \times 10^{-2}\ A$

$I_{ms} = \frac{I_0}{\sqrt{2}} = \frac{\sqrt{2} \times 10^{-2}}{\sqrt{2}} = 10^{-2}\ A = 10\ mA$

40. **(a)** We know, width of the central maxima

$\beta = \frac{2\lambda D}{d}$ ...(i)

Given, $\beta = 2\ d$ ...(ii)

$\Rightarrow$ Substituting (ii) in (i), we get:

$\Rightarrow 2\ d = \frac{2\lambda D}{d}$

$\Rightarrow \frac{d^2}{\lambda} = D$

$\Rightarrow D = \frac{d^2}{\lambda}$

41.(b) $3E = n \times (4\pi r^2 \times T) - (4\pi R^2 \times T)$

$3 \times (4\pi R^2 \times T) = n \times (4\pi r^2 \times T) - (4\pi R^2 \times T)$

$\therefore n = \frac{4R^2}{r^2}$

42. **(a)** For spectral series, $\frac{1}{\lambda} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$

For the Balmer series, $n_1 = 2, n_2 = \infty$

The wavelength for 2^{nd} line of the Balmer series is

$\frac{1}{\lambda_1} = R\left(\frac{1}{2^2} - \frac{1}{\infty^2}\right)$

$\frac{1}{\lambda_1} = R\left(\frac{1}{4}\right)$

$\frac{1}{\lambda_1} = \frac{R}{4} \Rightarrow \lambda_1 = \frac{4}{R}$

For the Balmer series, $n_1 = 3, n_2 = \infty$

The wavelength for the last line of the Balmer series is

$\frac{1}{\lambda_2} = R\left(\frac{1}{3^2} - \frac{1}{\infty^2}\right)$

$\frac{1}{\lambda_2} = R\left(\frac{1}{9}\right)$

$\lambda_2 = \frac{9}{R}$

$\therefore \frac{\lambda_2}{\lambda_1} = \frac{9}{R} \times \frac{R}{4} = \frac{9}{4}$

43. **(a)** The polaroid is arranged in order P_1, P_3 and P_2 Using law of Malus',

Intensity of light transmitted from P_1 will be reduced by half

$I_1 = \frac{I_0}{2}$

$\therefore$ Intensity of emergent light from P_2,

$I_2 = I_1 \cos^2 60^\circ = \frac{I_0}{2} \times \frac{1}{4} = \frac{I_0}{8}$

$\therefore$ Intensity of emergent light from P_3,

$I_3 = I_2 \cos^2 \theta \qquad \ldots(i)$

$\because$ P_1 and P_3 are perpendicular to each other,

$\therefore$ Angle between P_2 and $P_3, \theta = 90^\circ - 60^\circ = 30^\circ$

Substituting in equation (i),

$I_3 = I_2 \cos^2 30^\circ = \frac{I_0}{8} \times \frac{3}{4} = \frac{3I_0}{32}$

Substituting the value of I_0 in the equation above,

$I_3 = \frac{3 \times 256}{32} = 24 = 24\ W/m^2$

44. **(a)** The potential energy of a dipole in an electric field is given by

$W = -pE\cos\theta$

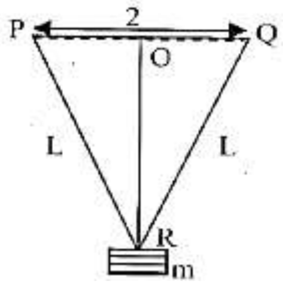
When $\theta = 0^\circ, W_1 = -pE\cos 0^\circ = -pE$

When $\theta = 90^\circ, W_2 = pE\cos 90^\circ = 0$

Work done $= W_2 - W_1 = 0 - (-pE) = pE$

45. **(c)** The saturation photoelectric current is directly proportional to the intensity of incident radiation, but it is independent of its frequency. Hence, saturation photoelectric current is doubled, when the intensity of the incident light is doubled, and the frequency is kept slightly greater than threshold frequency.

46. **(c)**



The motion of mass ' m ' is defined as a simple harmonic motion (S.H.M) with its length equal to OR:

From figure, $OR = \sqrt{L^2 - d^2}$

: Time period, $T = 2\pi\sqrt{\frac{(L^2-d^2)^{1/2}}{g}}$

47. **(c)** Given: $i = 3\sin\left[50\pi t + \frac{\pi}{4}\right]$ A

Peak value $i_0 = 3$ A

When $i = 3$ A, we have

$3 = 3\sin\left[50\pi t + \frac{\pi}{4}\right]$

$\therefore\ \sin\left[50\pi t + \frac{\pi}{4}\right] = 1$

$\therefore\ 50\pi t + \frac{\pi}{4} = \frac{\pi}{2}$ ... $(\sin 90° = 1)$

$\therefore\ 50\pi t = \frac{\pi}{4} \Rightarrow t = \frac{1}{200}$ s

48. **(c)** Fundamental frequency of pipe closed at one end is $n = \frac{v}{4l}$

Second overtone of pipe open at both the ends is $n' = 3\left(\frac{v}{2l'}\right)$

$n' = n$ ...(given)

$\therefore\ \frac{3v}{2l'} = \frac{v}{4l}$

$\therefore\ l' = 6l$

$\therefore\ X = 6$ ... $(l = L$ and $l' = XL)$

49. **(b)**

50. **(a)** Given: $I_g = 16\ \text{mA} = 16 \times 10^{-3}\Omega$

$V_g = 80\ \text{mA} = 80 \times 10^{-3}\Omega$

The galvanometer's internal resistance R_g will be:

$R_g = \frac{V_g}{I_g} = \frac{80 \times 10^{-3}}{16 \times 10^{-3}} = 5\Omega$

To increase the range of the galvanometer to 160 V , resistance needs to be connected in series to limit the current through the galvanometer to its full-scale deflection current when 160 V is applied.

This configuration converts the galvanometer into a voltmeter with a range of 160 V .

For a voltmeter, the total voltage (V) across the galvanometer and the series resistor is given by:

$V_g = I_g(R_g + R_s)$

Substituting the values, we get:

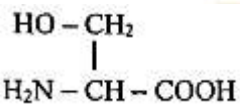
$160 = 16 \times 10^{-3}(5 + R_s)$

$R_s = \frac{160}{16 \times 10^{-3}} - 5 = 10000 - 5 = 9995\Omega$

Thus, the correct answer is 9995Ω in series.

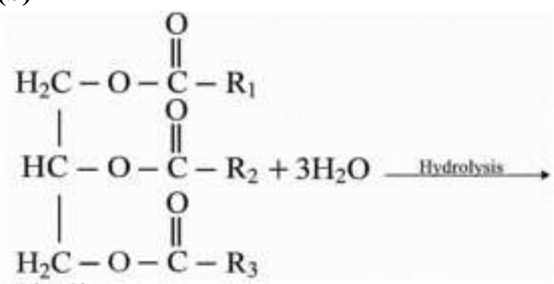
CHEMISTRY

51. **(d)**

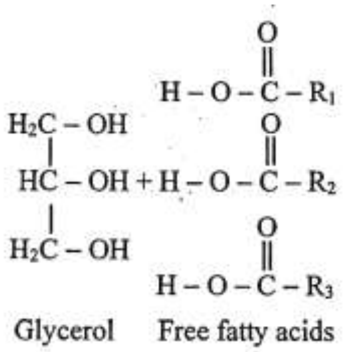


Serine

52. **(b)**



Triglyceride



53. **(b)** According to Fagan's rules, covalent nature is greater for small cations and large anions.

Li^+ is the smallest group-1 cation.

I^- is the largest group-17 anion.

$\therefore$ LiI has the highest covalent character.

