Physics

1. (b)

2. (d) The equation of displacement

$x_1 = 4\sin\left(10t + \frac{\pi}{6}\right)$

The energy of this equation

$E_1 = \frac{1}{2}m\omega_1^2 a_1^2$

$= \frac{1}{2}m \times 10 \times 10 \times 4 \times 4$

The second equation of displacement

$x_2 = 5\cos(\omega t)$

The energy of this equation

$E_2 = \frac{1}{2}m\omega_2^2 a_2^2$

$= \frac{1}{2}m\omega_2^2 \times 5 \times 5$

$\because E_1 = E_2$

$\therefore \frac{1}{2}m\omega_2^2 \times 5 \times 5$

$= \frac{1}{2}m \times 10 \times 10 \times 4 \times 4$

$\omega_2^2 = \frac{10 \times 10 \times 4 \times 4}{5 \times 5}$

$\omega_2 = 8$ unit

3. (c) (i) According to Hooke's law within the elastic limit, stress is proportional to the strain.

(ii) Shearing strain is defined as angle in radian through which a plane perpendicular to the fixed surface of the cubical body gets turned under the effect of tangential force.

(iii) The temporary loss of elastic properties because of the action of repeated alternating deforming force is called elastic fatigue.

Hence option (c) is correct.

4. (b) The excess pressure of soap bubble

$p = \frac{4T}{R}$

$h\rho g = \frac{4T}{R}$

$\therefore T = \frac{Rh\rho g}{4}$

$= \frac{1 \times 10^{-2} \times 2 \times 10^{-3} \times 0.8 \times 10^3 \times 9.8}{4}$

$= 3.92 \times 10^{-2}$ N/m

$= 0.0392$ N/m

5. (b) The equation of continuity

$A_1v_1 = A_2v_2$

$A \times 4 = \frac{2}{3}A \times v_2$

$v_2 = 6\ \text{ms}^{-1}$

From Bernoulli's theorem

$$p + \rho g h_1 + \frac{1}{2}\rho v_1^2 = p + \rho g h_2 + \frac{1}{2}\rho v_2^2$$

$$g(h_1 - h_2) = \frac{1}{2}(v_2^2 - v_1^2)$$

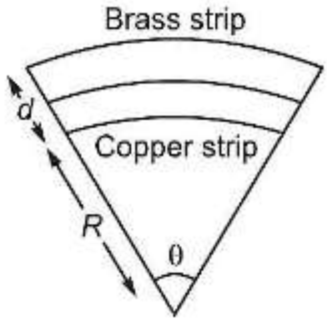
or

$$g \times h = \frac{1}{2}[(6)^2 - (4)^2][\because h_1 - h_2 = h]$$

$$10 \times h = \frac{1}{2}[36 - 16]$$

$$h = \frac{20}{20} = 1 \text{ m}$$

6. (b) Let L_0 be the initial length of each strip before heating. Length after heating will be



$$L_B = L_0(1 + \alpha_B \Delta T)$$

$$= (R + d)\theta$$

$$L_C = L_0(1 + \alpha_C \Delta T) = R\theta$$

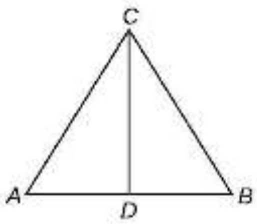
$$\frac{R + d}{R} = \frac{1 + \alpha_B \Delta T}{1 + \alpha_C \Delta T}$$

$$1 + \frac{d}{R} = 1 + (\alpha_B - \alpha_C)\Delta T$$

$$R = \frac{d}{(\alpha_B - \alpha_C)\Delta T}$$

$$R \propto \frac{1}{\Delta T}$$

7. (b)



$$CD^2 = (AC)^2 - (AD)^2 = l^2 - \left(\frac{l}{2}\right)^2$$

After small change in temperature

$$CD^2 = (AC)^2 - (AD)^2$$

$$= [l(1 + \alpha_2 t)]^2 - \left[\frac{l}{2}(1 + \alpha_1 t)\right]^2$$

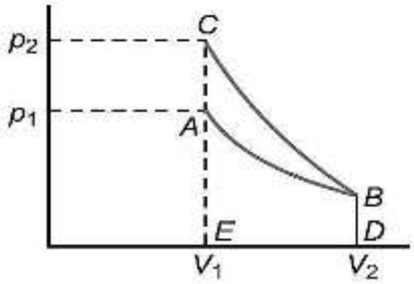
$$\therefore\ l^2 - \frac{l^2}{4} = l^2[1 + \alpha_2^2 t^2 + 2\alpha_2 t]$$

$$- \frac{l^2}{4}[1 + \alpha_1^2 t^2 + 2\alpha_1 t]$$

Neglecting $\alpha_2^2 t^2$ and $\alpha_1^2 t^2$, (because very small quantity)

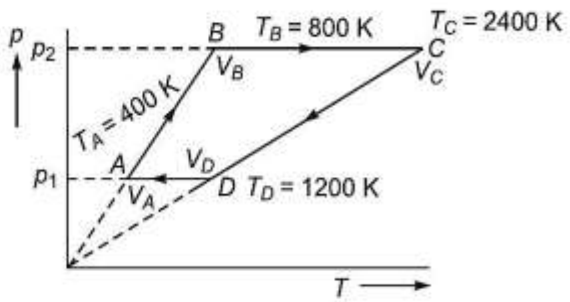
$0 = l^2(2\alpha_2 t) - \frac{l^2}{4}(2\alpha_1 t)$
or
$\alpha_1 = 4\alpha_2$

8. **(d)**



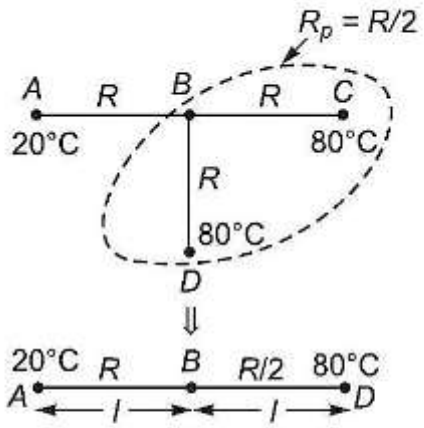
From graph it is clear that, $p_2 > p_1$.
Since, area under adiabatic process (BCED) is greater than that of isothermal process ($ABDE$). Therefore, net work done
$\because \quad W_A > W_1$
$\Rightarrow \quad W < 0$

9. **(b)**



Processes A to B and C to D are parts of straight- line graphs of form y = mx.
Also,
$p = \frac{\mu R}{V} T (\mu = 3)$
$p \propto T$
So, volume remains constant for the graphs AB and CD.
So, no work is done during processes for A to B and C to D.
$W_{AB} = W_{CD} = 0$
$W_{BC} = p_2(V_C - V_B)$
$= \mu R(T_C - T_B)$
$= 3R(2400 - 800)$
$= 3R \times 1600$
$= 4800R$
$W_{DA} = p_1(V_A - V_D)$
$= \mu R(T_A - T_D)$
$= 3R(400 - 1200)$
$= -2400R$
and
Work done in complete cycle
$W = W_{AB} + W_{BC} + W_{CD} + W_{DA}$
$= 0 + 4800R + 0 + (-2400)R$
$= 2400R$
$= 19.944\ J = 20\ kJ$

10. **(b)** Let the temperature of function θ. Since, rodBC and BD are parallel to each other (because both having the same temperature difference). Hence, given figure can be redrawn as follows (R be the resistance of each rod).



$$\because \frac{Q}{t} = \frac{(\theta_1 - \theta_2)}{R} \text{ and } \left(\frac{Q}{t}\right)_{AB} = \left(\frac{Q}{t}\right)_{BD}$$

$$\Rightarrow \frac{(80-\theta)}{R/2} = \frac{(\theta - 20^\circ)}{R} \Rightarrow \theta = 60^\circ$$

11. (d) Frequency of closed organ pipe for first harmonic $n_1 = \frac{v_1}{4l_1}$.

Frequency of open organ pipe for third harmonic

$$n_3 = \frac{3v_2}{2l_2}$$

At resonance, $n_1 = n_3$

or

$$\frac{v_1}{4l_1} = \frac{3v_2}{2l_2}$$

or

$$\frac{l_1}{l_2} = \frac{1}{6}\left(\frac{v_1}{v_2}\right)$$

$$\frac{l_1}{l_2} = \frac{1}{6}\sqrt{\frac{B}{\rho_1}} \times \sqrt{\frac{\rho_2}{B}}$$

$$= \frac{1}{6}\sqrt{\frac{\rho_2}{\rho_1}}$$

12. (d) If length of the wire between the two bridges is l, the frequency of vibration

$$n = \frac{1}{2l}\sqrt{\frac{T}{m}}$$

$$\therefore\ n \propto \frac{1}{l}$$

Therefore, $n_1 : n_2 : n_3 = \frac{1}{l_1} : \frac{1}{l_2} : \frac{1}{l_3}$

$$n_1 : n_2 : n_3 = \frac{1}{\frac{114 \times 1}{8}} : \frac{1}{\frac{114 \times 3}{8}} : \frac{1}{\frac{114 \times 4}{8}}$$

or

$$n_1 : n_2 : n_3 = 8 : \frac{8}{3} : 2$$

or $n_1 : n_2 : n_3 = 72 : 24 : 18$

13. (c) The range of incident angle

$$\sin\theta = \frac{\sqrt{\mu_1^2 - \mu_2^2}}{\mu_0}$$

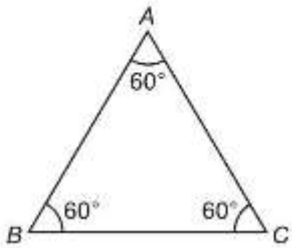
$$\theta = \sin^{-1}\sqrt{(\mu_1)^2 - (\mu_2)^2} \text{ [for air } \mu_0 = 1]$$

$$\theta = \sin^{-1}\sqrt{(1.5)^2 - (1.414)^2}$$

$$= \sin^{-1}(0.5006)$$

$$\theta = 30^\circ$$

14. (d)



Relation among the (i), (e), (A) and (δ)

$$i + e = A + \delta$$

$$\frac{3}{4} \times 60^\circ + \frac{3}{4} \times 60^\circ = 60^\circ + \delta$$

$$\text{or } \; 45^\circ + 45^\circ = 60^\circ + \delta$$

$$\text{or } \; \delta = 30^\circ$$

where, i = angle of incidence

e = angle of emergence

A = angle of prism

δ = angle of deviation

15. (a) Resolving limit

$$d\theta = \frac{1.22\lambda}{a}$$

$$= \frac{1.22 \times 4538 \times 10^{-10}}{1}$$

$$= 5.54 \times 10^{-7}\text{rad}$$

16. (c) The ratio of intensities of maxima and minima

$$\frac{I_{max}}{I_{min}} = \frac{\left(\sqrt{I_1} + \sqrt{I_2}\right)^2}{\left(\sqrt{I_1} - \sqrt{I_2}\right)^2}$$

$$= \frac{(\sqrt{64} + \sqrt{1})^2}{(\sqrt{64} - \sqrt{1})^2}$$

$$\frac{I_{max}}{I_{min}} = \frac{(8+1)^2}{(8-1)^2} = \frac{81}{49}$$

17. (b) In the sum and difference method of vibration magnetometer,

$$\frac{M_1}{M_2} = \frac{T_2^2 + T_1^2}{T_2^2 - T_1^2}$$

Here, $T_1 = \frac{1}{n_1} = \frac{1}{6}$ s

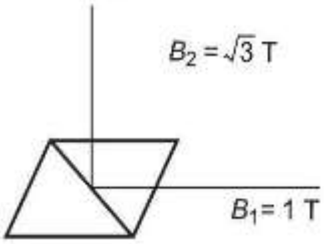
$$T_2 = \frac{1}{n_2} = \frac{1}{2} \text{ s}$$

$$\frac{M_1}{M_2} = \frac{\frac{1}{4} + \frac{1}{36}}{\frac{1}{4} - \frac{1}{36}} + \frac{\frac{9+1}{36}}{\frac{9-1}{36}}$$

$$= \frac{10}{8} = \frac{5}{4}$$

18. (d)

$M_1 : M_2 = 5 : 4$



In balance condition

or

$$B_2 = B_1 \tan\theta$$

$$\tan\theta = \frac{\sqrt{3}}{1} = \sqrt{3}$$

$$\theta = 60^\circ$$

19. (d) The force $F = \frac{eV}{d}$

$$= \frac{1.6 \times 10^{-19} \times 10^4}{0.5 \times 10^{-2}}$$

$$= 3.2 \times 10^{-13}\ \text{N}$$

20. (b) Capacitances 1μ F and $C\mu$ F are connected in series, then

$$C_{eq} = \frac{C}{1 + C}$$

Given, $V = 120$ V and $q = 80\mu C$

$\because \quad q = C_{eq}V$

$$80 = \frac{C}{C+1} \times 120$$

or $\quad C = 2\mu$ F

The energy stored in the capacitor of capacity C

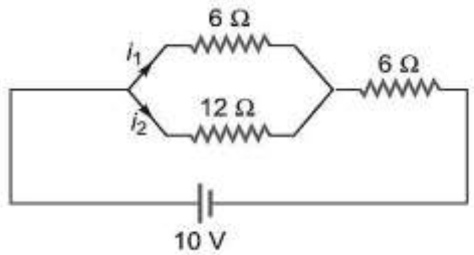
$$U = \frac{1}{2}\frac{q^2}{C}$$

$$= \frac{1}{2} \times \frac{(80 \times 10^{-6})^2}{2 \times 10^{-6}}$$

$$= \frac{1}{2} \times \frac{80 \times 10^{-6} \times 80 \times 10^{-6}}{2 \times 10^{-6}}$$

$$U = 1600\mu\ \text{J}$$

21. (a)



$$R = \frac{6 \times 12}{6 + 12} = \frac{6 \times 12}{18} = 4\ \Omega$$

Total resistance,

$$R_{eq} = 6 + 4 = 10\Omega$$

Current,

$$i = \frac{V}{R} = \frac{10}{10} = 1\ \text{A}$$

The current in 12Ω resistor is

$$i_2 = i\left(\frac{R_1}{R_1 + R_2}\right) = 1 \times \left(\frac{6}{6 + 12}\right)$$

$$i_2 = \frac{1}{3}$$

The potential difference in 12Ω resistor

$V = iR = \frac{1}{3} \times 12 = 4\ V$

22. (b)

$A = 0.3\ m^2$

$n = 2 \times 10^{25}/m^3$

$q = 3t^2 + 5t + 2$

$i = \frac{dq}{dt} = 6t + 5 = 17$

$i = neAv_d$

Drift velocity,

$$v_d = \frac{i}{neA}$$

$$= \frac{17}{2 \times 10^{25} \times 1.6 \times 10^{-19} \times 0.3}$$

$$= \frac{17}{0.96 \times 10^6}$$

$$= 1.77 \times 10^{-5}\ m/s$$

23. (d) At temperature of inversion

$E = 0$

$\because \quad E = aT + bT^2$

$\therefore \quad 0 = aT + bT^2$

or

$T = -\frac{a}{b} = 200°C$

Because the cold function is kept at 30°C, then the inversion temperature

$T = 200 - 30 = 170°C$

$T = 273 + 170 = 443\ K$

24. (a) The intensity of magnetic induction field

$$B = \frac{\mu_0 i}{2r}$$

$$= \frac{4\pi \times 10^{-7} \times 0.9}{2 \times 5 \times 10^{-2}}$$

$B = 36\pi \times 10^{-7}\ T$

25. (b) By equation of charging

$q = q_0(1 - e^{-t/CR})$

According to question

$\frac{q}{q_0} = \frac{1}{2} = 0.50$

$\therefore\ 0.50 = 1 - e^{-t/CR}$

$e^{-t/CR} = 1 - 0.50 = 0.50$

$e^{t/CR} = 2$

or $\frac{t}{CR} = \log_e 2$

or $\frac{t}{CR} = 2.3026 \log_{10} 2$

or $t = CR \times 2.3026 \log_{10} 2$

or $t = 0.1 \times 10^{-6} \times 10 \times 10^6 \times 2.3026 \log_{10} 2$

or $t = 2.3026 \times 0.3010$

or $t = 0.693\ s$

26. (a) Time constant $t = \frac{L}{R}$

$3 = \frac{L}{R}$ … (i)

When a 90Ω resistance is joined in series the time constant becomes 0.5 ms .

