Physics

1. **(d)** The time taken by the transverse wave

$$t = 2\sqrt{\frac{l}{g}} = 2\sqrt{\frac{2.45}{9.8}} = 1\ \text{s}$$

2. **(b)** We have, nl = constant

$$n_1 l_1 = n_2 l_2$$
$$(n+8)100 = (n-8)101$$
$$\frac{n+8}{n-8} = \frac{101}{100}$$
$$\frac{n}{8} = \frac{201}{1}$$
$$n = 1608$$

3. **(d)** From the lens maker formula

$$\frac{\mu_3}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R_1} + \frac{\mu_3 - \mu_2}{R_2}$$

$$\frac{4}{3f_o{}'} - \frac{1}{\infty} = \frac{1.5 - 1}{R} + \frac{\frac{4}{3} - 1.5}{-R}$$

$$\Rightarrow\ f_o' = R = 2f$$

Now, length $= f_o{}' + f_e{}'$

$$= 2f_o + 2f_e = 2(f_o + f_e)$$
$$= 2(L) = 2(16) = 32\ \text{cm}$$

4. **(d)** $P = P_1 + P_2 = \frac{q}{f_1} + \frac{q}{f_2}$

$$\frac{1}{60} = \frac{q}{f_1} + \frac{q}{f_2}$$
$$\frac{1}{60} = \frac{f_1 + f_2}{f_1 f_2} \ldots\ldots (i)$$

According to question,

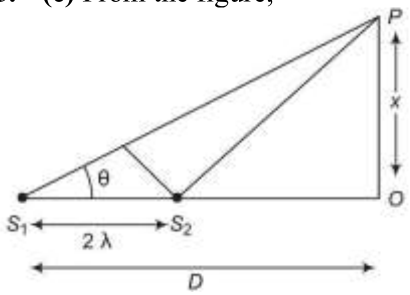
$$\frac{\omega_1}{\omega_2} = \frac{f_1}{f_2} \Rightarrow \frac{f_1}{f_2} = \frac{-4}{3}$$
$$f_1 = -\frac{4}{3}f_2$$

or

From Eq. (i)

$$\frac{1}{60} = \frac{\frac{4}{3}f_2 + f_2}{(f_2)^2\frac{4}{3}} \Rightarrow \frac{-\frac{1}{3}}{-\frac{4}{3}f_2}$$
$$f_2 = 15\ \text{cm}$$
$$\therefore\ f_1 = -\frac{4}{3} \times 15$$
$$= -20\ \text{cm}$$

5. **(c)** From the figure,



and

$(S_1P)^2 = D^2 + x^2$
$(S_2P)^2 = (D - 2\lambda)^2 + x^2$
$\text{so}(S_1P)^2 - (S_2P)^2 = D^2 + x^2$
$-[D^2 + 4\lambda^2 - 4D\lambda + x^2]$
$= 4D\lambda - 4\lambda^2$
$(S_1P + S_2P)(S_1P - S_2P) = 4D\lambda - 4\lambda^2$
$[D >> \lambda]$
Since, $[D >> \lambda]$
$\therefore\ S_1P = S_2P$
and neglecting higher power of λ
$\therefore\ 2S_1P(S_1P - S_2P) = 4D\lambda$
$2(S_1P)(\lambda) = 4D\lambda$
or
$S_1P = 2D$
or $\sqrt{D^2 + x^2} = 2D$
or $x^2 = 3D^2$
or
$x = \sqrt{3}D$
Alternative
$2\lambda\cos\theta = \lambda$
$\Rightarrow\ \cos\theta = \frac{1}{2} \Rightarrow \theta = 60°, \tan\theta = \frac{x}{D}$
$\sqrt{3} = \frac{x}{D} \Rightarrow x = \sqrt{3}D$

6. **(d)** $T = 2\pi\sqrt{\frac{I}{MB_H}}$

$\therefore\ \frac{T_1}{T_2} = \sqrt{\frac{(B_H)_2}{(B_H)_1}}$

$T_2 = T_1\sqrt{\frac{(B_H)_1}{(B_H)_2}}$

$n_1 = 30$ oscillation /min
$n_1 = \frac{1}{2}$ oscillation /s
$T_1 = 2$ s

$T_2 = 2\sqrt{\frac{B_H}{2B_H}}$

$= 2 \times \frac{1}{\sqrt{2}} = \sqrt{2}$ s

7. **(a)** Work done $W = MB(1 - \cos 60°)$
$= \frac{MB}{2}$
The torque required to maintain the magnetic needle
$\tau = MB\sin\theta$
$= MB\sin 60°$
$= MB\frac{\sqrt{3}}{2}$
$\tau = \sqrt{3}W$

8. **(a)** Charge that through the wire

$\Delta q = \Delta CV$
$= (C' - C)V$
$= (k - 1)\sigma V$
$= (20 - 1)(80 \times 10^{-6})(30)$
$= 4.56 \times 10^{-2}$
$= 45.6 \times 10^{-3} C$

9. **(a)** In this case, tension will be equal to force

The tension in each string

$T = F$

$$= \frac{1}{4\pi\varepsilon_0} \frac{Q^2}{(2L)^2}$$

$$= \frac{Q^2}{16\pi\varepsilon_0 L^2}$$

10. **(a)** For figure I,
Total resistance $= 3R$
I power $= I^2(3R)$
For figure II,
Resistance $= \frac{2R}{3}\Omega$
II power $= I^2 \frac{2R}{3}$
For figure III,
Total resistance $= \frac{R}{3}\Omega$
III power $= I^2 \frac{R}{3}$
For figure IV,
Total resistance $\frac{3}{2}\Omega$
IV power $= I^2 \frac{3}{2}R$

11. **(c)** Balancing condition of bridge balance

$$\frac{A}{B} = \frac{100}{x}$$

$$\frac{B}{A} = \frac{121}{x}$$

$$\therefore \frac{100}{x} = \frac{x}{121}$$

$$x^2 = 12100$$

$$x = 110\Omega$$

12. **(d)** Total emf produced in a thermocouple does not depend on the duration of time for which the current is passed through thermocouple.

13. **(b)** The magnetic field

$$dB = \frac{\mu_0 I \times r \times dl}{4\pi r^2}$$

14. **(b)** The induced emf in secondary coil.

$$e = M\frac{di}{dt} = 92 \times 10^{-6} \times \frac{25}{1 \times 10^{-3}} = 2.3\ V$$

15. **(d)** The ratio of kinetic energy of the electron to that of the photon.

$$= \frac{v}{2c}$$

$$= \frac{1.5 \times 10^8}{2 \times 3 \times 10^8} = \frac{1}{4}$$

16. **(a)** $\lambda_p = \lambda_\alpha$

$(mqV)_p = (mqV)_\alpha$
Potential difference
$V_\alpha = \frac{V}{8} \begin{bmatrix} \because m_\alpha = 4m_p \\ q_\alpha = 2q_p \end{bmatrix}$

17. (c) The number of atoms decay in one second

$$\frac{dN}{dt} = \lambda N$$
$$= \frac{0.693}{1620 \times 365 \times 86 \times 400} \times \frac{6.023 \times 10^{23}}{226}$$
$$= 3.61 \times 10^{10}$$

18. (b) After 4 half-lifes

$$\frac{N_0}{2^4} = \frac{N_0}{16}$$

19. (c) $\beta = \frac{\alpha}{1-\alpha}$

$$\beta_1 = \frac{\frac{20}{21}}{1 - \frac{20}{21}} = 20, \ \beta_2 = \frac{\frac{100}{101}}{1 - \frac{100}{101}} = 100$$

The value of β lies between $20 - 100$.

20. (c) D-layer - $65 - 75$ km
E-layer - 95 - 120 km
F_1-layer - 170 - 190 km
F_2-layer - $250 - 400$ km

21. (b) We have,

$$dV = -E \cdot dr = -(5\mathbf{i} + 12\mathbf{j}) \cdot (12\mathbf{i} + 15\mathbf{j})$$
$$= (60 + 60) = -120$$

The change in gravitational potential energy
$U = mdV = 2 \times (-120) = -240$ J

22. (c) Distance $y_1 = A\sin\frac{2\pi}{8} \times 1 = \frac{A}{\sqrt{2}}$

and $y_2 = A - \frac{A}{\sqrt{2}}$

The ratio of the distances travelled by it in the first and seconds.

$$\frac{y_1}{y_2} = \frac{1}{\sqrt{2} - 1}$$

23. (d) Young's modulus of materials

$$Y = \frac{F \times l}{A\Delta l}, \frac{\Delta l}{l} = \frac{F}{AY}$$
$$= \frac{22}{0.02 \times 10^{-4} \times 1.1 \times 10^{11}} = 10^{-4}$$

Poisson's ratio, $\sigma = \frac{\frac{\Delta l}{\Delta r}}{\frac{r}{r}}$

$$\frac{\Delta l}{l} = \sigma\frac{\Delta r}{r} = 0.32 \times \frac{\Delta r}{r}$$
$$= 0.32 \times 10^{-4} = 32 \times 10^{-6}$$

The decrease in cross-sectional area,

$$\frac{\Delta A}{A} = \frac{\Delta A}{0.02} = 0.32 \times \frac{\Delta l}{l}$$
$$\Delta A = 0.64 \times 10^{-6}\ \text{cm}^2$$

24. (c) According to the question,

$$\frac{4}{3}\pi r^3 dg = \frac{2}{3}\pi r^3 \rho g + T \times 2\pi r$$

Or $r = \sqrt{\frac{3T}{g(2d-\rho)}}$

25. (a) $\pi \frac{D^2}{2} Xv = n \frac{d^2}{2} Xv', \; v' = \frac{D^2 v}{nd^2}$

26. (d) We have,

$$\gamma a_1 + 3\alpha_1 = \gamma a_2 + 3\alpha_2$$

$$6 \times 10^{-6} + 3(18 \times 10^{-6}) = 24 \times 10^{-6} + 3\alpha_2$$

$$\alpha_2 = 12 \times 10^{-6} / {}^\circ C$$

27. (b) $\frac{\Delta I}{I} = 2\frac{\Delta k}{k} = 2\alpha \Delta T$

$$\frac{\Delta I}{I} = 2\alpha \Delta T$$

28. (c) $T^\gamma p^{1-\gamma} = \text{constant}$

or $\quad T^\gamma = p^{\gamma - 1}$

$$T = p^{\frac{\gamma - 1}{\gamma}}$$

$$\therefore \frac{\Delta T}{T} = \frac{\gamma - 1}{\gamma} \times \frac{\Delta p}{p}$$

$$\frac{\Delta T}{T} = \frac{1.5 - 1}{1.5} \times \frac{0.001}{1.013 \times 10^6}$$

$$\Delta T = 8.98 \times 10^{-8}\ K$$

29. (c) We have, $V \propto T^{2/3}$

and $p \propto \sqrt{V}$

$\therefore p = k\sqrt{V} \Rightarrow p = kV^{1/2}$

Work done, $W = \int pdV$

$$W = \int kV^{1/2}$$

$$W = \frac{kV^{3/2}}{3/2}$$

$$W = \frac{2}{3} \frac{p}{\sqrt{V}} V^{3/2} = \frac{2}{3} pV \;\; \because V = \frac{p}{\sqrt{V}}$$

$$= \frac{2}{3} nRT = \frac{2}{3} \times 1 \times 8.3 \times 30 = 166.2\ J$$

30. (d)

31. (c) Moment of inertia

$$I = \frac{2}{3} MR^2$$

$$\therefore \frac{I_A}{I_B} = \frac{\frac{2}{5} M_A R^2}{\frac{2}{5} M_B R^2} = \frac{\frac{4}{3}\pi R^3 \rho_A}{\frac{4}{3}\pi R^3 \rho_B} = \frac{\rho_A}{\rho_B}$$

32. (c) Moment of inertia

$I = MR^2$

i.e., the moment of inertia depends on the mass of the body.

