Physics

1. (d)

Using, the equation

$\frac{1}{2}mv_1^2 = hv_{0_1} - \phi_0$ .... (i)

and $\frac{1}{2}mv_2^2 = hv_{0_2} - \phi_0$ .... (ii)

Dividing Eq. (i) by Eq. (ii) we get

$$\frac{\frac{1}{2}mv_1^2}{\frac{1}{2}mv_2^2} = \frac{hv_{0_1} - \phi_0}{hv_{0_2} - \phi_0}$$

$$\frac{v_1^2}{v_2^2} = \frac{(2.5 - 1.5)}{(3.5 - 1.5)} = \frac{1}{2}$$

$$\therefore \frac{v_1^2}{v_2^2} = \frac{1}{2} \text{ or } \frac{v_1}{v_2} = \frac{1}{\sqrt{2}}$$

2. (a) Given, $z = 31$ and $a = 5 \times 10^7\ Hz^{1/2}$

$\sqrt{v} = a(z-1)$

or $v = a^2(z-1)^2$

$= (5 \times 10^7)^2(31-1)^2$

$= 25 \times 10^{14} \times 30 \times 30$

$= 2.25 \times 10^{18}$ Hz

$\therefore$

$v = \frac{c}{\lambda} \Rightarrow \lambda = \frac{c}{v}$

$= \frac{3 \times 10^8\ ms^{-1}}{2.25 \times 10^{18}\ Hz}$

$= 1.33 \times 10^{-10}$ m

$= 1.33$Å

3. (a) Energy released in the fission of one nucleus

$= 200$MeV

$= 200 \times 1.6 \times 10^{-13}$ J

$= 3.2 \times 10^{-11}$ J

Energy to be released

$= 1000$ J

3.2×10^{-11} J energy is released due to number of nucleus fissioned $= 1$

$\therefore$ 1000 J energy is released due to number of nucleus fissioned

$= \frac{1 \times 1000}{3.2 \times 10^{-11}}$

$= 3.125 \times 10^{13}$ nucleus

4. (c) Here, the barrier voltage, $V = 0.3$ volt and the width of depletion layer, $d = 2 \times 10^{-6}$ m

$\therefore$ Electric field at the junction,

$E = \frac{V}{d} = \frac{0.3\ V}{2 \times 10^{-6}\ m}$

$= 1.5 \times 10^5$ V/m

The direction of electric field is from n-type to p-type.

5. (c)

$[\mu_0] = [MLT^{-2}\ A^{-2}]$ and $[H] = [AL^{-1}]$

$\therefore \left.\frac{1}{2}\mu_0 H^2\right] = [MLT^{-2}\ A^{-2}][AL^{-1}]^2 = [ML^{-1}\ T^{-2}]$

6. (b) Here, $\mathbf{A} = 4\mathbf{i} + 10\mathbf{j}$

$\therefore |A| = \sqrt{(4)^2 + (10)^2} = \sqrt{16 + 100}$

$= \sqrt{116}$ m

Now, according to question,

$\mathbf{A} = 8\mathbf{i} + n\mathbf{j}$

So, $|\mathbf{A}| = \sqrt{(8)^2 + n^2}$

$\therefore \sqrt{116} = \sqrt{(8)^2 + n^2}$

squaring both sides

$116 = 64 + n^2$

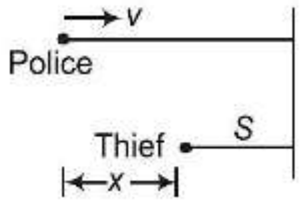
or $n^2 = 116 - 64 = 52$

or $n = 7.2$ m

7. **(c)** Let the police party catch the thief after t second.

$\therefore$ Distance travelled by police party in t second $= vt$

and distance travelled by thief $= x + \frac{1}{2}\alpha t^2$



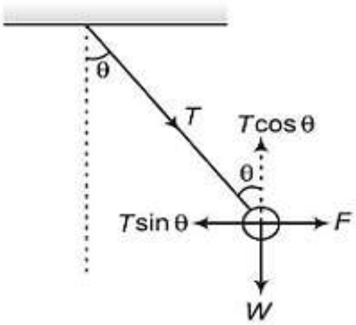
So, $x + \frac{1}{2}\alpha t^2 \leq vt$

or $\frac{\alpha t^2}{2} - vt + x = 0$

or $t = \frac{v \pm \sqrt{v^2 - 2\alpha x}}{\alpha}$

For t to be real, $v^2 \geq 2\alpha x$

8. **(c)** As the bob is in equilibrium under the effect of three forces - T, F and W



$\therefore \quad T + P + W = 0$

From, the figure $\quad T\cos\theta = W \ldots$ (i)

$T\sin\theta = F \ldots\ldots$ (ii)

From Eqs. (i) and (ii)

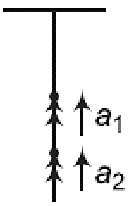
$T^2 = W^2 + F^2$

or $T = \sqrt{W^2 + F^2}$

$= \sqrt{(1)^2 + (2)^2}$

$= \sqrt{5}$

9. **(d)** 60 kg $-$ wt $= 20(g + a_1) + 30(g + a_2)$



$= 20\text{ kg} - \text{wt} + 30\text{ kg} - \text{wt} + 20a_1 + 30a_2$

$\Rightarrow \quad 20a_1 + 30a_2 = 10\text{ kg} - \text{wt}$

$\Rightarrow \quad 2a_1 + 3a_2 = 1$

$a_1 = \frac{1}{4}g,\ a_2 = \frac{1}{6}g$

$a_1 : a_2 = 6:4 = 3:2$

10. **(a)** The velocity of ball after nth rebound will be

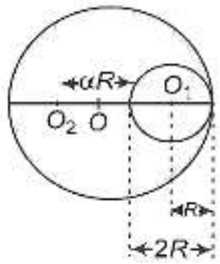
$v_n = e^n v_0$ (where $v_0 = \sqrt{2gh_0}$)

Therefore, the height after nth rebound will be

$$h_n = \frac{v_n^2}{2g} = \frac{e^{2n}v_0^2}{2g} = \frac{e^{2n} \times 2gh_0}{2g}$$

$$\Rightarrow e^{2n} = \frac{h_n}{h_0} \text{ or } e = \left[\frac{h_n}{h_0}\right]^{1/2n}$$

11. **(b)** Let centre O_1 of disc be the origin. Due to symmetry the centre of mass of remaining part will lie at x-axis. Let CM of new disc is at point O_2 where, $O_1O_2 = \alpha R$ (given).



here mass of cut off portion,

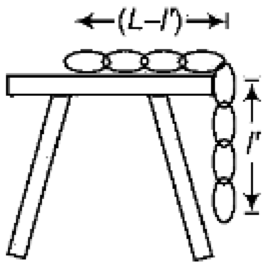
$$m = \frac{\pi(R)^2 \cdot M}{\pi(2R)^2} = \frac{M}{4}$$

and position of its centre of mass, $O_1O_1 = R$ Hence, for remaining part (new disc)

$$x_{CM} = \alpha R = \frac{M \times 0 - \left(\frac{M}{4}\right) \times R}{M - \frac{M}{4}}$$

$$= -\frac{R}{3} \Rightarrow \alpha = \frac{1}{3}$$

12. **(b)** Let l' part of the chain is hanging over the edge of table without sliding.



$$\therefore \mu = \frac{\text{Length hanging over the edge}}{\text{Length lying on the table}}$$

(As the chain have uniform linear density)

$$\therefore \quad \mu = \frac{l'}{(L - l')}$$

$$\Rightarrow \quad l' = \frac{\mu L}{(1 + \mu)}$$

13. **(a)** Moment of inertia of circular disc $= \frac{1}{2}MR^2$

$$= \frac{1}{2} M\left(\frac{M}{\pi t \rho}\right) = \frac{1}{2}\frac{M^2}{\pi t \rho}$$

$$\left(\text{As } M = \pi R^2 t\rho \Rightarrow R^2 = \frac{M}{\pi t \rho}\right)$$

If mass and thickness are same, then

$$I \propto \frac{1}{\rho}$$

So, the disc made from more dense material will have smaller rotational inertia.

14. (c) Total energy = KE + rotational KE

$$= \frac{1}{2}(2m)v^2 + \frac{1}{3}mv^2$$

$$= \frac{4}{3}mv^2$$

15. (d) Astronaut would not experience any gravitational force as it is balanced by centrifugal force.
So, A is false and R is true.

16. (c) Mass of the particle = m
spring constant = k

The time period of oscillator, $T = 2\pi\sqrt{\frac{m}{k}}$

As $k \propto \frac{1}{l}$ (where l is the length of spring)

$\because \quad k' = 2k$

$\therefore \quad T' = 2\pi\sqrt{\frac{m}{2k}} = \frac{1}{\sqrt{2}}T$

17. (c) Breaking stress $= \frac{F}{A}$

Here, breaking stress $= \frac{40}{3\pi} \times 10^6 \mathrm{Nm}^{-2}$

$$\Rightarrow \quad \frac{40}{3\pi} \times 10^6 = \frac{3 \times 10}{\pi r^2}$$

$$\Rightarrow \quad r^2 = \frac{9}{4} \times 10^{-6}$$

$$\text{or} \quad r = 1.5 \times 10^{-3}\ \mathrm{m} = 1.5\ \mathrm{mm}$$

18. (c) Air flows from smaller bubble to bigger as the pressure in smaller bubble is higher.

19. (a) By the principle of continuity

$A_1 v_1 = A_2 v_2$

According to question, $A_1 = L^2$

$$v_1 = \sqrt{2gy}$$

$$\text{and} \quad A_2 = \pi R^2$$

$$v_2 = \sqrt{2g4y}$$

$$L^2\sqrt{2gy} = \pi R^2\sqrt{2g4y}$$

$$\text{or} \quad L^2 = 2\pi R^2$$

$$\text{or} \quad R = \frac{L}{\sqrt{2\pi}}$$

20. (a) Coefficient of real expansion

$$\gamma_R = \frac{V_2 - V_1}{V_1(t_2 - t_1)}$$

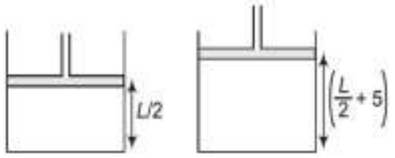
Here, $V_2 = \frac{3}{4}, V_1 = \frac{2}{3}$

and $(t_2 - t_1) = (100 - 20) = 80°\mathrm{C}$

$$\therefore\ \gamma_R = \frac{\left(\frac{3}{4}-\frac{2}{3}\right)}{\frac{2}{3}(80)} = \frac{1}{640}$$

$= 15.6 \times 10^{-4}\ °C^{-1}$

21. (b) According to question,



$T_1 = 0°C$ $\quad$ $T_2 = 100°C$

$= 273\ K$ $\quad$ $= 373\ K$

$$\therefore\ \frac{273}{L/2} = \frac{373}{(L/2)+5}$$

$$\Rightarrow\ 273\left(\frac{L}{2}+5\right) = 373\left(\frac{L}{2}\right)$$

or

$$273 \times 5 = 100\left(\frac{L}{2}\right)$$

or

$L = 27.3\ cm$

22. (a) The efficiency of engine, $\eta = \frac{T_1 - T_2}{T_1}$ According to question, $2\eta = \frac{T_1-(T_2-62)}{T_1}$

$$\text{or}\quad \frac{1}{2} = \frac{T_1 - T_2}{(T_1 - T_2) + 62}$$

or

$62 = (T_1 - T_2)$

Only option (a) satisfies the condition.