54. **(b)** Given: $m = 0.02$ m, $K_f = 1.86$ K kg mol^{-1}

Number of ions produced (n) = 2,

$\Delta T_f = 0.046$ K

$\Delta T_f = i \times K_f \times m$

$\alpha = \dfrac{i-1}{n-1}$

As $n = 1, \alpha = i - 1$

$\therefore$ $i = 1 + \alpha$ (α is degree of dissociation)

$\Delta T_f = (1+\alpha) \times K_f \times m$

$\therefore$ $0.046 = (1+\alpha) \times 1.86 \times 0.02$

$\therefore$ $0.046 = (1+\alpha) \times 0.0372$

$\therefore$ $1 + \alpha = \dfrac{0.046}{0.0372} = 1.236$

$\therefore$ $\alpha = 1.236 - 1 = 0.236$

$\therefore$ Percent degree of dissociation

$= \alpha \times 100\%$

$= 0.236 \times 100\% \approx 23.6\%$

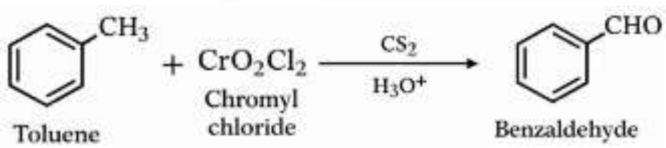
55. **(a)** $NO_{2(g)} + CO_{(g)} \longrightarrow NO_{(g)} + CO_{2(g)}$

Rate $= \frac{-\Delta[NO_2]}{\Delta t} = \frac{-\Delta[CO]}{\Delta t} = \frac{\Delta[NO]}{\Delta t} = \frac{\Delta[CO_2]}{\Delta t}$

$\therefore$ Rate of disappearance of $CO_{(g)}$

$= \dfrac{-\Delta[CO]}{\Delta t} = \dfrac{\Delta[NO]}{\Delta t} = Y\,mol\,dm^{-3}\,s^{-1}$

56. **(c)**



This is called as Étard reaction.

57. **(a)** Amylamines are weaker bases as their lone pair is not completely available for donation due to resonance. Aliphatic amines are stronger bases than ammonia. Among aliphatic amines, 2° amine is a stronger base than 1° amine.

Hence, N-Methylethanamine is the strongest base and its pK_b value is the lowest among the given amines as it is a 2° aliphatic amine.

58. **(b)** For the given structure, molecular formula is C_2H_6O.

$\therefore$ Molar mass of the compound

$= (\text{Mass of C} \times 2) + (\text{Mass of H} \times 6) +$
$(\text{Mass of O} \times 1)$
$= (12 \times 2) + (1 \times 6) + (16 \times 1)$
$= 24 + 6 + 16$
$= 46 \text{ g mol}^{-1}$

59. (a) $Zn_{(s)} + 2Ag^{+}_{(aq)} \longrightarrow Zn^{2+}_{(aq)} + 2Ag_{(s)}$

Given, $E_{cell} = E^{\circ}_{cell} - 0.0592 \quad \ldots(i)$

From Nernst equation,

$$E_{cell} = E^{\circ}_{cell} - \frac{0.0592}{n}\log\frac{[Zn^{+2}]}{[Ag^{+}]^{2}} \qquad \ldots(ii)$$

From equation (i) and (ii), we can write,

$$\frac{1}{n}\log\frac{[Zn^{+2}]}{[Ag^{+}]^{2}} = 1$$

$$\frac{1}{2}\log\frac{[Zn^{+2}]}{[Ag^{+}]^{2}} = 1 \ (\because n = 2)$$

$$\log\frac{[Zn^{+2}]}{[Ag^{+}]^{2}} = 2$$

$$\therefore \frac{[Zn^{+2}]}{[Ag^{+}]^{2}} = 10^{2}$$

Looking at the options, (A) is the correct answer.

$[Zn^{+2}] = 1M, [Ag^{+}] = 0.1M$

$$\therefore \frac{[Zn^{+2}]}{[Ag^{+}]^{2}} = \frac{1}{(0.1)^{2}}$$

$\therefore$ The possible values of $[Zn^{+2}]$ and $[Ag^{+}]$will be 1 M and 0.1 M respectively.

60. (c) For fcc unit cell,

$\therefore a = 2\sqrt{2}r$
$\therefore a = 2\sqrt{2} \times 141.4 \times 10^{-10}$ cm
$= 2\sqrt{2} \times 1.414 \times 10^{-8}$ cm
$= 2\sqrt{2} \times \sqrt{2} \times 10^{-8}$ cm $(\because \sqrt{2} \simeq 1.414)$
$\therefore a = 4 \times 10^{-8}$ cm
Volume of unit cell $= a^{3} = (4 \times 10^{-8} \text{ cm}^{3})$
$= 6.4 \times 10^{-23} \text{ cm}^{3}$

61. (c)

62. (b) $[A]_0 = 20$ millimol dm^{-3},

$[A]_t = 8$ millimol dm^{-3}, $t = 40$ minutes

For first order reaction,

$$K = \frac{2.303}{t}\log\frac{[A]_0}{[A]_t}$$

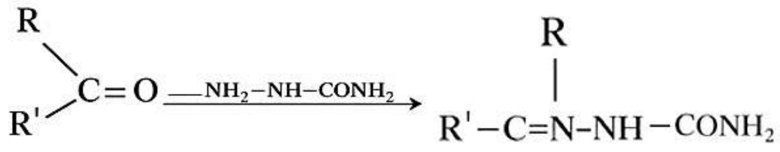
$$= \frac{2.303}{40}\log\frac{20}{8} = \frac{2.303}{40}\log\frac{5}{2}$$

$$= \frac{2.303}{40}(\log 5 - \log 2)$$

$$= \frac{2.303}{40}(0.6989 - 0.3010)$$

$= 0.023 \text{ minute}^{-1}$

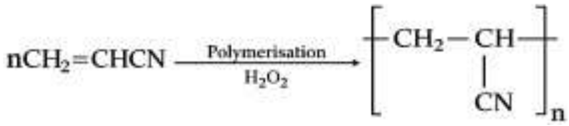
63. (b)



Ketone

Semicarbazone

64. (b)



Acrylonitrile Polyacrylonitrile (PAN)

65. (d) Transition elements have partially filled d-orbitals.

$Hg = [Xe]4f^{14}5\ d^{10}6\ s^2$

Among the given options, Hg has completely filled d-orbitals in its ground state. Hence, it is not regarded as transition element.

66. (a) Molar conductivity of an electrolyte

$\Lambda = \frac{1000 \times k}{C}$

∴ - The correct relationship among the given is

$k = \frac{\Lambda \times C}{1000}$

67. (a) Glass exhibits same magnitude for all properties like refractive index, conductivity, etc., in every direction. Hence, it is isotropic in nature.

68. (a) $\Delta S_{surr} = -\frac{\Delta H^\circ}{T} = -\frac{(-208.6\ kJ)}{298\ K}$

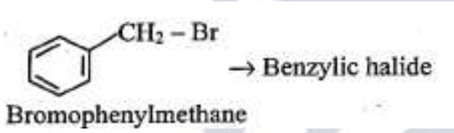
$= +0.7\ kJ\ K^{-1} = +700\ J\ K^{-1}$

$\Delta\ S_{total} = \Delta S_{sys} + \Delta S_{surr}$

$= -36\ J\ K^{-1} + 700\ J\ K^{-1}$

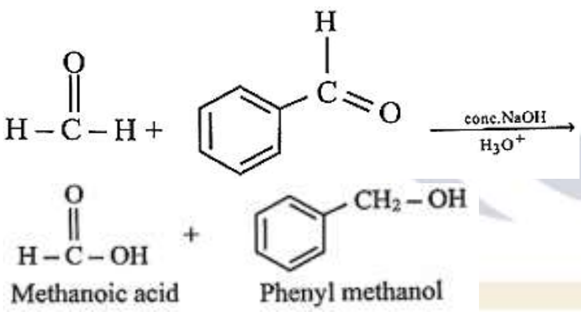
$= +664\ J\ K^{-1}$

69. (a)



70. (c) The duration of half-life ($t_{1/2}$) for first order reaction is independent of initial reactant concentration.

71. (c)



72. (a) N − H bonds in amines are less polar than O − H bond in alcohols. Thus, water solubilities of alcohols, amines and alkanes of comparable molar mass in water decreases in the order: alcohols > amines > alkanes.

73. (b) The ligand en is ethylenediamine.

74. (b) 1 mole CO_2 at STP occupies $22.4 dm^3$

∴ 0.5 mole CO_2 occupies

$= 0.5\ mol \times 22.4 dm^3\ mol^{-1} = 11.2 dm^3$

75. (b) Work is done on system.

Hence, $W = +ve = +20\ kJ$

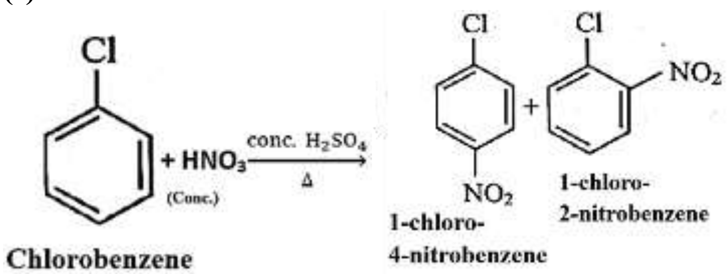
Heat is released.