So, $0.5 = \frac{L}{R+90}$ …. (ii)

From Eqs. (i) and (ii)

$$\frac{3\times10^{-3}}{0.5\times10^{-3}}=\frac{R+90}{R}$$
$$\frac{30}{5}=\frac{R+90}{R}$$
$$6R=R+90$$
$$R=18\Omega$$

This value put in Eq. (i)

$$3\times10^{-3}=\frac{L}{R}$$
$$3\times10^{-3}=\frac{L}{18}$$

or $L=54\text{mH}$

27. (c) The kinetic energy

$$K=\frac{1}{2}mv^2$$
$$v^2=\frac{2K}{m}$$
$$v^2=\frac{2\times45.5\times1.6\times10^{-19}}{9.1\times10^{-31}}$$
$$v^2=16\times10^{12}$$
$$v=4\times10^6$$

Again velocity, $v=\frac{E}{B}$

$$4\times10^6=\frac{1\times10^3}{B}$$
$$B=\frac{1\times10^3}{4\times10^6}$$
$$B=2.5\times10^{-4}\text{ Wb m}^{-2}$$

28. (b) By using $hv-hv_0=K_{max}$

$$h(v_1-v_0)=K_1$$
$$h(v_2-v_0)=K_2$$
$$\therefore\quad\frac{v_1-v_0}{v_2-v_0}=\frac{K_1}{K_2}=\frac{1}{n}$$
$$\text{or}\quad v_0=\frac{nv_1-v_2}{(n-1)}$$

29. (a) The deflection suffered by charged particle in an electric field is

$$y=\frac{qELD}{m\mu^2}=\frac{qELD}{p^2/m}\quad(p=mu)$$
$$\Rightarrow y\propto\frac{qm}{p^2}$$
$$\Rightarrow y_p:y_d:y_\alpha=\frac{q_pm_p}{p_p^2}:\frac{q_dm_d}{p_d^2}:\frac{q_\alpha m_\alpha}{p_a^2}$$

Since,

$p_\alpha=p_d=p_p$

(given)

$m_p:m_d:m_\alpha=1:2:4$

and $q_1:q_d:q_\alpha=1:1:2$

$\Rightarrow y_b:y_d:y_\alpha=1\times1:1\times2:2\times4$

$=1:2:8$

30. (d)

$$\beta=\left(\frac{\Delta i_C}{\Delta i_B}\right)$$
$$80=\frac{\Delta i_C}{250\times10^{-6}}$$

$\Delta i_C = 80 \times 250 \times 10^{-6}$
$\Delta i_C = 20000 \times 10^{-6}$
$= 20 \times 10^{-3} = 20 \text{ mA}$

31. (b) Dimension of at = Dimension of F

$[at] = [F]$

$$[a] = \left[\frac{F}{t}\right]$$

$$[a] = \left[\frac{MLT^{-2}}{T}\right]$$

$[a] = [MLT^{-3}]$

Dimension of bt^2 = Dimension of F

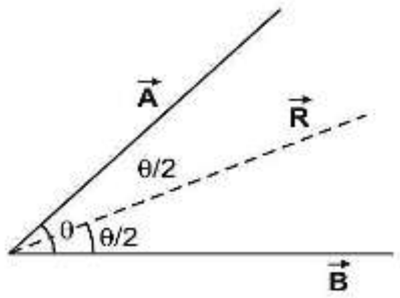
$[bt^2] = [F]$

$$[b] = \left[\frac{F}{t^2}\right]$$

$$[b] = \left[\frac{MLT^{-2}}{T^2}\right]$$

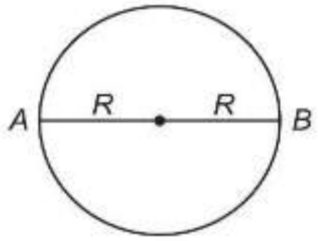
$[b] = [MLT^{-4}]$

32. (b)



If $\vec{A}$ and $\vec{B}$ having equal magnitudes they will make equal angles from resultant $\vec{R}$.
If angle between $\vec{A}$ and $\vec{B}$ is θ.
Then the angle between $\vec{A}$ or $\vec{B}$ with their resultant is $\frac{\theta}{2}$.

33. (b) The time = 2 min20 s
$= 2\text{min} + 20 \text{ s}$
$= 2 \times 60 \text{ s} + 20 \text{ s} = 140 \text{ s}$



In 40 s athelete completes = 1 round
In 140 s athelete will completes

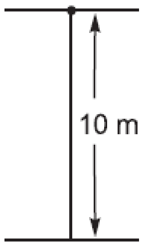
$$= \frac{140}{40} \text{ round}$$

$$= \frac{120}{40} + \frac{20}{40}$$

$$= 3 + \frac{1}{2} \text{ round}$$

The displacement in 3 round = 0
So displacement in $\frac{1}{2}$ round = AB = 2R

34. (c) At height 10 m , the kinetic energy is equal to potential energy then



$$\frac{1}{2}mv_0^2 = mgh$$
$$\frac{1}{2}v_0^2 = 9.8 \times 10$$
$$v_0^2 = 2 \times 9.8 \times 10$$
$$v_0 = \sqrt{196}$$
$$v_0 = 14 \text{ m/s}$$

35. (a) From conservation of linear momentum

$$5m(40\hat{\mathbf{i}} + 50\mathbf{j} - 25\hat{\mathbf{k}})$$
$$= m(200\hat{\mathbf{i}} + 70\hat{\mathbf{j}} + 15\hat{\mathbf{k}}) + 4m(x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}})$$
$$\Rightarrow x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}} = 45\hat{\mathbf{j}} - 35\hat{\mathbf{k}}$$

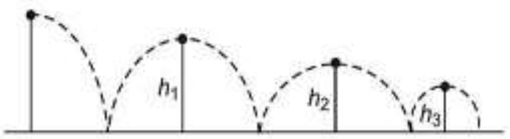
36. (c)

$$v_{CM} = \frac{m_1\frac{dr_1}{dt} + m_2\frac{dr_2}{dt}}{m_1 + m_2}$$
$$= \frac{4 \times 5\hat{\mathbf{i}} + 2 \times 10\hat{\mathbf{i}}}{4 + 2}$$
$$v_{CM} = \frac{40\hat{\mathbf{i}}}{6} = \frac{20}{3}\hat{\mathbf{i}}$$

The kinetic energy

$$K = \frac{1}{2}mv^2$$
$$= \frac{1}{2} \times (4 + 2) \times \frac{20 \times 20}{3 \times 3}$$
$$= \frac{1}{2} \times 6 \times \frac{20 \times 20}{3 \times 3}$$
$$K = \frac{400}{3} \text{ J}$$

37. (c)



Ball falls from height h then formula for height covered by it in nth rebound is given by

$$h_n = he^{2n}$$

where e = coefficient of restitution n = number of rebound

Total distance travelled by particle before rebounding has stopped.

$$H = h + 2h_1 + 2h_2 + 2h_n + \cdots$$
$$H = h + 2he^2 + 2he^4 + 2he^6 + 2he^8 + \cdots$$
$$= h + 2h(e^2 + e^4 + e^6 + e^8 + \cdots)$$
$$= h + 2h\left[\frac{e^2}{1 - e^2}\right]$$
$$= h\left[1 + \frac{2e^2}{1 - e^2}\right]$$
$$= h\left[\frac{1 + e^2}{1 - e^2}\right]$$

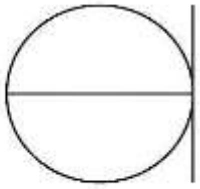
38. **(a)** The coefficient of kinetic friction between the object and the rough incline

$$\mu = \tan\theta\left(1 - \frac{1}{n^2}\right)$$

$$\mu = \tan 45^\circ\left(1 - \frac{1}{n^2}\right)$$

$$= 1 - \frac{1}{n^2}$$

39. **(d)** The moment of inertia of a disc about axis which is a tangent and parallel to its diameter is $\frac{5}{4}MR^2$.



40. **(b)**

$$\alpha = \frac{2\pi n}{t} = \frac{2\pi \times \frac{240}{60}}{20}$$

$$= \frac{2\pi \times 4}{20}$$

$$\alpha = \frac{2\pi}{5}$$

Torque,

$$\tau = I\alpha$$

$$\tau = MR^2\alpha$$

$$= 25 \times (0.04) \times \frac{2\pi}{5}$$

$$= 0.4\pi Nm$$

Chemistry

41. **(c)** Equivalent conductivity,

$$\Lambda_{eq} = \frac{\text{specific conductance } \times 1000}{\text{concentration (in g equiv /L)}}$$

$\because$ Specific conductance

$$= \frac{1}{\text{specific resistance}}$$

(Given, concentration = C g equiv/L)

$$\therefore \quad \Lambda_{eq} = \frac{1}{R} \times \frac{1000}{C}$$

$$\text{or} \quad \frac{1000}{RC}$$

42. **(c)** White tin, an allotropic form of tin is an example of tetragonal system and for tetragonal system, one side is not equal to other two while all the angles are equal, ie, $a = b \neq c$ and $\alpha = \beta = \gamma = 90^\circ$. Hence, Assertion (A) is true but Reason (R) is not.

43. **(b)** Arrhenius equation is

$$k = Ae^{-E_a/RT}$$

On taking log both sides,

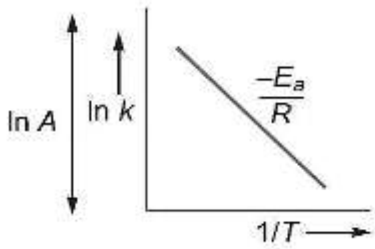
$$\ln k = \ln A - \frac{E_a}{RT}$$

$$\text{or} \quad \ln k = -\frac{E_a}{RT} + \ln A$$

Since, the reaction,

$$y = mx + c$$

when a graph is plotted between $\ln k$ and $\frac{1}{T}$ a straight line is obtained. The slope of the straight line is equal to $-\frac{E_a}{R}$.



44. (d)

45. (a) Given, pH of $0.01 M CH_3COOH$ solution
$= 5.0$
Concentration, C of the solution $= 0.01 M$
$[H^+] = 1 \times 10^{-pH}$
$= 1 \times 10^{-5}$ mol/L $= 1 \times 10^{-5} M$
Since, acetic acid is a weak acid and for weak acid,
$$[H^+] = \sqrt{K_a \cdot C}$$
$$[H^+]^2 = K_a \cdot C$$
$$\text{or } K_a = \frac{[H^+]^2}{C}$$
$$= \frac{(1 \times 10^{-5})^2}{0.01}$$
$$= 1 \times 10^{-8}$$

46. (a) From first law of thermodynamics,
$\Delta E = Q + W$
Since, heat is provided to the system,
$Q = +50$ J
and work is done on the system,
$W = +10$ J
$\Delta E = 50 + 10 = 60$ J

47. (b) Soap contains hydrophobic (ie, fat soluble or water repelling) as well as hydrophilic (water soluble) end. Hydrophobic end remains soluble in dust or grease particles while hydrophilic end is attracted towards water. Hence, several hydrophobic pocket surrounds the grease particles. This aggregate of soap and dirt is called micelle. On washing with plenty of water dust is removed along with soap.

48. (d) When an acidified solution of TiO_2 is treated with H_2O_2, an intense yellow orange colour is obtained due to the formation of H_2TiO_4.
$$TiO_2 + H_2O_2 \xrightarrow{H_2SO_4} H_2TiO_4$$
This reaction is used for detection of both Ti(IV) and H_2O_2.

49. (c) Solvay process is used in the manufacture of sodium carbonate, Na_2CO_3. This process involves following reactions.
$$NH_3 + H_2O + CO_2 \longrightarrow NH_4HCO_3$$
$$NH_4HCO_3 + NaCl \longrightarrow NaHCO_3 + NH_4Cl$$
$$NH_3 + H_2O + CO_2 + NaCl \longrightarrow NaHCO_3 + NH_4Cl$$
The sodium bicarbonate obtained on heating gives sodium carbonate.
$$NaHCO_3 \xrightarrow{\Delta} Na_2CO_3 + H_2O + CO_2$$
Note This process cannot be used in the manufacture of potassium carbonate as potassium bicarbonate is soluble in the solution of NH_4Cl and KCl .

50. (b) Diborane reacts with ammonia and gives different products under different reaction conditions as
$$B_2H_6 + \underset{\text{(excess)}}{NH_3} \xrightarrow[\text{temperature}]{\text{at low}} \underset{\text{addition product}}{B_2H_6 \cdot 2NH_3}$$
$$B_2H_6 + \underset{\text{(excess)}}{NH_3} \xrightarrow{\text{high temperature}} (BN)_n$$

boron nitride (resembles with graphite)

$$\underset{1\quad:\quad 2}{B_2H_6 + NH_3} \xrightarrow{\text{high temperature}}$$

$\underset{\text{borazine}}{B_3N_3H_6}$

(inorganic benzene)

Thus, $B_{12}H_{12}$ is not the product of reaction between diborane and ammonia.

51. (c)

Mineral	Formula
Galena	PbS
Cerussite	$PbCO_3$
Cassiterite	SnO_2
Anglesite	$PbSO_4$

Thus, cassiterite (tin stone) is a mineral for tin.

52. (b) By thermal decomposition of ammonium nitrate, nitrous acid (laughing gas) is obtained.

$$NH_4NO_3 \underset{250^\circ C}{\overset{\Delta}{\rightarrow}} N_2O + 2H_2O$$

53. (c) As the size of central atom, (order of size $O > S > Se > Te$) increases, the $H - A$ (where $A =$ central atom) bond length increases. Thus, the $H - A$ bond dissociation energy decreases. Consequently H_2Te gives a proton (H^+) more readily. Hence, it is most acidic among the given.

$$H_2Te + H_2O \longrightarrow H_3O^+ + HTe^-$$

54. (b) When chlorine reacts with hot concentrated sodium hydroxide, sodium chloride, sodium chlorate and water are obtained.

$$Cl_2(g) + \underset{\text{(hot conc)}}{6NaOH} \xrightarrow{70^\circ C} 5NaCl + \underset{\text{sodium chlorate}}{NaClO_3} + 3H_2O$$

55. (d) A mixture of helium and oxygen is used in the treatment of asthma as helium has low density and thus, this mixture flows easily through restricted passages.

56. (a)

(a) $Cu^+ = [Ar]3d^{10}$

↿⇂	↿⇂	↿⇂	↿⇂	↿⇂

(No unpaired electrons, so it is colourless)

$Zn^{2+} = [Ar]3d^{10}$

(No unpaired electrons, so it is colourless)

(b) $Cu^{2+} = [Ar]3d^9$

↿⇂	↿⇂	↿⇂	↿⇂	↿

(One unpaired electron is present, so it is coloured)

Zn^{2+} is colourless.

(c) The colour of Fe^{2+} is green while that of Fe^{3+} is blue.

(d) MnO_4^- contains no unpaired electrons, however, it is coloured due to charge transfer.

Thus, only statement given in option (a) is correct.

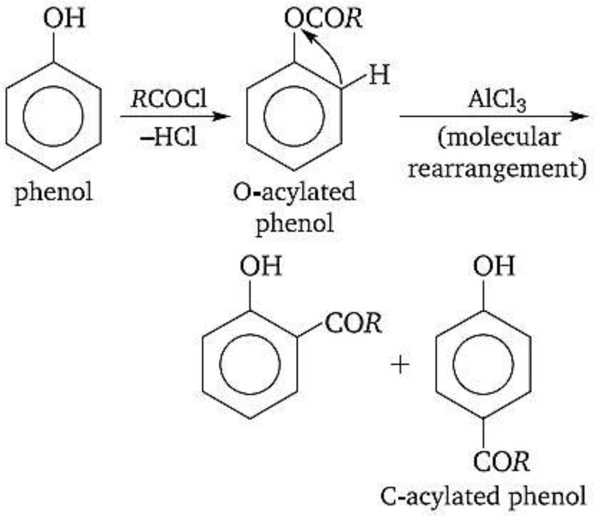
57. (c) Roasting is the process of heating of sulphide ores in excess of air to convert them into oxides. Thus, reactions,

$$2ZnS + 3O_2 \longrightarrow 2ZnO + 3SO_2$$

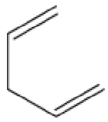
occurs during roasting process.

58. (a) The acceptable level of carbon monoxide gas (CO) in the atmosphere is $\approx$ 9ppm.

59. (c)

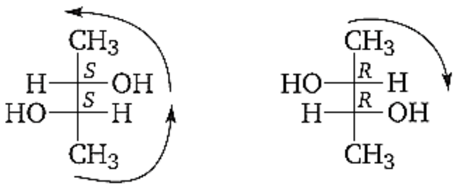


60. (a) In Diels-Alder reaction, conjugated dienes react with enes or olefinic compounds and give cycloalkenes.

Since,  (buta-1,4-diene) is not a conjugated alkene, it will not take part in Diels-Alder's reaction.

61. --- OH group is ortho/para directing and makes the benzene ring more active towards electrophilic substitution. Thus, ortho/para substitution in phenol by an electrophile is very facile.