33. (a) We have, $\mu_k mgs = \frac{p^2}{2M}$

or $s = \frac{p^2}{2M^2 \mu_k g}$

34. (d)

35. (c) Kinetic energy

$$K=\frac{1}{2}mu_1^2$$

$$u_1=\sqrt{\frac{2K}{m}}$$

$$\frac{1}{2}mv_1^2=\frac{K}{9}$$

$$v_1^2=\frac{2K}{9m}$$

Velocity, $v_1=\sqrt{\frac{2K}{9m}}$

Kinetic energy $K=\frac{p^2}{2m}$

From conservation of the momentum

$$mu_1=-mv_1+p_B$$

$$m(u_1+v_1)=p_B$$

Final momentum of B

$$p_B=m\sqrt{\frac{2K}{m}}+\sqrt{\frac{2K}{9m}}$$

$$=\sqrt{2mK}+\sqrt{\frac{2mK}{3}}=p+\frac{p}{3}=\frac{4p}{3}$$

36. (d) Induced emf is given by

$$e=Bvl\sin\theta$$

$$=0.1\times 10<4\sin 30=2\text{ V}$$

37. (b) $P=\frac{\frac{1}{2}mv^2}{t}\Rightarrow v^2=\frac{2Pt}{m}\Rightarrow v^2\propto\frac{2P}{m}$

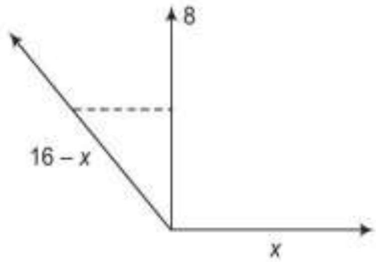
38. (b) $T_1=\frac{2u\sin\theta}{g}$

and $T_2=\frac{2u\cos\theta}{g}$

$$T_1T_2=\frac{2u^2\sin\theta\cos\theta}{g}=\frac{2r}{g}$$

$$\because R=\frac{v^2\sin 2\theta}{g}$$

39. (a) $(16-x)^2=8^2+x^2$



$$256+x^2-32x=64+x^2$$

$$32x=192$$

$$x=\frac{192}{32}=6$$

∴ Forces 6 N, 10 N.

40. (c) $\frac{\Delta g}{g}=\frac{\Delta l}{l}+\frac{2\Delta T}{T}$

We get, $\frac{\Delta g}{g}\%=10\%$

Chemistry

41. (a) Benzene-4-hydroxy acetanilide is also called paracetamol. It is an antipyretic (i.e., used for lowering body temperature) as well as analgesics (i.e., used to releive pain).

42. (c) Insulin is an hormone, secreted by pancreas. It is transported to different body parts by the blood stream. It maintains glucose level in blood and glucose metabolism. Thus, its site of action is plasma membrane.

43. (b) Chloroprene (or 2-chlorobuta-1,3-diene) is the monomer of synthetic rubber, neoprene.

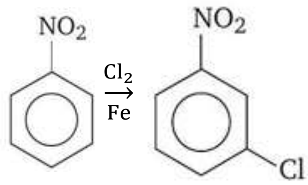
$$nCH_2{=}\underset{\substack{|\\ }}{\overset{\substack{Cl\\|}}{C}}{-}CH{=}CH_2 \xrightarrow{\text{polymerisation}}$$

2- chlorobuta- 1,3- diene

$$\left[CH_2{-}\overset{\substack{Cl\\|}}{C}{=}CH{-}CH_2 \right]_n$$

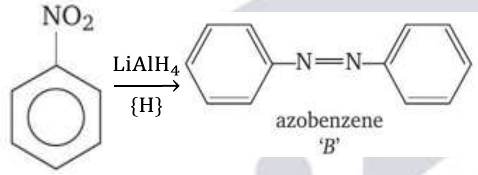
neoprene
(synthetic rubber)

44. (c) NO_2 being meta directing gives m -chloronitrobenzene, when treated with Cl_2, Fe.



m- chloronitrobenzene
A

On reduction with $LiAlH_4$, nitrobenzene gives azobenzene.



azobenzene
'B'

45. (a)

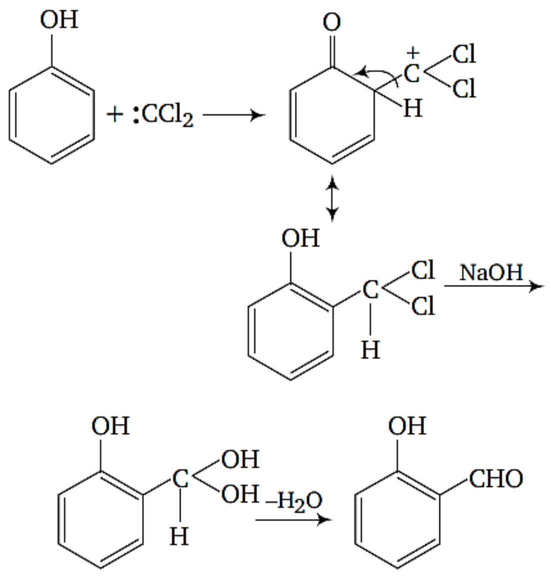
46. (a)

47. (b)

$$CHCl_3 + NaOH \longrightarrow \underset{\text{unstable}}{:CCl_3^-} + Na^+ + H_2O$$

$$\downarrow$$

$$\underset{\substack{\text{dichlorocarbene}\\ \text{(electrophile)}}}{:CCl_2} + Cl^-$$



salicylaldehyde
(o-hydroxy benzaldehyde)
product

This reaction is known as Reimer-Tiemann reaction.

48. (d) Cl exhibits $-$I and $+$M effect. Due to which it is ortho/para directing but ring deactivating.

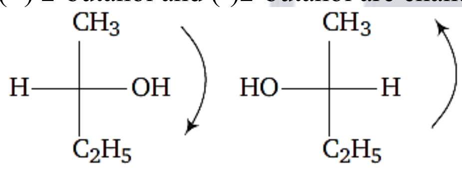
49. (b)

$$\underset{\text{acetaldehyde}}{CH_3CHO} \xrightarrow{\text{Tautomerisation}} \underset{\text{vinyl alcohol}}{CH_2} = CH - OH$$

Thus, these two are tautomers.

Eclipsed and staggered ethane are two conformations of ethane.

(+) 2-butanol and (-)2-butanol are enantiomers as these are non-superimposable mirror images.



(+)-2-butanol (–)-2-butanol

$CH_3NHC_3H_7$
and (methyl- n-propylamine)
$C_2H_5NHC_2H_5$
(diethylamine)

Due to difference in the nature of alkyl groups attached to the same functional group, these are called metamers.

50. (d) In benzene all the C $-$ C bond lengths are equal to 1.39Å ∘ due to resonance.

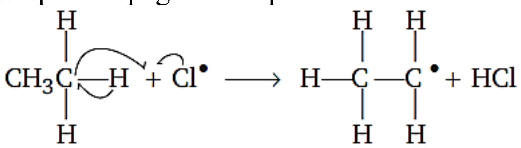
51. (d) Conformational isomers are also known as rotamers and the isomerism as rotamerism.

52. (c) Chlorination of ethane takes place by free radical substitution and involves the following steps

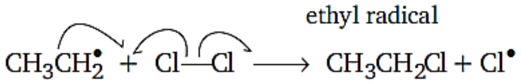
Step I Initiation step

$$Cl_2 \xrightarrow{h\nu} 2Cl^{\cdot}$$

Step II Propagation step



ethyl radical



Step III Termination step

$$CH_3CH_2^{\bullet} + Cl^{\bullet} \longrightarrow CH_3CH_2Cl$$
$$Cl^{\bullet} + Cl^{\bullet} \longrightarrow Cl_2$$
$$2CH_3CH_2^{\bullet} \longrightarrow CH_3CH_2CH_2CH_3$$

53. (c) Oxides of nitrogen and sulphur are responsible for acid rain.
$NO_2 + H_2O \rightarrow HNO_3$
$SO_2 \xrightarrow{[O]} SO_3 + H_2O \rightarrow H_2SO_4$
Due to the presence of HNO_3 and H_2SO_4 (strong acids) rain- water becomes acidic and rain is called the acid rain.

54. (b) As the size of atom (element) increases, energy required to overcome the attractive force on outermost electron decreases. Li, Na, K and Rb are the members of same group and have the following order of size
$Li < Na < K < Rb$
Thus, the energy in case of K is intermediate of Rb and Na , i.e.,
$2.30 < K > 2.09$
or the energy for K is 2.20 eV .

55. (a) Spin quantum number explains the line spectra observed as doublets in case of H and alkali metals and doublets and triplets in case of alkaline earth metals.

56. (d) Phosphorus being a non-metal always forms acidic oxides like P_2O_3, P_5O_{10}, etc. Al_2O_3, SnO and Sb_2O_3 are amphoteric oxides i.e., react with acids as well as bases.

57. (d) Formal charge (FC) $= V - lp - \frac{1}{2}bp$
[where, V = valence electrons, lp = lone pair and bp = bond pair]
For $:\ddot{O} = C = \ddot{O}:$
$$\text{FC of } O = 6 - 4 - \frac{1}{2} \times 4 = 0$$
$$\text{FC of } C = 4 - 0 - \frac{1}{2} \times 8 = 0$$

58. (c) The molecular orbital electronic configuration of O_2 is
$O_2(8+8=16e^-) = \sigma 1s^2, {}^*\sigma 1s^2, \sigma 2s^2, {}^*\sigma 2s^2,$
$\sigma 2p_z^2, \pi 2p_x^2, \approx \pi 2p_y^2, \overset{*}{\pi} 2p_x^1 \approx \overset{*}{\pi} 2p_y^1$
(Unstarred σ and π represent bonding orbitals.)
$\therefore$ number of bonding electrons $= 10$
and number of bonding electron pairs $= 5$

59. (c) $N_2H_4 - 10e^- \rightarrow X^{10+}$ (with two N atoms)
Total oxidation number of two N atoms in N_2H_4,
$\Rightarrow 2x + 4 = 0$
$2x = -4$
N_2H_4 loses 10 electrons, so total oxidation number of two N -atoms increases by 10 , i.e., the total oxidation number of two N -atoms in
$Y = -4 + 10 = +6$
$\therefore$ Oxidation number of each N atom in
$X^{10+} = +3$

60. (b) The variation of viscosity coefficient (η) with temperature (T) is given by the following expression $\eta = Ae^{E/RT}$

61. (c) Relative lowering of vapour pressure,
$$\frac{p^\circ - p_S}{p^\circ} = x_A$$
$$= \frac{\frac{w_A}{m_A}}{\frac{w_A}{m_A} + \frac{w_B}{m_B}}$$
(where, w_A and m_A are the mass and molar mass of solute and w_B and m_B are the mass and molar mass of water.)

$$0.02=\frac{\frac{x}{60}}{\frac{x}{60}+\frac{90}{18}}$$

$$0.02=\frac{\frac{x}{60}}{\frac{x}{60}+5}$$

$$\frac{1}{0.02}=\frac{\frac{x}{60}+5}{\frac{x}{60}}$$

$$50=1+\frac{5\times 60}{x}$$

$$49=\frac{300}{x}$$

$$x=\frac{300}{49}=6\text{ g}$$

62. **(c)** $BaCl_2$ being ionic in nature, ionises completely to give Ba^{2+} and $2Cl^-(1+2=3)$ ions, i.e., number of particles increases in the solution.