Hence, $Q = -ve = -10\ kJ$

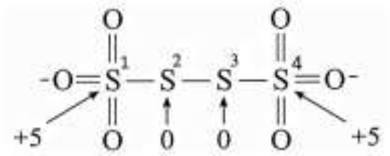
According to the first law of thermodynamics, $\Delta U = Q + W = -10 + 20 = +10\ kJ$

76. (a) $Li_2CO_3 \xrightarrow{\Delta} Li_2O + CO_2$

77. (c)



78. (b)



79. (b)

$CH_2 = \underset{\substack{| \\ CH_3}}{C} = CH - OH \Rightarrow 2°$ allylic alcohol

$CH_2 = CH - CH_2 - OH \Rightarrow 1°$ allylic alcohol

$CH_2 = CH - C(CH_3)_2 - OH \Rightarrow 3°$ allylic alcohol

$CH_3 - CH = CH - \underset{\substack{| \\ CH_3}}{CH} - OH \Rightarrow 2°$allylic alcohol

80. **(c)** Stachyose is a tetra saccharide consisting 2 units of galactose, 1 unit of fructose and 1 unit of glucose while raffinose is a trisaccharide consisting one unit each of glucose, fructose and galactose.

81. **(b)** $pH = -\log_{10}[H^+]$

$\therefore \log_{10}[H^+] = -pH = -3.76$

$\log_{10}[H^+] = -3.76$

$\therefore [H^+] = AL(-3.76)$

= AL(1 - 4.76)

$= AL(-\bar{4}.24) = 1.738 \times 10^{-4}$ $moldm^{-3}$

82. **(b)**

83. (d) The addition polymerization of tetrafluoroethene with a free radical or desulphated catalyst at high pressure leads to the formation of Teflon.

$$nCF_2 = CF_2 \xrightarrow[\text{Polymerisation}]{\textit{Catalys/High pressure}} [CF_2 - CF_2]_n$$

Tetrafluoroethylene (1,1,2,2-Tetrafluoroethene) — Teflon (PTFE)

84. **(c)** Ozonolytic is a reaction where ozone (O_3) reacts with unsaturated compounds containing double bonds to form addition products called ozonide's. These ozonide's are then decomposed by water or dilute acids, producing aldehydes or ketones.

[Note: The question has been modified to apply appropriate textual concepts.]

85. (c) $\frac{P_1^0 - P_1}{P_1^0} = \frac{\Delta P}{P_1^0} = \frac{W_2 M_1}{W_1 M_2}$

$$\frac{\Delta P}{P_1^0} = 0.02 = \frac{20\ g \times 18\ g\ mol^{-1}}{200\ g \times M_2}$$

$$M_2 = \frac{20\ g \times 18\ g\ mol^{-1}}{200\ g \times 0.02} = 90\ g\ mol^{-1}$$

86. (a) $W = -\Delta n_g RT$

For the equation,

$CH_{4(g)} + Cl_{2(g)} \longrightarrow CH_3Cl_{(g)} + HCl_{(g)}$

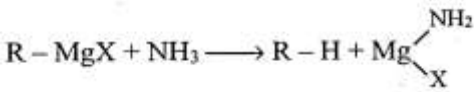
$\Delta n_g = 0$

$\therefore$ W = 0

87. (c) Alkyl halide $\xrightarrow[Dryether]{Mg} \underset{(A)}{RMgX} \xrightarrow{NH_3} \underset{(B)}{\text{Hydrocarbon}}$

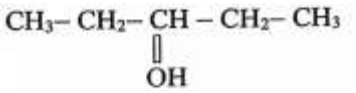
For example:

$R - X + Mg \xrightarrow[rther]{dry} R - MgX$



88. (b)

89. (c)



3-Hydroxypentane ($C_5H_{12}O$)

90. (a) From the ideal gas equation,

PV = nRT

Since V, R and T are constant,

$P \propto n \propto \frac{1}{M.W.}$

Order of molecular weight:

$H_2 < N_2 < O_2 < Cl_2$

$\therefore$ The gas with least molecular weight, H_2 will exert the highest pressure.

91. (c) $AX_{(s)} \rightleftharpoons A^+_{(aq)} + X^-_{(aq)}$

$\therefore\ x = 1$ and $y = 1$

$\therefore\ K_{sp} = x^x y^y S^{x+y} = S^2$

$\therefore\ S = \sqrt{K_{sp}} = \sqrt{4.9 \times 10^{-13}} = 7.0 \times 10^{-7}$ moldm^{-3}

92. (a) $V^{3+} \rightarrow 4\,s^0 3\,d^2$ (2 unpaired electrons)

$Cr^{3+} \rightarrow 4\,s^0 3\,d^3$ (3 unpaired electrons)

$Mn^{2+} \rightarrow 4\,s^0 3\,d^5$ (5 unpaired electrons)

$Fe^{2+} \rightarrow 4\,s^0 3\,d^6$ (4 unpaired electrons)

Lower the number of unpaired electrons, lower is the value of spin only magnetic moment.

93.(c) The required reaction is:

$2Cl^-_{(aq)} + 2e^- \longrightarrow Cl_{2(g)}$

Mole ratio $= \frac{1\ \text{mol}}{2\ \text{mole}^-}$

Moles of product formed

$= \frac{I \times t}{96500(C/mole^-)} \times$mole ratio

$0.5\ \text{mol} = \frac{100 \times t}{96500C/mole^-} \times \frac{1\ \text{mol}}{2\ \text{mole}^-}$

$\therefore\ t = \frac{0.5 \times 96500 \times 2}{100 \times 1} = 965$ seconds

94.(b) For bcc unit cell, n = 2.

No. of particles in x g $= \frac{xn}{\rho a^3}$

$\therefore$ Number of atoms in 1.58 g of metal

$= \frac{1.58 \times 2}{1.58 \times 10^{-22}} = 2.0 \times 10^{22}$ atoms

95.(c) The thermal stability of hydrides of group 16 elements decreases from H_2O to H_2Te.

96. (b) Longest wavelength corresponds to shortest wavenumber and minimum energy.

For Lyman series, $n_1 = 1$

The shortest wavenumber (longest wavelength) in the hydrogen spectrum of the Lyman series corresponds to the transition from $n = 2$ to $n = 1$. Therefore, $n_2 = 2$

$\bar{\nu} = R_H\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right] cm^{-1}$

$= 109677\left[\frac{1}{1^2} - \frac{1}{2^2}\right] cm^{-1}$

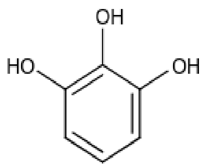
$= 109677\left[1 - \frac{1}{4}\right] cm^{-1}$

$= \frac{1}{\bar{\nu}} = \left[\frac{4}{3\times109677}\right] cm$

$\lambda = 109677\left[\frac{3}{4}\right] cm^{-1}$

$\lambda = \frac{4\times10^{-5}}{3\times1.09677} = 1.216\times10^{-5}$ cm

97. (d)



Pyrogallol
(Benzene-1,2,3-triol)

98. (a) % atom economy

$= \frac{\text{Formula weight of the desired product}}{\text{Sum of formula weight of all the reactants used in the reaction}} \times 100$

Formula weight of desired product

$= \frac{\text{\% atom economy} \times \text{Sum of formula weight of all the reactants}}{\text{used in the reaction}}$

$= \frac{50\times274}{100} = 137u$

99.(c) KNO_3 is a salt of strong acid (HNO_3) and strong base (KOH). Hence, it does not undergo hydrolysis.

100. (d) According to Sedef rule, in dehydrohalogenation reaction, the preferred product is that alkene which has greater number of alkyl groups attached to the doubly bonded carbon atoms.