62. (a) The pairs 2R, 3R-butane diol and 2S, 3S-butane diol are enantiomeric.



63. (b) Enantiomers have same physical properties but differ in specific rotation. Thus, the two enantiomers of secondary butyl chloride differ in specific rotation.

64. (b) $C_2H_5-O-C_2H_5+CO \xrightarrow[500\ \text{atm}]{BF_3/150^\circ C}$

$\underset{\text{ethyl propionate}}{C_2H_5COOC_2H_5}$

65. (a) Alcohols containing $CH_3CH(OH)-$ group and carbonyl compounds containing CH_3CO- group give yellow precipitate with iodine and NaOH solution. This reaction is called iodoform test.

Thus, CH_3CHO due to presence of CH_3CO group gives yellow ppt with I_2 and NaOH .

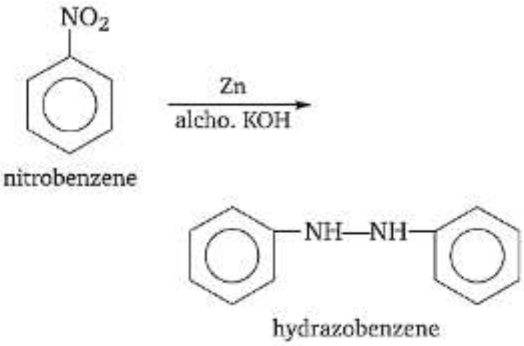
$CH_3CHO+3I_2+4NaOH \longrightarrow CHI_3$
$+HCOONa+3NaI+2H_2O$

66. (c)

$$CH_3Cl \xrightarrow{KCN} \underset{(A)}{\underset{\text{(ethyl cyanide)}}{CH_3CN}} \xrightarrow[H_3O^+]{\text{hydrolysis}}$$

$$\underset{(B)}{\underset{\text{ethanoic acid}}{CH_3COOH}} \xrightarrow[\Delta]{C_2H_5OH/H^+} \underset{(C)}{\underset{\text{ethyl ethanoate}}{CH_3COOC_2H_5}}$$

67. (a) Reduction of nitrobenzene with Zn and alcoholic KOH results in the formation of hydrazobenzene.



68. (a) Polydispersity index,

$PDI = \frac{\bar{M}_w}{\bar{M}_n}$

where, $\bar{M}_w$ = weight average molecular weight $\bar{M}_n$ = number average molecular weight

$PDI = \frac{60000}{40000} = 1.5$

ie, > 1

69. (b) The AT/GC ratio in human beings is 1.52 .

70. (b) Among the given drugs, ibuprofen is a non-narcotic (ie, not habit forming) analgesic.
Note Diazepam is hypnotic and sedative. Formalin and terpineol have antiseptic properties.

71. (c) As the value of n increases, the difference of energy between successive orbits decreases, ie,

$E_2 - E_1 > E_3 - E_2 > E_4 - E_3$ and so on...

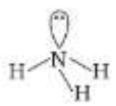
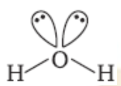
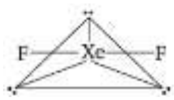
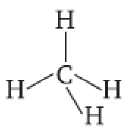
$\because \Delta E = \frac{hc}{\lambda}$

$\therefore \lambda$ is lowest for $n_2 = 2$ to $n_1 = 1$ transition.

72. (c) For a well- behaved wave function the Born's conditions are ψ must be finite, single valued and continuous.

73. (c) Elements having half or completely-filled orbitals, ie, having stable electronic configuration have very low negative value of electron affinity. Since, the electron affinity of B is lowest among the given, it has stable electronic configuration, ie, $3s^2, 3p^3$.

74. (c)

Molecule	Structure	Number of lone pairs
NH_3		One
H_2O		Two
XeF_2		Three
CH_4		Zero

Thus, the correct match is A-(5), B-(1); C—(2); D-(3).

75. (d) Given, the radius ratio of anion to cation

$= 10 : 9.3$

$\therefore$ The radius ratio of cation to anion

$= 9.3 : 10$

$= 0.93$

When the cation to anion ratio lie between 0.732 to 1.00 , the coordination number is 8 . Thus, the coordination number of the cation in the crystal is 8 .

76. (c)

$$C(s) + O_2(g) \longrightarrow \begin{array}{l} CO_2(g) \\ 1 \text{ mol} \\ = 6.023 \times 10^{23} \end{array}$$

$\because$ 1 mole of graphite on complete combustion gives CO_2
$= 6.023 \times 10^{23}$ molecules
$\therefore$ 0.1 mole of graphite will give CO_2

$$= \frac{6.023 \times 10^{23} \times 0.1}{1}$$
$$= 6.023 \times 10^{22}$$

77. (b) From Graham's law of diffusion,

$$\frac{r_{CH_4}}{r_X} = \sqrt{\frac{M_X}{M_{CH_4}}} \left(\text{ given, } r_{CH_4} = 2 \cdot r_X\right)$$

$$2 = \sqrt{\frac{M_X}{16}}$$

$$M_X = 16 \times 4 = 64$$

Thus, the molecular mass of gas X is 64 .
Number of molecules of gas X in 32 g gas

$$= \frac{32}{64} \times N = \frac{N}{2}$$

78. (a)

$$BaCl_2(s) \xrightarrow{H_2O} Ba^{2+} + 2Cl^-$$

1	0	0
$1-0.8$	0.8	2×0.8 In aq. solution

$$i = \frac{\text{No. of moles after reaction}}{\text{Initial number of moles}}$$
$$= \frac{1-0.8+0.8+(2 \times 0.8)}{1}$$
$$= 2.6$$

79. (d) When a non-volatile solute is added to a volatile solvent, the solute covers up some of the surface of solvent. Thus, less surface area is available for vaporisation of solvent and hence, vapour pressure decreases. As the amount of non-volatile solute increases, vapour pressure decreases accordingly.

$\because$ The order of concentration of X is

$0.01 < 0.10 < 0.25$

$\therefore$ The order of vapour pressure is

$p_3 > p_1 > p_2$

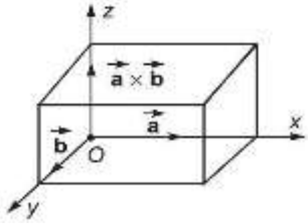
80. (c)

Mathematics

81. (a) Given that, $\vec{\mathbf{a}} = \hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$

$\vec{\mathbf{b}} = 2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - \hat{\mathbf{k}}$

and $\vec{\mathbf{c}} = \lambda\hat{\mathbf{i}} + \hat{\mathbf{j}} + (2\lambda - 1)\hat{\mathbf{k}}$



Also, since $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$ lies in the same plane, then $(\vec{\mathbf{a}} \times \vec{\mathbf{b}})$ is perpendicular vector to this plane. Given that vector $\vec{\mathbf{c}}$ is parallel to the plane
containing $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$, so vector $(\vec{\mathbf{a}} \times \vec{\mathbf{b}})$ also perpendicular to the vector $\vec{\mathbf{c}}$ ie, $(\theta = 90^\circ)$ So, $(\vec{\mathbf{a}} \times \vec{\mathbf{b}}) \cdot \vec{\mathbf{c}}$ should be equal to zero

$$(\vec{\mathbf{a}} \times \vec{\mathbf{b}}) \cdot \vec{\mathbf{c}} = 0$$

or $\quad = (2-9)\hat{\mathbf{i}} + (6+1)\hat{\mathbf{j}} + (3+4)\hat{\mathbf{k}}$

$$= -7\hat{\mathbf{i}} + 7\hat{\mathbf{j}} + 7\hat{\mathbf{k}}$$

Then from Eq. (i)

$$(-7\hat{\mathbf{i}} + 7\hat{\mathbf{j}} + 7\hat{\mathbf{k}}) \cdot (\lambda\hat{\mathbf{i}} + \hat{\mathbf{j}} + (2\lambda - 1)\hat{\mathbf{k}}) = 0$$

$\Rightarrow \quad -7\lambda + 7 + 7(2\lambda - 1) = 0$

$\Rightarrow \quad -7\lambda + 7 + 14\lambda - 7 = 0$

$\Rightarrow \quad 7\lambda = 0$

$\Rightarrow \quad \lambda = 0$

Hence, the value of λ is 0 .

82. (a) Given, condition is

$\vec{a} + \vec{b} + \vec{c} = \vec{0}$ .... (i)

and $\vec{\mathbf{a}}, \vec{\mathbf{b}}, \vec{\mathbf{c}}$ are the unit vectors.

Then $|\vec{\mathbf{a}}| = |\vec{\mathbf{b}}| = |\vec{\mathbf{c}}| = 1$

Let the angle between $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$ is θ.

Now, from Eq. (i)

$\vec{a} + \vec{b} + \vec{c} = 0$

$(\vec{a} + \vec{b}) = -\vec{c}$

Squaring on both sides

$$(\vec{\mathbf{a}} + \vec{\mathbf{b}})^2 = (\vec{\mathbf{c}})^2 \; [\because (\vec{\mathbf{c}})^2 = |\vec{\mathbf{c}}|^2]$$

$\Rightarrow \quad (\vec{\mathbf{a}})^2 + (\vec{\mathbf{b}})^2 + 2(\vec{\mathbf{a}}) \cdot (\vec{\mathbf{b}}) = |\vec{\mathbf{c}}|^2$

$\Rightarrow \quad |\vec{\mathbf{a}}|^2 + |\vec{\mathbf{b}}|^2 + 2\vec{\mathbf{a}} \cdot \vec{\mathbf{b}} = |\vec{\mathbf{c}}|^2$

$\Rightarrow \quad 1 + 1 + 2\{|\vec{\mathbf{a}}||\vec{\mathbf{b}}|\cos\theta\} = 1$

$\Rightarrow \quad [\because \vec{\mathbf{a}} \cdot \vec{\mathbf{b}} = |\vec{\mathbf{a}}||\vec{\mathbf{b}}|\cos\theta]$

$\Rightarrow \quad 2\{1 \cdot 1 \cdot \cos\theta\} = -1$

$\Rightarrow \quad \cos\theta = \dfrac{-1}{2}$

$\Rightarrow \quad \cos\theta = \cos\left(\dfrac{2\pi}{3}\right)$

$\Rightarrow \quad \theta = \dfrac{2\pi}{3}$

83. (c)

$(\vec{a} + 2\vec{b} - \vec{c}) \cdot (\vec{a} - \vec{b}) \times (\vec{a} - \vec{b} - \vec{c})$

$= (\vec{\mathbf{a}} + 2\vec{\mathbf{b}} - \vec{\mathbf{c}}) \cdot [\vec{\mathbf{a}} \times \vec{\mathbf{a}} - \vec{\mathbf{b}} \times \vec{\mathbf{a}} - \vec{\mathbf{a}} \times \vec{\mathbf{b}} + \vec{\mathbf{b}} \times \vec{\mathbf{b}}$

$-\vec{a} \times \vec{c} + \vec{b} \times \vec{c}]$

$= (\vec{\mathbf{a}} + 2\vec{\mathbf{b}} - \vec{\mathbf{c}}) \cdot [0 - (-\vec{\mathbf{c}}) - (\vec{\mathbf{c}}) + 0 - (-\vec{b}) + \vec{\mathbf{a}}]$

$[\because \vec{\mathbf{a}} \times \vec{\mathbf{a}} = \vec{\mathbf{b}} \times \vec{\mathbf{b}} = 0]$

$$= (\vec{\mathbf{a}} + 2\vec{\mathbf{b}} - \vec{\mathbf{c}}) \cdot (\vec{\mathbf{c}} - \vec{\mathbf{c}} + \vec{\mathbf{b}} + \vec{\mathbf{a}}) \left\{\begin{array}{l} \because \vec{\mathbf{a}} \times \vec{\mathbf{b}} = \vec{\mathbf{c}} \\ \vec{\mathbf{b}} \times \vec{\mathbf{c}} = \vec{\mathbf{a}} \\ \vec{\mathbf{c}} \times \vec{\mathbf{a}} = \vec{b} \end{array}\right.$$

$= (\vec{\mathbf{a}} + 2\vec{\mathbf{b}} - \vec{\mathbf{c}}) \cdot (\vec{\mathbf{b}} + \vec{\mathbf{a}})$

$= \vec{\mathbf{a}} \cdot \vec{\mathbf{b}} + 2\vec{\mathbf{b}} \cdot \vec{\mathbf{b}} - \vec{\mathbf{c}} \cdot \vec{\mathbf{b}} + \vec{\mathbf{a}} \cdot \vec{\mathbf{a}} + 2\vec{\mathbf{b}} \cdot \vec{\mathbf{a}} - \vec{\mathbf{c}} \cdot \vec{\mathbf{a}}$

$$= 0 + 2(1) - 0 + (1) + 2(0) - (0) \left\{\begin{array}{l} \because \vec{\mathbf{a}} \cdot \vec{\mathbf{b}} = 0 \\ \vec{\mathbf{b}} \cdot \vec{\mathbf{c}} = 0 \\ \vec{\mathbf{c}} \cdot \vec{\mathbf{a}} = 0 \end{array}\right.$$

$= 2 + 1 = 3 \cdot 1$

$$= 3 \cdot \{\vec{a} \cdot (\vec{b} \times \vec{c})\} \begin{cases} \because \vec{a} \cdot \vec{a} = 1 \\ \vec{b} \cdot \vec{b} = 1 \\ \vec{c} \cdot \vec{c} = 1 \end{cases}$$

$= 3[\vec{a}\vec{b}\vec{c}] \ \because [\vec{a}\vec{b}\vec{c}] = \vec{a} \cdot (\vec{b} \times \vec{c})$

84. (a) We have, $\vec{u} = \vec{a} - \vec{b}, \vec{v} = \vec{a} + \vec{b}$

$\Rightarrow \vec{u} \times \vec{v} = (\vec{a} - \vec{b}) \times (\vec{a} + \vec{b})$

$= 0 - \vec{b} \times \vec{a} + \vec{a} \times \vec{b} - 0$

$= 2\vec{a} \times \vec{b}$

$\Rightarrow |\vec{u} \times \vec{v}| = 2|\vec{a} \times \vec{b}|$

$$= 2\sqrt{|\vec{a} \times \vec{b}|^2}$$

$$= 2\sqrt{|\vec{a}|^2|\vec{b}|^2 \sin^2\theta |\hat{n}|^2}$$

$\{\because \hat{n} =$ unit vector $|\hat{n}| = 1\}$

$= 2\sqrt{4 \cdot 4 \cdot \sin^2\theta \cdot 1}$

$= 2\sqrt{16(1 - \cos^2\theta)}$

$= 2\sqrt{16 - 16\cos^2\theta}$

$$= 2\sqrt{16 - 16\left(\frac{\vec{a} \cdot \vec{b}}{|\vec{a}||\vec{b}|}\right)^2}$$

$\because \{\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}|\cos\theta\}$

$$= 2\sqrt{16 - 16\frac{(\vec{a} \cdot \vec{b})^2}{|\vec{a}|^2|\vec{b}|^2}}$$

$$= 2\sqrt{16 - 16\frac{(\vec{a} \cdot \vec{b})^2}{4 \cdot 4}}$$

$$\Rightarrow 2\sqrt{16 - (\vec{a} \cdot \vec{b})^2}$$

85. (a) Given, $\vec{a} = 2x^2\hat{i} + 4x\hat{j} + \hat{k}$

$\vec{b} = 7\hat{i} - 2\hat{j} + x\hat{k}$, also $90° < \theta < 180°$

We know that,

$$\cos\theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}||\vec{b}|}$$

$$\cos\theta = \frac{(2x^2\hat{i} + 4x\hat{j} + \hat{k}) \cdot (7\hat{i} - 2\hat{j} + x\hat{k})}{\sqrt{4x^4 + 16x^2 + 1} \cdot \sqrt{49 + 4 + x^2}}$$

$$\cos\theta = \frac{14x^2 - 8x + x}{\sqrt{4x^4 + 16x^2 + 1} \cdot \sqrt{53 + x^2}}$$

$$= \frac{14x^2 - 7x}{\sqrt{4x^4 + 16x^2 + 1} \cdot \sqrt{53 + x^2}}$$

$$\cos\theta = \frac{7x(2x - 1)}{\sqrt{4x^4 + 16x^2 + 1} \cdot \sqrt{53 + x^2}}$$

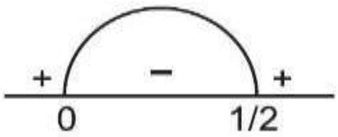
$\because \theta$ lies between $(90°, 180°)$

ie, $\cos\theta$ is negative in IInd quadrant.