23. (d) In adiabatic process

$pV^{\gamma} = k$ ….. (i)

Given, $p \propto T^3$ or $p = kT^3$

$$= k\left(\frac{pV}{R}\right)^3$$

$$= \left(\frac{k}{R^3}\right)p^3V^3$$

$$\text{or}\ \ p = \left(\frac{k}{R^3}\right)p^3V^3$$

$\Rightarrow\ p^2V^3 = k$

$\Rightarrow\ pV^{3/2} = k$

Comparing Eqs. (i) and (ii)

we get $\gamma = \frac{3}{2} = \frac{C_p}{C_V}$

24. (c) The given situation can be shown as

A	B

12^0 K_1 X K_2 0^0 C

Rate of flow of heat will be equal in both the slabs

$\therefore\ (12 - x)K_1 = K_2(x - 0)$

$$12 - x = 2x\ \left(\because K_1 = \frac{K_2}{2}\right)$$

$\Rightarrow\ x = 4°C$

The temperature difference across slab

$A = (12 - x) = (12 - 4)$

$= 8°C$

25. (a) Here, $v_1 = \frac{v}{\left(\frac{40}{195}\right)}$

and $\quad v_2 = \dfrac{v}{\left(\dfrac{40}{193}\right)}$

$\Rightarrow \quad v_1 = \dfrac{v(195)}{40}$

and $\quad v_2 = \dfrac{v(193)}{40}$

According to given condition

$\frac{v}{40}(195) - v = 9 \ldots . (i)$

and

$v - \frac{v(193)}{40} = 9 \ldots . (ii)$

Adding Eqs. (i) and (ii)

$$18 = \frac{v}{40}(195 - 193) = \frac{2v}{40} \Rightarrow 18 = \frac{v}{20}$$

$$\text{or } v = 360 \text{ m/s}$$

26. **(c)** Given, 1 st overtone of A = 2 nd overtone of B

$$\Rightarrow \frac{1}{l_1}\sqrt{\frac{T}{\pi r_1^2 d}} = \frac{3}{2l_2}\sqrt{\frac{T}{\pi r_2^2 d}}$$

$$\Rightarrow \frac{r_2}{r_1} = \frac{3}{2}\left(\frac{l_1}{l_2}\right)$$

But $r_1 = 2r_2$

$\therefore \ \dfrac{r_2}{2r_2} = \dfrac{3}{2}\left(\dfrac{l_1}{l_2}\right)$

or $\frac{l_1}{l_2} = \frac{1}{3} \Rightarrow l_1 : l_2 = 1 : 3$

27. **(d)** Here, $\omega_1 = 0.45$

$\omega_2 = 0.21$ and $f_2 = 84$ cm

As the lens are kept is contact it is achromatic condition.

$$\therefore \quad \frac{\omega_1}{\omega_2} = -\frac{f_1}{f_2}$$

$$\frac{0.45}{0.21} = -\frac{f_1}{84}$$

$$\Rightarrow \quad f_1 = -\frac{45 \times 84}{21} = -180 \text{ cm}$$

28. **(b)** In compound microscope both objective and eyepiece are convex lenses. An objective lens is of small focal length and eyepiece is of large focal length.

29. **(c)** As the light ray incident at critical angle,

$$\sin C = \frac{\mu_3}{\mu_2} = \frac{1.3}{1.8} \text{ and } C = r$$

$$\therefore \ \sin r = \frac{1.3}{1.8}$$

$$\Rightarrow \ \sin\theta = \frac{1.8}{1.6} \times \frac{1.3}{1.8} = \frac{13}{16}$$

$$\theta = \sin^{-1}\left(\frac{13}{16}\right)$$

30. **(c)** Given, $I_R = 75\%$ of I_{max}

$= \frac{3}{4} I_{max}$

$= \frac{3}{4}(4a^2) = 3a^2$

$\Rightarrow\ 4a^2 \cos^2 \frac{\phi}{2} = 3a^2$

$\cos^2 \frac{\phi}{2} = \frac{3}{4}$ or $\cos \frac{\phi}{2} = \frac{\sqrt{3}}{2}$

$\Rightarrow\ \frac{\phi}{2} = \frac{\pi}{6}$ or $\phi = \frac{\pi}{3}$

31. **(c)** Given, pole strength $= m$
and magnetic moment $= M$
When it is cut 5 times parallel to its axis,
pole strength, $m' = \frac{m}{6}$
When it is cut 3 times perpendicular to its axis, then length $2l' = \frac{2l}{4}$
$\therefore$ pole strength, $m' = \frac{m}{6}$
and magnetic moment,
$M' = \left(\frac{m}{6}\right)\left(\frac{2l}{4}\right) = \frac{M}{24}$

32. **(b)** Magnetic field intensity $- Am^{-1}$
Magnetic flux - Wb
Magnetic pole strength — A-m
Magnetic induction $- Wb - m^{-2}$

33. **(b)** Let V be the potential across the capacitor when it is fully charged, then, energy stored in the capacitor will be
$= \frac{1}{2} CV^2$
(where $C =$ capacitance of capacitor)
When the capacitor is fully discharged, loss of energy is in the form of heat $= \Delta H$
As the system is thermally isolated so,

$\Delta H = \frac{1}{2} CV^2$

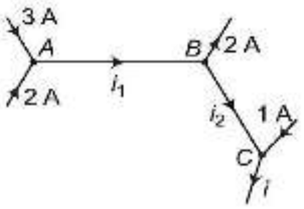
Here, $\Delta H = ms\Delta T$

$\therefore\ \frac{1}{2} CV^2 = ms\Delta T$

or $V = \left(\frac{2ms\Delta T}{C}\right)^{1/2}$

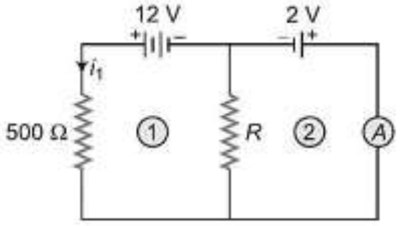
34. **(a)** Let the capacitance of condensor, $M = C$
The new capacitance of condensor, $C_M = 8C$
The capacitance of condensor, $N = C$
The new capacitance of condensor, $C_N = \frac{C'}{2}$
where $C' = \frac{A\varepsilon_0}{d/4} = \frac{4A\varepsilon_0}{d} = 4C$
$\therefore\ C_N = \frac{C'}{2} = \frac{4C}{2} = 2C$
Since the two condensers are connected in series
$\therefore\ V_M : V_N = \frac{1}{C_M} : \frac{1}{C_N}$
$= \frac{1}{8C} : \frac{1}{2C} = 1:4$
$\Rightarrow V_M : V_N = 1:4$

35. **(d)** Given, situation is predicted as



$i_1 = 3 + 2 = 5$ A
$i_2 = i_1 - 2 = 5 - 2 = 3$ A
$i = i_2 + 1 = 3 + 1 = 4$ A

36. (b) Given, circuit

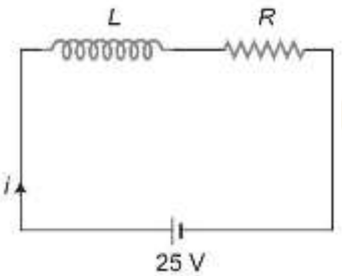


In loop (1)
$$12 - 500i_1 - Ri_1 = 0$$
$\Rightarrow \quad 12 = i_1(500 + R) \ldots\ldots (i)$
In loop (2)
$12 - 500i_1 - 2 = 0 \Rightarrow 10 = 500i_1$
Or $i_1 = \frac{1}{50}$ A ..... (ii)
From Eqs. (i) and (ii)
$$12 \times \frac{1}{i_1} = (500 + R)$$
$\Rightarrow \; 12 \times 50 = 500 + R \Rightarrow R = 100\Omega$

37. (c) Given, $E = a\theta + b\theta^2$
$$\therefore \; p = \frac{dE}{d\theta} = \frac{d}{d\theta}(a\theta + b\theta^2)$$
$= a + 2b\theta$
At $\theta = T_n, p = 0$
$$\Rightarrow T_n = -\frac{a}{2b} = -350°\text{C}$$

38. (b)

39. (a) At, $t = 0$, switch closes



Current $i = i_0\left[1 - e^{-tR/L}\right]$
So, at $t = 0, i = 0$
So, potential across inductor = 25 V
and across resistor = 0 V

40. (a) We have, $i_g = i\left(\frac{S}{G+S}\right)$
where i = total current
Sensitivity $= \frac{i_g}{i} = \frac{10}{60} = \frac{1}{6}$

$\therefore \quad \frac{1}{6} = \frac{S}{G+S}$

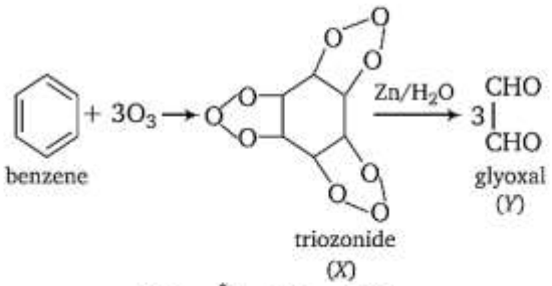
$\Rightarrow \quad G + S = 6S$

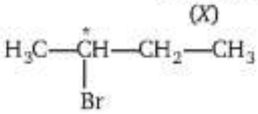
$\Rightarrow \quad G = 5S \text{ or } S = \frac{G}{5}$

$\Rightarrow \quad S = \frac{20}{5} = 4\Omega (\because G = 20\Omega)$

Chemistry

41. (c)

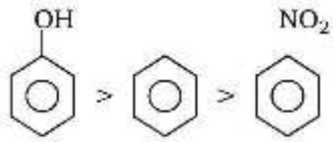


42. (b)  compound contains a chiral centre or an asymmetric carbon atom therefore, it is optically active compound and hence this compound exhibits enantiomerism.

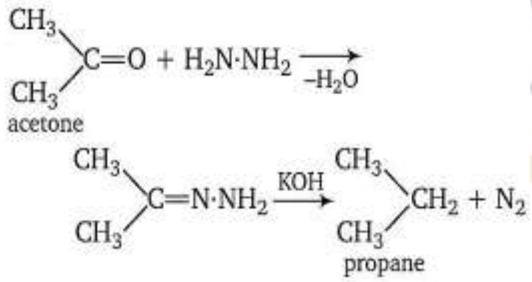
43. (a) In the presence of air and light, chloroform forms carbonyl chloride (also called phosgene) which is extremely poisonous.

$$\underset{\text{chloroform}}{CHCl_3} + \frac{1}{2}O_2 \xrightarrow{\text{Air and light}} \underset{\text{phosgene}}{COCl_2} + HCl$$

44. (c) Due to strong +R effect of -OH group, the electron density in phenol (I) is much more than in benzene (III) while due to the strong $-R$ effect of the $-NO_2$ group, the electron density in nitrobenzene (II) is much lower than in benzene. Hence, the order of reactivity towards nitration is as

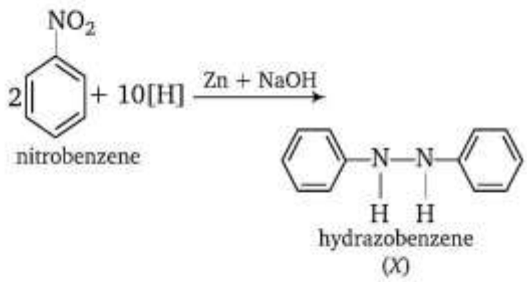


45. (a) Wolff-Kishner reduction is done by heating the carbonyl compound with a mixture of hydrazine and KOH in the presence of ethylene glycol.