$\therefore\ i=\frac{3}{1}=3=\frac{M_c}{M_o}$

(where, M_c = calculated molar mass and M_o = observed molar mass)

$3=\frac{M_c}{M_o}$

or $M_o=\frac{M_c}{3}$

i.e., $M_o<M_c$

or observed molecular mass of $BaCl_2$ is less than the calculated molecular mass.

63. **(a)** $Al^{3+}+\underset{3\text{ mol}}{3e^-}\rightarrow\overset{Al}{1\text{ mol}}=27\text{ g}$

$\because$ 27 g of Al is deposited by 3 moles of electrons

$\therefore$ 36 gAl will be deposited by electrons

$=\frac{3}{27}\times 36=4\text{ mol}$

64. **(d)** $E^\circ_{cell}=E^\circ_{Cu^{2+}/Cu^-}-E^\circ_{Zn^{2+}/Zn}$

$=+0.34-(-0.76)V$

$=1.1\text{ V}$

Further $E_{cell}=E^\circ_{cell}-\frac{0.059}{n}\log\frac{[\text{products}]}{[\text{reactants}]}$

For the reaction,

$CuSO_4+Zn\rightarrow ZnSO_4+Cu$

or $Cu^{2+}+Zn\rightarrow Zn^{2+}+Cu$

$E_{cell}=E^\circ_{cell}-\frac{0.059}{2}\log\frac{[Zn^{2+}]}{[Cu^{2+}]}$

$=1.1-\frac{0.059}{2}\log\frac{0.1}{0.01}$

$=1.1-\frac{0.059}{2}\log 10$

$=1.1-0.0295\times 1\ [\because \log 10=1]$

$=1.07\text{ V}$

65. **(c)** When silicon is doped with such an impurity valency of which is less than 4, p-type semiconductor is obtained. valency of As, P and Sb is +5 and of In is +3 . Thus, In is the impurity which when added to Si gives p-type semiconductor.

66. **(d)** For the reaction,

$CH_3COOC_2H_5(aq)+NaOH(aq)\rightarrow$
$CH_3COONa(aq)+C_2H_5OH(aq)$

the molecularity is two because two molecules are taking part.
Moreover, rate law for the reaction is,
rate (r) $= k[CH_3COOC_2H_5][NaOH]$
$\therefore$ Order of reaction is $1 + 1 = 2$

67. (c) In the Haber's process for the synthesis of ammonia from N_2 and H_2, Fe is used as catalyst and Mo works as activator.

$$N_2 + 3H_2 \overset{Fe,Mo}{\rightleftharpoons} 2NH_3 + 22.4\text{kcal}$$

68. (c) Cl^- is a Lewis base, not a Lewis acid as its octet is complete and it have a tendency to give electrons.

69. (a) Molar heat capacity is the amount of heat required to raise the temperature of 1 mole of a substance by 1°C.

$$\Delta H = nC_p\Delta T$$

$$\therefore \; 1000 = \frac{100}{18} \times 75 \times \Delta T$$

$$\Delta T = \frac{180}{75} = 2.4^\circ$$

70. (b) In gelly, dispersed phase is liquid and dispersion medium is solid. Thus, it is a colloidal solution of liquid in solid.

71. (b) H_2O_2 forms addition product with disodium hydrogen phosphate.

$$H_2O_2 + Na_2HPO_4 \rightarrow \underset{\text{addition product}}{Na_2HPO_4 \cdot H_2O_2}$$

72. (d) $Be_2C + 4H_2O \rightarrow \underset{\text{methane}}{CH_4} + 2Be(OH)_2$

73. (b)

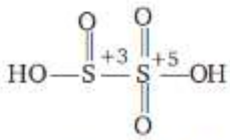
74. (c) NaOH when heated with coke, gets reduced to Na .

$$6NaOH + 2C \xrightarrow{\Delta} 2Na + 2Na_2CO_3 + 3H_2 \uparrow$$

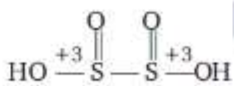
Here, C behaves as reducing agent.

75. (b) Treatment of cellulose with HNO_3 gives cellulose nitrate, which is a very important substance for the manufacture of explosives (gun powder), medicines, paints, lacquers, etc.

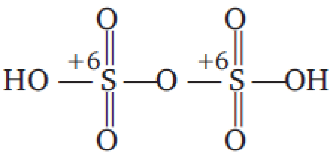
76. (a) The structure of the given oxoacids of sulphur are as



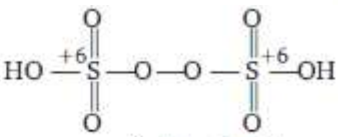
pyrosulphurous acid



hyposulphurous acid



pyrosulphuric acid



peroxodisulphuric acid

Thus, the oxidation states of two S atoms in pyrosulphurous acid are different.

77. (a) As the size of atom increases, A − A bond length increases and hence, A − A bond energy decreases. (where A = Cl, Br, I) $\therefore$ The order of bond energy is $Cl_2 > Br_2 > I_2$

78. (a) As the size of noble gases increases from He to Xe, van der Waals' forces increases and due to which boiling point increases.

79. (b) When the complex dissolved in water, it gives three ions, which shows that two Cl^- ions are present outside the coordination sphere.
$\therefore$ The formula of the complex is $[Co(NH_3)_5Cl]Cl_2$

$$[Co(NH_3)_5Cl]Cl_2 \xrightarrow{\text{Water}} \underbrace{[Co(NH_3)_5Cl]^{2+} + 2Cl^-}_{3 \text{ ion}}$$

80. (d) In Castner's process for the extraction of sodium, Ni anode is used.

Mathematics

81. (b) Let required equation of line is

$y = mx + c$ .... (i)

Since, angle between $y = mx + c$

and $y = 2x + 1$ is $\tan 45°$.

$\because \quad \tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$

$\Rightarrow \quad \tan 45° = \left|\frac{m - 2}{1 + 2m}\right|$

$\Rightarrow \quad 1 = \pm\frac{m - 2}{1 + 2m}$

$\Rightarrow \quad 1 + 2m = m - 2$

or $\quad 1 + 2m = -m + 2$

$\Rightarrow \quad m = -3$ or $3m = 1$

$\Rightarrow \quad m = -3$ or $\frac{1}{3}$

On putting $m = -3$ in Eq. (i), we get

$y = -3x + c$

Since, it passes through (1, 2).

$\therefore \quad 2 = -3 + c$

$\Rightarrow \quad c = 5$

$\therefore \quad y = -3x + 5$

$\Rightarrow \quad 3x + y = 5$

Again, putting $m = \frac{1}{3}$ in Eq. (i), we get

$y = -\frac{x}{3} + c$

Since, it passes through (1,2).

$\therefore\ 2 = -\frac{1}{3} + c \Rightarrow c = \frac{7}{6}$

$\therefore$ Equation of line is $y = -\frac{x}{3} + \frac{7}{6}$

$\Rightarrow\ 6y + 2x = 7$

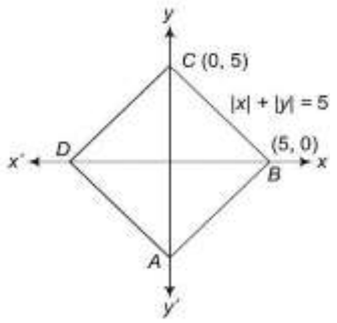
82. (c) In a figure, xx' and yy' are two perpendicular axes.

$\therefore\ |x| + |y| = 5$

$\therefore$ Coordinates of B and C are (5,0) and (0,5).

$\therefore$ Length of BC $= \sqrt{5^2 + 5^2} = 5\sqrt{2}$

$\therefore$ Area of square ABCD $= (5\sqrt{2})^2 = 50$



83. (c) Given equation is

$(x + 7y)^2 + 4\sqrt{2}(x + 7y) - 42 = 0$

Put $x + 7y = t$

$\Rightarrow t^2 + 4\sqrt{2}(t) - 42 = 0$

$\Rightarrow t = \dfrac{-4\sqrt{2} \pm \sqrt{32 + 168}}{2}$

$= \dfrac{-4\sqrt{2} \pm 10\sqrt{2}}{2}$

$= 3\sqrt{2}, -7\sqrt{2}$

$\therefore x + 7y = 3\sqrt{2}$

and $x + 7y = -7\sqrt{2}$

$\Rightarrow x + 7y - 3\sqrt{2} = 0$

and $x + 7y + 7\sqrt{2} = 0$

$\therefore$ Distance between parallel lines

$= \dfrac{|7\sqrt{2} + 3\sqrt{2}|}{\sqrt{1^2 + 7^2}} = \dfrac{10\sqrt{2}}{5\sqrt{2}} = 2$

84. (d) Given pair of lines is $8x^2 - 6xy + y^2 = 0$.

$\Rightarrow 8\frac{x^2}{y} - 6\frac{x}{y} + 1 = 0$

$\Rightarrow 4\frac{x}{y} - 1\ 2\frac{x}{y} - 1 = 0$

$\Rightarrow \frac{4x}{y} = 1$ and $\frac{2x}{y} = 1$

$\Rightarrow 4x = y$ and $2x = y$ ..... (i)

Also, other line is $2x + 3y = a$ .... (ii)

On solving Eqs. (i) and (ii), we get

$O(0,0), A\dfrac{a}{14}, \dfrac{2a}{7}$ and $B\dfrac{a}{8}, \dfrac{a}{4}$

$\therefore$ Area of $\triangle ABC = \dfrac{1}{2}\left\|\begin{matrix} 0 & 0 & 1 \\ \frac{a}{14} & \frac{2a}{7} & 1 \\ \frac{a}{8} & \frac{a}{4} & 1 \end{matrix}\right\|$

$\Rightarrow 7 = \dfrac{1}{2}\left|\dfrac{a^2}{56} - \dfrac{2a^2}{56}\right.$

$\Rightarrow 7 = \dfrac{1}{2}\dfrac{-a^2}{56}$

$\Rightarrow 14 \times 56 = a^2$

$\Rightarrow a = 28$

85. (b, d) Given pair of lines is

$(x^2 + y^2)\cos^2\theta = (x\cos\theta + y\sin\theta)^2$

$\Rightarrow (x^2 + y^2)\cos^2\theta = x^2\cos^2\theta + y^2\sin^2\theta$
$+2xy\sin\theta\cos\theta$

$\Rightarrow y^2(\cos^2\theta - \sin^2\theta) - 2xy\sin\theta\cos\theta = 0$

$\because$ Pair of lines are perpendicular to each other.