MATHEMATICS

101. (c) The equation of plane through A(7,8,6) is

$a(x-7) + b(y-8) + c(z-6) = 0$ …(i)

Since, the required plane is parallel to XY-plane

$\therefore$ Direction ratios are 0,0,1

$\therefore$ Equation of plane is

$0(x-7) + 0(y-0) + 1(z-6) = 0$

$\Rightarrow z - 6 = 0$

$\Rightarrow z = 6$

102. (c) 6 boys can seat at a round table in $(6-1)! = 5!$ ways. Now, no two girls sit together, i.e., 5 girls are to be arranged in 6 empty seats between two consecutive boys and number of such arrangements will be ${}^6P_5 = 6!$

Hence, required number of ways

$= 5! \times 6!$

$= 120 \times 720$

$= 86400$

103. (d) $[2\bar{p} - 3\bar{r} \quad \bar{q} \quad \bar{s}] + [3\bar{p} + 2\bar{q} \quad \bar{r} \quad \bar{s}]$

$= m[\bar{p} \quad \bar{r} \quad \bar{s}] + n[\bar{q} \quad \bar{r} \quad \bar{s}] + t[\bar{p} \quad \bar{q} \quad \bar{s}]$

$[2\bar{p} \quad \bar{q} \quad \bar{s}] - [3\bar{r} \quad \bar{q} \quad \bar{s}] + [3\bar{p} \quad \bar{r} \quad \bar{s}]$

$+ [2\bar{p} \quad \bar{r} \quad \bar{s}]$

$= m[\bar{p} \quad \bar{r} \quad \bar{s}] + n[\bar{q} \quad \bar{r} \quad \bar{s}] + t[\bar{p} \quad \bar{q} \quad \bar{s}]$

$3[\bar{p} \quad \bar{r} \quad \bar{s}] + 3[\bar{q} \quad \bar{r} \quad \bar{s}] + 2[\bar{q} \quad \bar{r} \quad \bar{s}]$

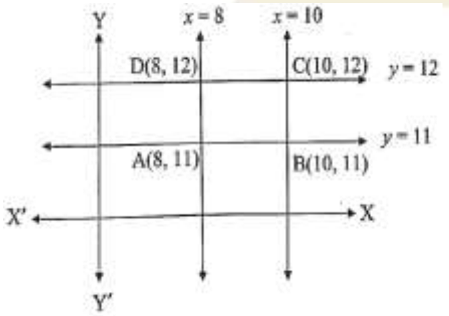
$+ [2\bar{p} \quad \bar{q} \quad \bar{s}]$

$= m[\bar{p} \quad \bar{r} \quad \bar{s}] + n[\bar{q} \quad \bar{r} \quad \bar{s}] + t[\bar{p} \quad \bar{q} \quad \bar{s}]$
$3[\bar{p} \quad \bar{r} \quad \bar{s}] + 5[\bar{q} \quad \bar{r} \quad \bar{s}] + 2[\bar{p} \quad \bar{q} \quad \bar{s}]$
$= m[\bar{p} \quad \bar{r} \quad \bar{s}] + n[\bar{q} \quad \bar{r} \quad \bar{s}] + t[\bar{p} \quad \bar{q} \quad \bar{s}]$
Comparing above equations, we get $m = 3, n = 5, t = 2$

104. **(c)** Let $P \equiv (-3,2,3)$
Equation of line passing through (4, 6, -2) and having d.r.'s $-1,2,3$ are
$$\frac{x-4}{-1} = \frac{y-6}{2} = \frac{z+2}{3}$$
Let $\frac{x-4}{-1} = \frac{y-6}{2} = \frac{z+2}{3} = \lambda$
$\Rightarrow x = 4 - \lambda, y = 2\lambda + 6, z = 3\lambda - 2$
Co-ordinates of point
$Q = (4 - \lambda, 2\lambda + 6, 3\lambda - 2)$
$\therefore$ the d.r's of PQ are
$(4 - \lambda - (-3), 6 + 2\lambda - 2, -2 + 3\lambda - 3)$
$\equiv 7 - \lambda, 4 + 2\lambda, -5 + 3\lambda$
Since PQ is perpendicular to given lines
$\therefore -1(7 - \lambda) + (2)(4 + 2\lambda) + 3(-5 + 3\lambda) = 0$
$\Rightarrow -7 + \lambda + 8 + 4\lambda - 15 + 9\lambda = 0$
$\Rightarrow \lambda = 1$
$\therefore$ Co-ordinates of Q are $(3,8,1)$
$\therefore PQ = \sqrt{(3-(-3))^2 + (8-2)^2 + (1-3)^2}$
$= \sqrt{76} = 2\sqrt{19}$

105. **(c)** The equation of a plane passing through
$(1, -2, 1)$ is
$a(x-1) + b(y+2) + c(z-1) = 0$ ...(i)
Plane (i) is perpendicular to planes
$2x - 2y + z = 0$ and $x - y + 2z = 4$.
$\therefore 2a - 2b + c = 0$, and ...(ii)
$a - b + 2c = 0$ ...(iii)
Solving (ii) and (iii), we get
$a = -3, b = -3, c = 0$
Substituting the values of a, b, c in equation (i), we get
$x + y + 1 = 0$
$\therefore$ The distance of this plane from $(1,2,2)$ is
$$d = \left|\frac{1+2+1}{\sqrt{1+1}}\right| = 2\sqrt{2}$$

106. **(b)**



Equation of diagonal AC
$$\frac{y - y_1}{y_1 - y_2} = \frac{x - x_1}{x_1 - x_2}$$
$$\Rightarrow \frac{y - 11}{11 - 12} = \frac{x - 8}{8 - 10}$$

$\Rightarrow \dfrac{y-11}{-1}=\dfrac{x-8}{-2}$

$\Rightarrow x-2y+14=0 \qquad \ldots(\text{i})$

Equation of diagonal BD is

$\dfrac{y-y_1}{y_1-y_2}=\dfrac{x-x_1}{x_1-x_2}$

$\Rightarrow \dfrac{y-11}{11-12}=\dfrac{x-10}{10-8}$

$\Rightarrow \dfrac{y-11}{-1}=\dfrac{x-10}{2}$

$\Rightarrow x+2y-32=0 \qquad \ldots(\text{ii})$

Solving equations (i) and (ii) we get

$x=9, y=\dfrac{23}{2}$

107. **(d)** Number of tickets $=9$

Number of odd numbered tickets $=5$

Number of even numbered tickets $=4$

Required probability

$=P\{\text{ odd, even, odd }\}+P\{\text{ even, odd, even }\}$

$=\dfrac{{}^5C_1}{{}^9C_1}\times\dfrac{{}^4C_1}{{}^8C_1}\times\dfrac{{}^4C_1}{{}^7C_1}+\dfrac{{}^4C_1}{{}^9C_1}\times\dfrac{{}^5C_1}{{}^8C_1}\times\dfrac{{}^3C_1}{{}^7C_1}$

$=\dfrac{5}{9}\times\dfrac{4}{8}\times\dfrac{4}{7}+\dfrac{4}{9}\times\dfrac{5}{8}\times\dfrac{3}{7}=\dfrac{80}{504}+\dfrac{60}{504}=\dfrac{140}{504}=\dfrac{5}{18}$

108. **(a)** $\lim\limits_{x\to 3}\frac{(84-x)^{\frac{1}{4}}-3}{x-3}$

Let $(84-x)^{\frac{1}{4}}=\text{t}$

$\Rightarrow (84-x)=\text{t}^4$

$\Rightarrow x=84-\text{t}^4$

As $x\to 3$ then $\text{t}\to 3$

$\therefore \lim\limits_{x\to 3}\dfrac{(84-x)^{\frac{1}{4}}-3}{x-3}=\lim\limits_{x\to 3}\dfrac{t-3}{84-t^4-3}$

$=\lim\limits_{t\to 3}\dfrac{t-3}{81-t^4}$

$=\lim\limits_{t\to 0}\dfrac{-1}{\dfrac{(t^4-81)}{t-3}}$

$=\dfrac{-1}{4\times(3)^3}=\dfrac{-1}{108}$

109. **(d)**

P	q	r	p → q	~ q	(p → q) ∧~ q	[(p → q) ∧ ~ q] → r
T	T	T	T	F	F	T
T	T	F	T	F	F	T
T	F	T	F	T	F	T
T	F	F	F	T	F	T
F	T	T	T	F	F	T

F	T	F	T	F	F	T
F	F	T	T	T	T	T
F	F	F	T	T	T	F

$\therefore$ $[(p \rightarrow q) \wedge \sim q] \rightarrow$ **r** is a tautology when all the entries in the last column are T, which is only possible when r $\equiv \sim$ q

110. **(d)** Let $\frac{\alpha+\beta}{2} = \theta, \frac{\alpha-\beta}{2} = \phi$

$\Rightarrow \alpha + \beta = 2\theta, \quad \ldots$(i)

$\alpha - \beta = 2\phi \quad \ldots$(ii)