46. (c) A stronger acid will have higher K_a value but smaller pK_a value while a weaker acid will have smaller K_a value but higher pK_a value. Hence among the given carboxylic acid the pK_a value of strongest acid is 0.23 .

47. (d) Nitrobenzene on reduction using zinc in alkaline medium gives hydrazobenzene.

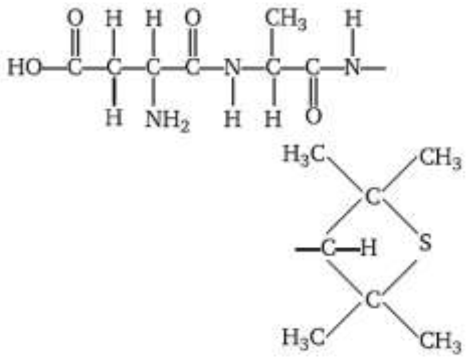


Hydrazobenzene (X) contains 27σ and 6π bonds.

48. (b) Polymers which control various life processes in plants and animals are called biopolymers. Cellulose, Insulin, DNA, starch, protein etc are examples of biopolymer. Nylon-6 is a synthetic polymer. It is not a biopolymer.

49. (b) At isoelectric point, the amino acids have the least solubility in water and this property is used for the separation of different amino acids obtained from the hydrolysis of proteins.

50. (d) Alitame is an artificial sweetening agent. It is 2000 times more sweet than cane sugar. Its structure is as



51. (b) Phosphorus acid (H_3PO_3) is a dibasic acid.

$0.3MH_3PO_3 = 0.6\ NH_3PO_3$

$N_1V_1 = N_2V_2$

$0.1 \times V_1 = 0.6 \times 100$

$V_1 = 600\ mL$

52. (c) Potential of hydrogen electrode at pH $= 10$ is -0.591 V .

Reduction potential of Cu^{2+}/Cu is 0.36 V .

Oxidation potential of Zn/Zn^{2+} is 0.76 V .

$$\frac{2.303RT}{F} = \frac{2.303 \times 8.314 \times 298}{96500} = 0.059$$

53. (a) $\Lambda_\infty NH_4Cl = \Lambda_\infty NH_4^+ + \Lambda_\infty Cl^- = 130$ …… (i)

$\Lambda_\infty NaOH = \Lambda_\infty Na^+ + \Lambda_\infty OH^- = 217$ …… (ii)

$\Lambda_\infty NaCl = \Lambda_\infty Na^+ + \Lambda_\infty Cl^- = 109$ ….. (iii)

On adding Eq. (i) and (ii) and subtracting the Eq. (iii)

$\Lambda_\infty NH_4^+ + \Lambda_\infty OH^- = 130 + 217 - 109$

$\Lambda_\infty NH_4OH = 347 - 109$

$= 238 ohm^{-1}\ cm^2\ equiv^{-1}$

54. (b) Potassium has bcc system

Number of mole $= 39/39 = 1$ mole

1 mole of atoms $= 6.022 \times 10^{23}$ atoms $= N$

∵ In bcc system 2 atoms are present in 1 unit cell.

∴ N number of atoms are present in $\frac{N}{2}$ unit cells.

55. (d) Activation energy for the forward reaction, E_f and the activation energy for the backward reaction, E_b are related to the enthalpy of the reaction by the equation.

$\Delta_rH = E_f - E_b$

For exothermic reaction

$\Delta_rH < 0$

$E_b > E_f$

56. (a)

$2AB \rightleftharpoons A_2 + B_2$

$$K_c = \frac{[A_2][B_2]}{[AB]^2}$$

again, $AB \rightleftharpoons \frac{1}{2}A_2 + \frac{1}{2}B_2$

$K_c' = \frac{[A_2]^{1/2}[B_2]^{1/2}}{[AB]}$

$K_c = (K_c')^2$

or $K_c' = \sqrt{K_c} = \sqrt{49} = 7$

57. (d)

$CH_3COOH \rightleftharpoons H^+ + CH_3COO^-$

$K_a = \frac{[H^+][CH_3COO^-]}{[CH_3COOH]} = \frac{[H^+]^2}{[CH_3COOH]}$

$[H^+] = \sqrt{K_a[CH_3COOH]}$

$= \sqrt{2 \times 10^{-5} \times 0.05}$

$= \sqrt{10^{-6}}$

$[H^+] = 10^{-3}$

$\because$ pH $= -\log[H^+]$

$= -\log[10^{-3}]$

$\therefore$ pH $= 3$

58. (d)

$\Delta S_f = \frac{\Delta H_f}{T_f}$

$= \frac{330 \times 27.3}{273}$

$= 33 JK^{-1}$

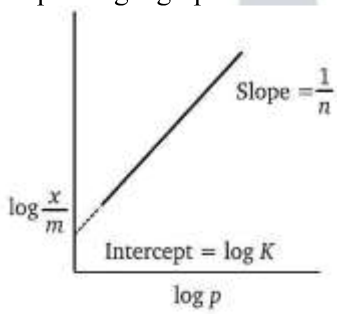
59. (a) Freundlich adsorption isotherm is as

$\frac{x}{m} = Kp^{1/n}$

Taking logarithms on both sides

$\log\frac{x}{m} = \log K + \frac{1}{n}\log p$

On plotting a graph between $\log x/m$ and $\log p$, a straight line will be obtained.



60. (d) Both hydrogen (H_2) and deuterium (D_2) molecules have same bond length but have different bond energy, melting point and boiling point.

Property	Hydrogen (H_2) 74 pm	Deuterium (D_2) 74 pm
Bond energy	436 kJ mol^{-1}	443.3 kJ mol
M.P.	0.00°C	3.81°C
B.P.	100.0°C	101.42°C

61. (b) Sodium hydroxide reacts with sulphur to give sodium sulphide and sodium thiosulphate.

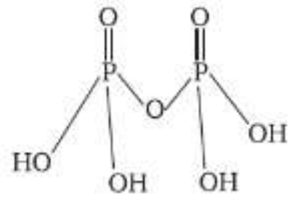
$4S + 6NaOH \rightarrow \underset{\text{sodium sulphide}}{2Na_2S} + \underset{\text{sodium thiosulphate}}{Na_2S_2O_3} + 3H_2O$

62. (b) Boron reacts with concentrated HNO_3 to give orthoboric acid and nitrogen dioxide.

$B + 3HNO_3 \longrightarrow H_3BO_3 + 3NO_2$

63. (d) Graphite has a two -dimensional sheet like structure. The adjacent layers of sheet are held together by weak van der Waals' forces. In graphite each carbon atom is present in sp^2 hybridised state.

64. (c) The structure of pyrophosphoric acid $(H_4P_2O_7)$ is as

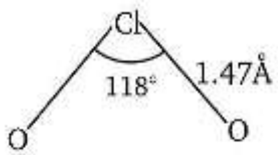


Hence, it contains 12σ and 2π-bonds

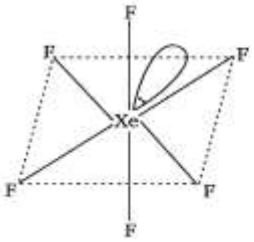
65. (a) When moist chlorine gas is reacted with hypo then colloidal sulphur, HCl and sodium sulphate products are formed.

$Na_2S_2O_3 + H_2O + Cl_2 \rightarrow Na_2SO_4 + 2HCl + S$

66. (b) The structure of ClO_2 is angular. The Cl -atom is sp^3-hybridised in the angular molecule with bond angle of $118°$ and $Cl - O$ bond length of 1.47Å.



67. (d) XeF_6 has distorted octahedral structure in which all the six position are occupied by fluorine atoms and one lone pair is present at the centre of one of the triangular faces. It involves sp^3d^3 hybridisation of xenon.



68. (d) Magnetic moment (μ) is given as

$\mu = \sqrt{n(n+2)}$

where, n is number of unpaired electrons.

Fe^{2+}: $[Ar]3d^6$; number of unpaired electrons $= 4$

Ti^{2+} :$[Ar]\ 3d^2$; number of unpaired electrons $= 2$

Cu^{2+} : $[Ar]\ 3d^9$; number of unpaired electrons $= 1$

V^{2+} : $[Ar]\ 3d^3$; number of unpaired electrons $= 3$

Hence, the increasing order of magnetic moment is as

$Cu^{2+} < Ti^{2+} < V^{2+} < Fe^{2+}$

69. (a) Nickel oxide is reduced by water gas to obtain the metal. Other oxides are reduced by other reducing agent.

$ZnO + C \xrightarrow{\Delta} Zn + CO$

$WO_3 + 3H_2 \xrightarrow{\Delta} W + 3H_2O$

$Fe_2O_3 + 3CO \xrightarrow{\Delta} 2Fe + 3CO_2$

$2NiO + \underbrace{CO + H_2}_{\text{Water gas}} \longrightarrow 2Ni + CO_2 + H_2O$

70. (c) Methyl isocyanate (MIC) gas is responsible for Bhopal gas tragedy in 1984. Methyl isocyanide reacts with O_3 on HgO to form methyl isocyanate.

71. (a) Percentage of S

$$= \frac{32 \times \text{ weight of } BaSO_4 \times 100}{233 \times \text{ weight of compound}}$$
$$= \frac{32 \times 0.233 \times 100}{233 \times 0.16}$$
$$= 20$$

72. **(a)** In cycloalkane the heat of combustion of cyclohexane ($657.9 \text{ kJ mol}^{-1}$) is the lowest hence it is the most stable cycloalkane. Cyclopropane and cyclobutane are less stable due to angle strain and torsional strain, therefore cyclopropane and cyclobutane are reactive cycloalkane.

73. **(d)**

$$v = \frac{c}{\lambda}$$
$$= \frac{3 \times 10^8}{600 \times 10^{-9}}$$
$$v = 5 \times 10^{14} \text{ Hz}$$

74. **(a)** According to Bohr's model, electrons revolve only in those orbits for which angular momentum is an integral multiple of $\frac{h}{2\pi}$ i.e., $mvr = \frac{nh}{2\pi}$.