$\therefore$ Coefficient of $x^2 +$ Coefficient of $y^2 = 0$

$\therefore 0 + \cos^2\theta - \sin^2\theta = 0$

$\Rightarrow \tan^2\theta = 1$

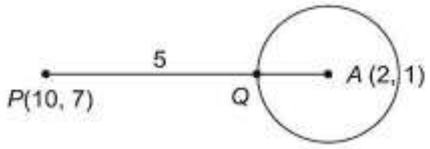
$\Rightarrow \tan\theta = \pm 1 \Rightarrow \theta = \dfrac{\pi}{4}$ or $\dfrac{3\pi}{4}$

86. (c)

87. (d) Given equation of circle is

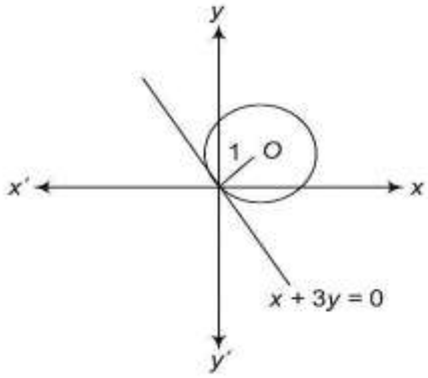
$x^2 + y^2 - 4x - 2y + c = 0$

whose centre is $A(2,1)$.



Now, AP $= \sqrt{(2-10)^2 + (1-7)^2}$
$= \sqrt{(-8)^2 + (-6)^2} = \sqrt{64+36} = \sqrt{100}$
$= 10$
$\therefore$ AQ = AP − PQ = 10 − 5 = 5
So, Q is the mid-point of AP
$= \frac{10+2}{2}, \frac{7+1}{2} = (6,4)$
Since, Q lies on a circle.
$\therefore \quad 6^2 + 4^2 - 4(6) - 2(4) + c = 0$
$\Rightarrow \quad 36 + 16 - 24 - 8 + c = 0$
$\Rightarrow \quad 20 + c = 0$
$\Rightarrow \quad c = -20$

88. **(d)** Given line is $x + 3y = 0$.
$\therefore$ Slope of a line $= -\frac{1}{3}$
Let the centres of circle be $(+g, +f)$.
We know that, the perpendicular drawn from the centre to the tangent is equal to radius.



Since perpendicular distance from (g, f) to the line is 1.
$\therefore \quad \frac{+g+3f}{\sqrt{1^2+3^2}} = 1$
$\Rightarrow \quad \frac{+g+3f}{\sqrt{10}} = 1$
Taking option (d) i.e., centre $= \frac{1}{\sqrt{10}}, \frac{3}{\sqrt{10}}$
$\therefore \quad \frac{+\frac{1}{\sqrt{10}} + 3\times\frac{3}{\sqrt{10}}}{\sqrt{10}} = \frac{10}{10} = 1$ (true)

89. **(b)** Since, the circle passes through (3,4) and cuts the circle $x^2 + y^2 = a^2$ orthogonally.
$\therefore \ (x-3)^2 + (y-4)^2 = 0$ .. (i)
Also, $x^2 + y^2 - a^2 = 0$ …. (ii)
$\therefore$ Equation of radical axis,
$(x-3)^2 + (y-4)^2 - (x^2 + y^2 - a^2) = 0$
$\Rightarrow x^2 - 6x + 9 + y^2 + 16 - 8y - x^2 - y^2 + a^2 = 0$
$\Rightarrow -6x - 8y + 25 + a^2 = 0$
$\Rightarrow 6x - 8y - 25 - a^2 = 0$ ….. (iii)
Now, the distance from (0,0) to Eq. (iii), we get
$\frac{|6(0) - 8(0) - 25 - a^2|}{\sqrt{36+64}} = 25$
$\Rightarrow \quad \frac{25 + a^2}{10} = 25$

$\Rightarrow 25 + a^2 = 250$
$\Rightarrow a^2 = 225$

90. (a) Given coaxial system of circle is

$$4x^2 + 4y^2 - 12x + 6y - 3 + \lambda(x + 2y - 6) = 0$$

or $x^2 + y^2 - 3x + \frac{3y}{2} - \frac{3}{4} + \frac{\lambda}{4}(x + 2y - 6) = 0$

Here, radical axis is $x + 2y - 6 = 0$. i.e., line of centre is
$2x - y + k = 0$ .... (i)

Here, centre of circle is $\frac{3}{2}, -\frac{3}{4}$ which lies on Eq. (i).

$$\therefore \quad 2\frac{3}{2} + \frac{3}{4} + k = 0$$

$$\Rightarrow \frac{12+3}{4} + k = 0$$

$$\Rightarrow \quad k = -\frac{15}{4}$$

$\therefore$ Required equation is

$$2x - y - \frac{15}{4} = 0$$

$$\Rightarrow 8x - 4y - 15 = 0$$

91. (b) Given equation of parabola is

$y^2 = 12x$

Here, $a = 3$

$\therefore$ Equation of normal is

$y = mx - 2am - am^3$ $(\because m = -1)$

$y = -x - 2(3)(-1) - 3(-1)^3$

$= -x + 6 + 3$

$X + y = 9$ .... (i)

But given normal is

$x + y = k$

$k = 9$

Focus of a given parabola is $S(3,0)$.

Now, perpendicular distance from $S(3,0)$ to the line (i) is

$$p = \frac{|3(1) + 0 - 9|}{\sqrt{1^2 + 1^2}} = \frac{6}{\sqrt{2}}$$

$$\therefore 4k - 2p^2 = 4(9) - 2\frac{6}{\sqrt{2}}^2$$

$$= 36 - 2 \times \frac{36}{2} = 0$$

92. (b)

93. (b) Given hyperbola is $5x^2 - y^2 = 5$

or It can be rewritten as

$$\frac{x^2}{1} - \frac{y^2}{5} = 1$$

Here, $a^2 = 1, b^2 = 5$

$\therefore$ Equation of tangent is

$$y = mx \pm \sqrt{a^2m^2 - b^2}$$

$$\Rightarrow \quad y = mx \pm \sqrt{1m^2 - 5} \text{ .... (i)}$$

But point (2,8) lies on it.

$$\therefore 8 = 2m \pm \sqrt{m^2 - 5}$$

$$\Rightarrow (8 - 2m) = \pm\sqrt{m^2 - 5}$$

On squaring both sides, we get

$$64+4m^2-32m=m^2-5$$
$\Rightarrow \quad 3m^2-32m+69=0$
$\Rightarrow \quad (3m-23)(m-3)=0$
$\Rightarrow \quad m=3,\dfrac{23}{3}$

On putting $m=3$ in Eq. (i), we get
$y=3x\pm\sqrt{3^2-5}=3x\pm 2$
$\Rightarrow y=3x+2$ and $y=3x-2$

94. (b) Given equation of hyperbola is $x^2-3y^2=3$.
$\therefore$ Equation of tangent at point $(\sqrt{3},0)$ is
$S_1=0$
$\Rightarrow \quad x\sqrt{3}-3y\times 0=3$
$\Rightarrow \quad x\sqrt{3}=3$
$x=\sqrt{3}$ .... (i)
The asympotes of given hyperbolas are
$x+\sqrt{3}y=0$ ... (ii)
and $x-\sqrt{3}y=0$ .... (iii)
On solving Eqs. (i), (ii) and (iii), we get
$(0,0),(\sqrt{3},-1)$ and $(\sqrt{3},1)$
$\therefore$ Area of triangle formed by joining the above points
$$=\frac{1}{2}\begin{vmatrix}0 & 0 & 1\\ \sqrt{3} & -1 & 0\\ \sqrt{3} & 1 & 0\end{vmatrix}$$
$$=\frac{1}{2}[1(\sqrt{3}+\sqrt{3})]=\frac{1}{2}\times 2\sqrt{3}=\sqrt{3}$$

95. (d) Given equation of circle is
$r=12\cos\theta+5\sin\theta$
Put $\cos\theta=\frac{x}{r}$ and $\sin\theta=\frac{y}{r}$, we get
$$r=12\times\frac{x}{r}+5\times\frac{y}{r}$$
$\Rightarrow \quad r^2=12x+5y$
$\Rightarrow \quad x^2+y^2=12x+5y$
$\Rightarrow \quad x^2+y^2-12x-5y=0$
$\therefore$ Centre is $6,\frac{5}{2}$.
$$\therefore \text{ Radius of circle }=\sqrt{(6)^2+\frac{5^2}{2}}$$
$$=\sqrt{36+\frac{25}{4}}=\sqrt{\frac{144+25}{4}}$$
$$=\sqrt{\frac{169}{4}}=\frac{13}{2}$$

96. (b) Let P divide $Q(2,2,1)$ and $R(5,1,-2)$ in the ratio of $m:1$.
$$\therefore \ P\frac{5m+2}{m+1},\frac{m+2}{m+1},\frac{-2m+1}{m+1}$$
But it is given $\frac{5m+2}{m+1}=4$.
$\Rightarrow \ 5m+2=4m+4$
$\Rightarrow \ m=2$
$\therefore$ Coordinate of $z=\frac{-2\times 2+1}{2+1}=\frac{-3}{3}=-1$

97. (b) Since, a line is equally inclined to the coordinate axes.

$\therefore \quad l = m = n$

$\because \quad l^2 + m^2 + n^2 = 1$

$\Rightarrow \quad 3l^2 = 1$

$\Rightarrow \quad l = \pm\frac{1}{\sqrt{3}}$

$\therefore$ Angle made by line with y-axis $= \cos^{-1}\frac{1}{\sqrt{3}}$

98. (b) The equation of a plane passing through $(1,2,3)$ is

$a(x-1) + b(y-2) + c(z-3) = 0$

Also, $(a, b, c) = (1-0, 2-0, 3-0)$

$= (1,2,3)$

$\therefore 1(x-1) + 2(y-2) + 3(z-3) = 0$

$\Rightarrow x + 2y + 3z - 1 - 4 - 9 = 0$

$\Rightarrow x + 2y + 3z = 14$

99. (a) Given points are $A(1,0,0), B(0,1,0)$ and $C(1,1,1)$.

$AB = \sqrt{(0-1)^2 + (1-0)^2 + 0^2} = \sqrt{2}$

$BC = \sqrt{(0-1)^2 + 0^2 + 1^2} = \sqrt{2}$

$CA = \sqrt{0^2 + 1^2 + 1^2} = \sqrt{2}$

$\therefore \triangle$ ABC is an equilateral triangle.