Solving (i) and (ii) we get

$\alpha = \theta + \phi, \beta = \theta - \phi$

Now,

$\sin(\theta + \phi) = \sin\theta\cos\phi + \cos\theta\sin\phi$

$\sin(\theta - \phi) = \sin\theta\cos\phi - \cos\theta\sin\phi$

Given equation is

$3\sin\alpha = 5\sin\beta$

$\Rightarrow 3\sin(\theta + \phi) = 5\sin(\theta - \phi)$

$\Rightarrow 3\sin\theta\cos\phi + 3\cos\theta\sin\phi$

$= 5\sin\theta\cos\phi - 5\cos\theta\sin\phi$

$\Rightarrow -2\sin\theta\cos\phi + 8\cos\theta\sin\phi = 0$

$\Rightarrow 4\cos\theta\sin\phi = \sin\theta\cos\phi$

$\Rightarrow \frac{\sin\phi}{\cos\phi} = \frac{\sin\theta}{4\cos\theta}$

$\Rightarrow \tan\phi = \frac{1}{4}\tan\theta$

$\Rightarrow \frac{\tan\theta}{\tan\phi} = 4$

$\Rightarrow \frac{\tan\left(\frac{\alpha+\beta}{2}\right)}{\tan\left(\frac{\alpha-\beta}{2}\right)} = 4 \quad \ldots$[From (i) and (ii)]

111. **(a)** Let $I = \int \frac{dx}{2e^{2x}+3e^{x}+1}$

$\therefore$ Let $e^x = t$

$e^x\, dx = dt$

$\Rightarrow dx = \frac{dt}{e^x} = \frac{dt}{t}$

$\therefore\ I = \int \frac{dt}{t(2t^2 + 3t + 1)}$

$= \int \frac{dt}{t(2t + 1)(t + 1)}$

Let $\frac{1}{t(2t+1)(t+1)} = \frac{A}{t} + \frac{B}{(2t+1)} + \frac{C}{t+1}$

$\Rightarrow I = A(2t + 1)(t + 1) + Bt(t + 1) + Ct(2t + 1) \quad \ldots$(i)

Put $t = -1$ in (i), we get

$C = 1$

Put $t = \frac{-1}{2}$ in (i), we get

$B = -4$

Put $t = 0$ in (i), we get

$A = 1$

$\therefore \int \frac{dt}{t(2t + 1)(t + 1)}$

$= \int \frac{1}{t}dt - 4\int \frac{1}{2t + 1}dt + \int \frac{1}{t + 1}dt$

$= \log t - \frac{1}{2} \times 4\log|2t+1| + \log|t+1| + c$
$= \log e^x + \log|e^x + 1| - 2\log|2e^x + 1| + c$
$= x + \log(e^x + 1) - 2\log(2e^x + 1) + c$

112. **(c)** Let $I = \int \frac{e^{2030\log x} - e^{2029\log x}}{e^{2028\log x} - e^{2027\log x}}\, dx$

$= \int \frac{e^{\log x^{2030}} - e^{\log x^{2029}}}{e^{\log x^{2028}} - e^{\log x^{2027}}}\, dx$
$= \int \frac{x^{2030} - x^{2029}}{x^{2028} - x^{2027}}\, dx$
$= \int \frac{x^{2027}(x^3 - x^2)}{x^{2027}(x-1)} dx$
$= \int \frac{x^2(x-1)}{(x-1)} dx$
$= \int x^2\, dx$
$= \frac{x^3}{3} + c$

113. **(b)** Let $I = \int_1^4 \log[x] dx$

$\int_1^4 \log[x]\, dx = \int_1^2 0 dx + \int_2^3 \log 2 dx + \int_3^4 \log 3 dx$
$= 0 + \log 2[3-2] + \log 3[4-3]$
$= \log 2 + \log 3$
$= \log(2 \times 3) = \log 6$

114. **(a)** Given equation of parabola

$x^2 = 4y$

The equation of tangent to parabola $x^2 = 4y$ is

$x = my + \frac{a}{m}$
$\Rightarrow x = my + \frac{1}{m} \qquad \cdots [\because a = 1]$
$\Rightarrow mx = m^2 y + 1$
$\Rightarrow \frac{dy}{dx} x = \left(\frac{dy}{dx}\right)^2 y + 1$

The order and degree of above differential equation is 1 and 2 .

115. **(c)** Since, $P(0) + P(1) + P(2) + P(3) + P(4) = 12k + k + 2k + 4k + k = 1$

$\therefore\ k = \frac{1}{10} \quad \ldots(i)$
$a = P(X < 3) = P(X = 0) + P(X = 1) + P(X = 2)$
$= 2k + k + 2k$
$= 5k$
$= \frac{5}{10} \qquad \ldots[\text{From (i)}]$
$= \frac{1}{2}$
$\cdot\, b = P(2 \le X < 4) = P(X = 2) + P(X = 3)$
$= 2k + 4k$
$= 6k$
$= \frac{6}{10} \ \ldots[\text{ From (i) }]$
$\therefore\ a = \frac{1}{2}, b = \frac{3}{5}$
$\Rightarrow a < b$

116. **(c)** $f(x) = \int_0^\infty \frac{k}{x^2+1}\, dx = 1$

$\Rightarrow k\int_0^\infty \frac{1}{x^2+1}\, dx = 1$
$\Rightarrow k[\tan^{-1} x]_0^\infty = 1$
$\Rightarrow k[\tan^{-1}\infty - \tan^{-1} 0] = 1$

$\Rightarrow k\left[\frac{\pi}{2}-0\right]=1$

$\Rightarrow k=\frac{2}{\pi}$

The c.d. f of x is

$$f(x)=\int_0^x \frac{\frac{2}{\pi}}{x^2+1}\,dx=\frac{2}{\pi}\tan^{-1}x$$

117. **(d)** $\left(\frac{2+\sin x}{1+y}\right)\frac{dy}{dx}=-\cos x$

$$\Rightarrow \frac{dy}{1+y}=\left(\frac{-\cos x}{2+\sin x}\right)dx$$

Integrating on both sides, we get

$$\int\frac{dy}{1+y}=-\int\frac{\cos x}{2+\sin x}\,dx$$

$\Rightarrow \log(1+y)+\log(2+\sin x)=\log c$

$\Rightarrow (1+y)(2+\sin x)=c \quad \ldots(i)$

Since $y(0)=2$, i.e. $y=2$ when $x=0$

$(1+2)(2+\sin 0)=c\Rightarrow c=6$

$\therefore\ (y+1)(2+\sin x)=6$ ...[From (i)]

$\Rightarrow y=\frac{6}{2+\sin x}-1$

$\therefore\ y\left(\frac{\pi}{2}\right)=\frac{6}{2+\sin\frac{\pi}{2}}-1$

$=\frac{6}{2+1}-1$

$=2-1$

$=1$

118. **(a)** Principal increases at a rate of $x\%$ per year

$$\therefore \frac{dP}{dt}=\frac{x}{100}\times P$$

$$\frac{dP}{p}=\frac{x}{100}dt$$

Integrating on both sides, we get

$\log P=\frac{x}{100}t+c \quad \ldots(i)$

When $t=0, P=100$

$\Rightarrow \log 100=c$

$\Rightarrow$ Also, when $t=10, P=200$

From (i)

$$\log 200=\frac{x}{100}\times 10+\log 100$$

$$\Rightarrow \log 200-\log 100=\frac{x}{10}$$

$$\Rightarrow \frac{x}{10}=\log 2$$

$\Rightarrow x=10\log 2$

$\Rightarrow x=6.931\%$

119. **(c)** Given that,

$3P(X=0)=P(X=1)$

$\Rightarrow 3\times {}^{33}C_0p^0q^{33}={}^{33}C_1p^1q^{32}$

$\Rightarrow 3\times 1\times 1\times q^{33}=33\times p\times q^{32}$

$\Rightarrow 3q=33p$

$$\Rightarrow p=\frac{1}{11}q$$

Now, $p+q=1$

$\Rightarrow \dfrac{1}{11}q + q = 1$

$\Rightarrow 12q = 11$

$\Rightarrow \mathrm{q} = \dfrac{11}{12}$

$\Rightarrow \mathrm{p} = \dfrac{1}{12}$

Now, variance $= \mathrm{npq} = 33 \times \frac{1}{12} \times \frac{11}{12} = \frac{121}{48}$

120. **(d)** $x^2 + y^2 - 6x = 0$

$\mathrm{C_1}(3,0), \mathrm{r_1} = 3$

$x^2 + y^2 + 6x + 2y + 1 = 0$

$\mathrm{C_2} = (-3,-1), \mathrm{r_2} = \sqrt{9+1-1} = 3$

$\mathrm{C_1C_2} = \sqrt{(-3-3)^2 + (-1-0)^2} = \sqrt{37}$

$\therefore\ \mathrm{C_1C_2} > \mathrm{r_1} + \mathrm{r_2}$

$\therefore$ Number of tangents $= 4$

121. **(a)** $\tan^{-1}\left(\frac{1}{3}\right) + \tan^{-1}\left(\frac{2}{9}\right) + \ldots + \left(\frac{2^{n-1}}{1+2^{2n-1}}\right) + \ldots$