75. **(c)** The electronic configuration of Na, Ne, Mg and Al after the removal of an electron is as

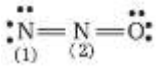
$Na^+: 1s^2, 2s^2, 2p^6$
$Ne^+: 1s^2, 2s^2, 2p^5$
$Mg^+: 1s^2, 2s^2, 2p^6, 3s^1$
$Al^+: 1s^2, 2s^2, 2p^6, 3s^2$

The ionisation energy decreases as the size of the atom/ion increases. Further ionisation energy increases with increase in nuclear charge. Since, Na^+ gives a stable electronic configuration hence its IE_2 is highest among the given species and therefore the correct order of their IE_2 is

$Mg < Al < Ne < Na$

76. **(b)** Formal charge $= \begin{bmatrix} \text{Total number of valence} \\ \text{electrons in the free atom} \end{bmatrix}$

$- \begin{bmatrix} \text{Total number of} \\ \text{non-bonding (lone} \\ \text{pair)electrons} \end{bmatrix} - \frac{1}{2} \begin{bmatrix} \text{Total number of} \\ \text{bonding (shared)} \\ \text{electrons} \end{bmatrix}$

In  structure the formal charge on...

the end N atom marked 1 ,

$$= 5 - 4 - \frac{1}{2}(4)$$
$$= 5 - 4 - 2 = -1$$

the central N atom marked 2,

$$= 5 - 0 - \frac{1}{2}(8)$$
$$= 5 - 4 = +1$$

the end O atom,

$$= 6 - 4 - \frac{1}{2}(4)$$
$$= 6 - 6 = 0$$

77. **(b)**

78. **(d)** Molecular weight of

$NaHCO_3 = 23 + 1 + 12 + 48 = 84$

Molecular weight of

$Na_2CO_3 = 46 + 12 + 48 = 106$

Hence, total weight $= 84 + 106 = 190$

$\because$ In 190 g of a mixture, weight of Na_2CO_3 is $= 106$

$\therefore$ In 19 g of a mixture weight of

$$Na_2CO_3 = \frac{106 \times 19}{190}$$
$$= 10.6\text{ g}$$

79. (c) Real gases approach the ideal gas behaviour at high temperature and low pressure.

80. (a) Lowering of vapour pressure is a colligative property. It depends on the number of particles of the solute.

$BaCl_2 \rightleftharpoons Ba^{2+} + 2Cl^-$; 3 ions

$NaCl \rightleftharpoons Na^+ + Cl^-$; 2 ions

$Al_2(SO_4)_3 \rightleftharpoons 2Al^{3+} + 3SO_4^{3-}$; 5 ions

Hence the ratio of lowering of vapour pressure is 3: 2: 5.

Mathematics

81. (c)

82. (b) Given, $f(x) = \begin{cases} \frac{a+2\cos x}{x^2} & ,x < 0 \\ b\tan\frac{\pi}{[x+4]} & ,x \geq 0 \end{cases}$

At $x = 0$

$$\text{LHL} = \lim_{x\to 0^-} f(x) = \lim_{x\to 0}\frac{a+2\cos x}{x^2}$$
$$= \lim_{x\to 0}\frac{a+2\left(1-\frac{x^2}{2!}+\frac{x^4}{4!}-\cdots\right)}{x^2}$$
$$= \lim_{x\to 0}\frac{(a+2)+2\left(-\frac{x^2}{x!}+\frac{x^4}{4!}-\cdots\right)}{x^2}$$
$$= \lim_{x\to 0}\frac{0+2\left(\frac{-x^2}{2!}+\frac{x^4}{4!}-\cdots\right)}{x^2} = -1$$

[$\because$ f(x) is continuous so we take $a + 2 = 0$]

$$\text{RHL} = \lim_{x\to 0^+} f(x) = \lim_{x\to 0} b\tan\frac{\pi}{[x+4]}$$
$$= b\tan\frac{\pi}{4} = b$$

But LHL = RHL

$\Rightarrow \; -1 = b$ and $a = -2$

83. (c) Given

$$y = \frac{(1+x)(1+x^2)(1+x^4)\ldots(1+x^{2n})\times(1-x)}{\times(1-x)}$$
$$= \frac{(1-x^2)(1+x^2)((1+x^4)\ldots(1+x^{2n})}{1-x}$$
$$\Rightarrow y = \frac{[1-(x^{2n})^2]}{1-x} = \frac{1-x^{4n}}{1-x}$$
$$\frac{dy}{dx} = \frac{(1-x)(-4nx^{4n-1})-(1-x^{4n})(-1)}{(1-x)^2}$$
$$\left(\frac{dy}{dx}\right)_{x=0} = \frac{1(-0)-(1-0)(-1)}{1^2}$$
$$= \frac{1}{1} = 1$$

84. (a) Given, $\cos^{-1}\left(\frac{x^2-y^2}{x^2+y^2}\right) = k$

$$\Rightarrow \frac{x^2-y^2}{x^2+y^2} = \cos k$$

On differentiating w.r.t. x, we get

$$\frac{(x^2+y^2)\left(2x-2y\frac{dy}{dx}\right)-(x^2-y^2)\left(2x+2y\frac{dy}{dx}\right)}{(x^2+y^2)^2}$$

$$\Rightarrow\ -4x^2y\frac{dy}{dx}+4xy^2=0$$

$$\Rightarrow\qquad \frac{dy}{dx}=\frac{y}{x}$$

85. **(c)** $f(x)=|x|+|\sin x|$

$$\text{LHD }=\lim_{h\to0}\frac{f(0-h)-f(0)}{0-h}$$

$$=\lim_{h\to0}\frac{|0-h|+|\sin(0-h)|-(0+0)}{0-h}$$

$$=\lim_{h\to0}\frac{h+\sin h}{-h}=-\lim_{h\to0}\left(1+\frac{\sin h}{h}\right)$$

$$=-(1+1)=-2$$

86. **(d)** Given, $y=\frac{\log_e x}{x}$ and $z=\log_e x$

$$\therefore\ y=\frac{z}{e^z}$$

On differentiating w.r.t. z, we get

$$\frac{dy}{dz}=\frac{e^z(1)-ze^z}{e^{2z}}=\frac{1-z}{e^z}$$

Again differentiating, we get

$$\frac{d^2y}{dz^2}=\frac{e^z(-1)-(1-z)e^z}{e^{2z}}$$

$$=\frac{-2+z}{e^z}$$

$\therefore$

$$\frac{d^2y}{dz^2}+\frac{dy}{dz}=\frac{1-z}{e^z}+\frac{-2+z}{e^z}=-\frac{1}{e^z}=-e^{-z}$$

87. **(c)** $\therefore\ \cos(60^\circ1')=\frac{1}{2}-\frac{\alpha\sqrt{3}}{120}$

88. **(c)** Given, $s=t^2-2t+5$

On differentiating w.r.t., t we get

$$\frac{ds}{dt}=2t-2$$

Again differentiating, we get

$$\frac{d^2s}{dt^2}=2$$

89. **(d)**

Given, $y=5^x$

$$\Rightarrow\ \frac{dy}{dx}=5^x\log 5$$

$$\Rightarrow\ \left(\frac{dy}{dx}\right)_{(x_1,y_1)}=5^{x_1}\log 5$$

$\therefore$ Length of subtangent

$$=y_1\frac{dx}{dy}=\frac{y_1}{5^{x_1}\log 5}=\frac{5^{x_1}}{5^{x_1}\log 5}=\frac{1}{\log 5}$$

90. **(a)** $u=\sin(y+ax)-(y+ax)^2\ \ldots$ (i)

On differentiating partially w.r.t. x, we get

$u_x=\cos(y+ax)a-2(y+ax)a$

Again, differentiating partially w.r.t. x, we get

$u_{xx}=-\sin(y+ax)\,a^2-2a^2\ \ldots\ldots$ (ii)

On differentiating partially Eq. (i) w.r.t. y, we get

$$u_y = \cos(y+ax) - 2(y+ax)$$
$$\Rightarrow \quad u_{yy} = -\sin(y+ax) - 2 \quad ..(iii)$$
$\therefore$ From Eqs. (ii) and (iii), we get
$u_{xx} = a^2 u_{yy}$

91. (b) Let $I = \int \left(\sqrt{\frac{a+x}{a-x}} + \sqrt{\frac{a-x}{a+x}}\right) dx$

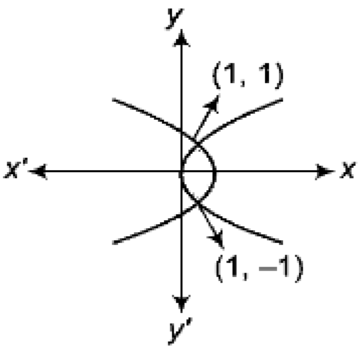
Put $x = a\cos 2\theta \Rightarrow dx = -2a\sin 2\theta d\theta$

$$\therefore I = -\int \left(\sqrt{\frac{a+a\cos 2\theta}{a-a\cos 2\theta}} + \sqrt{\frac{a-a\cos 2\theta}{a+a\cos 2\theta}}\right)$$
$\times\, 2a\sin 2\theta\, d\theta$
$$= -\int \left(\sqrt{\frac{2\cos^2\theta}{2\sin^2\theta}} + \sqrt{\frac{2\sin^2\theta}{2\cos^2\theta}}\right) 2a\sin 2\theta d\theta$$
$$= -2a\int \left(\frac{\cos\theta}{\sin\theta} + \frac{\sin\theta}{\cos\theta}\right)\sin 2\theta d\theta$$
$$= -2a\int \frac{1}{\sin\theta\cos\theta}\sin 2\theta d\theta$$
$$= -4a\int d\theta = -4a\theta + C_1$$
$$= -2a\cos^{-1}\left(\frac{x}{a}\right) + C_1$$
$$= -2a\left[\frac{\pi}{2} - \sin^{-1}\left(\frac{x}{a}\right)\right] + C_1$$
$$= 2a\sin^{-1}\left(\frac{x}{a}\right) + C_1 - \pi a$$
$$= 2a\sin^{-1}\left(\frac{x}{a}\right) + C, \text{ where } C = C_1 - \pi a$$

92. (a) Let $I = \int \frac{\sin^8 x - \cos^8 x}{1 - 2\sin^2 x\cos^2 x} dx$
$$= \int \frac{(\sin^4 x + \cos^4 x)/(\sin^4 x - \cos^4 x)}{(\sin^2 x + \cos^2 x)^2 - 2\sin^2 x\cos^2 x} dx$$
$$= \int (\sin^2 x + \cos^2 x)(\sin^2 x - \cos^2 x) dx$$
$$= -\int \cos 2x dx = -\frac{\sin 2x}{2} + B$$
But $I = A\sin 2x + B$
$$\therefore A = -\frac{1}{2}$$

93. (d) Let $I = \int \frac{1+\cos 4x}{\cot x - \tan x} dx$
$$= \int \frac{2\cos^2 2x}{\frac{\cos^2 x - \sin^2 x}{\sin x\cos x}} dx$$
$$= \int \frac{\sin 2x\cos^2 2x}{\cos 2x} dx$$
$$= \frac{1}{2}\int \sin 4x dx = -\frac{\cos 4x}{8} + C$$

94. (d) Given curves are $x = y^2$
and $x = 3 - 2y^2 \Rightarrow y^2 = -\frac{(x-3)}{2}$
Point of intersections are $(1,1)$ and $(1,-1)$