$\therefore$ Centre of sphere = Centroid of $\triangle$ ABC

$= C'\frac{2}{3}, \frac{2}{3}, \frac{1}{3}$

$\therefore$ Radius of sphere AC'

$$= \sqrt{\frac{2}{3} - 1^2 + \frac{2}{3}^2 + \frac{1}{3}^2}$$

$$= \sqrt{\frac{1}{9} + \frac{4}{9} + \frac{1}{9}} = \frac{1}{3}\sqrt{6}$$

$\therefore$ Equation of sphere is

$$x - \frac{2}{3}^2 + y - \frac{2}{3}^2 + z - \frac{1}{3}^2 = \frac{\sqrt{6}}{3}^2$$

$$\Rightarrow x^2 + y^2 + z^2 - \frac{4x}{3} - \frac{4y}{3} - \frac{2z}{3} + \frac{4}{9} + \frac{4}{9} + \frac{1}{9}$$

$$= \frac{6}{9} = \frac{2}{3}$$

$\Rightarrow 3(x^2 + y^2 + z^2) - 4x - 4y - 2z + 1 = 0$

100. **(c)** $\lim_{x\to\infty}\frac{x+6}{x+1}^{x+4} = \lim_{x\to\infty}1 + \frac{5}{x+1}^{x+4}$

$$= e^{5\lim_{x\to\infty}\frac{x+4}{x+1}} = x^{5\lim_{x\to\infty}\frac{1+\frac{4}{x}}{1+\frac{1}{x}}}$$

$$= e^5$$

101. **(b)**

102. **(a)** Given, $f(x) = \log e^x \frac{x-2}{x+2}^{3/4}$

$\Rightarrow f(x) = x + \frac{3}{4}[\log(x-2) - \log(x+2)]$

On differentiating w.r.t. x, we get

$$f'(x) = 1 + \frac{3}{4}\frac{1}{x-2} - \frac{1}{x+2}$$

$$= 1 + \frac{3}{4}\frac{4}{x^2-4} = 1 + \frac{3}{x^2-4}$$

$\Rightarrow f'(0) = 1 + \frac{3}{0-4} = \frac{1}{4}$

103. **(a)** Given, $x^m y^n = (x+y)^{m+n}$
On taking log on both sides, we get
$m\log x + n\log y = (m+n)\log(x+y)$
On differentiating w.r.t. x, we get

$$\frac{m}{x}+\frac{n}{y}\frac{dy}{dx} = \frac{m+n}{x+y}1+\frac{dy}{dx}$$

$$\Rightarrow \frac{m}{x}-\frac{m+n}{x+y} = \frac{dy}{dx}\frac{m+n}{x+y}-\frac{n}{y}$$

$$\Rightarrow \frac{my-nx}{x(x+y)} = \frac{dy}{dx}\frac{my-nx}{(x+y)y}$$

$$\Rightarrow \frac{dy}{dx} = \frac{y}{x}$$

104. **(a)** Given, $x^4+y^4 = t^2+\frac{1}{t^2}$

$$= t+\frac{1}{t}^2 - 2$$

$$= (x^2+y^2)^2 - 2\ (\because x^2+y^2 = t+\frac{1}{t}, \text{given}$$

$$\Rightarrow x^4+y^4 = x^4+y^4+2x^2y^2-2$$

$$\Rightarrow x^2y^2 = 1$$

$$\Rightarrow y^2 = \frac{1}{x^2}$$

On differentiating w.r.t. x, we get

$$2y\frac{dy}{dx} = -\frac{2}{x^3}$$

$$\Rightarrow x^3y\frac{dy}{dx} = -1$$

105. **(d)** Given, $f(x) = (x^2-1)^7$
When we expand above expansion it will give one term of x^{14} whose coefficient is 1 .
$\therefore\ f^{14}(x) = 14!$

106. **(b)** Given coordinate is
$x = a(\theta+\sin\theta), y = a(1-\cos\theta)$
On differentiating w.r.t. x, we get

$$\frac{dx}{d\theta} = a(1+\cos\theta), \frac{dy}{d\theta} = a(0+\sin\theta)$$

$$\therefore \frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{a\sin\theta}{a(1+\cos\theta)}$$

$$= \frac{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{2\cos^2\frac{\theta}{2}} = \tan\frac{\theta}{2}$$

$$\tan\frac{\pi}{4} = \tan\frac{\theta}{2}\ \because \frac{dy}{dx} = \tan x$$

$$\Rightarrow \frac{\pi}{4} = \frac{\theta}{2}$$

$$\Rightarrow \theta = \frac{\pi}{2}$$

$\therefore$ Coordinate of

$$P\ a\frac{\pi}{2}+\sin\frac{\pi}{2}, a\ 1-\cos\frac{\pi}{2}$$

$$= P\ a\ \frac{\pi}{2}+1, a$$

107. **(c)**

108. **(b)** Given, $\frac{dV}{dt} = 2\pi\ cm^3/s$

$\because$ Volume of sphere, $V = \frac{4}{3}\pi r^3$

On differentiating w.r.t. t, we get

$$\frac{dV}{dt} = \frac{4}{3}\pi \times 3r^2 \frac{dr}{dt}$$

$$\Rightarrow 2\pi = 4\pi r^2 \frac{dr}{dt}$$

$$\Rightarrow \frac{dr}{dt} = \frac{1}{2r^2} = \frac{1}{2 \times 6^2} = \frac{1}{72}\ cm/s$$

$$\because V = 288\pi = \frac{4}{3}\pi r^3 \Rightarrow 216 = r^3 \Rightarrow r = 6$$

109. **(c)** Given, $u = f(r)$ and $r^2 = x^2 + y^2$

$$\therefore\ u_x = f'(r)\frac{\delta r}{\delta x} = f'(r)\frac{x}{r}$$

$$\Rightarrow\ u_{xx} = f''(r) \times \frac{x^2}{r^2} + f'(r)\frac{r^2 - x \times x}{r^2}$$

$$= f''(r) \times \frac{x^2}{r^2} + f'(r)\frac{r^2 - x^2}{r^3}$$

$$u_y = f'(r)\frac{\delta r}{\delta y} = f'(r)\frac{(2y)}{2\sqrt{x^2 + y^2}}$$

$$= f'(r)\frac{y}{r}$$

and $u_{yy} = f''(r)\frac{y^2}{r^2} + f'(r)\frac{r^2 - y^2}{r^3}$

$$\therefore u_{xx} + u_{yy} = f''(r)\frac{x^2}{r^2} + \frac{y^2}{r^2}$$

$$+ f'(r)\frac{r^2 - x^2}{r^3} + \frac{r^2 - y^2}{r^3}$$

$$= f''(r)\frac{r^2}{r^2} + f'(r)\frac{2r^2 - (x^2 + y^2)}{r^3}$$

$$= f''(r) + f'(r)\frac{r^2}{r^3}$$

$$= f''(r) + \frac{f'(r)}{r}$$

110. **(c)** Let $I = \int \frac{dx}{x^2\sqrt{4+x^2}}$

$$= \int \frac{dx}{x^3\sqrt{\frac{4}{x^2} + 1}}$$

Put $\frac{4}{x^2} + 1 = t$

$$\Rightarrow\ -\frac{8}{x^3}dx = dt$$

$$\therefore\ I = \int \frac{dt}{-8\sqrt{t}}$$

$$= -\frac{1}{8} \times \frac{\sqrt{t}}{1/2} + C$$

$$= -\frac{1}{4}\sqrt{\frac{4}{x^2} + 1} + C$$

$$= -\frac{1}{4x}\sqrt{4 + x^2} + C$$

111. **(d)**

112. **(a)** Let $I = \int \frac{dx}{\sqrt{x-x^2}}$

$$= \int \frac{1}{\sqrt{x}} \times \frac{dx}{\sqrt{1-x}}$$

Put $\sqrt{x} = \sin\theta$

$$\Rightarrow \frac{1}{2\sqrt{x}} dx = \cos\theta d\theta$$

$$\therefore I = \int \frac{2\cos\theta d\theta}{\sqrt{1-\sin^2\theta}}$$

$$= \int \frac{2\cos\theta}{\cos\theta} d\theta$$

$$= \int 2d\theta = 2\theta + C$$

$=2\sin^{-1}\sqrt{x} + c$

113. **(a)** Let $I = \int_{-\pi}^{\pi} \frac{\sin^2 x}{1+a^x} dx$ .... (i)

Put $x = -x$, we get

$$I = -\int_{\pi}^{-\pi} \frac{\sin^2(-x)}{1+a^{-x}} dx$$

$\Rightarrow I = \int_{-\pi}^{\pi} a^x \frac{\sin^2 x}{1+a^x} dx$ .... (ii)

On adding Eqs. (i) and (ii), we get

$$2I = \int_{-\pi}^{\pi} \frac{(1+a^x)\sin^2 x}{(1+a^x)} dx$$

$$= \int_{-\pi}^{\pi} \sin^2 x dx$$

$$= 2\int_0^{\pi} \sin^2 x dx = \int_0^{\pi} (1-\cos 2x) dx$$

$$= x + \frac{\sin 2x}{2}\Big|_0^{\pi}$$

$$= \pi + \frac{\sin 2\pi}{2} - 0 - 0$$

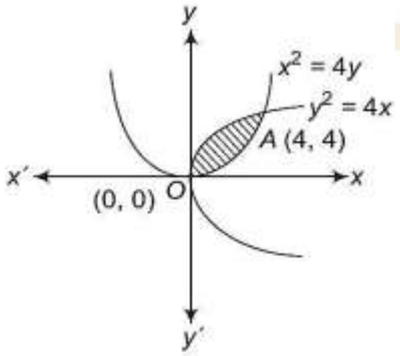
$$\Rightarrow 2I = \pi$$

$$\Rightarrow I = \frac{\pi}{2}$$

114. **(b)** Given curves are $y^2 = 4x$ and $x^2 = 4y$.

The intersection point is

$x^4 = 16y^2 = 16(4x)$



$\Rightarrow x(x^3 - 64) = 0$

$\Rightarrow x = 0,4$

$\Rightarrow y = 0,4$

Hence, intersection point is $O(0,0)$ and $A(4,4)$.