$= \tan^{-1}\left(\frac{2-1}{1+1\times 2}\right) + \tan^{-1}\left(\frac{4-2}{1+4\times 2}\right)$

$+ \ldots + \tan^{-1}\left(\frac{2^n - 2^{n-1}}{1+2^n 2^{n-1}}\right)$

$= \tan^{-1} 2 - \tan^{-1} 1$

$+ \tan^{-1} 4 - \tan^{-1} 2$

$+ \cdots + (\tan^{-1} 2^{n-1} - \tan^{-1} 2^{n-2})$

$+ (\tan^{-1} 2^n - \tan^{-1} 2^{n-1})$

$= \tan^{-1} 2^n - \tan^{-1} 1$

$= \tan^{-1} 2^n - \frac{\pi}{4}$

Infinite sum $= \lim\limits_{n\to\infty} \tan^{-1}(2^n) - \frac{\pi}{4}$

$= \tan^{-1}(\infty) - \dfrac{\pi}{4}$

$= \dfrac{\pi}{2} - \dfrac{\pi}{4}$

$= \dfrac{\pi}{4}$

122. **(c)** Let the common multiple be x

$\therefore$ Sides of triangle are $5x, 12x, 13x$

$\cos C = \frac{a^2+b^2-c^2}{2ab}$

$= \frac{(5x)^2+(12x)^2-(13x)^2}{2\times 5x \times 12x}$

$= \frac{25x^2+144x^2-169x^2}{120x^2}$

$= 0$

$\Rightarrow C = \frac{\pi}{2}$

$\therefore$ Area of right angled triangle

$= \frac{1}{2} \times$ product of sides containing right angle

$\Rightarrow 270 = \dfrac{1}{2} \times 5x \times 12x$

$\Rightarrow x^2 = \dfrac{270 \times 2}{5 \times 12}$

$\Rightarrow x^2 = 9$

$\Rightarrow x = 3$

$\therefore$ The sides of triangle are 15,36 and 39

123. **(c)** $4\sin^{-1} + \cos^{-1} x = \pi$

$\Rightarrow 3\sin^{-1}x + \sin^{-1}x + \cos^{-1}x = \pi$

$\Rightarrow 3\sin^{-1}x + \frac{\pi}{2} = \pi$

$\Rightarrow 3\sin^{-1}x = \frac{\pi}{2}$

$\Rightarrow x = \sin\frac{\pi}{6}$

$\Rightarrow x = \frac{1}{2}$

124. **(a)** $\int_{i}^{e} \frac{e^x}{x}(1 + x\log x)dx = \int_{1}^{e} e^x\left(\frac{1}{x} + \log x\right)dx$

$= [e^x \cdot \log x]_1^e$

$= e^e$

125. **(c)** $A_1 = \int_0^{\pi/3}\cos x\, dx = [\sin x]_0^{\frac{\pi}{3}} = \frac{\sqrt{3}}{2}$

$A_2 = \int_0^{\pi/3}\cos 2x\, dx = \frac{[\sin x]_0^{\frac{\pi}{3}}}{2} = \frac{\sqrt{3}}{4}$

$A_1 : A_2 = 2:1$

126. **(b)** Given equation is

$x\frac{d^2y}{dx^2} = 1$

$\Rightarrow \frac{d^2y}{dx^2} = \frac{1}{x}$

Integration on both sides, we get

$\frac{dy}{dx} = \log x + c$ …(i)

Given $\frac{dy}{dx} = 0$ at $x = 1$

$\therefore\ 0 = \log 1 + c$ ...[From (i)]

$\Rightarrow c = 0$ …(ii)

Again integrating (i), we get

$y = x\log x - x + cx + k$

$y = x\log x - x + k$ ...(iii)[From (ii)]

At $x = y = 1$

$1 = (1)\log(1) - 1 + k$

$\Rightarrow k = 2$

$\therefore$ From (iii)

$y = x\log x - x + 2.$

127. **(a)** Volume of tetrahedron $= \frac{1}{6}[\bar{a}\quad \bar{b}\quad \bar{c}]$

$\Rightarrow [\bar{a}\quad \bar{b}\quad \bar{c}] = 72$

$\Rightarrow \bar{a}\cdot(\bar{b}\times\bar{c}) = 72$ …(i)

Scalar projection of $\bar{a}$ on $\bar{b}\times\bar{c} = \frac{\bar{a}\cdot(b\times c)}{|\bar{b}\times\bar{c}|}$

$4 = \frac{\bar{a}\cdot(\bar{b}\times\bar{c})}{|\bar{b}\times\bar{c}|}$

$\Rightarrow 4 = \frac{72}{|\bar{b}\times\bar{c}|}$ ...[From (i)]

$\Rightarrow |\bar{b}\times\bar{c}| = \frac{72}{4} = 18$

128. **(b)** Given,

$\left(\sqrt{x^2+y^2}\right)^2 + \left(\sqrt{y^2+z^2}\right)^2 + \left(\sqrt{z^2+x^2}\right)^2 = 242$

i.e., $x^2 + y^2 + z^2 = 121$

$\therefore$ The distance from origin $= \sqrt{x^2+y^2+z^2}$

$= \sqrt{121}$
$= 11$

129. **(a)** $\overline{\text{AB}} = x\hat{\imath} - y\hat{\jmath}$

$\overline{\text{AC}} = x\hat{\imath} - \text{z}\hat{\text{k}}$

$\overline{\text{AD}} = (x-1)\hat{\imath} - \hat{\jmath} - \hat{\text{k}}$

A, B, C, D are coplanar.

$\Rightarrow [\overline{\text{AB}} \quad \overline{\text{AC}} \quad \overline{\text{AD}}] = 0$

$$\Rightarrow \begin{vmatrix} x & -y & 0 \\ x & 0 & -z \\ x-1 & -1 & -1 \end{vmatrix} = 0$$

$\Rightarrow x(-\text{z}) + y(-x + \text{z}(x-1)) = 0$

$\Rightarrow -x\text{z} - xy + xyz - \text{zy} = 0$

$\Rightarrow \dfrac{1}{x} + \dfrac{1}{y} + \dfrac{1}{z} = 1$

130. **(d)** $\frac{x-3}{-2} = \frac{y-\frac{2}{5}}{\frac{3\lambda+1}{5}} = \frac{z-5}{-1}$ and $\frac{x+2}{-1} = \frac{y-\frac{1}{3}}{-\frac{7}{3}} = \frac{z-4}{-2\mu}$

As these lines are at right angles, we get

$(-2)(-1) + \left(\frac{3\lambda+1}{5}\right)\left(\frac{-7}{3}\right) + (-1)(-2\mu) = 0$

$\Rightarrow 2 - \left(\frac{21\lambda+7}{15}\right) + 2\mu = 0$

$\Rightarrow 30 - 21\lambda - 7 + 30\mu 0$

$\Rightarrow 7\lambda - 10\mu = \dfrac{23}{3}$

131. **(c)** The d.r.s of line and plane are 1,2,2 and $2, -1, \sqrt{\lambda}$

$\therefore \sin\theta = \frac{2(1)+(-1)(2)+(\sqrt{\lambda})(2)}{\sqrt{1^2+2^2+2^2}\cdot\sqrt{2^2+(-1)^2+(\sqrt{\lambda})^2}}$

$\Rightarrow \frac{1}{3} = \frac{2\sqrt{\lambda}}{3\cdot\sqrt{5+\lambda}}$

$\Rightarrow \sqrt{5+\lambda} = 2\sqrt{\lambda}.$

$\Rightarrow \lambda = \frac{5}{3}$

$\Rightarrow \lambda + 1 = \dfrac{8}{3}$

132. **(a)** Feasible region lies on non-origin sides of $2x + 3y \geq 12$ and $y \geq 3$ and on origin side of $-x + y \leq 3$ and $x \leq 4$ in 1st quadrant.