So, RHS is also negative ie,

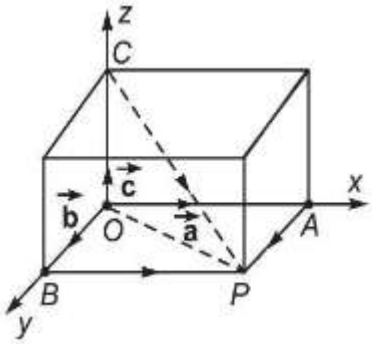
$$\frac{7x(2x - 1)}{\sqrt{4x^4 + 16x^2 + 1} \cdot \sqrt{53 + x^2}} < 0$$

$$7x(2x - 1) < 0$$



So, $x \in \left(0, \frac{1}{2}\right)$

86. (a) Here, OA, OB, OC are the co-terminal edges of a rectangular parallelopiped of volume V.



Also, we know that the volume of rectangular parallelopiped $= [\vec{\mathbf{a}}\vec{\mathbf{b}}\vec{\mathbf{c}}]$

ie, $V = [\overrightarrow{OA}\overrightarrow{OB}\overrightarrow{OC}]$ …. (i)

Let $\overrightarrow{OA} = \vec{a}, \overrightarrow{OB} = \vec{b}, \overrightarrow{OC} = \vec{\mathbf{c}}$

then from figure

$\overrightarrow{AP} = \vec{a} + \vec{b}, \overrightarrow{BP} = \vec{b} + \vec{c}, \overrightarrow{CP} = \vec{c} + \vec{a}$

$\because$ (By vector addition)

Now, we find

$[\overrightarrow{AP}\overrightarrow{BP}\overrightarrow{CP}] = [(\vec{a} + \vec{b})(\vec{b} + \vec{c})(\vec{c} + \vec{a})]$

$= (\vec{a} + \vec{b}) \cdot [(\vec{b} + \vec{c}) \times (\vec{c} + \vec{a})]$

$= (\vec{a} + \vec{b}) \cdot [\vec{b} \times \vec{c} + \vec{b} \times \vec{a} + \vec{c} \times \vec{c} + \vec{c} \times \vec{a}]$

$[\because \vec{\mathbf{c}} \times \vec{\mathbf{c}} = 0]$

$$\begin{cases} \vec{a} \times \vec{b} = \vec{c} \\ \vec{b} \times \vec{c} = \vec{a} \\ \vec{c} \times \vec{a} = \vec{b} \end{cases}$$

$= (\vec{\mathbf{a}} + \vec{\mathbf{b}}) \cdot [\vec{\mathbf{a}} - \vec{\mathbf{a}} \times \vec{\mathbf{b}} + 0 + \vec{\mathbf{b}}]$

$= (\vec{a} + \vec{b}) \cdot [\vec{a} - \vec{c} + \vec{b}]$

$= (\vec{a} + \vec{b}) \cdot \vec{a} - (\vec{a} + \vec{b}) \cdot \vec{c} + (\vec{a} + \vec{b}) \cdot \vec{b}$

$= \vec{a} \cdot \vec{a} + \vec{b} \cdot \vec{a} - \vec{a} \cdot \vec{c} - \vec{b} \cdot \vec{c} + \vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{b}$

$= 1 + 0 - 0 - 0 + 0 + 1$

$= 2 \cdot 1$

$\because [\vec{\mathbf{a}}\vec{\mathbf{b}}\vec{\mathbf{c}}] = \vec{\mathbf{a}} \cdot (\vec{\mathbf{b}} \times \vec{\mathbf{c}}) = \vec{\mathbf{a}} \cdot \vec{\mathbf{a}} = 1$

$= 2\{\vec{\mathbf{a}} \cdot (\vec{\mathbf{b}} \times \vec{\mathbf{c}})\}$

$= 2[\vec{\mathbf{a}}\vec{\mathbf{b}}\vec{\mathbf{c}}]$

$= 2[\overrightarrow{OA}\overrightarrow{OB}\overrightarrow{OC}]$

$= 2\ V$

[from Eq. (i)]

Hence, $[\overrightarrow{AP}\overrightarrow{BP}\overrightarrow{\mathbf{CP}}] = 2\ V$

87. (d) Case Ist Let a white ball be transfered from the first bag to the second bag.

The probability of selecting white ball from the first bag is $P_1 = \frac{3}{3+5} = \frac{3}{8}$

Now, the second bag has 7 white and 8 black balls. The probability of selecting a white ball from the second bag is $P_2 = \frac{7}{7+8} = \frac{7}{15}$

The probability that both these events take place simultaneously

$= P_1 \times P_2 = \frac{3}{8} \times \frac{7}{15} = \frac{7}{40}$

Case IInd Let a black ball be transfered from the first bag to the second bag.

Its probability is $P_3 = \frac{5}{3+5} = \frac{5}{8}$

Now, the second bag contains 6 white and 9 black balls.

The probability of drawing a white ball from the second bag is $P_4 = \frac{6}{6+9} = \frac{6}{15}$

$\therefore$ The probability of both these events taking place simultaneously

$= P_3 \times P_4 = \frac{5}{8} \times \frac{6}{15} = \frac{1}{4}$

$\therefore$ The required probability $= P_1P_2 + P_3P_4$

$= \frac{7}{40} + \frac{1}{4}$

$= \frac{7+10}{40} = \frac{17}{40}$

88. (d) The required probability

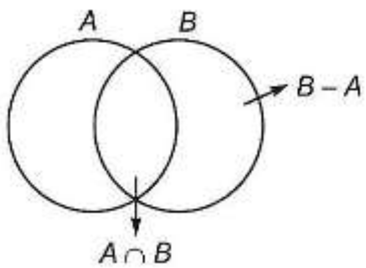
$= P(\bar{A}_1 \cap \bar{A}_2 \cap \dots \cap \bar{A}_n)$

$= P(\bar{A}_1)P(\bar{A}_2) \dots P(\bar{A}_n)$

$= \frac{1}{2} \cdot \frac{2}{3} \cdot \frac{3}{4} \dots \cdot \frac{n}{n+1}$

$= \frac{1}{n+1}$

89. (a)



Given, $P(A \cap B) = \frac{3}{25}$

$P(B - A) = \frac{8}{25}$

Then according to figure

$P(B) = P(A \cap B) + P(B - A)$

$P(B) = \frac{3}{25} + \frac{8}{25} \Rightarrow P(B) = \frac{11}{25}$

90. (c) Let λ be the mean of the poisson variate x.

Then, $P(X = r) = \frac{\lambda^r e^{-\lambda}}{r!}, r = 0,1,2, \dots$

Now, $P(X = 1) = P(X = 2) \Rightarrow \frac{\lambda e^{-\lambda}}{1!} = \frac{\lambda^2 e^{-\lambda}}{2!}$

$\Rightarrow \lambda = 2$

Hence, $P(X = 5) = \frac{\lambda^5 e^{-\lambda}}{5!} = \frac{2^5 e^{-2}}{120}$

$= \frac{32 \cdot e^{-2}}{120} = \frac{4e^{-2}}{15}$

91. (b) Mean $\Rightarrow np = 2$ and variance $\Rightarrow npq = 1$ then $q = \frac{1}{2}, p = \frac{1}{2}$ and $n = 4$ ($\because p = 1 - q$)

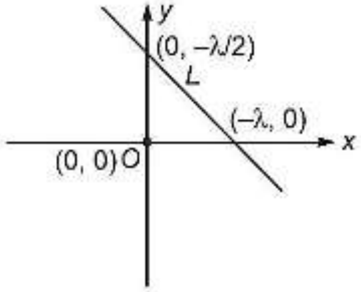
$\therefore$ Required probability

$= P(X \geq 1) = 1 - P(X < 1)$

$= 1 - \left(\frac{1}{16}\right)$

$= \frac{15}{16}$

92. (c) A straight line L is perpendicular to the line $4x - 2y = 1$ therefore the equation of the line is
$x + 2y + \lambda = 0 \quad \ldots$ (i)



Also, the line (i) form a Δ of area 4 unit ^{2} with the coordinate axes, then

$$\frac{1}{2}\begin{vmatrix} 0 & 0 & 1 \\ -\lambda & 0 & 1 \\ 0 & -\lambda/2 & 1 \end{vmatrix} = 4$$

$$(-\lambda)(-\lambda/2) = 8$$

$$\lambda^2 = 16$$

$$\lambda = 4$$

From Eq. (i)

$$x + 2y + 4 = 0$$

or

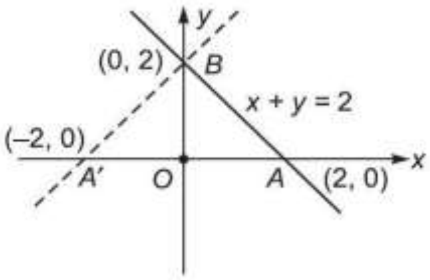
$$2x + 4y + 8 = 0$$

93. (a)

94. (a)

Equation of A′B is

$$(y - 0) = \frac{2-0}{0+2}(x+2)$$

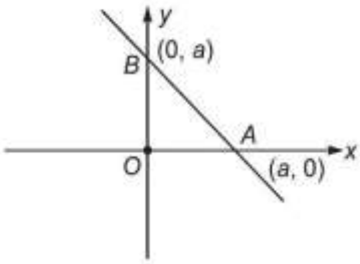


$$y - 0 = 1(x + 2)$$

$$x - y + 2 = 0$$

which is the image of the line AB with respect to y-axis.

95. (b) The equation of line AB which makes an equal intercepts on positive x and y axes are



$$\frac{x}{a} + \frac{y}{a} = 1 \quad \ldots \text{(i)}$$

$ie,\quad x + y = a$

Distance of Eq. (i) from origin = 1

$$\left|\frac{0+0-a}{\sqrt{1+1}}\right| = 1, \left|\frac{-a}{\sqrt{2}}\right| = 1$$

$$\Rightarrow a = \sqrt{2}$$

From Eq. (i), $x + y = \sqrt{2}$
Also given line,
$2x - y = -3 - \sqrt{2}$ ....(ii)
The intersection point of line (ii) and line (iii) is
$$(x_0, y_0) = (-1, \sqrt{2} + 1)$$
So, $2x_0 + y_0 = 2(-1) + \sqrt{2} + 1$
$$= -2 + \sqrt{2} + 1 = (\sqrt{2} - 1)$$
Hence, $2x_0 + y_0 = \sqrt{2} - 1$

96. (d)

97. (a) The intersection point of the curve $x^2 + y^2 = 4$ with $x + y = a$, where $(a > 0)$
$$x^2 + (a - x)^2 = 4$$
$$x^2 + a^2 + x^2 - 2ax = 4$$
$$2x^2 - 2ax + (a^2 - 4) = 0$$
$$x^2 - ax + \left(\frac{a^2}{2} - 2\right) = 0$$
$$x = \frac{+a \pm \sqrt{a^2 - 4\left(\frac{a^2}{2} - 2\right)}}{2}$$
$$x = \frac{a \pm \sqrt{a^2 - 2a^2 + 8}}{2} = \frac{a \pm \sqrt{8 - a^2}}{2}$$
Since, here the point of intersection should be real number, iff
$$8 - a^2 \geq 0$$
ie, $a^2 \leq 8, a \leq 2\sqrt{2}$
$a \leq 2.82$
Hence, the value is $a = 2$ according to option.

98. (b) The pair of straight lines is
$3x^2 - 11xy + 10y^2 - 7x + 13y + k = 0$
then on comparing with
$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$$
$$\Rightarrow \begin{cases} a = 3, h = -\frac{11}{2}, b = 10 \\ g = -\frac{7}{2}, f = \frac{13}{2}, c = k \end{cases}$$
Then, the point of intersection of the lines is
$$\left[\frac{hf - bg}{ab - h^2}, \frac{gh - af}{ab - h^2}\right]$$
$$= \left[\frac{\frac{-143}{4} + \frac{70}{2}}{30 - \frac{121}{4}}, \frac{\frac{77}{4} - \frac{39}{2}}{30 - \frac{121}{4}}\right]$$
$$= \left[\frac{-3}{120 - 121}, \frac{77 - 78}{120 - 121}\right]$$
$$= \left(\frac{-3}{-1}, \frac{-1}{-1}\right) = (3,1)$$

99. (c)
Let $S_1 \equiv x^2 + y^2 - x + 2y + \frac{18}{7} = 0$
and $S_2 \equiv x^2 + y^2 - \frac{7}{4}x + 2y + 5 = 0$
Now, the radical axis of the pair of circles are

$$S_1 - S_2 = 0$$

$$\Rightarrow \left(x^2 + y^2 - x + 2y + \frac{18}{7}\right)$$

$$-\left(x^2 + y^2 - \frac{7}{4}x + 2y + 5\right) = 0$$

$$\Rightarrow -x + \frac{7}{4}x + \frac{18}{7} - 5 = 0$$

$$\Rightarrow -x + \frac{7}{4}x - \frac{17}{7} = 0$$

$$\Rightarrow -28x + 49x - 68 = 0$$

$$\Rightarrow 21x - 68 = 0$$

100. **(c)** Let $P(x_1, y_1)$ be the point from which the tangents are drawn to the circles

$$S_1 \equiv x^2 + y^2 - 8x + 40 = 0$$

$$S_2 \equiv 5x^2 + 5y^2 - 25x + 80 = 0$$

$$S_3 \equiv x^2 + y^2 - 8x + 16y + 160 = 0$$

Since, the length of the tangent from P to the circle S_1, S_2, S_3 are equal

$$\therefore \sqrt{S_1} = \sqrt{S_2} = \sqrt{S_3}$$

$$\Rightarrow S_1 = S_2 = S_3$$

$$x_1^2 + y_1^2 - 8x_1 + 40 = 5x_1^2 + 5x_2^2 - 25x_1 + 80$$

$$= x_1^2 + y^2 - 8x_1 + 16y_1 + 160$$

Taking first and third part of above relation (i), we get

$$-40 + 16y_1 + 160 = 0$$

$$16y_1 + 120 = 0$$

$$y_1 = -\frac{120}{16} \Rightarrow -\frac{15}{2}$$

Taking first and second part of the relation (i).

$$-3x_1 + 24 = 0$$

$$x_1 = \frac{24}{3} \Rightarrow x_1 = 8$$

Hence, the point P is $\left(8, \frac{-15}{2}\right)$.

101. **(a)** The equation of the circle is

$$S \equiv x^2 + y^2 - 6x + 12y + 15 = 0$$

Let the equation of concentric circle of given circles, is

$$S_2 = x^2 + y^2 - 6x + 12y + 15 = 0$$

On comparing the circle S_1 with,

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

$$\Rightarrow g = -3, f = 6, c = 15$$

Then, radius of circle is

$$= \sqrt{g^2 + f^2 - c}$$

$$= \sqrt{9 + 36 - 15}$$

$$= \sqrt{45 - 15} = \sqrt{30} \text{ units}$$

and centre is $(-g, -f) = (3, -6)$

Now, the area of the circle S is $= \pi$ (radius) ^{2}

$$= \pi(\sqrt{30})^2$$

$$= 30\pi$$

Let the radius of the concentric circle is r_2.

$$r_2 = \sqrt{g^2 + f^2 - c}$$

$$= \sqrt{9 + 36 - c}$$

$$= \sqrt{45 - c}$$

Then, according to question, the area of concentric circle

$$= 2 \times \text{ area of S}$$
$$2 \times 30\pi = 60\pi$$
$$\pi r_2^2 = 60\pi$$
$$\Rightarrow (\sqrt{45 - c})^2 = 60$$
$$\Rightarrow 45 - c = 60$$
$$\Rightarrow c = -15$$

Hence, the equation of concentric circle is

$x^2 + y^2 + 2(-3)x + 2(6)y + (-15) = 0$

$$x^2 + y^2 - 6x + 12y - 15 = 0$$

102. **(b)** The equation of the circles are

$$S_1 \equiv x^2 + y^2 + 2x + 3y + 1 = 0$$

and $S_2 \equiv x^2 + y^2 + 4x + 3y + 2 = 0$

Since, the circles cuts each other at A and B then equation of AB is

$$S_1 - S_2 = 0$$
$$\Rightarrow (x^2 + y^2 + 2x + 3y + 1)$$
$$\Rightarrow -(x^2 + y^2 + 4x + 3y + 2) = 0$$
$$\Rightarrow -2x - 1 = 0$$
$$\Rightarrow x = \frac{-1}{2}$$

Putting the value $x = -\frac{1}{2}$ in S_2, we get

$$\frac{1}{4} + y^2 - 2 + 3y + 2 = 0$$
$$\Rightarrow 4y^2 + 12y + 1 = 0$$
$$\Rightarrow y = \frac{-12 \pm \sqrt{144 - 16}}{8}$$
$$\Rightarrow y = \frac{-12 \pm \sqrt{128}}{8} = \frac{-12 \pm 8\sqrt{2}}{8}$$
$$\Rightarrow y = \frac{-3}{2} \pm \sqrt{2}$$

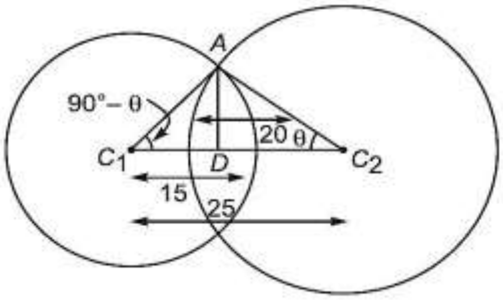
So intersection points are

$A\left(-\frac{1}{2}, -\frac{3}{2} + \sqrt{2}\right)$ and $\left(-\frac{1}{2}, -\frac{3}{2} - \sqrt{2}\right)$.