$\therefore$ Required area = shaded region of the curve

$= \int (y_2 - y_1)dx$

$= \int_0^4 \left(\sqrt{4x} - \frac{x^2}{4}\right) dx$

$= \left[\frac{2x^{3/2}}{3/2} - \frac{x^3}{12}\right]_0^4$

$= \left[\frac{4}{3}(4)^{3/2} - \frac{(4)^3}{12} - (0-0)\right]$

$= \left(\frac{32}{3} - \frac{64}{12}\right) = \frac{128-64}{12}$

$= \frac{64}{12} = \frac{16}{3}$ sq units

115. **(d)** Given integration is $\int_0^4 \frac{dx}{1+x^2}$ and $h = 1$.

x	0	1	2	3	4
f(x)	1	$\frac{1}{2}$	$\frac{1}{5}$	$\frac{1}{10}$	$\frac{1}{17}$

By using Trapezoidal rule,

$\int_0^4 f(x)dx = \frac{h}{2}[(y_0 + y_4) + 2(y_1 + y_2 + y_3)]$

$= \frac{1}{2}\left[\left(1 + \frac{1}{17}\right) + 2\left(\frac{1}{2} + \frac{1}{5} + \frac{1}{10}\right)\right]$

$= \frac{1}{2}\left[\frac{18}{17} + 2\left(\frac{5+2+1}{10}\right)\right]$

$= \frac{1}{2}\left(\frac{18}{17} + \frac{8}{5}\right) = \frac{1}{2}\left(\frac{90+136}{85}\right)$

$= \frac{1}{2}\left(\frac{226}{85}\right) = \frac{113}{85}$

116. **(c)** Given differential equation is

$\frac{dy}{dx} + 2x\tan(x-y) = 1$

Put $x - y = t$

$\Rightarrow \quad 1 - \frac{dy}{dx} = \frac{dt}{dx}$

$\Rightarrow \quad \frac{dy}{dx} = 1 - \frac{dt}{dx}$

$\therefore \quad 1 - \frac{dt}{dx} + 2x\tan t = 1$

$\Rightarrow \quad \frac{dt}{\tan t} = 2xdx$

$\Rightarrow \quad \cot t dt = 2xdx$

On integrating both sides, we get

$\log \sin t = x^2 + \log A$

$\Rightarrow \log \frac{\sin(x-y)}{A} = x^2$

$\Rightarrow \sin(x-y) = Ae^{x^2}$

117. **(d)** Given differential equation can be rewritten as

$\frac{dy}{dx} + \frac{xy}{1-x^2} = \frac{x^4\left(\sqrt{1-x^2}\right)^3}{(1+x^5)(1-x^2)}$

or $\frac{dy}{dx} + \frac{xy}{1-x^2} = \frac{x^4\sqrt{1-x^2}}{(1+x^5)}$

On comparing by linear differential equation is

$$\frac{dy}{dx}+Py=Q$$
$$\therefore\ P=\frac{x}{1-x^2}$$
$$\therefore\ IF=e^{\int Pdx}=e^{\int\frac{x}{1-x^2}dx}$$
$$=e^{-\frac{1}{2}\int\frac{-2x}{1-x^2}dx}=e^{-\frac{1}{2}\log(1-x^2)}$$
$$=e^{\log(1-x^2)^{-1/2}}=\frac{1}{\sqrt{1-x^2}}$$

118. **(c)** Given, $g\{f(x)\}=|\sin x|$

and $f\{g(x)\}=(\sin\sqrt{x})^2$

Let us consider $f(x)=\sin^2 x$ and $g(x)=\sqrt{x}$

$\therefore\ f\{g(x)\}=f(\sqrt{x})=\left(\sin^2\sqrt{x}\right)=(\sin\sqrt{x})^2$

and $g\{f(x)\}=g(\sin^2 x)=\sqrt{\sin^2 x}=|\sin x|$

119. **(a)**

120. **(a)** Given series can be rewritten as

$$\frac{1}{2}\left[\frac{1}{2}\quad-\frac{1}{4}+\frac{1}{4}-\frac{1}{6}+\cdots+\frac{1}{2n}-\frac{1}{2n+2}\right]$$
$$=\frac{1}{4}\left(1-\frac{1}{n+1}\right)$$
$$=\frac{1}{4}\left(\frac{n}{n+1}\right)$$

On comparing given equation, we get

$$k=\frac{1}{4}$$

121. **(c)** We know that,

$$\text{Number of diagonals}=\frac{n(n-3)}{2}$$
$$\therefore\quad 170=\frac{n(n-3)}{2}$$
$$\Rightarrow\quad 340=n(n-3)$$
$$\Rightarrow\quad 20\times 17=n(n-3)$$
$$\Rightarrow\quad n=20$$

122. **(b)** A committee of 12 members is to be formed when women are in majority.

Case I 9 women and 3 men

$\therefore$ Number of ways $={}^9C_9\times{}^8C_3=1\times\frac{8\times7\times6}{3\times2\times1}$

$=56$

Case II 8 women and 4 men

$\therefore$ Number of ways $={}^9C_8\times{}^8C_4$

$$=9\times\frac{8\times7\times6\times5}{4\times3\times2\times1}$$

$=630$

Case III 7 women and 5 men

$\therefore$ Number of ways $={}^9C_7\times{}^8C_5$

$$=\frac{9\times8}{2\times1}\times\frac{8\times7\times6}{3\times2\times1}$$

$=36\times56=2016$

$\therefore$ Required number of ways $=56+630+2016$

$=2702$

123. **(c)** There are two cases arise.

Case I When 5 questions are selected from first 6 questions and next 5 questions are selected from 7 questions.

$\therefore$ Number of ways $={}^6C_5\times{}^7C_5$

$= 6 \times \frac{7 \times 6}{2 \times 1}$

$= 126$

Case II When 6 questions are selected from first 6 questions and next 4 questions are selected from 7 questions.

$\therefore$ Number of ways $= {}^6C_6 \times {}^7C_4$

$= 1 \times \frac{7 \times 6 \times 5}{3 \times 2} = 35$

$\therefore$ Required number of ways $= 126 + 35 = 161$

124. **(d)** $\sum_{k=1}^{\infty} \sum_{r=0}^{k} \frac{1}{3^k} \left({}^kC_r\right)$

$= \sum_{k=1}^{\infty} \frac{1}{3^k} \left({}^kC_0 + {}^kC_1 + {}^kC_2 + \cdots + {}^kC_k\right)$

$= \sum_{k=1}^{\infty} \frac{2^k}{3^k} = \sum_{k=1}^{\infty} \frac{2^k}{3}$

$= \frac{2}{3} + \frac{2^1}{3} + \cdots \infty$

$= \frac{2/3}{1-\frac{2}{3}}$ $\because$ common ratio $= \frac{2}{3}$

$= 2$

125. **(d)**

126. **(b)** Given, $\frac{1}{x(x+1)\ldots(x+n)}$

$= \frac{A_0}{x} + \frac{A_1}{x+1} + \cdots + \frac{A_r}{x+r} + \cdots + \frac{A_n}{x+n}$

$\Rightarrow \frac{1}{x(x+1)\ldots(x+r-1)\cdot(x+r+1)\ldots(x+n)}$

$= \frac{x+r}{x} A_0 + \cdots + A_r + \cdots + \frac{x+r}{x+n} A_n$

$\lim_{x \to -r} \frac{1}{x(x+1)(x+2)\ldots(x+n)}$

$= 0 + \cdots + 0 + A_r + \cdots + 0$

$A_r = \frac{1}{-r(-r+1)\ldots(-1)\cdot 1 \cdot 2 \ldots(-r+n)}$

$= \frac{(-1)^r}{r(r-1)\ldots(1)[1\cdot 2\ldots(n-r)]}$

$\Rightarrow A_r = \frac{(-1)^r}{r!\,(n-r)!}$

127. **(b)** Given series is $1 + \frac{1}{3\cdot 2^2} + \frac{1}{5\cdot 2^4} + \frac{1}{7\cdot 2^6} + \cdots$

$= 2\left[\frac{1/2}{1} + \frac{(1/2)^3}{3} + \frac{(1/2)^5}{5} + \cdots\right]$

$= \log_e\left(\frac{1+\frac{1}{2}}{1-\frac{1}{2}}\right) = \log_e\left(\frac{3/2}{1/2}\right)$

$\left[\because \log_e\left(\frac{1+x}{1-x}\right) = 2\left(x + \frac{x^3}{3} + \frac{x^5}{3} + \cdots\right)\right]$

$\log_e 3$

128. **(a)** Given, $R = \frac{\pi}{4}$

Also, $P+Q+R=\pi$

$\Rightarrow \quad P+Q+\frac{\pi}{4}=\pi$

$\Rightarrow \quad P+Q=\frac{3\pi}{4}$

$\Rightarrow \quad \frac{P}{3}+\frac{Q}{3}=\frac{\pi}{4}$

$\Rightarrow \quad \tan\left(\frac{P}{3}+\frac{Q}{3}\right)=\tan\left(\frac{\pi}{4}\right)$

$\Rightarrow \quad \frac{\tan\frac{P}{3}+\tan\frac{Q}{3}}{1-\tan\frac{P}{3}\tan\frac{Q}{3}}=1$

Since, $\tan\frac{P}{3}$ and $\tan\frac{Q}{3}$ are the roots of the equation $ax^2+bx+c=0$.

$\therefore \tan\frac{P}{3}+\tan\frac{Q}{3}=-\frac{b}{a}$

and $\tan\frac{P}{3}\cdot\tan\frac{Q}{3}=\frac{c}{a}$

$\therefore$ From Eq. (i),

$\frac{-\frac{b}{a}}{1-\frac{c}{a}}=1$

$\Rightarrow \quad \frac{-b}{a-c}=1$

$\Rightarrow \quad a+b=c$

129. **(a)** Given equation is

$|x|^{6/5}-26|x|^{3/5}-27=0$

Put $|x|^{3/5}=t$

$\therefore t^2-26t-27=0$

$\Rightarrow t^2-27t+t-27=0$

$\Rightarrow t(t-27)+1(t-27)=0$

$\Rightarrow (t+1)(t-27)=0$

$\Rightarrow t=27$ or -1

$\Rightarrow |x|^{3/5}=27$

($\because |x|^{3/5}$ can not be negative)

$\Rightarrow |x|^3=(3^3)^5$

$\Rightarrow |x|=3^5$

$\Rightarrow x=3^5$ or -3^5

$\therefore$ Product of $x=3^5\times(-3)^5=-3^{10}$

130. **(b)**

131. **(d)** Given,

$A=\begin{vmatrix}2 & e^{i\pi}\\-1 & i^{2012}\end{vmatrix}$

$=\begin{vmatrix}2 & \cos\pi+i\sin\pi\\-1 & (1^4)^{503}\end{vmatrix}$

$=\begin{vmatrix}2 & -1\\-1 & 1\end{vmatrix}=2-1=1$

$C=\frac{d}{dx}\left(\frac{1}{x}\right)_{x=1}=\left(-\frac{1}{x^2}\right)_{x=1}=-1$

$D=\int_{e^2}^{1}\frac{dx}{x}$

$=[\log x]_{e^2}^{1}$

$=\log 1-\log e^2=0-2 \ldots\ldots$ (i)

$=-2$

and
Let α, β and γ are the roots of the equation

$$Ax^3 + Bx^2 + Cx + D = 0$$

$\therefore \quad x^3 + Bx^2 - x + 2 = 0 \quad \ldots (ii)$

$\therefore \quad \alpha + \beta + \gamma = -B$

$\Rightarrow \quad \gamma = -B$ ($\because \alpha + \beta = 0$ given)

$\therefore \quad -B$ satisfies the Eq. (ii),

$\therefore (-B)^3 + B^2 + B + 2 = 0$

$\Rightarrow \ B = 2$

132. (d)

Given, $A = \begin{bmatrix} i & -i \\ -i & i \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$

$$\therefore A^2 = \begin{bmatrix} i & -i \\ -i & i \end{bmatrix}\begin{bmatrix} i & -i \\ -i & i \end{bmatrix}$$

$$= \begin{bmatrix} i^2 + i^2 & -i^2 - i^2 \\ -i^2 - i^2 & i^2 + i^2 \end{bmatrix}$$

$$= \begin{bmatrix} -1-1 & 1+1 \\ 1+1 & -1-1 \end{bmatrix} = \begin{bmatrix} -2 & 2 \\ 2 & -2 \end{bmatrix} = -2B$$

$$B^2 = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+1 & -1-1 \\ -1-1 & 1+1 \end{bmatrix} = \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix}$$

$$= 2B \ \ldots \ldots (i)$$

Now, $A^8 = (A^2)^4 = (-2B)^4$

$= 16B^4 = 16(B^2)^2 16 \cdot (2B)^2$

$= 16 \times 4 \times B^2 = 16 \times 4 \times 2B$ [from Eq. (i)]