$\therefore$ Required area $= 2\int_0^1 (x_2 - x_1)dy$

$$= 2\int_0^1 [(3-2y^2) - y^2]dy = 2\left[\left(3y - \frac{3y^3}{3}\right)\right]_0^1$$

$$= 2[3-1] = 4$$

95. (c) Given, $I_n = \int_0^{\pi/4} \tan^n \theta d\theta$

$$\therefore\ I_{n-1} + I_{n+1} = \int_0^{\pi/4} \tan^{n-1}\theta d\theta$$

$$+ \int_0^{\pi/4} \tan^{n+1}\theta d\theta$$

$$= \int_0^{\pi/4} \tan^{n-1}\theta(1+\tan^2\theta)d\theta$$

$$= \int_0^{\pi/4} \sec^2\theta \tan^{n-1}\theta d\theta$$

Put $\tan\theta = \phi \Rightarrow \sec^2\theta d\theta = d\phi$

$$\therefore\ I_{n-1} + I_{n+1} = \int_0^1 \phi^{n-1} d\phi$$

$$= \left[\frac{\phi^n}{n}\right]_0^1 = \frac{1}{n}$$

96. (a) $\because h = \frac{b-a}{n}$

$$\Rightarrow\ h = \frac{2-0}{4} = 0.5$$

$\therefore$ Simpson's rule,

$$\int_a^b f(x)dx = \frac{h}{3}[(y_0+y_4) + 4(y_1+y_3) + 2(y_2)]$$

$$= \frac{0.5}{3}\left[(1+5) + 4\left(\frac{5}{4} + \frac{13}{4}\right) + 2(2)\right]$$

$$= \frac{0.5}{3}[6+18+4] = \frac{14}{3}$$

97. (b) Given, $\frac{dy}{dx} = \frac{y}{x} + \frac{\phi\left(\frac{y}{x}\right)}{\phi'\left(\frac{y}{x}\right)}$

Put $y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

$$\therefore\quad v + x\frac{dv}{dx} = v + \frac{\phi(v)}{\phi'(v)}$$

$$\Rightarrow\ \frac{\phi'(v)}{\phi(v)}dv = \frac{dx}{x}$$

On integrating, we get

$\log\phi(v) = \log x + \log k$

$\Rightarrow \phi(v) = kx \Rightarrow \phi\left(\frac{y}{x}\right) = kx$

98. (a) Given differential equation can be rewritten as

$$\frac{dy}{1+y} + \frac{\cos x}{2+\sin x}dx = 0$$

Put $2+\sin x = t \Rightarrow (\cos x)dx = dt$

$$\therefore \frac{dy}{1+y} + \frac{1}{t}dt = 0$$

On integrating, we get

$$\log(1+y) + \log t = \log C$$

$$\Rightarrow (1+y)t = C$$

$$\Rightarrow (1+y)(2+\sin x) = C$$

when $y(0) = 1$

$$(1+1)(2+0) = C \Rightarrow C = 4$$

$$(1+y)(2+\sin x) = 4$$

At $x = \frac{\pi}{2}$

$$\left[1+y\left(\frac{\pi}{2}\right)\right]\left[2+\sin\frac{\pi}{2}\right] = 4$$

$$\Rightarrow 1+y\left(\frac{\pi}{2}\right) = \frac{4}{3}$$

$$\Rightarrow y\left(\frac{\pi}{2}\right) = \frac{1}{3}$$

99. (b) Given, $f(x) = x^2 - 4x + 5$

On differentiating w.r.t. x, we get

$f'(x) = 2x - 4$

Put $f'(x) = 0 \Rightarrow x = 2$

For $x > 2, f'(x) > 0$, increasing

$\therefore$ Minimum value is $f(2) = 4 - 8 + 5 = 1$

$\therefore B \in [1, \infty)$

100. (d) Given, $f: R \to R$ and $f(x) = \left[\frac{x}{5}\right]$

Also,

$\{f(x): |x| < 71\} = \{f(x): -71 < x < 71\}$

$= \left\{\left[\frac{x}{5}\right]: -71 < x < 71\right\}$

$\left\{\left[\frac{-70-0.1}{5}\right], \dots, \left[\frac{-70}{5}\right], \dots, \left[\frac{0}{5}\right],\right.$

$\left.\dots, \left[\frac{65}{5}\right], \dots, \left[\frac{70}{5}\right], \dots, \left[\frac{70+0.999}{h}\right]\right\}$

$= \{-15, -14, \dots, 0, \dots, 13, 14\}$

101. (a) Here, we see that $(2n-1)$ is an odd number $\therefore a^{2n-1} + b^{2n-1}$ is divisible by $a+b$.

102. (b) According to the given condition

$$({}^nC_1\, {}^nC_1)({}^{n-1}C_1\, {}^{n-1}C_1)\dots({}^1C_1^1C_1) = 14400$$

$$\Rightarrow ({}^nC^{n-1}C_1 \dots {}^1C_1)^2 = (120)^2$$

$$\Rightarrow n(n-1)(n-2)\dots 1 = 120$$

$$\Rightarrow n! = 5!$$

$$\Rightarrow n = 5$$

103. (b) The five -digit number is formed only when unit place is either 0 or 5.

Case I. When 0 is in unit place, then rest of numbers can be selected in ${}^5P_4$ ways.

$= 5! = 120$

Case II. When 5 is in unit place, then rest of numbers can be selected in ${}^5P_4$ ways.

$= 5! = 120$

And we have to subtract those cases in which 0 is in thousand place.
The number of ways = ${}^4P_3 = 24$
$\therefore$ Required number of ways $= 120 + 120 - 24$
$= 216$

104. **(b)** Given, ${}^{15}P_8 = A + 8 \cdot {}^{14}P_7$

$\Rightarrow \frac{15!}{7!} = A + 8 \cdot \frac{14!}{7!}$

$\Rightarrow A = \frac{14!}{7!}(15-8) = \frac{14!}{7!}(7)$

$= \frac{14!}{6!} = {}^{14}P_8$

105. **(d)** Given, ${}^{n-1}C_3 + {}^{n-1}C_4 > {}^nC_3$

$\therefore \quad {}^nC_4 > {}^nC_3$

$[\because {}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r]$

$\Rightarrow \quad n - 4 > 3$

$\Rightarrow \quad n > 7$

$\therefore \quad n = 8$

106. **(d)** Coefficient of r th term

$=$ Coefficient of $(r+1)$ th term

$\therefore {}^{29}C_{r-1}(3)^{29-r+1}(7)^{r-1} = {}^{29}C_r(3)^{29-r}(7)^r$

$\Rightarrow {}^{29}C_{r-1}\frac{3}{7} = {}^{29}C_r$

$\Rightarrow \frac{29!}{(r-1)!\,(29-r+1)!} \times \frac{3}{7} = \frac{29!}{(29-r)!\,r!}$

$\Rightarrow \frac{3}{7(29-r+1)} = \frac{1}{r}$

$\Rightarrow 3r = 203 - 7r + 7$

$\Rightarrow 10r = 210$

$\Rightarrow r = 21$

107. **(d)** Given, $\frac{x^2+x+1}{(x-1)(x-2)(x-3)} = \frac{A}{x-1} + \frac{B}{x-2}$
$+ \frac{C}{x-3}$

$\Rightarrow x^2 + x + 1 = A(x-2)(x-3)$
$+ B(x-1)(x-3) + C(x-1)(x-2)$

Put $x = 1, 2$ and 3 respectively we get

$1 + 1 + 1 = A(1-2)(1-3)$

$\Rightarrow \quad 3 = 2A \Rightarrow A = \frac{3}{2}$

$2^2 + 2 + 1 = B(2-1)(2-3)$

$\Rightarrow \quad 7 = -B \Rightarrow B = -7$

and $\quad 3^2 + 3 + 1 = C(3-1)(3-2)$

$\Rightarrow \quad 13 = 2C \Rightarrow C = \frac{13}{2}$

$\therefore \quad A + C = \frac{3}{2} + \frac{13}{2} = 8$

108. **(a)** $\sum_{n=1}^{\infty} \frac{2n}{(2n+1)!} = \sum_{n=1}^{\infty} \frac{2n+1-1}{(2n+1)!}$

$$= \sum_{n=1}^{\infty} \left(\frac{1}{2n!} - \frac{1}{(2n+1)!}\right)$$
$$= \left(\frac{1}{2!} - \frac{1}{3!}\right) + \left(\frac{1}{4!} - \frac{1}{5!}\right) + \cdots$$
$$= 1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \frac{1}{5!} + \cdots$$
$$= e^{-1} = \frac{1}{e}$$

109. **(d)** Given, $y = ax^2 + bx + c$ Since, $a > 0$ and $b^2 - 4ac = 0$ Therefore, given curve touch the x-axis and lies above it.

110. **(d)** Since, $\tan A$ and $\tan B$ are the roots of the equation $x^2 - px + q = 0$

$\therefore \tan A + \tan B = p$ and $\tan A \tan B = q$

$$\therefore \tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$
$$= \frac{p}{1-q}$$
$$\Rightarrow \sin(A+B) = \frac{p}{\sqrt{p^2 + (1-q)^2}}$$
$$\therefore \sin^2(A+B) = \frac{p^2}{p^2 + (1-q)^2}$$

111. **(a)** Let $a = -2$,

$\therefore x^3 - 2x + 1 = 0$ and $x^4 - 2x^2 + 1 = 0$

Put $x = 1$, we get

$1 - 2 + 1 = 0$ and $1 - 2 + 1 = 0$

$\Rightarrow$ $0 = 0$ and $0 = 0$

Hence, required value is $a = -2$

112. **(d)** Let $y = \frac{x^2-3x+4}{x^2+3x+4}$

$\Rightarrow x^2(y-1) + x(3y+3) + (4y-4) = 0$

Since, x is real.