Then equation of circle with diameter AB is

$$\left(x + \frac{1}{2}\right)\left(x + \frac{1}{2}\right) + \left(y + \frac{3}{2} + \sqrt{2}\right)\left(y + \frac{3}{2} - \sqrt{2}\right)$$
$$= 0$$
$$\Rightarrow \left(x + \frac{1}{2}\right)^2 + \left(y + \frac{3}{2}\right)^2 - 2 = 0$$
$$\Rightarrow x^2 + \frac{1}{4} + x + y^2 + \frac{9}{4} + 3y - 2 = 0$$
$$\Rightarrow x^2 + y^2 + x + 3y + \frac{1}{2} = 0$$
$$\Rightarrow 2x^2 + 2y^2 + 2x + 6y + 1 = 0$$

103. **(c)**



Given, $r_1 = 15$ unit

$r_2 = 20$ unit
$C_1C_2 = 25$ unit
Let $\angle AC_2D = \theta$
Then, in right angled triangle ADC_2,
$\Rightarrow AD = r_2 \sin\theta$
$\frac{AD}{r_2} = \sin\theta$ …….. (i)

Now, in right angled triangle ADC_1
$\Rightarrow AD = r_1 \sin(90 - \theta)$
$\frac{AD}{r_1} = \cos\theta$….. (ii)

On squaring and adding Eqs. (i) and (ii), we get

$$AD^2\left[\frac{1}{r_1^2} + \frac{1}{r_2^2}\right] = 1$$

$$\Rightarrow AD^2\left(\frac{r_1^2 + r_2^2}{r_1^2 r_2^2}\right) = 1$$

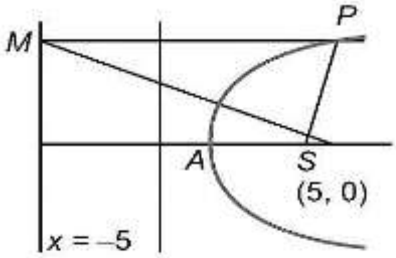
$$\Rightarrow AD^2 = \frac{r_1^2 r_2^2}{r_1^2 + r_2^2}$$

$$\Rightarrow AD^2 = \frac{225 \times 400}{225 + 400}$$

$$\Rightarrow AD = \frac{15 \times 20}{25} = 12$$

Thus, length of common chord $= 2AD$
$= 2 \times 12$
$= 24$ unit

104. **(c)** Given, that the $\triangle$ SPM (which is shown in figure) is equilateral.
Also, given parabola is $y^2 = 8(x - 3)$ focus of this parabola is $S(5,0)$ and vertex $A(3,0)$.



Let coordinate of $P(h + at^2, k + 2at)$
$= P(3 + 2t^2, 4t)$
Then, coordinate of $M(-5, 4t)$.
We know that the side of this equilateral triangle is $4a = 4 \times 2 = 8$
Now, $PS = 8$

$$\sqrt{(3 + 2t^2 - 5)^2 + (4t)^2} = 8$$

$$\Rightarrow \sqrt{(2t^2 - 2)^2 + (4t)^2} = 8$$

$$\Rightarrow \sqrt{(2t^2 + 2)^2} = 8$$

$$\Rightarrow 2t^2 + 2 = 8$$

$$\Rightarrow 2t^2 = 6$$

$$\Rightarrow t = \sqrt{3}$$

$$\therefore P(3 + 2 \times 3, 4 \times \sqrt{3}) = P(9, 4\sqrt{3})$$

105. **(c)** Since, the equation of the hyperbola differs from that of the joint equations of the tangents by a constant, therefore the equation of the hyperbola will be of the form
$(4x + 3y - 7)(x - 2y - 1) + k = 0$ ….. (i)
$\therefore$ Since, the hyperbola passes through the point (2,3).

$\Rightarrow \quad (8+9-7)(2-6-1)+k=0$

$\Rightarrow \quad (10)(-5)+k=0, -50+k=0$

$\Rightarrow \quad k=50$

Hence, from Eq. (i)

$(4x+3y-7)(x-2y-1)+50=0$

$4x^2+3xy-7x-8xy-6y^2+14y-4x$

$-3y+7+50=0$

$4x^2-5xy-6y^2-11x+11y+57=0$

106. **(b)** Let $(a\sec\theta, b\tan\theta)$ be any point on the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$

The equation of the asymptotes of the given hyperbola are

$\left(\frac{x}{a}+\frac{y}{b}\right)=0$ and $\left(\frac{x}{a}-\frac{y}{b}\right)=0$

Now, P_1 = length of the perpendicular from $(a\sec\theta, b\tan\theta)$ on

$\left(\frac{x}{a}+\frac{y}{b}\right)=0$

$P_1=\frac{\sec\theta+\tan\theta}{\sqrt{\frac{1}{a^2}+\frac{1}{b^2}}}$ .... (i)

P_2 = length of the perpendicular from $(a\sec\theta, b\tan\theta)$ on

$\left(\frac{x}{a}-\frac{y}{b}\right)=0$

$\Rightarrow P_2=\frac{\sec\theta-\tan\theta}{\sqrt{\frac{1}{a^2}+\frac{1}{b^2}}}$ ... (ii)

$\therefore \; P_1P_2=\frac{(\sec\theta+\tan\theta)}{\sqrt{\frac{1}{a^2}+\frac{1}{b^2}}}\cdot\frac{(\sec\theta-\tan\theta)}{\sqrt{\frac{1}{a^2}+\frac{1}{b^2}}}$

$=\frac{(\sec^2\theta-\tan^2\theta)}{\left(\frac{1}{a^2}+\frac{1}{b^2}\right)}=\frac{1}{\frac{(a^2+b^2)}{a^2b^2}}$

$\Rightarrow \; P_1P_2=\frac{a^2b^2}{a^2+b^2}$

107. **(c)** Given, conjugate lines are

$2x+3y+12=0$ … (i)

and $x-y+k=0$ … (ii)

We know, that two lines are said to be conjugate with respect to a curve, if each passes through the pole of the polar of that curve.

Let (x_1, y_1) be the pole of parabola

$y^2=8x$

It's polar is $yy_1=4(x+x_1)$

$\Rightarrow \; 4x-y_1y+4x_1=0$

$\Rightarrow \; 2x-\left(\frac{y_1}{2}\right)y+2x_1=0$ .... (iii)

On comparing Eqs. (i) and (iii), we get

$\frac{-y_1}{2}=3\Rightarrow y_1=-6$ and $2x_1=12\Rightarrow x_1=6$

$\therefore$ Pole $(x_1, y_1)=(6,-6)$

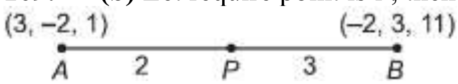
Eq. (ii) also passes through pole $(6,-6)$

$\therefore \quad 6-(-6)+k=0$

$\Rightarrow \quad k=-12$

108. **(c)**

109. **(b)** Let require point is P, then by section formula the coordinate of point P are



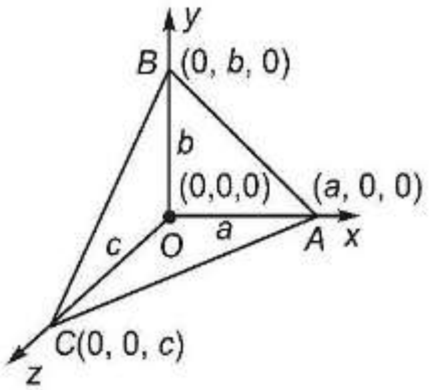
$\Rightarrow P\left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}, \frac{mz_2+nz_1}{m+n}\right)$
$\Rightarrow \ P\left(\frac{-4+9}{2+3}, \frac{6-6}{2+3}, \frac{22+3}{2+3}\right)$
$\Rightarrow \ P\left(\frac{5}{5}, \frac{0}{5}, \frac{25}{5}\right)$
$\Rightarrow \ P(1,0,5)$

110. **(b)** Given equation $x^3-6x^2+11x-6=0$
has the roots $\alpha, \beta\gamma$.
Given,

$a=\alpha^2+\beta^2+\gamma^2 \quad \ldots.(i)$
$b=\alpha\beta+\beta\gamma+\gamma\alpha \quad \ldots..(ii)$
$c=(\alpha+\beta)(\beta+\gamma)(\gamma+\alpha) \quad \ldots(iii)$

In cubic equation the sum of the roots
$\alpha+\beta+\gamma=-\left(\frac{-6}{1}\right)=6$
$\alpha\beta+\beta\gamma+\gamma\alpha=\left(\frac{11}{1}\right)=11$
product of the roots
$\alpha\cdot\beta\cdot\gamma=-\left(\frac{-6}{1}\right)=6$
From Eq. (ii), $b=11$
From Eq. (i), $a=\alpha^2+\beta^2+\gamma^2$
$\Rightarrow \quad a=(\alpha+\beta+\gamma)^2-2(\alpha\beta+\beta\gamma+\gamma\alpha)$
$\Rightarrow \quad a=(6)^2-2(11) \Rightarrow 36-22$
$\Rightarrow \quad a=14$
From Eq. (iii)
$c=(\alpha+\beta)(\beta+\gamma)(\gamma+\alpha)$
$=(\alpha\beta+\beta^2+\alpha\gamma+\beta\gamma)(\gamma+\alpha)$
$=\alpha\beta\gamma+\beta^2\gamma+\alpha\gamma^2+\beta\gamma^2+\alpha^2\beta$
$+\alpha\beta^2+\alpha^2\gamma+\alpha\beta\gamma$
$\Rightarrow c=[(\alpha+\beta+\gamma)-\gamma][(\alpha+\beta+\gamma)-\alpha]$
$[(\alpha+\beta+\gamma)-\beta]$
$=(6-\gamma)(6-\alpha)(6-\beta)$
$=(36-6\gamma-6\alpha+\alpha\gamma)(6-\beta)$
$=216-36\gamma-36\alpha+6\alpha\gamma-36\beta+6\gamma\beta$
$+6\alpha\beta-\alpha\beta\gamma$
$=216-\alpha\beta\gamma+6(\alpha\beta+\beta\gamma+\gamma\alpha)-36(\alpha+\beta+\gamma)$
$=216-6+6(11)-36(6)$
$=210+66-216=60$
Hence, $c=60$
$\Rightarrow \ b<a<c$

111. **(b)** The equation of the plane meets the coordinate axes at A, B, C is



$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$ ...... (i)

Let $OA=a, OB=b, OC=c$

Also, the centroid of $\triangle ABC$ is $\left(\frac{a}{3},\frac{b}{3},\frac{c}{3}\right)$.

which is equal to (1,2,4)

ie,

$$\frac{a}{3}=1 \Rightarrow a=3$$

$$\frac{b}{3}=2 \Rightarrow b=6$$

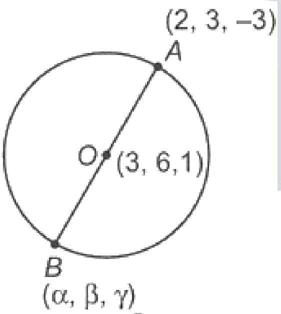
$$\frac{c}{3}=4 \Rightarrow c=12$$

From Eq. (i), $\frac{x}{3}+\frac{y}{6}+\frac{z}{12}=1$

or $4x+2y+z=12$

112. **(b)** The equation of the sphere is

$$x^2+y^2+z^2-6x-12y-2z+20=0$$



So, the centre of the sphere is

$(-u,-v,-w)=(3,6,1)$

given the one end of the diameter is $(2,3,-3)$, let the other end of the diameter is (α,β,γ). Since, O is the mid point of the diameter, then $(3,6,1)=\left[\frac{\alpha+2}{2},\frac{\beta+3}{2},\frac{\gamma-3}{2}\right]$

$$\Rightarrow \frac{\alpha+2}{2}=3, \alpha=4$$

$$\Rightarrow \frac{\beta+3}{2}=6, \beta=9$$

$$\Rightarrow \frac{\gamma-3}{2}=1, \gamma=5$$

Hence, the other end is (4, 9, 5).

113. **(a)** $\lim_{x\to 0}\frac{\tan x-\sin x}{x^2}$, $\left(\text{form } \frac{0}{0}\right)$

By 'L' Hospital Rule

$$\lim_{x\to 0}\frac{\sec^2 x-\cos x}{2x}, \left(\text{form } \frac{0}{0}\right)$$

Again, by 'L' Hospital Rule

$$\lim_{x\to0}\frac{2\sec x\cdot\sec x\cdot\tan x+\sin x}{2}$$
$$=\frac{2\cdot1\cdot1\cdot0+0}{2}=\frac{0}{2}=0$$

114. (b)

$$f(x)=\begin{cases}\dfrac{1+3x^2-\cos 2x}{x^2}, \text{ for } x\neq 0\\ \quad k, \text{ for } x=0\end{cases}$$

RHL

$$f(0+h)=\lim_{h\to0}\frac{1+3(0+h)^2-\cos 2(0+h)}{(0+h)^2}$$
$$=\lim_{h\to0}\frac{1+3h^2-\cos 2h}{h^2}$$
$$=\lim_{h\to0}\frac{1+3h^2-(1-2\sin^2 h)}{h^2}$$
$$=\lim_{h\to0}\frac{1+3h^2-1+2\sin^2 h}{h^2}$$
$$=\lim_{h\to0}\left\{3+2\left(\frac{\sin^2 h}{h^2}\right)\right\}$$
$$=3+2\cdot\lim_{h\to0}\left(\frac{\sin h}{h}\right)^2$$
$$=3+2\cdot(1)^2\left\{\because \lim_{x\to0}\frac{\sin x}{x}=1\right\}$$
$$=3+2=5$$

LHL

$$f(0-h)=\lim_{h\to0}\frac{1+3(0-h)^2-\cos 2(0-h)}{(0-h)^2}$$
$$=\lim_{h\to0}\frac{1+3h^2-\cos 2h}{h^2}$$
$$=5$$

Since, the function is continuous at $x=0$, then
$LHL=RHL=f(0)\ \because f(0)=k\Rightarrow k=5$

115. (b)

116. (c)

$$y=\cos^{-1}\left(\frac{a^2-x^2}{a^2+x^2}\right)+\sin^{-1}\left(\frac{2ax}{a^2+x^2}\right)$$

Put $x=a\tan\theta$

$$\Rightarrow\ \theta=\tan^{-1}\left(\frac{x}{a}\right)$$

$$\Rightarrow\ y=\cos^{-1}\left(\frac{a^2-a^2\tan^2\theta}{a^2+a^2\tan^2\theta}\right)$$

$$+\sin^{-1}\left(\frac{2a^2\tan\theta}{a^2+a^2\tan^2\theta}\right)$$

$$\Rightarrow y=\cos^{-1}\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right)+\sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right)$$

$$\Rightarrow y=\cos^{-1}(\cos 2\theta)+\sin^{-1}(\sin 2\theta)$$

$$\left[\begin{array}{l}\because \cos 2\theta=\dfrac{1-\tan^2\theta}{1+\tan^2\theta}\\ \sin^2\theta=\dfrac{2\tan\theta}{1+\tan^2\theta}\end{array}\right]$$

$$\Rightarrow\ y=2\theta+2\theta$$

$$\Rightarrow\ y=4\theta$$

$$\Rightarrow\ y=4\tan^{-1}\left(\frac{x}{a}\right)$$

$$\Rightarrow\frac{dy}{dx}=4\cdot\frac{1}{1+\frac{x^2}{a^2}}\cdot\frac{1}{a}=4\cdot\frac{a^2}{a^2+x^2}\cdot\frac{1}{a}$$

$$\Rightarrow\ \frac{dy}{dx}=\frac{4a}{a^2+x^2}$$

117. **(b)**

$f(x)=\sin x+\cos x$

$f'(x)=\cos x-\sin x$

$f''(x)=-\sin x-\cos x$

$f'''(x)=-\cos x+\sin x$

$f''''(x)=\sin x+\cos x$

So, $f\left(\frac{\pi}{4}\right)=f''''\left(\frac{\pi}{4}\right)=\sin\left(\frac{\pi}{4}\right)+\cos\left(\frac{\pi}{4}\right)$