$= 128B$

133. (a) Given, f(x)

$$= \begin{vmatrix} 1 & x & x+1 \\ 2x & x(x-1) & x(x+1) \\ 3x(x-1) & x(x-1)(x-2) & (x-1)(x)(x+1) \end{vmatrix}$$

Taking x and $x(x-1)$ common from R_2 and R_3 and $(x+1)$ common from C_3

$$= x \times x(x-1) \times (x+1) \begin{vmatrix} 1 & x & 1 \\ 2 & (x-1) & 1 \\ 3 & (x-2) & 1 \end{vmatrix}$$

(applying $C_2 \rightarrow C_1 + C_2$

$$= x^2(x^2-1) \begin{vmatrix} 1 & x+1 & 1 \\ 2 & x+1 & 1 \\ 3 & x+1 & 1 \end{vmatrix}$$

$$= x^2(x^2-1)(x+1) \begin{vmatrix} 1 & 1 & 1 \\ 2 & 1 & 1 \\ 3 & 1 & 1 \end{vmatrix} = 0$$

$\Rightarrow \ f(x) = 0$

$\therefore \ f(2012) = 0$

134. (c)

Given, $A = \begin{bmatrix} -1 & -2 & -3 \\ 3 & 4 & 5 \\ 4 & 5 & 6 \end{bmatrix}$

$\therefore |A| = -1(24-25) + 2(18-20) - 3(15-16)$

$= 1 - 4 + 3 = 0$

Now, $\begin{vmatrix} 4 & 5 \\ 5 & 6 \end{vmatrix} = 24 - 25 = -1 \neq 0$

$\therefore$ Rank of $A = a = 2$

$B = \begin{bmatrix} 1 & -2 \\ -1 & 2 \end{bmatrix}$

$|B| = 2 - 2 = 0$

$\therefore$ Rank of $B = b = 1$

and $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

R $|C| = 2(4-0) = 8 \neq 0$

$\therefore$ Rank of $C = 3, c = 3$

$b < a < c$

135. **(b)** Given system of equations is

$(a\alpha + b)x + ay + bz = 0$

$(b\alpha + c)x + by + cz = 0$

and $(a\alpha + b)y + (b\alpha + c)z = 0$

For non-trivial solution,

$$\begin{vmatrix} a\alpha + b & a & b \\ b\alpha + c & b & c \\ 0 & a\alpha + b & b\alpha + c \end{vmatrix} = 0$$

Applying $R_3 \to R_3 - \alpha R_1 - R_2$

$$\Rightarrow \begin{vmatrix} a\alpha + b & a & b \\ b\alpha + c & b & c \\ -(a\alpha^2 + 2b\alpha + c) & 0 & 0 \end{vmatrix} = 0$$

$\Rightarrow -(a\alpha^2 + 2b\alpha + c)(ac - b^2) = 0$

$\Rightarrow ac - b^2 = 0 \; (\because a\alpha^2 + 2b\alpha + c \neq 0)$

$\Rightarrow ac = b^2$

Hence, a, b and c are in GP.

136. **(a)** Given, $(a + ib)^2 = (c + id)^2(x + iy)$

$\Rightarrow |a + ib|^2 = |c + id|^2 |x + iy|$

$\Rightarrow a^2 + b^2 = (c^2 + d^2)\left(\sqrt{x^2 + y^2}\right)$

$\Rightarrow 4 = 2\left(\sqrt{x^2 + y^2}\right)$

($\because a^2 + b^2 = 4$ and $c^2 + d^2 = 2$ given)

$\Rightarrow \sqrt{x^2 + y^2} = 2$

$\Rightarrow x^2 + y^2 = 4$

137. **(d)** Given, $\left|z - \frac{4}{z}\right| = 2$

$\therefore |z| = \left|z - \frac{4}{z} + \frac{4}{z}\right|$

$\leq \left|z - \frac{4}{z}\right| + \left|\frac{4}{z}\right|$

$\Rightarrow |z| \leq 2 + \frac{4}{|z|}$

$\Rightarrow |z|^2 - 2|z| \leq 4$

$\Rightarrow (|z|^2 - 1)^2 \leq 5$

$\Rightarrow (|z| - 1) \leq \sqrt{5}$

$\Rightarrow |z| \leq \sqrt{5} + 1$

138. **(d)**

139. **(b)** Given trigonometrical equation is
$27\tan^2\theta + 3\cot^2\theta$

(using AM $\geq$ GM)

$$\therefore \frac{27\tan^2\theta + 3\cot^2\theta}{2} \geq \sqrt{27\tan^2\theta \cdot 3\cot^2\theta}$$

$$\Rightarrow \frac{27\tan^2\theta + 3\cot^2\theta}{2} \geq \sqrt{81}$$

$$\Rightarrow 27\tan^2\theta + 3\cot^2\theta \geq 18$$

Hence, minimum value of given equation is 18 .

140. **(b)**

$$\cos 36^\circ - \cos 72^\circ = -2\sin\left(\frac{36^\circ + 72^\circ}{2}\right)$$

$$\sin\left(\frac{72^\circ - 36^\circ}{2}\right)$$

$$= -2\sin 54^\circ \sin(-18^\circ)$$

$$= 2\sin 54^\circ \sin 18^\circ$$

$$= 2 \times \frac{\sqrt{5}+1}{4} \times \frac{\sqrt{5}-1}{4} = 2 \times \frac{5-1}{4\times 4} = \frac{4}{8}$$

$$= \frac{1}{2}$$

141. **(c)**

$$\text{LHS } = \tan x + \tan\left(x + \frac{\pi}{3}\right) + \tan\left(x + \frac{2\pi}{3}\right)$$

$$= \tan x + \frac{\tan x + \sqrt{3}}{1 - \sqrt{3}\tan x} + \frac{\tan x - \sqrt{3}}{1 + \sqrt{3}\tan x}$$

$$= \tan x + \frac{\begin{bmatrix}(\tan x + \sqrt{3})(1 + \sqrt{3}\tan x) \\ +(\tan x - \sqrt{3})(1 - \sqrt{3}\tan x)\end{bmatrix}}{1 - 3\tan^2 x}$$

$$= \frac{\begin{bmatrix}\tan x(1 - 3\tan^2 x) + \tan x + \sqrt{3} \\ +\sqrt{3}\tan^2 x + 3\tan x + \tan x - \sqrt{3} \\ -\sqrt{3}\tan^2 x + 3\tan x\end{bmatrix}}{1 - 3\tan^2 x}$$

$$\Rightarrow \frac{\tan x(1 - 3\tan^2 x) + 8\tan x}{1 - 3\tan^2 x} = 3 \text{ (given)}$$

$$\Rightarrow \tan x(1 - 3\tan^2 x) + 8\tan x$$

$=3$ $(1 - 3\tan^2 x)$

$$\Rightarrow \tan x(9 - 3\tan^2 x) = 3(1 - 3\tan^2 x)$$

$$\Rightarrow \tan x(3 - \tan^2 x) = 1 - 3\tan^2 x$$

$$\Rightarrow \frac{3\tan x - \tan^3 x}{1 - 3\tan^2 x} = 1$$

$$\Rightarrow \tan 3x = 1$$

142. **(b)** Given, $3\sin x + 4\cos x = 5$

$$\Rightarrow 3\left(\frac{2\tan\frac{x}{2}}{1 + \tan^2\frac{x}{2}}\right) + 4\left(\frac{1 - \tan^2\frac{x}{2}}{1 + \tan^2\frac{x}{2}}\right) = 5$$

$$\Rightarrow 6\tan\frac{x}{2} + 4 - 4\tan^2\frac{x}{2} = 5\left(1 + \tan^2\frac{x}{2}\right)$$

$$\Rightarrow 6\tan\frac{x}{2} - 9\tan^2\frac{x}{2} = 1$$

143. **(b)**

$$\therefore \cos^{-1}x+\cos^{-1}\left(\frac{x}{2}+\frac{\sqrt{3}}{2}\sqrt{1-x^2}\right)$$

$$=\cos^{-1}x+\cos^{-1}\left(x\times\frac{1}{2}+\frac{\sqrt{3}}{2}\times\sqrt{1-x^2}\right)$$

$$=\cos^{-1}x+\cos^{-1}\left(\frac{1}{2}\right)-\cos^{-1}x$$

$$=\cos^{-1}\left(\frac{1}{2}\right)=\frac{\pi}{3}$$

144. **(c)** Since, a, b and c form a geometric progression

$\therefore\ a=a, b=ar, c=ar^2$

Therefore, given line becomes

$$ax+ary+ar^2=0$$

$\Rightarrow\ x+ry+r^2=0$

$\Rightarrow\ x=-ry-r^2\ \ldots(i)$

On putting $x=-ry-r^2$ in given curve $x+2y^2=0$, we get

$-ry-r^2+2y^2=0$

$\Rightarrow\ 2y^2-ry-r^2=0$

$\therefore$ Sum of ordinates $=\frac{r}{2}$

145. **(c)** Given point is (3,2).

(i) Reflection of point (3,2) about the line $y=x$ is (2,3).

(ii) Translation of a point through 1 unit distance in the positive direction of x-axis is (3,3).

(iii) $X=-x\cos\theta+y\sin\theta$

$=-\frac{3}{\sqrt{2}}+\frac{3}{\sqrt{2}}=0$

and $Y=x\sin\theta+y\cos\theta$

$=\frac{3}{\sqrt{2}}+\frac{3}{\sqrt{2}}=3\sqrt{2}=\sqrt{18}$

Hence, final position is $(0,\sqrt{18})$.

146. **(b)** Given X is a poisson variate such that

$\alpha=P(X=1)=P(X=2)$

$\therefore\ \frac{e^{-\lambda}\lambda}{1!}=\frac{e^{-\lambda}\lambda^2}{2!}\Rightarrow\lambda=2$

$\therefore\ \alpha=P(X=1)$

$=e^{-2}\times 2=\frac{2}{e^2}\ \ldots(i)$

$\therefore\ P(X=4)=\frac{e^{-\lambda}\lambda^4}{4!}=\frac{e^{-2}(2)^4}{24}$

$=\frac{e^{-2}\times 16}{24}=\frac{2}{3}e^{-2}=\frac{1}{3}\alpha$ [from Eq. (i)]

147. **(a)** Given $P(X=r)={}^nC_rp^rq^{n-r}$

$$\therefore\frac{P(X=r)}{P(X=n-r)}=\frac{{}^nC_rp^rq^{n-r}}{{}^nC_{n-r}p^{n-r}q^r}$$

$$=\left(\frac{q}{p}\right)^{n-2r}$$

For independent of n, $\frac{q}{p}=1$

$\Rightarrow\ q=p$

$\because\ p+q=1$

$\therefore\ 2p=1\Rightarrow p=\frac{1}{2}$

148. **(b)** Let E_1 : The event that the students knows the answer and

E_2 : The event that the student guesses the answer

$\therefore\ P(E_1) = \frac{9}{10}$ and $P(E_2) = 1 - \frac{9}{10} = \frac{1}{10}$

Let E : The answer is correct.