$\therefore$ $D \geq 0$

$\Rightarrow$ $B^2 - 4AC \geq 0$

$\Rightarrow$ $(3y+3)^2 - 4(y-1)(4y-4) \geq 0$

$\Rightarrow$ $9y^2 + 9 + 18y - 16(y^2 + 1 - 2y) \geq 0$

$\Rightarrow$ $7y^2 - 50y + 7 \leq 0$

$\Rightarrow$ $(7y-1)(y-7) \leq 0$

$\Rightarrow$ $\frac{1}{7} \leq y \leq 7$

113. **(b)** Given, $A(\alpha, \beta) = \begin{bmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & e^{\beta} \end{bmatrix}$

Now, $|A(\alpha, \beta)| = e^{\beta}(\cos^2\alpha + \sin^2\alpha) = e^{\beta}$

$C_{11} = e^{\beta}\cos\alpha, C_{12} = e^{\beta}\sin\alpha, C_{13} = 0$
$C_{21} = -e^{\beta}\sin\alpha, C_{22} = e^{\beta}\cos\alpha, C_{23} = 0$
$C_{31} = 0, C_{32} = 0, C_{33} = \cos^2\alpha + \sin^2\alpha = 1$

$$\therefore \operatorname{adj}(A(\alpha,\beta)) = \begin{bmatrix} e^{\beta}\cos\alpha & -e^{\beta}\sin\alpha & 0 \\ e^{\beta}\sin\alpha & -e^{\beta}\cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\therefore [A(\alpha,\beta)]^{-1} = \frac{1}{e^{\beta}} \times e^{\beta} \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & e^{-\beta} \end{bmatrix}$$

$$= \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & e^{-\beta} \end{bmatrix} = A(-\alpha,-\beta)$$

114. **(d)** Let $A = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

$$\therefore \begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} [1 \quad 1]$$

$$= \begin{bmatrix} 2x_1 + x_2 \\ 3x_1 + 2x_2 \end{bmatrix} [1 \quad 1]$$

$$= \begin{bmatrix} 2x_1 + x_2 & 2x_1 + x_2 \\ 3x_1 + 2x_2 & 3x_1 + 2x_2 \end{bmatrix}$$

$$\therefore \begin{bmatrix} 2x_1 + x_2 & 2x_1 + x_2 \\ 3x_1 + 2x_2 & 3x_1 + 2x_2 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$$

$\Rightarrow 2x_1 + x_2 = 1$ and $3x_1 + 2x_2 = 0$

On solving, we get

$x_1 = 2$

and $x_2 = -3$

$\therefore \quad A = \begin{bmatrix} 2 \\ -3 \end{bmatrix}$

115. **(b)**

116. **(c)** Let $\Delta = \begin{vmatrix} 24 & 25 & 26 \\ 25 & 26 & 27 \\ 26 & 27 & 27 \end{vmatrix}$

Applying $R_2 \to R_2 - R_1$
and $R_3 \to R_3 - R_2$

$$\Delta = \begin{vmatrix} 24 & 25 & 26 \\ 1 & 1 & 1 \\ 1 & 1 & 0 \end{vmatrix}$$

$= 1[25 - 26] - 1[24 - 26]$
$= -1 + 2 = 1$

117. **(b)** Given, $z = a - \frac{i}{2}$

$$\therefore |i + z|^2 - |i - z|^2 = \left|a + \frac{i}{2}\right|^2 - \left|-a + \frac{3i}{2}\right|^2$$

$$= a^2 + \left(\frac{1}{2}\right)^2 - \left(a^2 + \left(\frac{3}{2}\right)^2\right)$$

$$= \frac{1}{4} - \frac{9}{4} = -2$$

118. **(a)** Let $z = x + iy$

$$\therefore \arg\left(\frac{z-2}{z+2}\right)=\frac{\pi}{3}$$

$$\Rightarrow \arg(z-2)-\arg(z+2)=\frac{\pi}{3}$$

$$\Rightarrow \arg((x-2)+iy)-\arg((x+2)+iy)=\frac{\pi}{3}$$

$$\Rightarrow \tan^{-1}\left(\frac{y}{x-2}\right)-\tan^{-1}\left(\frac{y}{x+2}\right)=\frac{\pi}{3}$$

$$\Rightarrow \tan^{-1}\left[\frac{\frac{y}{x-2}-\frac{y}{x+2}}{1+\frac{y}{x-2}\times\frac{y}{x+2}}\right]=\frac{\pi}{3}$$

$$\Rightarrow \left[\frac{4y}{x^2-4+y^2}\right]=\tan\frac{\pi}{3}=\sqrt{3}$$

$$\Rightarrow x^2-4+y^2=\frac{4y}{\sqrt{3}}$$

$$\Rightarrow$$

$$x^2+y^2-\frac{4y}{\sqrt{3}}-4=0$$

Hence, it represents a equation of circle.

119. (d)

$$\frac{(1+i)^{2011}}{(1-i)^{2009}}=\frac{(\sqrt{2})^{2011}\left(\frac{1}{\sqrt{2}}+\frac{i}{\sqrt{2}}\right)^{2011}}{(\sqrt{2})^{2009}\left(\frac{1}{\sqrt{2}}-\frac{i}{\sqrt{2}}\right)^{2009}}$$

$$=\frac{(\sqrt{2})^2\left(e^{\frac{i\pi}{4}}\right)^{2011}}{\left(e^{-\frac{i\pi}{4}}\right)^{2009}}=2\left(e^{\frac{i\pi}{4}}\right)^{4020}$$

$$=2\left(e^{i\pi}\right)^{1005}=2(-1)^{1005}=-2$$

120. (d) Since, period of $\cos(5x+3)=\frac{2\pi}{5}$

$\therefore$ Period of $f(x)$ is also $\frac{2\pi}{5}$.

121. (d) $16\left[2\sin\frac{A}{2}\sin\left(\frac{5A}{2}\right)\right]=16[\cos 2A-\cos 3A]$

$$=16[2\cos^2 A-1-(4\cos^3 A-3\cos A)]$$

$$=16\left[2\left(\frac{3}{4}\right)^2-1-\left(4\times\frac{27}{64}-3\times\frac{3}{4}\right)\right]$$

$$=16\left[\frac{1}{8}-\left(\frac{27}{16}-\frac{9}{4}\right)\right]$$

$$=16\left[\frac{2-27+36}{16}\right]$$

$$=11$$

122. (c)

123. (b) Given trigonometric equations are

$$\tan\theta=-1$$

and $\cos\theta=\frac{1}{\sqrt{2}}$

$$\Rightarrow \sin\theta=-1/\sqrt{2}$$

$$\Rightarrow \sin\theta=\sin\frac{7\pi}{4}$$

$$\therefore \theta=2n\pi+\frac{7\pi}{4}$$

124. (a) Given equation is

$$(\tan^{-1}x)^2+(\cot^{-1}x)^2=\frac{5\pi^2}{8}$$

$$(\tan^{-1}x+\cot^{-1}x)^2-2\tan^{-1}x\cdot\cot^{-1}x=\frac{5x^2}{8}$$

$$\left[\because \tan^{-1}x+\cot^{-1}x=\frac{\pi}{2}\right]$$

$$\Rightarrow \frac{\pi^2}{4}-2\tan^{-1}x(\pi/2-\tan^{-1}x)=\frac{5\pi^2}{8}$$

$$\Rightarrow -2\cdot\frac{\pi\tan^{-1}x}{2}+2(\tan^{-1}x)^2=\frac{5\pi^2}{8}-\frac{\pi^2}{4}$$

$$\Rightarrow (\tan^{-1}x)^2-\frac{\pi\tan^{-1}x}{2}=\frac{3\pi^2}{16}$$

$$\therefore \tan^{-1}x=\frac{\frac{\pi}{2}\pm\sqrt{\frac{\pi^2}{4}+\frac{3\pi^2}{4}}}{2}$$

$$\Rightarrow \tan^{-1}x=\frac{\frac{\pi}{2}\pm\pi}{2}$$

$$=\frac{3\pi}{4},-\frac{\pi}{4}$$

$$\Rightarrow x=\tan\left(\frac{3\pi}{4}\right),\tan\left(-\frac{\pi}{4}\right)$$

$$\Rightarrow x=-1,-1$$

125. **(a)** We know that

$$\sinh^{-1}(y)=\log\left(y+\sqrt{1+y^2}\right)$$

Put $y=\cot x$

$$\Rightarrow \sinh^{-1}(\cot x)=\log\left(\cot x+\sqrt{1+\cot^2x}\right)$$

$$=\log\left(\cot x+\sqrt{\operatorname{cosec}^2x}\right)$$

$$=\log(\cot x+\operatorname{cosec}x)$$

$$=\log\left(\frac{1+\cos x}{\sin x}\right)$$

$$=\log\left(\frac{2\cos^2x/2}{2\sin x/2\cdot\cos x/2}\right)$$

$$=\log(\cot x/2)$$

126. **(a)** In $\triangle ABC, a\cos^2\frac{c}{2}+a\cos^2\frac{A}{2}=\frac{3b}{2}$

$$\Rightarrow a\left\{\sqrt{\frac{s(s-c)}{ab}}\right\}^2+c\left\{\frac{\sqrt{s(s-a)}}{bc}\right\}=\frac{3b}{2}$$

$$\Rightarrow a\cdot\frac{s(s-c)}{ab}+c\cdot\frac{s(s-a)}{bc}=\frac{3b}{2}$$

$$\Rightarrow \frac{s(s-c)}{b}+\frac{s(s-a)}{b}=\frac{3b}{2}$$

$$\Rightarrow \frac{s}{b}(s-c+s-a)=\frac{3b}{2}$$

$$\Rightarrow 2s(2s-a-c)=3b^2$$

$$\Rightarrow 2s(a+b+c-a-c)=3b^2$$

$$\Rightarrow [\because 2s=a+b+c]$$

$$\Rightarrow 2s\cdot b=3b^2$$

$$\Rightarrow 2s=3b$$

$$\Rightarrow a+b+c=3b$$

$$\Rightarrow 2b=a+c$$

$\Rightarrow$ a, b, c are in arithmetic progression.

127. **(c)** In $\triangle$ ABC, $\frac{\cos A}{a} = \frac{\cos B}{b} = \frac{\cos C}{c}$

By sine rule,

$$\Rightarrow \frac{\cos A}{k\sin A} = \frac{\cos B}{k\sin B} = \frac{\cos C}{k\sin C}$$

$$\Rightarrow \cot A = \cot B = \cot C$$

$$\Rightarrow A = B = C$$

Hence, all angles are equal so the $\triangle$ ABC is an equilateral.

128. **(c)** In $\triangle$ ECD

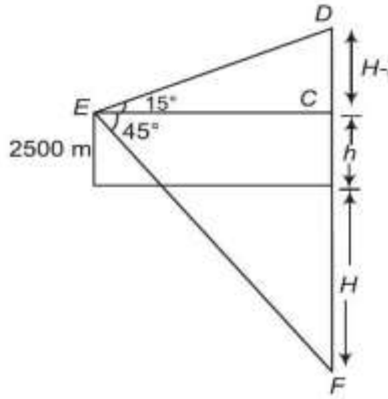
$$\cot 15^\circ = \frac{EC}{CD}$$

$$\Rightarrow EC = CD\cot 15^\circ$$

$$\Rightarrow EC = CD(2+\sqrt{3}), \text{ given}$$

EC= (H -h) $(2+\sqrt{3})$ .... (i)

In $\triangle$ ECF, $\cot 45^\circ = \frac{EC}{CF}$



$\Rightarrow$ $EC = CF\cot 45^\circ = (H+h)$ .... (ii)

From Eq. (i)

$$(H-h)(2+\sqrt{3}) = H+h$$

$$\Rightarrow H(2+\sqrt{3}) - h(2+\sqrt{3}) = H+h$$

$$\Rightarrow H(1+\sqrt{3}) = h(3+\sqrt{3})$$

$$\Rightarrow H = h\left(\frac{3+\sqrt{3}}{1+\sqrt{3}}\right) = 2500\left(\frac{3+\sqrt{3}}{\sqrt{3}+1}\right)$$

$$\Rightarrow H = 2500\sqrt{3}\text{ m}$$

129. **(b)** Let the vector **v** make an angle α with each of the three axes, then direction cosine of **v** are $< \cos\alpha, \cos\alpha, \cos\alpha >$

Also, $\cos^2\alpha + \cos^2\alpha + \cos^2\alpha = 1$

Hence, direction cosine of **v** are $< \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} >$ or $< -\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}} >$

So, the required line which makes equal angle with the coordinate axes is

$$\mathbf{v} = \pm\frac{1}{\sqrt{3}}\mathbf{i} \pm \frac{1}{\sqrt{3}}\mathbf{j} \pm \frac{1}{\sqrt{3}}\mathbf{k}$$

Now, the magnitude of the projection of the vector $\mathbf{a} = 4\mathbf{i} - 3\mathbf{j} + 2\mathbf{j}$ on line **v**.