$=\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}=\frac{2}{\sqrt{2}}=\sqrt{2}$

Then, $f\left(\frac{\pi}{4}\right)f''''\left(\frac{\pi}{4}\right)=\sqrt{2}\times\sqrt{2}=2$

118. **(b)** $y=\sin(m\sin^{-1}x)$

$$y_1=\cos(m\sin^{-1}x)\cdot m\cdot\frac{1}{\sqrt{1-x^2}}$$

where $\left(y_1=\frac{dy}{dx}\right)$

$$y_1\sqrt{1-x^2}=m\cos(m\sin^{-1}x)$$

$$\Rightarrow\ y_2\sqrt{1-x^2}+y_1\frac{1}{2\sqrt{1-x^2}}\cdot(-2x)$$

$$=-m\sin(m\sin^{-1}x)\cdot\frac{m}{\sqrt{1-x^2}}$$

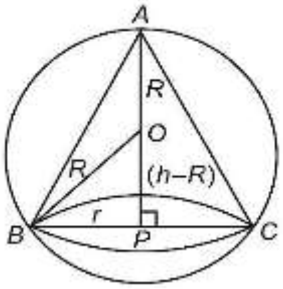
$$\left(\because y_2=\frac{d^2y}{dx^2}\right)$$

$$\Rightarrow y_2\sqrt{1-x^2}-\frac{xy_1}{\sqrt{1-x^2}}$$

$$=\frac{-m^2}{\sqrt{1-x^2}}\sin(m\sin^{-1}x)$$

$$\Rightarrow\ y_2(1-x^2)-xy_1=-m^2y$$

119. **(c)** Let the height of the cone = h and the radius of the cone = r Given, radius of the sphere = R Now, In △ OPB



$\Rightarrow \ R^2 = r^2 + (h - R)^2$
$\Rightarrow \ r^2 = R^2 - (h - R)^2$
$= (R + h - R)(R - h + R)$
$\Rightarrow \ r^2 = h(2R - h)$

The volume of the cone is

$$V = \frac{1}{3}\pi r^2 h$$

$$\Rightarrow \quad V = \frac{1}{3}\pi h(2R - h)h$$

$$\Rightarrow \quad V = \frac{\pi}{3}(2Rh^2 - h^3)$$

Differentiating with r to h

$$\frac{dV}{dh} = \frac{\pi}{3}(4Rh - 3h^2)$$

For maximum or minimum value of volume

$$\frac{dV}{dh} = 0$$

$$\Rightarrow \quad \frac{\pi}{3}(4Rh - 3h^2) = 0$$

$$\Rightarrow \quad h(4R - 3h) = 0$$

$$\Rightarrow \quad h = 0, h = \frac{4R}{3}$$

(Not possible)

Now, $\frac{d^2V}{dh^2} = \frac{\pi}{3}(4R - 6h)$

$$\left(\frac{d^2V}{dh^2}\right)_{\left(\text{at } h=\frac{4R}{3}\right)} = \frac{\pi}{3}\left(4R - 6 \cdot \frac{4R}{3}\right)$$

$$= \frac{\pi}{3}(4R - 8R) = -\frac{4\pi}{3}R \Rightarrow \text{ Negative}$$

ie., Maximum

Hence, the height of the cone of maximum volume is $\left(\frac{4R}{3}\right)$.

120. **(c)** Given, curve is

$$2x^2 + y^2 = 2x$$

$$2x^2 - 2x + y^2 = 0$$

$$\Rightarrow \ 2\left(x - \frac{1}{2}\right)^2 + y^2 = \frac{1}{2}$$

$$\Rightarrow \ \frac{\left(x - \frac{1}{2}\right)^2}{\frac{1}{4}} + \frac{y^2}{\frac{1}{2}} = 1$$

which represents an ellipse.

Here, $a = \frac{1}{2}, b = \frac{1}{\sqrt{2}}, h = \frac{1}{2}, k = 0$

Consider a point $P(h + a\cos\theta, k + b\sin\theta)$

$= P\left(\frac{1}{2} + \frac{1}{2}\cos\theta, \frac{1}{\sqrt{2}}\sin\theta\right)$ on the ellipse from which the distance of point $(a, 0)$ is maximum. let $Q(a, 0)$

Now,

$$PQ = \sqrt{\left(\frac{1}{2}+\frac{1}{2}\cos\theta - a\right)^2 + \left(\frac{1}{\sqrt{2}}\sin\theta - 0\right)^2}$$

$$= \sqrt{\begin{gathered}\frac{1}{4}+\frac{1}{4}\cos^2\theta + a^2 + \frac{1}{2}\cos\theta \\ -a\cos\theta - a + \frac{1}{2}\sin^2\theta\end{gathered}}$$

$$PQ = \sqrt{\frac{1}{2} + a^2 - a + \left(\frac{1}{2} - a\right)\cos\theta + \frac{1}{4}\sin^2\theta}$$

$\Rightarrow$ Let $y = PQ^2$

$$= \frac{1}{2} + a^2 - a + \left(\frac{1}{2} - a\right)\cos\theta + \frac{1}{4}\sin^2\theta$$

For maxima and minima, put $\frac{dy}{d\theta} = 0$

$$\Rightarrow -\left(\frac{1}{2} - a\right)\sin\theta + \frac{1}{4}\cdot 2\sin\theta\cos\theta = 0$$

$$\Rightarrow \sin\theta\left(-\frac{1}{2} + a + \frac{1}{2}\cos\theta\right) = 0$$

$$\Rightarrow \sin\theta = 0 \text{ or } -\frac{1}{2} + a + \frac{1}{2}\cos\theta = 0$$

$$\Rightarrow \theta = 0 \text{ or } \cos\theta = 1 - 2a$$

$$\Rightarrow \sin^2\theta = 1 - \cos^2\theta$$

$$= 1 - (1 - 2a)^2$$

$$= -4a^2 + 4a$$

Now, $\frac{d^2y}{d\theta^2} < 0$ for $\cos\theta = 1 - 2a$

Thus, distance PQ is maximum, when

$\cos\theta = 1 - 2a$ and $\sin^2\theta = -4a^2 + 4a$

Now, required longest distance is

$$= \sqrt{\frac{1}{2} + a^2 - a + \left(\frac{1}{2} - a\right)(1 - 2a) + \frac{1}{4}(-4a^2 + 4a)}$$

$$= \sqrt{\frac{1}{2} + a^2 - a + \frac{1}{2} - a - a + 2a^2 - a^2 + a}$$

$$= \sqrt{2a^2 - 2a + 1}$$

$$= \sqrt{1 - 2a + 2a^2}$$

121. **(b)** Since, m_1 and m_2 are the roots of the equation $x^2 + (\sqrt{3} + 2)x + (\sqrt{3} - 1) = 0$.

$$m_1 + m_2 = -(\sqrt{3} + 2),$$

$$m_1 m_2 = \sqrt{3} - 1$$

then $\therefore m_1 - m_2 = \sqrt{(m_1 + m_2)^2 - 4m_1m_2}$

$$= \sqrt{(3 + 4 + 4\sqrt{3} - 4\sqrt{3} + 4)}$$

$$= \sqrt{11}$$

and coordinates of the vertices of the given triangles are $(0,0)$, $(c/m_1, c)$ and $(c/m_2, c)$.

Hence, the required area of triangle

$$=\frac{1}{2}\left\|\begin{vmatrix} 0 & 0 & 1 \\ \frac{c}{m_1} & c & 1 \\ \frac{c}{m_2} & c & 1 \end{vmatrix}\right\| = \frac{1}{2}c^2\left|\left(\frac{1}{m_1}-\frac{1}{m_2}\right)\right|$$

$$=\frac{1}{2}c^2\frac{m_2-m_1\mid}{m_1m_2}$$

$$=\frac{1}{2}c^2\frac{\sqrt{11}}{(\sqrt{3}-1)}$$

$$=\frac{1}{2}c^2\cdot\frac{\sqrt{11}\cdot(\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}+1)}$$

$$=\left(\frac{\sqrt{33}+\sqrt{11}}{4}\right)\cdot c^2$$

122. **(d)** $u=\sin^{-1}\left(\frac{x^4+y^4}{x+y}\right)$

Let $v=\sin u=\frac{x^4+y^4}{x+y}$, here degree is homogeneous, so $n=4-1=3$

By Euler's theorem, we have to prove that,

$$x\frac{\partial v}{\partial x}+y\frac{\partial v}{\partial y}=3v$$

$$x\frac{\partial}{\partial x}(\sin u)+y\frac{\partial v}{\partial y}(\sin u)=3\sin u$$

$$x\cos u\cdot\frac{\partial u}{\partial x}+y\cos u\frac{\partial u}{\partial y}=3\sin u$$

$$x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=3\frac{\sin u}{\cos u}$$

$$x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=3\tan u$$

123. **(a)** $\int\frac{7x^8+8x^7}{(1+x+x^8)^2}dx=f(x)+c$

here better way to solve this integration we will check the options ie,
we will differentiate the option one by one and check the integration function
Taking option (a) we see that,

$$f(x)=\frac{d}{dx}\left\{\frac{x^8}{(1+x+x^8)}\right\}$$

$$=\frac{[(1+x+x^8)\cdot 8x^7-x^8(1+8x^7)]}{(1+x+x^8)^2}$$

$$f(x)=\frac{(8x^7+8x^8+8x^{15}-x^8-8x^{15})}{(1+x+x^8)^2}$$

$$f(x)=\frac{(7x^8+8x^7)}{(1+x+x^8)^2}$$

Hence, $\int\frac{7x^8+8x^7}{(1+x+x^8)^2}dx=\frac{x^8}{(1+x+x^8)}+c$

124. **(a)** $f_n(x)=\log\cdot\log\cdot\log\ldots\cdot\log x$ (upto n times)

$f_1(x)=\log x$

$f_2(x)=\log\log x$

$f_3(x)=\log\log\log x$

$f_{n-1}(x)=\log\log\log\ldots\ldots\log x$

(up to ($n-1$) times)

Now, $\int(xf_1(x)f_2(x)\ldots f_n(x))^{-1}dx$

$$=\int\frac{dx}{[xf_1(x)f_2(x)\ldots f_n(x)]}$$

$$= \int \frac{[xf_1(x)f_2(x) \dots f_{n-1}(x)]dt}{[xf_1(x)f_2(x) \dots f_{n-1}(x)] \cdot t} = \int \frac{dt}{t}$$

$= \log t + c$

$$\left[\begin{array}{ll} \text{Put,} & f_n(x) = t \\ \dfrac{dt}{dx} & = \dfrac{1}{[f_{n-1}(x)f_{n-2}(x) \dots f_1(x) \cdot x]} \\ dx & = [xf_1(x) \cdot f_2(x) \dots f_{n-1}(x)]dt \end{array}\right]$$

$= \log f_n(x) + c$

$= f_{n+1}(x) + c$

Hence,

$$\int (xf_1(x)f_2(x) \dots f_n(x))^{-1}dx = f_{n+1}(x) + c$$

125. (a)

$\int (1 - \cos x)\operatorname{cosec}^2 x dx$

$$= \int \left(2\sin^2\frac{x}{2}\right) \cdot \frac{1}{\sin^2 x}dx$$

$\left[\because \cos x = 1 - 2\sin^2\frac{x}{2}\right]$

$$= 2\int \frac{\sin^2\frac{x}{2}}{\left(2\sin\frac{x}{2} \cdot \cos\frac{x}{2}\right)^2}dx$$

$\left[\because \sin x = 2\sin\frac{x}{2} \cdot \cos\frac{x}{2}\right]$

$$= 2\int \frac{\sin\frac{x}{2} \cdot \sin\frac{x}{2}}{2 \cdot 2\sin^2\frac{x}{2} \cdot \cos^2\frac{x}{2}}dx$$

$$= \frac{1}{2}\int \sec^2\frac{x}{2}dx = \frac{1}{2}\tan\frac{x}{2} \cdot 2 + c$$

$$= \tan\frac{x}{2} + c = f(x) + c$$

$$\Rightarrow f(x) = \tan\frac{x}{2}$$

126. (c) $I_n = \int_0^{\pi/4} \tan^n xdx$

We have, $I_{r+2} = \int_0^{\pi/4} \tan^{r+2} xdx$

$$= \int_0^{\pi/4} \tan^r x \cdot \tan^2 xdx$$

and

$$I_r = \int_0^{\pi/4} \tan^r xdx$$

Then, $I_r + I_{r+2} = \int_0^{\pi/4} \tan^r xdx$

$$+ \int_0^{\pi/4} \tan^r x \cdot \tan^2 xdx$$

$$= \int_0^{\pi/4} \tan^r x(1 + \tan^2 x)dx$$

$$= \int_0^{\pi/4} \tan^r x \cdot \sec^2 xdx$$

[Put $t = \tan x \Rightarrow dt = \sec^2 xdx$]

$= \int_0^1 t^r dt \Rightarrow \left[\frac{t^{r+1}}{r+1}\right]_0^1$

$= \left(\frac{1}{r+1}\right)$

So,

$I_r + I_{r+2} = \frac{1}{r+1}$

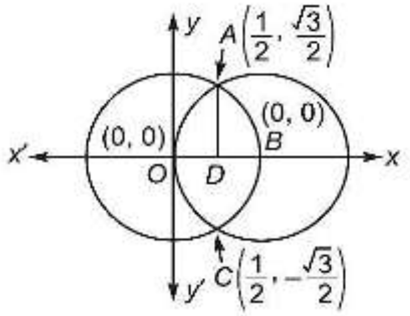
ie, $I_2 + I_4 = \frac{1}{3}$

$I_3 + I_5 = \frac{1}{4}$

$I_4 + I_6 = \frac{1}{5}$

which are clearly in HP.

127. **(c)** Intersection point of two circles



$x^2 + y^2 = 1$ .... (i)

$(x-1)^2 + y^2 = 1$ .... (ii)

is given by

$(x-1)^2 + (1-x^2) = 1$

$\Rightarrow x^2 + 1 - 2x - x^2 = 0$

$\Rightarrow x = \frac{1}{2}$

From Eq. (i), $\frac{1}{4} + y^2 = 1$

$y^2 = 1 - \frac{1}{4} \Rightarrow y = \pm\frac{\sqrt{3}}{2}$

Point A $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ and C $\left(\frac{1}{2}, \frac{-\sqrt{3}}{2}\right)$

So, Area of region OABCO

$=$ 2 × Area of region OABDO ... (i)