133. **(c)** $y = ax^{n+1} + bx^{-n}$

Differentiating w.r.t. x, we get

$\dfrac{\text{dy}}{\text{dx}} = \text{a}(\text{n}+1)x^{\text{n}+1-1} + \text{b}(-\text{n})x^{-\text{n}-1}$

$\Rightarrow \dfrac{\text{dy}}{\text{d}x} = \text{a}(\text{n}+1)x^{\text{n}} - \text{bn}x^{-\text{n}-1}$

Again differentiating w.r.t. x, we get

$\dfrac{d^2y}{dx^2} = a(n+1)nx^{n-1} - bn(-n-1)x^{-n-1-1}$

$= a(n+1)nx^{n-1} + bn(n+1)x^{-n-2}$

$= n(n+1)\dfrac{ax^n}{x} + n(n+1)b\dfrac{x^{-n}}{x^2}$

$\dfrac{d^2y}{dx^2} = \dfrac{n(n+1)}{x^2}[ax^{n+1} + bx^{-n}]$

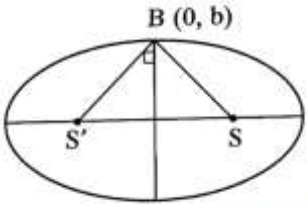
$\therefore x^2\dfrac{d^2y}{dx^2} = n(n+1)y$

134. **(b)** $\text{f}(x) = (\cos x + \text{i}\sin x)(\cos 3x + \text{i}\sin 3x)$

$\ldots[\cos(2n-1)x + i\sin(2n-1)x]$
$= e^{ix}\cdot e^{i3x}\ldots e^{i(2n-1)x}$
$= e^{i(x+3x+\cdots(2n-1)x)}$
$= e^{i(n^2x)}$ ... [Sum of n odd terms $= n^2$]
$f(x) = e^{in2x}$ ...(i)
Differentiating w.r.t. x, we get
$f'(x) = in^2e^{in^2x}$
Again differentiating w.r.t. x, we get
$f''(x) = i^2n^2 \times n^2 \times e^{i^2x}$
$= -1 \times n^4 \times e^{i^2x}$
$= -n^4 f(x)$ ...(ii)

135. **(c)** $p(t) = 1000 + \frac{1000t}{100+t^2}$
$\Rightarrow p'(t) = \frac{(100+t^2)(1000)-1000t(2t)}{(100+t^2)^2}$
$= \frac{1000(100-t^2)}{(100+t^2)}$
For maximum
$p'(t) = 0$
$\Rightarrow \frac{1000(100-t^2)}{(100+t^2)^2} = 0$
$\Rightarrow t = \pm 10$
Also,
$$\mathbf{p''(t)} = 1000\frac{\begin{bmatrix}(100+t^2)^2\cdot(-2t)\\-(100-t^2)2(100+t^2)(2t)\end{bmatrix}}{(100+t^2)^2}$$
$$\mathbf{p''}(10) = \frac{1000[(40000)(-20)-0]}{40000}$$
$= -2000 < 0$
$$\therefore\ p(\max)\text{ at }(t=10) = 1000 + \frac{1000(10)}{100+(10)^2}$$
$$= 1000 + \frac{10000}{200} = 1050$$

136. **(b)**



$S = (ae, 0), S' = (-ae, 0)$ and $b = ae$
Now, $a^2e^2 = a^2(1-e^2)$
$$\Rightarrow e = \frac{1}{\sqrt{2}}$$

137. **(b)** Since $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$
$\therefore\ \cos^2 45° + \cos^2 60° + \cos^2\gamma = 1$
$$\Rightarrow \cos^2\gamma = 1 - \frac{1}{2} - \frac{1}{4} = \frac{1}{4}$$
$$\Rightarrow \cos\gamma = \pm\frac{1}{2}$$
$\Rightarrow \gamma = 60°$ or $120°$

138. **(b)** Let $y = \tan^{-1}\left(\sqrt{1+x^2}-1\right)$
Differentiating w.r.t. x, we get
$$\frac{dy}{dx} = \frac{1}{1+\left(\sqrt{1+x^2}-1\right)^2} \times \frac{d}{dx}\left(\sqrt{1+x^2}-1\right)$$

$= \frac{1}{1+2+x^2-2\sqrt{1+x^2}} \times \frac{1}{2\sqrt{1+x^2}} \times 2x$

$= \frac{x}{(\sqrt{1+x^2})(x^2-2\sqrt{1+x^2}+3)}$

139. **(c)** Given, the rate of increasing the radius

$= \frac{dr}{dt}$

$= 2.1\ cm/sec.$

Area $= A = \pi r^2$

$\therefore \frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

$\Rightarrow \frac{dA}{dt} = 2\pi \times 10 \times 2.1 \ldots [\because r = 10\ cm]$

$= 2 \times \frac{22}{7} \times 10 \times 2.1$

$= 132\ cm^2/$ second

140. **(a)** Solving $xy = 6$ and $x^2y = 12$, we get $x = 2, y = 3$

$\therefore$ Consider, $xy = 6$

$\Rightarrow y = \frac{6}{x}$

$\Rightarrow \frac{dy}{dx} = \frac{-6}{x^2}$

$m_1 = \left(\frac{dy}{dx}\right)_{at x=2} = \frac{-6}{4} = \frac{-3}{2}$

Now, $x^2y = 12$

$\Rightarrow y = \frac{12}{x^2}$

$\Rightarrow \frac{dy}{dx} = \frac{12(-2)}{x^3} = \frac{-24}{x^3}$

$m_2 = \frac{dy}{dx_{at x=2}} = \frac{-24}{8} = -3$

Using, $\tan\theta = \left|\frac{m_1-m_2}{1+m_1 \cdot m_2}\right|$

$\Rightarrow \theta = \tan^{-1}\left|\frac{m_1-m_2}{1+m_1 \cdot m_2}\right|$

$= \tan^{-1}\left|\frac{\frac{-3}{2}+3}{1+\left(\frac{-3}{2}\right)(-3)}\right|$

$= \tan^{-1}\left(\frac{3}{11}\right)$

141. **(c)** $f(x) = x(x-1)(x-2)$

$f\left(\frac{1}{2}\right) = \frac{1}{2}\left(\frac{1}{2}-1\right)\left(\frac{1}{2}-2\right)$

$= \frac{3}{8}$

$f(0) = 0$

$\therefore$ Using LMVT

$f'(c) = \frac{f\left(\frac{1}{2}\right)-f(0)}{\frac{1}{2}-0}$

$= \frac{\frac{3}{8}-0}{\frac{1}{2}}$

$= \frac{3}{4}$

Now, $f(x) = x(x-1)(x-2)$

$= x^3 - 3x^2 + 2x$

$f'(x) = 3x^2 - 6x + 2$

$f'(c) = 3c^2 + 6c + 2$

$\Rightarrow 3c^2 - 6c + 2 = \frac{3}{4}$

$\Rightarrow 3c^2 - 6c + \frac{5}{4} = 0$

$\Rightarrow c = \frac{6\pm\sqrt{36-15}}{6} = \frac{6\pm\sqrt{21}}{6} = 1 \pm \frac{\sqrt{21}}{6}$

$c \in \left(\frac{1}{2}, 0\right)$

$\therefore\ c = 1 - \frac{\sqrt{21}}{6}$

142. **(c)** Let $I = \int \frac{\sin 2x}{(a+b\cos x)^2} dx$

$I = \int \frac{2\sin x\cos x}{(a+b\cos x)^2}\ dx$

Let $(a + b\cos x) = t$

$\cos x = \frac{t-a}{b}$

$\Rightarrow -\sin x\ dx = \frac{dt}{b}$

$\Rightarrow \sin x\ dx = \frac{-dt}{b}$

$\therefore\ I = \frac{-2}{b^2}\int \frac{t-a}{t^2} dt$

$= \frac{-2}{b^2}\left[\int \frac{1}{t} dt - \int \frac{a}{t^2} dt\right]$

$= \frac{-2}{b^2}\left[\log t - a\left(\frac{-1}{t}\right)\right] + c$

$= \frac{-2}{b^2}\left[\log(a + b\cos x) + \frac{a}{(a + b\cos x)}\right] + c$

143. **(b)** If the slopes of lines

$ax^2 + 2hxy + by^2 = 0$ are in the ratio $m_1 : m_2$ then

$(m_1 + m_2)^2 ab = 4\ m_1\ m_2\ h^2$ ...(i)