$\therefore$ Projection of **a** along $\mathbf{v} = \frac{\mathbf{a}\cdot\mathbf{v}}{|\mathbf{v}|}$

$$= \frac{3/\sqrt{3}}{1} = \sqrt{3}$$

130. **(a)** Since, vectors are orthogonal.

$$\therefore (\mathbf{i} - 2x\mathbf{j} - 3y\mathbf{k}) \cdot (\mathbf{i} + 3x\mathbf{j} + 2y\mathbf{k}) = 0$$

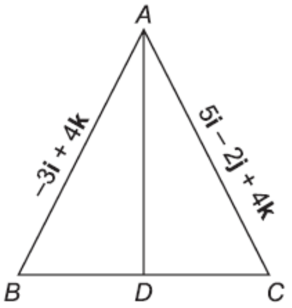
$$\Rightarrow 1 - 6x^2 - 6y^2 = 0 \Rightarrow x^2 + y^2 = \frac{1}{6}$$

Hence, the locus of a point is a circle.

131. **(b)** Now, $i \times (r \times i)$

$= (i \cdot i)r - (i \cdot r)i$
$= r - r_1 i$
Similarly, $j \times (r \times j) = r - r_2 j$
and $k \times (r \times k) = r - r_3 k$
$\therefore (i \times (r \times i)) + (j \times (r \times j)) + (k \times (r \times k))$
$= 3r - (r_1 i + r_2 j + r_3 k)$
$= 3r - r = 2r$

132. **(b)** $\therefore$ Position vector of AD



$$= \frac{1(-3i + 4k) + 1(5i - 2j + 4k)}{1 + 1}$$
$= i - j + 4k$
$\therefore |AD| = \sqrt{1 + 1 + 16} = \sqrt{18}$

133. **(d)** $\therefore \{(\mathbf{a} + 3\mathbf{b}) \times (3\mathbf{a} - \mathbf{b})\}^2$
$= \{3\mathbf{a} \times \mathbf{a} - \mathbf{a} \times \mathbf{b} + 9\mathbf{b} \times \mathbf{a} - 3\mathbf{b} \times \mathbf{b}\}^2$
$= \{-10\mathbf{a} \times \mathbf{b}\}^2$
$= 100[|\mathbf{a}||\mathbf{b}|\hat{\mathbf{n}}\sin 120°]^2$
$$= 100\left[1 \times 2 \times \hat{\mathbf{n}} \times \frac{\sqrt{3}}{2}\right]^2 = 100 \times 3 = 300$$

134. **(c)** Since, $[uvw] = u \cdot (v \times w)$
$\Rightarrow [uvw] \le |u||v \times w|$
$\Rightarrow [uvw] \le |v \times w|$
Now, $v \times w = \begin{vmatrix} i & j & k \\ 2 & 1 & -1 \\ 1 & 0 & 3 \end{vmatrix} = 3i - 7j - k$
$\therefore\ v \times w = \sqrt{9 + 49 + 1} = \sqrt{59}$
$\therefore\ [u\ v\ w] \le \sqrt{59}$

135. **(a)** $\therefore$ Required probability $= \frac{{}^{15}C_2 \times {}^5C_1}{{}^{20}C_3}$
$$= \frac{15 \times 7 \times 5}{\frac{20 \times 19 \times 18}{3 \times 2}}$$
$$= \frac{7 \times 25}{20 \times 19} = \frac{35}{76}$$

136. **(b)** Firstly, we fix the seven white balls in alternate position in 7 !. In out of 8 positions 3 black balls can be placed in ${}^8P_3$ ways.
$\therefore$ Required probability $= \frac{7! \times {}^8P_3}{10!}$
$$= \frac{7! \times 8!}{5! \times 10!}$$
$$= \frac{8 \times 7 \times 6}{10 \times 9 \times 8} = \frac{7}{15}$$

137. **(b)** Given, $P(A) = 0.5, P(B) = 0.4$
and $P(A \cup B) = 0.6$
(i) $\because P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow \quad 0.6 = 0.5 + 0.4 - P(A \cap B)$

$\Rightarrow \quad P(A \cap B) = 0.3$

(ii) $P(A \cap \bar{B}) = P(A) - P(A \cap B)$

$= 0.5 - 0.3 = 0.2$

(iii) $P(\bar{A} \cap B) = P(B) - P(A \cap B)$

$= 0.4 - 0.3 = 0.1$

(iv) $P(\overline{A} \cap \bar{B}) = P(A \cup B)^c$

$= 1 - P(A \cup B) = 1 - 0.6$

$= 0.4$

138. **(a)** Mean, $m = \sum_{i=1}^{4} p_i x_i$

$$= 0 \times \frac{1}{10} + \frac{1 \times 2}{10} + \frac{2 \times 3}{10} + \frac{3 \times 4}{10}$$

$$= 0 + \frac{2}{10} + \frac{6}{10} + \frac{12}{10}$$

$$= \frac{20}{10} = 2$$

$$\therefore \text{ Variance } = \sum_{i=1}^{4} p_i (x_i - m)^2$$

$$= \frac{1}{10}(0-2)^2 + \frac{2}{10}(1-2)^2 + \frac{3}{10}(2-2)^2 + \frac{4}{10}(3-2)^2$$

$$= \frac{4}{10} + \frac{2}{10} + 0 + \frac{4}{10}$$

$$= \frac{10}{10} = 1$$

139. **(d)** Given, $p = 0.001, n = 2000$

$\therefore$ Mean $= np = 2000 \times 0.001 = 2$

Using poisson distribution,

$$P(X = x) = \frac{\lambda^x e^{-x}}{x!}$$

$$\therefore \; P(X = 3) = \frac{2^3 e^{-3}}{3!} = \frac{4}{3e^2}$$

140. **(c)** Let $P(x_1, y_1)$ be any point, then

$$\sqrt{(x_1 - 0)^2 + (y_1 - 2)^2} + \sqrt{(x_1 - 0)^2 + (y_1 + 2)^2} = 6$$

$$\Rightarrow \sqrt{x_1^2 + (y_1 - 2)^2} = 6 - \sqrt{x_1^2 + (y_1 + 2)^2}$$

$$\Rightarrow x_1^2 + (y_1 - 2)^2 = 36 + (x_1^2 + (y_1 + 2)^2) - 12\sqrt{x_1^2 + (y_1 + 2)^2}$$

$$\Rightarrow -8y_1 = 36 - 12\sqrt{x_1^2 + (y_1 + 2)^2}$$

$$\Rightarrow 3\sqrt{x_1^2 + (y_1 + 2)^2} = 2y_1 + 9$$

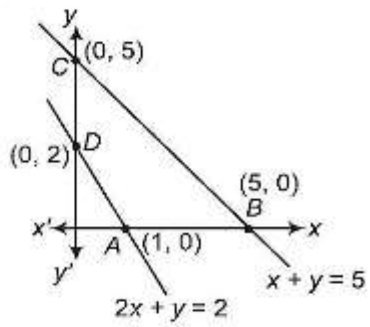
$$\Rightarrow 9(x_1^2 + (y_1 + 2)^2) = 4y_1^2 + 81 + 36y_1$$

$$\Rightarrow \quad 9x_1^2 + 5y_1^2 = 45$$

Hence, locus of a point is

$9x^2 + 5y^2 = 45$

141. **(c)** Given equations are $2x + y = 2, x = 0, y = 0$ and $x + y = 5$.



The total integral points lies inside the quadrilateral ABCD is
$(1,1), (1,2), (2,1), (2,2), (1,3), (3,1)$.

142. **(c)** The equation of perpendicular line on $x+3y=7$ is $3x-y+\lambda=0$.

Since, it is passes through (3,8).

$\therefore \quad 9-8+\lambda = 0$

$\Rightarrow \quad \lambda = -1$

$\therefore \quad 3x-y-1 = 0$

The point of intersection of lines $x+3y=7$ and $3x-y-1=0$ is (1,2), which is the foot of a point.
Let the image of a point (3,8) be (x_1, y_1).

$\therefore \quad \frac{3+x_1}{2}=1, \frac{8+y_1}{2}=2$

$\Rightarrow \quad x_1=-1, y_1=-4$

143. **(a)** Slope of AB $=\frac{1-0}{3-2}=1$

Therefore, $\angle BAX = 45°$

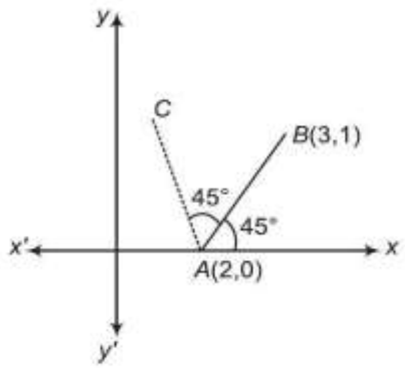
But $\quad \angle BAC = 45°$

$\Rightarrow \quad \angle CAX = 90°$

So, the equation of AC is

$\frac{x-2}{\cos 90°}=\frac{y-9}{\sin 90°}=r$ (say)

We have $AB=\sqrt{(3-2)^2+(1-0)^2}=\sqrt{2}$



As AC is the new position of AB, therefore $AC=AB=\sqrt{2}$ thus, the coordinates of C are given by

$\frac{x-2}{\cos 90°}=\frac{y-0}{\sin 90°}=\sqrt{2}$

$\Rightarrow x=2, y=\sqrt{2}$

Hence, the coordinates of C are $(2,\sqrt{2})$.

144. **(c)** Since, one of the lines represented by $ax^2+2hxy+ky^2=0$ bisect the angle between the axes therefore its equation is $y=x$.