Area of OABDO

$=$ Area of OADO + Area of ABDA

$$= \int_0^{1/2} \sqrt{1-(x-1)^2}dx + \int_{1/2}^1 \sqrt{1-x^2}dx$$

$$= \left[\frac{1}{2}\cdot(x-1)\sqrt{1-(x-1)^2} + \frac{1}{2}\sin^{-1}\left(\frac{x-1}{1}\right)\right]_0^{1/2}$$

$$+ \left[\frac{1}{2}x\sqrt{1-x^2} + \frac{1}{2}\sin^{-1}\left(\frac{x}{1}\right)\right]_{1/2}^1$$

$$= \left[-\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{\sqrt{3}}{2} + \frac{1}{2}\sin^{-1}\left(\frac{-1}{2}\right) - \frac{1}{2}\cdot 0 - \frac{1}{2}\sin^{-1}(-1)\right]$$

$$+ \left[\frac{1}{2}\cdot 0 + \frac{1}{2}\sin^{-1}(1) - \frac{1}{4}\cdot\frac{\sqrt{3}}{2} - \frac{1}{2}\sin^{-1}\left(\frac{1}{2}\right)\right]$$

$$= \left(-\frac{\sqrt{3}}{8}\right) - \frac{1}{2}\sin^{-1}\left(\frac{1}{2}\right) + \frac{1}{2}\sin^{-1}(1)$$

$$+\frac{1}{2}\sin^{-1}(1) - \frac{\sqrt{3}}{8} - \frac{1}{2}\sin^{-1}\left(\frac{1}{2}\right)$$

$$= \sin^{-1}(1) - \sin^{-1}\left(\frac{1}{2}\right) - \frac{\sqrt{3}}{4}$$

$$= \frac{\pi}{2} - \frac{\pi}{6} - \frac{\sqrt{3}}{4}$$

$$= \left(\frac{\pi}{3} - \frac{\sqrt{3}}{4}\right)$$

from Eq. (i),

$$\text{Area of region OABCO} = 2 \times \left(\frac{\pi}{3} - \frac{\sqrt{3}}{4}\right)$$

$$= \left(\frac{2\pi}{3} - \frac{\sqrt{3}}{2}\right)$$

128. **(c)**

x	0	1	2	3	4	5
f(x)	2	3	6	11	18	27

h = difference of two values of x

Take value of $f(x)$ as $(y_0, y_1, y_2, \ldots, y_5)$

Then by Trapezoidal rule

Now, $\int_{x_0}^{x_0+nh} f(x)dx$

$$= \frac{h}{2}[(y_0 + y_5) + 2(y_1 + y_2 + y_3 + y_4)]$$

$$= \frac{1}{2}[(2+27) + 2(3+6+11+18)]$$

$$= \frac{1}{2}[29 + 2\cdot 38] = \frac{1}{2}(29+76)$$

$$= \frac{1}{2} \times 105 = 52.5$$

Approximate area = 52.5 sq unit

129. **(b)** $\tan y \frac{dy}{dx} = \sin(x+y) + \sin(x-y)$

$$\tan y\frac{dy}{dx}=2\cdot\sin\left(\frac{2x}{2}\right)\cdot\cos\left(\frac{2y}{2}\right)$$
$$\left[\because \sin C+\sin D=2\sin\left(\frac{C+D}{2}\right)\cdot\cos\left(\frac{C-D}{2}\right)\right]$$
$$\Rightarrow \tan y\frac{dy}{dx}=2\sin x\cdot\cos y$$
$$\Rightarrow \frac{\sin y}{\cos y}\frac{dy}{dx}=2\sin x\cos y$$
On integrating
$$\Rightarrow \int\frac{\sin y}{\cos^2 y}dy=\int 2\sin x dx$$
$$\Rightarrow \begin{bmatrix}\text{Let } t=\cos y\\ \frac{dt}{dy}=-\sin y\\ -dt=\sin y dy\end{bmatrix}$$
$$\Rightarrow -\int\frac{dt}{t^2}=2(-\cos x)+c$$
$$\Rightarrow -\left(-\frac{1}{t}\right)=-2\cos x+c$$
$$\Rightarrow \frac{1}{\cos y}=-2\cos x+c$$
$$\Rightarrow \sec y=-2\cos x+c$$
130. (d)
$$xy\frac{dy}{dx}=2y^2-x^2$$
$$\frac{dy}{dx}=\frac{2y}{x}-\frac{x}{y}$$
$$\frac{dy}{dx}-\frac{2y}{x}=\frac{-x}{y}\Rightarrow y\frac{dy}{dx}\frac{-2y^2}{x}=-x \quad \text{....(i)}$$
Put, $v=y^2$ .... (ii)
$$\Rightarrow\left(\frac{dv}{dx}=2y\frac{dy}{dx}\right)\Rightarrow\left(\frac{1}{2}\frac{dv}{dx}=y\frac{dy}{dx}\right)$$
From Eq. (i) $\frac{1}{2}\frac{dv}{dx}-\frac{2v}{x}=-x$
$$\frac{dv}{dx}-\frac{4v}{x}=-2x$$
$$\text{IF }=e^{\int p dx}=e^{\int -\frac{4}{x}dx}=e^{-4\log x}$$
$$=e^{\log x^{-4}}=x^{-4}$$
Complete solution is
$$x^{-4}\cdot v=\int(-2x)\cdot x^{-4}dx+c$$
$$\frac{v}{x^4}=-2\int\frac{dx}{x^3}+c\Rightarrow\frac{v}{x^4}=\frac{1}{x^2}+c$$
$v=x^2+cx^4$
$y^2=cx^4+x^2$ [from Eq. (ii)]
which is the required family of curves.
131. (d) $f(0)=0, f(1)=1, f(2)=2$
Given,
$f(x)=f(x-2)+f(x-3), x=3,4,5,\ldots$
The given function is known as "Reccurrence function".

put $x=3, f(3)=f(1)+f(0)$

$=1+0=1$

put $x=4, f(4)=f(2)+f(1)$

$=2+1 \Rightarrow 3$

put $x=5, f(5)=f(3)+f(2)$

$=1+2 \Rightarrow 3$

put $x=6, f(6)=f(4)+f(3)$

$=3+1 \Rightarrow 3+1 \Rightarrow 4$

put $x=7, f(7)=f(5)+f(4)$

$=3+3 \Rightarrow 6$

put $x=8, f(8)=f(6)+f(5)$

$=3+4 \Rightarrow 7$

put $x=9, f(9)=f(7)+f(6)$

$=6+4 \Rightarrow 10$

Hence, $f(9)=10$

132. (c)

Given, $f(x)=-x^2, f: A \rightarrow B$

where, A and $B \in R$

$R \rightarrow$ Real No's $x \in A$

Here, the domain of the function is positive real number only

ie, Domain $=A \in R^+$

Example $\{1,2,3, \ldots\}$ and $B=\{1,4,9, \ldots\}$

Both set have $f: A \rightarrow B$ unique image.

But, $A=\{-1,1,2, \ldots\}$ and $B=\{1,1,4, \ldots\}$

In $f: A \rightarrow B$ have not unique image and for Range $x^2=y, x=\pm\sqrt{y}$

ie, (Range $\in R^+$)

In onto function the (Range = Co-domain = B) ie, $(B \in R^+)$

So,

(A) f is one-one and onto, if $A=B=R^+$

(B f is one-one but not onto, if $A=R^+, B=R$

(C) f is onto but not one-one, if $A=R, B=R^+$

(D) f is neither one-one nor onto, if $A=B=R$ Hence, the answer is

$(A) \rightarrow 4,(B) \rightarrow 1,(C) \rightarrow 3,(D) \rightarrow 2$

133. (d) Given, $a_n=6^n-5n, n=1,2,3, \ldots$

We take; $6^n=(1+5)^n$

Expand with binomial expansion

$6^n={ }^nC_0+{ }^nC_1 5+{ }^nC_2 5^2+{ }^nC_3 5^3+\cdots$

$6^n=1+n \cdot 5+{ }^nC_2 25+{ }^nC_3 5^3+\cdots$

$(6^n-5n)=1+25\{{ }^nC_2+{ }^nC_3 \cdot 5+\cdots\}$

$(6^n-5n)=1+25 \cdot k$

where $k=$ positive integer.

Hence, $a_n=6^n-5n$ divided by 25 and leave the remainder $=1$

134. (b) Given, $n=1!+4!+7!+\cdots+400!$

$1!=1, 4!=24, 7!=5040, 10!=3628800$

and further the terms has last two digits

$=00$

So,

$1!+4!+7!+\cdots+400!=\cdots \ldots .65$

Here ten digit is 6 .

Hence, 6 is the answer.

135. (c) Given, $a_n=\frac{10^n}{n!}, n=1,2,3, \ldots$, here we see that when we increase the value of n like as 1,2,3, ... the value of a_n increases but when we reach at $n=9$ or 10 the value of a_n remain unchanged, ie, minor difference in after decimal places and when we cross the value $n=10$ie, $n=11$, then we see that the value of a_n is monotonically decreasing.

Hence, a_n have its maximum value at $n = 9$ or 10.

136. **(d)** Number of diagonals in polygon = 54

Let the number of sides in polygon = n

Then,

The number of diagonal in any polygon

$$= \frac{n(n-3)}{2}$$

$$54 = \frac{n(n-3)}{2}$$

$$n^2 - 3n = 108$$

$$n^2 - 3n - 108 = 0$$

$$n^2 - 12n + 9n - 108 = 0$$

$$n(n-12) + 9(n-12) = 0$$

$$(n+9)(n-12) = 0$$

$$n = -9 \text{ or } n = 12$$

(not possible)

Hence, the number of sides in polygon is $n = 12$

137. **(a)**

$(1 + 2x + 3x^2)^{10}$

$= a_0 + a_1x + a_2x^2 + \cdots + a_{20}x^{20}$

$= \{1 + x(2 + 3x)\}^{10}$

Then by binomial expansion

$= {}^{10}C_0 + {}^{10}C_1x(2 + 3x) + {}^{10}C_2x^2(2 + 3x)^2 + \cdots$

Now, the coefficient of x in this expansion

$= 2 \cdot {}^{10}C_1$

ie, $a_1 = 2 \cdot 10 = 20$

and the coefficient of x^2 in this expansion

$= 3 \cdot {}^{10}C_1 + 4 \cdot {}^{10}C_2$

ie, $a_2 = 3 \cdot 10 + 4 \cdot 45$

$= 30 + 180$

$= 210$

So, $\frac{a_2}{a_1} = \frac{210}{20} = 10.5$

138. **(a)** $|x| < \frac{1}{5}$ and $\frac{1}{(1-5x)(1-4x)}$

$= (1 - 5x)^{-1} \cdot (1 - 4x)^{-1}$

then by binomial expansion

$=\{1 + 5x + 25x^2 + 125x^3 + \cdots\}$

$\cdot \{1 + 4x + 16x^2 + 64x^3 + \cdots\}$

the coefficient of x^3 in this expansion is

$= (125 + 100 + 80 + 64)$

$= 369$

139. **(b)**

140. **(d)**

$\log_4 2 - \log_8 2 + \log_{16} 2 - \cdots$

$$= \frac{1}{\log_2 4} - \frac{1}{\log_2 8} + \frac{1}{\log_2 16} - \cdots.$$

$$\left\{\because \log_b a = \frac{1}{\log_a b}\right\}$$

$$= \frac{1}{\log_2 (2)^2} - \frac{1}{\log_2 (2)^3} + \frac{1}{\log_2 (2)^4} - \cdots.$$

$$= \frac{1}{2\log_2 2} - \frac{1}{3\log_2 2} + \frac{1}{4\log_2 2} - \cdots.$$

$(\because \log_2 2 = 1)$

$$= \left(\frac{1}{2} - \frac{1}{3} + \frac{1}{4} - \cdots.\right)$$

$$\because \log_e(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots.$$

(Put $x = 1$)

$\Rightarrow \log_e(1+1) = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \cdots.$

$\Rightarrow \frac{1}{2} - \frac{1}{3} + \frac{1}{4} - \cdots = 1 - \log_e 2$

$= 1 - \log_e 2$

141. (d)

Let $f(x) = \frac{x^2-6x+5}{x^2+2x+1}$ for $x \in R$

Let $y = \frac{x^2-6x+5}{x^2+2x+1}$

$$yx^2 + 2yx + y = x^2 - 6x + 5$$

$$(y-1)x^2 + (2y+6)x + (y-5) = 0$$

$$x = \frac{-2(y+3) + \sqrt{4(y+3)^2 - 4(y-1)(y-5)}}{2 \times (y-1)}$$

Since, x is real.

Then, its discriminant should be ≥ 0.

$\therefore\ 4(y+3)^2 - 4(y-1)(y-5) \geq 0$

$\Rightarrow\ (y+3)^2 - (y-1)(y-5) \geq 0$

$\Rightarrow y^2 + 9 + 6y - (y^2 - 6y + 5) \geq 0$

$\Rightarrow\ y^2 + 9 + 6y - y^2 + 6y - 5 \geq 0$

$\Rightarrow\ 12y + 4 \geq 0$

$\Rightarrow\ 4(3y+1) \geq 0$

$\Rightarrow\ y \geq -1/3$

So, least value of given expression is $\left(\frac{-1}{3}\right)$.

142. (d)

$$\left\{x \in R: \frac{14x}{x+1} - \frac{9x-30}{x-4} < 0\right\}$$

$$\frac{14x(x-4) - (9x-30)(x+1)}{(x+1)(x-4)} < 0$$

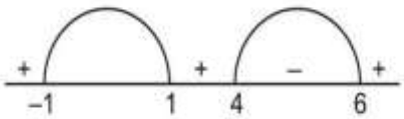
$$\frac{14x^2 - 56x - (9x^2 - 30x + 9x - 30)}{(x+1)(x-4)} < 0$$

$$\frac{(14x^2 - 56x - 9x^2 + 30x - 9x + 30)}{(x+1)(x-4)} < 0$$

$$\frac{(5x^2 - 35x + 30)}{(x+1)(x-4)} < 0$$

$$\frac{(x^2 - 7x + 6)}{(x+1)(x-4)} < 0, \frac{(x-1)(x-6)}{(x+1)(x-4)} < 0$$

Drawn number line,



Hence,

$x \in (-1,1) \cup (4,6)$

143. **(a)** Equation, $x^3 - bx^2 + cx - d = 0$

Let the roots of this cubic equation in GP are $\left(\frac{a}{r}, a, ar\right)$.

Then, Sum of the roots

$$\frac{a}{r} + a + ar = -\left(\frac{-b}{1}\right) \Rightarrow b$$

$$\Rightarrow \quad a\left(\frac{1}{r} + 1 + r\right) = b \quad \ldots (i)$$

and $\quad \frac{a}{r} \cdot a + a \cdot ar + ar \cdot \frac{a}{r} = \frac{c}{1}$

$$\Rightarrow \quad a^2\left(\frac{1}{r} + r + 1\right) = c \ldots (ii)$$

Product of the roots

$$\frac{a}{r} \cdot a \cdot ar = -\left(\frac{-d}{1}\right) = d$$

$$\Rightarrow \quad a^3 = d \ldots (iii)$$

Eq. (ii) divided by Eq. (i), we get $a = c/b$, put the value of ' a ' in Eq. (iii)

$$(c/b)^3 = d$$

$$\Rightarrow c^3 = b^3 d$$

144. **(c)** Equation $x^3 - ax^2 + ax - 1 = 0$ and $\alpha \neq 1, a \neq -1$.

we put, $\left(x = \frac{1}{y}\right)$ in given equation

Then, $\left(\frac{1}{y}\right)^3 - a\left(\frac{1}{y}\right)^2 + a\left(\frac{1}{y}\right) - 1 = 0$

$$\frac{1}{y^3} - \frac{a^2}{y^2} + \frac{a}{y} - 1 = 0$$

$$1 - a^2 y + ay^2 - y^3 = 0$$

$$\Rightarrow y^3 - ay^2 + a^2 y - 1 = 0$$

Since, the reduced equation is same as original equation by replacing $\left(x = \frac{1}{y}\right)$ ie, reciprocal root of the given equation.

Hence, $\left(x = \frac{1}{\alpha}\right)$ is a root of the given equation

145. **(b)**

$$f(x) = \begin{vmatrix} 2\cos x & 1 & 0 \\ x - \frac{\pi}{2} & 2\cos x & 1 \\ 0 & 1 & 2\cos x \end{vmatrix}$$

$$f'(x) = \begin{vmatrix} -2\sin x & 1 & 0 \\ 1 & 2\cos x & 1 \\ 0 & 1 & 2\cos x \end{vmatrix}$$

$$+\begin{vmatrix} 2\cos x & 0 & 0 \\ x-\frac{\pi}{2} & -2\sin x & 1 \\ 0 & 0 & 2\cos x \end{vmatrix}$$

$$+\begin{vmatrix} 2\cos x & 1 & 0 \\ x-\frac{\pi}{2} & 2\cos x & 0 \\ 0 & 1 & -2\sin x \end{vmatrix}$$

$$f'(\pi)=\begin{vmatrix} -2\sin\pi & 1 & 0 \\ 1 & 2\cos\pi & 1 \\ 0 & 1 & 2\cos\pi \end{vmatrix}$$

$$+\begin{vmatrix} 2\cos\pi & 0 & 0 \\ \pi-\frac{\pi}{2} & -2\sin\pi & 1 \\ 0 & 0 & 2\cos\pi \end{vmatrix}$$

$$+\begin{vmatrix} 2\cos\pi & 1 & 0 \\ \pi-\frac{\pi}{2} & 2\cos\pi & 0 \\ 0 & 1 & -2\sin\pi \end{vmatrix}$$

$$f'(\pi)=\begin{vmatrix} 0 & 1 & 0 \\ 1 & -2 & 1 \\ 0 & 1 & -2 \end{vmatrix}$$

$$+\begin{vmatrix} -2 & 0 & 0 \\ \pi/2 & 0 & 1 \\ 0 & 0 & -2 \end{vmatrix}+\begin{vmatrix} -2 & 1 & 0 \\ \pi/2 & -2 & 0 \\ 0 & 1 & 0 \end{vmatrix}$$

$$\begin{bmatrix} \because \sin\pi=0 \\ \cos\pi=-1 \end{bmatrix}$$

$$f'(\pi)=-(-2+0)=2$$

146. (a)

$$\begin{vmatrix} x & x^2 & 1+x^3 \\ y & y^2 & 1+y^3 \\ z & z^2 & 1+z^3 \end{vmatrix}=0, x\neq y\neq z$$

$$\begin{vmatrix} x & x^2 & 1 \\ y & y^2 & 1 \\ z & z^2 & 1 \end{vmatrix}+\begin{vmatrix} x & x^2 & x^3 \\ y & y^2 & y^3 \\ z & z^2 & z^3 \end{vmatrix}=0$$

$$\begin{vmatrix} x & x^2 & 1 \\ y & y^2 & 1 \\ z & z^2 & 1 \end{vmatrix}+\begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix}xyz=0$$

$C_1 \leftrightarrow C_2$ and $C_2 \leftrightarrow C_3$

$$\begin{vmatrix} x & x^2 & 1 \\ y & y^2 & 1 \\ z & z^2 & 1 \end{vmatrix}+\begin{vmatrix} x & x^2 & 1 \\ y & y^2 & 1 \\ z & z^2 & 1 \end{vmatrix}xyz=0$$

$$(1+xyz)\begin{vmatrix} x & x^2 & 1 \\ y & y^2 & 1 \\ z & z^2 & 1 \end{vmatrix}=0$$

$\because R_2 \to R_2 - R_1$

$R_3 \to R_3 - R_1$

$$(1+xyz)\begin{vmatrix} x & x^2 & 1 \\ y-x & y^2-x^2 & 0 \\ z-x & z^2-x^2 & 0 \end{vmatrix}=0$$

Expand with r to C_3

$$(1+xyz)\{(y-x)(z-x)(z+x)-(z-x)(y-x)(y+x)\}=0$$

$$(1+xyz)(y-x)(z-x)(z+x-y-x)=0$$

$$(1+xyz)(x-y)(y-z)(z-x)=0$$

$$\because x\neq y\neq z \Rightarrow xyz+1=0$$

147. (b)

148. (a)

Given, $|I+A| \neq 0$

ie, $(A+I)$ is a non-singular matrix.