Comparing (i) with $16\ h^2 = 25ab$, we get

$(m_1 + m_2)^2 = 25$ and $m_1 m_2 = 4$

$\Rightarrow m_1 + m_2 = \pm 5$ ...(ii)

and $m_1\ m_2 = 4$

Solving (ii) and (iii), we get

$m_1 = 4$ or $m_1 = -4$

$m_2 = 1$ or $m_2 = -1$

$\therefore\ m_1 = 4\ m_2$

144. **(d)** Conjugate of $-7 + 24i$ is $-7 - 24i$

Let $\sqrt{-7 - 24i} = a + ib$

Squaring on both sides,

$-7 - 24i = a^2 + b^2 i^2 + 2abi$

$\therefore\ -7 - 24i = (a^2 - b^2) + 2abi$

$\therefore\ a^2 - b^2 = -7,$

$\therefore\ a^2 - b^2 = -7$ ...(i)

$2ab = -24$

$ab = -12$ ...(ii)

Solving (i) and (ii), we get

$a = \pm 3$ and $b = \mp 4$

$\therefore \sqrt{-7 - 24i} = \pm(3 - 4i)$

Modulus of $\pm(3 - 4i) = \sqrt{(3)^2 + (-4)^2}$

$\frac{\text{or}}{\sqrt{(-3)^2+4^2}}$

$= \sqrt{25} = 5$

145. **(a)** $x + \log_{15}(5+3^x) = x\log_{15}5 + \log_{15}24$

$\Rightarrow x\log_{15}15 + \log_{15}(5+3^x) = \log_{15}5^x + \log_{15}24$

$\Rightarrow \log_{15}15^x + \log_{15}(5+3^x) = \log_{15}5^x + \log_{15}24$

$\Rightarrow \log_{15}15^x(5+3^x) = \log_{15}(5^x \times 24)$

$\Rightarrow 15^x(5+3^x) = 5^x \times 24$

$\Rightarrow 3^x \times 5^x(5+3^x) = 5^x \times 24$

$\Rightarrow 3^x(5+3^x) = 24$

$\Rightarrow 3^{2x} + 5 \times 3^x - 24 = 0$

$\Rightarrow 3^x = 3$ or $3^x = -8$

$3^x \neq -8$

$\therefore\ 3^x = 3$

$\therefore\ x = 1$

146. **(b)** Since $\mathrm{f}(x)$ is continuous at $x = 0$.

$\therefore\ f(0) = \lim_{x\to 0^-} f(x)$

$\frac{2}{\log 2} = \lim_{x\to 0^-} \frac{3\sin x + 5\tan x}{a^x - 1}$

$= \lim_{x\to 0} \frac{\frac{3\sin x+5\tan x}{x}}{\frac{a^x-1}{x}}$

$= \frac{3+5}{\log a}$

$= \frac{8}{\log a}$

$\Rightarrow \frac{2}{\log 2}$

$\Rightarrow \log a = \frac{8\times\log 2}{2}$

$\Rightarrow \log a = \log 2^4$

$\Rightarrow a = 16$ a

Now, $\mathrm{f}(0) = \lim_{x\to 0^+} \mathrm{f}(x)$

$\frac{2}{\log 2} = \lim_{x\to 0} \frac{8x+2x\cos x}{b^x-1}$

$= \lim_{x\to 0} \frac{\frac{8x+2x\cos x}{x}}{\frac{b^x-1}{x}}$

$= \frac{8+2(\cos 0)}{\log b}$

$\Rightarrow \frac{2}{\log 2} = \frac{10}{\log b}$

$lo\,g\,b = \frac{10\times lo\,g\,2}{2}$

$\Rightarrow lo\,g\,b = 5lo\,g\,2$

$\Rightarrow b = 2^5 = 32$

$\therefore\ a = 16$ and $b = 32$

147. **(b)** Let $a = 6 + \sqrt{12}, b = \sqrt{48}, c = \sqrt{24}$

$co\,s\,C = \frac{\mathrm{a}^2+\mathrm{b}^2-\mathrm{c}^2}{2\mathrm{ab}}$

$= \frac{(6+\sqrt{12})^2+(\sqrt{48})^2-(\sqrt{24})^2}{2\times(6+\sqrt{12})\times\sqrt{48}}$

$= \frac{48+24\sqrt{3}+48-24}{4(3+\sqrt{3})\times 4\sqrt{3}} = \frac{\sqrt{3}}{2}$

$\therefore\ \mathrm{C} = \cos^{-1}\frac{\sqrt{3}}{2} = \frac{\pi}{6}$.

148. **(c)** $\mathrm{p}: \forall\, \mathrm{n} \in \mathrm{N}, 10\mathrm{n} - 3$ is prime number, when n is not divisible by 3 .

For $\mathrm{n} = 1, 10\mathrm{n} - 3 = 7$, which is prime.

For $\mathrm{n} = 2, 10\mathrm{n} - 3 = 17$, which is prime

For n $= 4, 10n - 3 = 37$, which is prime
For n $= 5, 10n - 3 = 47$ which is prime
For n $= 8, 10n - 3 = 77$ which is composite
$\therefore$ Statement does not hold for all n .
$\therefore$ p is false.
q: $l = \frac{2}{\sqrt{3}}, m = \frac{-2}{\sqrt{3}}, n = \frac{-1}{\sqrt{3}}$
$l^2 + m^2 + n^2 = \left(\frac{2}{\sqrt{3}}\right)^2 + \left(\frac{-2}{\sqrt{3}}\right)^2 + \left(\frac{-1}{\sqrt{3}}\right)^2$
$= \frac{4}{3} + \frac{4}{3} + \frac{1}{3}$
$= 3 \neq 1$
$\therefore \frac{2}{\sqrt{3}}, \frac{-2}{\sqrt{3}}, \frac{-1}{\sqrt{3}}$ cannot be direction cosines of a line.
$\therefore$ q is false.
r: $\sin x$ is an increasing in interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
$f(x) = \sin x$
$f'(x) = \cos x$
$\cos x \geq 0$ in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
$\therefore$ $\sin x$ is increasing in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
$\therefore$ r is true.
From all options, $(\sim p \vee q) \wedge r \equiv (T \vee F) \wedge T \equiv T$

149. **(a)** $A = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$A_{21} = (-1)^{2+1} M_{21} = (-1)^3 \begin{vmatrix} \sin\theta & 0 \\ 0 & 1 \end{vmatrix} = -\sin\theta$
$A_{22} = (-1)^{2+2} M_{22} = (-1)^4 \begin{vmatrix} \cos\theta & 0 \\ 0 & 1 \end{vmatrix} = \cos\theta$
$A_{23} = (-1)^{2+3} M_{23} = (-1)^{2+3} \begin{vmatrix} \cos\theta & \sin\theta \\ 0 & 0 \end{vmatrix} = 0$
$\therefore a_{21} A_{21} + a_{22} A_{22} + a_{23} A_{23}$
$= -\sin\theta(-\sin\theta) + \cos\theta\cos\theta + 0$
$= \sin^2\theta + \cos^2\theta$
$= 1$

150. **(a)** Let $\frac{b+c}{11} = \frac{c+a}{12} = \frac{a+b}{13} = k$
$\therefore b + c = 11k$ …(i)
$c + a = 12k$ …(ii)
and $a + b = 13k$ ……(iii)
From (i) + (ii) + (iii), $2(a + b + c) = 36k$
$\therefore a + b + c = 18k$ ……..(iv)
Now, (iv) - (i) gives, $a = 7k$
(iv) - (ii) gives, $b = 6k$
(iv) - (iii) gives, $c = 5k$
Now,
$\cos C = \frac{a^2+b^2-c^2}{2ab} = \frac{(7k)^2+(6k)^2-(5k)^2}{2\times(7k)\times(6k)}$
$= \frac{49k^2+36k^2-25k^2}{84k^2}$
$= \frac{60k^2}{84k^2}$
$= \frac{5}{7}$
$\cos B = \frac{a^2+c^2-b^2}{2ac}$
$= \frac{(7k)^2+(5k)^2-(6k)^2}{2\times7k\times5k}$

$= \frac{49k^2+25k^2-36k^2}{70k^2}$

$= \frac{38}{70} = \frac{19}{35}$

$\cos A = \frac{b^2+c^2-a^2}{2bc}$

$= \frac{(6k)^2+(5k)^2-(7k)^2}{2\times6k\times5k}$

$= \frac{36\mathrm{k}^2+25\mathrm{k}^2-49\mathrm{k}^2}{60\mathrm{k}^2}$

$= \frac{12\mathrm{k}^2}{60\mathrm{k}^2}$

$= \frac{1}{5}$

$\therefore\ \cos \mathrm{A} : \cos \mathrm{B} : \cos \mathrm{C} = \frac{1}{5} : \frac{19}{35} : \frac{5}{7}$

$= 7 : 19 : 25$