Since, $y=x$ satisfies $ax^2+2hxy+by^2=0$, therefore

$ax^2+2hx^2+bx^2 = 0$

$(a+b)^2=44h^2$ ….. (i)

Now for equation $4x^2+6xy+ky^2=0$

Here, $a=4, h=3, b=k$

Now from Eq. (i)

$\Rightarrow \quad (4+k)^2 = 4(3)^2 = 36$
$\Rightarrow \quad 4+k = \pm 6$
$\Rightarrow \quad k = 2, -10$
Hence, $k \in \{-10, 2\}$
145. **(b)** If $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ represents a pair of parallel lines, then

$$\sqrt{\frac{g^2 - ac}{a(a+b)}} = \sqrt{\frac{f^2 - bc}{b(a+b)}}$$

$$\Rightarrow \sqrt{\frac{g^2 - ac}{f^2 - bc}} = \sqrt{\frac{a}{b}}$$

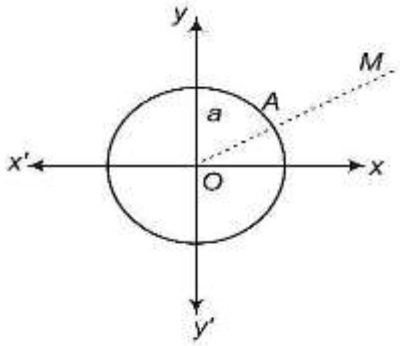
146. **(b)** Given pair of lines is
$3x^2 - 2xy - 15y^2 = 0$
on compare with,
$ax^2 + 2hxy + by^2 = 0$
$\Rightarrow a = 3, h = -1, b = -15$
Now, sum of slopes,

$$s = m_1 + m_2 = \frac{-2h}{b} = \frac{-2(-1)}{-15} = \frac{-2}{15}$$

and product of slopes

$$p = m_1 m_2 = \frac{a}{b} = \frac{3}{-15} = \frac{-3}{15}$$

Then, $s : p = \frac{-2}{15} : \frac{-3}{15} = 2 : 3$
147. **(d)** Given line, $y = 2x + c \ldots$ (i)
Here, $m = 2$ and $c_1 = c$
Given equation of circle is
$x^2 + y^2 = 5$
Here radius $a = \sqrt{5}$
Since, the line (i) is tangent to the circle.
$\therefore \quad c = a\sqrt{m^2 + 1} = \sqrt{5}\sqrt{(2)^2 + 1}$
$\Rightarrow \quad c = \sqrt{5} \cdot \sqrt{5} = 5$
148. **(a)** The given equation of circle,
$x^2 + y^2 = a^2$
and length of $AM = a$



Now, total length $OM = OA + AM = 2a$
Since, point A moves along the circle, so it describes a circle with radius $OM = 2a$, whose equation is
$x^2 + y^2 = (2a)^2$
$\Rightarrow \quad x^2 + y^2 = 4a^2$
149. **(c)** Given, lines
$3x + 4y - 14 = 0 \ldots$ (i)
and $6x + 8y + 7 = 0 \ldots$ (ii)
are both tangents to a circle we observe that, both lines are parallel
i.e., $3x + 4y - 14 = 0$

and $3x + 4y + \frac{7}{2} = 0$
So, radius of circle
$= \frac{1}{2}$ (distance between two parallel lines)
$$= \frac{1}{2}\left\{\frac{|7/2 + 14|}{\sqrt{9+16}}\right\}$$
$$= \frac{1}{2}\left\{\frac{35/2}{5}\right\} = \frac{1}{2} \times \frac{7}{2}$$
$$= \frac{7}{4}$$

150. **(b)**

151. **(b)** Given equation of circles are
$S_1 \equiv x^2 + y^2 + 2x + 2y + 1 = 0$
and $S_2 \equiv x^2 + y^2 - 2x + 2y + 1 = 0$
In circle S_1, centre $c_1 = (-1, -1)$, radius $r_1 = \sqrt{1+1-1} = 1$
In circle S_2, centre $C_2 = (1, -1)$, radius $r_2 = \sqrt{1+1-1} = 1$
Now, distance between two centre $C_1C_2 = \sqrt{(-1-1)^2 + 0}$
$= 2$
Here, we see that
$C_1C_2 = r_1 + r_2$
i.e., both circle touch externally.
So, point of contact of circle S_1 and S_2
$=$ Mid-point of C_1 and C_2
$$= \left(\frac{-1+1}{2}, \frac{-1-1}{2}\right) = (0, -1)$$

152. **(c)** Let $P(t^2, 2t)$ be the one end of a focal chord PQ of the parabola $y^2 = 4x$, the coordinate of the other end Q are $\left(\frac{1}{t^2}, \frac{-2}{t}\right)$.
$[\because tt' = -1]$
$$\therefore\ PQ = \sqrt{\left(t^2 - \frac{1}{t^2}\right)^2 + \left(2t + \frac{2}{t}\right)^2}$$
$$= \left(t + \frac{1}{t}\right)\sqrt{\left(t - \frac{1}{t}\right)^2 + 4}$$
$$PQ = \left(t + \frac{1}{t}\right)^2 \quad \ldots..(i)$$

Given, the chord makes a θ with positive direction of x-axis
$$\Rightarrow\ \tan\theta = \frac{-2/t - 2t}{1/t^2 - t^2} = \frac{-2}{(1/t - t)}$$
$$= (t - 1/t) = 2\cot\theta$$
Now from Eq. (i)
$PQ = (t + 1/t)^2$
$PQ = (t - 1/t)^2 + 4$
$= (2\cot\theta)^2 + 4$
$= 4(\cot^2\theta + 1)$
$= 4\operatorname{cosec}^2\theta$

153. **(d)** Given equation of parabola, $y^2 = lx$
Here, length of latusrectum $= l$
Since, point $\left(\frac{c^2}{8}, c\right)$ is the point of intersection of parabola and the line $y = mx + c$ then it will satisfy the equations.

$\Rightarrow \quad y^2 = lx$

$\Rightarrow \quad (c)^2 = \frac{lc^2}{8} \Rightarrow l = 8$

$\therefore$ Length of latusrectum $(l) = 8$

154. **(a)** Equation of ellipse

$$x^2 + 4y^2 + 2x + 16y + 13 = 0$$

$$\Rightarrow (x^2 + 2x) + (4y^2 + 16y) + 13 = 0$$

$$\Rightarrow (x^2 + 2x + 1) + 4(y^2 + 4y + 4) - 16$$

$$\Rightarrow +13 - 1 = 0$$

$$\Rightarrow (x+1)^2 + 4(y+2)^2 = 4$$

$$\Rightarrow \frac{(x+1)^2}{4} + \frac{(y+2)^2}{1} = 1$$

$$\Rightarrow \frac{X^2}{4} + \frac{Y^2}{1} = 1$$

where, $X = x + 1$ and $Y = y + 2$

Here, $a^2 = 4$ and $b^2 = 1$

i.e., $a > b$

The eccentricity,

$$\Rightarrow b^2 = a^2(-e^2 + 1)$$

$$\Rightarrow 1 = 4(-e^2 + 1) = -4e^2 + 4$$

$$\Rightarrow e^2 = \frac{3}{4}$$

$$\Rightarrow e = \frac{\sqrt{3}}{2}$$

155. **(c)** Given equation of hyperbola is

$$x^2 - 3y^2 = 3$$

$$\Rightarrow \frac{x^2}{3} - \frac{y^2}{1} = 1$$

Here, $a^2 = 3$ and $b^2 = 1, a > b$.

Now, the equation of asymptote of this hyperbola is,

$$y = \pm\frac{b}{a}x$$

$$\Rightarrow y = \pm\frac{1}{\sqrt{3}}x$$

$$\Rightarrow y = \frac{x}{\sqrt{3}} \; \dots\dots (i)$$

$$\text{and} \quad y = \frac{-x}{\sqrt{3}} \; \dots\dots (ii)$$

Let slope of asymptote (i) is, $m_1 = \frac{1}{\sqrt{3}}$

and slope of asymptote (ii) is, $m_2 = \frac{-1}{\sqrt{3}}$

Let θ be the angle between both asymptote, then

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right| = \left|\frac{1/\sqrt{3} + 1/\sqrt{3}}{1 - 1/3}\right|$$

$$\Rightarrow \tan\theta = \left|\frac{2/\sqrt{3}}{2/3}\right| = \sqrt{3}$$

$$\Rightarrow \tan\theta = \tan 60^\circ$$

$$\Rightarrow \theta = \pi/3$$

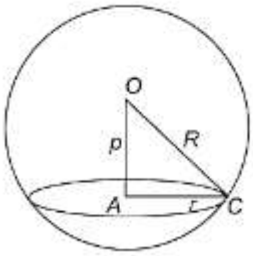
156. **(a)** Given polar equation of line

$\sin\theta - \cos\theta = 1/r \; \dots\dots (i)$

and point $(2, \pi/6)$

Let $x = r\cos\theta = 2 \cdot \cos\pi/6 = \sqrt{3}$

and $y = r\sin\theta = 2 \cdot \sin\pi/6 = 1$
The cartesian point is $(\sqrt{3}, 1)$.
Now, we change the polar of line into cartesian form
i.e., $r\sin\theta - r\cos\theta = 1$
$\Rightarrow \quad y - x = 1 \ldots\ldots (ii)$
Equation of perpendicular line to Eq. (ii) is
$y + x = \lambda \ \ldots (ii)$
which passes through $(\sqrt{3}, 1)$
$\lambda = \sqrt{3} + 1$
From Eq. (iii), we get
$x + y = \sqrt{3} + 1$
Now, we convert this into polar form
$r\cos\theta + r\sin\theta = \sqrt{3} + 1$
$$\Rightarrow \sin\theta + \cos\theta = \frac{\sqrt{3}+1}{r}$$
157. **(c)** Let the ratio is $\lambda : 1$.
Equation of plane
$3x + 2y + z = 4 \ \ldots (i)$
By section formula, coordinates of
$$P = \left(\frac{-4\lambda+2}{\lambda+1}, \frac{5\lambda-4}{\lambda+1}, \frac{-6\lambda+3}{\lambda+1}\right)$$
which lies on the plane
$$3\left(\frac{-4\lambda+2}{\lambda+1}\right) + 2\left(\frac{5\lambda-4}{\lambda+1}\right) + \left(\frac{-6\lambda+3}{\lambda+1}\right) = 4$$
$\Rightarrow -12\lambda + 6 + 10\lambda - 8 - 6\lambda + 3 = 4\lambda + 4$
$\Rightarrow \ -8\lambda + 1 = 4\lambda + 4$
$\Rightarrow \ 12\lambda = -3 \Rightarrow \lambda = -1/4$
$\Rightarrow \ \lambda : 1 = -1 : 4$
158. **(c)** We know that, if α, β, γ are angles of a line which makes from the coordinate axes
Also, $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$ .....(i)
But, given $\alpha = \alpha, \beta = \frac{\pi}{2} - \alpha, \gamma = \beta$
From Eq. (i)
$$\cos^2\alpha + \cos^2\left(\frac{\pi}{2} - \alpha\right) + \cos^2\beta = 1$$
$$\Rightarrow \quad (\cos^2\alpha + \sin^2\alpha) + \cos^2\beta = 1$$
$$\Rightarrow \quad \cos^2\beta = 0$$
$$\Rightarrow \quad \cos\beta = 0 = \cos\frac{\pi}{2}$$
$$\Rightarrow \quad \beta = \frac{\pi}{2}$$
159. **(d)**
160. **(b)** Equation of sphere,
$x^2 + y^2 + z^2 + 2x - 2y - 4z - 19 = 0 \ldots (i)$
and equation of plane
$x + 2y + 2z + 7 = 0 \ \ldots. (ii)$
centre of the sphere $(-1,1,2)$ and radius of the sphere
$R \ = \sqrt{1 + 1 + 4 + 19}$
$\quad = \sqrt{25} = 5$
Now, p = perpendicular distance from the centre to the plane
$$p = \frac{|-1 + 2 + 4 + 7|}{\sqrt{1 + 4 + 4}} = \frac{12}{\sqrt{9}} = \frac{12}{3} = 4$$



In $\triangle AOC$

$R^2 = p^2 + r^2$

$\Rightarrow r^2 = R^2 - p^2$

$\Rightarrow r^2 = 25 - 16 = 9$

$\Rightarrow r = 3$