$O \rightarrow$ Null Matrix

$I \rightarrow$ Unit Matrix

$\because\ I^3 = I$

$\Rightarrow\ A^3 = 0$

$\Rightarrow\ A^3 + I = O + I$

$\Rightarrow\ A^3 + I^3 = O + I$

$\Rightarrow\ (A+I)(A^2 - A + I) = (O + I)$

$(A+I)(A^2 - A + I) = I\ \because (O + I = I)$

Operate $(A+I)^{-1}$ on both sides

$\{(A+I)^{-1}(A+I)\}(A^2 - A + I)$

$= (A+I)^{-1} \cdot I$

$\Rightarrow\ I \cdot (A^2 - A + I) = (A+I)^{-1}$

$(\because I \cdot (A+I)^{-1} = (A+I)^{-1})$

$\Rightarrow\ (A+I)^{-1} = (A^2 - A + I)$

or

$(I+A)^{-1} = (I - A + A^2)$

149. (d) $z = 1 + i\sqrt{3}$

Let $1 + i\sqrt{3} = r(\cos\theta + i\sin\theta)$

Then, $r\cos\theta = 1 \ldots$ (i)

$r\sin\theta = \sqrt{3} \ldots.$ (ii)

Eq. (i) + Eq. (ii), we get

$r^2(\sin^2\theta + \cos^2\theta) = 3 + 1$

$(\because \sin^2\theta + \cos^2\theta = 1)$

$r = 4$

r = 2

Eq. (ii) ÷ Eq. (i), we get

$\tan\theta = \sqrt{3}$

$\Rightarrow\ \tan\theta = \tan\frac{\pi}{3}$

$\Rightarrow\ \theta = \frac{\pi}{3}$

So, $z = (1 + i\sqrt{3}) = 2\left\{\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right\}$,

Then $\bar{z} = (1 - i\sqrt{3}) = 2\left\{\cos\frac{\pi}{3} - i\sin\frac{\pi}{3}\right\}$

$= 2\left\{\cos\left(\frac{-\pi}{3}\right) + i\sin\left(\frac{-\pi}{3}\right)\right\}$

So, $\operatorname{Arg}z = \frac{\pi}{3}$

$\operatorname{Arg}\bar{z} = -\frac{\pi}{3}$

$|\operatorname{Arg}z| + |\operatorname{Arg}\bar{z}| = \left|\frac{\pi}{3}\right| + \left|\frac{-\pi}{3}\right|$

$= \frac{\pi}{3} + \frac{\pi}{3}$

$= \frac{2\pi}{3}$

150. (a) $\omega \rightarrow$ cube root of unity

ie, $\omega^3 = 1$ and $1 + \omega + \omega^2 = 0$

Then, $(x+1)(x+\omega)(x-\omega-1)$

$= (x+1)(x^2 + \omega x - \omega x - \omega^2 - x - \omega)$
$= (x+1)\left(x^2 - x - (\omega + \omega^2)\right)$
$= (x+1)(x^2 - x - (-1))$
$= (\because \omega + \omega^2 = -1)$
$= (x+1)(x^2 - x + 1)$
$= (x+1)(x^2 + 1 - x)$
$= (x^3 - 1)$

151. **(c)** $(\sqrt{3}+i)^7 + (\sqrt{3}-i)^7$

Let $\sqrt{3} + i = z$

$z = (\sqrt{3}+i) = r(\cos\theta + i\sin\theta)$

Then, $r\cos\theta = \sqrt{3}$ .... (i)

$r\sin\theta = 1$ ..... (ii)

Eq. (i) + Eq. (ii), we get

$$r^2(\sin^2\theta + \cos^2\theta) = 3 + 1$$

$\Rightarrow \quad r^2 = 4$

$\Rightarrow \quad r = 2$

Eq. (ii) ÷ Eq. (i), we get

$\Rightarrow \quad \tan\theta = \dfrac{1}{\sqrt{3}} = \tan\dfrac{\pi}{6}$

$\Rightarrow \quad \theta = \dfrac{\pi}{6}$

Then, $z = (\sqrt{3}+i) = 2\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$

$\Rightarrow \bar{z} = (\sqrt{3}-i) = 2\left(\cos\dfrac{\pi}{6} - i\sin\dfrac{\pi}{6}\right)$

Then, $(\sqrt{3}+i)^7 + (\sqrt{3}-i)^7$

$= \left[2\left\{\cos\dfrac{\pi}{6} + i\sin\dfrac{\pi}{6}\right\}\right]^7$

$+ \left[2\left\{\cos\dfrac{\pi}{6} - i\sin\dfrac{\pi}{6}\right\}\right]^7$

$= 2^7\left\{\cos\dfrac{7\pi}{6} + i\sin\dfrac{7\pi}{6}\right\} + 2^7$

$\left\{\cos\dfrac{7\pi}{6} - i\sin\dfrac{7\pi}{6}\right\}$

(By De-moiver theorem)

$= 2^7\left\{\cos\dfrac{7\pi}{6} + i\sin\dfrac{7\pi}{6} + \cos\dfrac{7\pi}{6} - i\sin\dfrac{7\pi}{6}\right\}$

$= 2^7 \cdot 2\cos\dfrac{7\pi}{6}$

$= 2^8 \cdot \cos(\pi + \pi/6)$

$= -2^8 \cdot \cos\dfrac{\pi}{6}$

$= -2^8 \cdot \dfrac{\sqrt{3}}{2}$

$= -2^7 \cdot \sqrt{3} = -128\sqrt{3}$

152. **(a)**

$\left(\tan\theta - \dfrac{1}{3}\tan^3\theta\right)\left(\dfrac{1}{3} - \tan^2\theta\right)^{-1}$

where, $\tan^2\theta \neq \frac{1}{3}$

$= \dfrac{\left(\tan\theta - \frac{1}{3}\tan^3\theta\right)}{\left(\frac{1}{3} - \tan^2\theta\right)} = \dfrac{3(3\tan\theta - \tan^3\theta)}{3(1 - 3\tan^2\theta)}$

$= \tan 3\theta$

$\because \tan 3\theta = \left(\frac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}\right)$
$= \tan(\pi + 3\theta)$
$= \tan 3\left(\frac{\pi}{3} + \theta\right)$

So, the period of the given function is $\frac{\pi}{3}$.

153. (b)

$a\sin^2\theta + b\cos^2\theta = c$

On dividing both sides by $\cos^2\theta$

$a\tan^2\theta + b = c\sec^2\theta$

$\Rightarrow a\tan^2\theta + b = c(1 + \tan^2\theta)$

$\Rightarrow a\tan^2\theta + b = c + c\tan^2\theta$

$\Rightarrow b = c + c\tan^2\theta - a\tan^2\theta$

$\Rightarrow (c - a)\tan^2\theta = (b - c)$

$\Rightarrow \tan^2\theta = \frac{b - c}{c - a}$ or $\frac{c - b}{a - c}$

154. (a) $\cos(x - y), \cos x, \cos(x + y)$ are in HP.

Then, $\cos x = \frac{2\cos(x-y)\cos(x+y)}{\cos(x+y)+\cos(x-y)}$

$\cos x = \frac{\cos 2x + \cos 2y}{2\cos x \cdot \cos y}$

$\cos x = \frac{2\cos^2 x + 2\cos^2 y - 2}{2\cos x \cdot \cos y}$

$\cos^2 x \cdot \cos y = \cos^2 x + \cos^2 y - 1$

$\cos^2 x(\cos y - 1) = (\cos^2 y - 1)$

$\cos^2 x(1 - \cos y) = (1 - \cos^2 y)$

$\cos^2 x\left(2\sin^2\frac{y}{2}\right) = \sin^2 y$

$\cos^2 x\left(2\sin^2\frac{y}{2}\right) = \left(2\sin\frac{y}{2} \cdot \cos\frac{y}{2}\right)^2$

$\cos^2 x\left(2\sin^2\frac{y}{2}\right) = 4\sin^2\frac{y}{2} \cdot \cos^2\frac{y}{2}$

$\cos^2 x = \left(2\cos^2\frac{y}{2} - 1\right) + 1$

$\cos y + 1 = \cos^2 x$

or $1 + \cos y = \cos^2 x$

155. (a) $(\sqrt{3} - 1)\sin\theta + (\sqrt{3} + 1)\cos\theta = 2$

$\frac{\sqrt{3}-1}{2}\sin\theta + \frac{\sqrt{3}+1}{2}\cos\theta = 1$ .... (i)

Comparing with $a\sin\theta + b\cos\theta = 1$.

ie, $a = \frac{\sqrt{3} - 1}{2}, b = \frac{\sqrt{3} + 1}{2}$

$\sqrt{a^2 + b^2} = \sqrt{\frac{(\sqrt{3} - 1)^2}{4} + \frac{(\sqrt{3} + 1)^2}{4}}$

$= \frac{1}{2}\sqrt{3 + 1 - 2\sqrt{3} + 3 + 1 + 2\sqrt{3}}$

$= \frac{1}{2}\sqrt{8} = \frac{1}{2} \cdot 2\sqrt{2} = \sqrt{2}$

Dividing on both sides by $\sqrt{2}$ in Eq. (i), we get $\left(\frac{\sqrt{3}-1}{2\sqrt{2}}\right)\sin\theta + \left(\frac{\sqrt{3}+1}{2\sqrt{2}}\right)\cos\theta = \frac{1}{\sqrt{2}}$

Let $\sin\alpha = \frac{\sqrt{3}-1}{2\sqrt{2}}$

Then, $\cos\alpha = \sqrt{1 - \left(\frac{\sqrt{3}-1}{2\sqrt{2}}\right)^2}$

$$= \sqrt{1 - \frac{(4 - 2\sqrt{3})}{8}}$$
$$= \sqrt{\frac{8 - 4 + 2\sqrt{3}}{8}} = \sqrt{\frac{4 + 2\sqrt{3}}{8}}$$
$$= \sqrt{\frac{\sqrt{3} + 1}{4}} = \sqrt{\frac{(\sqrt{3} + 1)^2}{8}}$$
$$= \left(\frac{\sqrt{3} + 1}{2\sqrt{2}}\right)$$

So, $\sin\alpha \cdot \sin\theta + \cos\alpha \cdot \cos\theta = \frac{1}{\sqrt{2}}$
$$\cos(\theta - \alpha) = \frac{1}{\sqrt{2}} = \cos\frac{\pi}{4}$$
$$\theta - \alpha = 2n\pi \pm \frac{\pi}{4}, \theta = 2n\pi \pm \frac{\pi}{4} + \alpha$$
$$\because\ \cos 15 = \frac{\sqrt{3} + 1}{2\sqrt{2}}$$
ie, $\cos\alpha = \cos 15^\circ = \cos\frac{\pi}{12}$
$$\Rightarrow\ \alpha = \frac{\pi}{12}$$
From Eq. (ii), $\left[\theta = 2n\pi \pm \frac{\pi}{4} + \frac{\pi}{12}\right], n \in Z$

156. (b)

$$\tan^{-1} x + \tan^{-1} y + \tan^{-1} z = \frac{\pi}{2}$$
$$\Rightarrow (\tan^{-1} x + \tan^{-1} y) + \tan^{-1} z = \frac{\pi}{2}$$
$$\Rightarrow\ \tan^{-1}\left(\frac{x + y}{1 - xy}\right) + \tan^{-1} z = \frac{\pi}{2}$$
$$\Rightarrow\ \tan^{-1}\left[\frac{\frac{(x + y)}{1 - xy} + z}{1 - z\left(\frac{x + y}{1 - xy}\right)}\right] = \frac{\pi}{2}$$
$$\Rightarrow\ \frac{(x + y) + z(1 - xy)}{(1 - xy) - z(x + y)} = \tan\frac{\pi}{2} = \infty$$
ie, $(1 - xy) - z(x + y) = 0$

$\Rightarrow\ 1 - xy - zx - yz = 0$

157. (c) $\tanh^{-1} x = a\log\left(\frac{1+x}{1-x}\right), |x| < 1$ .. (i)

But $\tanh^{-1} x = \frac{1}{2}\log\left(\frac{1+x}{1-x}\right)$ ….. (ii)

From Eqs. (i) and (ii), we get
$$a = \frac{1}{2}$$

158. (b)

$\Delta = a^2 - (b - c)^2$

$\Rightarrow \quad \Delta = (a + b - c)(a - b + c)$
$\Rightarrow \quad \Delta = (2s - c - c)(2s - b - b)(2s = a + b + c)$
$\Rightarrow \quad \Delta = (\because 2s - 2c)(2s - 2b)$
$\Rightarrow \quad \Delta = 4(s - b)(s - c)$
$\because \Delta = \sqrt{s(s-a)(s-b)(s-c)}$
$\Rightarrow \quad \sqrt{s(s-a)(s-b)(s-c)} = 4(s-b)(s-c)$
$\Rightarrow \quad \sqrt{s(s-a)} = 4\sqrt{(s-b)(s-c)}$
$\Rightarrow \quad \frac{1}{4} = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}}$
$\because \quad \tan\frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}}$
$\Rightarrow \quad \tan\frac{A}{2} = \frac{1}{4}$
$\therefore \quad \tan A = \frac{2\tan\frac{A}{2}}{1 - \tan\frac{A}{2}} = \frac{2 \times \frac{1}{4}}{1 - \frac{1}{16}} \Rightarrow \frac{\frac{1}{2}}{\frac{15}{16}}$
$\Rightarrow \quad \tan\frac{8}{15}$

159. (b)

$\angle C = 90°, \frac{a^2-b^2}{a^2+b^2} \quad \ldots..(i)$
$\because \quad \cos c = \frac{a^2 + b^2 - c^2}{2ab} = \cos 90°$
$\Rightarrow \quad \frac{a^2 + b^2 - c^2}{2ab} = 0$
$\Rightarrow \quad a^2 + b^2 - c^2 = 0$
$c^2 = a^2 + b^2$

From Eq. (i), we get
$$\frac{a^2 - b^2}{a^2 + b^2} = \frac{a^2 - b^2}{c^2}$$
$\because \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = R(\text{ constant })$
$= \frac{(R\sin A)^2 - (R\sin B)^2}{(R\sin C)^2} = \frac{\sin^2 A - \sin^2 B}{\sin^2 90°}$
$(\because \sin 90° = 1)$
$= (\sin A + \sin B)(\sin A - \sin B)$
$= 2 \cdot \sin\frac{A+B}{2} \cdot \cos\frac{A-B}{2} \cdot 2\cos\frac{A+B}{2}$
$\sin\frac{A-B}{2}$
$= 2\sin\frac{\pi}{4} \cdot \cos\frac{A-B}{2} \cdot 2\cos\frac{\pi}{4} \cdot \sin\frac{A-B}{2}$
$\because A + B = \pi - C$
$= \pi - \frac{\pi}{2} = \frac{\pi}{2}$
$= 2 \cdot 2 \cdot \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} \cdot \sin\frac{A-B}{2} \cdot \cos\frac{A-B}{2}$
$= 2 \cdot \sin\left(\frac{A-B}{2}\right) \cdot \cos\left(\frac{A-B}{2}\right)$
$\because \sin 2A = 2\sin A \cdot \cos A$

$= \sin\left[2 \times \frac{(A-B)}{2}\right]$
$= \sin(A-B)$
Hence, $\frac{a^2-b^2}{a^2+b^2} = \sin(A-B)$
160. (c)