The probability that the student answered correctly, given that he knows the answer

i.e., $P\frac{E}{E_1} = 1$

The probability that the students answered correctly, given that he guessed is $\frac{1}{4}$.

i.e., $P\frac{E}{E_2} = \frac{1}{4}$

By using Baye's theorem,

$$P\left(\frac{E_2}{E}\right) = \frac{P\left(\frac{E}{E_2}\right)P(E_2)}{P\left(\frac{E}{E_1}\right)P(E_1) + P\left(\frac{E}{E_2}\right)P(E_2)}$$

$$= \frac{\frac{1}{4}\times\frac{1}{10}}{1\times\frac{9}{10}+\frac{1}{4}\times\frac{1}{10}}$$

$$= \frac{\frac{1}{40}}{\frac{9}{10}+\frac{1}{40}} = \frac{\frac{1}{40}}{\frac{36+1}{40}} = \frac{1}{37}$$

149. **(a)** Required probability = P (In first two test either both are faulty or both are not faulty)

= P (First two are faulty)+ P (First two are not faulty)

$$= \frac{2}{4}\times\frac{1}{3}+\frac{2}{4}\times\frac{1}{3}$$

$$= \frac{4}{12} = \frac{1}{3}$$

150. **(a)** Probability of getting a tail in a single toss

$p = \frac{1}{2}$ and not getting tail, $q = \frac{1}{2}$.

Using Binomial distribution,

$\therefore$ Required probability

$$= P(X=1) + P(X=3) + P(X=5) + \cdots + P(X=99)$$

$$= {}^{100}C_1\left(\frac{1}{2}\right)^1\left(\frac{1}{2}\right)^{99} + {}^{100}C_3\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^{97} + {}^{100}C_5\left(\frac{1}{2}\right)^5\left(\frac{1}{2}\right)^{95} + \cdots + {}^{100}C_{99}\left(\frac{1}{2}\right)^{99}\left(\frac{1}{2}\right)$$

$$= \left(\frac{1}{2}\right)^{100}\left({}^{100}C_1 + {}^{100}C_3 + {}^{100}C_5 + \cdots + {}^{100}C_{99}\right)$$

$$= \frac{1}{2^{100}}(2^{99})$$

$$= \frac{1}{2}$$

151. **(b)** Given, $\mathbf{a} = \mathbf{i} + \mathbf{j} - 2\mathbf{k}$

$$\therefore \sum [(\mathbf{a}\times\mathbf{i})\times\mathbf{j}]^2 = \sum [(\mathbf{a}\cdot\mathbf{j})\mathbf{i}]^2$$

$$= \mathbf{a}^2 = |\mathbf{i}+\mathbf{j}-2\mathbf{k}|^2$$

$$= (\sqrt{1+1+4})^2 = 6$$

152. **(d)** Given, $p = \frac{b\times c}{[abc]}, q = \frac{c\times a}{[abc]}$

and $r = \frac{a\times b}{[abc]}$

Now, $(a+b)\cdot p = (a+b)\cdot \frac{(b\times c)}{[abc]}$

$= \frac{[abc]}{[abc]} + \frac{[bbc]}{[abc]}$

$= 1 + 0 = 1$

Similarly, $(b+c)\cdot q = 1$

and $(c+a)\cdot r = 1$

$\therefore\ (a+b)\cdot p + (b+c)\cdot q + (c+a)\cdot r$

$= 1 + 1 + 1 = 3$

153. **(d)** Since, vectors a and b are in a same plane.

$\therefore \mathbf{r} = \mathbf{a} + t\mathbf{b}$

$= (\mathbf{i} + 2\mathbf{j} + \mathbf{k}) + t(\mathbf{i} - \mathbf{j} + \mathbf{k})$

$= (1+t)\mathbf{i} + (2-t)\mathbf{j} + (1+t)\mathbf{k}\ \ldots\ldots(\mathbf{i})$

$\therefore$ Projection of $\mathbf{r}$ on $\mathbf{c} = \frac{\mathbf{r}\cdot\mathbf{c}}{|\mathbf{c}|}$

$$\frac{1}{\sqrt{3}} = \frac{\left[\begin{array}{c}\{(1+t)\mathbf{i} + (2-t)\mathbf{j} + (1+t)\mathbf{k}\} \\ \cdot(\mathbf{i}+\mathbf{j}-\mathbf{k})\end{array}\right]}{\sqrt{1^2+1^2+(-1)^2}}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{1+t+2-t-(1+t)}{\sqrt{3}}$$

$\Rightarrow 1 = 2 - t$

$\Rightarrow t = 1$

On putting $t = 1$ in Eq. (i), we get

$\mathbf{r} = 2\mathbf{i} + \mathbf{j} + 2\mathbf{k}$

154. **(c)** Put $r = xi + yj + zk$

$\therefore xi + yj + zk = (i - 6j + 2k) + t(i + 2j + k)$

$\therefore$ Any point on line is $P(1+t, -6+2t, 2+t)$

is satisfied the second equation of line.

$\therefore\ (1+t)i + (-6+2t)j + (2+t)k$

$= 2ui + (4+u)j + (1+2u)k$

On equating the coefficients of i, j and k, we get

$1 + t = 2u$

$t - 2u + 1 = 0\ \ldots\ldots(i)$

$-6 + 2t = 4 + u$

$2t - u - 10 = 0\ \ldots.(ii)$

and $2 + t = 1 + 2u$

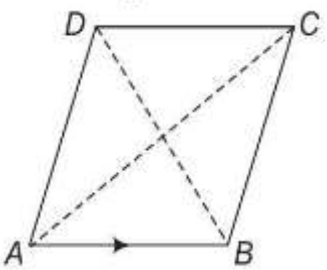
$t - 2u + 1 = 0\ \ldots\ldots(iii)$

On solving Eqs. (i) and (ii), we get

$t = 7,\ u = 4$

$\therefore\ P(1+7, -6+2\times 7, 2+7) = P(8,8,9)$

155. **(d)** Given, $AB = 3i - 2j + 2k$



and $BC = i - 2k$

Diagonals $AC = AB + BC$

$= 3i - 2j + 2k + i - 2k$

$= 4i - 2j$

and $BD = BC - AB$

$= i - 2k - (3i - 2j + 2k)$
$= -2i + 2j - 4k$

$\therefore \cos\theta = \dfrac{AC \cdot BD}{|AC||BD|}$

$= \dfrac{(4i - 2j) \cdot (-2i + 2j - 4k)}{\sqrt{4^2 + (-2)^2}\sqrt{(-2)^2 + (2)^2 + (-4)^2}}$

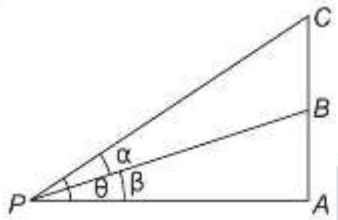
$= \dfrac{-8 - 4}{\sqrt{20}\sqrt{24}} = -\dfrac{12}{4\sqrt{30}} = -\dfrac{3}{\sqrt{30}} = -\sqrt{\dfrac{3}{10}}$

$\Rightarrow \theta = \cos^{-1} - \sqrt{\dfrac{3}{10}}$

Or $\pi - \cos^{-1} - \sqrt{\frac{3}{10}}$

156. **(b)**

157. **(c)** Let AC be a pole and point P be the position on of the ground.



Given, $\theta = \tan^{-1}\frac{1}{2}$

$\Rightarrow \tan\theta = \dfrac{1}{2}$

Also, $\theta = \alpha + \beta$

$\Rightarrow \tan\theta = \tan(\alpha + \beta)$

$\Rightarrow \dfrac{1}{2} = \dfrac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$

(a) When $(\tan\alpha, \tan\beta) = \frac{1}{4}, \frac{1}{5}$

$\therefore$ RHS $= \dfrac{\frac{1}{4} + \frac{1}{5}}{1 - \frac{1}{4} \times \frac{1}{5}} = \dfrac{\frac{9}{20}}{\frac{19}{20}}$

$= \dfrac{9}{19} \neq \dfrac{1}{2}$, not true

(b) When $(\tan\alpha, \tan\beta) = \frac{1}{5}, \frac{2}{9}$

RHS $= \dfrac{\frac{1}{5} + \frac{2}{9}}{1 - \frac{1}{5} \times \frac{2}{9}} = \dfrac{\frac{19}{45}}{\frac{43}{45}}$

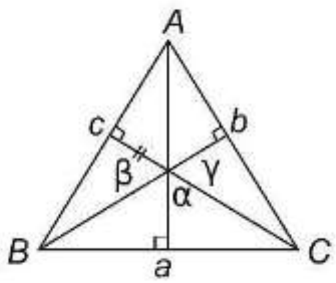
$= \dfrac{19}{43} \neq \dfrac{1}{2}$, not true

(c) When $(\tan\alpha, \tan\beta) = \frac{2}{9}, \frac{1}{4}$

RHS $= \dfrac{\frac{2}{9} + \frac{1}{4}}{1 - \frac{2}{9} \times \frac{1}{4}} = \dfrac{\frac{17}{36}}{\frac{34}{36}} = \dfrac{1}{2}$, true

158. **(a)** $\because$ Area of triangle $= \frac{1}{2}$ base $\times$ altitude

$\therefore \Delta = \dfrac{1}{2}a\alpha = \dfrac{1}{2}b\beta = \dfrac{1}{2}c\gamma$



$\Rightarrow \alpha = \frac{2\Delta}{a}, \beta = \frac{2\Delta}{b}$ and $\gamma = \frac{2\Delta}{c}$

$\therefore \frac{\Delta^2}{R^2}\left(\frac{1}{\alpha^2}+\frac{1}{\beta^2}+\frac{1}{\gamma^2}\right)$

$= \frac{\Delta^2}{R^2}\left(\frac{a^2}{4\Delta^2}+\frac{b^2}{4\Delta^2}+\frac{c^2}{4\Delta^2}\right)$

$= \frac{1}{4R^2}(4R^2\sin^2 A + 4R^2\sin^2 B + 4R^2\sin^2 C)$

$\left[\because \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin c}{c} = \frac{1}{2R}\right]$

$= \sin^2 A + \sin^2 B + \sin^2 C$

159. **(c)** In $\triangle ABC$,

$A + B + C = 180°$

$\Rightarrow A + B = 180° - C$

$\Rightarrow \cot(A+B) = \cot(180° - C)$

$\Rightarrow \frac{\cot A \cot B - 1}{\cot B + \cot A} = -\cot C$

$\Rightarrow \cot A \cot B + \cot B \cot C + \cot C \cot A = 1$

160. **(d)**

Given, $x = \log\left(\frac{1}{y} + \sqrt{1+\frac{1}{y^2}}\right)$

$\therefore x = \operatorname{cosech}^{-1} y$

$\Rightarrow y = \operatorname{cosech} x$